Experimental Probability in Elementary School

Perusing the 5th grade Heath mathematics textbook *Connections*, we find the concept of probability introduced as a way to predict the result of an experiment. For example, suppose 1 yellow, 2 red, 2 green, and 5 blue connecting cubes are placed in a paper bag. If one of the cubes is drawn out, what color will it be? Of course, there is no way of knowing for sure which will be drawn, but by conducting experiments with the objects in the bag, we might be able to make a prediction which is “better.” An example of an experiment which would help a person make a better prediction would be to draw a cube from the bag, record what color it is, than return the cube to the bag, mix up the cubes, and repeat the process. Repeating this process, say ten times, would give a person an idea of a color that comes out frequently. This color, then, could be predicted in answer to the question: If one of the cubes is drawn out, what color will it be?

This type of problem is a somewhat weak use of experimental probability, for it seems obvious that the blue cubes would come out of the bag most frequently. A more impressive use of experimental probability is framed by a problem in *Elementary and Middle School Mathematics* (Van de Walle, 2004, p. 405): Toss a paper cup in the air and let it land on the floor. There are three possible ways for the cup to land: upside down, right side up, or on its side. If the cup were tossed this way 100 times, about how many times do you think it would land in each position? The strategy recommended by the text is to do this experiment with the cups enough times until you can make a reasonable guess at the answer. This context yields a more powerful example of experimental probability because there is no real way of knowing what the probability is without trying it with the particular cups in the person’s possession. In other words, there is no way to calculate the theoretical probability in this situation, as there was in the cube and paper bag situation.

Understanding the experimental probability associated with certain events allows a student to better understand what probability truly means. In particular, (1) performing experiments helps students differentiate between possible outcomes (sample space) and outcomes which actually occur (successful outcomes), (2) performing an experiment a large number of times can demonstrate to students the “unlikelihood” of certain types of outcomes, and (3) matching experimental results with their corresponding theoretical probabilities strengthens the meaning of theoretical probability.

Children can understand “concepts of chance” (Van de Walle, 2004, p. 387) in the earliest grades. They can make judgements (on a scale of 0 to 100, for example) about what events are likely to occur and which are not as likely by common sense and past experiences. But a more systematic approach to predicting chance events is soon needed, and fractions are used as a device to keep track of and compare results obtained by collecting data (NCTM, 2000). Once fractions are introduced, the idea of a sample space and a successful outcome become important.

For instance, suppose the following problem is posed: If a student in your school was randomly chosen, what is the probability their hair is black? In solving this problem, students are put in the position of differentiating students who have black hair (successful outcome) from those with other types of hair color. If students conducted an experiment in which they tallied the hair color of 40 or 50 students, they would soon discover all the possible outcomes (the sample space) and how frequent a success was (black hair).

Edwards and Hensien (2000) note that simple probability experiments can foster a great deal of student discourse in the classroom. With this communication, students have the opportunity to grapple with comparing the number of students with black hair to those who have other colors of hair. Students would be in the position of having real “mathematical experience” (Thelfall, 2004). They can evaluate different comparisons among the numbers obtained by their experiment, struggle for consistency and fairness, and create meanings grounded in real situations.

Suppose 18 students were found to have black hair, and 22 did not. Some students might express the probability as 18/22. But if classroom discourse is truly open, this answer will not remain an option because this fraction is equivalent to .81 or 81%, which is intuitively wrong. The classroom’s common sense would bring the answer to around 50%, and students would have to understand eventually that the number of black-haired children needs to be compared with the results from the entire sample space. As one can see, experimental probability is deeply connected to number sense and different representations of numbers such as percent, decimals, ratios and fractions (Van de Walle, 2004). These concepts can be strengthened when students are allowed to experiment and discover true meaning, as was demonstrated by the work of Kiernan (2001).

Van de Walle (2004) states many students have a naïve concept of probability in the beginning, and that understanding probability is progressive. Some children believe an event will happen “because it’s my favorite color” or because “it did it that way last time” (p. 407). These notions can be readily dispelled through experimentation. After these misconceptions are worked out, students become more and more adept at guessing ratios of successes to trials. As students begin to notice that repeating a trial a large number of times eliminates unlikely results (such as flipping a coin and getting tails every time), they can learn to depend on each other by having groups all perform multiple trials of the same experiment, and then tallying all the results. This leads students to begin to abstract the process of an event as outside of their control. In fact, Van de Walle encourages students to work with simulators of probability instead of conducting the actual event.

For example, suppose a person wanted to find the most likely sum when two ten-sided dice are rolled. To simulate this event, students might number ten strips of paper 1-10 to represent one of the die, and more strips of paper for the other die, and after placing the papers in two bags and drawing, tabulate the experimental probability. This further abstracts the idea of probability in a student's mind, and demonstrates the isomorphic nature of certain events. Other simulations are possible when technology is used. The TI-83 calculator, for example, comes with a coin flipping and dice rolling simulator which can quickly conduct thousands of trials. Several websites also offer interactive simulations involving lotteries, coin tossing, dice, the Monte Carlo problem, and an assortment of probability games (e.g. Drew, 1997; Lee's Summit R7 School District, 2006).

Elementary students eventually progress in their understanding of probability to crave a theoretical, or paper and pencil process for calculating the probability of certain events. This desire hopefully comes from a foundation of direct limited trial experimentation, leading into experimentation consisting of large number trials (trials which are often shared between groups of students). Simulation devices which replicate the experiment are also relied on more and more. Thus, multiple experimental approaches help students see that there could be an “ideal” probability external to temporal events which a person could find if they wanted to conduct the experiment a million times or more. This is the notion of theoretical probability. When students first learn about paper and pencil formulas for computing probability, the best approach is to link these answers to the experimental probability (Van de Walle, 2004). Students are then more likely to attribute real meaning to theoretical probabilities, grounding them in concrete experience.

Experiments are not always possible, nor are they time efficient. Students naturally begin to lose their need to see the experiment conducted as they think of probability as a more abstract measure. This decreased reliance on experimentation is superior to over reliance on theoretical probability with little experimental backing. Threlfall’s (2004) report on probability as it was implemented in England and Wales in the 1980’s and 1990’s supports this view. He argues that students during those years experienced extensive exposure to a probabilistic curriculum, without having any meaningful experience with the true ideas underlying probability. This lack of understanding impacted test scores and the teaching of probability was soon taken out of the curriculum altogether—an unfortunate mistake. Care needs to be taken therefore in what experiences students have with the fundamental ideas of chance.

Experimental probability underlies more theoretical notions of probability and participation in such activity helps a person understand fundamental principles used in chance problems. Probability might also help students overcome their fear of fractions because if they really understand probability, they can appreciate how two numbers, the numerator and denominator, are compared and what that means in real world situations.

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