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A Comparison Between The Use of Beta Weights and Structure Coefficients In Interpreting Regression Results

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Abstract

An extensive body of researches has favored the use of regression over other parametric analyses that are based on OVA. In case of noteworthy regression results, researchers tend to explore magnitude of beta weights for the respective predictors. This is plausible when the predictor variables are perfectly uncorrelated. However, as other researchers suggested, the failure to compute structure coefficients may cause erroneous interpretation in regression, when multicollinearity is present between predictors. This paper provides an explanation by using two heuristic examples, so as to emphasize the importance of balancing attention to both beta weights and structure coefficients, so that valid interpretations of regression results are formulated.

A Comparison Between The Use of Beta Weights and Structure Coefficients In Interpreting Regression Results

An extensive body of researches has favored the use of regression over other parametric analyses that are based on OVA, when the objective of the research project is to explore the relationship between a set of independent variables and one dependent/outcome/criterion variable (Cohen, 1968; Courville & Thomspon, 2001; Friedrich, 1991; Pedhazur, 1997; Thompson, 1992; Thompson & Borrello, 1985; Wilson, 1980). The advantage of regression lies in that it can handle both intervally and nominally scaled predictor variables, whether or not they are correlated. Furthermore, the univariate feature makes the results straightforward and explicit by taking the following mathematical model:

$$Y \leftarrow\!\!-\!-\!- \quad \hat{Y} = a + b_1 X_1 + b_2 X_2 + ... + b_j X_j \quad . \tag{1}$$

$\hat{Y}$ is the predicted value of the criterion upon given predictor X (s). Researchers have argued that all statistical analyses are correlational and generate additive and multiplicative weights applied to measured or observed variables that can be used to estimate the scores on latent or synthetic variables (Fan, 1996; Thompson, 1991). Supposed in a simple regression case where there is a single predictor, the model can be simplified into:

$$Y \leftarrow\!\!---\;\; \hat{Y} = a + bX \;. \qquad (2)$$

In Equation 2, the additive weight a (sometimes referred to as the Y-intercept of the fit line) serves to help control the mean of $\hat{Y}$ to be equal to the mean of Y; while the multiplicative weight b (usually termed as unstandardized regression coefficient or slope of the regression) functions to also convert the "'spreadoutness' of the predictor variable into the same metric as the 'spreadoutness' of the dependent variable" (Perry, 1990, p. 4). In terms of regression, the primary interest in any research practice is to minimize the distance (i.e. e score) between the observed score Y and the predicted score on $\hat{Y}$ so as to yield a perfect prediction for each case by weighting the predictor variables. Therefore, in a regression model, there are at least two measured variables (X and Y), and always two latent variables ($\hat{Y}$ and e). Because b weights are sensitive to the existence of different scaling in predictor variables, and the variance of measured variables, they are usually transformed into standardized coefficients, the beta weights, by the following computation:

$$\beta = b(SD_X / SD_Y) \;. \qquad (3)$$

By doing so, the influence of the variance of the measured variables has been washed out. As a result, the magnitude of the predictor's beta weight can be employed to identify the contribution of that variable in the prediction; at least from one perspective.

Uncorrelated Predictors: Heuristic Example #1

In case when there are noteworthy results occurring, researchers tend to explore the source of the effect size by emphasizing β weights of the respective predictors (Courville & Thompson, 2001). This is reasonable in simple regression or in multiple regression (MR) with two or more predictors, all of which are completely uncorrelated with each other; but is reasonable only in these cases.

INSERT TABLE 1 ABOUT HERE.

As reflected in Table 1, the biviarate correlation between X_1 and X_2 is 0, which indicates that each of the predictor variable shares a non-overlapping area in the variance of the Y scores. According to formula (4) (which is always applicable for two predictor variables), the beta weights of X_1 and X_2 equal to their respective Pearson r with the outcome variable (and thus ranging from -1 to +1):

$$\beta_1 = \frac{r_{YX_1} - (r_{YX_2})(r_{X_1X_2})}{1 - r^2_{X_1X_2}} . \qquad (4)$$

$$\beta_2 = \frac{r_{YX_2} - (r_{YX_1})(r_{X_1X_2})}{1 - r^2_{X_1X_2}} .$$

Since

$$r_{X_1X_2} = 0$$

$$\beta_1 = r_{YX_1}, \quad \beta_2 = r_{YX_2}$$

Hence

$$\beta_1 = .385, \quad \beta_2 = .859$$

Moreover, the effect size multiple R^2 is .887, which corresponds to the following calculation:

$$R^2_{Y(X_1,X_2)} = \beta_1(r_{YX_1}) + \beta_2(r_{YX_2})$$

$$= r^2_{YX_1} + r^2_{YX_2}$$

$$= .385^2 + .859^2$$

$$= .887$$

It is evident that X_1 explains 14% of the criterion whereas X_2 contributes 75%. By examining the beta weights, we can claim that X_2 supercedes X_1 in accounting for a larger proportion of the Y information and thus does a better job of predicting score on the latent variable $\hat{Y}$. This is illustrated by the Figure 1 Venn diagram.

INSERT FIGURE 1 ABOUT HERE.

Multicollinearity

However, it is fallacious (and erroneous) to believe that once the job of interpreting beta weight has been accomplished, the interpretation of the analysis will be free of suspicion. As many other scholars have suggested, the beta weight is not the only issue that counts. In fact, structure coefficient (or the Pearson *r* between predictors and the dependent variable, both of which will yield identical results) is equally important in terms of interpretation, if not more (Cooley & Lohnes, 1971; Pedhazur, 1997; Thompson & Borrello, 1985). When multicollinearity (or collinearity) is present between the predictors in multiple regression, both beta weights and structure coefficients must be interpreted.

Considering the reality that not all predictors are highly uncorrelated or "researchers purposely introduce collinearity when using multiple measures of variables in which they have greater interest or which are more important from a theoretical point of view" (Thompson & Borrello, 1985, p. 204), the sole reliance on beta weights may create serious interpretation problems. As Pedhazur (1982) stated:

> It should be clear now that high multicollinearity may lead not only to serious distortions in the estimations of regression coefficients but also to reversals in their signs. Therefore, the presence of high collinearity poses serious threats to the interpretation of the regression coefficients as indices of effects. (p. 246)

Consequently, the advocacy for the interpretation of structure coefficient in regression research has emerged (Cohen, 1968; Cooley & Lohnes, 1971; Dunlap & Landis, 1998).

A structure coefficient is defined as the zero-order bivariate correlation between a given predictor variable X and the latent (or synthetic) variable $\hat{Y}$. It should be noted that a structure coefficient differs from a Pearson r between a given predictor and the measured variable Y by the formula:

$$r_S = r_{YX} / R$$

Heuristic Example #2

In presence of three predictor variables which are all correlated to each other, the interpretation of regression becomes complicated.

As shown in Table 2, if we only center our attention on beta weights of the three predictors, we might assert that X_2 has little or no effect on the outcome variable because β_2 equals to zero (or near zero depending on how many rounding decimals we chose). Therefore, in this regression model, X_2 contributes little in producing the effect size. Similarly, we might judge that X_3 is the best predictor among the three ($\beta_3 = .880827$), followed by X_1 ($\beta_1 = .117911$). However, when the attempt is to investigate the respective relationship of each X with $\hat{Y}$, it is surprising to find that X_2 has a near perfect relationship with the latent variable ($r_s = .9108653$), while X_1 only has a moderate correlation ($r_s = .57610974$), which contradicts with the previous conclusion.

INSERT TABLES 2 AND 3 ABOUT HERE.

A further exploration of this special case reveals that X_2 and X_3 are highly correlated with each other with a Pearson r of .946. The correlation matrix in Table 3 also demonstrates that the three predictors are actually 'combined' with each other ($r_{X_1X_2} = .221$; $r_{X_1X_3} = .483$). Hence, the sum of the r^2 of each predictor with Y will be much greater than the multiple R square (.890). An explanation can be reached by consulting Figure 2.

INSERT FIGURE 2 ABOUT HERE.

Obviously, X_2 and X_3 account for most of the common area, and the structure coefficient of X_3 is bigger than that of X_2, which has simply arbitrarily not been granted credit to yield part of the effect size (the overlapping area with Y is circumscribed within that of X_1 and X_3). However, this doesn't mean X_2 assumes no predictive impact in explaining the variance of Y. To the contrary, X_2 turns out to be a very qualified predictor! In fact, when holding the other two predictors constant, X_2 serves as a predictor almost as significant as X_3 (r_S=.9939596), but more predictive than X_1.

This example illustrates that the sole reliance on beta weights could have caused serious interpretive problems. Because beta weights only work appropriately in cases when predictors are perfectly uncorrelated with each other, the computation of structure coefficients becomes a must in any multicollinearity situation when the research interest is to assess the predictive impact of each predictor.

Summary

Beta weights can be addressed to identify the merit of any predictor uncorrelated with other independent variables. On the other hand, structure coefficients can be applied to evaluate the relative predictive importance of a single predictor on the latent variable $\hat{Y}$, to indicate how strongly each independent variable influences the criterion variable. Structure coefficients are not affected by multicollinearity.

Even though some scholars have denied the use of structure coefficients in multiple regression (see Harris, 1992), Thompson (1992) proposed that "… thoughtful researcher should always interpret either (a) both the beta weights and the structure coefficients or (b) both the beta weights and the bivariate correlations of the predictors with *Y*" (p. 14). By consulting both beta weights and structure coefficients, researchers can report unbiased and valid regression results by balancing their attention to both interpretation perspectives. Chapters 8 and 9 in Thompson (2006) provide more detail on these various issues.

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Table 1

Correlation Matrix for Example #1

	Y	X_1	X_2
Y			
X_1	0.385		
X_2	0.859	0.000	1.000

Note. R^2 =.877.

Table 2

Beta Weights and Structure Coefficients of Example #2

Predictors	Standardized Coefficients β	Structure Coefficients r_s
X_1	0.117911	0.57610974
X_2	8.680040E-15	0.9108653
X_3	0.880827	0.99399596

Table 3

Correlation Matrix for Example #2

	Y	X_1	X_2	X_3
Y				
X_1	0.554			
X_2	0.859	0.221		
X_3	0.938	0.483	0.946	1.000

Note. R^2 =.89.

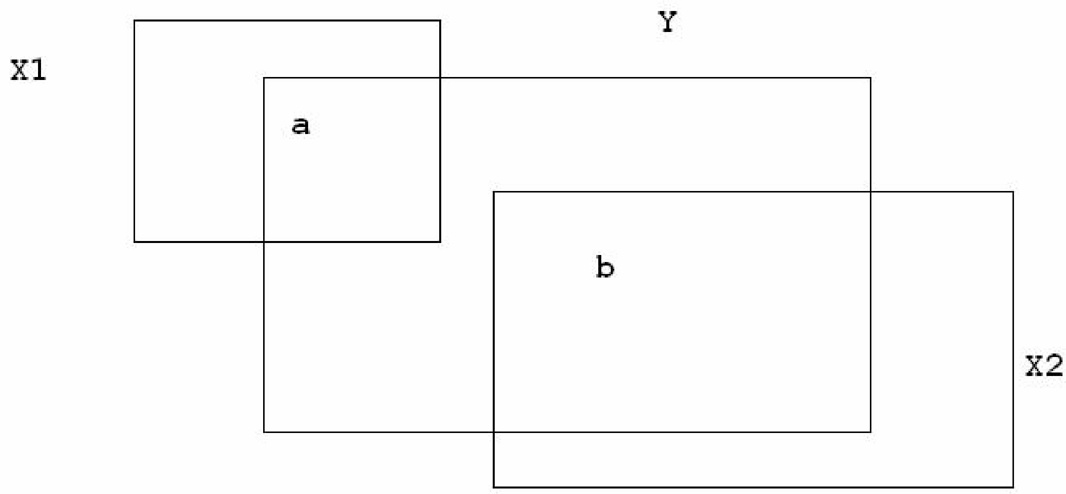

Figure 1. Venn diagram for two uncorrelated predictors.

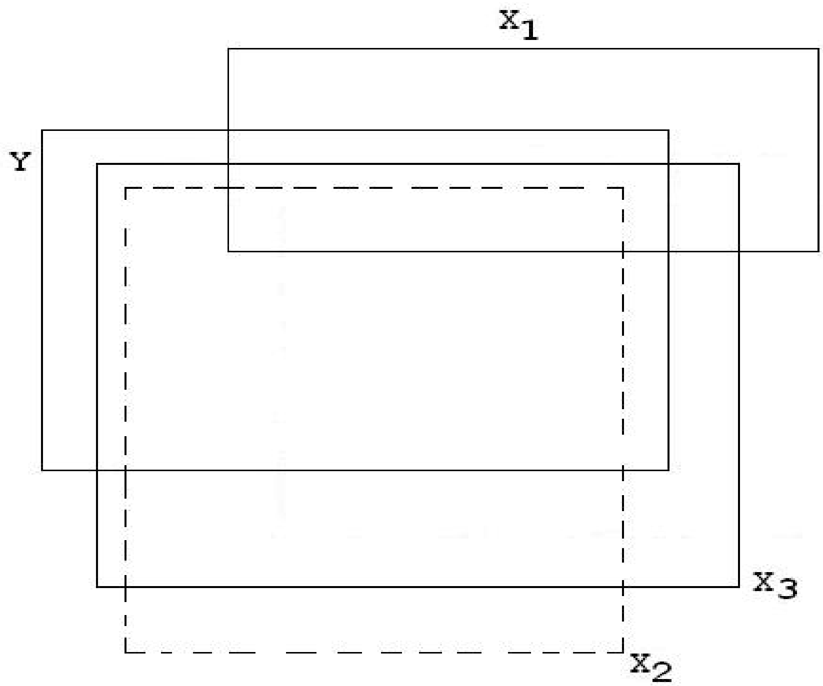

Figure 2. Venn diagram for three correlated predictors.