Algebra Rules Object Boxes as an Authentic Assessment Task of Preservice Elementary Teacher Learning in a Mathematics Methods Course

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Abstract

The purpose of this study was to describe elementary preservice teachers' difficulties with understanding algebraic generalizations that were set in an authentic context. Fifty-eight preservice teachers enrolled in an elementary mathematics methods course participated in the study. These students explored and practiced with authentic, hands-on materials called "object boxes," then created sets of their own object box materials. Each algebra rules object box contained materials to illustrate and describe four different algebraic generalizations, or "rules." The variables "n" and "z" were used in each of the generalizations. For each generalization, there was a set of objects attached to a piece of mat board that showed three cases of the generalization for different values of "n." Two sets of cards accompanied these objects, giving word problems, defining variables, stating equations, and explaining the algebraic generalizations. Students matched word problems to the object sets, defined variables and checked their work, then wrote algebraic generalizations for the object sets and used the reverse sides of the equation cards to check their work. Projects were graded with a rubric. Students were then surveyed about their difficulties. Results of the analysis showed that students were able to make an assortment of authentic materials in a variety of contexts and enjoyed the creative aspects of the project, but found the algebraic content challenging. The most common mathematical difficulties were being able to define the variable, and identify the pattern. Examples of effective student materials are provided.

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Introduction and Literature Review

Because it is critical for elementary students to be taught algebraic concepts (National Council of Teachers of Mathematics [NCTM], 2000; Rand Mathematics Study Panel [RAND], 2003), it is equally critical that pre-service methods courses include instruction on how to teach algebra effectively. The NCTM (2000) advocates that algebraic concepts be presented in a meaningful and authentic context, and be relevant to students lives. While recent initiatives call for further study on developing and promoting effective algebraic teaching practices for elementary teachers (RAND, 2003), there is an insufficient body of research on teachers' knowledge in the area of algebraic instruction (Doerr, 2004, Kieran, 2006) including the use of authentic assessment and manipulatives (NCTM, 2000). Therefore, three bodies of literature informed the conceptualization of this study, research on effective teaching and learning of algebra generalizations, authentic assessment practices, and use of concrete manipulatives.

The purpose of this study was to inform and improve the teaching and learning of algebraic generalizations with preservice teachers. A previous study (Hallagan, Rule, & Carlson, in review) investigated the effect of making algebra rules object boxes on preservice teacher knowledge of algebra, finding a significant improvement. Our current study focuses on the types of difficulties preservice teachers encountered while engaged in this project, the errors they made in the materials they created, and the successful sets of materials they devised. These objectives enabled us to understand preservice teachers' development of algebraic generalizations through this authentic assessment.

Preservice Teachers' Conceptions of Algebra

In her seminal review, on the learning and teaching of algebra, Kieran (1992) noted an "enormous gap in the existing literature on teaching regarding how algebra teachers interpret and deliver that content" (p. 356) and also "the scarcity of research emphasizing the role of the classroom teacher in algebra instruction" (p. 395). To date, there is scant research that reports on the practices of algebra teachers (Doerr, 2004; RAND, 2003), and in particular preservice teachers. Current studies demonstrate that elementary level students are capable of algebraic reasoning (Kaput & Blanton, 2001a), yet many elementary preservice teachers (Zizkas and Liljedahl, 2002) may not appreciate the role of algebraic generalizations in the elementary curriculum nor do they understand ways to connect generalizations to an authentic context. Bishop and Stump (2000) examined preservice elementary and middle school teachers' conceptions of algebra. In a semester course, the preservice teachers engaged in college-level algebraic experiences involving generalization, problem solving, modeling, and functions. They also explored algebraic activities for children involving variables, functions and pattern generalization. Bishop and Stump found that many preservice teachers did not understand what distinguishes arithmetic from algebra, and of those that did make the distinction, a majority held a procedural perspective even at the end of the semester course. The few preservice teachers that held a conceptual view of algebra in the beginning of the semester valued algebraic generalizations at the end. A similar finding was reported by Goulding, Suggate, and Crann (2000) who examined differences between preservice teachers. The preservice teachers were assigned weekly research to read on elementary mathematics involving algebraic proof. Then, they reported on the problem in class. During the preservice teachers' presentations, they asked the other preservice teachers to try the activities themselves before discussing the solutions. The weaker presentations reflected the pre-service teachers' inability to think deeply about the actual responses.

Authentic Assessment in Mathematics

Assessment practices are central to effective teaching. The current reform movement in mathematics education calls for teachers to simultaneously improve the quality of classroom assessment practices while increasing instructional emphasis on problem solving. Leaders in educational reform argue the form and content of assessments must change to better represent thinking and problem solving skills; additionally, the way assessment is used in the classroom needs to change in a corresponding manner (NCTM, 2000). Despite the reformed vision of assessment in mathematics classrooms, vast differences exist in how teachers use and view classroom assessment (Stiggins, 1999). Traditional assessment practice is pervasive as evidenced by a comparative study of TIMSS data, where most lessons in the United States and Great Britain emphasized procedures (Stigler & Hiebert, 1997) as well as others (Battista, 1999; Manouchehri, 1997; NCTM, 2000). In contrast, reformed models of assessment are interactive between teachers and students, and between teaching and learning.

A reformed perspective of assessment includes a process of constant development and structured, purposeful experiences where teachers conceive of assessment as a way to understand and enhance students' learning rather than just checking for mastery (NCTM, 2000). Preservice teachers cannot be expected to carry out a broad based authentic assessment task with their future students unless they have this experience themselves. Many elementary education majors have weak, fragmented knowledge of teaching mathematics (Ma, 1999; Hill, Schilling, & Ball, 2004), yet when preservice elementary teachers engage in reform-based curriculum materials and related assessments, Lloyd and Frykholm (2000) found that those who struggled the most, learned the most and also were able to identify ways in which they could help future elementary students. Hence, it is significant that preservice teachers have experiences in reform-based mathematical tasks and assessments.

Using Object Boxes in Teaching Mathematics

Manipulatives have been shown to be useful in motivating students, focusing their attention, and helping them to conceptualize abstract mathematical concepts (NCTM, 2000). The use of manipulatives can help students make connections between abstract mathematics and a concrete representation. Manipulatives can increase student interest and understanding, serving as a bridge to successful mathematical learning.

Our study focuses on a specific type of manipulative material called an "object box." An "object box" is a set of objects and corresponding cards that are housed in a box and used for instruction. Montessori (1964) devised the first object boxes for teaching reading and writing of words. Rule (2001) then expanded this concept to include many phonological awareness exercises. Object boxes have been used very successfully in science to increase descriptive vocabulary (Rule, 1999; Rule, Barrera & Stewart, 2004), to teach form and function concepts (Rule & Barrera, 1999; Rule & Furletti, 2004; Rule & Rust, 2001), and science words with multiple meanings (Rule & Barrera, 2003; Rule, Graham, Kowalski, & Harris, 2006).

Recently, object boxes have been created for reviewing/ teaching mathematical concepts with preservice teachers and their elementary students. Rule, Grueniger, Hingre, McKenna, and Williams (2006) reported that preservice teachers significantly improved their knowledge of numeration, algebra, geometry, and measurement through making "mathematical mystery object boxes" for elementary students. These materials included a set of items and corresponding clue cards that described numeration, algebraic, geometric or measured aspects of an object. The student read the clues and attempted to locate the object that satisfied them.

Algebra Rules Object Boxes

In the investigation by Rule, Grueniger, Hingre, McKenna, and Williams (2006), preservice teachers reviewed mathematical concepts through planning and creation of an object box for use with elementary students, thereby improving their knowledge of mathematics. Our current study drew upon this idea by asking preservice elementary teachers to devise a new type of mathematical object box called an "algebra rules object box" for use with elementary students.

Each algebra rules object box contained materials to illustrate and describe four different algebraic generalizations, or "rules." The variables "n" and "z" were used in each of the generalizations. For instance, four generalizations from one of the sets created by the authors and used by preservice teachers as an example were: $z = n^2$, $z = 5n^2$, $z = 2n$, and $z = 6n + 2$. For each generalization, there was a set of objects attached to a piece of mat board that showed three cases of the generalization for different values of "n."

There were two sets of cards that accompanied these object sets. The first set of cards was in the box with the objects and the second set was in an envelope for later use. The front of each card in the first card set showed a word problem referring to one of the sets of objects. The student read each word problem and matched the card to the corresponding object set. Then the student defined what the variables "n" and "z" represented. These definitions were listed on the reverse side of each card for self-checking.

Students then attempted to write an algebraic generalization using "n" and "z" for each of the four object sets. After writing these algebra rules, they removed the remaining second card set from the envelope and tried to match the algebraic equations written on the card fronts to the object sets. Sometimes the equations provided did not match those the students devised and allowed the students to revise their thinking. After all cards had been paired with object sets, students examined the reverse sides of the cards to determine the correct corresponding object set and to read a brief explanation of the algebraic generalization. In this manner, the cards and object sets guided students through the process of devising an algebra rule for the sets of objects.

Example sets of cards for algebra rules object boxes created by the authors are included in Appendix1.

Authentic Learning and Assessment

This project involved students in an authentic learning experience. Authentic learning has four major components (Rule, 2006): real-world problems that engage learners in the work of professionals; inquiry activities that practice thinking skills and metacognition; discourse among a community of learners; and student empowerment through choice.

Students were involved in making a useful curriculum material that many were able to immediately use during their practicum experiences and others will use in the future. Orion and Kali (2005) identified three teacher factors that positively influenced student conceptual understanding: “a) openness towards innovative teaching methods, b) scientific background and c) enthusiasm, and willingness to invest time and effort in teaching” (p. 392). Therefore, enthusiastic investment of time in making innovative curriculum materials is an important and real-world part of becoming an effective teaching professional.

Inquiry, problem solving, and critical thinking occurred as students devised algebraic equations for objects and as they found/ created objects for new problems. Metacognition occurred as students reflected on aspects of algebra and of the project with which they encountered difficulties. Group work allowed students to discuss their ideas among a community of learners and conference presentations along with publication of results of this work allowed preservice teachers and their professors to share ideas with other mathematics educators. Finally, students chose the objects and algebraic generalizations they wished to illustrate, thereby feeling ownership in the project. The assignment given to preservice teachers to work in small groups to construct their own algebra rules object boxes constituted an authentic assessment of their knowledge consistent with Newmann (1993) in that the students were active in their own construction of knowledge, and the problems extended value beyond the school environment.

Method

Participants

Fifty-eight preservice elementary teachers (48 females, 10 males; 58 Euro-American) who were college juniors or seniors and who were enrolled in a mathematics methods course at a mid-sized college in central New York State participated in the study. Preservice teachers worked in groups of four students in general, but two groups had only three students, for a total of fifteen groups.

Procedure

Students explored and practiced with sample algebra rule object boxes provided by the course instructor, then worked in small groups of four students each to create their own. Each preservice teacher supplied a set of objects representing three cases of an algebraic generalization and the two corresponding cards. Preservice teachers were required to check the work of other members of their group, thereby increasing the amount of discussion among members and providing opportunities for informal peer evaluation of products. Additionally, when the projects were completed, groups examined the work of other groups and used the given rubric to score them, although the instructor scored all projects and her scores were used as the final grades.

The project integrated several important aspects of the course: knowledge of mathematics, mathematics pedagogy, group work, and technology integration. Technology integration into the project had the following components:

- Students were given a PowerPoint template to write over and create their own fronts and backs of clue cards for the objects.
- Students used digital cameras to photograph the objects and learned to insert images into PowerPoint.
- Students learned how to crop and improve lightness and contrast of images.
- Students learned how to use superscripts for exponents.

Assessments

Students were provided with the rubric with which their projects would be scored. The rubric is shown in Table 1.

Table 1. Rubric for scoring preservice teacher algebra rules object box projects.

Criteria	Yes	Border	No
Labeled box with 4 objects and 8 cards?	1	½	0
Cards neat and durable?	1	½	0
Set contains several types of algebraic expressions?	2	1	0
Are the algebra rules mathematically correct?	4	2	0
3 examples of different cases included in each object?	1	½	0
Printout of PowerPoint card file included?	1	½	0
Total Points out of 10			

As students worked with the example algebra rules object boxes, they completed a questionnaire that asked about their reactions to the materials and their difficulties during this initial exploration. The results of this survey were compiled and analyzed. Additionally, the final projects were examined for errors and trends.

Results and Discussion

Questionnaire Responses

Groups were asked to complete a short survey the second day of working with the object boxes, which asked preservice teachers to tell their initial reactions to the algebra rules object boxes and to describe their difficulties in solving the algebraic generalizations using the object boxes. These were open-ended questions and the resulting responses were tallied according to general category of response. Table 2 shows reactions to the materials.

Table 2. Preservice teacher reactions to the algebra rules object boxes during the second day of use of example sets of materials. There were 15 groups responding in total.

Reaction	Frequency
Materials are interesting	15
Materials are colorful/ attractive	11
Can touch/ manipulate the materials	6
The materials help me figure it out	4
The materials keep my attention	3
A pattern can be seen in the materials	3
Fear – what is all this stuff?	3

Preservice teacher reactions to the materials indicated that the hands-on materials increased their interest in the work (unanimous response of fifteen groups) and that they found the sets of materials colorful and attractive. This observation is important because Rule, Sobierajski, and Schell (2005) showed that preservice teachers who viewed hands-on materials for mathematics as “beautiful” or “attractive” *performed better mathematically* with the exercise as well as noting that they felt more motivated. Several groups remarked that the materials helped them understand and figure out the concepts through manipulation, seeing patterns, and focusing attention. That was the initial intent of the authors in creating these algebra rules object boxes. Focus of attention on the activity through touching and manipulating the objects is important because research shows that attention is more important than time on task (Wittrock, 1986).

A few groups had an initial reaction of concern as to what to do with the materials because this approach to algebra was very new to them. Related to this fear are the difficulties preservice teachers indicated they encountered while working with the example object boxes. The group responses to this open-ended question are shown in Table 3.

Although the students had done some initial work with identifying and creating patterns with color tiles and sets of printed symbols before using the algebra rules object boxes, they were somewhat less familiar with defining variables from story problems. Therefore, they noted this difficulty in using the new materials. Some also had trouble seeing the pattern in some sets of materials and interpreting the word problems. Some preservice teachers were unfamiliar with or confused terms such as area, volume, perimeter and diameter. The instructor took time to review these terms for them and rulers were provided so that preservice teachers could measure the objects rather than rely on estimation.

These difficulties reveal the incomplete knowledge base many preservice elementary teachers have regarding mathematics. Concrete activities, such as the object box described here, help preservice teachers build a stronger foundation that supports more abstract reasoning.

Table 3. Difficulties preservice teachers encountered in working with the example algebra rules object boxes.

Mathematical Difficulty	Frequency
Difficult to define variables	13
Difficult to identify the pattern	7
Interpreting the wording of the problem	5
Determining what is wanted	4
Understanding the terms: area, volume, perimeter, diameter	4
All of algebra is difficult	4
Estimating the inches without a ruler	3

Algebra Rules Projects Created by Preservice Teachers

Table 4 shows mean scores on different aspects of the projects. In general, most groups of preservice teachers produced quality algebra rules object boxes containing the required components. However, there were some mathematical errors made that bear discussion because they shed light on the most difficult aspects of the project for preservice elementary teachers.

Four of the five group projects that contained mathematical errors had errors related to squaring. Early on in the lessons leading to this project, several preservice teachers expressed their confusion in understanding what squaring meant. Many were helped by using square color tiles to make squares of different sizes and therefore "see" square numbers as square shapes.

Two of the story problems with accompanying sets of objects preservice teachers devised for the equation "$z = n^2$" did not show a repeating square arrangement of items (as other more successful sets made by other groups did). Instead, preservice teachers attempted to fabricate a story of items being added that just happened to work for the equation, but the scenario had no repetitive basis to allow other values for "n" to be substituted. This indicates that the preservice teachers in these groups did not truly grasp the idea of squaring.

In two other projects, the equation "$z = 2n^2$" caused confusion with some preservice teachers interpreting it a $(2n)^2$ rather than $2 \times n^2$.

Finally, the last mathematical error occurred when preservice teachers misinterpreted the equation "$z = 4n + 3$" as "$z = 4n + n$" with $n = 3$. The fact that they only supplied an object for n=3 and not a set of objects for n of different values probably allowed them to overlook their mistake.

Table 4. Mean preservice teacher scores on different aspects of the project. Standard deviations are shown in parentheses.

Criteria	Mean Score
Labeled box with 4 objects and 8 cards?	1.0 (0.0)
Cards neat and durable?	0.95 (0.1)
Set contains several types of algebraic expressions?	2.0 (0.0)
Are the algebra rules mathematically correct?	3.74 (0.4)
3 examples of different cases included in each object?	0.92 (0.2)
Printout of PowerPoint card file included?	0.93 (0.3)
Total Points on Project out of 10	9.55 (0.5)

Examples of Effective Materials

Preservice teachers created many sets of clever, creative, and effective materials to illustrate algebraic generalizations. Examples of their work are shown in the following sections with comments.

$Z = n^2$. Although, as noted previously, several groups had difficulty with squared variables, many preservice teachers were able to produce effective examples for $z = n^2$. Here are four examples.

Bowling pins. Boy Scouts are bowling and are allowed to set up as many pins as they like as long as they are arranged in a square. Write a rule for the pins in their games. The set of objects that accompanied this story is shown in Figure 1.

Figure 1. Sets of bowling pins to illustrate $Z = n^2$.

Basketballs in cages. Mr. McNamara asks his gym students to place basketballs into three different cages. The number of basketballs that fit in a cage depends on the size of the cage. Write a rule for the basketballs and cages. The set of objects that accompanied this story is shown in Figure 2.

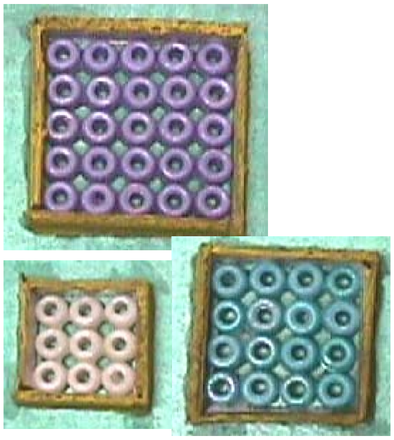
Figure 2. Sets of basketballs in cages to illustrate $Z = n^2$.

Marbles in boxes. Bobby is storing marbles in compartmentalized boxes. Write a rule for the number of marbles each box can hold if each marble needs one square inch. The set of objects that accompanied this story is shown in Figure 3.

Figure 3. Sets of marbles to illustrate $Z = n^2$.

Donut boxes. Daylight Donuts sells their yummy product in square boxes. Write a rule for the size of the boxes if different numbers of donuts are placed flat inside. The set of objects that accompanied this story is shown in Figure 4.

Figure 4. Sets of donuts in boxes to illustrate $Z=n^2$.



Bears in cages. A zookeeper is cleaning bears' habitat areas and needs to put the bears in cages. If the number of bears that can fit in a temporary cage depends on the area of the cage bottom, write a rule for the number of bears a cage can hold. The set of objects that accompanied this story is shown in Figure 5.

Figure 5. Bears in cages used to illustrate $Z=n^2$.

Z = a n. All projects showing a generalization of this form were done correctly, indicating the familiarity of preservice teachers with mathematical expressions for multiplication. Some interesting examples follow.

Toilet tissue. A fancy hotel has a strict policy for the number of toilet tissue rolls provided for each guest. For every guest, the maid supplies two rolls of toilet tissue. Write a rule for this. The set of objects that accompanied this story is shown in Figure 6.

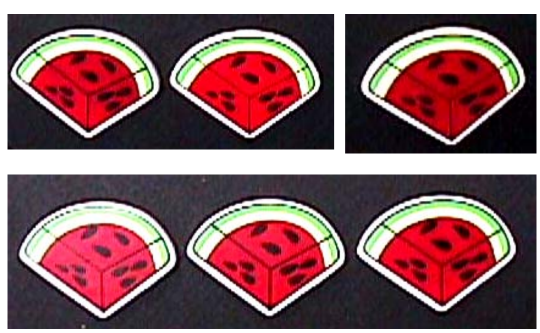
Figure 6. Sets of toilet tissue rolls used to illustrate $Z = 2n$.

Carnations. A florist uses three purple carnations for every rose in flower arrangements for a wedding. Write a rule for the number of carnations in arrangements of different sizes. The set of objects that accompanied this story is shown in Figure 7.

Figure 7. Sets of flowers used to illustrate $Z=3n$.

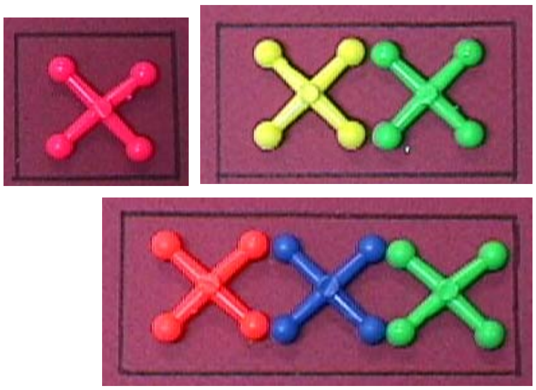
Watermelon seeds. Joe eats watermelons all day but picks out the seeds. If each slice of watermelon has 9 seeds, write a rule for the number of seeds in different numbers of slices. The set of objects that accompanied this story is shown in Figure 8.

Figure 8. Watermelon slices with seeds illustrating $Z = 9n$.



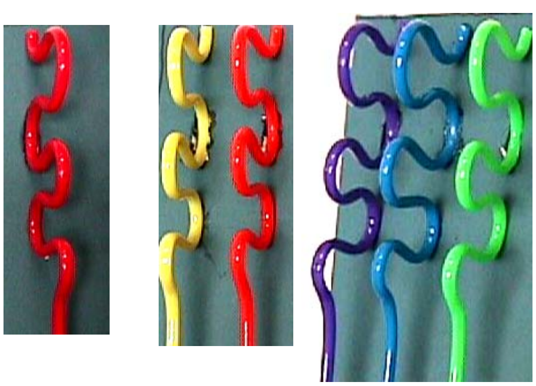
Propeller blades. Planes at an airplane show were arranged in groups of different sizes. Write a rule for the number of propeller blades in a group if all planes have four propeller blades. The set of objects that accompanied this story is shown in Figure 9.

Figure 9. Propeller sets with blades to illustrate $Z = 4n$.



Rollercoaster hills. Claire likes to raise her hands when she rides over rollercoaster hills. She wonders how many times she would do this on different groups of rollercoasters. Write a rule for this. The set of objects that accompanied this story is shown in Figure 10.

Figure 10. Sets of rollercoasters, cleverly made from drinking straws, used to illustrate $Z=3n$.



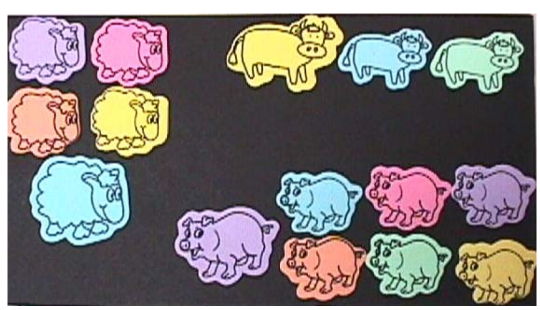
$Z = an + b$. Preservice teachers thought of clever ways to illustrate equations of this type. These are shown in the following sections.

Racing tires. A special race has teams composed of cars with one motorcycle. Write an equation to determine the total number of tires in each race. The set of objects that accompanied this story is shown in Figure 11.

Figure 11. Sets of vehicles used to illustrate the “racing tires” problem with the equation $Z= 4n + 2$.

Farm animals. In the spring, each animal mother at the farm gave birth to twins. If one mother always watches over one or more groups of twin babies, write a rule for different-sized groups. The set of objects that accompanied this story is shown in Figure 12.

Figure 12. Sets of animal groups to illustrate $Z=2n+1$.

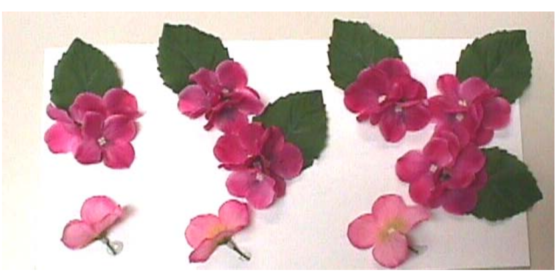


Egg fieldtrips. When a class of eggs goes on a fieldtrip, student eggs must always travel in three's. Each class of eggs needs two adult chaperones. Write a rule for the total number of eggs on a fieldtrip. The set of objects that accompanied this story is shown in Figure 13.

Figure 13. Sets of eggs to illustrate Z=2n+1.

Flower leis. Chelsea was making leis for her Hawaiian luau. Dark pink flowers come in bundles of three, but light pink flowers are purchased singly. If each lei has one light pink flower and a variable number of dark pink bundles, write the rule for the total number of flowers. The set of objects that accompanied this story is shown in Figure 14.

Figure 14. Sets of flowers to illustrate Z=3n+1.



Other Equations. Students produced a variety of additional algebraic generalizations. Two examples are shown here.

Land parcels. A real estate company is selling land parcels. The number of farm animals that a parcel of land can accommodate depends upon its area. Write a rule for the number of animals that can live on parcels of land. The set of objects that accompanied this story is shown in Figure 15.

Figure 15. Plots of land used to illustrate the land parcels story problem and the equation $Z=2n^2$.

Packages. The Postal Service changed its fees to a volume-based system. They now charge $1 per cubic foot plus a $10 delivery fee. Write the formula that the Postal Service uses. The set of objects that accompanied this story is shown in Figure 16.

Figure 16. Packages used to illustrate $Z=n^3+10$.

Conclusion

Students' work and comments during the practice with object boxes showed they enjoyed the work but found it very challenging. Understanding variables, square numbers, and interpreting wording of problems proved difficult. However, students were able to create correct sets of materials with algebraic generalizations in most cases. Students enjoyed the creative aspects of the assignment and analyzing/ critiquing other groups' work.

We recommend that projects such as this one be part of project-work in mathematics for preservice teachers. Effective projects are: 1) complex; 2) require transference of abstract concepts to concrete materials; 3) allow personal expression and creativity through choice of topic or materials; and 4) require analysis of the work of others.

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Appendix 1. Example sets of cards for the algebra rules object boxes. These are cards that were used with the example boxes that preservice teachers in our study explored. Card fronts are shown on the left; the reverse sides of cards are shown on the right.

$$z = n^2$$

Servings on square wedding cake layers
Each layer is one unit tall, n units wide and n units long.

Wedding Cake Story Problem

A catering company makes wedding cakes with layers that are one unit tall. In their line of square-prism-shaped cakes, a cubic unit equals one serving. They want to determine a rule for the number of servings in a layer so that they can easily calculate the number of servings in a cake chosen by a customer.

Let z be the total number of cubic unit servings in a layer.
Let n be the number of units along one of the square sides of the layer.

$$z = 5\,n^2$$

Values of nickel squares
Each square is made with nickels. The value of each square is the number of nickels along one side times the number of nickels along an adjacent side times five cents for: n x n x 5 or $5n^2$

Nickel Squares Story Problem

A company is making decorative coasters, trivets, and mats for coin collectors and others who enjoy seeing coins as decoration. If the items are each decorated with a square array of nickels, what is the cost (coin value) of the nickels used in squares of different sizes?

Let z be the total value (cost) of the nickels in a square.
Let n be the number of nickels along one side of the square.

$$z = 2n$$

Gumballs in compartments of plastic liners
Each row, n, of a box liner has two compartments.
The number of gumballs in plastic liner is 2n.

Giant Gumball Story Problem

A small company makes fancy giant gumballs in exotic flavors. They pack their gumballs in boxes with plastic liners with compartments to hold and protect the gumballs. If each row of a plastic liner has 2 compartments, write a rule for the number of gumballs a plastic liner holds.

Let z be the total number of gumballs.
Let n be the number of rows in a plastic liner.

$$z = 6n + 2$$

Suction cups on shower treads
Each shower tread has n holes. There are six suction cups around each hole, plus two additional cups per tread, giving 6n + 2 cups per tread.

Shower Treads Story Problem

A company makes non-slip rubber treads for showers. The treads have loop designs with suction cups arranged around holes. The suction cups hold the treads to the shower stall floor. Determine a rule for the number of suction cups needed depending upon the number of holes in a tread design.

Let z be the total number of suction cups needed.
Let n be the number of holes in the looped design.