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To Disagree, We Must Also Agree: How Intersubjectivity Structures and Perpetuates Discourse in a Mathematics Classroom[1]

Mitchell J. Nathan, Billie Eilam, and Suyeon Kim

Theories of learning in social settings, no matter their philosophical orientation, must address issues of intersubjectivity (IS). Sociologists such as Schegloff (1992) place IS foremost in addressing any and all aspects of social interaction: "[T]he problem of intersubjectivity (or cognitive order) is theoretically anterior to whatever formulations of problems of order or conflict are part of the tradition of social theory" (p. 1296). Without IS, Schegloff argues, the entire enterprise of social science stands without any reference to the world it purports to identify or describe. Psychologists such as Herb Clark (1996) regard all forms of communication as a way to ground meaning in both the cognitive and social realms. IS also plays a significant role for sociocultural theorists such as Lerman (1996, 2000) and Vygotsky (1986), who consider it to be at the heart of learning and of consciousness itself. With such notable scholars affording such a prominent place to IS, it is important to clarify what IS refers to, how it is manifest, and how it functions as an influential force for the structure and dynamics of classroom discourse.

The archetypal account of IS appears to be the story *The Blind Men and the Elephant*. Although the origin of the story is in some dispute (it is reported as an ancient tale stemming from both Buddhist and Jain cultures) and there are many versions to be found, the essential elements of the story are these: A group of men, blind since birth, encounter an elephant, though each experiences only a part of the animal. Each man asserts that the entire elephant is as its part: the elephant is a rope (tail), a spear or ploughshare (tusk), a tree (leg), and so on. In his rendition, the American poet John Godfrey Saxe (1816–1887; Saxe, 2005) wrote,

> Each in his own opinion
> Exceeding stiff and strong,
> Though each was partly in the right,
> And all were in the wrong!

The story is, of course, an allegory of IS. Consensus is thwarted because of the men's limited and differing perspectives—the "blindness" that we all have. All that is needed, seemingly, is a shared understanding of the elephant, made possible if the blind men could just "see" the elephant as it really is.

This paper starts with the premise that IS is a fundamental and unavoidable aspect of social interaction, and that understanding its nature is necessary for developing reliable theories of socially mediated learning and for designing the next generation of effective learning environments. In this paper, we show that IS can be regarded as broader than agreement or consensus (Matusov, 1996, 2001) and can provide insights into participants' interactions more generally, including their disagreements, divergence of ideas and solutions, and misunderstandings in the constructivist classroom. Our central hypothesis is that IS acts as more than a point of convergence toward a common idea or solution, but that the dynamics toward and

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away from convergent ideas appears to be instrumental in fostering sustained and engaging discourse and influencing the representations that students propose during problem solving. We use discourse analysis techniques to show how IS is manifest in the classroom and explore its role in structuring and perpetuating participants' intellectual interactions.

Intersubjectivity

Traditional views tend to equate IS with consensual agreement and present IS as an attribute of a group activity or discussion that a group either succeeds or fails to achieve. Success, in this view, means that participants have acquired a shared understanding (Cole, 1991; Stahl, 2006) or univocality (Lotman, 1988). Efforts by interlocutors, such as conversational repair, constitute a normal and critical aspect of dialogue, as participants strive to address obstacles to their mutual understanding (Schegloff, 1992). In the traditional view, IS is reduced to a single subjectivity among participants (Matusov, 1996).

More contemporary views of IS examine measures of convergence among interlocutors as the overall movement toward or away from a common goal (e.g., Kapur, Voiklis, Kinzer, & Black, 2006). Typically, evidence for IS within traditional and more contemporary views shows movement from a state of disagreement or misunderstanding to one of agreement or "symmetry" (e.g., Wertsch, 1979).

Accounts of IS have received some reevaluation. For example, Steffe and Thompson (2000), articulating the radical constructivist view, take a more nuanced approach to the process and outcome of IS. On the one hand, their perspective is consistent with the idea that interlocutors reach some form of convergence as part of the process of establishing IS. On the other hand, they distinguish their perspective from the traditional one by emphasizing the reciprocal interactions that are achieved: "By *reaching mutual agreement* we do not mean that the interacting individuals end up with the identical conceptual structures. Rather, we mean only that their conceptual structures are sufficiently compatible for successful reciprocal assimilation" (p. 193, italics from original).

There are also several challenges to the view that consensual agreement and convergence toward a common idea capture the essence of IS. First, some writers have expressed concern about the strong value judgment that deems agreement as favorable and disagreement as unfavorable (Smolka, de Goes, & Pino, 1995). This bias is problematic, since the important role of disagreement in cognitive development and socially mediated learning is well established e.g., Johnson & Johnson, 1989; Piaget, 1975/1985; Vygotsky, 1978; Posner, Strike, Hewson, & Gertzog, 1982). Second, some contest the traditional view of disagreement and agreement, which sees these as separate states or phases along a developmental progression (e.g., Wertsch, 1979) and dismisses their common and complementary nature (Smolka et al., 1995) and frequent coexistence (e.g., Matusov, 1996). Third, it can be argued that the traditional view is too narrow in suggesting that the processes that are unique to IS are no longer in play in failed IS. This traditional perspective further distances those processes that mediate disagreement and disequilibrium from those that mediate agreement.

The participatory view of IS (Matusov, 1996) can be seen as complementary to the traditional view. It focuses on "the coordination of individual participation in joint sociocultural activity rather than as a relationship of correspondence of individuals' actions to each other" (p. 26). Within the participatory view, agreement and disagreement are considered aspects of a common set of processes that mediate collective activity. Interlocutors need not reach consensus to exhibit IS. They can converge on some aspects and diverge on others (Matusov & White, 1996). For example, a speaker may appropriate the representation of a peer but regard it through an alternative interpretive frame. In this way, the participatory view distinguishes between establishing a shared space of interaction and establishing consensus.

Socially Mediated Discourse in the Classroom

Constructivist approaches to classroom instruction draw heavily on students' own conceptions. For this reason, IS is evident in several studies of socially mediated learning and practice that operate within the constructivist paradigm (e.g., Cobb, Yackel, & Wood, 1993). Lerman (1996) argued that IS is constituted through social practices and socially mediated activity. In Vygotsky's (1978) theory of social development, speech, writing, and other social "tools" serve to mediate social interaction. Such tool usage also serves as a mediator of participants' cognitive development (Wertsch & Sohmer, 1995), as people internalize the tools' physical and cognitive functions, which then contribute to the construction of higher mental processes.

Recent education reform has adopted some of the principles of socially mediated learning as a means to promote higher order thinking in all subject areas, including reading (Palinscar & Brown 1984), science (van Zee & Minstrell, 1997; Songer, 2004; Brown & Campione, 1994; Palincsar & Magnusson, 2000), teacher education and professional development (Matusov, 2001; Palincsar, Magnusson, Marano, Ford, & Brown, 1998; Grossman, Wineburg, & Woolworth, 2001), and mathematics (Ball, 1996; Cobb et al., 1993; Cognition and Technology Group at Vanderbilt, 1997; Lehrer, Strom, & Confrey, 2002). Current mathematics education standards, for example, call for an emphasis on communication as one of the five process standards considered essential to acquiring and using mathematical knowledge (National Council of Teachers of Mathematics [NCTM], 2000). Teachers struggle with their role as facilitators within this new learning environment (e.g., Nathan & Knuth, 2003; Rittenhouse, 1998). However, as teachers come to develop facility in their new role and learn how to manage classrooms that draw heavily on peer interactions and student-led presentations, they do see benefits (Cobb et al., 1993; French & Nathan, 2006).

It was within a setting of the early adoption of principles and practices of socially mediated classroom learning that we came to observe sixth graders and their teacher engaged in a spirited dialog about a spatial reasoning task posed by one of the students. We call this task the Pie Problem: How do you cut a pie into eight equal-sized pieces making only three cuts? For most of the double period we observed, students worked out solutions on their own, discussed them with peers, and then publicly presented their ideas, offered alternatives, and critiqued and elaborated their proposed solutions to the Pie Problem.

We focused considerable attention on the representations produced by the class participants because it was through these that students conveyed their analytical ideas about the problem and about their reasoning and interpretations of the problem context. From a pedagogical perspective, the public display of solution representations supports tenets of social constructivism that acknowledge the collaborative co-creation of mathematics. It also provided occasions for “teaching moments” to address mathematical ideas that may have been presented either correctly or incorrectly in the course of the group interaction.

The examination of representations also was a natural way to consider whether convergence toward a common solution representation, as would be expected from a traditional view of IS, was the proper way to describe the discourse. However, rather than convergence, we observed students refining their ideas and uses of representations to suit their interpretive frames. In the end, there was no clear convergence. Yet, we will argue, there was a great deal of IS among participants, and this was a major force shaping the extended discourse.

Analysis of Classroom Discourse

We used discourse analysis methods at multiple levels to understand the nature of the classroom discussion and to identify the elements of the ensuing interactions and the dynamics that drove them.

Discourse Methods

There are a number of approaches for analyzing discourse, though certain methods seem to be prominent when studying naturally occurring interaction (Levinson, 1983). We focus here on conversation analysis, the method formally called *discourse analysis*, and content analysis.

Conversation analysis grew out of sociological studies of how people interact with one another through talk. Hutchby and Wooffitt (1998) defined conversation analysis as “the study of recorded, naturally occurring talk-in-interaction” that can manifest how interactants understand each other’s actions in subsequent turns (p. 14). Seedhouse (2004) summarized the two main aims of conversation analysis as (a) to describe in detail how talk-in-interaction is co-constructed by interactants and show how the interactional structures display emic perspectives; and (b) to pursue the development of IS, focusing on how participants develop a mutual understanding.

Institutional talk such as classroom conversation (Drew & Heritage, 1992) shares many characteristics with ordinary conversation, but it also exhibits some unique properties. For example, in ordinary conversation, self-initiated self-repair by the speaker is preferred (Schegloff, Sacks, & Jefferson, 1977). But a trouble source produced by a student during classroom conversation is commonly identified by the teacher (McHoul, 1990; Nathan & Kim, 2006).

Discourse analysis takes a linguistic perspective on people’s talk and typically focuses on speech events and speech acts, such as declaratives, directives, imperatives, and representatives (Richards & Schmidt, 2002). Sequences of speech acts are produced in subsequent turns and build into a coherent discourse. One of the coding schemes for depicting sequences of classroom discourse is known as *initiation-response-evaluation* (IRE) cycles (Mehan, 1979; Sinclair & Coulthard, 1975). In the typical IRE pattern, the teacher asks a closed question or invites student input (I), which elicits a reaction (R) from a student, whose response is then evaluated (E) by the teacher, often in a way that terminates the interaction (“That is incorrect” or “Correct!”). In one common modification, IRF, the evaluation phase is replaced with a follow-up question that tends to perpetuate the IRF/IRE pattern.

Another discourse analysis approach is *critical discourse analysis*, which focuses on the organization of language and its use associated with the ideologies and values in social contexts (Kress, 1990). Gee (2005) identified *discourse* (with a "little d") as language that is used "on site to enact activities and identities," and *Discourse* (with a "big D") as language that is integrated with non-language aspects (e.g., values) to "enact specific activities and identities" (p. 7). The ways people use words, values, beliefs, and actions allow them to display characteristic *whos* ("a socially situated identity") doing characteristic *whats* ("a socially situated activity") (p. 22).

Gee (2004) argues that language use is always part of some social practice, and as such, always has a situated meaning that is context-specific. One example is the use of the term *proof* in a math classroom. The situated meanings in a specific context are relative to a specific Discourse, and the Discourse of specialists has situated meanings different from those of the ordinary word in everyday life.

Meaning and usage are useful determiners of the unitization of extended discourse. Gee (2005) has suggested using *stanzas* to describe these units. Stanzas in a transcript can be regarded as similar to paragraphs in an essay. As Gee (2005) described:

> Each stanza is a group of lines about one important event, happening, or state of affairs at one time and place, or it focuses on a specific character, theme, image, topic, or perspective. When the time, place, character, event, or perspective changes, we get a new stanza. (p. 109)

In addition to qualitative systems of discourse analysis such as those discussed above, one can draw on methods that strive to integrate hermeneutic aspects of rich and "messy" verbal data with aspects of quantitative data analysis methods that reduce the subjective aspects of interpretation. *Content analysis* makes the statistical patterns of words and phrases the analytic focus of research (Carley, 1990; Chi, 1997; Krippendorf, 2004). This method allows one to pursue hypothesis testing as well as descriptive approaches of inquiry.

Multilevel Perspectives of Discourse

Studies of classroom discourse can convey the complex and adaptive nature of the interactions that shape group learning and collaborative problem solving (e.g., Lampert & Blunk, 1998; Peressini & Knuth, 1998; Schoenfeld, 1998). Classroom discourse can be studied at several interdependent but partially decomposable levels (Matusov, 1996; Nathan, Knuth, & Elliott, 1998). For example, Nathan and Knuth (2003) looked at how considerations such as teacher beliefs and goals, current education reform demands, and opportunities to reflect on one's emerging teaching practices influenced classroom instruction over a 3-year period. The details of this complex relation were not apparent, however, without also examining mutually constraining levels of analysis that addressed (a) the moment-to-moment (micro-level) flow of information among the members of the discourse community, (b) the nature and purposes of classroom scaffolding (at the meso level), and (c) global patterns of interaction that occurred across an entire discourse. Analyses along mutual but partially decomposable levels allow one to focus on certain phenomena, while still providing a relatively integrated account of the behavior captured by the data.

Research Focus

Although the existence and importance of IS is well documented, the way in which IS transpires in discourse-based classrooms and the role it plays in shaping social interactions are less well understood. We draw on a multilevel framework in organizing our current views on the data and establishing our research questions. However, we are responsive to the particularities of this data set in determining the levels of analysis that structure our inquiry. The *global level* considers changes over the entire discourse. As mentioned, one of our central foci was the changing use of solution representations. We asked: How does the discourse unfold over the course of the class? How does representation use change over the discourse? What role does IS appear to play in these changes?

At the *meso level*, sitting between descriptions of the global progression of the discourse and the micro-level view of individual turns and actions, we are interested in identifying and describing the discourse events among participants that reveal the nature of their dynamics. How is the discourse structured? What perpetuates the discourse? What role does IS play in influencing students' interpersonal interactions, including their uses of public solution representations and the subsequent reactions of the other participants?

We provide more details of the various unitization schemes in the next section. In a companion paper, we will report on analyses at the *micro level*, where the focus is on individual students' actions and utterances as they occur during each turn of the discussion. The focus in this paper is on the global and meso levels, with the aim of analyzing the structure and dynamics of the discourse where IS appears to emerge in the classroom discourse most explicitly.

Method

Participants and Setting

Sixth-grade students in a middle-class community in the western U.S. engaged in solving the Pie Problem that is the focus of this study. One of the students, Manisha,[2] posed the problem as follows: "How do you cut a pie into eight equal-sized pieces making only three cuts?" Manisha presented the problem to the teacher during the customary class warm-up activity. Finding the problem appropriate, the teacher invited Manisha to present it to the entire class. Students spent over an hour out of a 90-min double period of their mathematics class solving this problem and discussing its solutions, first working individually, then in pairs, and then with the class as a whole. Our focus here is on the whole class discussion.

The class normally had 24 students. We observed 20 students participating in the discussion, with 13 playing a particularly active role. The other 7 students involved were observed verbalizing their views clearly but indirectly, as part of a chorus of students.

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[2] All participants' names have been replaced by pseudonyms to ensure confidentiality.

The mathematical performance of the students in the class varied widely, with performance on the California Achievement Test (CAT) ranging from the 5th to the 99th percentile. Five students in the class received special education support for physical and cognitive disabilities. A paraprofessional came once a week to help the teacher meet these students' special needs, though the aide was not present during the lesson under investigation.

The teacher had taught middle school mathematics for 16 years at the time of the session we observed, and soon thereafter became the senior mathematics teacher at the school. The class session took place in late October, when school had been in session about 2 months. By this time, students were familiar with classroom norms for group participation and had spent considerable time publicly presenting their own mathematical ideas, posing questions to their peers, and practicing active listening skills in both mathematical and non-mathematical contexts.

Analytic Approach

Transcription and unitization. The classroom discourse was captured on video and digitized, then imported into Transana, a computer application for discourse analysis (Fassnacht & Woods, 2005; *www.Transana.org*). The video was first transcribed generally for utterances. At this time, we also used Transana to create an audio waveform file that visually illustrated the amplitude of sound over the time course of the video (Figure 1). The waveform is particularly useful for identifying pauses and especially active parts of the discourse. This initial "rough" transcript served as the record for viewing the videotape and fostering early hypothesis generation. The video and transcript were then analyzed over multiple passes in the manner suggested by Duncan (n.d.).

Multiple passes through the data. During the first pass, we unitized the transcript at the stanza level, identifying principal interactions between participants. As we conceptualized the criteria for determining stanza boundaries, following Gee (2005), each stanza started with the initiation of a new speaker's turn to present ideas about the problem and continued until a substantively new idea or line of discussion was introduced. Stanzas in the transcript were bracketed by video time codes that allowed us to coordinate movement through one medium (e.g., video) with movement through the other (e.g., transcript). (Transana visually highlights the corresponding region of the transcript during video playback.) This allowed us to easily track the speech with the videotaped actions, and vice versa.

The second pass extended the initial transcript by including information on gestures and representation use. We also corrected any initial transcription errors and included any utterances that seemed unintelligible during the first pass. The third pass focused on the particular representations used and coded them along three dimensions: (a) use of the principles of perspective, (b) effort to disambiguate the representation, and (c) internal consistency.

The fourth pass took a discourse analysis perspective, examining speech events within stanzas that were appropriate for our research questions at the meso level. Here, we were guided by prior work on classroom discourse that has identified common triadic sequences among interlocutors, such as the previously mentioned initiation-response-evaluation (IRE) patterns (Mehan, 1979; Sinclair & Coulthard, 1975; Lemke, 1990). Whereas stanzas may be regarded as paragraphs, addressing one complete interaction, events in our analyses may be likened to sentences.

The fifth pass identified utterances that conveyed IS, both convergent (IS^+) and divergent (IS^-) (Matusov, 1996). We coded IS^+ whenever there was evidence that speakers shared a common frame of reference, such as speaking about a common representation or stating agreement. IS^- was coded when speakers showed disagreement, alternative interpretations, or confusion.

Reliability. We established inter-rater reliability for our unitization and coding practices. An individual who was familiar with video coding and the Transana software package, but was unaffiliated with the research team and unfamiliar with our specific research questions, served as a reliability check. Codes were accompanied by video exemplars for training purposes. The external rater assigned codes and drew unit boundaries for approximately 10% of the video clips. We report Cohen's Kappa measure of inter-rater reliability and percent agreement within each of the Results sections.

Results

The unitization process revealed 36 stanzas during the whole class discussion of the Pie Problem, which ran for about 37 min and 30 sec (with a classroom break midway through). The inter-rater reliability for stanza divisions based on the criteria stated above was found to be 100% agreement and no disagreement. As mentioned, stanzas were themselves composed of more basic events of participant interaction that distinguished between the initiation of a new topic, participant responses, and participant evaluations of those responses. Prior research on classroom communication showed the prevalence of IRE sequences and their variants (Greenleaf & Freedman, 1993; Lemke, 1990; Mehan, 1979; Sinclair & Coulthard, 1975). Several common events were observed within stanzas during our fourth pass through the transcript:

- *Initiation-closed:* A known-answer question (Matusov, Bell, & Rogoff, 2002)—asked to involve or assess other speakers—that has a fixed or "closed" set of responses, often with a "best" response known by the speaker.

- *Initiation-open:* An information-seeking question (Matusov et al., 2002), with no expected or "best" response known by the speaker.

- *Response:* Student response to a known-answer question, as in an IRE sequence (Mehan, 1979).

- *Demonstration:* The presentation of an idea in response to an initiation-open event, often presented directly and externally in the form of a drawing, string of gestures, or manipulation of objects.

- *Evaluation:* Value judgments in reference to a response or demonstration.

- *Elaboration:* Additions, modifications, and queries about a preceding demonstration or response.

Table 1 shows the frequencies of occurrence of these events.

As can be expected in a socially mediated problem-solving setting such as the classroom we studied, demonstration, elaboration, and evaluation phases were common throughout the discourse and took up a majority of the time. In a surprisingly large number of cases ($n = 24$), the elaboration and evaluation phases were interlaced, often by the same speaker, leading us to apply both codes to the same events 85.7% of the time. For this reason, we opted to combine elaboration and evaluation events into a single *E event* category.

Initiation Events: Who Directs the Discourse

An essential part of understanding the basic structure of a discourse is knowing who directs it. Initiation (I) events indicate when a speaker begins a new thread or invites others to contribute to the discourse. As noted above, initiation can be open or closed. Closed I events are common in certain instructional settings, but they made up only a small portion (5%) of the I events observed in this corpus. The prevalence of open I events is indicative of the discursive nature of this class. Most of the time (78.6%), the agent of initiation of a stanza was the teacher. This is to be expected, since one significant role of the teacher in a discourse-based learning environment is to orchestrate participation through social scaffolding (Nathan & Knuth, 2003). However, students also contributed to the social scaffolding role, co-initiating new stanzas with the teacher about 30% of the time. For example, in Stanza #2, we see:[3]

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T: Well, why don’t you go (.) draw yours on the board?
S1: Oh yeah, that’s it. (This refers to the previous drawing.)
S2: Ben, draw yours on the board then.
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Topics were initiated solely by a student 14% of the time and by an attending researcher 6% of the time.

Response Events

Response (R) events were coded only when a participant responded to a closed I event, in keeping with prior conventions of IRE research (Mehan, 1979; Sacks, Shegloff, & Jefferson, 1974). We observed only two R events in the data set (Stanzas #8 and #28), consistent with the small number of known-answer I events that prompted them (Table 1).

Demonstration Events: Analysis of Students’ Solution Representations

We look next at the nature of the problem solutions that students offered and the ensuing discussions that took place in response. Students’ demonstrations of their solution representations took numerous forms; in fact, we coded 28 unique demonstration representations in our corpus. To make sense of the range of solution representations proposed, we coded each as being in one of four categories: (a) drawings, (b) single objects (e.g., a bowl or tabletop), (c) aggregate objects (e.g., blocks), and (d) representational gestures. For record keeping, representational gestures were coded when students gestured without an accompanying medium

[3] For a list of transcription conventions, see the Appendix.

such as a drawing or object. We follow Alibali and colleagues (Alibali, Heath, & Myers, 2001) in distinguishing representational gestures that address the content of speech, from others, such as beats, that “do not present a discernible meaning” (McNeill, 1992, p. 80). Representational gestures are those “in which the hand shape or motion trajectory of the hand or arm represented some object, action, concept or relation” (Alibali & Nathan, in press, p. 8). The category of representational gesture as used here collapses the categories of iconic and metaphoric gestures as described by McNeill (1992) and others. Representational gestures were coded if solutions were provided through hand and arm shapes without use of any object or drawing.

Drawing representations were produced on the classroom whiteboard using pens. Some involved the use of multiple colors, word labels, speech, or gestures to identify constituent parts. Object representations were coded when students utilized or referred to a prefabricated item. If construction or deconstruction operations involving (or creating) multiple objects were used (including assembly or cutting), then we coded this representation as an aggregate object. If, however, the item was involved only in integrity-preserving transformations (such as rotations or pointing), it was coded as use of a single object.

Table 2 shows the types of representations along with examples, and their frequency. The most common representational forms exhibited by students were drawings (39%) and gesture-only (39%). Drawings varied tremendously in how they conveyed in two dimensions the spatial relations of the problem and how they communicated actions and their consequences within this static form. Sometimes labels were used. However, it was more common for students to annotate the drawings with speech and gesture. This annotation was reflected in the extensive use of multimodal forms of communication[4] of the solutions (e.g., Alibali, 2005; Engle, 1998).

Sometimes students moved away from drawing, relying on more spatially and temporally oriented representational forms. Object use (21%) was the next most common representational form used by students to demonstrate their solutions. In the next section, we report on the interpretations of these representations by participants and the artists themselves. The relative sophistication of students’ solution representations will be taken up in a later section.

Demonstration Events: Analysis of Students’ Interpretive Frames

Students’ demonstrations conveyed more than solution methods. These demonstrations also shed light on the interpretive frames that students brought to the problem. The frames, in turn, provided a conceptual point of view from which to analyze students’ interpretations of the representations proposed by others.

A *frame* is the enveloping meta-message that conveys the orientation of the participants to the ensuing discourse and thereby influences the interpretation of the actions and utterances (Bateson, 1972). Notably, a frame need not be made explicit at the outset. Two prominent and fundamentally different interpretations were voiced by the students in this study. The *literal view* of the problem emphasized the properties of a pie: its round structure, the filling, the asymmetry of top and bottom layers of crust (to some, the “bottom” crust was actually seen as coming up the

[4] Specific analyses of the multimodal forms of communication are beyond the scope of this paper, though they will be the topic of a future article.

“sides” of the pie because both are made from the bottom flat of dough), and so on. In contrast, the *geometric view* considered the “pie” to be abstract, used for purposes of mathematical convenience. The latter view most closely conforms to the situated meaning (Gee, 2004) of mathematical practice, in that the exact nature of the pie was arbitrary; it was (or could be) uniform in its composition, and it could be cast in any geometric form to suit the demonstration.

Students often (93% of the time) provided solutions that conformed to either the literal (32%) or the geometric (61%) perspective. Table 3 shows examples of demonstration events generated by students that were coded as *literal*, *geometric*, and *neutral*, along with their frequencies of occurrence.

The two contrasting views contributed to the classroom dynamics as students interpreted one another’s demonstrations. For example, in Excerpt 1, we see a student named Dave, who has a geometric stance, offer his solution.

Excerpt 1 (from Stanza #6)

Dave: Well this is the top ((pointing to the top circle of the pie)) and this is the side ((pointing to the middle portion of the pie drawing)).
Roger: So you cut through the tin, or you take it out of the tin.
Dave: You took it out of the tin.
S: (Indecipherable)
Manisha: So, are you, you’re cutting it diagonal here, right?
Dave: Yeah, (   )
T: Up or down would be...
Manisha: So you’re doing it now and that won’t work, that’s not totally equal.
Draper: I know. There’s, they won’t. There’s, the top’s on top and the bottom is on the table. And, and you know that third. You know how here’s the pie. If you made cut to it like that, it wouldn’t be the same number of pieces. That line going though the middle ...
Manisha: Yeah.
Draper: ... that separates the two parts, that’s not a cut, that’s the side of the pie.
Manisha: Yeah.
Dave: Well, I know that.
Draper: So that wouldn’t work. (Indecipherable)
T: Uhhh it’s Bob’s turn.
Bob: Like I mean... who would want to have a pie that doesn’t have like a bottom dress thing, the thing would like fall off and like I think it’s just weird that you cut it through the middle.
S: This is just a demonstration of like how you’d see it from that perspective.

Dave’s solution includes cuts that go through the top and sides (Lines 1–3; see Figure 2). Voicing a literal concern, Roger asks about the pie tin (Line 4), and Dave complies (Line 5). Students who also have a geometric view have trouble understanding how the cuts (meant to be in 3D) are drawn (Line 7) and whether these would actually result in equal-sized pieces (Lines 10–11). Another student, Draper, raises an issue about the interpretation of all the lines in the drawing (Lines 17–18). However, Bob raises a different issue: he cannot fathom why one would dissect a pie in this manner, since it would lose its integrity. He concludes “it’s just weird” (Lines 23–25) and, in a later interaction, challenges the idea since “if you have the top crust and you like lift it up, all the stuff’s gonna fall out” (Stanza #37). In defense of Dave’s solution, a third student takes the situated stance that is common in mathematical practice, arguing that the specifics of a pie are inconsequential and are just for purposes of demonstration (Lines 26–27).

Dynamics of the Discourse: Emergence of IDE Sequences and Cycles

In addition to exploring the nature of the events that constitute the discourse, we are interested in the dynamics of the discourse itself. In this section, we investigate the ways in which the discourse institutes change. In subsequent sections, we look at the way forms of IS that pull participants toward or away from shared interpretations interact, and the role these interactions play in sustaining the discourse.

Triadic dialogue—a question, followed by a response, and a subsequent evaluation—is the most common form of discourse pattern observed in classrooms (Lemke, 1990). Our analysis of the pattern of event occurrences reveals that stanzas are often composed of sequences of initiation (I), demonstration (D), and evaluation and elaboration (E). In all, 28 out of the 36 stanzas (77.8%) were made up of IDE sequences. IRE sequences were evident in only 2 of the stanzas (5.5%). Three stanzas were interpreted exclusively as “meta” topics, while one received both an IRE and a meta code. Meta topic stanzas focused on the wording of the Pie Problem itself and followed an alternative structure. The remaining 3 stanzas did not fit any particular pattern.

The preponderance of IDE sequences was evident not only in the high proportion of total stanzas that followed IDE patterns, as noted above, but also in the total amount of time participants spent interacting within an IDE sequence. The set of 36 stanzas ran 37:30. Of that, 84.3% of the time (31:38) was spent within IDE sequences.

We used the keyword mapping feature within Transana to show the locations of I, D, and E events over the time course of the transcript. Figure 3 reveals the combinatorial nature of the events. The first pattern to observe is that I, D, and E events occurred in sequence quite often (21 times). On only two occasions (Stanzas #2 and #9) did an E event fail to directly follow the referenced D event. In both cases, this occurred when a student interjected another demonstration before students could evaluate the current one. Otherwise, the recurrent pattern from I to D to E is seen as one moves along the time line.

The IDE sequence provides a useful way to characterize the discourse structure. Someone, usually the teacher, initiates a new IDE sequence by asking a question or inviting ideas. Participants usually respond to the invitation with some demonstration of their ideas about the solution, most often by making a drawing. Some evaluation ensues, often based on the geometric or literal interpretative frame within which participants perceive the demonstration. The evaluation is often accompanied by some elaboration, such as further explanation or even modification of the representation used to convey the solution.

E Events: Establishing Intersubjectivity Among Participants

We also investigated the occurrence of IS as a property of participants' interactions and the ways in which seemingly opposing forces interacted. In this analysis, we drew from the participatory view of IS (Matusov, 1996) reviewed earlier. In this view, as described above, IS^+ is cast more broadly than in the traditional view, so that IS^+ was considered in evidence not only when students reached agreement, but also whenever students operated within a shared conceptual space. In practice, this often meant that we were evaluating whether we thought interlocutors shared or even appropriated one another's language and representations. IS^- was considered in evidence when students disagreed or presented divergent interpretations. Based on these coding criteria for IS, we achieved perfect inter-rater reliability (Kappa = 1.0). If, in addition to sharing a common representational space, students disagreed, misinterpreted one another, or expressed divergent views, we considered this evidence for *both* IS^- and IS^+.

Instances of IS were ubiquitous in our data set. All 28 E events—events that exhibited elaboration or evaluation—received at least one of the two IS codes. We also found numerous instances of the co-occurrence of IS^- and IS^+ codes. In all, IS^+ and IS^- co-occurred in 23 out of 28 E events. Only 2 cases of IS^+ and 3 cases of IS^- occurred in isolation.

In one case of IS^-, a student created a folded paper model that, when cut, was to yield eight pieces. This model had the potential to convey many difficult aspects of the solution, such as its three dimensionality and the creation of multiple pieces over time. However, the student had added additional folds so that more than eight pieces were produced (another student commented that it looked like "about 20"). She also had difficulty handling the pieces and scissors and dropped the pieces partway through her cutting demonstration. Participants voiced disagreement with her conclusions and confusion about her method, earning an IS^- code. In the end, no one picked up on the demonstration in subsequent speech acts.

As expected from the participatory view of IS, seemingly conflicting aspects of IS commingled to create rich discourse, as evidenced in Excerpt 2.

<u>Excerpt 2 (from Stanza #20)</u>

```
Bob: ((Drawing 2 circles one above the other.)) Alright here is the top and here is the bottom. Just say that they're like if you really look at it like, and like if you cut it like this
Manisha: Then it would have to go all the way through.
S2: Yeah, it would have to go all the way through. Right here is like... wait is this like the bottom?
S3: That's the side.
Manisha: And then you'd have to cut it in half (using Bob's drawing as reference).
S: (Indecipherable)
Researcher: Can you guys speak up?
T: Guys you need to talk louder okay?
Bob: You cut all the way down. That wouldn't make eight pieces.
Manisha: If you cut it in half it would.
```

In Excerpt 2, Bob provides a drawing (Lines 1–3) that shows two separated layers of the pie (one top view of a circle drawn above another; see Figure 4). While this captures the three-dimensional aspect of the problem for some students, it reinforces the Bob's assertion that there are only *four* pieces made (to Bob, the four shown on top and again on the bottom are the *same* pieces; Line 13). Manisha points to the board to address the representation, thereby adopting Bob's representational frame. This indicates a shared discourse space characteristic of IS^+. However, she also conveys an alternative interpretation, arguing that the cuts shown in each layer have to "go all the way through" from one layer to the other (Line 4). She actually physically enters Bob's space at the board and reframes his drawing (Lines 8–9), while using her hand gestures to refer to the space between the two layers that, in her mind, captures the third and crucial cut that yields eight (rather than four) pieces. Bob challenges her interpretation (Line 13), but Manisha reasserts her initial point (Line 14) that if someone cut all four pieces in half horizontally, it would yield eight equal pieces.

The frequent co-occurrence of IS^+ with IS^- during E events is in keeping with the participatory view that maintains that these two forms of IS are not mutually exclusive within the discourse, and may even be necessary for substantive disagreements to occur.

Sustaining of the Discourse: IDE Cycles and the Role of Intersubjectivity

One further insight from the keyword map of Figure 3 is that IDE sequences tend to be cyclical, chaining one to another. Most (81%) IDE sequences followed directly from a previous IDE sequence (ignoring the first stanza and the stanza immediately following the classroom break in this calculation). In contrast, we observed no chaining of IRE sequences. In the typical IRE pattern, the teacher asks a closed question that invites a relatively narrow range of student input. This question elicits a response (R) from a student, which is then evaluated (E), usually by the teacher, in a way that typically terminates the interaction ("That is incorrect" or "Correct!"). Some chaining has been seen in the IRF (follow-up question) variant, but it was not observed in this corpus.

Students had many sources of difficulty interpreting one another's (and their own!) line drawings. Several IS^- instances were the apparent result of inadequate drawing as students tried to convey their three-dimensional ideas using two-dimensional drawings on the white board. For example, Dave's drawing in Figure 2 showed three lines all intersecting in a way that violates principles of perspective (e.g., foreshortening).

Several other points of conflict centered on the interpretation of the curved, convex edge (see Figure 2 and Excerpt 1) that is formed through use of perspective when the flat, top layer intersects the curved side (Waltz, 1975). In some cases, students (even the artists themselves) would (re-)interpret the curved line as a cut when they counted the resulting pieces.

Excerpt 3 (from Stanza #1)

```
1  Manisha: That makes... four pieces on the bottom and four pieces on
2     top.
3  Bob: What's wrong with mine?
4  S: Yours is, you didn't circle.
5  Bob: Who cares?
```

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6   Roger: That's six pieces.
7   Bob: Fine.
8   S: No!
```

In Excerpt 3 (Stanza #1), a student with a literal view actually uses the individually bounded regions shown on the board to count the number of “pieces” that result from making the cuts and concludes, “That’s six pieces” (Excerpt 3, Line 6). This conclusion conflicts with the geometric interpretation offered by another student that leads to eight pieces when interpreted in three dimensions (Line 1).

While IS codes were common throughout this discussion, they also appeared in key places in the discourse. All of the 26 IDE cycles that followed a prior stanza (i.e., excluding the first one and the one following the classroom break) were preceded by statements coded for IS. Most of these (85%) showed the co-occurrence of IS^+ and IS^-. Furthermore, IS was the predecessor of all but three IDE sequences.

The role of IS, then, appears to be substantial in this extended classroom discussion. In its convergent or positive form, it identifies the common discursive space within which participants can meet about their problem-solving ideas. In its divergent or negative form, it reflects the challenges that speakers face in communicating to others, particularly when the representations are ambiguous and when interlocutors have differing interpretive frames and limited skill with 3D drawing. Furthermore, in its general form, IS appears to serve as the impetus for further triadic event sequences that structure the group problem-solving interactions. Thus, IS appears to be a major influence in the cycling of the IDE event sequences that are so prevalent in these data.

Structure of the Discourse

From these meso-level (event-level) analyses, we conclude that the discourse was largely structured by IDE sequences. IDE becomes central to our study because I events tend to mark the boundaries of discourse units (stanzas) and then serve to elicit through open-ended invitations rich responses from students in the form of representation-laden demonstrations (D events). It is within D events that students often show their mathematical thinking. D events, in turn, trigger E events, in which the proposed solutions are evaluated and elaborated upon, often by new speakers. It is within the IDE triads that discourse participation is enabled and supported.

The analyses showed that the recurrence of IDE sequences was preceded by, and perhaps even driven by, IS—through disagreement, inadequate representational skills, and conflicting interpretations among members of the class (IS^-), on the one hand, and through agreement, elaboration, and use of shared representations (IS^+), on the other. Regular prompting by the teacher and students served as a catalyst for students’ engagement with the task and with each other. But the discourse was actually sustained through IDE cycles reflecting students’ engaged pursuit of ideas and solution representations. However, at this level of analysis, we can only describe this process as sequences of events. To see the landscape that this discussion actually traverses, we need to step back and examine the discourse from a global level.

Direction of the Discourse: Trends Toward Representation Standardization

The preceding findings paint a portrait of the discourse as seen at the event level. We see the particulars of students' interactions and the frequencies of their occurrence. However, as with a pointillist painting viewed up close, it is difficult to determine the nature of the picture. Thus, we pull back a bit so we can look at the discourse at a coarser grained level. At the macro level of analysis, we are principally interested in the terrain the discourse covers. In other words, where does it start and where does it go? To assay whether the interactions are productive, we examine changes in the discourse, as speakers receive feedback and monitor their own views.

Demonstrations of solution representations operate as objects (sometimes literally) around which IDE sequences organize. One way to characterize the discourse is to examine the evolution of the uses of representations over time. Toward this end, we examined the first and last uses of representations by one of the students—in this case, one with a literal view of the Pie Problem. Bob was a fairly vocal member during this session, although he was generally not a central participant in math class and typically scored near the bottom of his class. He appeared to be very engaged in this lesson, however, and provided some of the earliest as well as some of the latest contributions to the discussion. For this reason, he makes for an important case. Later, we show the changes across all of the representations used over the entire discourse.

Figure 5 shows Bob's first demonstration (Stanza #3). In addition to the drawing he made, Bob used verbal explanation and gestures to elaborate his intentions, as shown in Excerpt 4.

Excerpt 4 (from Stanza #3)

Bob: OK, here's the pie (sigh).
T: Let's listen to Bob now please.
S: I drew them like square bodies... like that. And they're curving...
Bob: Well, fine rou:::nd.
Bob: Yeah, Dude. It does... cuz you're cutting it in half.
S: No.
Mary: No... No... you're, it usually goes to the bottom (Indecipherable) pie,... usually goes to the bottom.
T2: Bob, Bob how many slices are there?
Bob: What?
T2: How many pieces are there?
Roger: There's 12.
Mary: ((Counting each piece))
Bob: No, not there ((touching the extraneous middle triangle)).
Bob & Mary: ((Pointing inside each bounded region of the drawing while counting)) One, two, three, four, five, six, seven, eight.
S: No.
T2: No, no, no, no, no.
Mary: ((Pointing at each region while counting)) One, two, three, four... eight. No.
Mary: ((Erases Bob's drawing))
S: Yeah, but they're not equal.

The drawing is a pie (Excerpt 4, Line 1), and Bob completes its three-dimensional depiction before he adds the lines that represent the cuts. The pie is to be cut up by three straight lines: The first is a completely straight vertical line through the top ellipse and the side, while the other two lines form an *X*. It was the artist's intention that the three cuts meet in the middle, and when they formed an unintended triangular shape in the center of the ellipse, it was discounted (Line 14). To Bob, and another student up at the board, it was natural to equate the separate regions with pieces, and so Bob and Mary each count to eight, pointing and touching each region as they go (Lines 15–16). Not all participants agree with this (Lines 17–18), but a second count by Mary, who is skeptical of the solution, lends further support to the view that there are eight pieces, given the region-as-pieces interpretation (Lines 19–20). Still, Mary is unwilling to believe her count and she concludes "No" and erases the drawing (Lines 20–21).

Bob's initial demonstration can be evaluated with respect to three criteria:

1. *External consistency:* The adherence of the demonstration to the principles of perspective drawing;

2. *Internal consistency:* The uniformity with which elements of the representation take on certain meaning or roles in the solution; and

3. *Ambiguity:* The amount of effort and elaboration needed to interpret the drawing in an unambiguous manner.

On the first criterion, the representation violates some of the principles of perspective drawing. The diagram does not convey a sense of depth or of a "vanishing point" (often referred to as linear perspective) and lacks proper size and shape variation. Thus, we see the top plane of the pie from the top view (as fairly circular), but the sides from a side view. The drawing also does not apply notions of modeling that would, for example, lead one to expect the vertical slice to bend at the side of the pie as it changed planes. On the second criterion, we see inconsistencies within the drawing itself. The artist misinterprets the curved, convex edge, as evident by his counting method. Finally, there are aspects that are ambiguous (such as the addition of the triangular region, and the square edges) and require further explanation from the student.

<u>Excerpt 5 (from Stanza #33)</u>

```
Bob: Can I go up?
T: Bob, since people seem to be directing at you, Bob, I think it's only fair you have a chance to speak out.
Bob: Okay ((walking up to the board with a hand full of blocks)).
T: We're gonna spend five more minutes on this and then we have to move on. And we can come back to it, but for today, five more minutes.
Bob: Okay, um... ((places eight blocks into his right hand in a cube formation))
Bob: Okay, say I'm... this is a pie, and you cut it like right there ((using hand like knife makes a cutting motion to top of cube perpendicular to his chest)) and right there ((using hand like knife makes a cutting motion to top of cube parallel to his chest)).
Bob: And then you cut it at the bottom ((using flat hand, palm up, like cutting the cube in two layers)).
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Bob: That is still going to be four pieces because you cut it at the bottom.
```

In contrast to his early solution, Bob's final demonstration (Excerpt 5; also see Figure 6) uses eight wooden cubic blocks to form a 2 × 2 × 2 cube. He intends for the cube to be the pie (Line 9), and his hand motions to represent the slices (Lines 9–14). He uses this representational form to argue that the horizontal layer cut (Lines 13–14) is not legitimate since it does not really result in doubling the number of pieces that one would expect to get when eating a real slice of pie with both a top and bottom crust. To Bob, these three cuts (like those proposed earlier by several students) result in only four pieces of pie (Line 15).

Bob takes issue with the geometric interpretation that all cuts are reasonable and that all cuts make more pieces. He does not abandon the literal view that he had initially. However, his use of representation shows a rather significant shift. It is more refined, more conventional, and more explicit, suggesting he is more aware of the perspectival needs of others (Greeno & van de Sande, in press; Greeno & MacWhinney, 2006). The three-dimensional object is consistent with principles of perspective (Criterion 1), and the components of the representation maintain a stable meaning throughout his demonstration (Criterion 2). Furthermore, the use of an object with gestures shows the spatial and temporal aspects of his solution in an explicit manner (Criterion 3). While one may disagree with his interpretation of the layer cut, his position is consistent and relatively unambiguous.

The changing nature of representation use seen here gives a sense of the global dynamics of the discourse. To place these contrasting demonstrations within the larger corpus, we applied a four-level rubric based on the three criteria above (external consistency, internal consistency, and ambiguity) to the entire discourse (inter-rater reliability produced 83% agreement and Kappa = 0.77 across the levels). As Table 4 shows, we identified 46 representations across all of the demonstration and elaboration/evaluation events over the 36 stanzas. Level 1 identified the most impoverished representations, and they were observed with the greatest frequency. Levels 3 and 4, the most refined representations, were relatively rare. Figure 7 shows the distribution of the level codes over time. Rows represent the four levels from the rubric. Colors designate video clips that received codes, with unique colors assigned to each clip and the length of a clip proportional to its duration. As Figure 7 shows, codes for the more idiosyncratic and ambiguous representations (Levels 1 and 2) were scattered throughout the discourse, though they appear to be in more frequent use early on. Often, these were casual drawings that paid little attention to accuracy or conventions of perspective. Some were representational gestures that demonstrated spatial and temporal relations in an idiosyncratic and ephemeral manner, and addressed only part of the solution. Later in the discourse, it was far more common for students to propose solutions that were more standardized (Levels 3 and 4). For example, in the second half of the discourse, students introduced objects that they could manipulate in space to show three-dimensional relationships that inherently addressed issues of perspective; some actually yielded eight equal-sized pieces. In the latter half of the discourse, we also observed more frequent use of multiple viewpoints with written or verbal labels to address the limits of the 2D medium (e.g., see the example in Table 4 for Level 4).

It appears that the quality of the representations, as gauged by our three criteria, actually improved over time. As a statistical check that this pattern was in evidence, we split the transcript in half and compared the coded levels for the first half of the discourse to those for the second half. We found that it was reliably more likely for students' representations to receive higher levels in the second half of the discourse, $t(40) = 3.27$, MS = .35, $p < .005$. Indeed, Level 4 representations ($n = 5$) were observed only in the second half of the discourse.

This change in the use of representations appears to be motivated by earlier interactions, and, perhaps, a sense that one can be more persuasive with the right kind of demonstration. Early on, students were willing to use solution representations of any sort. They invented them readily and executed them casually. But as the discourse continued without resolution, students became more refined in their methods and drew more often on standard ways of depicting the problem and the important mathematical relationships. As we look across time, we can see that the discourse took a discernible direction even though there was no central agent or explicitly stated goal guiding it. As explored earlier at the event level, IS clearly served an important role in perpetuating the discourse. Here at the more global level, we begin to see how participants developed their uses of representations in service of IS. In this way, we see how knowledge socially accumulates, mediated by the interactions of the whole classroom discourse.

Discussion

This paper identifies several elements that appear to be central for advancing our understanding of socially mediated learning. We have seen how the classroom discourse under investigation naturally decomposed into a set of smaller discussions around key occurrences or ideas—stanzas, in Gee's (2005) terminology—and how it tended to move from stanza to stanza as students' ideas and their understandings of each another's viewpoints changed. At the global level, for example, we have noted important shifts in the nature of the representations used by students to articulate their positions.

Stanzas themselves exhibited an internal structure—event sequences, in our terminology—that were specific to these circumstances. In traditional classrooms, the IRE triad is a direct manifestation of a particular social relationship between the students and a central authoritative figure, and shows little self-perpetuation. The specific structure of events differed in this discourse-centered classroom. As we saw, IDE sequences often led to subsequent IDE sequences. (In fact, the teacher had to demand that the discussion of the Pie Problem end so other class matters could be attended to.)

While the importance of IS in social interaction is well documented, its role in shaping discourse and socially mediated learning is only beginning to be understood. We found that IS played a central role. At the event level, it was consistently the precursor to new IDE triads, and in this way seemed to help perpetuate the discourse. Challenges to IS were largely attributed to (a) fundamentally different interpretative frames of the problem and solution representations exhibited by those with literal and geometric views and (b) inadequacies within the representations themselves. Over time, the demonstrations evolved from relatively casual and ambiguous representations to representations that were more principled and explicit about the spatial and temporal relations of the proposed solutions. This shift to more widely accessible representations is the kind of change we expect among speakers striving for a shared understanding.

These analyses revealed aspects of the structure of the discourse as well as its dynamics, as class members used communication and representation to explore the Pie Problem. Communication and representation are central aims of reform mathematics instruction in their own rights, expanding the range of mathematical competencies beyond calculation and fact retrieval (NCTM, 2000). Classroom discourse provides other benefits as well. Teachers gain great insights when students share their thinking. Student discourse informs teachers' assessments of students' current thoughts and also contributes to longer-term models of their conceptual development (Nathan, Elliott, Knuth, & French, 1997). For students to participate and be engaged, they must share a great deal. However, teachers working to adopt reform practices also acknowledge that they must conduct their classes in new ways, and this places large and unfamiliar demands on them, as they must also monitor their progress against curricular and administrative objectives. Here lies one of the greatest challenges of managing the discursive classroom. In the class studied here, the teacher attended to many things in her design of the learning environment, including establishing classroom norms for listening and presenting, mathematical notation, vocabulary of problem-solving strategies, and so on. The set of shared knowledge and practices is never bounded, however, and each activity creates new challenges for manifesting engaging and transformative discussion.

When sustained discourse occurs, the perceptive instructor tries to cultivate it and give it space, circumstances permitting. So it was with the instructor of the class we observed. The teacher played a significant role, but one that could be described as catalytic rather than central. Her efforts were directed mainly at social scaffolding of the discourse (Yackel & Cobb, 1996; Nathan & Knuth, 2003). Student talk clearly dominated the room. The presentations of mathematical ideas and their evaluations were largely left up to the student participants. What emerged was a healthy, sustained mathematical discourse. Students posed solutions, asked questions, critiqued one another, and reformulated ideas in hopes that the next round would be better—more accurate, more widely understood, and more persuasive. Indeed, students' desires to make themselves understood and convincing appeared to play a critical role in the dynamics of this discourse.

In their longitudinal analysis of mathematics classroom discourse, Nathan and Knuth (2003) defined *productive discourse* as "forms of social exchange which provide participants with an avenue to construct and build upon mathematically correct conceptions through their interactions with other class members" (p. 204). In reviewing the current classroom interaction, we see several indicators that lead us to believe that this was a productive discourse. First, participants worked together, continually reflecting on each other's ideas. In this sense, students were engaged in a *recursive communication process*: they listened to one another and were genuinely interested in each other's ideas and contributions (Rommetveit, 1989, as cited in Matusov 2001). Students built on each other's ideas, even if they did not agree. Moreover, in several cases, students appropriated one another's representations. This finding is reminiscent of Latour's (1996) notion of *interobjectivity*, where people with divergent points of view can still exhibit coordinated interaction through the use of shared objects and representations.

Students also evaluated and reflected on the activity itself. For example, there were several meta stanzas in which students commented on the nature of the problem statement and offered ways to reword it in accordance with their own understanding. These occurred only after protracted discussion of the solutions and the different interpretive frames.

Throughout the discourse, students disagreed and challenged each other, but did so respectfully and productively, pushing peers to be clearer and more mindful of different interpretations. And in the end, the discourse did not seem to convert many students from their initial interpretations to new ones. Rather, disagreements during the discourse spawned clarifications and standardization of solution representations. In this way, the disagreements fostered critical dialogue (Bakhtin, 1990) and led students to articulate their disparate positions in more sophisticated ways.

While these analyses reveal the influences of IS, IS does not tell the whole story about sustained and productive classroom discourse. For, if IS is the objective in constructivist classrooms, then representation is the means by which it is to be achieved. By working across interpretive frames, those with different views had to refine their ideas, forms of communication, and representation, in much the same way scientists do when working across traditional disciplinary boundaries (Hall, Stevens, & Torralba, 2002). It is also through the presentation of one's ideas that acceptance and substantive disagreement can occur. As the students in this classroom revealed, such acceptance and disagreement are not easily achieved through casual presentations. Too often, idiosyncratic representations meant little to the audience, and they were sometimes even misinterpreted by the artists themselves, as in Excerpt 1 (Line 15; also see Figure 2), in which the curved edge was taken to be a drawn cut. Instead, students came to adopt representational forms that carried common meanings that were assayed with minimal effort. This finding is similar to Schwartz's (1995) insights about the solutions generated by students operating in dyads. Schwartz found that working across multiple representations was a demand unique to students working in teams. The dyadic work process required more communication than that of students working on their own, and it fostered greater production of abstract representations, which contributed to dyads' superior problem-solving performance. Similarly, in the study reported here, students adjusted their demonstrations to suit the group's needs.

The study of intersubjectivity is growing in importance as the bounds of cognitive science expand (Stahl, 2006). There is greater awareness of the essentially social nature of human thought and learning, as well as a growing appreciation of the complexities of designing and managing socially mediated learning environments. Within the maturing fields of embodied cognition and cognitive neuroscience, basic interpersonal processes such as imitation, empathy, and the ability to impute the intentions of others—all behaviors that hinge on IS—are being considered vital to advancing our understanding of fundamental mechanisms of both individual and social behavior.

Researchers studying imitation and the comprehension of *observed* actions and emotional facial expressions have found that participants (primates, in many of the studies) necessarily engage their *own* motoric and emotional processes through a system of *mirror neurons* when they observe the actions and feelings of others (Rizzolati, Fogassi, & Gallese, 2001). Currently, the claim is that mirror neurons are specially evolved and selected areas of brain circuitry "that allow us to appreciate, experience and understand the actions we observe, the emotions and the sensations we take others to experience" (Gallese, 2003, p. 525), by constituting them in intersubjective relation to our own actions and feelings. This process is called *embodied simulation*. More recently, investigators are drawing on embodied simulation to explain behaviors that are considered more complex than imitation and comprehension of actions; it can also explain findings on the comprehension of emotionally laden text (Havas & Rink, in press).

The embodied simulation model of empathy and imitation does not necessitate that participants experience others' actions and emotions in the same way that they experience their own. Rather, the model stresses that to appreciate others, participants must share a common interpersonal space—the *shared manifold of intersubjectivity*—within which the embodied simulations operate and interpret the world around them (Gallese, 2003). This notion is consistent with both the participatory and the radical constructivist views of IS reviewed earlier. While large gaps between neuroscience and educational practices still persist (Bruer, 1997, 2006), theoretical and empirical advances in the study of empathy, imitation, and embodied cognition contribute to our appreciation of the role that social factors play in shaping individual behavior and perhaps even basic aspects of our cognitive architecture. As our understanding of socially mediated learning and communication continues to develop, we expect to see a greater exchange among these previously disparate fields of inquiry.

Students engaged in collaborative problem solving and substantive mathematical argument are a sight to behold, and such a collaborative learning process has become one of the critical markers of successful reform-based classrooms (e.g., Strom, Kemeny, Lehrer, & Forman, 2001; Nathan & Knuth, 2003). As teachers permit—and even invite—students to publicly share their multiple perspectives, the need to understand more completely the nature and dynamics of intersubjectivity increases. As Bakhtin (1990) pointed out, dialogic exchanges of this form are necessary; without them, we learn nothing, and do little to advance and refine our understanding and our means of communicating our understandings to others.

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Appendix

Transcription Conventions

[	Point of overlap onset
]	Point of overlap termination
=	No interval between adjacent two turns
(2.3)	Interval between utterances (in seconds)
(.)	Very short untimed pause
<u>word</u>	Speaker emphasis
the:::	Lengthening of the preceding sound
?	Rising intonation, not necessarily a question
,	Low-rising intonation, suggesting continuation
.	Falling (final) intonation
CAPITALS	Especially loud sounds relative to surrounding talk
° °	Utterances between degree signs are noticeably quieter than surrounding talk
↑ ↓	Marked shifts into higher or lower pitch in the utterance following the arrow
( )	A stretch of unclear or unintelligible speech
(( ))	Nonverbal actions

Table 1
The Frequency of Events Coded From the Whole Classroom Discourse

	Codes	*N*	Examples
Total stanzas		36	
Events			
	Initiation (I)	30	
	Closed	2	T: Who would respond to Janet about nobody said the pieces are equal?”
	Open	28	S2: Ben, draw yours on the board then.
	Response (R)	2	S: Well, it says on the board how you ‘cut a pie into eight equal-sized pieces.’
	Demonstration (D)	28	S: Like I mean... who would want to have a pie that doesn’t have like a bottom dress thing ((making a small circle with his right hand and then putting his left hand at the bottom with his right hand hitting the left hand several times))
	Elaboration/ Evaluation (E)	30	
	Elaboration*	24	S: If you eventually cut all the pieces, those pieces will fall into the cuts and then it’ll be like a cut.
	Evaluation*	24	S: Well, cutting on the top of the pie that wouldn’t be right. Well, ours is right.
IDE stanzas		28	
IRE stanzas		2	

* Of the 24 evaluation events and 24 elaboration events, 24 co-occur. Elaboration occurs uniquely only four times, and evaluation occurs uniquely twice.

Table 2
Categories and Frequencies of Representations Used in D Events, With Examples

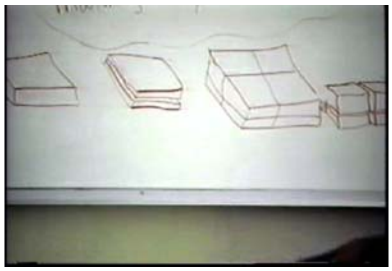
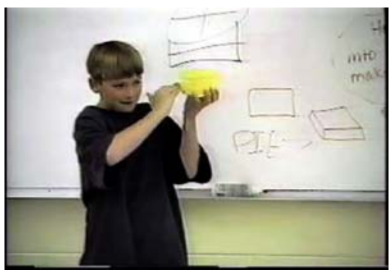
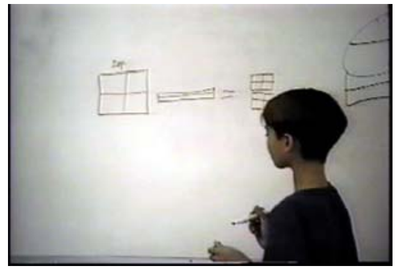
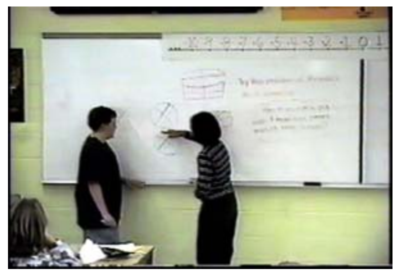
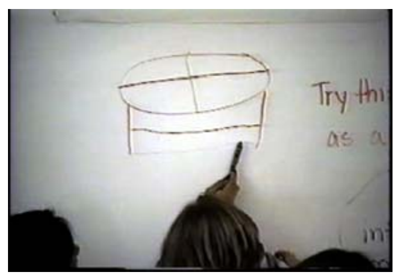
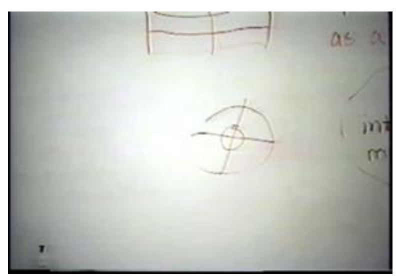
Type	Example	*N*
Gesture-only		11
Drawing		11
Object: Single		3
Object: Aggregate		3
Total		28

Table 3
D Events (*N *= 28) Coded as Depicting Literal, Geometric, and Neutral Interpretations

Code	***N***	**Comment**
Literal	9	"So you cut through the tin, or you take it out of the tin…" "Yeah, yeah... if you have the top crust and you like lift it up, all the stuff's gonna fall out, and what if it's an apple pie. How could you...?"
Geometric	17	"This is just a demonstration of like how you'd see it from that perspective." "Well, like before he was saying like... well, then, who's going to eat a pie with just the bottom and I mean just the top and I mean this is just the example. It's just a diagram. I mean nobody's just going to come up here and eat the dry erase board." "Here's a pie (she is holding eight pieces of blocks on her hand). It's not round…"
Neutral	2	"They're not equal, they're not equal." "Well, I don't really think that the directions were good."

Table 4

Frequency, Description, and Visual Examples of Each Coded Level of Standardization Representations in Both D and E Events (**N** ***= 46)***

Level	*N*	Description	Examples
4	5	• The representation is completely consistent with the principles of perspective. • The meaning or role of each component of a representation is applied uniformly. • Ambiguity of interpretation is kept to a minimum, and resolving it requires little effort or elaboration.	
3	6	• The representation is somewhat consistent with the principles of perspective, though there is one violation or omission. • The meaning or role of a minor component of a representation is not applied uniformly. • There is some ambiguity of interpretation, but it is resolved with a small amount of conscious effort and/or elaboration.	
2	10	• There is an attempt to be consistent with the principles of perspective, though there is more than one violation. • The meaning or role of an important component of a representation is not applied uniformly. • There is a great deal of ambiguity of interpretation that is resolved with a large amount of conscious effort and/or elaboration.	
1	25	• The representation disregards the principles of perspective. • The meaning or role of an important component of a representation is not applied uniformly. • Interpretation of the representation is highly ambiguous and requires a substantial effort and/or elaboration.	
Total	46		

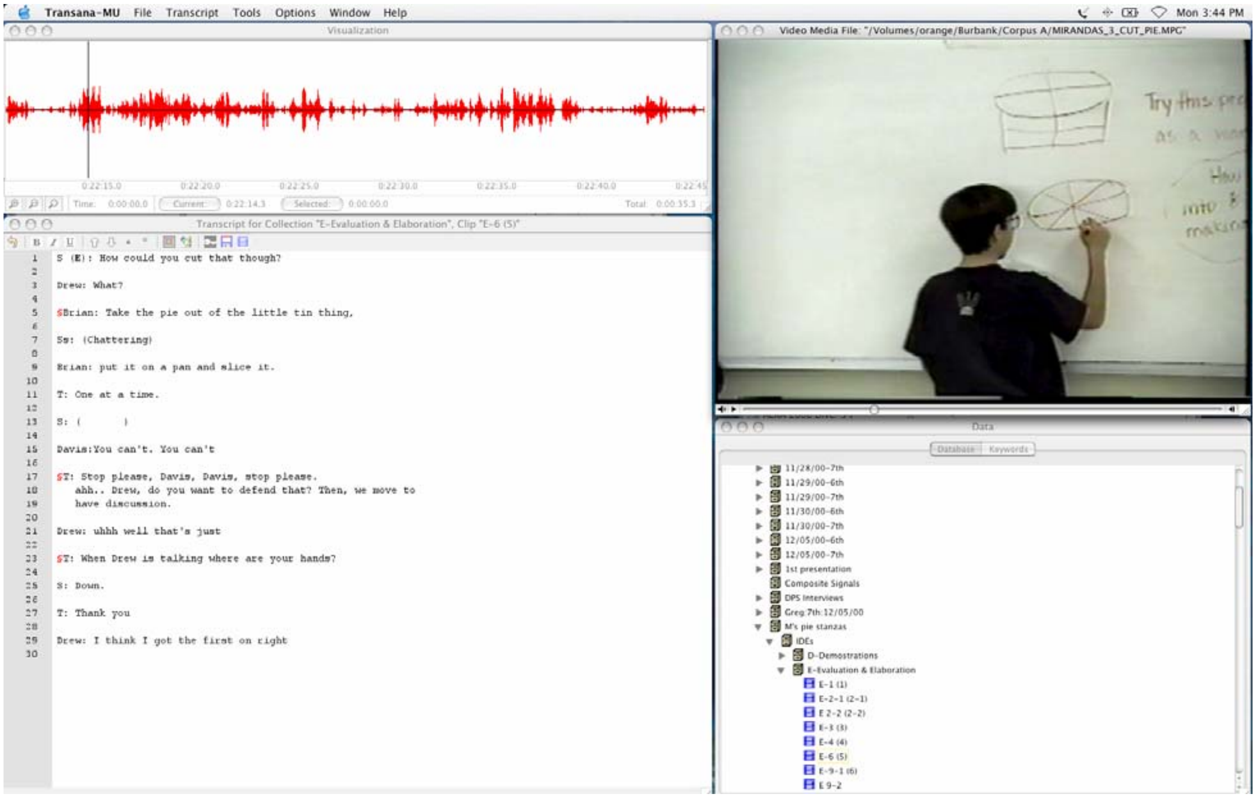


Figure 1. **Screen image of the Transana software package depicting (clockwise from top left) the audio wave form, video, hierarchical database, and transcript windows.**

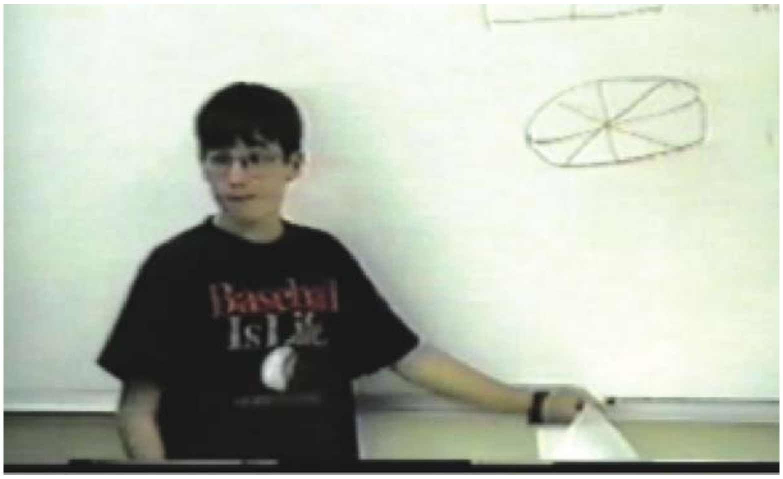


Figure 2. **An example solution representation (from Stanza #6; see Excerpt 1) that shows a horizontal cut that separates the pie in layers.**

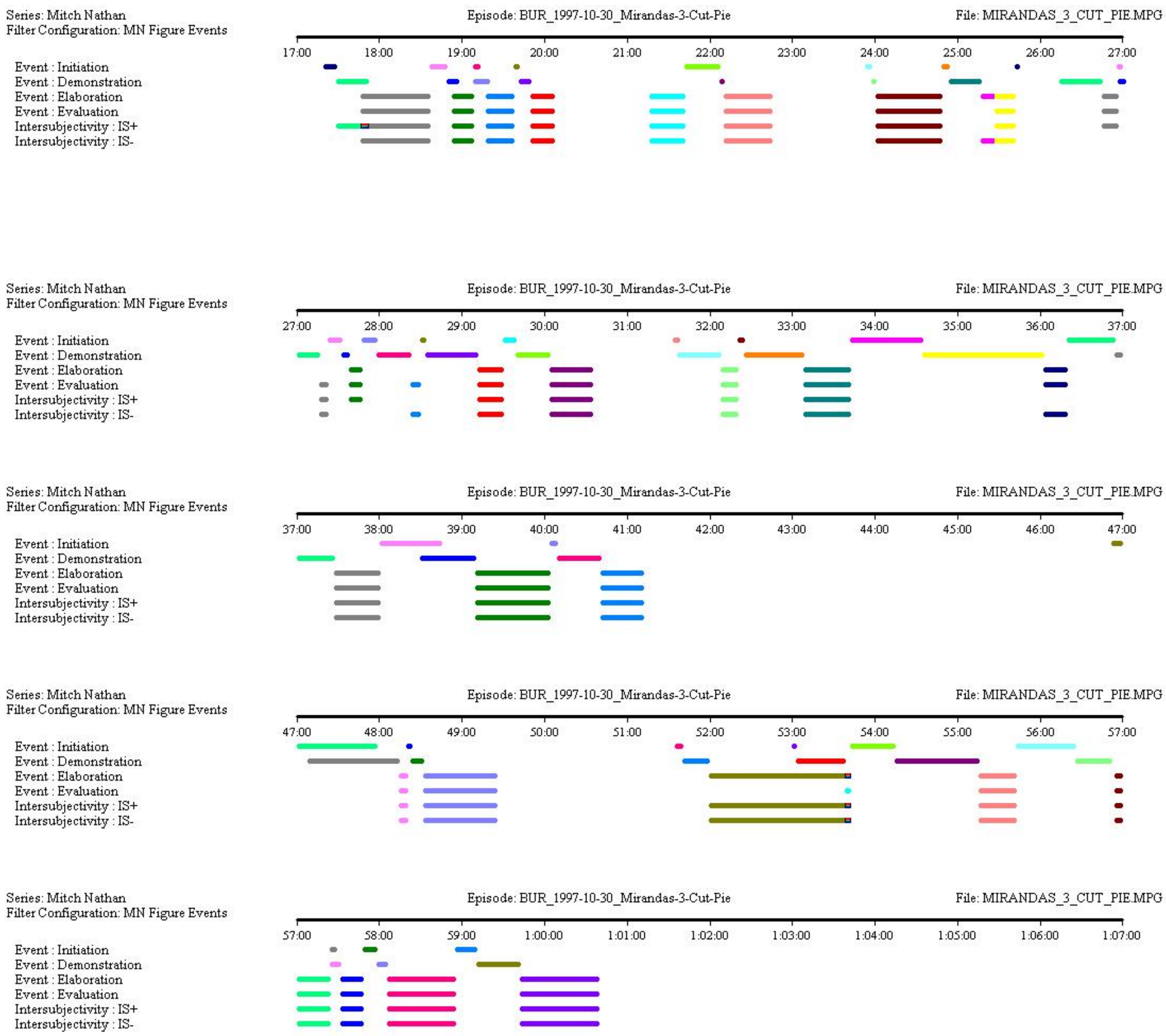


***Figure 3.* The keyword mapping feature in Transana.**
The feature shows the distribution of selected codes along a time line. Color denotes the events to which the codes were assigned. Each time line shows 10 min of discourse. I, D, and E events (the first four rows in each time line) tend to follow in sequence. The IDE sequence also tends to recur cyclically. Occurrence of IS (bottom two rows of each time line) tends to align with E events and precede the subsequent IDE sequence.

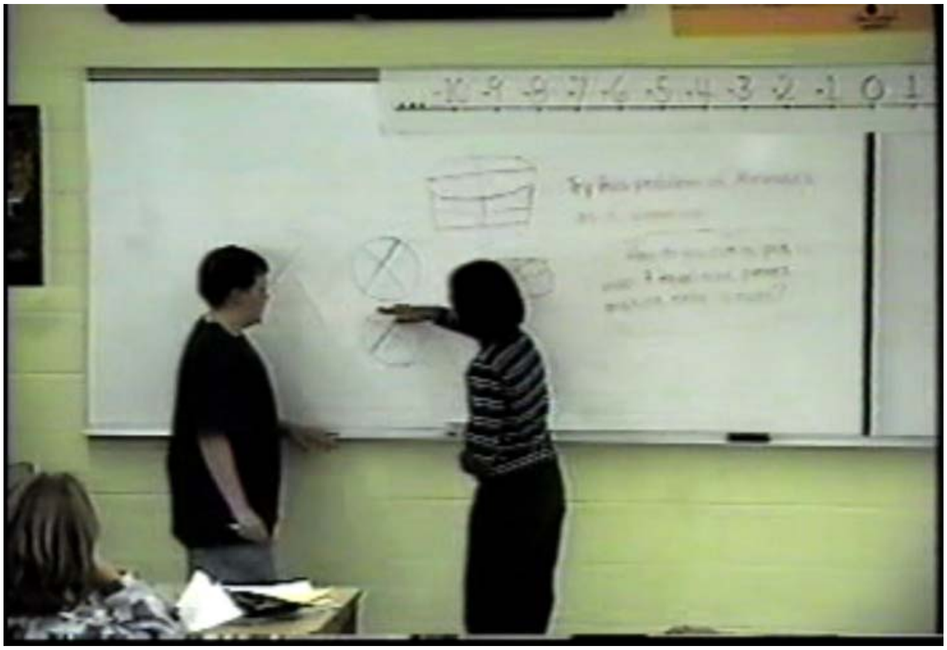


Figure 4. **A student provides a drawing that shows two separated layers of the pie that are then each cut twice (from Stanza #20; see Excerpt 2).**

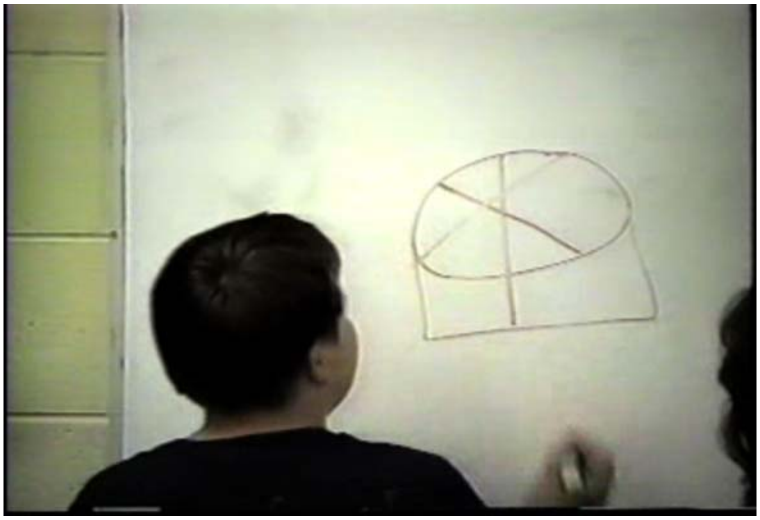


Figure 5. **Early in the discussion (Stanza #3; see Excerpt 4), Bob demonstrates his drawn solution to the Pie Problem.**

Figure 6. **Near the end of the discussion (Stanza #33; see Excerpt 5), Bob demonstrates a solution to the Pie Problem using aggregate objects (blocks).**

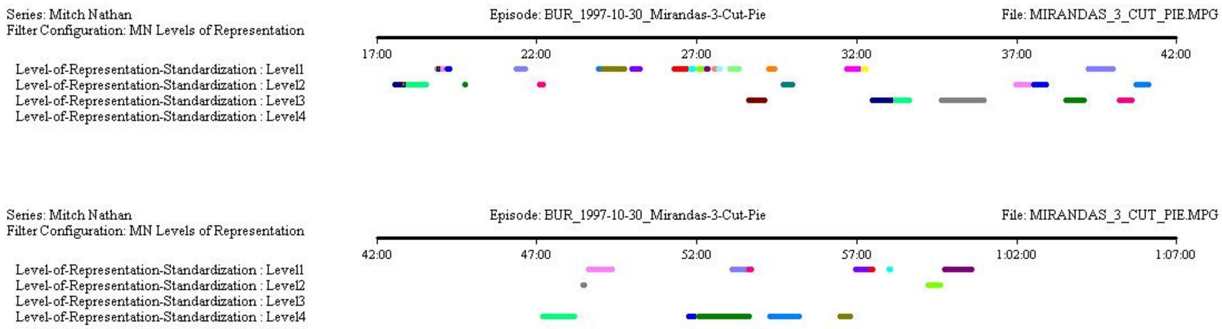


Figure 7. **Transana keyword map showing the distribution of the four representational levels as assigned to each solution representation (*N* = 46) over the time course of the discourse. Each time line spans 25 min.**