# Conceptions of Mathematics in Different Ability and Achievement Groups among 7th Grade Students

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***Abstract***

*This report deals with 7th grade pupils´ conceptions of mathematics, its learning and teaching. The report focuses on the identification and comparison of views expressed by pupil groups of different mathematical ability and achievement. The analysis is based on the results of the ability tests, subject tests and a questionnaire conducted among the 7th grade pupils of Estonian schools.*

## Introduction

Pupils' conceptions of mathematics and themselves as learners of mathematics are of fundamental importance in their learning of and performance in mathematics. Previous researches have shown that differences in pupils' conceptions are much bigger between countries than within one country. This implies that each country has its own teaching style and teaching culture, which do not change very easily. The biggest differences in the conceptions of pupils of different countries are found in the understanding of the role of the teacher. For example, Estonian pupils are more teacher-centred (Pehkonen, 1996). Compared to their counterparts in other countries (such as Sweden, Finland, Hungary, Germany), they prefer a teacher who explains well, helps them immediately and always tells them exactly what they ought to do to solve a mathematical problem. Previous researches on pupils' conceptions have also shown no significant differences between the conceptions of girls and boys. Some mathematics educators (Schoenfeld, Silver) have pointed out that pupils' conceptions of mathematics may prove to be an impediment to the solving of non-routine problems and to effective learning. However, the solving of such problems is important with respect to pupils' ability and creativity.

The purpose of our study was to answer the following question: How is the student's conception of mathematics teaching connected with his or her ability and academic achievement?

We also tried to answer the following sub-questions in our research:

- What characterises the ways of mathematics learning of more talented and less talented pupils?
- In what way do the conceptions differ between students achieving better results and those achieving poorer results?

## Data Gathering and Tests

This paper summarises Year 1 results of a [three-year] research project aimed at analysing the progress made by and the changes in the conceptions of mathematics teaching and mathematics learning of a cohort of pupils proceeding from Grade 7 to Grade 9.
In Year 1 of the project (1998), an ability test (*Potential*), three achievement tests (*Numbers, Algebra*, *Shape & Space)* and a questionnaire including 33 questions about the teaching and learning of mathematics were carried out among the 7th grade pupils in 14 schools in Estonia. The ability and achievement tests used were prepared within the framework of the Kassel-Exeter

(*Kassex)* project (1994-1998). The general coordinator of the project was *Centre for Innovation in Mathematics Teaching* – CIMT of the University of Exeter (*Anon. a), b), 1994).* The questionnaire was prepared by E. Pehkonen and B. Zimmermann and, like the *Kassex* project tests, has been used in several countries.
The initial test *(Potential)* consisted of 26 questions and the subject tests of 50 questions each. The answers were graded on a dichotomous scale as follows: “1” - true, “0” - false. Thus, the highest possible score was 26 points on the initial test and 50 points on each of the subject tests.
The questionnaire (responses according to the following pattern: “-2” – I strongly disagree; “-1” – I disagree; "0" - I do not know; “1” – I agree; “2” – I strongly agree) addresses four different conceptual domains: *Conceptions of mathematics (*questions of this type in the questionnaire are below marked with a C)*; Conceptions of the way of doing mathematics* (D)*; Conceptions of mathematics learning* (L) *and Conceptions of mathematics teaching* (T). The number of pupils who completed at least one test or the questionnaire was 414. Of these, 198 (47.8%) were girls and 216 (52.2%) were boys. The average age of the respondents was 13.5 years. Those pupils who did not complete all the tests and the questionnaire were left out of the following analysis.
The average results of the tests are presented in Table 1. Since the *Kassex* project required the tests *Numbers, Algebra and Shape & Space* to be applicable to the testing of the same pupils for three consecutive school years (ages 13+, 14+ and 15+), the average test results for this age group (13+, Grade7) are, for natural reasons, relatively low.

Table 1

| Test | N | Possible maximum | Minimum | Maximum | Mean | Std. Deviation |
|---|---|---|---|---|---|---|
| *Potential* | 396 | 26 | 2 | 24,0 | 12,7 | 3,6 |
| *Numbers* | 399 | 50 | 7 | 40,0 | 24,1 | 6,7 |
| *Algebra* | 396 | 50 | 3 | 24,0 | 11,2 | 4,2 |
| *Shape and Space* | 395 | 50 | 0 | 29,0 | 12,4 | 5,1 |

We assessed the reliability of the tests using two methods: as internal consistency by Cronbach Alpha, which, due to the dichotomous assessment of the test results, is here equivalent to the Kuder-Richardson Formula 20 (KR20), and by finding the Guttman’s lower bound for the true reliability. It appeared that the reliability of our tests is at least satisfactory, remaining within the range of 0.68 (*Potential* and *Algebra*) to 0.86 (*Numbers)* for Cronbach $\alpha$ and 0.68 *(Shape and Space)* to 0.87 *(Numbers)* for Guttman’s coefficient.
For grouping the pupils by their **mathematical ability,** we divided them into three groups on the basis of the results of the ability test *(Potential)* as follows: *Low ability group* (17% of the pupils, total score 2 - 9 points), *Medium ability group* (68%, 10 - 16 p) and *High ability group* (15%, 17 - 24 p).
For grouping the pupils by their **mathematical achievement**, we used the arithmetic mean of the results of the three subject (achievement) tests (*Numbers, Algebra, Shape and Space).* Since the arithmetic mean ranged from 0.0 … 30.0 points, we divided the pupils into three achievement groups as follows: *Low achievement group* (13% of the pupils tested, the mean being 0.0 – 10.0 points; *Medium achievement group* (69%, mean 10.1 ... 20.0) and *High achievement group* (18%, mean 20.1 ... 30.0). The following presentation of some of the results of our research on the pupils’ conceptions includes only the high and low ability and achievement groups, leaving aside the respective medium groups.

**The Conceptions of the Pupils in Different Ability and Achievement Groups**

It appeared that in most cases (79% of all 33 statements) it was impossible to prove the existence of differences between the conceptions of the pupils of the two extreme ability groups and the two extreme achievement groups at the significance level of $p < 0.05$. This is evidenced by the statistically significant correlations (for Pearson´s $r$ and Spearman´s $\rho$ the $p < 0.000$) and by the similarity of the respective divisions ($\chi^2$-test, $p < 0.999$) between the evaluations of pupils of different groups along the means of points in the questionnaire.

**A. Conceptual domain – *the content of mathematics***

This conceptual domain was measured by the following 9 items:

Good mathematics teaching includes:

C1 doing calculations by heart
C2 mechanical calculations
C3 drawing figures (e.g. triangles)
C4 doing word problems
C5 using calculators
C6 different topics … taught … separately
C7 problems … have practical applications
C8 calculating areas and volumes
C9 constructing concrete objects

There are no significant differences (t-test, p<0.05) in the conceptions of the content of mathematics between the low and high ability groups. Both groups consider calculating to be of the highest importance. The importance of calculating with the help of a calculator is significantly lower than calculating by heart or in writing both for the high-ability and low-ability pupils. The use of a calculator is considered necessary or highly necessary by only 50% of the low-ability and 51% of the high-ability students. In this respect, the conceptions of Estonian pupils have undergone considerable changes: in 1990, calculating with the help of a calculator was the most preferred method (Lepmann 2000). Surprisingly, there are also no differences of opinion with regard to non-routine tasks: 76% of the low-ability students and 77% of the high-ability students think that textual tasks (C4) are an integral part of mathematics teaching.

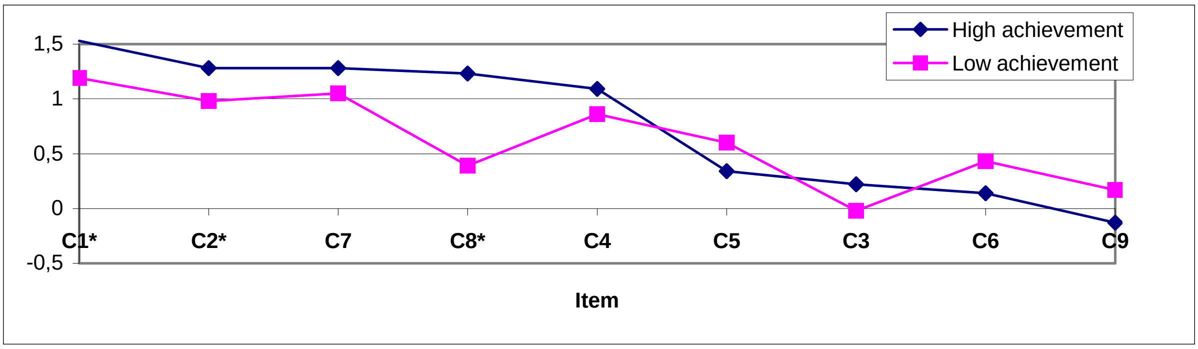


Figure 1

In the achievement groups, statistically relevant differences are only evidenced in the responses to three questions related to calculating skills (C1, C2, C8). The responses reveal that calculating skills with their different aspects are considered significantly more important by the pupils in the high achievement group than those in the low achievement group (Figure 1). Strikingly, the conceptions on this domain of the high ability and the high achievement groups coincide fully – of the highest importance is the skill of calculation by heart while the skills related to the construction of concrete objects are ranked lowest. The respective opinions on this domain of the low ability and the low achievement groups, however, are less coincidental ($r=0.71$, $p>0.05$ and $\rho=0.63$, $p>0.05$). Nevertheless, in both cases the pupils attribute the lowest importance to the statements related to geometry. An analysis of the responses to items C3 and C9 given by the low group pupils reveals that in both instances these students are slightly more in favour of enactive representation than iconic representation. Among high-ability pupils, it is on the contrary.

**B. Conceptual domain** – *the way of doing mathematics*
This conceptual domain was measured by the following 5 items:

D1 right answer… more important than the way
D2 everything … expressed … exactly
D3 there is … procedure … to exactly follow
D4 everything … reasoned exactly
D5 there is … more than one way of solving

In this domain, no significant differences could be observed in the conceptions of the extreme groups. It is only worth mentioning that both extremes considered exact reasoning (D4) to be of the highest importance. All groups of pupils liked solving the same task in different ways. Encouraging from the perspective of promoting constructivist approach is the fact that all the groups responded negatively to Statement D1: the right answer is more important than the way to get it. However, the low-ability students were more in favour of the statement that everything ought to be expressed as exactly as possible (D2). Probably, they understand the material presented in this fashion better

**C. Conceptual domain** – *mathematics learning*
This conceptual domain was measured by the following 8 items:

L1 all pupils understand
L2 much will be learned by heart
L3 as much repetition as possible
L4 only … talented pupils can solve it
L5 learning is not always fun
L6 it demands much effort
L7 as much practice as possible
L8 all the material … will be understood

With respect to mathematics learning, both ability groups (Figure 2) consider understanding the material (L1, L8) to be of the highest importance. Interestingly, the low-ability pupils oppose learning by heart (mental arithmetic) in the same way as by the high-ability pupils.

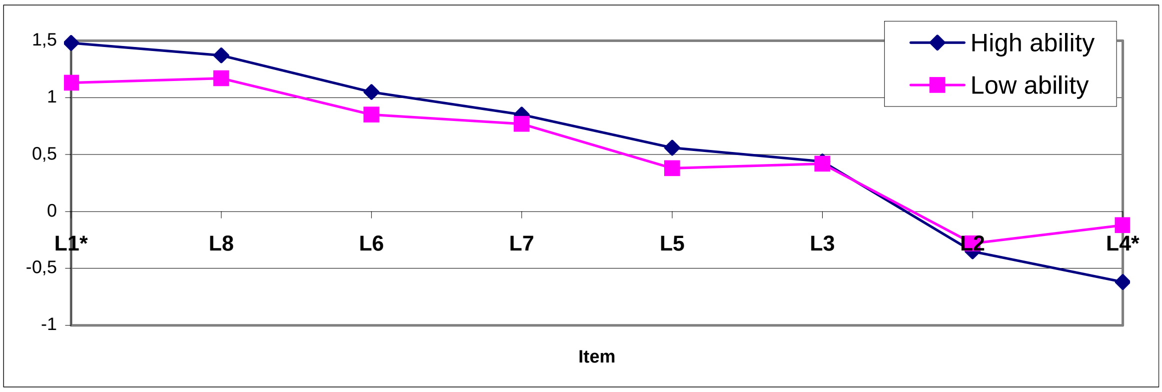


Figure 2

A closer analysis (Figure 3) reveals that: both of the groups include more than 40% of those who are opposed to learning by heart, and more than 20% of those who consider it necessary. However, there are significant differences in the attitude of the pupil groups towards the relative importance of ability. Namely, both the low ability and the low achievement groups (Figure 4) tend to agree with statement L4 - that only talented pupils can solve most of the tasks. Nevertheless, the average opinion on this statement stays fairly close to neutral in both cases, with 38% of the low-achievement pupils and 32% of the low-ability pupils having responded to this question with the option "0"- “I do not know”.

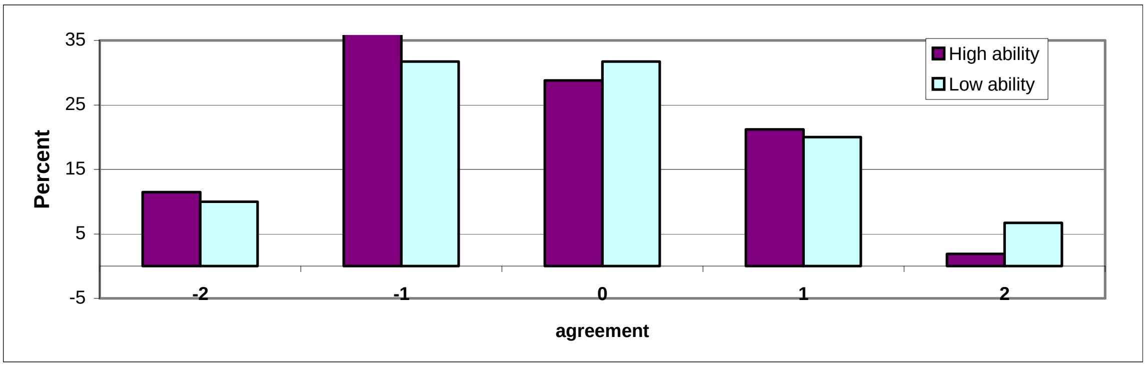


Figure 3

A tendency can be observed that the pupils of the highest ability believe more than those of low ability that all the pupils are able to learn mathematics and that success in learning is possible if effort is made. They have a somewhat higher degree of self-perceived ability and desire to learn mathematics (L5, L6). The low-ability pupils, however, are more disposed to give up.

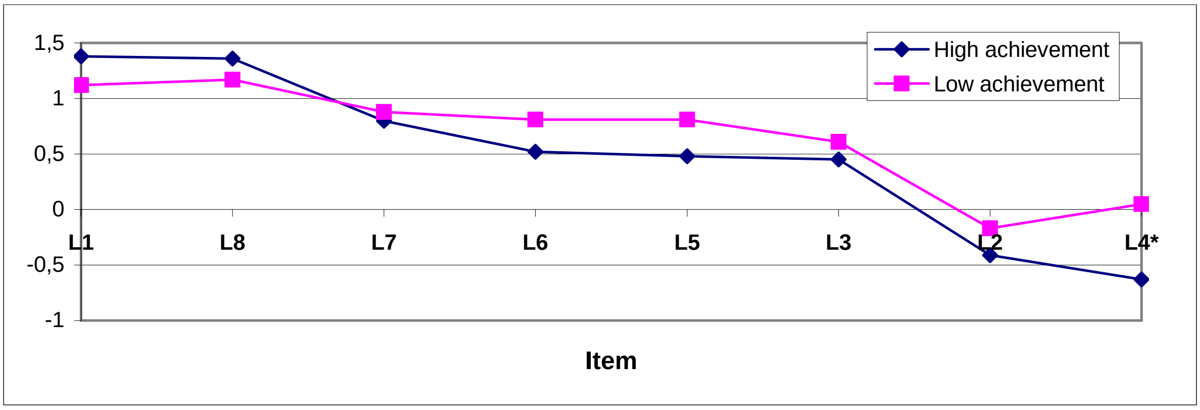


Figure 4

**D. Conceptual domain -** *mathematics teaching*

T1 pupils … guess and ponder
T2 right answer quickly
T3 strict discipline
T4 pupils put … own questions
T5 teachers help … when in difficulty
T6 learning games
T7 teachers explain every stage
T8 pupils solve … independently
T9 working in small groups
T10 teachers tell exactly what to do
T11 pupils can work according to abilities

In this domain, the opinions of different ability groups differ the most (Figure 5). The greatest differences for the entire questionnaire are manifested in the responses to statement T11: high-ability pupils are considerably more willing to agree to the view that everyone should be able to work according to his or her ability. Thus, it is the more able pupils that need a greater differentiation of teaching. Surprisingly, the views of the two extreme achievement groups on statement T11 are identical, whereas those of the two extreme ability groups who differ significantly in this respect. Both the ability and the achievement groups approve of process-orientated teaching, but their conceptions concerning some aspects of this process are different. The high-ability and the high-achievement pupils appreciate significantly more the fact that pupils are allowed to (try to) solve problems as independently as possible (T8); they want to make their own guesses at the solution of a problem (T1). At the same time, the low-ability and the low-achievement pupils are more expectant to receive help from the teacher (T7, T10).

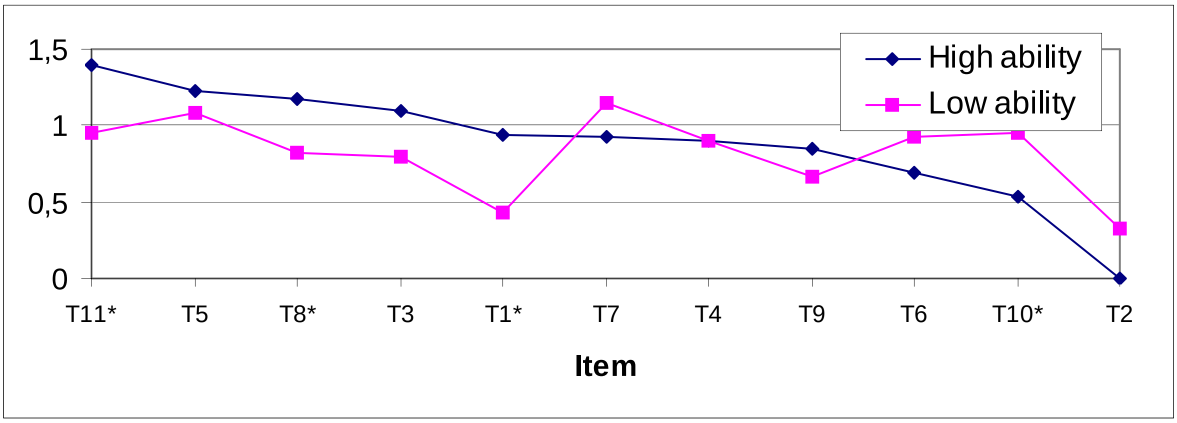


Figure 5

**Conclusions and Discussion**

The findings of the study support the view laid out in the introduction that pupils' conceptions of mathematics and the learning of mathematics are generally correlated with not so much their individual ability and achievements as the general paradigm of mathematics teaching prevalent in a given country. However, certain differences based on achievement and ability may still be observed among the pupil groups examined in our study.

The study revealed that high-ability pupils have considerably greater faith in achieving success in mathematics learning than low-ability pupils. Compared to other pupils, high-ability students are considerably more desirous of each pupil being able to work according to his or her ability. They want to develop their ability and are ready to do more work in the name of success. Low-ability pupils, however, are more disposed to give up than pupils with high ability.

The greatest differences between the students of low and high ability lie in the determination of a pupil's independence and activity by the pupil himself. Low-ability pupils are considerably more disposed to receiving explanations from the teacher while high-ability students desire to work independently. Nevertheless, several studies indicate that all students must be actively involved in their learning. Under such organization of the learning process, students are able to achieve higher grade point averages than under traditional learning methods (Alper 1997). As students work a task, the teacher must give them freedom to solve the problem in their way rather than in some predetermined way. However, the teacher's duty is to organize independent work in such a fashion that each pupil (more able and less able alike) receives instructions from the teacher to the extent necessary for him or her.

Understanding the subject is considered equally important both by less able and more able pupils. This finding was also confirmed by an analysis of the part of the questionnaire where free answers were allowed. Sometimes, however, teachers do not believe that each pupil is able to understand the material. For instance, C. Römer writes that in the opinion of a pre-service teacher only 50% of the students have the chance to learn something in mathematics lessons by understanding. The rest have to learn by heart only to survive his time at school (Römer, 1977). Nevertheless, the pupils´ responses demonstrate how important is for the teacher to look for a suitable way for less able pupils also to be able to understand the subject they learn. For instance, one of the low-ability students in our study claims that *mathematics can be very interesting if I understand everything*.

The conceptions of the extreme achievement groups broadly coincide with those of the extreme ability groups. Significant differences were manifested in two domains: a) mathematics teaching and b) the content of mathematics.

Regarding the ways of mathematics teaching there were no significant differences in the conceptions of the extreme achievement groups whereas the extreme ability groups evidenced

statistically significant differences in four of the eleven items. All these items were related to the teaching methods encouraging independent learning: in this respect, the high-ability students preferred teaching methods granting them greater freedom. This may imply that teaching methods presupposing freedom of activity are particularly necessary for mathematically talented pupils. Pupils whose mathematical achievement is based not only on innate ability but also other factors are less enthusiastic about teaching methods that give them a free hand.
Regarding the content of mathematics, however, the only significant differences were those between the two achievement groups. They concerned the importance of calculation skills in mathematics. The high achievement group considered these skills an important feature of good mathematics teaching. The establishment of the type of high-achievement pupils who consider the above feature particularly important would require further research.

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