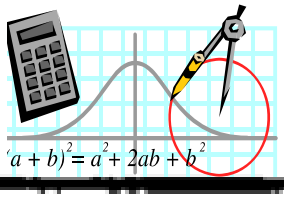
2007 Mississippi Mathematics Framework

2 + 3 = 5

$8 \times 2^2 + 7 \times 5 =$
$8 \times 4 + 7 \times 5 =$
$32 + 35 = 67$



Mississippi Department of Education
2007

Mississippi Mathematics Framework

2007

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Approved by the Mississippi State Board of Education, February 2006

ACKNOWLEDGEMENTS

The Mississippi Department of Education gratefully appreciates the hard work and dedication of the following educators for developing a quality document to improve mathematics education.

John Bakelaar, Jackson Public School District
Marilyn Bingham, Covington County School District
Libby Chance, Forrest County School District
Martha Charlwood, East Union School District
Amanda Cross, Meridian Public School District
Kathy Dedwylder, Enterprise School District
Dana Franz, Mississippi State University
Linda Gater, Jackson Public School District
Faith Gibson, Rankin County School District
Jennifer Halfacre, Mississippi University for Women
Amanda Hanegan, Meridian Public School District
David Jay Herbert, Delta State University
Pamela Hilton, Natchez-Adams School District
Brad Johns, Rankin County School District
Nita Johnson, Grenada School District
Vicki Kibodeaux, Hattiesburg School District
Joe Knight, Desoto County School District
Phillip Knight, Wesson Attendance Center
Genny Lindsey, Rankin County School District
Pat Luscomb, Rankin County School District
Cathy Lutz, Madison County School District
Shauneille Mason, Holly Springs School District
Felicia McCardle, Richton School District
Stephanie McCullough, Gulfport School District
Aisha McGee, Mississippi Department of Education
Wayne McGee, Copiah County School District
Jan Metzger, Oxford School District
Clif Mims, University of Mississippi
Viola Mixon, McComb School District
Cathey Orian, Mississippi Valley State University
Mary Phinisey, Columbus Municipal School District
Gwenda Purnell, Pascagoula School District
Debbie Ray, Pontotoc School District
Terry Richardson, Columbus Municipal School District
Joan Roberts, Corinth School District
Tina Scholtes, Starkville School District
Ruth Ann Striebeck, Greenville School District
Emily Thompson, McComb School District
Anita Waltman, East Jasper School District
Amy Zitta, Starkville School District

Special thanks to those individuals who served on the Mathematics Advisory Team and provided feedback in developing this document.

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MISSION STATEMENT

The Mississippi Department of Education is dedicated to student success including the improvement of student achievement in mathematics in order to produce citizens who are capable of making complex decisions, solving complex problems, and communicating fluently in a technological society. Through the utilization of the *2007 Mississippi Mathematics Framework*, teachers will challenge their students to think more deeply about the mathematics content, thus improving student understanding of mathematics. This document is based on premises that all children can learn, and that high expectations produce high achievement.

PURPOSE

The primary purpose of the *2007 Mississippi Mathematics Framework* is to provide a basis for curriculum development for K-12 teachers. The framework provides an outline of what students should learn through competencies and objectives/benchmarks. Suggested teaching strategies and assessments for those strategies are correlated to the competencies. The *2007 Mississippi Mathematics Framework* replaces the *2000 Mississippi Mathematics Framework.* The content of the framework is centered on the areas of **number and operations**, **algebra**, **geometry**, **measurement**, and **data analysis & probability**. Instruction in these areas is designed to expose students to experiences, which reflect the value of mathematics, to enhance students' confidence in their ability to do mathematics, and to help students communicate and reason mathematically. The *2007 Mississippi Mathematics Framework* provides teachers with the systematic progression across grade levels and is written to ensure the development of essential mathematical concepts that students will utilize as they pursue a career or continue their education.

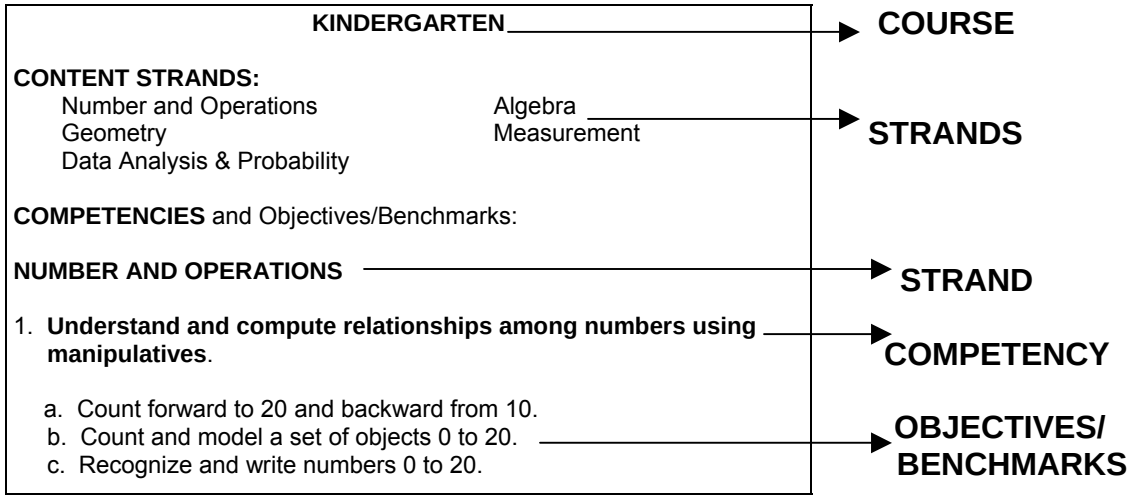
CYCLE

All Mississippi content area frameworks are revised on a six-year cycle. Approximately three years after a framework is implemented, a writing team is selected to review the current framework and make modifications based on best practices in the teaching of content areas as reflected in state and national trends. The revision process is approximately two years.

The pilot (optional) year for the *2007 Mississippi Mathematics Framework* is school year 2006-2007. The implementation (required) year for the framework is school year 2007-2008.

ORGANIZATION

The framework is organized by grade level (K-8) and by secondary courses (grades 9 – 12). A general description that includes the purpose, overview, and prerequisites is found preceding each curriculum outline for the grade level/course. Following each curriculum outline is a curriculum guide that provides suggested teaching strategies and suggested assessments. To enhance the implementation of the framework, a section of Literature Connections, Technology Connections, a Glossary, and Resources are included at the end of the framework. The curriculum outline for the *2007 Mississippi Mathematics Framework* is formatted as follows:



STRANDS

The *2007 Mississippi Mathematics Framework* is comprised of five content strands: **Number and Operations, Algebra, Geometry, Measurement, Data Analysis & Probability** and five process strands: **Problem Solving, Reasoning & Proof, Communication, Connections,** and **Representations**. The five interrelated content strands along with the five process strands combine to provide continuity to the teaching of K-12 mathematics. The strands overlap and should be integrated throughout the framework. This continuity provides the necessary foundation for successful completion of high school mathematics requirements. Each competency covers at least one content strand. The objectives/benchmarks and suggested teaching strategies assist in the incorporation of the process strands. Even though the process strands are not listed throughout the framework, these strands should be incorporated when presenting the content of the curriculum. The ten strands help to assure that appropriate processes are used and important concepts are learned throughout each grade level and secondary course.

COMPETENCIES

The competencies, printed in bold face type, are required to be taught to all students. Competencies do not have to be taught in the order presented in the framework. The order is for consistency and for easy reference throughout the framework. Competencies are intentionally broad in order to allow school districts and teachers the flexibility to create a curriculum that meets the needs of their students. They may relate to one, many, or all of the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills. The competencies are not intended to be a list of content skills that are taught and recorded as "mastered."

OBJECTIVES/BENCHMARKS

The objectives/benchmarks serve as a guide, indicating how competencies can be fulfilled through a progression of content and concepts at each grade level and course. Many of the objectives/benchmarks are interrelated rather than sequential, which means that **objectives/benchmarks are not intended to be taught in the specific order in which they are presented. Multiple objectives/benchmarks can and should be taught at the same time.** Each school district may adopt the objectives/benchmarks and are encouraged to write additional objectives/benchmarks that meet the needs of the students in their district. *Please note, the objectives/benchmarks are used as a guide to develop the Mississippi Curriculum Tests and the Algebra I Subject Area Test.*

SUGGESTED TEACHING GUIDE

The purpose of the "Teaching Guide" following the curriculum outline of each grade level and secondary course is to assist school districts and teachers in the development of possible methods of organizing the competencies and objectives/benchmarks to be taught. The suggested teaching strategies provide generic suggestions on how to introduce or reinforce each competency and objective/benchmark.

The teaching guide is a set of teaching strategies and assessments designed to be only a starting point for innovative teaching. There may not be enough time to utilize every strategy in the teaching guide. Most strategies in the teaching guide are not fully developed and should be developed further by the school district and teachers.

The Revision Process For The Mathematics Framework

From nominations by school district superintendents and others, the Mississippi Mathematics Curriculum Writing Team was selected in January 2003. The purpose of the team was to draft a new mathematics framework to include a teaching guide. The 37 member team was composed of teachers, administrators, and university professors throughout Mississippi.

All nominated, but not selected to the Mississippi Mathematics Curriculum Writing Team, were asked to serve on the Mathematics Curriculum Advisory Team. The Advisory Team was composed of teachers, administrators, university professors, and other professionals interested in mathematics education.

In order to gain a sufficient understanding of the direction of mathematics education, the writing team reviewed the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics (2000),* the National Assessment of Educational Progress (NAEP) *Mathematics Framework for 2005*, current literature and research. These resources served as a foundation for the development of the framework.

Drafts were distributed to the writing team and advisory team in March 2005 and to superintendents and curriculum coordinators in November 2005 as a part of the Administrative Procedures Act. Revisions were made in response to the submitted suggestions and feedback. The Mississippi Department of Education solicited further comment from the Norman Webb Group (See glossary, page 232), and other outside evaluators to assure a vertical flow of mathematics from First Grade to Algebra II with emphasis on rigorous mathematical content and alignment with national standards. Final approval was given by the Mississippi State Board of Education (February 2006).

SEQUENCE

Students will progress according to grade level through the sixth grade. Beginning in the seventh grade, students are given course sequence options. Below are proposed secondary course sequence options:

Proposed Secondary Course Sequence Options

Grade Level	OPTION 1	OPTION 2	OPTION 3	OPTION 4
7	7th grade Math	7th grade Math	Pre-Algebra	Pre-Algebra
8	Pre-Algebra	Pre-Algebra	Transition to Algebra	Algebra I
9	Transition to Algebra	Algebra I	Algebra I	Geometry or Algebra II
10	Algebra I	Geometry or Algebra II	Geometry or Algebra II	Geometry or Algebra II
11	Geometry or Algebra II	Geometry or Algebra II	Geometry or Algebra II	Advanced Algebra, Trigonometry, or Elective
12	Geometry or Algebra II	Advanced Algebra, Trigonometry, or Elective	Advanced Algebra, Trigonometry, or Elective	Pre-Calculus, Calculus, Statistics, or Elective

The following secondary mathematics electives have been included in the *2007 Mississippi Mathematics Framework*:

- *Advanced Algebra, Pre-Calculus, Trigonometry, Discrete Mathematics, and Statistics*, which are designed for students who have successfully completed Algebra II; and
- *Calculus*, which provides a survey of Calculus without the theory and rigor necessary to receive advanced placement credit. This course is designed for the student who has a thorough knowledge of college preparatory mathematics.

The following secondary elective has been included in the *2007 Mississippi Mathematics Framework*:

- Survey of Mathematical Topics and Introduction to Engineering which may not be included in the four mathematics courses required for graduation, however, these courses may be included in the 4 ½ general electives required for graduation.

TECHNOLOGY

The Mississippi Department of Education strongly encourages the use of technology in **all** mathematics classrooms. The learning and teaching of mathematics can be greatly enhanced when quality instructional technology is appropriately used.

The appropriate use of instructional technology is integrated throughout the *2007 Mississippi Mathematics Framework*. Suggested teaching strategies at each grade level and in every secondary course incorporate technology in the form of calculators, software, or on-line internet resources. The graphing calculator is an integral part of all secondary mathematics courses beginning with Algebra I.

The MDE believes strongly in NCTM's *Principles and Standards for School Mathematics* Technology Principle):

> "Electronic technologies – calculators and computers – are essential tools for teaching, learning, and doing mathematics. They furnish visual images of mathematical ideas, they facilitate organizing and analyzing data, and they compute efficiently and accurately. They can support investigation by students in every area of mathematics, including geometry, statistics, algebra, measurement, and number. When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving."
>
> "Students can learn more mathematics more deeply with the appropriate use of technology. Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics." (NCTM, 2000, page 24-25)

KINDERGARTEN

Kindergarten is the foundation for all formal learning experiences. Students will explore different representations of numbers 0 to 20, two- and three-dimensional shapes, patterns, units of measurement, and data analysis. The emphasis is on mathematical language development, representations, and concrete learning. Mathematics instruction at this level should revolve around the use of manipulatives and cooperative learning experiences.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands: **problem solving, reasoning & proof, communication, connections, and representation**. The competencies may relate to one, many, or all of the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

KINDERGARTEN

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Identify relationships among numbers and compute using manipulatives.

a. Count forward to 20 and backward from 10.
b. Count and model a set of objects 0 to 20.
c. Recognize and write numbers 0 to 20.
d. Estimate quantities fewer than or equal to 20.
e. Determine "first" through "tenth" (ordinal numbers), "next", and "last" positions.
f. Demonstrate an understanding of fractional concepts including the vocabulary (whole, all, part, some, and none).
g. Explore addition and subtraction of numbers through 10 with manipulatives.
h. Apply mathematical language by telling when a certain number is "too many," "not enough," "just right," "more than," "less than," or "equal to" for a given situation.

ALGEBRA

2. Reproduce and describe patterns using concrete objects.

a. Sort a set of objects by attributes including color, size, and shape.
b. Recognize and identify patterns.
c. Reproduce and extend visual, auditory, and physical patterns.
d. Describe patterns using a wide variety of materials and activities.

GEOMETRY

3. Identify and classify two- and three- dimensional shapes.

a. Trace, cut, and manipulate shapes.
b. Recognize open and closed figures.
c. Recognize and identify two-dimensional/plane figures such as square, rectangle, triangle, and circle.
d. Investigate and explore three-dimensional/solid figures.
e. Demonstrate an understanding of positional words, (for example: in, above, below, over, under, beside, etc.).
f. Understand and apply lines of symmetry through real-world models and artwork.

MEASUREMENT

4. Identify measurable attributes.

a. Model and discuss terms of comparison (for example: more/less, taller/shorter, heavier/lighter, and before/after).
b. Measure the length, weight, and capacity of objects using nonstandard units.
c. Investigate capacity (for example: holds more, less, or about the same) using different shaped containers and materials.
d. Recognize the clock (analog and digital) as a measurement of time.
e. Create a clock and identify the minute and hour hands.
f. Recognize the calendar as a measurement of time (days, weeks, months).
g. Sort and name coins (penny, nickel, dime, quarter) by physical characteristics.
h. Determine the difference between the concepts of hot and cold.
i. Recognize the thermometer as a tool for determining hot or cold temperatures.

DATA ANALYSIS & PROBABILITY

5. Collect, organize, and interpret data.

a. Collect data and construct graphs using real objects.
b. Interpret and analyze data in graphical form (for example: bar graphs and pictographs).
c. Explore, discuss and demonstrate an understanding of the terms “always,” “maybe,” and “never” events.

KINDERGARTEN

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Practice rote counting forward and backward with motor activities. (For example: Students "grow" as they count forward and "shrink" as they count backward. Students clap as they count forward and snap as they count backward. Students take baby steps forward as they count forward and baby steps backward as they count backward.)	Teacher observation; Student response
1	a	Read stories with counting forward and backward patterns such as *Ten in the Bed* by Penny Dale, *Five Little Monkeys* by Eileen Christelow, and *Mouse Count* by Ellen Stoll Walsh. Students predict the number that will be on the next page. Record and illustrate numbers as they appear in the book. Extension: Have students use manipulatives to model each number in the story.	Teacher observation; Student response
1	a,b,c	Create number spinners or number cubes. Have students spin the spinner, identify the number, and use manipulatives to represent the number.	Teacher observation; Student response
1	a,b,c	Display a set of objects on the overhead. Students count the objects and write the number the set represents on a dry erase board.	Teacher observation; Student response
1	a,b,d	Place 0 to 20 objects in a "Guessing Container." Students estimate how many objects are in the container. Count the objects to compare the estimation with the actual and discuss "reasonable estimates." Repeat with same sized container and different objects and different sized containers and same objects.	Teacher observation; Student response
1	b	Use manipulatives such as bear counters, unifix cubes, buttons, etc., to build and count sets of 20.	Teacher observation; Student response
1 5	b d	After counting sets of objects, compare the sets using the terms too much, not enough, just right, more than, less than, and equal to.	Teacher observation; Student response
1	e	Place ten students in a line and give instructions to particular students (For example: "If you are second in line, pat your head."). Demonstrate how to find that student using ordinal numbers. Also incorporate the vocabulary (next, last, before, after).	Teacher observation; Student response

KINDERGARTEN

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	e	After reading a story such as *I Know an Old Lady Who Swallowed a Fly* by Mary Ann Hoberman to students, chart the sequence of the events using terms first, second, third, last, etc.	Teacher observation; Student response
1	f	Read *Give Me Half* by Stuart J. Murphy. Discuss whole and fair/equal shares. Cut real food (such as muffins, moon pies, apples, bananas, poptarts, etc.) to illustrate unequal and equal parts. Discuss vocabulary (whole, all, part, some, none).	Teacher observation; Student response
1	f	Using manipulatives (for example: unifix cubes, Fruit Loops, Skittles, etc.) model and describe fractions, and illustrate expressions (whole, all, part, some, and none). For example: Some of them are red. All of them are blue. None of them are orange.	Teacher observation; Student response
1	g	Model a story problem using Goldfish, Cheerios, or Fruit Loops. Model story events by adding or subtracting the appropriate number of objects.	Teacher observation; Student response
1	g	Tell "People Problems," (such as "There Were Five Children Playing Ring-Around-the-Rosies. One child came to join them. How many children are playing?"; "There are four girls standing. The girl with pigtails sat down. How many girls are left standing?"; etc.). Students act out story and the teacher writes the corresponding addition or subtraction equation, discussing the meaning of the operations.	Teacher observation; Student response
1	g	Provide students with a two-column work mat and five unifix cubes of one color and five unifix cubes of another color to work guided addition problems. The teacher rolls a 0 to 5 number cube. The students build the first addend on the left side of the work mat with one color. The teacher records the first addend. The teacher rolls the number cube again. The students build the second addend on the right side of work mat with the other color. The teacher records the second addend. The students find the sum of the two addends. The teacher will record sum, completing the equation. Repeat.	Teacher observation; Student response
1	g	Provide students with a work mat and ten unifix cubes of the same color to work guided subtraction problems. The teacher rolls a 5 to 10 number cube. The students build the minuend on the work mat. The teacher records the minuend. The teacher rolls a 0 to 4 number cube. The students take away the subtrahend, covering it with their hand. The teacher records the subtrahend. The students count what is not covered/taken away to find the difference. The teacher records the difference, completing the equation. Repeat.	Teacher observation; Student response

KINDERGARTEN

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	a	Provide students with sorting trays and a variety of objects (pattern blocks, buttons, shells, etc.). Students sort by attribute and explain reasoning. Encourage students to sort in more than one way.	Teacher observation; Student response
2	a	Read *Gray Rabbit's Odd One Out* by Alan Baker. Discuss how objects are sorted and why one object did not belong. Present a bag of objects (For example: A bag filled with a paintbrush, a crayon, a marker, a pencil, and a container of play dough). The students determine how the bag was sorted and which item does not belong (For example: The play dough does not belong). Repeat with other bags of objects.	Teacher observation; Student response
2 5	a a	Read *Shoes* by Elizabeth Winthrop. Discuss the different kinds of shoes in the book. Have each student take off a shoe. Sort the shoes by attribute (For example: shoes with laces and shoes without laces). Use shoes to create a real graph. Discuss and analyze data, making comparisons. Repeat sorting by another attribute.	Teacher observation; Student response
2 5	a a,b	Ask each student to bring a favorite fruit to school. Discuss and sort fruit. Create a real graph using the fruit. Discuss and compare data. Create a pictograph and discuss using terms more, less, and same.	Teacher observation; Student response
2	b,c,d	Lead students in a game of "Pattern Copy Cat." The teacher will play a simple pattern with rhythm sticks and have students echo. The teacher will clap a simple pattern and have students echo.	Teacher observation
2	b,c,d	Create "People Patterns" using students. Discuss pattern and what would come next if the pattern was continued (for example: boy, boy, girl, boy, boy, girl, boy, boy, girl...).	Teacher observation; Student response
2	b,c,d	Provide pattern starter cards and a variety of manipulatives such as (cereal, unifix cubes, buttons, bottle tops, etc.). The students model, discuss, and extend a pattern.	Teacher observation; Student response
3	a	Students trace a pattern of a shape using inside stencils. The students count and write the number of sides on the shape. Repeat with other shapes. Cut out shapes to make a mobile.	Teacher observation; Student response and Product
3	b	Model open and closed figures using a piece of yarn. (Open means ends of yarn do not touch. Closed means ends of yarn touch.)	Teacher observation; Student response

KINDERGARTEN

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	c	Provide students with string. The students work together to create different two-dimensional shapes. Discuss how the shapes were created and their characteristics.	Teacher observation; Student response
3	c	The students explore the environment to find examples of squares, rectangles, circles, and triangles. Discuss the characteristics of each shape.	Teacher observation; Student response
3	c,e	Use precut shapes to create designs on the overhead. Students replicate designs on work mat following directions using positional words.	Teacher observation; Student response
3	d,f	Collect real-world three-dimensional objects such as milk cartons, boxes, cardboard rolls, etc., to create a class village. Use popsicle sticks and other materials to represent people and scenery for the village. Discuss characteristics of three-dimensional shapes in artwork and how they show symmetry.	Teacher observation; Student response and Product
3	e	Play "Simon Says" using a theme related object such as a block. Students follow directions with positional words to manipulate object. (For example: "Put the block over your head."; "Put the block behind your back."; etc.)	Teacher observation; Student response
3	f	Students use real-world objects and sort them as symmetrical or asymmetrical.	Teacher observation; Student response
3	f	Students create a shape on a geoboard with a geoband. Students use a straw as the line of symmetry to divide the shape into two equal parts to determine if the shape is symmetrical.	Teacher observation; Student response
4	a	Using adding machine tape, measure the height of all students. Arrange the lengths in order from shortest to tallest. Compare and discuss using the terms shorter/taller.	Teacher observation; Student response
4	a	Allow students to compare the weight of classroom objects (For example: a pencil and a book). Compare and discuss using the terms lighter/heavier.	Teacher observation; Student response
4	a,b	Investigate and compare the weights of objects (such as feather, block, box of crayons, pencil, glue, etc.) using a balance scale. Discuss heavier/lighter and more/less. Extension: Use ceramic tiles to weigh an object on a balance scale.	Teacher observation; Student response

KINDERGARTEN

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	b	Explore measuring the length of objects using non-standard units of measurement (such as unifix cubes, popsicle sticks, tiles, etc.). Help students align non-standard units correctly to measure. Students will place units end to end, not overlapping units, and leaving no gaps.	Teacher observation; Student response
4	c	At the sand table, explore and compare how much different containers hold using terms “more,” “less,” and “about the same.” Use a small scoop to fill assorted containers with rice. Compare the number of scoops using terms “more,” “less,” and “about the same.”	Teacher observation; Student response
4	d,e	Use a large Judy Clock to discuss how an analog clock is divided into minutes and hours and how the minute and hour hand work to tell time. Simulate clockwise motion. Provide students with a blank clock face with missing numbers. Provide students with number pieces. Students will fill in the numbers correctly. Students will identify minute and hour hand.	Teacher observation; Student response
4	g	Students use real coins to sort into categories based on physical characteristics. Create rubbings of each coin and discuss their characteristics.	Teacher observation; Student response
4	g	Play “Who Am I?” The teacher chooses a coin and gives at least three clues about the coin. Students may look at real coins to help solve the riddle. Repeat.	Teacher observation; Student response
4	g	Provide the students with a money spinner that has a picture of both heads and tails of each coin. (The spinner can be made by taping or contact papering real coins to a spinner.) Provide the student with a sock that has one of each coin inside. The students work with a partner. Partner 1 spins and Partner 2 finds the coin through their sense of touch. Repeat with Partners swapping roles.	Teacher observation; Student response
4	h,i	Have students sort seasonal pictures according to hot and cold. Provide students with pictures of thermometers marked with temperatures. Match thermometer with picture. Use real thermometers in the classroom to measure cold water and hot water, noting what happens to the mercury when measuring different temperatures. Place outdoor thermometers in the shade and in the sun and compare temperatures. Discuss why they are different.	Teacher observation; Student response

KINDERGARTEN

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	b,c	Chart the weather each day of the week with calendar math. Use symbols to represent weather of the day to create a pictograph. Discuss patterns and trends in weather. Discuss always, maybe, and never to predict what the weather is likely and unlikely to do. (For example, it is unlikely for Mississippi to get a lot of snow in winter; whereas, Montana is likely to get a lot of snow in winter.)	Teacher observation; Student response
5	c	Using a fruit graph, discuss terms always, maybe, and never. If the activity was repeated in another classroom what would be the likelihood of the results turning out the same? Make predictions. Have another class collect data. Compare and see if there are trends in the results. Survey another class and use the two data samples to help predict what is likely and unlikely to occur.	Teacher observation; Student response

FIRST GRADE

The First Grade mathematics framework is an extension of Kindergarten concepts. Students should explore patterns, shapes, place value concepts 0 to 100, gathering and organizing data, addition and subtraction of whole numbers, measurement, time, temperature, and value of money. Students should develop and investigate mathematical concepts through manipulative exploration. Throughout these processes, students should explore concepts through concrete, abstract, and symbolic representations.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability**, and the process strands: **problem solving, reasoning & proof, communication, connections and representation**. The competencies may relate to one, many, or all mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

FIRST GRADE

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships among numbers and compute operations (addition and subtraction) with and without manipulatives.

a. Count forward and backward 0 to 100.
b. Identify, model using manipulatives, and write numbers 0 to 100.
c. Associate names and numerals that accompany two-digit numbers.
d. Explain how to compare two-digit numbers using the terms “more,” “less,” “greater than,” “less than,” “equal to,” and “almost.” Explain reasoning.
e. Sequence two-digit numbers using the terms “before,” “after,” and “in between.”
f. Identify place value of a given digit in a two- and three-digit number.
g. Given a group of objects, skip count by 2’s, 5’s, and 10’s.
h. Identify and explain even and odd numbers.
i. Demonstrate various concepts of addition and subtraction with and without manipulatives. Write equations to accompany them.
j. Demonstrate that addition and subtraction are inverse operations.
k. Develop strategies (such as doubles, plus/minus 1 and fast tens) to find the sums and differences of basic facts 0 to 18 with and without manipulatives.
l. Find the sums of 3 single-digit addends (for example: 3 + 6 + 2= 11).
m. Find the sums and differences of two-digit whole numbers without regrouping (for example: 21 + 34 = 55 or 58 – 32 = 26).
n. Explore problems that involve a missing addend, subtrahend, or minuend.
o. Model, draw and discuss representations of story problems. Write number sentences to accompany them using addition and subtraction.
p. Compare a whole to fractional parts by dividing a whole into fractional parts (halves and fourths) using drawings and manipulatives.
q. Represent fractional parts by using models, drawings and numerical representations.
r. Estimate quantities less than or equal to 100.
s. Identify the value of coins (penny, nickel, dime, quarter).
t. Determine the value of like coins up to $1.00.
u. Explore the value of mixed coins up to $1.00.
v. Find equal money amounts with different coin combinations up to $.25.

ALGEBRA

2. Model, recognize, create and extend patterns.

a. Represent and explain visual, physical and auditory patterns using various representations and manipulatives.
b. Identify, describe, extend, create, and record growing and repeating patterns using a variety of materials and representations.
c. Using a pattern rule, translate patterns from one pattern representation to another.
d. Extend a pattern to a given ordinal position.
e. Formulate and explain patterns of addition and subtraction.
f. Interpret and describe number patterns using a 100's chart.

GEOMETRY

3. Identify and classify properties of two- and three-dimensional shapes.

a. Recognize and identify two-dimensional/plane figures such as the triangle, square, rectangle, circle, trapezoid, hexagon, and rhombus.
b. Identify and classify three-dimensional/solid figures (cube, rectangular prism, sphere, cone, pyramid, and cylinder) according to their characteristics.
c. Identify symmetrical objects and their lines of symmetry (horizontal, vertical, and diagonal.
d. Model, name, and follow directions using positional words including "over," "under," "beside," "in front of," "above," "below," "in," and "near to."

MEASUREMENT

4. Identify and apply measurable attributes.

a. Use nonstandard units (paper clips, unifix cubes, etc.) and standard units (inches, centimeters) to measure length.
b. Compare weight of objects using a balance scale with and without nonstandard units.
c. Compare and estimate capacity of various containers in nonstandard units.
d. Tell time to hour and half-hour intervals using both digital and analog clocks.
e. Use a calendar to name the months of the year and to find a particular date.
f. Interpret concepts of hot and cold using a thermometer.

DATA ANALYSIS & PROBABILITY

5. Collect, organize, and interpret data in graphical form. Investigate concepts of probability.

a. Gather data, construct and interpret bar graphs and pictographs.
b. Make a prediction, collect data, and compare results.
c. Investigate concepts of probability through explorative activities using the terms "always," "maybe," "sometimes," "never," "likely," "unlikely," "certain," and "impossible."

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Students practice rote counting on a daily basis. When counting forward, clap on every change in tens. Mastery of counting forward is a prerequisite to counting backward. With counting backward, count from 10 to 0. When students have mastered this, count from 20 to 0. Add a new decade until counting from 100 to 0 has been mastered.	Teacher observation
1	a,b	Students use both a 0-99 chart and a 1-100 chart, pointing at the numbers as they count forward and backward. Students fill in a blank 10-by-10 grid (both 0-99 and 1-100.) The grid may be tailored such that tens are added upon mastery.	Teacher observation; Student response
1	a,b,c,d	Call out a number from 0-99 or 1-100. Have students locate the number on the corresponding chart. Students then write the number on a dry erase board. Use vocabulary terms “more,” “less,” “greater than,” “fewer than,” “equal to,” “almost”, etc., to compare numbers on the grid.	Teacher observation; Student response
1	a,b,c,d,e	Have boxes or bags with 1-100 of the same items in them. Items may include small manipulatives or small objects (such as buttons, bottle tops, old postage stamps, shells, etc.). Students will determine how many items are in a box/bag by counting by tens and ones. Use portion cups to place groups of tens. Students count portion cups by tens and the extras by ones. Students name and write number in standard and expanded form. For example: 6 portion cups (tens) and 3 extras (ones) equal the number 63. 6 tens + 3 ones = 63 60 + 3 = 63 After students have worked with three different boxes/bags, have the students compare the numbers and explain how they knew which was less, more, equal to, etc. Students sequence numbers using terms before, after, between.	Teacher observation; Student response
1	a,b,d,e	Prepare a large 0-99 or 1-100 chart and laminate it. Play “Guess My Number.” The teacher chooses a number. Students must ask yes and no questions to determine special number. Encourage questions that use before, after, between, greater than, less than, or knowledge of patterns on the chart. (For example: Is your number greater than 50?, Does your number have a 3 in the tens place?, Is your number in the fifty family?, etc.) Cross out numbers that can be eliminated by questions explaining why they can be crossed out. Continue until the special number is found.	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a,b,c,d,e,f	Display a place value board that shows compartments for ones, tens, and hundreds. Use base ten blocks to model two-digit and three-digit numbers. Students pull a number out of a bag and build. Students count base ten blocks and discuss the number, attaching meaning to the digits. Students name and write number.	Teacher observation; Student response
1	a,g	Compare numbers using comparison terms. Students do "Math Exercises" by selecting an exercise such as knee bends, toe touches, jumping jacks, and counting by 1's, 2's, 5's, and 10's while performing the exercises.	Teacher observation
1	c	Create student task bags by dividing index cards in half. On the top half, write two-digit numeral name and stamp with corresponding base ten block stamps. On the bottom half, write the numeral. Cut the index cards in half. Have the students empty their bag and match the numeral with its correct name and representation. Extension: Divide index cards into thirds. On a third of the cards, write the numeral name and stamp with corresponding base ten block stamps. On the second third, write the numeral. On the last third, write the expanded notation form. Have students match.	Teacher observation; Student response
1	d,e	Write numbers 0-100 on tag board cards. Give to students in sets of 3. Students sequence the numbers in order from least to greatest or from greatest to least. Ask before, after, and in between questions.	Teacher observation; Student response
1	g	Provide students with a page of sets of objects. Circle groups of 2's, 5's, and 10's. Students will count in at least two different ways explaining and/or writing about the methods they used.	Teacher observation; Student response; work sample
1	h	When students understand that even numbers are in the counting by 2's pattern, students can look at the ones place to determine if it is even or odd. Students will draw a number card from a bag and identify the number drawn as even or odd. Students may refer to highlighted pattern on 0-99 and/or 1-100 chart. Extension: Roll number cubes. Identify the numbers rolled as odd or even.	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	i	Choose a certain number to work with. For example, provide students with 5 red unifix cubes and 5 blue unifix cubes. Students will combine the cubes to create sets that equal five. Encourage students to find and record all the different combinations and their equations. 5 red + 0 blue = 5 4 red + 1 blue = 5 3 red + 2 blue = 5 2 red + 3 blue = 5 1 red + 4 blue = 5 0 red + 5 blue = 5 Have students note resulting pattern that arises and the commutative property. Repeat with other numbers.	Teacher observation; Student response
1	i	Provide spoons and cereal to play "Subtraction Munchies." Students scoop a spoonful of cereal and count out total on workspace, recording minuend. Students determine how many to "eat" (take away), recording subtrahend. Students find difference and record, writing a complete subtraction equation. Repeat.	Teacher observation; Student response
1	i	Create workspace out of blue construction paper. Use the left half of the paper for "water." On the right half, draw a fishing net. Laminate. Provide students with a workspace paper, goldfish crackers, and a die to play "Goldfish Take Away." Students place (6-10) goldfish in "water," recording minuend. Students roll die to determine how many fish got caught in the net, recording subtrahend. Students count how many fish are left swimming in the "water" to find difference, recording the complete subtraction equation. Repeat.	Teacher observation; Student response
1	i	Create Task Cards to play "Sum or Difference." Create 5 "Sum or Difference" key cards by labeling each with a number from 0-10. Create five task cards for each key card, writing both addition and subtraction number names. For example: If the key card was 4, the task cards could be $4 - 0$, $3 + 1$, $2 + 2$, $6 - 2$, $10 - 6$. To play, students will sort task cards under corresponding sum or difference key card.	Teacher observation; Student response
1	i,j	Place the same number of red and blue objects in a paper bag. Draw a designated number of objects from the bag. Write the corresponding addition and subtraction number sentences. For example, draw 6 cubes. Four cubes drawn are red and two are blue, resulting in the following equations: $4 + 2 = 6$ $2 + 4 = 6$ $6 - 4 = 2$ $6 - 2 = 4$	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	i,j	Provide students with dominos. Students choose a domino and write corresponding addition and subtraction number sentences. For example, a domino with 6 dots on the left and 0 dots on the right would have the following equations: 6 + 0 = 6 0 + 6 = 6 6 – 6 = 0 6 – 0 = 6 Include doubles to illustrate that doubles have only 1 addition and 1 subtraction fact. For example, a double 4 domino would have the following equations: 4 + 4 = 8 8 – 4 = 4	Teacher observation; Student response
1	j,k	Create a chart that has been divided into 3 sections labelled “Less Than 10,” “Exactly 10,” and “More Than 10.” Provide partners with playing cards where face cards have been removed and aces have been turned into ones. Also provide partners with a chart. Students will draw 2 cards at a time, recording addition sentence under appropriate column. Repeat until all cards have been used. Students use strategies such as fast tens and doubles plus and minus 1 to help solve. Extension: Collect data from partnerships and tally how many times sums were less than ten, exactly ten, and more than ten. Discuss results and explain why results occurred the way they did. Predict if results would happen the same again. Repeat activity to see and discuss.	Teacher observation; Student response
1	k	Provide students with unifix cubes. Each student should receive two trains of 9 in two different colors. Call out a 9 + __ fact, for example, 9 + 4. Students show with one 9 train and 4 extras. Students take one of the 4 extras and place it on 9 train to form a “fast ten.” Now, the students have 10 + 3, making the equation easier to solve because they know teen numbers are 10 + __. 10 + 3 = 13	Teacher observation; Student response
1	k,l	Take an egg carton and write a number in each space (0-5 or 0-9). Place 3 small beans in the carton and close. Students shake carton and open to reveal where the three beans landed. Students write a number sentence with three addends and solve.	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	m	Create 2-digit addition task cards. (Make sure that you do not have to regroup when adding ones.) Students choose a task card. Students "build" the first addend with base ten blocks on a place value board. Then the students "build" the second addend with base ten blocks on the place value board. Have the students pull down the ones and find the sum. Then have them pull down the tens and find the sum to determine total. (This can be easily adapted for subtraction).	Teacher observation; Student response
1	n	Provide students with lima beans. Choose a particular number to work with to explore algebra with missing addends and subtrahends as you play "The Hand Game." For example, choose the number 10. Students will work with a partner. Partner 1 will be the "hand person." Partner 2 will be the "problem-solver." Partner 1 takes 10 lima beans and places some in left hand and some in right hand. For example Partner 1 puts 6 lima beans in left hand and 4 lima beans in right hand. Partner 1 opens left hand to reveal 6 lima beans, keeping right hand closed. Partner 2 determines how many lima beans are in right hand using knowledge of the total number of lima beans provided. Repeat.	Teacher observation; Student response
1	n	Provide a teacher-made folder game that looks like the following □ ○ □ = □ The rectangles will hold the number cards drawn. The circle will hold the appropriate operation sign (+ or –) depending upon how the students use the numbers. Provide a deck of cards numbered 0-10. Also provide a circle + card and a circle - card. Students work with a partner. Each player will draw a card. The students will use the two numbers drawn to make equations that will explore a missing addend, sum, minuend, subtrahend, or difference. For example: The cards drawn are 5 and 11. The students could do the following: 5 + ___ = 11 5 + 11 = ___ 11 – 5 = ___ 11 – ___ = 5 ___ – 5 = 11 ___ – 11 = 5	Teacher observation; Student response; Student work sample

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	n,o	Use commercial manipulatives (pattern blocks, unifix cubes, soft counters, etc.) or consumable manipulatives (Teddy Grams®, Cheerios, Goldfish, etc.) to create, model, and act out a variety of story problems involving addition, subtraction, missing addend, minuend, and subtrahend.	Teacher observation; Student response; Student work sample
1	n,o	Provide students with examples of problems such as Jenny has 4 dolls. Together Jenny and Pam have 10 dolls. How many dolls does Pam have? or Mom baked 3 pans of cookies with 10 cookies on each pan. How many cookies did she bake in all? Have students draw examples of the problems and write the corresponding equations and bind for a class book. Have students present how they solved the problem.	Teacher observation; Student response; Student work sample
1	p	Fold paper to show one-half. Cut along the fold and place these pieces on top of the whole sheet of paper to compare the whole to the fractional parts. Repeat with one-fourth.	Teacher observation; Student response; Student work sample
1	p	Provide students with pattern blocks. Students will explore whole and fractional parts. Take a yellow hexagon (whole). Find the shape that represents halves (2 red trapezoids).	Teacher observation; Student response
1	p,q	Provide students with commercial manipulatives such as Fraction Circles or Fraction Bars. Students explore whole and fractional parts. Students may record findings by gluing teacher-made construction paper models onto paper, writing corresponding fractional notation. Have students discuss which fractional parts are smaller and larger.	Teacher observation; Student response; Student work sample
1	p,q	Provide students with teacher made key cards denoting fractional notation ($\frac{1}{2}$, and $\frac{1}{4}$) Also provide 5 task cards for each fraction showing shapes that have been divided into equal parts with 1 of those equal parts shaded. Students match task cards to corresponding key card.	Teacher observation; Student response
1	p,q	Provide a set of items such as unifix cubes of one color. Have the students determine how to divide the set into halves, or fourths. For example, the students have 8 yellow unifix cubes. How can they divide 8 into halves--two equal groups? $\frac{1}{2}$ of 8 is 4.	Teacher observation; Student response

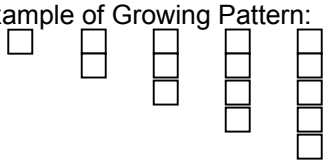
FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	q	Provide students with a paper bag. Place the same number of two different colored cubes in the bag. Students will play "Grab Bag Fractions." Students grab a handful of cubes and count the total. This is the denominator of the fraction—how many parts in the whole. The student records this information. Then the student finds the fractional part for each color writing the corresponding numerator—how many parts of the whole for each color. For example: The student drew 9 cubes (denominator) 5 were green and 4 were yellow (numerator.) The fractional notations would be $\frac{5}{9}$ green and $\frac{4}{9}$ yellow.	Teacher observation; Student response;
1	r	Place 0 – 100 objects in a "Guessing Jar." Students estimate the number of objects in the jar. Record guesses. Count objects in the jar (making groups of ten may be helpful.) Discuss estimates and actual number of objects in jar. (Was estimate too large? too small? etc.) Repeat activity with same jar and different objects. Students use knowledge of previous guesses to make future guesses. (For example: The jar will hold fewer large objects and more small objects.)	Teacher observation Student response
1	s	Provide students with magnifying glasses and real coins (1 each--penny, nickel, dime, and quarter.) Provide students with a recording sheet. Students work with a partner to find all the characteristics of each coin, recording words, drawings, and descriptions on recording sheet. Discuss in whole group and chart on class chart. Discuss how the coins are alike and different.	Teacher observation; Student response Student work sample
1	s,t	Demonstrate that counting pennies, nickels, and dimes is the same as counting by 1's, 5's, 10's respectively and that 100 pennies equals $1.00, 20 nickels equals $1.00, and 10 dimes equals $1.00. Make a chart showing the value of one, two, three, and four quarters.	Teacher observation

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	s,t,u	Play "Coin in My Pocket." The teacher places a coin in her pocket. The students ask yes and no coin characteristic questions to discover what coin is in the teacher's pocket. Students may not ask, "Do you have a quarter?" Students could ask, "Does your coin have rough edges?" "Does your coin have a building on the back?" "Does your coin have a torch and flame on the back?" Students use questions to eliminate coins. Play continues until the correct coin has been discovered. This game can be made more challenging by presenting the following situations: I have 20 cents in my pocket. Ask me interesting questions to discover what combination of coins I have. I have three silver coins in my pocket that are worth 25 cents. Ask me interesting questions to discover what coins are in my pocket.	Teacher observation; Student response
1	s,t,u	Play "Spoonful of Money." Provide students with a bag of assorted real coins and different sized measuring spoons. (The larger the size of the spoon, the more challenging the activity becomes.) The students select a spoon and scoop a spoonful of money from the bag. The student sorts the coins from greatest to least value. The student records how many of each type of coin they have. The student counts to find total value and records. Repeat.	Teacher observation; Student response; Student work sample
1	s,t,u,v	Play "Make the Price." Provide students with a choice of small items (toys, household items, etc.) that have been labelled with a price. Provide students with a bag of real coins and a recording sheet that has been divided into four sections. Students choose a toy and label price on recording sheet. Students create at least 3 different coin combinations that equal the price, recording by drawing combinations or by using money stamps.	Teacher observation; Student response; Student work sample
2	a	Model and extend patterns (with units such as AB, ABB, ABC, ABA, etc.) using large motor skills (such as stomp, clap, slap, snap, jump, etc.) A unit must be repeated at least two times to be a pattern. Explain patterns and show with real objects.	Teacher observation; Student response;

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	a,b	Prepare a variety of pattern starter cards for unifix cubes and pattern blocks. For unifix cubes, color pattern on tag board. For pattern blocks, glue construction paper pattern blocks on tag board or use pattern block stamps and color. Repeat pattern unit at least three times for repeating patterns. For growing patterns, show progression of pattern at least three times. (Note: Growing patterns may show numbers in increasing or decreasing order or show buildings increasing or decreasing in size.) Example of Repeating Pattern: ○ △ ○ △ ○ △ ○ △ ○ △ Example of Growing Pattern:  Students choose pattern starter card. Students identify the unit of the given repeating pattern or the growing pattern rule. Students copy, extend and explain pattern.	Teacher observation; Student response; Student product
2	b,d	Give each student a prepared strip with a letter pattern (such as ABCABCABC__ __ __) Strips could also have patterns created with drawings, construction paper, pattern blocks or pattern blocks stamps, etc. Have the students extend the pattern and identify the attribute that would be in a given ordinal position. (For example: "What would be in the 13^{th} position?") Students could continue to draw pattern out or build with manipulatives, or students could identify unit of pattern and use this knowledge to count on to the find the missing piece.	Teacher observation; Student response
2	b,e,f	Add small beans, two at a time on the overhead. Count the beans and circle the correct number on 0-99 or 1-100 grid with each addition. Discuss the resulting pattern that every other number is circled, that you are counting on or adding 2 each time, which is the counting by 2's pattern. Repeat the activity with 5's and 10's.	Teacher observation; Student response
2	b,e,f	Give each student a 0-99 or 1-100 chart and colored overhead discs. (Overhead discs are translucent and allow students to still see numbers. Have students cover a variety of number patterns.	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	b,f	Have students build a unifix cube train one cube at a time, stopping between additions to discuss the pattern of adding one and writing corresponding equations. Have students build a unifix cube train of ten. Remove one cube at a time, stopping to discuss the pattern of subtracting one and writing corresponding equations. Repeat activity with different adding and subtracting rules such as plus/minus 2, plus/minus 3, etc.	Teacher observation; Student response
2	b,f	Use cumulative growing patterns in children's literature to develop patterns of addition and subtraction. Use books such as *Fish Eyes* by Lois Ehlert, *One Cow Moo, Moo* by David Bennett, and *Ten in the Bed* by Penny Dale.	Teacher observation; Student response
2	b,f	Play "What's My Rule?" Using a commercial 10-by-10 grid pocket chart, fill in with numbers 0-99 or 1-100. Using colored squares cut from transparencies, cover a pattern (such as: "All the numbers have a four in the ones place;" or "All the numbers are two-digit numbers that are made with the same digit;" or "All the numbers are even numbers;" etc.) Students discuss pattern and explain the rule followed.	Teacher observation; Student response; Student product
2	b,f	Play "Show Me the Pattern." Give students a rule (such as: "The numbers all have a five in them;" or "I began with 6 and added 10 each time." Students use transparency squares to highlight pattern. Students explain how they know the numbers covered follow the pattern rule.	Teacher observation; Student response; Student product
2	c	Create 5 key cards each labeled with a different letter pattern. Create 5 task cards for each letter pattern, translating with stickers, stamps, number patterns, drawings, etc. (Each unit should be repeated at least three times on key cards and task cards.) Students get key cards and study letter patterns and their unit of pattern. Students then sort task cards under appropriate letter pattern key card. As students sort like patterns into groups have them explain why they are the same.	Teacher observation; Student response; Student product
3	a	Review two-dimensional shapes by going on a "Shape Hunt." Have students record the real-world objects they find. Create charts for basic shapes and classify the real world objects on charts. Discuss characteristics of each shape and discuss how the shapes are alike and different. Extension: Prepare pieces of construction paper with a cut out shape glued to each. Give each student one paper. Ask students "What can you make with a shape." Have students refer back to charts. Students use pencil, crayons, and markers to create something from the shape. (For example: The student might create a Christmas tree from a triangle, a wagon from a rectangle, etc.)	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	a	Students will use pattern blocks and sort them according to attributes such as four sides and four angles, three sides and three angles, etc. Describe and compare triangle, square, trapezoid, rhombus, parallelogram, and hexagon.	Teacher observation; Student response
3	a	Use attribute blocks to review characteristics of shape.	Teacher observation
3	b	Use three-dimensional shape models to identify and classify. Discuss characteristics and how the three-dimensional shapes are alike and different (For example: sort by like faces.) Have students bring real objects from home that are three-dimensional. Sort into spheres, cubes, rectangular prisms, cylinders, cones, and pyramids. Discuss which three-dimensional shapes there are more of and discuss possible reasons why. Extension: Use three-dimensional shapes students brought in and three-dimensional shape models. In a large circle, place one three-dimensional shape for each student. Play "Musical Three-Dimensional Shapes." Turn music on. Students must walk clockwise around the circle until the music stops. When the music stops, each student stands by a three-dimensional shape. The teacher will call out a direction such as "Give a thumbs up if you are standing by a shape that has a circle face." Ask other questions like "Raise your hand if you have a shape with 4 triangle faces and 1 square face."	Teacher observation; Student response
3	c	Use real objects or pictures of real objects such as a person, a teddy bear, and a butterfly. Use a yardstick or ruler as the "line of symmetry" and divide the objects into two equal parts. Show other pictures that will allow you to show other lines of symmetry.	Teacher observation; Student response
3	c	Provide die cut alphabet letters and die cut pictures. Give partnerships or teams a collection of shapes. Have the students experiment and define shapes as symmetrical or asymmetrical. Sort accordingly. Have students define lines of symmetry for symmetrical shapes.	Teacher observation; Student response;
3	c	The teacher can model positional words with a cube and a bag. (For example: The teacher will put the cube "in" the bag, "over" the bag, "beside" the bag, etc.) Extension: Play "Hidden Secret Object." The students work in teams as detectives. The teacher gives directions to the team to help them find a hidden secret object. (For example: "The secret object is near the front of the room. It is beside the cubbies. It is below the science table. It is inside the animal food tub.")	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	a	Measure classroom objects with nonstandard units such as popsicle sticks and unifix cubes. Have students make estimates and check by measuring. Students record findings. Help students use nonstandard units to accurately measure, lining them up from end to end without overlapping or leaving gaps. Discuss why the measurements of the same object are different when using popsicle sticks versus unifix cubes. Note: Experiences and discussions like these with non-standard units will help students understand the need for standard units and measuring units, helping students develop the awareness that different measuring tools will yield different numerical measurements.	Teacher observation; Student response; Student work sample
4	a	Read the story *How Big is A Foot*? by Rolf Myller. Discuss why the bed for the queen did not end up the right size. (The king measured with his "big" foot and the apprentice measured with his "little" foot.)	Teacher observation; Student response; Student work sample
4	b	Provide students with a balance scale and small objects (such as a box of crayons, a bottle of glue, a marker, a box of raisins, a pencil, an eraser, etc.) Have students predict which objects are lighter and heavier. Students will use a balance scale to compare objects. Students may order objects from lightest to heaviest or from heaviest to lightest.	Teacher observation; Student response
4	c	Provide students with various size containers, a scoop, rice, and a recording sheet. Students will choose a container and estimate how many scoops of rice it will take to fill the container. Students will find the actual measurement and compare to estimate.	Teacher observation; Student response; Student work sample
4	d	Use a large Judy Clock to model the direction and movement relationship of the clock hands, discussing seconds, minutes, and hours. Model telling time to the hour and half-hour. Relate counting how many minutes past an hour by counting by fives, discussing how the clock is divided into five-minute intervals from 0 to 60. Relate thirty-minutes past an hour to half-past. Show students a digital clock time and have the students place hands on small Judy Clocks to tell corresponding analog time.	Teacher observation; Student response

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	d	Create task cards on index cards. On the top half of cards, stamp and draw hands to show analog time. On the bottom halves, write the corresponding digital times. Cut into pieces. Students match puzzle pieces.	Teacher observation Student response
4	e	On a daily basis, use a calendar display to discuss the days of the week, the months of the years, holidays, etc. Discuss how to find a week after a specific date. (For example: If today is the 8th day of the month and John's cousin will arrive in a week, what date will John's cousin arrive?) Demonstrate how to find a specific day and date of a month. (For example: What is the 3rd Monday of this month?)	Teacher observation Student response
4	f	Provide pairs with classroom thermometers, a bowl of ice water, and a bowl of warm water. Pairs will use thermometer to measure hot and cold, recording findings through drawings and writing. The teacher will guide a class discussion with a manipulative thermometer to show students' findings. The teacher will move the "mercury" to show that when the "mercury" rises, the temperature is hotter and when the "mercury" falls, the temperature is colder.	Teacher observation Student response
4	f	Record the temperature daily, keeping a chart for each day. At the end of the week, discuss highest and lowest temperature. At the end of the month, discuss patterns in the temperatures. Compare different months and discuss patterns that emerge. Use emerging pattern knowledge to make predictions about what temperature is "likely" and "unlikely" to occur. Also, discuss the types of activities you would enjoy on days with similar temperatures. Discuss the types of clothing you would wear on days with a particular temperature.	Teacher observation Student response
5	a,b,c	Ask students to bring a favorite toy to school. Have students predict what kind of toy most will bring. Categorize/sort toys to make a real graph. Analyze graph and discuss which toy there was more of, less of, how many more of this toy than that toy, etc. Extension: Use the real graph to create a bar graph, tally graph, or pictograph.	Teacher observation Student response
5	b	Have each student draw an illustration of his or her favorite type of weather. Create a pictograph from illustrations and compare data. Have students analyze data and write at least three facts they learned from the graph. Share findings.	Teacher observation Student response Student work sample

FIRST GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	b,c	Have students predict which color apple most students in the class will like best: red, yellow, or green. Have a "Taste test." Make a tally and/or bar graph to show which apple students like best. Extension: Have students predict if it is "likely" or "unlikely" for another class to like the same apple best. Have a neighboring class repeat activity and share results. Discuss in terms of "always," "maybe," "sometimes," "never," etc. Poling several classes (getting a larger sample) will enable students to see patterns and trends that will help them analyze data to make more reliable predictions.	Teacher observation; Student response
5	c	Create different spinners (such as: spinner with one half yellow and one half purple, a spinner with one fourth yellow and three fourths purple) for students to experiment with. Provide partnerships with a spinner and a recording graph. Students predict which color will be spun more often/least often. Partner 1 spins and Partner 2 colors in a space on the graph beginning at the bottom. Play continues until 1 color fills all spaces on the graph to the top, indicating the color that was spun most often. Compare with classmate's results. Students analyze and discuss data, relating to their predictions. Discuss in terms of "always," "maybe," "sometimes," "never," "likely," "unlikely," etc.	Teacher observation; Student response; Student product
5	c	Write "events" on sentence strips such as "George Washington will visit our class today;" "We will have pizza for lunch Friday;" "We will play outside during recess;" "The principal will visit our class this week;" etc. Have students classify sentences according to terms, "always," "maybe," "sometimes," "never," "likely," "unlikely," "certain," "impossible," etc. Extension: Students will divide a journal page into two sections labeling them "Likely" and "Unlikely." Students write about at least two events under each category. Share with class and discuss.	Teacher observation; Student response; Student product

SECOND GRADE

The Second Grade mathematics framework is an extension of concepts learned in kindergarten and first grade, in order to begin development of individuality among students. Concepts should include an understanding of patterns, algebraic thinking, data analysis, prediction, measurement, and geometric concepts. Many new concepts should be explored, discovered, and modeled by students. The development of these concepts will be enhanced through the use of whole and/or small groups, manipulatives, pictorial representations, exploration, and technology.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability**, and the process strands: **problem solving, reasoning & proof, communication, connections, and representation**. The competencies may relate to one, many, or all the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather, the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

SECOND GRADE

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships among numbers and operations (addition, subtraction, multiplication). Compute fluently using effective strategies or rote memory.

a. Add and subtract basic facts 0 to 18.
b. Solve problems to find missing addends, minuends and subtrahends.
c. Analyze the inverse relationship of addition and subtraction in fact families.
d. Add and subtract two- and three-digit whole numbers with and without regrouping.
e. Model multiplication problems by drawing, writing explanations and using repeated addition.
f. Multiply one factor (0-5) and the other factor (0-12).
g. Explain how to skip count by 2's, 3's, 5's, and 10's to a given total.
h. Solve one- and two-step addition and/or subtraction problems by drawing, discussing, modeling, and writing explanations.
i. Model division with the concept of "fair shares" of sets.
j. Identify, model and write numbers to 1,000 in word form, standard form, and expanded form.
k. Compare three-digit numbers using the terms "before," "after," and "between."
l. Identify place value of a given digit in a four-digit number.
m. Identify, discuss, and model representations of fractions (halves, thirds, fourths, fifths, sixths, etc).
n. Round two-digit whole numbers to the nearest ten.
o. Round three-digit whole numbers to the nearest hundred.
p. Explain how to compare and order three-digit numbers utilizing <, >, and =.
q. Estimate quantities less than 1000. Check to determine reasonableness of results.
r. Identify and apply patterns to count pennies, nickels, dimes, quarters, and half dollars.
s. Determine and compare the value of money up to $5.00. Select and use the appropriate symbols for dollars and cents.
t. Find equal money amounts with different coin combinations to $1.00.
u. Make change from $1.00.

ALGEBRA

2. Analyze patterns, relationships and functions.

a. Explain and extend repeating patterns.
b. Explain and extend growing patterns (including 2's, 3's, etc.).
c. Identify and describe qualitative changes (such as temperature changes: getting hotter).
d. Identify and describe quantitative changes (such as the temperature increase five degrees).

GEOMETRY

3. Describe, classify and sort geometric figures according to their properties.

a. Recognize and identify polygons according to the number of sides (rhombus, square, triangle, trapezoid, rectangle, pentagon, hexagon, octagon, and decagon).
b. Recognize transformations using the terms translations, rotations, and reflections (slides, turns, and flips respectively).
c. Compare and contrast the characteristics of shapes using various resources (for example: manipulatives, software, etc.).
d. Use manipulatives or grid regions to show perimeter and area of squares and rectangles.
e. Compare and contrast the relationships between plane and solid geometric shapes (for example: the cube has square faces).
f. Locate and describe points in a coordinate system, such as maps.
g. Recognize that different shapes can be made from plane figures when they are cut up and rearranged (for example: a square can be cut into two triangles).
h. Explore congruent shapes.

MEASUREMENT

4. Estimate, identify, and apply measurable attributes.

a. Select appropriate tools to measure length, capacity, and weight.
b. Estimate and measure length, weight, and capacity using standard units of measurement (inch, foot, yard, centimeter, meter, ounce, pound, gram, kilogram, cup, pint, quart, gallon and liter).
c. Categorize measurement units according to length, weight, and capacity, and justify (oral/written) the selection of a specific unit of measure for a given item.
d. Collect and compare temperatures using a Fahrenheit thermometer.
e. Identify terms for time (before, after, until, a.m., p.m.).
f. Read and write time to the hour, half-hour, quarter-hour, and five-minute intervals using digital and analog clocks.
g. Use time (time line) to sequence events of the day.
h. Use the calendar to determine past and future days.

DATA ANALYSIS & PROBABILITY

5. Investigate and demonstrate the concepts of probability. Collect, organize and interpret data in graphical form.

a. Tally, record, interpret, and predict outcomes based on given information.
b. Create line, bar, and picture graphs using data collected from students and other resources.
c. Interpret graphical data in terms of “more,” “less,” “same,” “most,” and “least”.
d. Interpret graphical data with scales of 2, 5, and 10 units.
e. Predict the likelihood of events happening using the terms: likely, unlikely, certain, impossible, always, maybe, sometimes, and never.

SECOND GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Use vocabulary words to form addition and subtraction sentences. Select two words or student's names and count the number of letters in each word with the students. An addition and subtraction problem can be formed from these words. Example: Amy + Lorenzo ↓ ↓ 3 + 7 = 10 Create addition and subtraction problems to be shared and evaluated by classmates.	Teacher observation; Student response
1	b	The teacher will create a set of word problems designed to identify missing addends, minuends and subtrahends. Example 1 : If Johnny and Alisha had ten cookies all together. Alisha had four. How many does Johnny have? Example 2 : Pretend there were ten cookies in the cookie jar before you went to school. There were seven cookies in the jar when you got home from school. How many cookies did your mom eat while you were gone?" The students will use work mats/story boards and manipulatives to discover the answers to the problems.	Teacher observation; Student response
1	c	Give each student two paper plates and two different kinds of beans. Tell an addition story using an addition fact such as, "Mom put 7 beans on my plate and 6 beans on my sister's plate. How many beans did she serve?" Have students write the addition sentence and their related subtraction sentence that corresponds with the story. 6 + 7 = 13 and 13 – 7 = 6	Teacher observation; Student response
1	d	Demonstrate how to add and subtract numbers using vertical and horizontal format. Use grid paper to practice keeping numbers lined up. The teacher will write various addition problems containing two and three digit whole numbers on the board. (There should be a mixture of problems with and without regrouping.) The student will demonstrate and model the problems using base ten manipulatives. Practice using base ten blocks and base ten work mats.	Teacher observation; Written evaluation
1	e	Give students inch grid paper and crayons. Have students color two rows of 5 squares. Ask what addition this shows 5 + 5 = 10. Tell the students that two groups of five is also 2 x 5 = 10. Have students color three rows of five – show 5 + 5 + 5 = 15 or 3 x 5 = 15. Have students repeat the activity coloring rows of twos and tens using different numbers of rows.	Teacher observation

SECOND GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	f	Recite daily the multiplication facts of 0 to 5 and listen to a rap or other musical multiplication tape.	Teacher observation
1	g	Model skip counting using a number line 0 to 20. Tape the number line to the floor. Demonstrate and explain how to count by 2's, 3's, 5's, and 10's. Circle the numbers as they are demonstrated. Practice using the number line and instruct students to "skip to 1's", "hop to 2's", "stretch to 3's", "jump to 5's" and "clap to tens". Repeat this activity several times.	Teacher observation; Student participation
1	h	Use manipulatives and role play to demonstrate and discuss one and/or two step problems involving addition, subtraction, and multiplication. Draw a representation of the problem, write number sentences, and explain how the problem was solved.	Teacher observation; Written evaluation
1	i	Present each team or group with a bag of individually wrapped candy. The students must distribute the candy one piece at a time, demonstrating "fair shares". (May use counters such as 16 counters shared between two students or four students.)	Teacher observation; Peer evaluation
1	j	Play "Concentration" with numbers in standard, word and expanded forms.	Teacher observation; Written evaluation
1	j	The teacher will write a three-digit number on the board. The students will model the number using base ten blocks and a base ten mat. The students will record their findings on a recording sheet in expanded form. (ex. 300+ 20 + 9 = 329)	Teacher observation; Student product
1	k	Prepare number cards 100 – 999 and distribute them to different students. Have them line up in front of the classroom and arrange themselves in order. Discuss "before," "after," and "between".	Teacher observation
1	l	Use an Abacus constructed from a shoebox lid to show place value using four-digit numbers. Construct work mats and use unifix cubes or base ten blocks to demonstrate place value.	Teacher observation
1	m	Use a sheet of notebook paper to show one whole. Fold the paper into two equal parts, labeling each part as half. Repeat the activity to show one-fourth. Using the folded fraction parts as examples, have the students draw a shape (rectangle) and divide it into halves and fourths. (Repeat demo using a paper plate.)	Teacher observation; Peer evaluation

SECOND GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	n,o	Use a number line to help with rounding.	Student observation
1	p	Have groups of students roll number cubes to generate three digit numbers. Write the numbers on dry erase boards with a middle space to fill in the appropriate symbol of equality or inequality.	Teacher observation
1	q	Provide each student with a large pre-drawn sunflower. Have the students predict to the nearest multiple of ten how many sunflower seeds it will take to fill the inner part of the sunflower. Then have the students place seeds on the sunflower. When it is full, have the students divide the seeds into groups of ten to check their estimations. Finally, the students can glue the seeds on the sunflower for a work of art.	Teacher observation; Peer evaluation; Student product
1	r,s	Use real money or facsimiles to count coins. Have students arrange a variety of coins from highest to lowest value. Count coins from greatest to least value. Remind students to count by ones, fives, and tens when counting pennies, nickels, and dimes.	Teacher observation; Student response
1	r,s	Display a penny, nickel, dime, quarter, and half dollar. Students take turns naming the coins and their value. Introduce a dollar bill. Explain that $1.00 is the same as 100 cents. Write $1.00 on the chalkboard. Point out the dollar sign and the decimal point. Explain that the decimal point separates the dollars from cents. Give each student an envelope with different money amounts up to $5.00. Have the students count the money inside the envelope and determine who has the greater amount.	Teacher observation; Student response
1	t	Given a money amount up to $1.00, have students use play money or money stamps to show the same amount in different ways. While working in pairs, have one student write an amount of money less than $1.00 on a card, and the other student will use coins or stamps to illustrate the amount shown on the card.	Teacher observation; Student response
1	u	Set up a toy store and let students go shopping with a $1.00 bill. One student will be the storekeeper and will be responsible for making change. Students will choose one or more items less than $1.00. The storekeeper will make necessary change for each student. Allow students to take turns being the storekeeper.	Teacher observation; Peer evaluation

SECOND GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	a,b	Show a beginning pattern, such as 12, 15, 18, __, __. Divide the class into small groups. Have the students discuss the pattern and take turns extending the pattern.	Teacher observation; Student response
2	b	Use unifix cubes to show a stair step pattern. Have the students how the pattern grows. Have students continue the pattern. Read the book <u>There was An Old Woman Who Swallowed a Fly</u> to reinforce the growing pattern concept.	Teacher observation; Student response
2	b	Give students a table with rules and have them complete the table. Example: 5 .. 9 6..10 The rule is adding 4. 7..11 8.. ?	Teacher observation; Student response
2	c	Plot daytime and nighttime temperatures to show "getting hotter" and "getting colder".	Teacher observation; Student response
2	d	Plot specific temperatures on a grid or chart from one day to the next over a period of time to show change in degrees.	Teacher observation
3	a	The teacher models and identifies shapes. Students should construct the same shapes on their geo-boards with rubber bands. Read <u>The Greedy Triangle</u> to discuss characteristics of shapes.	Teacher observation; Student response
3	b	Use construction paper to cut out various patterns. Model how these patterns can be used to show reflections, translations, and rotations (flips, slides, and turns).	Student Model; Teacher observation
3	c	Provide each group of four to five students approximately 6 feet of yarn. Have them work together to form a rhombus, square, hexagon, triangle, trapezoid, parallelogram, quadrilateral, and circle. Ask, "How many corners and sides will the shape have?" "Will all sides be the same length?" Find similar shaped items in the classroom.	Teacher observation; Peer evaluation
3	d	Demonstrate the perimeter of certain shapes using a piece of yarn and a ruler. Measure the yarn to determine the perimeter in standard units. Draw a square or a rectangle on a grid. Have the students count the blocks to demonstrate the area.	Teacher observation
3	e	Display geometric solids on the overhead. Have the students trace the faces of geometric solids to identify the two-dimensional shapes. Compare/contrast the different solids.	Teacher observation

SECOND GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	f	Take a picture of each student. Create a grid with spaces large enough for each picture. Label the coordinates. Place the pictures on the grid. Have students locate their picture by telling you the coordinates. (Remember – o before u – over before up.)	Teacher observation; Student response
3	g	Provide construction paper shapes for each student. Have them fold to discover new shapes that can be created.	Teacher observation; Student product
4	a,b	Put a list of measures on separate pieces of paper in two rows across a bulletin board: 1 inch 2 inches 4 inches 1 foot 2 feet 4 feet Show students how to measure objects to the nearest inch and to the nearest foot. When the measurement of an object matches one of the measurements on the bulletin board, have students write that object under the corresponding measure.	Student response; Teacher observation
4	b	Use appropriate units to estimate, measure, and compare lengths of various classroom objects, capacity using colored water or beans and a variety of containers and weights of classroom objects. (For example: apple in grams, chair in pounds)	Teacher observation; Peer evaluation; Student response
4	c	As a whole group, use three empty boxes labeled “length” “weight” and “capacity”. Sort items such as piece of yarn, book, and can according to the appropriate measurement term. Complete a recording sheet and discuss why these choices were made.	Teacher observation; Student response
4	d	Using a Fahrenheit thermometer, collect, chart, and compare temperature readings taken at different times during the year.	Teacher observation Project Rubric
4	e	Display and discuss a schedule of classroom activities. Draw or use clock stamps to show the time for each activity. Write the time under the clocks, being sure to include a.m. or p.m. Discuss activities using the terms before, after and until.	Teacher observation; Student response
4	f	Display a demonstration clock and identify the hands. Discuss the meaning of the twelve numbers and the lines between them. Model with the students how to skip count by 5’s to show that there are 60-minutes in an hour. Read and write different times by having student pairs take turns writing a time on a dry erase board and showing the correct time on a demonstration clock.	Teacher observation; Student response

SECOND GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	f,g	From a collection of index cards with an assortment of analog clock times and digital clock times, have students choose five cards. Punch two holes across the top of each card and have students arrange their five cards in chronological order beginning at 12:00 a.m. Have students string their cards on yarn in order. Have the class check. Ask students to write a short story about events that occurred at each time displayed.	Teacher observation; Student product
4	h	Using a calendar display, discuss the days of the week. Demonstrate how to find specific information from the calendar. Have students answer questions such as "Cathy's friend will visit her on the 3rd Wednesday of the month. What is the day of this visit?"	Student response
5	a,b	Create a line graph to record temperature over a period of time. Compare and interpret the temperature over that period of time.	Teacher observation; Student project
5	a,b,c	Conduct a survey of students' favorite day of the week. Organize and create a bar graph. Interpret and compare data using the terms "more", "less", "same", "most" and "least."	Teacher observation; Student response; Student project
5	a,b,d	List the months of the year on a chart. Each student will indicate the month he/she was born by making a tally mark under the appropriate month. Create a picture graph using data from the birthday chart.	Teacher observation; Student project
5	e	In small groups or pairs, conduct probability experiments using color tiles, marbles, spinners or coins. Record predictions and results on a chart. Use the terms "always," "maybe," "sometimes," "likely," "unlikely," "certain," "impossible," and "never" to compare results. Have half of the class create pictures of situations that will probably never happen, (like a green dog with wings), and the other half of the class create pictures of situations that are likely to happen, (like a brown dog sitting). Compare and discuss the different pictures using the words "likely, unlikely, maybe, never, etc."	Teacher observation; Student response; Student product

THIRD GRADE

The Third Grade competencies and objectives/benchmarks are designed to be an extension of those concepts learned in kindergarten through second grade. These concepts include the properties of the four basic operations, measurement gathering, organizing and interpreting data, and geometric and money concepts. Students should also explore estimation and the use of variables.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability**, and the process strands: **problem solving, reasoning & proof, communication, connections, and representation**. The competencies may relate to one, many, or all the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather, the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

THIRD GRADE

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships among numbers and the four basic operations. Compute fluently and make reasonable estimates.

a. Read, model, and write numbers up to five-digits in word and standard form.
b. Identify the value of a given digit in a five-digit number.
c. Explain how to compare and order five-digit numbers using <, >, and =. .
d. Write numbers up to five-digits in expanded form.
e. Identify points on a number line.
f. Explain how to round numbers to the nearest thousand.
g. Recognize, draw, model and order fractions with fourths, fifths, sixths, and eighths.
h. Recognize, draw, and model equivalent fractions.
i. Choose appropriate operational symbols to complete number sentences.
j. Add (up to three addends) and subtract five-digit numbers with and without regrouping.
k. Recall multiplication and division facts 1 to 12.
l. Multiply up to three-digit factors by one-digit factors.
m. Divide two-digit numbers by one-digit divisors with and without remainders.
n. Add and subtract fractions with like denominators.
o. Model and identify decimals to hundredths.
p. Add and subtract numbers with decimals to hundredths.
q. Estimate and add amounts of money less than $1000 represented with decimal notation.
r. Estimate and subtract amounts of money from $100.00 or less.
s. Count change from $5.00 or less.
t. Use mental math strategies to solve problems.
u. Construct fact families for the basic operations.
v. Apply problem-solving techniques to solve one- or two-step problems involving addition, subtraction, multiplication or division.

ALGEBRA

2. Explain, analyze, and generate patterns, relationships, and functions using algebraic symbols.

a. Recognize, describe, and identify a pattern unit within a given pattern in order to extend patterns to a given position.
b. Create and explain numerical patterns.
c. Determine the value of variables within number sentences.
d. Using appropriate manipulatives, model and identify the following properties:
   - Zero property of multiplication;
   - Associative property of addition and multiplication;
   - Commutative property of addition and multiplication; and
   - Identity properties of addition and multiplication.
e. Model and identify the inverse operations of addition/subtraction and multiplication/division.

GEOMETRY

3. Describe, compare, and contrast two- and three-dimensional shapes and relationships.

a. Identify, compare, and contrast among parallel, intersecting, and perpendicular lines.
b. Identify, compare, and contrast right, acute and obtuse angles.
c. Use grid regions to determine perimeter and area of regular geometric figures.
d. Locate ordered pairs in the first quadrant of the coordinate plane.
e. Classify, describe, and model the results of translations, reflections, and rotations (slides, flips, and turns) on figures.
f. Create polygons up to ten sides.
g. Identify congruent figures.

MEASUREMENT

4. Measure and explain the measurable attributes of objects, units, systems, and processes.

a. Identify and compare differences in length, weight/mass, and capacity/volume within measurement system (English or metric). Choose appropriate unit of measurement.
b. Using various types of measuring tools, estimate, read and measure:
   - length in centimeters, meters;
   - length in inches, feet, yards;
   - length to the nearest half-inch;
   - weight in grams and kilograms;
   - weight in ounces and pounds;
   - capacity in liters;
   - capacity in cups, pints, quarts, and gallons;

- time to nearest minute; and
- temperature in Celsius and Fahrenheit.

c. Compute elapsed time to the hour and half hour to solve problems.
d. Use the calendar to determine specified dates.

DATA ANALYSIS & PROBABILITY

5. Interpret and analyze data. Explore basic concepts of probability.

a. Compare data and interpret quantities represented on charts, tables and different types of graphs (line, pictograph, and bar) and make predictions based on the information.
b. Analyze, predict, and model the number of different combinations of two or more objects.
c. Determine simple probabilities by using manipulatives such as spinners.

THIRD GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Given a five-digit number, write the number in word form. Count orally and write the five counting numbers before or after the given number.	Teacher observation
1	b,c	Each student draws up to five blanks on a piece of paper. Have each student roll a number cube to produce a digit that will be placed in one of the five blanks – continue until a number is generated. Working in pairs, have students use symbols of equality or inequality to read and compare numbers. Identify the value of each digit.	Peer evaluation, Student work sample
1	e	Create a number line using yarn and number cards to identify points. Let students take turns placing numbers on the line.	Teacher observation
1	g	Cut candy bars or pizza to demonstrate fourths, fifths, sixths, and eighths or equivalent fractions. Compare the relationship between the fraction part and the whole. Model adding and subtracting fractions using these manipulatives.	Teacher observation;
1	h	Have students divide six paper plates into equal pie wedge parts: halves, thirds, fourths, sixths, eighths, and twelfths. Write $\frac{1}{2}$ = __, $\frac{1}{3}$ =__, $\frac{1}{4}$ =__ on the board. Ask students to find as many equivalent fractions as they can using the plates. Continue with other fractions.	Teacher observation; Student product
1	i	Have students cut a sheet of paper into four parts and write an operational sign on each part. Call out a number sentence and have students hold up the correct operational symbol to complete the number sentence. Ex. 4 $\square$ 2 = 6	Teacher observation
1	j	Use base ten blocks to review adding and subtracting .	Student work sample
1	l	Use base ten blocks to illustrate multiplication of a three-digit number by a one-digit number.	Teacher observation

THIRD GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	m	Provide students with graph paper. Have students cut the graph paper to represent various division problems. Start with division equations that the students are likely to know and proceed to problems with two-digit numbers divided by one-digit numbers. Ask students to write the equation that the graph paper strips represent. Ex: Put 6 grid blocks into groups of 2 . The quotient is the number of groups.	Student work sample
1	n	Display a rectangle divided into eight equal parts. Write $\frac{3}{8} + \frac{2}{8}$. Have a student shade $\frac{3}{8}$ of the rectangle and have another shade $\frac{2}{8}$ of the rectangle. Have students tell how much of the rectangle is shaded – $\frac{5}{8}$. Follow the same procedure for other fractions.	Student work sample
1	o,p	Provide a blank hundreds chart to use as a decimal grid . Write sample problems on the board, such as 0.23 + 0.45 = __. Have students shade the decimal numbers on their grid using a different color for each number. Have students count the shaded squares to find the total.	Teacher observation; Student response
1	q	Solve problems using information such as the following: Science Museum tickets: Adults…$3.95 Seniors…$1.75 Children…$1.50 Example: Two adults and one child are going to the Science Museum together. How much does it cost?	Student work sample
1	r	Use $1, $10 and $100 paper bills to represent ones, tens, and hundreds. Use dimes and pennies to represent tenths and hundredths of a dollar. Add and subtract money amounts using these manipulatives.	Teacher observation; Peer observation
1	s	Set up a store in the classroom filled with items costing less than $5.00. Students will pretend to buy item(s) at the store using a five-dollar bill. Select students to count the correct change.	Teacher observation; Peer observation

THIRD GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	u	Make two spinners with the numbers one through twelve on them. Students will play a game in pairs. Each player spins a spinner. Both players write as many facts (addition, subtraction, multiplication and division) as they can that include the numbers . Example: 4 & 5 were spun. 4 + 5 = 9, 5 + 4 = 9, 9 – 5 = 4, 9 – 4 = 5, etc.	Teacher observation; Student work sample
1	v	Use the following activities to solve word problems: • Role-play the problem presented • Illustrate basic elements of the story problem • Identify information necessary for solution • Reword and paraphrase to clarify meaning • Write and solve student generated word problems	Teacher observation; Peer observation
2	a	Use colors, symbols, beads, or other objects to analyze and describe patterns.	Teacher observation
2	b	Generate and extend number patterns. Analyze the patterns and explain the pattern by relating it to skip counting, number properties, or place value.	Teacher observation
2	d	Use manipulatives such as M&M's® or linking cubes to demonstrate properties of basic operations. Ex: Commutative property of addition – 2 red cubes + 1 blue cube = 1 blue cube + 2 red cubes	Teacher observation
2	e	Have students draw on centimeter graph paper a 4 x 3 grid. Ask, "How many rows are there? How many columns? What multiplication sentence shows the total number of units?" "How can you use division to find the number of units in each row?" Be sure to point out that the same grid can be used to show both the division and the multiplication sentences. Repeat the activity using a 5 x 3 grid.	Teacher observation
3	a	Glue uncooked spaghetti to poster board to illustrate parallel, intersecting, and perpendicular lines.	Teacher observation; Rubric
3	b	Make right angle models using the corner of note cards for angle comparison. Using opened scissors, trace the inside of the blades to identify angles less than, equal to, or greater than right angles. Introduce acute, obtuse, and right angles.	Teacher observation

THIRD GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	c	Use an overhead projector to display a 6 unit by 4 unit rectangle on centimeter graph paper. Demonstrate how to find the perimeter and area. To determine the perimeter, count the total number of length units around the four sides. To determine area, count the total number of unit rectangles. Provide each student with centimeter graph paper. Have students draw several regular polygons with specific perimeters and areas.	Teacher observation; Student product
3	d	Arrange the desks in the classroom in an array format. Identify the location (0,0). Place the names of students on slips of paper and place them in a box. Have students take turns coming to the front of the room, taking a slip of paper from the box, and identifying the location of that student by using an ordered pair.	Teacher observation; Student participation
3	e	Have all students trace a triangle on a piece of paper so that they are all in the same position. Give students the following directions, one at a time. 1. Slide the triangle one time and trace. (translation) 2. Turn the triangle from a given point to the right and trace. (rotation) 3. Flip the triangle to the right two times and trace it each time. (reflection)	Teacher observation; Student product
3	f	Have pairs of students spin a spinner numbered three through ten. Have partners work together to glue toothpicks onto paper that represents a closed figure that has the number of sides shown on the spinner. Pairs of students can share and compare figures.	Teacher observation; Student product
4	a,b	Perform an activity using dry and liquid measurements. Use various tools to weigh ingredients and compare amounts.	Teacher observation
4	b	Use Gallon Man or Big G for demonstration purposes. Gallon Man: Cut a large rectangle for **body** – **gallon**; cut four smaller rectangles for **arms** – **quarts**; cut eight even smaller rectangles for fingers and **toes** – **pints**; cut sixteen very small triangles for **fingernails/toenails** – **cups**. Paste together and draw head and face. Big G: Draw a big G. Inside the G, write four Q's; Inside each Q, write two P's; inside each P, write two C's.	Teacher observation; Student response
4	b	After students practice using thermometers, have them complete a chart filling in the temperature in both Fahrenheit and Celsius: Room Temperature ___F ___C Water Freezes ___F ___C Body Temperature ___F ___C Cool Day ___F ___C Warm Day ___F ___C	Student response

THIRD GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	c	Have students work in pairs. Teacher provides beginning and ending times for events. Have students use an analog clock with moveable hands to determine elapsed time.	Teacher observation; Student product
4	d	Students create a calendar of their favorite month, filling in appropriate dates. Have students answer questions about specific dates. Example: "Tom's first basketball game is on the second Saturday of the month. What is the date?"	Student product
4	d	Provide students with a copy of the following: **Thirty days have September,** **April, June, and November;** **All the rest have thirty-one,** **Except for February alone,** **It has twenty-eight we hear,** **And twenty-nine in each leap year.** Have the students complete a chart showing the number of day in each month.	Teacher observation; Student work sample
5	a	Collect data based on students' interests (outside temperature, favorite books, movies, etc). Construct appropriate graphs – line, bar, and pictograph. Display findings and interpret data using student-generated questions.	Teacher observation
5	b	Use menu entrees and side items to show all possible combinations.	Teacher observation; Student product
5	c	Use spinners with different colors, paper bags with different colored cubes and a collection of coins to predict, experiment, and report probability results.	Teacher observation

FOURTH GRADE

The Fourth Grade framework is designed to expand concepts and processes learned in kindergarten through third grade. Students should explore numerical relationships and will utilize the four basic operations. Students should explore the processes of data analysis, geometric concepts, number patterns, measurement and related concepts. Students should be provided with learning experiences that enable them to select appropriate strategies to solve real-world problems. Actively investigating and discussing mathematical ideas using a variety of tools will help students become confident problem solvers.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability,** and the process strands: **problem solving, reasoning & proof, communication, connections, and representation.** The competencies may relate to one, many, or all the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather, the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities or skills.

Objectives/benchmark indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

FOURTH GRADE

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships among numbers, relate to real life situations, use the four basic operations, compute fluently and make reasonable estimates.

a. Add and subtract up to seven-digit whole numbers with and without regrouping.
b. Add and subtract decimals through hundredths to include money amounts.
c. Multiply whole numbers by two- and three- digit factors.
d. Divide four-digit dividends by one- and two-digit divisors, with and without remainders.
e. Model and identify factors and multiples of whole numbers.
f. Count change from $10.00.
g. Add and subtract fractions with like and unlike denominators.
h. Apply problem-solving techniques to solve one- and two-step problems involving the basic operations.
i. Read and write seven-digit whole numbers, decimal numbers through hundredths, and fractions in word form and standard form.
j. Write seven-digit numbers in word, standard and expanded form.
k. Order and compare seven-digit whole numbers, decimal numbers through hundredths, and fractions with denominators of twelve or less.
l. Explain how to round whole numbers to one hundred thousand and round decimal numbers through hundredths.
m. Identify, draw, and model equivalent fractions with denominators up to twelve.
n. Represent, draw, and explain the relationship between fractions and decimals using real-life objects.
o. Estimate sums, differences, products, and quotients using a variety of techniques. Determine whether estimated answers are reasonable.
p. Estimate and use mental computation to solve real-life problems where exact answers are not required.

ALGEBRA

2. Explain, analyze, and generate patterns, relationships, and functions using algebraic symbols. Demonstrate an understanding of the properties of the basic operations.

a. Recognize, describe, and extend a given pattern to an identified ordinal position.
b. Analyze a given numeric pattern and generate a similar pattern.
c. Determine the value of variables in a whole number sentence (i.e., equations and inequalities).
d. Construct and complete input/output function tables with whole numbers using basic operations.
e. Demonstrate and explain the properties of the basic operations using numbers and variables:
   - Zero property of multiplication;
   - Associative property of addition and multiplication;
   - Commutative property of addition and multiplication; and
   - Identity property of addition and multiplication.
f. Demonstrate and explain the inverse operations of addition/subtraction and multiplication/division.

GEOMETRY

3. Analyze characteristics and properties of two- and three- dimensional geometric shapes and relationships.

a. Identify, describe, classify, and compare two- and three- dimensional geometric shapes, figures, and models using specific vocabulary.
b. Identify, model, and extend figures using the terms translations, rotations, and reflections (slides, turns and flips).
c. Identify and compare points, lines (including parallel, perpendicular, and intersecting), line segments, and rays.
d. Identify, model, and compare right, acute, and obtuse angles. Locate angles in geometric shapes.
e. Define and label the parts of a circle: center, radius, diameter, and chord.
f. Identify and create congruent figures.
g. Plot and locate ordered pairs in the first quadrant coordinate plane.

MEASUREMENT

4. Evaluate and justify measurable attributes of objects, units, systems, and processes. Perform measurements.

a. Measure a given object to the nearest centimeter and fourth of an inch.
b. Select, use, and compare length, weight/mass, capacity, and volume across the appropriate standard (English and metric) system of measurement:
   - Centimeters and inches;
   - Meters and yards;
   - Kilograms and pounds; and
   - Liters and quarts.
c. Convert capacity, weight/mass, and length within the English System of measurement involving real-life situations.
d. Convert capacity, weight/mass, and length within the metric system of measurement involving real-life situations.
e. Solve problems with elapsed time using hours, minutes, days, weeks, months, and years.
f. Determine the scale of various thermometers in increments of two, five, and ten degrees.
g. Use formulas to compute the perimeter and area of geometric figures and circumference of circles.

DATA ANALYSIS & PROBABILITY

5. Formulate and analyze data and concepts of probability. Evaluate inferences and predictions.

a. Draw, label, and interpret bar graphs, line graphs, and pictographs.
b. Formulate and solve problems that involve data analysis and prediction.
c. Demonstrate finding the probability using various manipulatives. Express probability in terms of a ratio ($\frac{a}{b}$).
d. Find and discuss the mean, mode, median, and range of a set of data (using one- and two-digit numbers).
e. Interpret graphical data with scales of 2, 5, 10, and 25.

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a,b	Use graph paper to assist students with adding and subtracting whole numbers and decimals.	Teacher observation
1	c	Use colored chalk or colored overhead pens to differentiate the digits and subproducts. Ex: 321 x23 The 3 might be blue as would the product of 3 x 321 would be blue. The 2 might be red as would the product of 20 x 321.	Teacher observation; Student work sample
1	d	Have students turn their lined notebook paper horizontal to form columns in which to place the digits when dividing numbers.	Student work sample
1	d	Use the mnemonic device "Does McDonalds Sell Cheese Burgers?" to help students remember the acronym DMSCB and division algorithm: **D**ivide, **M**ultiply, **S**ubtract, **C**ompare, and **B**ring-down.	Teacher observation
1	e	Using hundred chart and markers, identify multiples of numbers by placing a dot of a designated color on each multiple. For example, multiples of 2, red dots are on 2, 4, 6, etc.	Teacher observation; Student work sample
1	f	Divide the students into pairs and distribute $10.00 in play money to each group. Make sure there is a variety of coins and bills. One student will make a "purchase" and the other will "make change." Roles will be reversed after several purchases.	Peer observation
1	g	Create fraction strips to aid in the addition of fractions with like and unlike denominators up to 12. The fraction strips may be "stacked" to show equivalent fractions when adding or subtracting with unlike denominators.	Teacher observation; Student work sample
1 2	h c	Write word problems on a 3 x 5 index card which require 2 steps (operations) to solve. After modeling, have the students write the value and operation above each corresponding part of the problem in order to develop a number sentence (equation). **Example:** Bob went to the fair and came home with $6.00 after paying $3.00 for a ticket and riding 6 rides which cost $1.00 each. How much money did Bob take to the fair? The student would write + above "paying", + above "riding", then (6 x $1.00) and = after "each". This would yield the equation of: $6.00 + $3.00 + (6 x $1.00) = m	Student observation; Student work sample

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	i, j	Write the digits 0 through 6 on 6 x 9 index cards. Write a comma on two index cards. Give one card to 9 students. Have them stand in a line across the front of the classroom. Call out instructions such as, "move the 6 to the hundreds place, the 4 to the hundred thousands place, the 1 to the millions place, etc. Have the rest of the class write the number in word, standard, and expanded form. The activity may be enhanced by writing a decimal point on an index card to create decimal numbers.	Student work sample
1	k	Have the students arrange a given group of numbers from least to greatest or greatest to least by "stacking" them in a vertical column and then by comparing the digits in the greatest position down to the least as far as is necessary to determine the order.	Student work sample
1	l	Have students look for large numbers and decimal numbers in a newspaper or magazine. Round the numbers to the greatest position. Have students justify answer verbally or in writing . Students should be aware that in decimal numbers, the digits to the *right* of the rounded position are dropped. For example, rounding 0.328 to the nearest hundredth = 0.33 **not** 0.330.	Student work sample
1	m	Using grid paper, have students construct fraction bars showing $\frac{1}{2}$ through $\frac{12}{12}$ using factors of 12. From this construction, the students will create a list of equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{6}{12}$ and $\frac{1}{3} = \frac{2}{6} = \frac{4}{12}$, etc.	Teacher observation; Student work sample
1	n	Using play money coins, the students can show fractional parts of a dollar and its equivalent decimal: 1 quarter = $\frac{1}{4}$ of a dollar = $\frac{25}{100}$ = .25 1 nickel = $\frac{1}{20}$ of a dollar = $\frac{5}{100}$ = .05 The students can create a table of coins to show fractional parts of a dollar and the equivalent decimal.	Teacher observation; Student work sample
1	o	Students are provided with a given amount of play money to spend on dinner. Have students make selections from a menu and then estimate the cost. Tip should also be calculated. Discuss menu items they can "afford."	Teacher observation

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	o	Have students estimate quotients using compatible numbers. Example: For 652 divided by 84, 652 is close to 640 and 84 is close to 80. 640 and 80 are compatible numbers. The estimated quotient is 8. Have students discuss how they determine their estimates. (The actual answer is 7.7)	Teacher observation; Student work sample
1	o	Students can estimate products by rounding the factors to some multiple of 10. Multiply the counting digits and add the total number of zeros in both factors. Example: 3,456 x 57 would yield 3,000 x 60 or 3 x 6 =18 with 4 zeros = 180,000.	Teacher observation; Student work sample
1	o	Fractions may be estimated by comparing them to 0, $\frac{1}{2}$, or 1. For example: $\frac{3}{4} + \frac{4}{5}$ would round to 1 + 1 = 2.	Teacher observation; Student work sample
1	p	Give students problems such as: The fourth grade wants to raise $1000 for landscaping the school. They want to sell tickets to a spaghetti dinner. The tickets are $4.75 each. About how many should they sell to have enough money?	Student work sample
2	a	Form a pattern using a line of students. For example, "girl, girl, boy, girl, girl, boy." Have students describe and extend the pattern.	Teacher observation; Student work sample
2	b	Explain that numbers can form patterns. Give examples such as multiples of numbers and families of numbers.	Teacher observation; Student work sample
2	c	Students should be given the opportunity to reason by solving problems such as: 234 – *n* = 198 *n* = _____ $\frac{12}{n} = 3$ *n* = _____	Teacher observation; Student work sample

FOURTH GRADE

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>2</td><td>d,f</td><td>The students should experience input/output boxes. For example:

The rule is 3x (That is, 3 multiplied by IN = OUT). Therefore if the IN, is not given, the OUT can be divided by 3 to obtain the IN.

<table><tr><td>IN</td><td>OUT</td></tr><tr><td>4</td><td>12</td></tr><tr><td>7</td><td>21</td></tr><tr><td>X</td><td>45</td></tr><tr><td>12</td><td>y</td></tr></table></td><td>Teacher observation; Student work sample</td></tr>
<tr><td>3</td><td>a</td><td>Have students use a geo-board or grid paper to make quadrilaterals. List like attributes.</td><td>Student work sample</td></tr>
<tr><td>3</td><td>a, b</td><td>Visit www.nctm.org/illumination. Students can create different shapes, identify shapes, and explore transformational results. The students can also explore three-dimensional solid figures.</td><td>Teacher observation</td></tr>
<tr><td>3</td><td>a,b</td><td>Visit http://www.AAAmath.com for self-checking quizzes with two- and three-dimensional figures.</td><td>Teacher observation; Student work sample</td></tr>
<tr><td>3</td><td>a,b</td><td>Have students draw, cut out and identify various two-dimensional shapes. Glue shape to a card listing their attributes and create a mobile attaching each card with yarn and a clothes hanger.</td><td>Student work sample</td></tr>
<tr><td>3</td><td>a,d</td><td>Give each student or group of students a selection of pattern blocks . Have the students name the shape of the pattern block and identify angles on the shapes.</td><td>Student work sample</td></tr>
<tr><td>3</td><td>c</td><td>Students can use city maps, state maps, or other illustrations to identify points, lines, segments and rays.</td><td>Student work sample</td></tr>
<tr><td>3</td><td>d</td><td>Use uncooked spaghetti to create acute, right, and obtuse angles.</td><td>Teacher observation</td></tr>
<tr><td>3</td><td>d</td><td>Using an analog clock, the teacher will move the hands of the clock so that students can identify angles.</td><td>Teacher observation; Student work sample</td></tr>
</table>

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	e	Provide students with a circle . Have the students fold the circle in half twice to find the center. Label the circle with a letter at the center point. Have the students measure a crease passing through the center point, label this crease "diameter" and record the measurement. Measure this segment and label it "radius". Note that the radius is exactly $\frac{1}{2}$ the diameter. Connect any two points on the circumference and label this "chord".	Teacher observation; Student work sample
3 4	e g	Provide student pairs with non-stretchy string, ruler, five assorted lids and a calculator. Have students measure the circumference and diameter of each lid. Record results in a chart. Use a calculator to figure out how many diameters fit into each circumference. Students will discover the relationship between circumference and diameter and the meaning of *pi* (π).	Teacher observation; Student work sample
3	f	To illustrate **congruency** – **same shape, same size**: give each student two square "post-it™" notes of different colors. Have the student place one post-it™ over the other. One completely covers the other. Since they are of the same size, shape, and they are congruent.	Teacher observation
3	g	Students should recognize that the Cartesian coordinate plane is 4 quadrants made from two intersecting perpendicular line segments. The upper right $\frac{1}{4}$ is the first quadrant. Where the segments intersect is point (0,0) or ***origin***. Have students locate ordered pairs. The **horizontal** component is located before the **vertical**. There are several methods for students to remember this: **H** comes before **V** Over first, then up (**O** comes before **V**) Right then Up (**R** comes before **U**)	Teacher observation
3	g	Give students grid paper and have them number the vertical and horizontal lines making sure that the lower left hand corner is labeled 0. Call out ordered pairs and have the students plot the coordinate with a dot.	Teacher observation; Student work sample

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	b	There are certain parallels between English and metric units of measure. Students should understand the following parallels before selecting appropriate metric units. Centimeters correspond to inches kilometers correspond to miles meters correspond to feet or yards grams correspond to ounces kilograms correspond to pounds liters correspond to quarts or gallons Prepare task cards labeled with items: weight of a box of rice …… .grams(g)/ounces(oz) length of a tennis court … .feet(ft)/yards(yd)/meters(m) volume of a soda ……. ounces(oz)/milliliters(ml) weight of a piece of gum … ounces(oz)/grams(g) capacity of a gas tank …..gallons(gal)/liters(l) length of a pencil ……..inches(in)/centimeters(cm) weight of a bag of sugar…..pounds(lb)/kilograms(kg) Students match the task cards to appropriate units of measure cards.	Teacher observation; Student work sample
4	b	Display various items and measuring tools. Ask students to select the appropriate tool to measure the various objects.	Student work sample
4	b	Have students create benchmarks for metric measurement, i.e., kilometer – $\frac{1}{2}$ mile meter – height of door knob from floor decimeter – width of the palm centimeter – width of a little finger at the nail millimeter – diameter of paper clip wire or thickness of a dime liter – $\frac{1}{2}$ of a 2 liter soft drink bottle kilogram – 2.2 pounds (weight of ordinary English textbook) gram – weight of a dime centigram – weight of a half dollar milligram – weight of a toothpick Using these benchmarks, appropriate metric units can be chosen to use in measurement.	Student work sample

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	c	Provide standard English measuring sets or household measuring containers, scales, water and/or dry rice for measuring. Students will predict correct conversions and experiment to verify predictions and complete a table such as: _____ cups = $\frac{1}{2}$ pint _____ pints = $1\frac{1}{2}$ quarts _____ quarts = 1 gallon _____ pints = 1 gallon _____ fluid ounces = $1\frac{1}{2}$ cups	Teacher observation; Student work sample
4	c,d	As a basis for comparison between metric and English units of length, provide two pieces of strong cotton string 60 feet each. Tie a 4-inch length of colored yarn every meter on one of the 60 feet strings and every yard on the other string. Use these non-standard instruments to measure a hallway, room, gym, drive way, etc., and compare the number of English units to metric units for the same item measured. Provide students with task cards having problems such as: It takes Juan 45 minutes to walk to school. If school begins at 7:30, what time must he leave home? It takes Jim 2.5 minutes to solve each problem. The teacher gives him 24 problems to solve beginning at 10:05 a.m. If the class is over at 11:00 a.m., will he have enough time to finish? Explain. Prepare an overhead of a thermometer (either Fahrenheit or Celsius) marked in this manner. _____ 20 _____ 15 ⇐ _____ 10 _____ 5 _____ 0 What is the temperature at the arrow?	Student work sample

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	f	Prepare a thermometer with intervals of 2 degrees. Example: _____ 30 ____ ____ ____ ____ _____ 20 ____ ____ ____ ____ _____ 10 ~~~~~~~~~~~~~~~ Have the student duplicate this on grid paper and place an arrow at 16 degrees. (Prepare thermometers or use commercial overhead thermometers marking them in intervals of 2, 5, and 10 degrees and alternate with Celsius and Fahrenheit).	Teacher observation; Student work sample
4	f	Provide copies of simple floor plans and have the students find the perimeter and area of different rooms.	Student work sample
4	g	Using an overhead projector and pattern block squares, illustrate an irregular shape (such as a cross) and have the students calculate the perimeter by counting only outside edges.	Student work sample
4	g	Use square shapes to make a rectangular array, e.g., 4 squares by 3 squares. Have a student find the perimeter (14 units). Now, using the concept of the yard inside the fence, have a student count the squares inside the outer edges (12). This is the **area**. Discuss formula Area = length x width or A = l x w.	Student work sample
5	g	Have students collect temperature data over a given time period and prepare a chart of the temperatures. Use the chart to create a bar graph.	Student work sample
5	a	Have the students design a bar graph about their favorite fast food restaurant.	Student work sample
5	a,c	Display a bag of M&M®s. Poll the students to determine their predictions as to the number of each color in the bag. Record their predictions and have the students graph the results on a pictograph. Have students count each color and record on a tally chart. Display and graph results. Have students compare estimate with actual.	Student work sample

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	a,c,d	With the students in smaller groups, provide them with spinners marked with equal red, green, and blue spaces. Student groups will predict how many times they will land on red if they spin 30 times. Have groups spin the spinner 30 times and record the results using a tally chart. Graph the results of each group's results. Repeat for the colors blue and green. Record and discuss the similarities.	Student work sample
4	a,c	Have the students in pairs discuss flipping a coin for heads or tails. Ask the following discussion questions: Would the coin be more likely heads or tails? If they toss it 50 times, will the coin come up heads almost ½ the time and tails almost ½ the time? Have the pairs toss the coin 50 times and record the results on a tally chart. Graph the class results on a bar graph or pictograph. Discuss the data. Ask discussion questions about the data such as: Does the data show a heads and tails frequency of almost 50% each? If they tossed the coin 100 times would the results generalize closer to 50 – 50 for heads or tails?	Student work sample
5	b,e	In order for students to realize that the same data can appear differently in a graph, prepare bar graphs of the same data but with different scales. Ask the students to explain how the graphs are the same (same data). Ask how they are different (different scales). Generate a discussion as to what makes the data look different and how this may cause the viewer to misinterpret the data. Have the students generalize the discussion in writing.	Student responses
5	c	Students working in groups are given a standard deck of 52 playing cards. They predict the number of times they will draw a black card, a diamond, king, etc., out of 26 draws. After making 26 draws (replacing the drawn card each time), record the results. (The probability of drawing a black card is $\frac{26}{52}$ as $\frac{1}{2}$ the deck are black – 13 clubs and 13 spades. Prompt a discussion asking if at least 13 of the 26 draws should be black since $\frac{1}{2}$ the deck is black).	Student work sample Student response
5	c	Discuss with students that probability is a ratio and since all fractions can be expressed as ratios then all probabilities must also be fractions. Probability = $\frac{\text{Desired outcome (What you want to happen)}}{\text{Possible outcomes (What can happen)}}$	Student work sample

FOURTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	c	With the students in small groups, provide them with spinners marked with equal red, green, blue spaces. Ask the following discussion questions: 1) Will you more likely land on one color than another? (no) 2) What is the probability of landing on red? ($\frac{1}{3}$) 3) What is the probability of landing on blue? ($\frac{1}{3}$) 4) What is the probability of landing on green? ($\frac{1}{3}$) 5) What is the probability of landing on red or blue? ($\frac{2}{3}$) 6) What is the probability of landing on red or green? ($\frac{2}{3}$) 7) What is the probability of landing on yellow: ($\frac{0}{3}$) 8) What is the probability of landing on red, blue, or green? ($\frac{3}{3}$)	Student responses
5	d	Have students use data sets consisting of one- and two-digit numbers to determine the mode, median, range and mean. .	Student work sample

FIFTH GRADE

The Fifth Grade competencies and objectives build upon kindergarten through fourth grade. Students should discover relationships of whole numbers, fractions, and decimals involving the four basic operations. Students should explore percents, classify polygons, and solve multi-step problems. Students should continue to use manipulatives and incorporate appropriate technology whenever possible. A variety of problem-solving strategies should be a major focus throughout this grade level. Also included are concepts of measurements, algebra and data interpretations.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability,** and the process strands: **problem solving, reasoning & proof, communication, connections, and representation.** The competencies may relate to one, many, or all the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather, the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

FIFTH GRADE

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Analyze relationships among numbers and the four basic operations. Compute fluently and make reasonable estimates.

a. Compare and order nine-digit whole numbers, decimals to the nearest thousandth, like and unlike fractions, and mixed numbers using >, <, and =. Explain rationale.
b. Read and write numbers in word, standard and expanded forms through hundred millions.
c. Use divisibility rules of 2, 3, 5, 6, and 10 to identify factors and multiples of whole numbers to 500.
d. Model and distinguish between prime and composite numbers, factors and common factors including Greatest Common Factor multiples and common multiples including Least Common Multiple.
e. Model and show relationships among common fractions, decimals, and percents (10%, 20%, 25%, and 50%).
f. Compute percentages (10%, 20%, 25%, and 50% of a number.
g. Model, identify, and write equivalent fractions including improper fractions and mixed numbers.
h. Multiply up to four-digit factors by two-digit factors to include whole and decimal numbers.
i. Divide by two-digit whole-number divisors with and without remainders.
j. Convert between mixed numbers and improper fractions.
k. Add, subtract, multiply, and divide fractions and mixed numbers.
l. Add, subtract, and multiply decimal numbers up to three decimal places and divide decimal numbers by two-digit whole number divisors.
m. Determine unit rate and unit price.
n. Incorporate appropriate technology and manipulatives to explore basic operations of whole numbers, fractions, mixed numbers, and decimals.
o. Solve multi-step word problems to include money using the four basic operations with computation and/or estimation.
p. Locate numbers including fractions, decimals and whole numbers on a number line.

ALGEBRA

2. Explain, analyze, and generate patterns, relationships, and functions using algebraic symbols, demonstrate an understanding of the properties of the basic operations, and analyze change in various contexts.

a. Determine the value of variables in addition, subtraction, multiplication, and division equations.
b. Construct, with a given rule, and complete input/output function tables using basic operations with decimals, fractions, and whole numbers.
c. Determine a given pattern and extend to an identified ordinal position.
d. Apply and explain the properties of basic operations using variables:
   - Zero property of multiplication;
   - Commutative property of addition and multiplication;
   - Associative property of addition and multiplication; and
   - Identity properties of addition and multiplication.
e. Apply and explain inverse operations of addition/subtraction and multiplication/division.

GEOMETRY

3. Develop mathematical arguments about geometric relationships and describe spatial relationships using coordinate geometry.

a. Draw, label, describe, classify, and identify points, lines (parallel, intersecting and perpendicular), line segments, and rays. Use appropriate symbols.
b. Identify, compare, and classify polygons to include congruent, regular, and irregular shapes.
c. Utilize appropriate terminology and manipulatives to classify two- and three-dimensional geometric figures using the attributes of faces, vertices, and edges.
d. Use transformations (rotations, reflections, and translations) of two-dimensional figures to tessellate on a plane.
e. Draw, label, and classify angles, quadrilaterals, and triangles based on their properties.
f. Create a simple design in the first quadrant of the coordinate plane, listing each ordered pair.

MEASUREMENT

4. Develop concepts and apply appropriate tools and techniques to determine units of measure.

a. Measure length to nearest millimeter in the metric system and one-eighth inch in the English system.
b. Determine and compare appropriate units for measurement of weight/mass, capacity, temperature, length, distance and volume, in English and metric systems and time in real life situations.

c. Use appropriate tools to measure weight, capacity, temperature, and volume in the English and metric systems.
d. Convert units within a given measurement system to include length, capacity, weight/mass, and volume.
e. Estimate measurements of various objects in English and metric systems.
f. Estimate the measure of angles and verify with a protractor.
g. Determine the perimeter and area of squares, rectangles, and triangles using formulas.
h. Determine the radius, diameter, and circumference of a circle using manipulatives. Use the relationship of circumference to diameter to determine *pi* (approximately 3.14).
i. Solve two-step problems using measurement tools and calculations (Example: measure the length of the sides of a figure to determine the perimeter/area).

DATA ANALYSIS & PROBABILITY

5. Collect, organize, interpret, analyze and display data. Apply basic concepts of probability.

a. Demonstrate finding the probability of simple events and express as fractions (ratios).
b. Collect, organize, construct and interpret data using bar graphs, line graphs, line plots, pictographs, tables, and charts (using a variety of scale units). Incorporate appropriate technology and manipulatives.
c. Interpret circle graphs using percentages.
d. Analyze a set of data to find the mean, median, mode, and range.

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a,b	Give groups of students nine index cards with a single digit written on each. (Digits may be used more than once.) Arrange the cards to form the greatest and least value. Use symbols > or < to compare numbers. Have the students write both numbers in standard and expanded form.	Teacher observation; Student work sample
1	a,b	Give groups of students three index cards with a single digit written on each and one index card with a decimal point. Arrange the cards to make the greatest and least decimal numbers possible. Use symbols > or < to compare numbers. Have the students write the displayed numbers in all three forms.	Teacher observation; Student work sample
1	a,e,p	Stretch a strong cord such as small diameter clothesline across the room. Prepare ten 5 x 9 index cards with whole numbers, fractions, and decimals written on them. Using paper clips or clothespins, place four or five cards on the line in correct order from least to greatest. Pass out the remaining cards to a group of students and have them pin or clip their card in the proper position on the line. The observing students should check the line and explain reasoning. As the school year progresses, mixed numbers and percentages can be added to the mix when using the number line.	Student observation; Student work sample
1	c	Explain the divisibility rules by first stating that "divisibility" means able to be divided without having a remainder. Example: Divisible by 2 means that a number can be divided by 2 without having a remainder. **These rules are helpful when reducing fractions and finding prime numbers.** Display these rules and test a variety of numbers to 500.	Teacher observation; Student response
1	c	Students should realize that multiples can be a result of "skipping counting". For example, when skip counting by 5s, (5, 10, 15, 20, 25, …) 15 is the 3^{rd} multiple of 5 as 25 is the 5^{th} multiple. Provide opportunities for students to find missing multiples, such as "What is the 7^{th} multiple of 3? (21)	Teacher observation; Student response

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	c,d	Provide a copy of a 100s chart to each student and prepare a "**Sieve of Eratosthenes**" to define the **prime** numbers between 1 and 100. Prime numbers have only 2 factors (1 and itself). A **composite** number has *more* than 2 factors. **To prepare the Sieve:** First, cross out 1 because it is neither prime nor composite. Circle 2, 3, 5, and 7. These are the prime numbers between 1 and 10. (**Note that 2 is the *only* <u>even</u> prime number**.) Using different colors, shade a portion of each square that contains multiples of 2, 3, 4, 5, 6, 7, 8, 9, 10. Circle all the numbers that are left. These circled numbers are the **prime** numbers between 1 and 100. Have the students write observations on what they discover. For example, all the multiples of 2 include the multiples of 4, 6, 8, and 10. All the multiples of 5 include the multiples of 10. Common multiples of 2 and 3 are also multiples of 6. The next step is to make the connection between this and the divisibility rules. Rule of 2 – The ones digit is 0, 2, 4, 6, 8 Rule of 3 – The sum of the digits is divisible by 3 Rule of 5 – The ones digit is 0 or 5 Rule of 6 – The number is divisible by 2 and 3 Rule of 10 – The ones digit is 0	Teacher observation; Student response
1	d	**"Factors are Few, Multiples are Many"** To find **factors** and **common** factors of numbers, use a chart like the example below. Use divisibility rules to determine factors. Start with 1 since all numbers have 1 and itself as factors. Check to see if 2 is a factor and so on. Ex: The factors of 24 are: 1,2,3,4,6,8,12,24 The factor of 32 are: 1,2,4,8,16,32 Stress to students that there are only a certain number of factors for a given number – Factors are **FEW**. All factors of 24 and 32 are listed. Common factors between 24 and 32 are: 1,2,4, and 8. The **Greatest Common Factor (GCF)** is 8.	Teacher observation; Student response
1	f	Provide student groups with a menu. Have them select a meal and compute mentally a 15% tip and a 20% tip.	Student response
1	f	Provide student groups with sales flyers showing a "percent off" and have them compute final costs.	Student work sample

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	g	Use overhead and student sets of fractions strips to model commonly used fractions such as $\frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \frac{1}{3}$, etc. Stack the fraction strips to show equivalent fractions.	Student response
1	g	Use fraction "pizzas" or circles to model equivalent fractions. Have student write their relationships, e.g., $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=\frac{6}{12}$.	Student response
1	i	Have them use grid paper or turn the lined paper vertically when doing division problems.	Teacher observation
1	i	Provide students with division problems such as: "An egg carton holds $1\frac{1}{2}$ dozen eggs. Today, Miguel gathered 79 eggs. How many cartons will he need to crate <u>all</u> the eggs?" Explain answer. 79÷18 = 4 r7 **5 cartons**, 4 full and one with 7 in it.	Student work sample
1	j	Use pattern blocks to demonstrate fractions and mixed numbers.	Student work sample
1	k	Use fraction strips to demonstrate addition and subtraction of like and unlike fractions.	Student work sample
1	k,n	Use pattern blocks to show students how to add and subtract like and unlike fractions and mixed numerators. Use fraction calculators such as TI – Explorer to verify results. These calculators will also simplify fractions and mixed numbers.	Student work sample
1	k,n	Shade the appropriate regions of grid paper to represent the addition and subtraction of decimal numbers.	Teacher observation: Student work sample
1	l,m	Using store advertisements, students will determine the best buy when given the price and package quantity of items. For example: Is a 16-ounce box of cereal @ $3.29 a better buy than a 24-ounce box for $4.19?	Student work sample

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	l,n	Prepare index cards with decimal numbers. Have the students select two index cards. Using the calculator, find the product and record. Select a second card and find the product using the calculator. Continue in this matter until the rule for multiplying decimal numbers is discovered. The product will have the same number of decimal places as the two factors combined. Discuss products that end in zero as most calculators will simplify the product by dropping the zero.	Student response
1	l,n	Students will be given problems with decimal dividends that will not result in a remainder. They will solve them using the calculator. They should continue until the discovery is made that the quotient has the same number of decimal places as the dividend.	Student; response; Teacher observation
1	m	Present students with problems such as the following: • A pound box of raisins costs $4.80. What is the cost per ounce? • A dozen eggs cost $1.08. What is the cost per egg? • A set of 4 tires costs $222.00. What is the cost per tire? • A gross (144) of pencils costs $21.60. What is the per pencil? • A 48-piece box of candy costs $4.80 per box. What is the cost per piece? • At what rate was an employee filling cartons if he filled 30 cartons in 1 hour? • At what average rate of speed was a car traveling if it went 240 miles in 4 hours?	Student work sample
1	n	With students in groups, give each group a catalog or sales flyer. Have them make a list of things (4-5) they would like for their birthday. Give the students an order form to complete. When completed, give them problems to solve: 1. "You have $50.00 to spend for your birthday. Does your list exceed this amount? If yes, which item(s) will you drop?" 2. "If your items do not exceed $50.00, which additional item(s) will you pick and not exceed $50.00?" 3. "What is your final total?"	Student response; Student work sample
1	n	Have students in groups prepare problems requiring the use of 1 of the 4 basic operations per problems ad give to another member in the group to solve. Work is checked with a calculator.	Student work sample

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	n,o	Have the groups plan the snack food for a party with 18 people attending. Establish a budget for the party of approximately $2.00 per person or about $40.00. Use food ads or a menu. Have the groups compare their menus for the party and discuss.	Student response; Student work sample
1	n,o	Students will spend $500.00 by placing orders from catalogs. Estimate the total order first. Add to get the exact total (check addition with a calculator). Compare the results of estimation with the exact total. Discuss the results: What estimation method was used? Did one method give a closer estimate? Was the actual total over or under the $500.00? How could a closer estimate be obtained?	Student response; Student work sample
1	n,o	Provide groups of students with a grocery ad. Have them select items for 7 days for a family of four, breakfast, lunch, and dinner. Instruct them to run a total on the groceries and add 10% for tax. Check the total with a calculator.	Student work sample
1	o	Students will solve word problems that require division with a remainder. They will decide the necessity of either dropping the remainder, use it as part of the answer, or use to round the quotient up by 1.	Student response; Student work sample
1	o	Students will prepare word problems, one each of addition, subtraction, multiplication, division, and estimation. The teacher will check the problems and place them into a “problem bank”. Students will draw a problem each day or week and solve.	Student work sample
2	a,b,e	Provide students with a blank input/output function table. Have them draw a rule card that will determine the output. The rule cards should be with addition, subtraction, multiplication, and division. Pass out cards with input values to include whole numbers, fractions, and decimals. Students will record the input values and compute the output by applying the rule. As a variation, cards can be prepared with some input values and some output values. This will require the student to compute the missing values using the inverse operation.	Student work sample
2	a,c	The teacher will prepare number sequence cards for half of the class. The other half will have cards prepared that show the rule that explains the sequence. An example of this would be 2, 4, 8, 16. The rule is double each number. Have students match the rule with the corresponding sequence card.	Student response

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	a,e	Use manipulatives to model balancing equations. For example, if there are 15 items on the right side of the equal sign and 8 on the left side of the equal sign, how many more are needed on the left to balance with the right side? Have students justify answer. Ex: 15 = 8 + n; therefore, 15 – 8 = n.	Teacher demonstration
2	c	Place the students in groups and present them with the beginning 7 values in the Fibonacci sequence – 0, 1, 1, 2, 3, 5, 8 …. Have students discuss the sequence and find the rule that explains each successive number.	Student response
2	c	Display the first 4 triangular numbers in diagram form: 1 2 3 4 . . . . **(1) (3) (6) (10)** Have them describe thepattern that determines the pattern of dots in each successive number. (+2, +3, +4, etc.) From this, predict and compute the number of dots in the n^{th} triangular number.	Student response; Student work sample
2	d	A rectangular array can be used to demonstrate the distributive property. Construct a rectangular array to represent 6 x 7; 6 down, 7 across. Separate this into two arrays of 6 down, 3 across and 4 across. 7 3 + 4 ####### ### #### ####### ### #### 6 ####### 6 ### #### ####### ### #### ####### ### #### ####### ### #### This shows that 6 x 7 = 6 · 3 + 6 · 4.	Teacher observation; Student response
3	a,b	Cards will be prepared with representations of points, lines (parallel, intersecting, and perpendicular), segments, rays, and different polygons from real life. Some examples might include a beam from a flash light (ray), stop sign (octagon), baseball diamond (rhombus), street intersection (intersecting lines) to name a few. Students will tell the name for the representation and give characteristics that support their answer. In the case of line, segment, ray, point, and plane, they will give the appropriate symbol.	Student response

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	a,b,e	Students can go on a "Geometry Scavenger Hunt" looking for any representation of various types of segments, angles, shapes, etc. This may occur in the classroom, inside the school building, or both indoors and outdoors. They will list the representation found and its description including attributes. Extension: A digital camera can be used to photograph the examples and compiled into a "Book of Geometry".	Teacher observation; Student work sample
3	b	Have students use centimeter grid paper to draw congruent polygons . Draw examples of congruency using irregular shapes.	Teacher observation; Student work sample
3	b	Use pattern blocks to demonstrate some common shapes, i.e., hexagon, equilateral triangle, square, rectangle, rhombus, and trapezoid. Have students name the shapes and list their attributes.	Student response
3	b,e	Display a square, rectangle, parallelogram, rhombus, and trapezoid. Have the students identify them as quadrilaterals. Working in groups, the students will discuss the shapes and list attributes. Using this list of attributes, they will determine which shapes can share a name and justify in writing. (For example, a square is always a rectangle, but a rectangle is not always a square.)	Student work sample
3	c	Students can use plastic drinking straws and string threaded through them to tie them together and make skeletal models of polyhedra such as tetrahedrons. After constructing, they will list the faces, vertices, and edges.	Student response
3	c	Using examples of polyhedra from magazines, newspapers and/or photos, students will make a collage of solid shapes. For example: cereal box (rectangular prism), Egyptian pyramid (square pyramid), soup can (cylinder), basketball (sphere), etc.	Teacher observation; Student work sample
3	c	Use three-dimensional shapes to identify faces, edges, and vertices. Complete a chart to show the number of faces, vertices, and edges. Extension: Students can determine the relationship among number of faces, edges, and vertices (Euler's Formula).	Student response
3	d	Have students use pattern blocks to explore rotations reflections, and translations. Trace around their shapes on grid paper and perform one of the transformations and draw the shape again. Label each transformation as it is drawn. Repeat the process for each of the transformations.	Teacher observation; Student work sample

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	d	Use geoboards and colored rubber bands to demonstrate transformations. The student will use one color band to represent the original figure and another color to represent the transformation specified by the teacher.	Student response
3	e	Use a protractor to measure angles and determine types of triangles.	Student work sample
3	f	Provide students with a copy of the first quadrant of the coordinate plane. Design a simple figure and list the coordinates.	Student work sample
4	a,b,c	Measure the dimensions and weight of several objects and the capacity or volumes of various containers. Express each measurement using units of measure in both the metric and English system of measurement.	Student work sample
4	a,b,c,e	Place students in groups and have them consider objects such as width of the math textbook and the length of the classroom. Have students determine the best unit of measure to use in metric and in the English system. Have students measure the textbook to the nearest centimeter and one-eighth of an inch and the length of the classroom to the nearest meter and yard.	Student response
4	b	Have students solve problems with time using calendars or clock time. Examples: 1. Calculate the date of the 3rd Tuesday from today's date. 2. A movie begins at 7:45 p.m. and lasts 2 hours and 10 minutes. If it takes 50 minutes to get home and Sam has to be home by 11:00 p.m., does he have enough time to see the movie?	Student work sample
4	b	Place a thermometer that shows the temperature in Celsius and Fahrenheit outside the classroom in sight from a widow. At a given time, have a student read the thermometer and record both the Celsius and Fahrenheit temperature. Have the students note the difference between Celsius and Fahrenheit. Explain that Celsius is a metric measurement and is based on a 100-degree difference between the freezing point and the boiling point (100°) of water. Fahrenheit is based on freezing at 32° and the boiling point at 180°. Display the conversion formulas and have students convert Celsius and Fahrenheit temperatures. $C = \frac{5(F - 32)}{9}$ $F = (9/5c) + 32$	Student work sample

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	c,g	Use trundle wheels displaying the measurements in metric and English units to measure the perimeter of the classroom, hall, etc. Students can derive a formula for the perimeter of a rectangular shape, i.e., P = 2*l* + 2*w* where *l* = length and *w* = width	Student work sample
4	f	In order to estimate angles, have the students fold a square post-it™ note diagonally. This forms a 45, 45, 90 right isosceles triangle. Have them label the right angle 90° and the other angles 45°. They can use these benchmarks to estimate angles. An angle that is about $\frac{1}{2}$ of 45° would be about 22°, $\frac{1}{2}$ of this about 11°. Half way between 45 and 90 would be about 67°. From a teacher-made worksheet, have the students estimate angle measurements. Check estimates with a protractor.	Student work sample
4	g	Display of formulas: Area of rectangle = length x width (A = *l* x *w*) Area of triangle = $\frac{1}{2}$ base x height (A = $\frac{b \times h}{2}$) Place students into groups and have them use geoboards and bands to verify the formulas. Describe a 2 x 4 rectangle with one color rubber band. Note that its area is a 2 x 4 or 8 sq. units. With another color band, divide the rectangle diagonally. Note that the rectangle is separated into two congruent triangles. If the two triangles are the same size, they must have the same area. If the area of the rectangle is 8, then the area of one triangle must be 4. The triangle's base is 2 units and the height is 4 units. A = $\frac{1}{2}$ b x h so $\frac{1}{2}$ (2 x 4) = 4.	Student work sample
4	g	Provide the students with centimeter grid paper. Have them create a figure with a perimeter of a given number of units.	Student work sample
4	h	With the students in groups, give each group a round object (jars, cans, etc.) and a flat ribbon. Have the groups measure the diameter of their round item being careful to measure across the widest point of the circumference. Instruct them to multiply this number by 3.14. (Explain that 3.14 is the approximate value for *pi*. *Pi* is the term for the ratio of the circumference to the diameter of a circle.) Have them record the product obtained. Using the flat ribbon, wrap it around the item and carefully mark the distance around. Measure this length and the results should approximately be the product of pi times the diameter. Provide practice by supplying a list of diameters and have the students calculate the circumference.	Student response; Teacher observation

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	i	Provide students with grid paper and problems with missing measures: Example: 1. Length = 6 units Perimeter = 20 units 2. Width = 5 units Area = 25 square units Instruct the students to create the rectangles on the grid paper and find the area in #1 and the perimeter in #2.	Student work sample
5	a	Display a standard number cube numbered 1 – 6. Ask the students to predict the probability of tossing a 2, 3, 4, 5, 6. After the students have predicted for all numbers, lead them into setting up a probability chart.	Teacher observation Student work sample
5	a	Probability will be explained using coins. When a single coin is tossed, there are two possible results (heads or tails). There is a 1 out of 2 (1/2 or 50%) chance of tossing a head or tail. This is the theoretical probability. Probability = $\frac{\text{favorable outcome}}{\text{possible outcomes}}$ In groups, discuss tossing a coin. What is the probability of heads occurring? Of tails occurring? What percentage is this? If a coin was tossed 10 times, predict the number of heads and tails that would occur. Have groups toss a coin 25 times and record the number of heads and tails in chart form. Express each as fractions and percents. Do the events meet the probability 50% for each? Repeat the process for 25 more tosses. Record results in the chart. Does the average probability come closer to meeting the theoretical probability? Measure and record heights of students in the class. Organize and chart the information. Prepare a line plot (a diagram showing frequency of data on a number line) to graphically show the information. Analyze the line plot to determine gaps and clusters, if any, in the data. Repeat twice more with 25 tosses each time and record the results after each 25 tosses. (100 tosses total) The average should approach 50 each for heads and tails. Ask the students to predict the number of heads and tails after 1000 tosses.	Teacher observation Student response Student work sample

FIFTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	b	Measure and record heights of students in class. Organize and chart the information. Prepare a line plot to determine gaps and clusters, if any, in the data.	Student work sample
5	b	Students will survey 50 students in their grade level to determine their favorite fast food restaurant. A tally chart will be used to record results of the survey. They will construct a frequency table for the information in the tally chart. Utilizing word processing and spreadsheet programs, students will present the data in at least two formats (table, bar graph, pictograph, etc.). Additionally, students will compose a paragraph explaining the results of the data collection and display.	Rubric
5	b,c	Supply students with different types of data. Ask students to tell which type of graph would be best for each data set. ❑ Bar graph/pictograph – compare data side by side ❑ Line graph – compare data over time	Student response
5	d	Provide students with a catalog. Students will be "given" $100.00 to shop. After deciding what will be purchased and the amount to be "spent" on each item, they will construct a circle graph. The information will be shown using percentage. (Example: $25.00 spent for a shirt will be shown as $\frac{1}{4}$ the circle as it is 25%.) Divisions on the circle may be approximations if the dollar amount is an "odd" amount.	Student work sample
5	e	Students will compute the mean (average) of their last 6 daily grades	Student work sample

SIXTH GRADE

The Sixth Grade mathematics framework continues the emphasis on the study of rational numbers greater than and equal to zero. Students should expand their study of percents by applying this knowledge to everyday problems such as sales tax and discounts. Also included are concepts of geometry, data analysis, algebra, measurement, and number theory. The sixth grade framework incorporates the use of technology to assist students in problem solving decisions that relate to real-world situations.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands: **problem solving, reasoning & proof, communication, connections and representation.** The competencies may relate to one, many, or all the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather, the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

SIXTH GRADE

CONTENT STRANDS:

Number and *Operations* — *Algebra*
Geometry — *Measurement*
Data Analysis & Probability

COMPETENCIES and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Analyze numbers using place value, prime factorization, and exponents. Solve problems involving basic operations of rational numbers greater than or equal to zero.

a. Read, write, and round decimal numbers to the nearest ten-thousandths.
b. Compare and order rational numbers using =, <, and >.
c. Use patterns to develop and illustrate the concept of exponents.
d. Write and compare whole numbers in exponential form.
e. Determine the Greatest Common Factor (GCF) and Least Common Multiple (LCM) of a given set of numbers.
f. Solve problems using order of operations with whole numbers, including parentheses and exponents.
g. Divide whole or decimal dividends by whole or decimal divisors in problem solving situations and explain the meaning of both the quotient and the remainder in terms of the problem.
h. Estimate and solve one- and two-step problems involving addition, subtraction, multiplication, and division of rational numbers in real-life situations.
i. Add, subtract, multiply, and divide fractions and mixed numbers. Express answers in simplest form.
j. Use models to illustrate percent.
k. Convert among fractions, decimals, and percents.
l. Identify and demonstrate the following:
    - Zero property of multiplication;
    - Inverse operations of addition, subtraction, multiplication, and division;
    - Commutative and associative properties of addition and multiplication;
    - Identity properties of addition and multiplication; and
    - Distributive property of multiplication.

ALGEBRA

2. Use algebraic functions, patterns, and language across a variety of context.

a. Solve one-step equations involving addition and subtraction with non-negative rational numbers and solutions.
b. Use patterns and sequences to investigate and draw conclusions.
c. Complete a function table based on a given rule using non-negative rational numbers.

GEOMETRY

3. Analyze geometric relationships of lines, angles, two- and three-dimensional shapes, and transformations.

a. Construct and classify parallel and perpendicular lines.
b. Construct, classify and measure angles.
c. Construct polygons, circles, and semi-circles from written or oral descriptions.
d. Compare, classify, and construct transformations (reflections, translations, and rotations).
e. Construct three-dimensional figures using manipulative materials. Compare and contrast vertices, faces and edges of objects.

MEASUREMENT

4. Apply geometric formulas and standard (English and Metric) units of measurement in mathematical and real-life situations.

a. Measure length to the nearest one-sixteenth inch and nearest millimeter.
b. Select and apply appropriate units for measuring length, mass, volume, and temperature in the standard (English and metric) systems.
c. Convert units within a given measurement system.
d. Use estimation to solve problems in the standard (English and metric) systems.
e. Calculate the perimeter and area of parallelograms and triangles.
f. Calculate the circumference and area of a circle with and without manipulatives.
g. Predict, calculate and compare the volume of cubes and rectangular prisms.

DATA ANALYSIS/PROBABILITY

5. Collect, organize, interpret, analyze, and display data. Apply concepts of probability to solve problems.

a. Construct and explain a frequency table.
b. Construct and interpret line graphs, bar graphs and histograms.
c. Interpret circle graphs and explore construction of circle graphs.
d. Determine how changes in data affect mean, median, mode, and range.
e. Determine sample space of a given situation and use it to determine the probability of a simple or compound event.

SIXTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Have students draw blank spaces on a sheet of paper, placing a decimal point as directed by their teacher. Randomly give digits to be placed in blanks. Round numbers to the placevalue indicated.	Teacher observation; Discussion
1	a	Have students write down the decimal quotient and place an arrow under the place value to be rounded to help the student round to the appointed place value.	Teacher observation; Student work sample
1	b	Use index cards with rational numbers greater than or equal to zero. Students will draw two or more cards at random and compare numbers using =, < and >.	Teacher observation; Student response; Teacher Test
1	b	Students will record numbers as instructed on index cards. The teacher will call on groups of two to five students . Students will arrange themselves in the correct order and display their cards for the class to see.	Teacher observation; Student response
1	c	Students will expand on the idea that multiplication is repeated addition and exponentiation as repeated multiplication (with and without calculators). Students will look at various powers of single digit numbers and notice how powers of even numbers are always even and powers of odd numbers are always odd.	Teacher observation; Student response; Teacher Test
1	c,d	Have students use repeated division to demonstrate that large numbers can often be represented as powers of smaller numbers, for example, $125 = 5^3$ or that $64 = 4^3$.	Teacher observation; Student response; Teacher Test
1	e	Have students use a hundred chart to find the LCM of numbers. For example, to find the LCM for 4, 6 and 8, use a red pen to cross out multiples of 4, a blue pen to cross out multiples of 6, and a green pen to cross out multiples of 8. Numbers crossed out with all three colors represent common multiples. The least of these numbers is the LCM.	Teacher observation; Student work sample
1	e	Have students, with and without calculators, use repeated division to determine all of the prime factors of two or more given numbers. Have students circle common prime factors in all the numbers. The circled number(s) represent the GCF.	Teacher observation; Student work sample; Teacher test
1	f	Provide students with expressions to simplify using the order of operations and properties of mathematics. Have students justify each step.	Teacher observation; Student response

SIXTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	g	Given a problem to solve involving division with both whole number and decimal quotients and divisors, ask the student to explain the meaning of the quotient and remainder in context of the problem. Example: If 14 people are all going to the movies and each car can hold, at most, 4 people, how many cars are needed for everyone to get to the movies? 14 divided by 4 = 3 remainder 2. From this answer the student should see that the quotient of 3 indicates the number of full cars needed and that 2 people (remainder) are left. Thus, 4 cars are needed.	Teacher observation; Student work sample; Teacher made test
1	h	Use prices from local stores to produce real world problems about shopping at a store involving one and two-step problems with addition, subtraction, multiplication and division. Have students begin with a specific amount of money and estimate what they may be able to purchase from the list.	Teacher observation; Student work sample
1	h	Use recipes to have students estimate, solve and compare solutions to problems involving one and two-step fraction problems.	Student work sample
1	i	Construct three decks of cards, two with numbers in either fraction or mixed number form, and the third consisting of operators (+, x). Have students pick a card from each pile and compute the final result in simplest form.	Teacher observation; Student work sample
1	j	Using graph paper, have students shade in appropriate numbers of squares to represent different percentages of a given amount of squares.	Student work sample; Teacher-made test
1	k	Using graph paper, have students shade in appropriate numbers of squares to represent different fractions, decimal amounts and percentages of a given amount of squares. Have the students provide answers in all three forms.	Student work sample; Teacher-made test
1	k	Given a partially completed chart of fractions, decimals, and percents, students will complete the chart.	Student work sample; Teacher made-test
1	l	Divide the class into two teams. On an overhead, write problems that can be solved easier by using properties. For example: (25 x 6 x 4) = (25 x 4) x 6. One person from each team races to get the correct answer. Have students explain the use of properties with each problem.	Teacher observation

SIXTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	a	Use manipulatives such as Algebra Tiles™ to model the process of solving equations.	Teacher observation; Discussion
2	b	Provide sets of numbers or geometric shapes that are in a sequence. Determine numbers or shapes that will continue the sequence by determining a rule to explain the pattern.	Teacher observation; Discussion; Student Work Sample; Teacher Made Test
2	c	The teacher writes down a rule such as 2x + 5 and covers it up. Have students give the teacher a number to apply to the rule. The teacher gives an answer. Have students guess the rule.	Teacher observation
3	a	Have students use string or yarn to construct "line segments" across the room to represent parallel and perpendicular positioning.	Teacher observation; Discussion
3	b	Slit two paper plates along a radius. Use the slit to overlap the paper plates. Have students create given angles by turning the plates. Students should check the measure of each angle using protractors.	Teacher observation; Discussion
3	c	Divide the class into groups of two. One student reads a polygon's description and the other student attempts to draw the correct figure based on what he/she heard. Teams discuss and then rotate roles.	Teacher observation; Discussion
3	c	Fold paper plates to locate the center of the circle. Cut the plate in half along a diameter to form semi-circles, noting that only by cutting through the center of circle are semi-circles formed.	Teacher observation
3	d	Have students use cut out shapes and graph paper to investigate transformations by telling the students where to place two congruent figures. The students should then by telling what type of transformation is used to move one shape on top of the other.	Teacher observation; Discussion; Teacher Made Test
3	e	Using straws and paper clips have students construct various three-dimensional shapes. Cut out paper to form faces and attach to the vertices and edges.	Teacher observation; Student Work Sample
4	a,c	Using appropriate instruments, measure various objects in and around the school to the nearest sixteenth of an inch and the nearest millimeter.	Teacher observation; Student work sample

SIXTH GRADE

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>4</td><td>b</td><td>Identify appropriate units of measurement and tools for measuring items such as water in a glass, mass of a book, or weight of a penny.</td><td>Teacher observation; Discussion</td></tr>
<tr><td>4</td><td>b,c,d</td><td>Have students determine appropriate units of measurement for various items, such as volume of a box, length of a paper clip, the distance to their home, etc.</td><td>Teacher observation; Discussion;</td></tr>
<tr><td>4</td><td>b,c,d</td><td>Have students estimate using appropriate units of length, mass, and volume of various objects. Have students estimate the temperature inside and outside the classroom. After estimation, have students measure with an appropriate instrument to check the estimation.</td><td>Teacher observation; Student work sample</td></tr>
<tr><td>4</td><td>d,e</td><td>Given dimensions of a wall, students will calculate the area to determine the amount of paint needed to cover the wall. Given the dimensions of a picture, students will determine the size of the frame needed to go around the painting.</td><td>Teacher observation; Student work sample; Discussion</td></tr>
<tr><td>4</td><td>e</td><td>Have students build as many rectangles as possible using square tiles that have the same perimeter. Students should be able to discover that rectangles with the same perimeter could produce different areas. Thus, the generalization is that perimeter and area are not related.</td><td>Teacher observation; Student work sample; Discussion</td></tr>
<tr><td>4</td><td>e</td><td>Use grid paper to demonstrate the different ways that a number can be modeled as a rectangle. Make a table from the different lengths and widths that produce the given number. Discuss that different factors will produce the same product.
Example:
<table><tr><td>4</td><td>1x4, 2x2, 4x1</td></tr><tr><td>6</td><td>1x6, 2x3, 3x2, 6x1</td></tr><tr><td>8</td><td>1x8, 2x4, 4x2, 8x1</td></tr></table></td><td>Teacher observation; Student work sample</td></tr>
<tr><td>4</td><td>f</td><td>Have students use string and ruler or measuring tape to measure the circumference and diameter of several circular objects. The diameter can be measured by placing the round object on a sheet of paper and tracing it. Cut out the circle, and fold it to get increased accuracy of the diameter. Using a calculator, divide the circumference by the diameter. Discuss results from all of the objects to discover the meaning of pi. (All of the student's findings of pi could be examined using mean median and mode)</td><td>Teacher observation; Student work sample; Discussion</td></tr>
<tr><td>4</td><td>g</td><td>Using boxes, have students predict the volume of the box, then measure the dimensions of the box to calculate the volume. Have the students perform the same task with the classroom using various units of measurement.</td><td>Teacher observation; Discussion</td></tr>
</table>

SIXTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	a,b	Use student heights to construct a frequency table. From a frequency table have the students construct histograms or bar graphs.	Teacher observation; Discussion
5	a,b	Use a bag of candy such as M&Ms® or Skittles® and separate the candy by color to construct a frequency table and bar graph.	Teacher observation; Discussion; Student work sample
5	c	Using a bag of candy and a paper plate, have the students arrange the candy along the edge of the plate by color. Have the students draw marks on the plate where one color begins and the other ends. Remove the candy and write on the plate where the colors had been. Fold the plate to determine the center of the plate. Use a ruler to draw the various radii that connect the center to the marks that separated the colored regions. Use this information to introduce the idea of constructing circle graphs and to explain the meaning of circle graphs.	Teacher observation; Student work sample
5	d	Provide students with a data set to compute mean, median, and mode. Add another data point or change one of the values re-compute the mean, median, and mode. Discuss the effect the additional value or changed value has on the measures of central tendency.	Teacher observation; Discussion
5	d	Have students develop a set of data for a predetermined mean, median, or mode.	Teacher observation; Discussion;
5	e	Provide students with two collections of objects that are distinct. Have the students make all possible combinations of the two collections, i.e., with dice that are red, blue, and green, and with tiles that are yellow and white. Have students count and list all possible outcomes (sample space).	Teacher observation; Discussion; Student work sample
5	e	Use a tree diagram to illustrate all possible outcomes for tossing a coin three times. Have students list all possible outcomes, and determine the probability of each event.	Teacher observation: Discussion; Teacher-made test

SEVENTH GRADE

The Seventh Grade mathematics framework features an in-depth, integrated preparation for algebra and geometry. Concepts include number sense, measurement, data interpretation, geometry, integers, algebraic concepts, probability, proportions, and percents. A variety of problem-solving techniques, real-world applications, and technology should be used when applying these concepts. This course is designed to prepare students for Pre-Algebra.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies may relate to one, many, or all the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

SEVENTH GRADE

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Apply concepts of rational numbers and perform basic operations emphasizing the concepts of ratio, proportion, and percent. Implement concepts with and without the use of calculators.

a. Identify and use the commutative, associative, distributive, and identity properties with and without variables.
b. Use the order of operations to simplify and/or evaluate numerical expressions (including exponents and grouping symbols).
c. Solve real-life problems involving addition, subtraction, multiplication, and division of rational numbers (whole numbers, fractions, decimals, and mixed numbers).
d. Convert among decimals, fractions, mixed numbers, and percents.
e. Solve problems using decimals, fractions, and/or percents.
f. Convert between standard form and scientific notation.
g. Use the inverse relationship to develop the concept of roots and perfect squares.
h. Find greatest common factor (GCF) and least common multiple (LCM).
i. Explore equivalent ratios and express in simplest form.
j. Solve problems involving proportions and scale drawings.
k. Calculate and apply unit rates to real-life situations.
l. Use proportions and equations to solve problems with rate, base and part.
m. Solve real-life problems involving sales tax, discount, and simple interest.
n. Determine percent of increase and decrease.
o. Model and write integers including opposites and absolute value .
p. Compare and order integers and apply integers to real life situations (for example: temperature).
q. Illustrate addition and subtraction of integers with and without the use manipulatives.

ALGEBRA

2. Develop and apply the basic operations of non-negative rational numbers with non-negative solutions. Create and apply algebraic expressions and equations.

a. Recognize, describe, and state the rule of generalized numerical and geometric patterns using tables, graphs, words and symbols.
b. Translate between simple algebraic expressions and verbal phrases.
c. Write and solve one-step equations that represent real-world problems using the properties of equality.
d. Formulate an original real-world situation from a given equation.
e. Complete a function table based on a given rule and vice versa.

GEOMETRY

3. Apply geometric relationships of angles, two- and thee-dimensional shapes and transformations.

a. Classify and compare polygons using their properties (including parallel sides, congruent sides, and angles).
b. Identify three-dimensional figures and describe their faces, vertices and edges from a two-dimensional drawing (nets).
c. Determine if figures are congruent or similar.
d. Perform transformations on two-dimensional figures using the coordinate plane.
e. Use manipulatives to analyze the relationship between the lengths of the legs of a right triangle to the length of the hypotenuse to verify the Pythagorean theorem.
f. Use the Pythagorean theorem to determine if a triangle is a right triangle.
g. Define and determine complementary and supplementary angles.
h. Identify *x*-axis, *y*-axis, origin, and quadrants of the coordinate plane.
i. Graph ordered pairs, which when connected will create a given geometric shape on a coordinate plane.

MEASUREMENT

4. Apply appropriate techniques, tools, and formulas to determine measurements with a focus on real-world problems.

a. Convert, perform basic operations, and solve word problems using standard (English and metric) measurements.
b. Measure the dimensions of given items using standard (English and metric) measurements.
c. Find the perimeter and area of polygons and circumference, area, radius, and diameter of circles.
d. Use geometric formulas for cylinders and prisms to solve a variety of mathematical and real-world problems involving volume and surface area.

DATA ANALYSIS & PROBABILITY

5. Collect, organize, interpret and display data. Analyze data to make predictions.

a. Interpret and construct circle graphs from real-world data.
b. Interpret and construct frequency tables, bar graphs, line graphs, histograms and stem-and-leaf plots from real-world data.
c. Interpret data and make predictions from statistical graphs.
d. Determine probabilities of independent and dependent events and express in an appropriate form (such as ratios, decimals and percents).
e. Determine the probability of simple and compound events through experimentation, simulation or calculation.
f. Determine how outliers affect mean, median, mode, or range.
g. Use various methods (such as organized lists, tree-diagrams and the Fundamental Counting Principle) to solve probability problems.

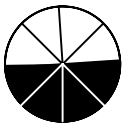
SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Visit NCTM's Illuminations website: http://illuminations.nctm.org/index.asp and see the lesson – "Algebraic Transformations: Investigating the Identity, Inverse, Commutative, and Associative Property using a geometric model".	Teacher observation; Student work samples
1	a	Create one set of cards with the property names written on them and another set with an example of a property. Have students play "Memory®" by matching the property with example.	Teacher observation; Test
1 4	a c	On graph paper, draw rectangles with the dimensions 4 x 7 and 4 x 5. Find and record the number of squares (area) in each rectangle. Put the two rectangles end to end so the sides of equal length touch. What is the length and width of the new rectangle? [4 x 12] Find the number of squares (area) in the new rectangle. How does the number relate to the numbers for the two smaller rectangles? [They are the same.] This illustrates the distributive property: 4 x 7 + 4 x 5 = 4(7 + 5).	Student work sample; Teacher observation
1	b	Present students with an order of operations problem which has been worked showing all steps. Have students determine if the answer is correct and explain why it is or isn't correct. If there is an error, identify the error and correct it.	Teacher observation; Student work samples; test
1	c,e	Using recipes, double, triple and quadruple the ingredients. Calculate ingredients needed to serve the class.	Student work sample; Teacher observation
1	c,e	Set up a display of several priced items. Set a spending limit of $20. Mentally determine if there is enough money to purchase sets of items. Verify answers with calculators.	Student work sample; Teacher observation; Written response; test
1 4	c a	The followings may help students remember the metric chart "**K**ing **H**ector **D**ied **M**onday; **D**on't **C**all **M**e," or "**K**ing **H**enry **D**ied **M**onday **D**rinking **C**hocolate **M**ilk". Liters or grams can be used as base units.	

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1 4	c,a	Provide the student with a chart of equivalent measures. Show students how to use cancellation to find answers. **Example:** Convert 5 gal. to quarts. $5\text{ gal} \bullet \frac{4\text{qt}}{1\text{gal}} = 5 \times 4\text{qt} = 20\text{qt}$ Explain to students that whatever unit they begin with must be the unit in the denominator of the second fraction and the numerator is its equivalent measure. Multiply numerators. If a number is greater than 1 in the denominator, divide by that number. Use in real-world situations. Use unlike measures within a problem; i.e., a problem is in inches and solution must be in feet.	Teacher observation; Student work samples
1	c,e	After teaching various estimation methods such as rounding, compatible numbers, and front-end estimation, use newspaper ads and have students choose five items. Estimate the cost of the items using different estimation methods. Calculate the exact cost of the items. Compare the estimation answers to the actual. Determine which estimation method was closer to the actual. Discuss when to use each method.	Presentation; Student work samples; Teacher observation
1 4	c,j,a	Convert by solving a proportion. 5 gal = ____qt. $\frac{5\text{ gal}}{n\text{ qt}} = \frac{1\text{ gal}}{4\text{ qt}}$	Student work samples; test
1 4	c,j,k,a	Have the students work in pairs or small groups. Give each group a map, plan a trip and select the kind of car to be used. The teacher will list several makes and models along with a list of miles per gallon of gas each car will hold. The students will estimate the total number of miles they will travel and estimate the cost of the gas. Have students find the exact travel distance and calculate the exact cost of gas. A calculator may be used.	Presentation; Student work samples; Written response; Rubric
1 4	c,j,k,m,a	Have students work in pairs or small groups to estimate the cost of a multi-day vacation. Students will calculate the mileage, gas needed, hotel costs, food costs, entertainment, etc. Use the computer to determine the correct tax amounts, hotel rates, etc.	Presentation; Student work samples; Written response; Rubric
1	c,k	Set up a class store with produce , meat , and canned good sections. Have students bring pictures of items that are found in these sections. The students will cut out the pictures and glue them to construction paper. Provide prices by each item, such as, bananas cost 4/$1.00, apples cost 3/$0.89, etc. Set up the items in centers. Have the students rotate from one item to the next to determine the unit rate. Determine the cost of sets such as 5 apples, etc. Have the students work with a partner to compare and discuss.	Student work sample; Teacher observation; Written and verbal response; Presentation

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	c,k	Have the students bring in grocery ads from newspapers. Have the students figure and compare unit prices on given items at two different stores to find the best buy.	Teacher observation; Student work samples
1 2	c, d	Give the students an equation such as $3 + ($2.50) x = $11.50. Have the students write a story problem based on the equation. Sample Answer: The Rollo Skating Rental charges an entrance fee of $3 and an additional $2.50 per hour. If Sam paid $11.50 to skate, how many hours did he skate?	Teacher observation; Student work samples
1 4	c,j,k, a	Provide the students with standard English and metric units of measure and calculators. Provide a series of questions and require students to work in pairs to answer the questions. Examples: 1. A beauty shop has about 66 customers per day, keeping 3 beauticians busy for most of a 9-hour day. How long does each beautician average with each customer? Give answer in minutes. 2. Dominick is recovering from surgery and can't lift more than 25 pounds. He works in a grocery store as a stocker. A bag of flour weighs 5 pounds. Each case contains 6 bags. Can he carry the cases to the aisle and stock the shelf? Explain.	Teacher observation; Student work samples; Written response; Presentation
1 4	c a,b,c	Provide several circular items. Have the students measure and record the circumference and the diameter of each item. Have students use the formula for circumference of a circle with the measured diameter. Discuss differences between measured and calculated circumference.	Teacher observation; Student work samples
1 4	c a,b,c	Using grid paper, draw a circle with a diameter of 4 cm. Cut the circle into eight equal wedges. Color half of the wedges and fit the wedges together to form a shape that looks like a parallelogram. Example:   Use the formula for area of a parallelogram to justify the formula for area of a circle.	Teacher observation; Student work samples
1 4	c a,b,c,d	Have students calculate the volume of boxes, i.e., tissue box, cereal box, snack food box.	Teacher observation; Student work samples

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1 4	c a,b,c d	Present students with the idea of working in the marketing department of a candy company. The task is to design the best box to hold fudge. The fudge needs to be shipped in quantities of one dozen. Each piece of fudge is 2 square inches. The fudge can be stacked. Design at least two boxes to hold the fudge. Which box requires the least material?	Student work sample; Teacher observation; Written response; Presentation; Rubric
1 4	c a,b,d	Collect several different sizes of gift boxes. Have students measure the boxes and determine the minimum amount (surface area) of wrapping paper needed to wrap each box.	Teacher observation; Student work samples; Presentation; Written response
1	c,l	Have students look at sale advertisements. Select one item and write the specific size, brand, and other characteristics. From several stores find the original price of the same item. Compare prices to see if the sale item is actually a good buy. Calculate the discount, sale price, sales tax, and percent decrease for the item.	Student work sample; Teacher observation; Written and verbal response; Presentation
1	c,m	Create a menu or provide one. Each student is to order a meal from the menu. Determine the total cost of the meal. Find the amount of sales tax at (9%). Gratuity (tip) of 15% should be determined and added to the final cost.	Student work sample; Teacher observation; Written response; Presentation; Test
1	c,m	Have students get a sales advertisement from a department store. Pretend each student has $100 to spend. Have the students make a purchase and determine the total cost including 7% sales tax without overspending.	Student work sample; Teacher observation; Written response; Presentation; Test
1	c,m	Have the students look through the newspaper car sales. The students should select a car and use the formula for simple interest to determine the total cost with the given rate of interest and time period.	Teacher observation; Student work samples; Test

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	c,m	Have students research interest rates at local banks for savings accounts. Use the simple interest formula to calculate the amount of interest for various deposits based on interest rates over set periods of time.	Student work samples;
1	c,n	Have students use base ten blocks and/or cm grid paper to model percent of increase. For example, students put a hundred squares on their desks and identify this as 1 whole. Ask the students what fraction is one (1) square. ($\frac{1}{100}$). Then ask what percent is this (1%). Have the students put a 1-square next to the hundred squares. By doing this, the hundred square is increased by 1%. Have students add a ten-strip to increase the hundred-square by 10%. Ask what fraction is 1 ten-strip. (10/100) and ask what percent is this. (10%). Inform students that adding a ten-strip increase the hundred-square by 10%. Have students use the blocks to increase the hundred-square by various percents. Have students count the squares subtracted from the hundred grid to determine percent of decrease.	Student work samples; Presentation; Teacher observation; Written response;
1	d,i	Place students in groups of 3 or 4. Give each group a set of index cards with numbers written on them (fractions, decimals, mixed numbers, and percents). Typically, use numbers between 0 and 2 and include halves, thirds, fourths, fifths, sixths, tenths, etc. Some of the cards may be equivalents. Be sure the cards have been well mixed. Students are to sort the cards in order. Hang a clothesline in the room and allow each group to clip their cards to the clothesline. If there are cards with equivalent numbers, clip them underneath each other, making sure all cards can be seen.	Teacher observation; Student work samples
1 5	d d	Use only the face cards from a deck of cards. Lay the cards face-up in random order. If three cards are chosen at random, what is the probability of various series of events if the cards are returned after each draw and if the cards are not returned after each draw, i.e., P (Queen, Jack, or King)? Write each probability as a decimal, ratio, and percent.	Teacher observation; Student work samples
1	d	Play the "Concentration Game". Students work with a partner. Each group is given a stack of 24 cards. Eight of the cards will have a ratio written on them, eight will have the decimal form, and the remaining eight will have the percent form, i.e., ¾, 0.75, 75%. Shuffle the cards and place face down in rows of four. Play "Concentration" with three cards to match a ratio with its decimal and percent. The student with the most matches at the end of the game is the winner.	Teacher observation

SEVENTH GRADE

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>1
4</td><td>d,j
e</td><td>Have students work with a partner. Each group will be given a page from the local newspaper and a centimeter ruler. Students will estimate the total area of the newspaper page, excluding margins, and determine the area of each section. The sections may consist of local news, national news, advertisements, entertainment, sports, photographs, weather, obituaries, etc. Have students express the area of the article to the area of the page as a fraction, decimal and percent. The class records all findings. The total area of each of the sections is calculated and compared to the area of the entire newspaper page.</td><td>Student work sample; Teacher observation; Written response; Presentations</td></tr>
<tr><td>1
5</td><td>d,j
e</td><td>Have the student roll two number cubes and find their sum. Record results in a frequency table. Use results to determine experimental probability of getting two sums of 2 through 12. Express probability as a decimal, ratio, and percent.</td><td>Student work sample; Teacher observation;</td></tr>
<tr><td>1</td><td>d,j,l</td><td>Compare the number of boys and girls in various classrooms and convert boy/girl ratios to percent. Use proportions to predict the number of boys or girls in the school. Use school records to confirm. Discuss the differences and why they occur (sampling.)</td><td>Student work sample; Teacher observation; Written and verbal response</td></tr>
<tr><td>1</td><td>f</td><td>Research distances from each planet to the sun. Write each distance in standard form and in scientific notation.</td><td>Teacher observation; Student response</td></tr>
<tr><td>1</td><td>g</td><td>Use cm squares or graph paper, illustrate various squares. Discuss the sides and note representation of square root. These squares represent “perfect squares.” Tell how many pieces it takes to make the squares. Note the square of one side is equal to the area. Since the sides of a square are the same, the root is expressed as the measure of one of the sides. Thoroughly discuss the inverse relationship between square roots and squared numbers.
Ex. $3 \times 3 = 9$; $3^2 = 9$; $\sqrt{9} = 3$</td><td>Student work sample; Teacher observation; Written response;</td></tr>
<tr><td>1</td><td>g</td><td>Provide the students with a set of numbers. Have the students predict which numbers are perfect squares. Have the students use the calculator to check their answers. Example:
<table><tr><th>Number</th><th>Prediction</th><th>Calculator Check</th></tr><tr><td>1</td><td>Yes</td><td>$\sqrt{1} = 1$ yes</td></tr><tr><td>10</td><td>No</td><td>$\sqrt{10} = 3.2$ no</td></tr><tr><td>18</td><td>Yes</td><td>$\sqrt{18} = 4.2$ no</td></tr></table></td><td>Student work samples; Teacher observation; Written responses</td></tr>
</table>

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	g	Write the prime factors of both numbers – one over the other as if they were fractions. Draw a line through the factors that are the same in both. The factors that are the same in numerator and denominator are the GCF. To find the LCM multiply the GCF by the remaining numbers that are not in common. Ex. Find the GCF and LCM of 9 and 12 $\frac{9}{12} = \frac{3 \times \cancel{3}}{2 \times 2 \times \cancel{3}} = \frac{3}{4}$; GCF = 3; LCM = (the GCF) x 3 x 2 x 2 = 36	Student work sample; Teacher observation
1	h	Multiply the two numbers together. Divide by the GCF. The result is the least common multiple of the two numbers. Ex: 50 and 30: 50 x 30 = 1500; 1500 ÷ 10 = 150; LCM is 150. 24 and 36: 24 x 36 = 864; 864 ÷ 12 = 72; LCM is 72.	Student work sample; Teacher observation; Written response;
1	i	On a sheet of paper, list several ratios. On the opposite side, in random order, give equivalent ratios. Have the students connect a ratio from the left to a ratio on the right. Give more ratios on the right side so that some will be left over.	Student work samples; Teacher observation
1	i	Make two lists of equivalent ratios (Set A will not be simplified. Set B will be simplified). Use a calculator that will simplify fractions to match each fraction in Set A with its equivalent form in Set B.	Student work samples; Teacher observation
1	i	Provide students cards with ratios written on them. Have the students match a ratio card with another card that is an equivalent ratio in simplest forms.	Student work sample; Teacher observation
1	i	Give students several pairs of equivalent ratios. Have students draw a rectangle to show the ratio and then draw a rectangle the same size showing the equivalent ratio. Compare the shaded part of the two rectangles. It should be the same. Example: $\frac{1}{2}, \frac{2}{4}$ 	Student work samples; Written responses; Presentation; Teacher observation

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment

1	j	Teach students how to solve proportions using factor of change or unit rate. Factor of Change: $\frac{20\,\text{min.}}{4mi.} = \frac{n}{12mi.}$ Multiply by 3 $\frac{20}{4} \times \frac{3}{3} = \frac{60}{12}$ n = 60 min. Unit Rate: $\frac{20\,\text{min}}{4mi} = \frac{n}{12mi}$ It takes 20 minutes to go 4 miles. 20 ÷ 4 = 5, so it takes 5 minutes to go 1 mile. 12 miles x $\frac{5\text{ min}}{1\text{ mi.}}$ = 60 minutes.	Student work samples; Presentation; Test; Written responses
1	j	Investigate the connections between test grades (percents), total problems, and number correct. Example 1: Use proportions to determine how many problems would have to be correct on a 25 problem test to make a grade of A, B, C, D, F. Example 2: Determine how many items were on the test if the student made a grade of 80 and got 12 correct.	Student work samples; Presentation; Test; Written responses
1	j	Have students create a scale drawing of the classroom. Show the proportions used for the measures. Have students create scale drawings of their bedrooms.	Teacher observation; Student work samples; Presentation
1	j	Have the students set up a proportion using real world situations. Example: "Sheila wants to buy a CD player that costs $240. She earns $25 in 5 hours for babysitting. How many hours will Sheila have to work in order to buy the CD player? $\frac{5(hrs)}{25(earnings)} = \frac{n(hrs)}{240(earnings)}$	Student work samples
1 3	j c	Give students two different sizes of the same shape. Have students identify the corresponding sides and corresponding angles. Use ratios to prove similarity.	Written response; Student work samples
1 3	j c,f	Make sets of cards showing congruent figures and similar figures. Indicate side measures on all figures. Pass cards out to students and have them find the matching congruent card and/or similar card. Students must be able to justify the match.	Student work samples; Written/Verbal Response
1 3	j c,f	Have students go outside and measure their height, the length of their shadow, and the length of the shadow of the flagpole. Using their understanding of similar figures, ask students to determine the height of the flagpole.	Student work samples; Teacher observation

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	l	Use 10-by-10 cm grids for students to model such problems as: o 10% of 100 o 30% of 60 o 50% of 75	Presentation; Student work samples
1	o	Use a number line to discuss and demonstrate absolute value.	Teacher observation
1	o,p	Make a large thermometer on the board/overhead. Ask the students to locate various temperatures and write an inequality to compare them.	Teacher observation; Student work samples
1	o,p	Research the high and low temperatures for five cities in different regions of the United States or the world for a week. Compare and order the temperatures.	Student work sample; Teacher observation; Written response; Presentation
1	o,p	Make a set of index cards with one number on each card using positive and negative numbers including zero. Give each student a card and have him/her form a line from least to greatest value.	Teacher observation; Student responses
1	p	Place students into groups of four or less. Provide the students with a set of cards with integers and >, <, =, and absolute value signs written on them. Have the students arrange the cards from least to greatest. Example: -96 -50 -31 0 30 48 101	Teacher observation; Student responses; test

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	q	Provide two colored counters to model addition and subtraction. Let red be negative and yellow be positive. Pull out any zero pairs. Zero pairs are one red and one yellow, since a positive one and a negative one equal zero. Whatever is left is the answer. Example: 3 + ⁻5 Y Y Y R R R R R There are three Y/R pairs which are equivalent to zero. Two reds are left; therefore, the answer is -2. Have students relate the actions to the algorithm. Example: 3 – (⁻4) There are no negatives (R) to subtract so zero pairs (Y/R) must be added. Y Y Y YR YR YR YR When the four negatives are removed, seven positives (Y) remain. 3 – (⁻4) is the same as 3 + 4 = 7	Teacher observation; student work samples
1	q	Give a deck of cards to a pair of students. Let the black cards be positive and the red cards negative. Place eight cards face up on the desk. The objective of the game is to make zero using as many cards as possible. Example 1: A red 5 and a black 5 represent negative 5 and positive 5 that equal zero. Example 2: red 4, red 2, red 10, and black 10, black 6 equal zero.	Teacher observation
2	a	Have the students study the pattern below: 999 x 2 = 1998 999 x 3 = 2997 999 x 4 = 3996 999 x 5 = 4995 Have the students explain the patterns they see. Have the students extend the pattern to find 999 x 9. An extension to this activity would be for the students to use the pattern to find 9,999 x 7 and 99,999 x 8.	Teacher observation Student work sample
2	a	The teacher will think of a rule. One student at a time will call out a number. The teacher applies the rule to the number and tells the class the solution. Repeat the process until someone determines the rule the teacher is applying.	Teacher observation Student oral sample

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	a	Describe and extend patterns in sequences • Use pattern blocks to create a tessellation. • Use pencil and paper to illustrate patterns. • Observe sets of numbers or geometric shapes that are in a sequence. Determine numbers or shapes that will continue the sequence by determining a rule to explain the pattern. • • Explore patterns in the Fibonacci sequence.	Student work sample; Teacher observation; Written response; Project; Presentation; Rubric
2	b	Create cards using algebraic expressions and verbal phrases. Have the students match the expression to the corresponding phrase. Example: Clue — Answer A number (x) increased by five — x + 5 Twice a number (n) — 2n Six minutes less than Bob's time (t) — t – 6 3 years younger than Seth (s) — s – 3	Student work sample; Teacher observation
2	b	Give the students a number. Have the students perform the instructed operations on that number. Example: Start with 12 — 12 Twice that number — $12 \times 2 = 24$ Less 8 — $24 - 8 = 16$ Divide by 4 — $16 \div 4 = 4$ Increase by 2 — $4 + 2 = 6$ Square that number — $6^2 = 36$	Teacher observation; Student work samples; Student responses
2	c	Read a sentence such as as 2 times what number is 10. Have students write the algebraic equation. A correct response will be 2x = 10. Repeat this activity several times using sentences involving all four basic operations.	Student work sample; Teacher observation; Written response
2	c	Provide students with checking account scenarios such as "I have money in my checking account. I wrote a check for $43.00. I had $25.00 left in my account. How much did I have in my account before I wrote the check?" Write an equation such as x – 43 = 24 to solve.	Student work sample; Teacher observation
2	d	Relate equations to sports. Give students an equation and have them tell a sports situation that could represent the equation. Example: 14 + p = 30. John scored 14 points in the basketball game on Tuesday. By the end of the second game on Friday, his two-game total was 30 points. How many points did John score in the second game? Solution: Tuesday's game + Friday's game = total points or 14 + p = 30.	Student work sample; Teacher observation; Written and oral responses

SEVENTH GRADE

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>2</td><td>d</td><td>Choose several occupations and write problem-solving situations for each occupation on index cards. Divide the class into groups according to the occupations chosen and distribute the cards for that occupation to the appropriate group. Each group will write and explain equations that describe the real world situation. Group presentations can be made demonstrating how various occupations use equations.
Example: Carpenters
One sheet of 4x8 plywood will cover 32 sq. ft. How many sheets will be needed to cover 640 sq. ft.?
32 p = 640</td><td>Student work sample; Teacher observation; Written response; Project; Presentation; Rubric</td></tr>
<tr><td>2</td><td>e</td><td>Have the student create a function table for the following example.

The average American family wastes approximately 30,000 gallons of water a year. F is the number of families and W is the amount of water wasted in a year.

Down the Drain<table><tr><td>F</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>10</td><td>100</td></tr><tr><td>W</td><td>30,000</td><td>60,000</td><td>90,000</td><td>120,000</td><td>---</td><td>---</td><td>---</td></tr></table>The students will find the pattern and write a rule. Before using the rule to extend the pattern, have the students check the rule with the values that they already know.</td><td>Teacher observation; Student work samples; Project; Presentation; Rubric</td></tr>
<tr><td>3</td><td>a</td><td>Visit NCTM’s Illuminations website: http://illuminations.nctm.org/index.asp and use the lesson – “Polygon Capture: A Geometry Game”.</td><td>Teacher observation; Student work samples</td></tr>
<tr><td>3</td><td>a</td><td>Draw different polygons on the board. Have students draw a card from a deck naming one of the polygons drawn. Students will match the card to the polygon.</td><td>Teacher observation</td></tr>
</table>

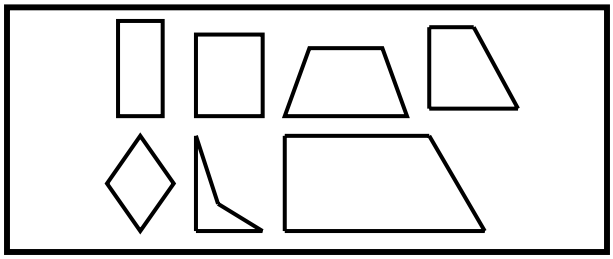
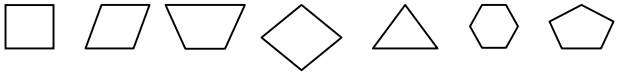
SEVENTH GRADE

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>3</td><td>a</td><td>Students will create polygons on individual dry erase boards, pegs or geoboards, dot paper, etc. and exchange with partners. Partners will then identify the polygons and describe the characteristics of the polygon using correct terminology. Use a variety of polygons including all quadrilaterals and triangles (isosceles, scalene, equilateral, etc.) Have students use geoboard to demonstrate flips, slides, and turns.</td><td>Presentation; Student work samples; Teacher observation</td></tr>
<tr><td>3</td><td>a</td><td>Give students sets of pictures of polygons with several characteristics in common. Have students give at least one similarity and one difference in each set.

Example: Rectangle Square
  </td><td>Student work sample; Teacher observation; Written response; Presentation</td></tr>
<tr><td>3</td><td>a</td><td>Similarity: 4 Right angles Difference: Length of sides (Note: A square is a rectangle, but not all rectangles are squares.)</td><td>Student work sample; Teacher observation; Written response</td></tr>
<tr><td>3</td><td>a</td><td>Use a circular geoboard template to connect the points of a triangle listed below. Measure the length of each side of the figure with a ruler. What type of triangle is represented?

<table><tr><th colspan="4">Triangle Points</th><th>Type of Triangle</th></tr><tr><td>1.</td><td>P7</td><td>P15</td><td>P23</td><td></td></tr><tr><td>2.</td><td>P1</td><td>P6</td><td>P11</td><td></td></tr><tr><td>3.</td><td>P11</td><td>P15</td><td>P20</td><td></td></tr><tr><td>4.</td><td>P20</td><td>P6</td><td>P10</td><td></td></tr></table></td><td>Student work samples; Student responses; Teacher observation</td></tr>
</table>

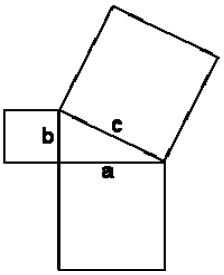
SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	a	Provide the student with pictures of the five kinds of quadrilaterals (trapezoid, parallelogram, rectangle, rhombus, square). Use a Venn diagram to name, compare and contrast the quadrilaterals below. Have students explain why they should be grouped in a particular way. Have the student explain the properties of quadrilaterals that are alike and the properties of each set that are different (number of sides, types of i angles, congruent sides, length of sides, parallel sides, diagonals). 	Student work sample; Teacher observation; Written response; Presentation; Rubric
3	a	Use the geoboard to investigate if the following triangles are possible 1. an acute scalene 2. an obtuse scalene 3. an obtuse isosceles 4. an obtuse equilateral 5. an acute equilateral 6. a scalene right 7. an isosceles right 8. an equilateral right 9. an acute isosceles Students should justify why a triangle is not possible.	Student work sample; Teacher observation; Written response; Presentation; Rubric
3	a	Arrange the students in groups of four. Ask the students to discuss the set of shapes below and begin sharing sentences that involve ALL of the shapes, SOME of the shapes, and NONE of the shapes. The recorder will post on chart paper and groups will present to the class. 	Student work sample; Teacher observation; Written response; Oral Response; Presentation; Rubric
3	b	Visit NCTM's Illuminations website: http://illuminations.nctm.org/index.asp and use the lesson – "Building a Box: Using Nets a Three-Dimensional Visualization."	Teacher observation; Student work samples
3	b	Recognize and identify faces, vertices, and edges of objects found in the classroom and on the campus.	Student work samples

SEVENTH GRADE

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>3</td><td>b</td><td>Give students different three-dimensional shapes and have them describe the faces, vertices, and edges. Have students draw the shape on paper and/or select its corresponding two-dimensional drawing (nets).</td><td>Student work samples</td></tr>
<tr><td>3</td><td>b</td><td>Cut pictures out of magazines and have students identify the three-dimensional shapes.</td><td>Student response</td></tr>
<tr><td>3</td><td>b</td><td>Give students three-dimensional drawings of various space figures. Ask students to draw the corresponding net for each figure on grid paper and cut out their net to check for correctness.</td><td>Student work samples</td></tr>
<tr><td>3</td><td>b</td><td>Working with both three-dimensional shapes and two-dimensional drawings (nets), select strategies such as: make a table, use a formula, draw a diagram, or guess and check to complete the table below.
N - The number of sides in the base
F - The number of faces
V - The number of vertices
E - The number of edges
Identify the relationship among the number of faces, vertices and edges. Look for equivalent expressions or equations.
<table><tr><th>Figures</th><th>N</th><th>F</th><th>V</th><th>E</th></tr><tr><td>Triangular Prism</td><td></td><td></td><td></td><td></td></tr><tr><td>Rectangular Prism</td><td></td><td></td><td></td><td></td></tr><tr><td>Pentagonal Prism</td><td></td><td></td><td></td><td></td></tr></table></td><td>Teacher observation; Student work samples; Completed chart; Written responses; Test</td></tr>
<tr><td>3
4</td><td>c
c,d</td><td>Visit NCTM’s Illuminations website: http://illuminations.nctm.org/index.asp and use the following lessons:
- “Linking Length, Perimeter, Area, and Volume”
- “Patterns and Function”</td><td>Teacher observation</td></tr>
<tr><td>3</td><td>d</td><td>Research the art of M. C. Escher.Model rotations, reflections, and translations using graph paper. Draw tessellations.</td><td>Teacher observation; Student work samples; Written Response; Project; Rubric</td></tr>
<tr><td>3</td><td>d,h,i</td><td>Using a coordinate grid, name coordinates and create patterns or figures (e.g., butterfly, umbrella, sailboat) by plotting points. Perform transformations on these patterns. Name and label coordinates of the transformed image.</td><td>Teacher observation; Student work samples; Written Response</td></tr>
</table>

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	d,h,i	Plot the following points on grid paper and translate (slide) the point and name the new coordinates. Use colored pencils to mark new points. Example: point A: (-2, 4) Slide 4 units right to point A_1:(2, 4) point B:(4, 0) Slide 3 units down to point B_1:(4, -3)	Teacher observation; student work samples
3 4	e c	Using a geoboard, construct a right triangle that has legs of 3 units and 4 units long. Form two squares using the length of each leg as the side of each square. Also form a square using the hypotenuse as one side. The area of square "a" plus the area of square "b" = the area of square "c". Show students the relationship of the lengths squared to the hypotenuse squared.  	Student work samples; Teacher observation
3	e	Use grid paper to draw and justify the Pythagorean theorem $a^2 + b^2 = c^2$. Give the measures of two legs and find the hypotenuse and/or give a leg and the hypotenuse and find the missing leg.	Student work sample; Teacher observation; Written response; Presentation
3	f	Give students three side measures of a triangle. Students cut strips of paper to the given 3 lengths and attempt to assemble them into a right triangle. Hold up figures and see if anyone constructed a right triangle. Have students prove if the triangle could be a right triangle using the Pythagorean theorem.	Student work sample; Teacher observation; Written response; Presentation
3	f	Challenge students to find the Pythagorean triples (measures of right triangles using whole numbers). Note to teachers: If students find a set of triples, they can multiply each side by the same constant to find another set of triples (similar figures). Example: 3, 4, 5 is a Pythagorean triple. Multiply all sides by two and 6, 8, 10 is also a triple.	Teacher observation; Student work samples; Written response
3	g	Using index cards, draw angles with their measures marked. Have students select index cards which are complements and/or supplements. Put angles together to prove.	Student work samples; Teacher observation

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	g	Working in pairs, one student draws a pair of complementary or supplementary angles with one of the angle measures indicated. The partner must identify the pair as complements or supplements and state the missing measure.	Student work sample; Teacher observation; Written response; Presentation
3	h, i	Provide the students with a coordinate plane on grid paper. Discuss the x-axis and y-axis and the ordered pair of numbers which indicate the coordinates of a point (x, y). Have the students plot several coordinates on the graph and identify the quadrant of each point. Examples: (5, 2), (3, 4), (0,-3), (-4, -2).	Teacher observation; Student work samples
3	h,i	Given the following set of ordered pairs, plot the points on grid paper and connect the line segments between each of the coordinates given to create a polygon. (-1,3) (8,3) (8, -2) (-1,-2)	Teacher observation; Student work samples
3	h,i	Create a given polygon by connecting points represented by (x, y) coordinates. Give all but one point and have students identify the necessary point to correctly complete the polygon.	Student work samples; Teacher observation
4	a,b	Visit NCTM's Illuminations website: **http://illuminations.nctm.org/index.asp** and use the lesson "Measuring Up: Concepts in Measurement."	Student work samples; Teacher observation
4	a	Draw a very large capital G on the chalkboard/ overhead. Explain to the students that the G stands for one gallon. Within the G write four capital letter Q's. This stands for 4 Quarts. Inside each of the 4 Q's, write two capital letter P's. This stands for 2 pints in 1 quart. Within each of the P's write two capital letter C's. This stands for 2 cups in 1 pint. Within each of the C's place 8 dots. These dots represent fluid ounces. There are 8 fluid ounces in 1 cup. Apply to real-world problems.	Student work sample; Teacher observation; Written response; Presentation
4 5	b b,f	Have students measure their height in cm. Make a frequency table. Find the mean, median, mode and range of their heights. Ask students what would happen to the mean, median, mode, and range if a very tall person or very short person were included in the group.	Teacher observation; Student work samples; Student oral and written responses
4 5	b b	Have the students measure their height and arm span to the nearest centimeter. Construct into two frequency tables. Have groups of students construct a stem-and-leaf plots from both sets of data. .	Student work sample; Teacher observation; Written response; Presentation; Rubric

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	b	Students will estimate the length of one of their shoes in inches, centimeters, and millimeters. Measure to check for accuracy.	Teacher observation; Student work samples
4	b	Given objects found in any classroom (e.g., books, paper clips, desktops), estimate and measure to find the dimensions of these objects.	Teacher observation; Student work samples
5	a	Have the students estimate how many hours per day they spend doing various activities. Collect data and create a circle graph.	Teacher observation; Student work samples; Project; Rubric; Graphs
5	b	Visit NCTM's Illuminations website: http://illuminations.nctm.org/index.asp and use the following lessons: -"Accessing and Investigating Data using the World Wide Web"; - "Application: Using Graphs and Patterns in Everyday Life"; - "Exploring Histograms"; - "Mathematics as Communication: Graphing Information Collected Over Time"; -"State Names: Investigating Real World Data"; - "Uses of Numbers". .	Teacher observation; Student work samples
5	b	List the grades made on a particular test in random order. Students will place in a frequency table and construct stem-and-leaf and/or bar graph.	Student work samples; Test
5	a,b	Construct a frequency table to find the favorite football team of the class. Divide the class into 3 groups. Assign each group a different type of graph to construct from the data (circle, bar, stem and leaf plot, etc). Have the groups present their graphs to the class. Conduct a class discussion to help recognize the benefits of the different representations of the same data.	Student work sample; Teacher observation; Written response; Presentation; Rubric
5	a,b,c	Provide the class with several examples of bar graphs, circle graphs, and histograms. Working in pairs or small groups, have the students select a graph. The students will write a variety of questions that can be answered from the data given in their graph including questions on the key, the intervals used, the type of graph used and what the graph represents. Groups exchange graphs with questions. Have each group answer the questions.	Student work sample; Teacher observation; Written response; Presentation; Rubric

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	c	Use the Internet to track the progress of a given stock for the past year. Predict patterns or trends for the next month.	Teacher observation; Student work samples
5	e	Visit NCTM's Illuminations website: http://illuminations.nctm.org/index.asp and use the lesson "Simulating Probability Situations Using Box Models" and "Sticks and Stones: Investigating Probability with a Native American Game."	Teacher observation; Student work samples
5	e	Discuss why a town of 20,000 households with one phone each must have multiple prefixes in addition to the 4 digit phone numbers. Example: 10 x 10 x 10 x 10 = 10,000 possible numbers < 20,000 households Extension: Why did Mississippi have to split the state into three area codes? Show how prefixes could determine 400,000 different phone numbers.	Teacher observation; Student work samples; Written response
5	e	Have the students roll a number cube to find the probability of rolling an even number, odd number, prime number, composite number, and a given number.	Teacher observation; Student work samples
5	f	Give the student the following test scores: 85, 80, 93, 85, 80, 88, 65, 50, 93, 90, and 57. Have the student place the test scores in order from least to greatest and find the mean, mode, median and range. Ask what would happen to the mean, median, mode, and range if someone made a zero. What would happen if someone made 110?	Teacher observation; Student work samples; Student oral and written responses
5	g	Present students with a menu from a sandwich shop. Have students consider the number of different lunches that could be ordered from the menu if a lunch consists of a beverage, a sandwich, and a salad. Have students make guesses at the number of different lunches. Ask each student to describe different lunch options until all options are listed. Show them how a tree diagram can be used to create the same list and how the Fundamental Counting Principle can be used to obtain the total number of possibilities.	Teacher observation; Student work samples; Student oral and written responses

SEVENTH GRADE

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	g	Present the scenario: Ned has a 1-6 number cube and three cards: a jack, a queen, and a king. Ask the students to use an organized list or tree diagram to name every possible outcome and use the Fundamental Counting Principle to confirm the total possible if he rolls the number cube and selects one card at the same time.	Teacher observation; Student work samples; Student oral and written responses

PRE-ALGEBRA

The Pre-Algebra mathematics framework serves as a bridge between elementary mathematics and Algebra. This course will build a foundation of algebraic concepts through the use of manipulatives and collaborative/cooperative learning. Concepts include real numbers, algebraic expressions, linear equations, polynomials, inequalities, geometry, ratio, proportion, percents, number theory, measurement, data analysis, statistics, and graphing. A variety of problem-solving techniques and technology will be used when applying these concepts, which will enable students to solve real life problems. This course is designed to prepare students for Transition to Algebra or Algebra I.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies may relate to one, many, or all of the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

PRE-ALGEBRA

CONTENT STRANDS:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Apply concepts and perform basic operations using real numbers in real-world context.

a. Use a diagram to describe and classify real numbers and their subsets.
b. Solve real-life problems involving addition, subtraction, multiplication, and division of rational numbers (i.e., integers, decimals, fractions, and mixed-numbers).
c. Classify, compare, and order rational numbers (i.e., integers, decimals, fractions, and mixed numbers).
d. Add, subtract, multiply, and divide rational numbers (i.e., integers, decimals, fractions, and mixed numbers) with and without calculators.
e. Simplify and evaluate expressions using order of operations and justify solutions.
f. Use the rules of exponents to multiply or divide like bases.
g. Convert between standard form and scientific notation in real-world situations. Multiply and divide numbers written in scientific notation.
h. Evaluate and estimate powers, square roots, and cube roots with and without calculators.
i. Solve proportions, including unit rate, scale, and measurement. Apply proportional reasoning to real-world problems.
j. Find commission, rates of commission, discount, sale price, sales tax, and simple interest, and apply to real-world situations.
k. Write, solve, and apply real-life problems using percents with and without calculators.
l. Use estimation to determine the reasonableness of results in a variety of situations.

ALGEBRA

2. Use properties to formulate and simplify algebraic expressions, solve linear equations and inequalities and apply principles of graphing.

a. Translate real-life situations into algebraic expressions, equations, or inequalities, and vice versa.
b. Simplify and evaluate numerical and algebraic expressions and justify solutions.
c. Identify and apply properties of real numbers with an emphasis on distributive property.
d. Solve and check one and two-step equations and inequalities using one variable.
e. Graph solutions to inequalities on a number line.
f. Using function tables, graph simple linear equations and common non-linear equations (i.e., $y=x^2$.)
g. Given a linear graph, identify its slope as positive, negative, undefined, or zero, and interpret slope as rate of change.
h. Determine slope and y-intercept from a graph and/or equation in y-intercept form. Graph a line given its slope and y-intercept.
i. Classify and determine degree of a polynomial and arrange polynomials in ascending or descending order of a variable.
j. Use manipulative models to add, subtract, and multiply monomials and polynomials.
k. Use the rules of exponents to multiply and divide monomials and to multiply monomials by polynomials.

GEOMETRY

3. Apply geometric principles to polygons, angles, and two- and three-dimensional figures.

a. Locate and identify angles formed by parallel lines cut by a transversal (e.g., adjacent, vertical, complementary, supplementary, corresponding, alternate interior, and alternate exterior).
b. Find missing angle measurements for parallel lines cut by a transversal(s).
c. Classify polygons by sides and angles and find the missing vertex angle measure.
d. Use two-dimensional representations (nets) of three-dimensional objects to describe objects from various perspectives.
e. Use and apply the Pythagorean theorem to solve problems, with and without a calculator.
f. Apply congruent and similar figures to real-world problems.

MEASUREMENT

4. Understand measurable attributes of objects and apply various formulas in problem solving situations.

a. Convert, perform basic operations, and solve real-world application problems using standard measurements.
b. Apply appropriate strategies and tools to determine measurements.
c. Determine the area, perimeter and/or circumference of polygons, circles and composite figures (figures made up of several different shapes).
d. Use given geometric formulas to solve a variety of mathematical and real world problems including volume and surface area of cylinders, prisms, spheres, cones, pyramids, and composite figures (figures made up of several different shapes).

DATA ANALYSIS & PROBABILITY

5. Interpret, organize, and make predictions about a variety of data using concepts of probability.

a. Construct and interpret histograms, bar graphs, line graphs, frequency tables, circle graphs, stem-and-leaf plots, box-and-whisker plots, and scatter plots from given data.
b. Predict patterns or generalize trends based on given data.
c. Explain the role of fair and bias sampling and its effect on data.
d. Calculate basic probability of simple, compound, independent, and dependent events.
e. Apply the concepts of basic probability to real-world problems.
f. Construct and use a variety of methods (i.e., organized lists, tree-diagrams, Fundamental Counting Principle) to solve real-world problems.
g. Collect data. Select and justify the most appropriate representations to organize, record and communicate data.
h. Use a given mean, mode and/or median of a set of data to construct a data set and determine missing values from a data set.

PRE-ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Use yarn to create a Venn diagram of real numbers (natural, whole, integer, rational, and irrational). Choose an index card with a number on it and place it in the correct place.	Student work sample; Observation
1 5	b,c,d h	Using a weather map, compare temperatures around the country. Discuss differences. Find mean and median weekly temperatures.	Discussion; Student presentation
1	b,d	Use maps, bus/plane schedules and fares, hotel rates, etc., to plan a vacation. Estimate total expenses.	Project; Rubric; Observation
1	c	Give students a set of index cards with a different rational number. Students use a number line to locate and compare the numbers, including absolute values and additive inverses.	Student work sample; Teacher-made test
1	e	Divide the class into two groups to play tic-tac-toe. A student from each group goes to the board to work an order of operations problem. The first correct answer wins and marks the tic-tac-toe board. Continue until one group wins.	Student work sample; Observation
1 2	e b,c	Divide class into groups of two. Ask each student to write a step-by-step solution to simplify or evaluate an algebraic expression. Have each student exchange work with partner. Partners study the work to determine if it is correct. If incorrect, label the errors and discuss.	Student work sample; Discussion; Teacher-made test
1	f	Distribute problems involving multiplying and dividing like bases or powers of ten. Work in groups to discuss and determine a rule for solving the problems.	Constructed response; Discussion; Observation
1	g	Use articles to find examples of very large and very small numbers. Using the examples, write the numbers in scientific notation. Select two of these numbers to multiply and divide indicating the answer in scientific notation and standard form. Discuss the advantage of writing and using numbers in scientific notation.	Discussion; Constructed response; Teacher-made test
1	h	Play “Jeopardy” with powers, squares, square roots and cube roots. From an overhead transparency, select a category and point value (equations for 200). Allow calculators.	Observation; Discussion; Performance-based

PRE-ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1 5	i b	Count and record the number of boys and girls in the class. Write the ratios of boys to girls, girls to boys, girls to total, etc. Convert the ratios to decimals and percents. Use these ratios to write and solve proportions to estimate numbers in larger populations (i.e., How many girls in the school city? state? nation?). Compare estimates to published data and discuss.	Student work sample; Discussion; Teacher-made test
1	i,j,k	Use store advertisements to calculate unit rate sale price, original price and discount. Compare results to determine the best buy.	Student work sample; Teacher-made test
1	i,l	Use map's scale to write a proportion and solve it to determine actual distance.	Student work sample; Teacher-made test
1	j	Use real estate booklets to determine the commission a realtor will make on the sale of a home at asking price. Calculate commission on various offers on the home. Given the sale price and the realtor's fee, find the realtor's commission rate.	Student work sample; Teacher-made test
2	a	Have pairs of students write an algebraic expression. Put these into a hat. Each pair draws out one algebraic expression and writes a real-life situation, which corresponds to the algebraic expression they drew.	Student work sample
2	a,b,c,d,e	Play Algebraic Jeopardy. From an overhead transparency, choose a category and point value (e.g., expressions for 400). Categories include expressions, word phrases/ sentences, properties, equations, inequalities, or algebraic phrases/sentences. Points range from 10 to 50 based on level of difficulty. Answers must be in the form of a question. The team with the most points when the board is completed or when time runs out wins.	Student work sample; Observation

PRE-ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	c	Divide the class into two teams. On an overhead, write problems that can be solved by using properties. For example: (25 x 6 x 4) = (25 x 4) x 6. One person from each team races to get the correct answer. (Explain the use of properties with each problem.) The team with the most number of correct answers wins.	Observation; Discussion
2	d,e	Prepare a set of index cards containing equations and/or inequalities. The set should contain pairs that have the same solution. Distribute one card to each student. Have students solve and graph their equation/inequality on a number line. Identify the classmate with the same solution.	Observation; Discussion
2	d,f,h	Write a linear equation on the overhead. Give each row of the class different x or y values to use in solving the equation. Let one row choose their own values. When all have finished, have each row plot their points on a wall coordinate grid. Discuss the reasons that all points fall on the same line. If any points are not on the line, look for mistakes in calculations. Determine the line's slope and y-intercept.	Observation; Student work sample Observation; Discussion
2	g	Give each student a pipe cleaner. Call out a type of slope (positive, negative, undefined, or zero) and ask students to hold the pipe cleaner up in a way which represents that slope.	Observation; Discussion
2	g,h	Divide the class into groups of two. Ask each student to draw a line with a given slope and y-intercept and compare his or her line with partner's line and discuss findings.	Observation; Discussion
2	g,h	Divide the class into groups of two. Given a coordinate plane, ask each student to draw a line on the front and write its slope and y-intercept on the back. Exchange papers. Each partner will determine the line's slope and y-intercept. Flip the paper to compare/discuss answer.	Student work sample; Observation
2	h	Give each student an individual dry erase board with a pre-marked coordinate grid or graph paper. Call out a y-intercept and slope and ask students to graph the corresponding line on his/her board/paper and hold it up for teacher observation.	Student work sample; Observation
2	i	Make a set of 24 cards. The set should contain 12 cards with polynomials, all with different degrees. The other 12 cards should have the numerical degrees. Turn all cards face down and play "Concentration™" to form pairs that match a polynomial to its corresponding degree.	Student work sample Observation

PRE-ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	i	Make a classroom set of cards. Each card will have a polynomial written on it with various degrees. Assign each student a number and then give each student one of the polynomial cards. Randomly call out a group of numbers (2,12,7) and have those students arrange themselves in ascending or descending order of the variable.	Observation
2	j,k	Use algebra tiles™ to develop the rules of exponents and apply the rules to adding, subtracting, multiplying, and dividing monomials and polynomials.	Observation; Teacher test
3	a	Use masking tape on the floor to create parallel lines cut by a transversal. Number the interior and exterior angles 1 to 8. Play "Twister™" by placing hands and feet on indicated pairs of angles.	Observation
3	b	Ask students to draw a pair of parallel lines and cut them with a transversal. Number the angles 1 to 8 and use a protractor to measure them. Each student presents findings to the class. After each presentation, students analyze results to search for similarities (patterns) that work on each drawing. Continue presentations until students can reliably calculate the 7 missing angle measures when given one angle measure.	Discussion; Observation; Student work sample; Teacher-made test
3	c	Use manipulatives (e.g., geoboards, D-Stix, plastic straws, flat spaghetti, or connectors) to construct polygons. Discuss the characteristics of each.	Performance-based; Student work sample
3	c	Create a table to determine the relationship between the number of angles, sides, diagonals, total triangles, and the total degrees in polygons. Then given an n-sided regular polygon, students use their found patterns to determine the number of degrees in each of its angles.	Observation; Student work sample; Teacher-made test
3	d	Use Tinkertoys™, Connects™, Legos™ or other building manipulatives to create a three-dimensional figure from a two-dimensional drawing. Describe how the figure looks from the top, bottom, left, right, front and back.	Discussion; Observation
3	d	Give students a two-dimensional net drawing of a three-dimensional object. Ask them to draw how the figure looks from the top, bottom, front, back, left and right. Give students the three-dimensional object and have them compare their drawings to how it actually looks from those six perspectives.	Discussion; Observation; Student work sample
3	e	Have students sketch right triangles on grid paper. Use the Pythagorean theorem to find the measure of the hypotenuse. Verify the measure with a ruler or by counting the squares on the grid paper.	Performance-based; Student work sample; Discussion; Presentation; Rubric

PRE-ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	e,f	Students go outside and measure their height, the length of their shadow and the length of the shadow of the flagpole. Using their understanding of similar figures, ask students to determine the height of the flagpole. Using the flagpole's calculated height and measured shadow length, apply the Pythagorean theorem to determine the distance from the top of the flagpole to the end of its shadow.	Student work sample
4	a,b,c	Design decks of various shapes to be added to a patio. Use basic formulas to find perimeter, area, and amount of materials needed for the job. Select appropriate units of measurement.	Project; Rubric
4	a,b,c	Measure length and width of the classroom in feet. Calculate the number of square yards needed to cover the room with carpet. Given the conversion factor, estimate the number of square meters needed.	Student work sample
4	b	Determine measurements of given objects found in any classroom (e.g., books, paper clips, trash can).	Performance-based assessment
4	d	Students will design a can to hold soup. They must determine the surface area of their can (the metal), the lateral area of their can (the label), and the volume of their can (the soup).	Project; Rubric
5	a	Interview classmates to determine their favorite foods. Construct a circle graph from the data.	Project; Student work sample
5	a	Organize the height of each student into a stem-and-leaf plot. Analyze this information by creating a box-and-whisker plot.	Observation; Student work sample
5	a	Given a salary, plan a monthly budget. Construct a circle graph to display monthly budget.	Rubric; Student work sample
5	a	In pairs, have students listen to a selection of 20 song excerpts. Each student rates the song on a scale from –5 to 5 based on whether or not he/she likes the song. Using the ratings, partners form an ordered pair. Plot the ordered pairs to create a scatter plot. Discuss if partners are musically compatible.	Student work sample
5	a,b	Cut out various charts and graphs from magazines and/or newspapers. Ask students to predict patterns or generalize trends based on the charts and graphs.	Student work sample

PRE-ALBEGRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	a,b,g	For each family member, record age, height in inches, and birth month. Construct various plots from data (e.g., stem-and-leaf graphs and scatter plots). Make predictions based on the height/age plots. Use the graphing calculator to construct histograms, scatter plots, and box-and-whisker graphs.	Performance-based; Project; Student work sample; Discussion; Observation
5	a,c	The teacher will divide the class into groups and give questions to the students to conduct a survey. Each group will be assigned a different sample of a population to survey (girls only, boys only, honor society only, etc.). Students use their survey results to construct a frequency table and circle graph. Each group then presents their findings to the class. Differences are discussed with an emphasis on how sampling can affect data. Extension: Discuss fair and biased sampling.	Performance-based; Discussion; Student work sample
5	b	Take students outside to record the colors of cars that go by. Using this data, ask students to predict the color of the next ten cars.	Student work sample; Discussion
5	d	Use marbles, coins, number cubes and spinners to calculate basic probability of simple, compound, independent and dependent events.	Performance-based; Discussion; Student work sample
5	f	Given a packet of clothes (such as pants and shirts in different colors), arrange clothes and determine possible number of outfits.	Presentation; Student work sample; Observation; Discussion
5	f	Imagine a certain group of students eating at a restaurant. Use an organized list, a tree diagram, or the Fundamental Counting Principle to determine the number of different meals available or the different ways the group can be seated.	Student work sample; Teacher-made test
5	g	Divide the class into groups. Each group is given the same data but is assigned a different vertical/horizontal scale. Ask each group to display the data in a bar graph using their specified horizontal/ vertical scale. Groups present their graphs to the class. Discuss how horizontal/vertical scales may affect the shape of the graph, cause a graph to be misleading, or cause data to appear differently.	Observation; Discussion; Student work sample

PRE-ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	g	Divide the class into groups. Groups are asked to design and conduct an experiment of their choice. Groups will be required to collect/organize data, create a visual of their findings, and present their findings to the class.	Observation; Discussion; Student work sample; Rubric
5	h	Give the students a set of data (such as test scores). Determine what the next data item must be in order to achieve a desired mean.	Student work sample; Teacher-made test
5	h	Give the students some information about a set of data and ask them to reconstruct one or more possible data set(s) that fit the given requirements. (Example: There are 5 items and the mean is 12.)	Student work sample; Teacher-made test

TRANSITION TO ALGEBRA

Transition to Algebra is designed to give students an additional opportunity to develop foundational skills required to be successful in Algebra I. Students should enter Transition to Algebra with fluency in computing with the four basic operations with rational numbers and a basic understanding of linear equations and graphing. In Transition to Algebra, students will maintain and build upon their understanding of basic operations with rational numbers, the use of formulas, proportional reasoning, linear equations, and basic functions.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies may relate to one, many, or all of the mathematics framework strands and may be combined and taught with other competencies throughout the school year. Competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

TRANSITION TO ALGEBRA

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships between numbers and their properties and perform operations fluently.

a. Compare and contrast the subsets of real numbers.
b. Add, subtract, and perform scalar multiplication on matrices.
c. Evaluate expressions with real numbers using order of operations to include absolute value.
d. Evaluate powers and simplify radicals including square and cube roots.
e. Express, interpret, and compute numbers using scientific notation in meaningful context.
f. Apply ratios and use proportional reasoning to solve real-world algebraic problems.
g. Classify and determine degree of a polynomial.
h. Use the rules of exponents to multiply and divide monomials, to multiply monomials by polynomials, and to multiply binomials by binomials.

ALGEBRA

2. Understand, represent, and analyze patterns, relations, and functions.

a. Recognize and demonstrate the difference among “evaluate”, “simplify”, and “solve”.
b. Given a literal equation, solve for a specified variable of degree one.
c. Explain and illustrate how changes in one variable may result in a change in another variable.
d. Solve and check multi-step equations and inequalities, including distributive property, variables on both sides, and rational coefficients.
e. Use real-world data to express slope as a rate of change.
f. Compare and contrast the solutions of linear equations and inequalities.
g. Graph solutions to linear inequalities.
h. Graph linear equations using slope and y-intercept or two points.
i. Examine the graph of a linear function and identify domain, range, slope, and intercepts.
j. Use appropriate technology to investigate families of graphs (linear, quadratic, and absolute value) and develop generalizations to characterize the behaviors of graphs.

GEOMETRY

3. Understand geometric principles of polygons, angles and two and three-dimensional figures.

a. Identify angles formed by parallel lines cut by a transversal to include alternate interior, alternate exterior, and corresponding angles.
b. Determine missing angle measurements for parallel lines cut by transversal(s).
c. Apply the Pythagorean theorem to solve problems.
d. Apply proportional reasoning to determine similar figures and find unknown measure.

MEASUREMENT

4. Demonstrate and apply various formulas in problem-solving situations.

a. Use manipulatives to model polynomial operations for problems involving perimeter and area.
b. Solve real-world problems involving measurements (i.e., circumference, perimeter, area, volume, distance, temperature, etc.).

DATA ANALYSIS & PROBABILITY

5. Interpret data and apply concepts of probability.

a. Construct and interpret data involving histograms, bar graphs, line graphs, scatter plots, box-and-whisker plots, circle graphs, stem-and-leaf plots, frequency distributions, and tables.
b. Analyze situations that involve the probability of independent/dependent events.
c. Determine and use the sample space of events to determine the probability of an event.
d. Use and apply the concepts of probability to real-world problems.

TRANSITION TO ALGEBRA

<table>
<tr><th>Comp.</th><th>Obj.</th><th>Suggested Teaching Strategies</th><th>Suggested Assessment</th></tr>
<tr><td>1</td><td>a</td><td>Show relationships using visual organizers (Venn Diagrams) among the subset(s) of the set of real numbers.</td><td>Observation; Rubric</td></tr>
<tr><td>1</td><td>a</td><td>Distribute cards containing a rational or irrational number. Have students arrange cards in order on a number line and justify placement.</td><td>Observation</td></tr>
<tr><td>1</td><td>b</td><td>Have teams comprised of four students form numerical expressions to represent the numbers 1 to 26. Teams will use grouping symbols, the digits 1,2,3, and 5 only once, and the basic operations to create each expression.</td><td>Self-assessment using graphing calculator</td></tr>
<tr><td>1</td><td>c</td><td>Create foursomes such as:
3(x + 4) = 3x + 12 3x +2x = x(3 + 2)
Distributive property 5x + 3 = 3 + 5x
Which one does not belong?</td><td>Teacher test; Constructed response</td></tr>
<tr><td>1</td><td>d</td><td>Create a table with three columns.
<table><tr><td>Numbers
1
2
•
•
15</td><td>Square
1
4
•
•
225</td><td>Square root=number
$\sqrt{1}$ = 1
$\sqrt{4}$ = 2
•
•
$\sqrt{225}$ = 15</td></tr></table>
Use the table to estimate the square root of non-perfect squares.</td><td>Self-assessment using a calculator</td></tr>
<tr><td>1</td><td>e</td><td>Use the graphing calculator in scientific model to discover rules for multiplying and dividing numbers in scientific notation.</td><td>Self-assessment using graphing calculator</td></tr>
<tr><td>1</td><td>f</td><td>Use scale drawings to determine actual measurements.</td><td>Teacher test; Rubric</td></tr>
<tr><td>1</td><td>f</td><td>Given a recipe that serves no more than six people, convert it to serve the entire class.</td><td>Rubric</td></tr>
<tr><td>1</td><td>h</td><td>Use expanded notation to multiply or divide monomials. For example:
$\frac{x^6}{x^4} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x} = x^2$</td><td>Teacher observation</td></tr>
</table>

TRANSITION TO ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	b	Working in pairs, one student runs a specified distance while another uses a stopwatch to measure time. Use the formula d = rt to determine rate.	Observation; Student response
2	b,d	Given a solved equation with mistakes, verify and explain why the process is incorrect.	Observation; Rubric
2	c	Using algebra tiles™ to show differences among "x+ x" and "x • x", "x + y", "x • y", "(+ 1) + x" and "(y + 1) • x" Given a rectangle of specific length and width, extend length and width by a variable and calculate new perimeter and area in terms of the variable.	Observation
2	c	Using equations involving rational numbers, such as y = .5x Use a T- chart to graph the relation and verify with graphing calculator.	Rubric
2	d	Use manipulatives (e.g., algebra tiles or blocks) to model processes used to solve equations. Check solutions using the graphing calculator or substitution.	Observation; Constructed response
2	j	Using a graphing calculator, enter y = x, y = 2x, y = $\frac{1}{2}$x, and y = $\frac{1}{4}$x, one at a time. Explore what happens with the steepness of each line.	Observation
3	c	Plot two points on a coordinate plane. Use the Pythagorean theorem to find distance. Show how the distance formula is derived from the Pythagorean theorem.	Observation
3	d	Given two similar triangles, use highlighters to color code corresponding parts; set up ratios and proportions to find unknown measures.	Teacher test

TRANSITION TO ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	b	Using a coffee can, calculate volume of the can. Calculate circumference and area of one of the bases.	Teacher Observation; Student response
5	a	In groups, assign each a topic from which to design and conduct a survey. Compile, graph, and interpret results and present to class.	Teacher observation
5	b	Using a bag of marbles containing 2 red, 6 yellow, 3 green, and 4 purple. Find the probability of drawing a yellow marble and replacing it and then drawing a green marble.	Teacher observation; Student response
5	b	Have students discuss what it means to be independent or dependent. Have students write ten relationships that are dependent/independent of each other.	Teacher observation Student response
5	b,c	Define “events” and “sample space” for experiments involving number cubes, spinners, coins, and cards.	Observation

ALGEBRA I

The Algebra I framework provides the minimum competencies required for students to be successful in higher-level math courses. Algebra I provides students the opportunity to develop and communicate an understanding of algebraic representation that is a prerequisite for all higher mathematical concepts. Students must enter Algebra I with fluency in computing with all four basic operations using rational numbers, knowledge and understanding of how to use formulas to solve real-world problems, and an understanding of solving and graphing linear equations.
The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

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ALGEBRA I

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships between numbers and their properties and perform operations fluently.

a. Utilize properties of equivalence to simplify variable expressions.
b. Simplify and perform basic operations on square roots.
c. Add and subtract matrices containing rational numbers and apply the concept of scalar multiplication.

ALGEBRA

2. Understand, represent and analyze patterns, relations and functions.

a. Solve, check and graph multi-step linear equations and inequalities in one variable, including rational coefficients in mathematical and real-world situations.
b. Solve and graph absolute value equations and inequalities in one variable.
c. Analyze the relationship between *x* and *y* values, determine whether a relation is a function, and identify domain and range.
d. Explain and illustrate how a change in one variable may result in a change in another variable and apply to the concepts of independent and dependent variables.
e. Write, graph, and analyze linear and nonlinear functions (such as quadratic, greatest integer), and absolute value and inequalities in two variables given a table of values, slope, intercept, or rule.
f. Use algebraic and graphical methods to solve systems of linear equations and inequalities in mathematical and real-world situations (i.e., graphing, elimination, substitution).
g. Add, subtract, multiply, and divide polynomial expressions.
h. Factor polynomials by using Greatest Common Factor (GCF) and factor quadratics that have only rational roots.
i. Utilize the quadratic formula and factoring to determine real number solutions to quadratic equations algebraically and graphically.
j. Use graphic and symbolic means to justify why some polynomials are prime over the rational number system.

GEOMETRY

3. Understand how algebra and geometric representations interconnect and build on one another.

a. Given the equations of two lines, determine graphically and/or algebraically whether the lines are parallel, perpendicular, or neither and justify answer.
b. Use algebraic and geometric representations to compare slopes as rates of change.

MEASUREMENT

4. Demonstrate and apply various formulas in problem-solving situations.

a. Solve real-world problems involving formulas (e.g., circumference, perimeter, area, volume, interest, distance, rate, work, etc.).
b. Apply the appropriate formula to determine length (distance formula, Pythagorean Theorem), midpoint, and slope of a segment in a coordinate plane.
c. Apply polynomial operations to problems involving perimeter and area.

DATA ANALYSIS & PROBABILITY

5. Represent, analyze and make inferences based on data with and without the use of technology.

a. Collect, organize, graph, and interpret data sets.
b. Draw conclusions and make predictions from scatter plots. Perform the analysis of data including line-of-best fit.
c. Interpret how changes in one-variable data sets affect mean, median, mode, and range.
d. Analyze data and apply appropriate scale to the graph of the data.

ALGEBRA I

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Have students play expression Concentration by preparing card with expressions and their simplifications on another card. Students must match the original expression with its correct simplification	Teacher observation
1	b	Use the Box Method to demonstrate how to multiply radical expressions.	Teacher observation
1	b	Have students to find the perimeter and area of polygons using radical terms as dimensions.	Teacher observation; Student response
2	a	Use two-column notes or an algebraic proof to solve multi-step equations and inequalities. Students must justify each step used to solve the equation or inequality using a real number property. Assign each student or student pair an equation to solve using two-column proofs. Then, utilize a Gallery Walk to review the problems and discuss any errors.	Teacher observation; Teacher-made informal assessment
2	a	Play Equation Bingo with the students. Give students the scrambled answers to equations on the overhead for them to randomly place on a Bingo card. Then, you may call out the equations or utilize a Power-point presentation to display the problems. The winner must have the correct answers in a row, column or diagonal. Give a prize as an additional incentive.	Teacher observation
2	a	Reinforce solving multi-step inequalities by placing the steps for solving inequalities on sentence strips. Cut apart each step and place in an envelope. Make enough inequality envelopes so that small groups of students or student pairs must solve at least four of them and order them on chart paper. Have each group report out or do student presentations of their solutions.	Teacher observation ; Teacher-made assessment
2	b	Prepare Absolute Value Inequality Cards that contain the inequality, the graphed solution, and the solution set. As a whole class activity or in small student groups, have the students play concentration with them matching the inequality, the graph and its solution set in set notation.	Teacher-made assessment; Teacher observation
2	c	Give students a relation presented in several forms: a table of values, a set of ordered pairs, and a graph and ask them to identify whether the relation is a function. As an extension, if the relation is not a function, have them tell you how to modify the relation to make it a function, if possible.	Teacher observation; Teacher-made assessment

ALGEBRA I

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	d	Using equations involving rational numbers, such as $y = 0.05x$ to represent the value of x nickels, explore how changes in x affect y. Identify domain as nickels and range as value. Use a T-table to graph the relation and verify with graphing calculator.	Teacher-made assessment
2	e	Give real- world examples of linear functions such as car rental costs vs. mileage; error ranges in polling numbers; and postage rates at various years.	Teacher observation
2	e	As an aid to remembering that the step function can be either a rounding up or rounding down function, use the terms ceiling (rounding up) or floor function (rounding down).	Teacher observation
2	e	. Demonstrate how to write linear equations when given a table of values, the slope and an ordered pair, the x and y intercepts, or when given a graph.	Teacher observation
2	f	Demonstrate all three methods for solving systems of equations e. g. substitution, elimination, and graphing so that students understand when to utilize the methods.	Teacher-made assessments
2	g	Use algebra tiles™ to demonstrate adding, subtracting, and multiplying polynomial expressions. When multiplying expressions, utilize the Box Method.	Teacher-made assessment Teacher-made assessment
2	h	Utilize algebra tiles™ to factor polynomials.	Teacher observation
2	h	Have students to expand the terms of a polynomial into prime factors in order to help determine the GCF.	Teacher observation Teacher-made assessment
2	i,j	Utilize the graphing calculator to help students connect the factors with the x intercepts. Use the quadratic formula to solve quadratic equations. Be sure to link the terms solutions, x intercepts, and factors	Teacher observation Teacher-made assessment

ALGEBRA I

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	j	Use the discriminant to help students determine when a quadratic equation will have rational number solution(s).	Teacher-made assessment
2	j	Have students make up 2 quadratic equations which they must graph, identify the *x* and *y* intercepts, and determine if the solutions are rational or irrational solutions.	Teacher observation
3	a	Relate systems of equations to parallel lines, perpendicular lines or ither by having students to re-write the equations in slope-intercept form and analyze the slope of the lines.	Teacher-made assessment
3	b	Relate income to the number of hours worked in equations such as: $y = 5.25x$ and $y = 15.85x$	Teacher-made assessment
3	b	Use the graphing calculator to compare the change of income (y) as it relates to the change in hourly wage (slope)	Teacher observation; Student response
4	a	Relate formulas such as $C = \pi d$ to the slope by having students measure the circumference and diameter of various container lids. Have the students divide the circumference by the diameter and place the information in a table format.	Teacher observation; Student response
4	a	Measure off a given distance such as 100 ft. With students in groups of three, have them walk the distance and time themselves walking the given distance. Students are to calculate their walking speed by dividing the distance by the time.	Teacher observation; Student response
4	a	Mark off distances such as 100 ft, 60 ft, and 30 ft. Let students use a stopwatch to calculate the time it takes to walk the given distances. Use the data to calculate the students' walking speed.	Teacher observation
4	a	Have students to measure the dimensions of the classroom and use the dimensions to calculate the perimeter and the area of the classroom.	Teacher observation
4	a	As a project, have students to design a floor plan for their dream house. They must calculate the total square footage of the house.	Rubric

ALGEBRA I

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	b	Give pairs of students several sets of coordinates that will form different plane figures (parallelograms, triangles, etc.). Instruct students to utilize the distance formula, the Pythagorean theorem, and the slope formula to determine the type of plane figure involved. They must justify their answer algebraically as well as graphically.	Teacher observation; Rubric
4	c	Use Algebra Tiles™ to solve problems involving perimeter and area.	Teacher-made assessment; Observation
5	a,b,c	Have students collect data such as height, shoe size, arm span, forearm length, foot length, etc. for 10-25 people. Have them to utilize the data to calculate the mean, median, and mode. Relate this to linear equations by having them take two of the gathered data such as height and shoe size, and graph it in a scatter-plot. They can use spaghetti to approximate the line of best fit. Then, use the linear regression feature of the graphing calculator to have them find the slope and y intercept of the line. Have them to write the equations in slope-intercept form, point-slope form, and standard form.	Teacher-made assessment; Observation; Rubric if a long term project
5	c	Give students various data sets to calculate the mean, median, mode, and range. Have them to remove the outliers and then re-calculate the mean, median, mode, and range. Discuss how removing the outliers change the measures of central tendency, if any.	Student work sample; Teacher-made test

GEOMETRY

The Geometry framework provides the minimum competencies required for students to be successful in higher-level mathematics courses. Students should enter Geometry with an understanding and the ability to solve linear equations and graph results, be familiar with quadratic equations, understand the Pythagorean Theorem, be able to identify 2- and 3-dimensional shapes, and be familiar with the basic geometric formulas.

Geometry provides a graphical and visual representation of the mathematical world around us. Students should be given an opportunity to develop spatial sense and an understanding of proofs. Students should also continue to utilize formulas and graph lines.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, and measurement** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district. This course is designed for students who have successfully completed Algebra I. This is a one-credit course.

NOTE: Geometry is essential for college bound students. Many geometric skills evaluated on college entrance exams (SAT/ACT) and the National Assessment of Educational Progress (NAEP) 12th grade assessments are developed in this course.

GEOMETRY

CONTENT STRANDS:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Compute and determine the reasonableness of a result in mathematical and real-world situations with and without technology.

a. Use proportional reasoning to solve for unknown measures in similar polygons.
b. Given exact irrational solutions, determine the best estimation.

ALGEBRA

2. Understand relations, functions, and patterns and analyze change using various geometric properties.

a. Solve problems in the coordinate plane using geometric properties and algebraic formulas such as: identifying parallel lines, writing equations of circles, finding midpoint and distance, and exploring relationships among quadrilaterals.
b. Analyze the effect of changing various parameters on geometric figures.
c. Draw conclusions from investigating the effect of scale factors on perimeter, area, and volume.
d. Recognize and write the equation of a circle in standard form $(x-h)^2+(y-k)^2=r^2$ and identify the center and radius.

GEOMETRY

3. Investigate, apply, and prove properties and theorems from postulates and definitions related to angles, lines, circles, polygons, and two- and three-dimensional figures. Explore applications of patterns and transformational geometry.

a. Explore and determine the relationship between inductive and deductive reasoning using logic and geometric concepts.
b. Develop and evaluate mathematical arguments and proofs to include paragraphs, two-column, and flow chart forms.
c. Analyze properties and determine attributes (including congruence and similarity) of two- and three- dimensional figures (including the Platonic Solids).
d. Identify, classify, explore, and apply angle relationships formed by parallel lines cut by transversals.
e. Examine the relationships between arcs, angles and circles.

f. Identify and investigate altitude, median, angle bisectors, and perpendicular bisectors of triangles.
g. Classify triangles and apply postulates and theorems to test for triangle inequality, congruence, and similarity.
h. Use protractor, compass, straight edge and technology to investigate and construct geometric figures and drawings.
i. Create designs using point, line, and rotational symmetry.
j. Use various representations to understand transformations (reflections, translations, rotations, dilations, and combinations thereof).
k. Use translations and rotations in the creation of tessellations.
l. Identify conic sections based on modeling with a cone.
m. Graph a vector and determine the magnitude and direction of a given vector.

MEASUREMENT

4. Select and apply various strategies, tools, and formulas to calculate length, surface area, volume, and angle measurements.

a. Use arc, angle, and segment relationships to find unknown measures related to circles.
b. Use the appropriate definition, postulate, or theorem to find angle measurements associated with polygons.
c. Select the appropriate formula from special right triangle relationships, geometric means, or trigonometric functions to find missing measurements in right triangles.
d. Solve real-world or mathematical problems including perimeter, circumference, and area.
e. Find surface area and volume of three-dimensional figures, including spheres (with great circles) and hemispheres.

GEOMETRY

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Given two similar polygons, use highlighters to color code corresponding parts. Set up ratios and proportions to find unknown measures.	Teacher observation
2	b	Form a square with string. Measure a side and calculate perimeter and area. Cut the string in half and repeat procedure. Record results and determine the relationship between change in perimeter and resulting area.	Teacher observation Rubric Rubric
3 4	c d	Have students measure the dimensions of the classroom and calculate the length of the diagonal from an upper corner. Then measure the distance and compare with the calculation.	Teacher observation
3	f	Fold different types of triangles to demonstrate medians, altitudes, and bisectors. (Patty paper would work well for this.)	Teacher observation
3	j	Take a picture overlaid on a grid. Have the students dilate their individual grid piece to a specified size so that when the enlarged grids are laid out, the picture has been accurately enlarged.	Project rubric; Observation
3	k	Investigate several designs by M.C. Escher and other tessellation sites and use them to create original tessellations.	Project rubric
4	c	Students will construct a clinometer using a protractor and a plum line. They will then calculate the height of a flagpole or basketball goal using right triangle trigonometry.	Observation and accuracy of the measurement
4	d	Use sample floor plan sketches to calculate the area (and perimeter) of each room. The rooms have unusual shapes to include triangular portions and other polygons. Using a price chart provided, the students complete a cost analysis for flooring and wall covering in each room.	Project rubric

ALGEBRA II

Algebra II builds on earlier experiences with linear equations and functions. The genre of functions expands to include polynomial, exponential, rational, and radical examples. Attention is given to inverses, composition of functions, and families of graphs. Technology is utilized to investigate properties of functions in greater detail. Graphing calculators, spreadsheets, and software aid students in modeling real-world applications.

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Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

Algebra II, a one-credit course, is to be taken by students who have successfully completed Algebra I.

ALGEBRA II

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand relationships among numbers and compute fluently. Verify with technology.

a. Diagram the relationship among the subsets of the complex number system.
b. Find the sum, difference, and product of radical expressions.
c. Examine the cyclic property of the powers of the imaginary unit, *i*.
d. Find the sum, difference, product and quotient of complex numbers and express in simplest form.
e. Add, subtract, and multiply matrices.
f. Calculate the determinant and inverse of a matrix.

ALGEBRA

2. Use algebraic concepts to identify patterns, use multiple representations of relations and functions, and apply operations to expressions, equations, and inequalities.

a. Solve compound and absolute value inequalities and represent solutions using interval notation and graphing.
b. Solve systems of linear, absolute value, and quadratic equations using various methods, such as substitution, elimination, and matrices. Verify with technology.
c. Translate a word problem into a system of algebraic equations.
d. Given the constraints, find the maximum and minimum value(s) of a system of linear inequalities.
e. Add, subtract, multiply, divide, and simplify polynomial functions.
f. Perform operations on polynomial expressions including both long division and synthetic division.
g. Write an equation of a quadratic using sum and product of roots.
h. Write the equation of a polynomial when given its roots including complex roots.
i. Use the discriminant to classify solutions of quadratic equations and justify your answer.
j. Select the appropriate rule to factor perfect square trinomials, sums and differences of cubes, and polynomials by grouping.
k. Solve quadratic equations by factoring, completing the square, and using the quadratic formula.
l. Solve radical equations.

m. Write equivalent forms of rational expressions using real and complex conjugates.
n. Simplify and perform operations on rational expressions including complex fractions.
o. Solve equations involving rational expressions and verify solutions.
p. Convert between exponential and logarithmic forms.
q. Select the appropriate exponent rule or logarithmic property to simplify exponential and logarithmic expressions.

GEOMETRY

3. Use coordinate geometry to specify locations, describe relationships, and apply transformations to analyze algebraic relationships.

a. Determine domain and range from the graphs of relations (including functions).
b. Determine whether the inverse of a relation or function exists and justify the answer.
c. Recognize and sketch graphs of functions such as linear, quadratic, absolute value, greatest integer, piece-wise, etc.
d. Sketch and describe transformations of functions such as linear, quadratic, absolute value, etc., using technology.
e. Graphically represent complex numbers and the sum of complex numbers in a complex coordinate plane.
f. Find the composition of two or more functions.
g. Identify and sketch the essential graphs of the four conic sections: circle, parabola, ellipse, and hyperbola.

MEASUREMENT

4. Understand measurable attributes of objects and apply appropriate techniques and formulas to determine measurements.

a. Use dimensional analysis to solve real-world problems.
b. Solve mathematical or real-world problems involving plane and solid figures.
e. Solve absolute value inequalities to describe the accuracy of measurements in real-world situations.

DATA ANALYSIS & PROBABILITY

5. Use technology to represent, analyze, and make inferences based on data.

a. Use scatter plots and apply linear and quadratic regression analysis to data using technology.
b. Solve simple combinations.
c. Apply the appropriate formula to basic application problems involving probability.
d. Use permutations and combinations.
e. Collect, analyze, and compare data sets.

ALGEBRA II

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	In groups, students will write on index cards one number in each set and subset of the complex number system including complex, real, pure imaginary, rational, irrational, integer, whole, and natural. Shuffle the cards and create a tree diagram for display.	Teacher observation
1	b	Use the Box Method to demonstrate how to multiply radical expressions.	Teacher observation
1	b	Prepare cards with radical expressions on them. Pairs of students are to pull two cards and perform operations such as +, -, x with the two cards. Have each student pair to present their solutions to the class. Have students to find the perimeter and area of polygons using radical terms as dimensions.	Teacher-made assessment; Teacher observation
1	c	Simplify powers *i* through at least three cycles. Let student pairs explain and discuss the pattern. Let students simplify powers of *i* and construct a power of *i* to given specifications (Ex. Find a power of *i* that is equal to -*i* with an exponent between 240 and 250. There may be more than one answer.).	Teacher observation; Student response
1	d	In small groups or pairs compare and contrast the multiplication of binomials with the multiplication of complex numbers. Groups/pairs will list the similarities and differences and share out in class. Prepare cards with complex number expressions on them. Pair students and instruct them to pull two expressions and perform all four operations on the expressions. Have the pairs to chart the problems and present to the class.	Teacher observation
1	e	Give students the values for matrices A, B, and C. In pairs, let students perform the following operations: A+B and B-A, AB and BA, (A+B)+C and A+ (B+C), and A(BC) and (AB)C. Utilize these matrices to teach/reinforce that all real number properties do not apply to matrix operations. Verify with technology using the matrix operation of the graphics calculator.	Teacher observation

ALGEBRA II

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	f	Draw a rectangle in the coordinate plane with one vertex at the origin. Label the vertices of the rectangle. Set up a 2 x 2 matrix using the two points adjacent to the origin with the x-values in the top row and the y-values in the bottom row. Find the determinant of the matrix using the following formula: $\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$ Find the area of the rectangle and compare to the determinant. Try with a parallelogram (let one side of the parallelogram be on the x-axis).	Student work; Teacher observation
1	f	Use the graphing calculator to find the inverse of a 2 x 2 matrix and verify using the following formula: $A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	Student work
2	a	Prepare cardstock sets of compound inequalities with their solutions presented in three forms: interval notation, graphs, and set notation. Play Concentration where students must match the inequality with its solution set, interval, and graph.	Teacher observation
2	a	Emphasize the definition of absolute value as the "distance from zero". Ex. $\lvert 2x + 3 \rvert < 4$ is translated to '2x + 3 is less than 4 units from zero'. Less than 4 units from zero must be between –4 and 4 which translates to $-4 < 2x + 3 < 4$. Likewise, $\lvert 2x + 3 \rvert > 4$ is translated to '2x + 3 is more than 4 units from zero'. More than 4 units from zero must be less than –4 or greater than 4 which translates to $2x + 3 < -4$ or $2x + 3 > 4$. Compare the graphic interpretation of the solution to the interval notation.	Teacher observation; Student work sample
2	b	Use the graphing calculator to examine the graphic solutions to a variety of problems. Graph both sides of the equation as separate functions and examine the points of intersection. When graphing absolute value equations, graph both sides of the equation and also graph the inequality using the TEST menu.	Teacher observation

ALGEBRA II

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	c	Divide students into groups. Use index cards with the words or phrases of a problem on one side and the algebraic expressions on the other to translate from words to symbols.	Teacher observation
2	e	Groups of four will be given 2 sets of index cards. Set 1 will contain 8 cards with simple functions; each person will receive 2 cards. Set 2 will contain the function operations add, subtract, multiply, divide, simplify and compose functions. An operation card will be turned over. Each person will perform the function operation with his/her 2 cards. Each person with a correct answer will get a point. Function cards may be redealt after each hand.	Teacher observation; Teacher-made assessment
2	f	Be sure to stress the process of division using the following jingle: Divide, multiply, subtract, bring down, and repeat. Have students to check their division by multiply the quotient times the divisor and add the remainder. This is good reinforcement for multiplication of polynomials and combining like terms.	Teacher-made assessment
2	g,h	Make index cards that contain the sum and products of roots. Have student pairs to write the polynomial equation that is exemplified by the sum and product of the roots. Have them check their answers using synthetic division.	Teacher-made assessment
2	i	Use a Semantic Feature Analysis chart that contains various quadratic equations that the students must classify the number of roots and whether the roots are complex, real, rational, irrational, etc.	Teacher-made assessment
2	j	Emphasize methods such as Tic-Tac-Toe for factoring polynomials.	Teacher-made assessment
2	k	Utilize the graphing calculator to demonstrate why some polynomials are prime and to verify algebraic methods such as the use of the quadratic formula.	Teacher observation; Teacher-made assessment
2	l	Solve radical equations on sentence strips. Cut the strips apart, mix them up, and place them in envelopes. In small groups, have students to order the steps.	Teacher observation assessment
2	m,n	Prepare a set of 10 rational expressions to include all four basic operations. Set up a relay system with groups of four students with one student working one step of the process and passing it on to the next student. When completed, the group leader brings the completed problem to the teacher for verification. If the answer is incorrect, the group must collaborate and figure out the error without teacher direction.	Teacher observation

ALGEBRA II

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	o	Compare solving equations with rational expressions to solving proportions. For example, use the proportion $\frac{7}{4}=\frac{x}{2}$ and compare the steps to solving rational equations such as $\frac{x-3}{x+1}=\frac{x-2}{x-1}$. Have the students to utilize the same steps to solve for x in the rational equation. Point out to students that more complex rational equations such as $\frac{6}{x-1}-\frac{3}{x+1}=\frac{3}{x}$ involve simplification of the left hand side of the equation prior to utilizing the cross multiplication exemplified in solving proportions.	Teacher observation; Teacher-made assessment
2	p	To emphasize the relationship between logarithmic and exponential forms, prepare a table containing exponential forms and logarithmic forms where the students must fill in either the exponential form when given the logarithmic form and vice-versa.	Teacher observation; Student response
2	q	Use technology to reinforce logarithmic rules. For example, in $\log_a x + \log_a y = \log_a (xy)$, assign values for *a*, *x* and *y* like a = 3, *x* = 9 and *y* = 27. Then use calculators to show that $\log_3 9 + \log_3 27 = \log_3 3^2 \cdot 3^3 = 2 + 3$.	Teacher observation
3	a	Prepare examples of the four view of functions e.g. table, graph, set of ordered pairs, and rule so that students can determine the domain and range in different views.	Teacher observation
3	b	Use the graphing calculator to verify if two functions are inverses by inputting *f(g(x))* in *y* = to see if you obtain *y=x*. If not, the two functions are not inverses.	Teacher observation
3	c,d	Given the transformation of parent graphs, match the graphs of functions with the word descriptions of the functions.	Teacher observation
3	e	Using the Cartesian coordinate plane, plot 2 complex numbers, add them and plot the sum. Then draw three vectors for the origin to each complex number and complete a parallelogram. The sum is the diagnosed of the parallelogram.	Teacher observation
3	f	To find the composition of two functions *R(x)* and *G(x*), where *R(x)* is written in red and *G(x)* is written green colored markers in the board, ask the students to find *R(G(x))* and *G(R(x))* coded in the appropriate colors.	Teacher observation; Student response

ALGEBRA II

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	g	Given a list of quadratic equations, determine the type of conic section. Write each equation in standard form and identify specific characteristics. Graph each conic section with and without graphing calculator.	Teacher observation; Teacher-made test
4	a	Use dimensional analysis to solve conversion problems or to verify the solution for a problem that involves conversions.	Teacher observation; Teacher-made test
4	c	Write statements involving inequalities and absolute values that model finding the gas tank capacity, average city miles per gallon, and highway miles per gallon of a car.	Rubric
5	a	Make cards with 3 non-collinear pints on them so that each pair or trio of students has at least three different cards. Use the STAT menu of the graphing calculator to perform the quadratic regression analysis for the three points.	Teacher observation; Teacher-made assessment
5	a	Make cards with 3 collinear points so that each pair or trio of students has at least three different cards. Use the STAT menu of the graphing calculator to perform the linear regression analysis for the three points.	Teacher observation; Teacher-made assessment
5	b,d	Give students 5 cubes of different colors. Have them make as many combinations of 2 colors as possible. (10) Have students draw two blank cards to symbolize the cubes to be chosen. Ask them how many choices for the 1^{st} cube (5) and for the 2^{nd} cube (4). Write in the blank: 5 x 4 = 20. Point out to students that a red/blue combination is the same as a blue/red combination (since with combinations order is not important) so we divided by 2 to obtain 10.	Teacher observation
5	c	Utilize the random number generator function of the graphing calculator to simulate tossing coins with a group of 3. Put students in groups of 3 with one student, operating the calculator, another tallying the results of the calculator and with the third keeping track of the size of the trial. Have each group randomly generate 0 and 1 by entering int (2*rand) and pressing enter. Each group records the number generated and keeps a tally of the size of the trial. You may want to assign different group different size trials. Lead a group discussion of the relationship between the sample size and the theoretical probability.	Teacher observation
5	e	Get the height and ring finger size of each member of the class. Organize the data and use the statplot feature on the graph to find the correlation of the data, choose a couple of students to compare the line of best fit.	Teacher-made assessment

ADVANCED ALGEBRA

Advanced Algebra requires skills developed in earlier courses to investigate advanced topics such as conic sections, higher order polynomials, matrices, functions, and probability. The use of technology, especially graphing calculators, should be an integral part of this course. Prerequisites for this course include Geometry and Algebra II. This is a one-half credit course.

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ADVANCED ALGEBRA

Content Strands:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Understand and perform computations with different representations of numbers.

a. Express a series using summation notation.
b. Evaluate the sum of a series.
c. Find the determinant of a 4 x 4 matrix using expansion by minors.
d. Find the inverse of a matrix using technology.

ALGEBRA

2. Use algebraic concepts to identify patterns, use multiple representations of relations and functions, and apply operations to expressions and equations.

a. Determine domain and range of functions.
b. Find the sum, difference, product, and quotient of functions noting any restrictions on the domain.
c. Perform the composition of functions.
d. Write the inverse of a function.
e. Determine if two functions are inverses of each other.
f. Expand binomials using Pascal's triangle and the Binomial Theorem.
g. Write the equations of conic sections given essential information.
h. Solve a system of equations using inverse matrices, augmented matrices, and Cramer's Rule.
i. Solve real-world applications using linear programming.
j. Solve linear-quadratic and quadratic-quadratic systems of equations and inequalities.

GEOMETRY

3. Recognize, analyze, and graph conic sections.

a. Recognize conic sections by their graphs and equations.
b. Identify the essential features of each conic section from standard form.
c. Sketch the graph of each conic section from its equation.

DATA ANALYSIS & PROBABILITY

4. Apply simple probability and curve fitting to data.

a. Solve problems involving permutations and combinations.
b. Determine quadratic and cubic regression equations for given data using technology.

ADVANCED ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	c	Place students in small groups. Have each student to write the rules on how to find determinates using expansion by minors. Once you are sure their rules are correct, give them 4x4 matrices on index cards and tell each member of the group to complete a step in the group rules. Have them use different colored highlighters/pens to identify their step, and then have them review each other's work to ensure correctness. Follow up with having them write a summary of the rules and their group's experiences working on their project. Use graphing calculators to find determinants of matrices, allow students to work in groups. Have each student write the process of how the graphing calculator was utilized.	Teacher observation; Student work
1	d	Have students use a graphing calculator to input a matrix, and then have students use the graphing calculator to find the inverse of the matrix. Have students write about the changes in the matrix and it's inverse and what part does the determinant play in this change. Then define an inverse matrix.	Student work
2	a	Place students in small groups, and then give each group a set of functions. (Be sure to include functions that do not have all real numbers for domain/range.) Have each group predict and write down what they think the graph of each function looks like. Then, have students enter the functions into the graphing calculators. Once the group has viewed the graph in the calculator and the table of values, have them write down any changes or similarities they see for each function, compared to the predictions they made earlier. Have each group address questions such as, "Are you viewing the entire graph?", "What values are not part of the domain and range?", "How can you predict what the domain and range of an equation are?", "Are all real numbers included in the domain and range of each function? Why/Why not?"	Teacher observation; Student work

ADVANCED ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	b,c	Have students write what they know about substitution and combining like terms. Review those rules with them, and allow them to make any corrections to their writing. Have them create a few problems of their own, and ask a few to share or put theirs on the board. Then, put them in small groups and give each group several index cards with functions on them, and several cards with basic operations on them. Have each group to randomly pull two functions from the function stack and randomly pull an operation from the operations stack. Have them to evaluate and discuss their answers, and then repeat the process without replacing the cards to their stacks. (Be sure to have enough functions and at least the four basic operations).	Teacher observation; Student work
2	d,e	Place functions that do/do not have inverses on index cards or small strips of paper. Ask students to work in small groups to determine which functions do/do not have inverses. Then, have the group prepare a report on their findings.	Teacher observation; Student work
2	f	Give example involving baseball player's batting average. In small groups, have the students work problems involving batting averages and coin tossing using the Binomial Theorem and Pascal's Triangle. (Have them determine the probability of getting at least 3 hits in next 5 times at bat)	Teacher observation; Student work
2	g	On index cards, describe conics (parabola, circle, ellipse, hyperbola). Have students to write the equation about each.	Teacher observation; Student work
2	h	Have students use Cramer's Rule to show how each determinate is computed using a graphing calculator to confirm each determinate. Have students show steps in solving augmented matrices, and use calculators for inverse matrices.	Teacher observation; Student work
2	i	Create, construct, and solve a linear programming problem with at least four equations.	Teacher observation; Student work

ADVANCED ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	j	Give students systems of linear equations to solve, encourage/require use of graphing calculators to confirm answers. Then give students systems of linear-quadratic equations and quadratic-quadratic equations and inequalities to solve by use of a graphing calculator. Have the students write the steps used in the calculator to solve the systems of equations.	Teacher observation; Student work
3	a,b	Given a list of quadratic equations, have students determine the type of conic section and justify their reasons. Given several graphs, have students determine the type of conic section and justify their reasons. Write each equation in standard form and identify specific characteristics of each equation/graph.	Teacher observation; Student work
3	c	Graph parent conic sections and identify their equations and predict translations, verify using a calculator.	Student work sample
3	c	Give students conic sections by equations and ask them to sketch the graph on graph paper. Verify using a graphing calculator.	Teacher observation; Student work
4	a	Give students the lunch menu for the day, and ask them to determine the number of possible combinations of meals. Verify using graphing calculators.	Teacher observation; Student work
4	a	Have students create their own “deli” and determine the number of meal choices from their created menus. Verify using graphing calculators.	Teacher observation; Student work
4	a	Ask students to pick a committee of five members from their classmates and to record the order in which they were picked. Then ask the students to pick the same five students for the committee, but in a different order and to record the order. Ask the students if the order the students were picked changed the committee members. Then ask the students to pick five students to place in line for front row seating at their favorite concert. Have them record the order they placed them in line. Then have them pick the same five students again, but in a different order and record that order in line for tickets. Ask the students if the order the students were picked changed the place in line that they were, and did it matter to those five in line? What if they were the last five in line and not the first five in line, would order matter to them now? Have students discuss and write why they think order matters/doesn’t matter and how they can tell when it does/doesn’t. Have the students exchange papers and pair up for discussions of their writings.	Teacher observation; Student work

ADVANCED ALGEBRA

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	a	Give the students problems on permutations and combinations, and allow them to use the calculator to solve them. Have them write the steps used in the calculator to solve the problems.	Teacher observation; Student work
4	b	Have students work in small groups and assist each other in using the graphing calculators to determine quadratic and cubic regression equations for given data. Require the students to discuss their decisions and justify their answers.	Teacher observation
4	b	Using a graphing calculator, use curve fitting to find the equation of the curve of best fit containing three or more non-linear points. Make predictions using the equation and the graph.	Teacher observation; Student work sample
4	b	Students will plot their shoe size and wrist measurement on a large graph. After drawing the line of best fit, predict a professional athlete's wrist size based on a given shoe size.	Teacher observation; Student work sample

TRIGONOMETRY

Trigonometry builds on a well-developed geometry and algebra background to explore the study of unit circles and triangles. Trigonometric functions, their properties, and graphs are analyzed and studied in the context of real and complex numbers. Graphing calculators and software aid students in the analysis and application of concepts. Trigonometry, a one-half credit course, is taken by students who have successfully completed Algebra II and Geometry and is a pre-requisite for Calculus.

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Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

TRIGONOMETRY

CONTENT STRANDS:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Represent and compare numbers in various forms and perform operations.

a. Convert from polar to rectangular coordinates and vice versa.
b. Convert from rectangular to trigonometric form and vice versa.
c. Determine the product and quotient of complex numbers in trigonometric form.
d. Determine the power and roots of complex numbers using DeMoivre's Theorem.

ALGEBRA

2. Compare and produce equivalent forms of trigonometric expressions and solve trigonometric equations.

a. Convert between degree and radian measurements of angles.
b. State and utilize trigonometric identities.
c. Verify identities analytically and with technology.
d. Solve trigonometric equations using both radians and degrees.

GEOMETRY

3. Use geometric modeling to analyze trigonometric relationships.

a. Identify and locate angles in radians and degrees based on the unit circle.
b. Define and apply the six trigonometric functions in relation to a right triangle.
c. Find exact values of trigonometric functions of special angles in the unit circle.
d. Recognize, sketch, and interpret the graphs of the six trigonometric functions and their inverses to include restrictions on their domains.
e. Recognize, sketch, and interpret graphs illustrating transformations of trigonometric functions.
f. Solve for unknown parts of triangles using the Law of Sines and the Law of Cosines.
g. Graph polar coordinates and equations.

MEASUREMENT

4. Select and apply formulas to determine length and area.

a. Find arc length and sector area of a circle.

b. Find the area of a triangle using Heron's Formula, $(\frac{1}{2})ab\sin C$, and/or $(\frac{1}{2})c^2\frac{\sin A\sin B}{\sin C}$.

TRIGONOMETRY

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Use a transparency for a polar graph and place a rectangular graph with common units on top of it to illustrate.	Teacher observation
1	b	Show complex numbers in the form $x + yi$ and remind students that the cosine function relates to x and the sine function relates to y. Therefore $x + yi = r (\cos \theta + i \sin \theta)$	Demonstration
1	c	Review laws of exponents to relate products and quotients in trigonometric forms. For example in $(2x^3)(3x^4)$ we multiply numbers and add the exponents. In $3(\cos 30° + i \sin 30°) \bullet 2 (\cos 60° + i \sin 60°)$ we multiply radii and add angles. (Of course we simplify the product to rectangular form if necessary.)	Teacher observation
1	d	Have students expand and simplify $(3 + 2i)^6$ algebraically. Then to emphasize the value of De Moivre's Theorem, change the problem to polar form and simplify using DeMoivre's Theorem.	Demonstration and observation
2 3	a a	Using a protractor and a paper plate, show the multiples of 30°, 45°, 60°, and the quadrantals for all key angles in the circles. Label in degrees and radians.	Student work sample
2	c	Using graphing calculators have students pick several angles in the domain of given functions. Substitute each angle into given identities to show that the identities are true.	Student work sample
2	d	Use technology to verify answers. For example in the equation $\cos 2x + 3\cos x = -2$, Let $y_1 = \cos 2x + 3 \cos x$ and $y_2 = -2$. Graph and use the calculate intersect feature to compare the "paper and pencil" solution to the graphic solution. Use degrees instead of radians on the interval [0°, 360°).	Demonstration
3	b	Review the 30°- 60°- 90° and the 45°- 45° - 90° special right triangles. Define the 6 trigonometric functions for each special triangle using the acronym Oscar Had A Heap Of Apples.	Demonstration
3	c	Use the acronym, All Students Take Calculus, for finding the signs of the six trigonometric functions in all four quadrants. Begin with "A" in the first quadrant, all functions are positive. With "S" in the second quadrant, only sin x and its reciprocal are positive. With"T" in the third quadrant, only tan x and its reciprocal are positive. With "C" in the fourth quadrant, only cos x and its reciprocal are positive.	Demonstration

TRIGONOMETRY

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	d,e	Use graphing calculators to show the basic graphs, their inverses, and to illustrate transformation.	Demonstration
3	f	Have students go throughout the building or outside and illustrate non-right triangles. Then use Law of Sines and Law of Cosines to solve for unknown parts. Afterward have them measure all parts to verify.	Student work sample
3	g	Have students research and give examples of things that can be modeled with polar coordinates. (like satellites). Illustrate the path using polar graph paper.	Rubric
4	a	Bring 2 or 3 pies of different radii. Use the arc length formula to find the crust length and the area of a sector formula for several pieces of pie of different sizes. (Estimate the central angle in radians.) Eat the pie.	Demonstration

PRE-CALCULUS

Pre-Calculus covers those skills and objectives necessary for success in calculus. Topics of study include sequences and series, functions, and higher order polynomials. These topics are addressed from a numeric, graphical, and analytical perspective. Technology is to be used to enhance presentation and understanding of concepts. Pre-Calculus, a one-half-credit course, is taken by students who have successfully completed Algebra II and Geometry and is a prerequisite for Calculus.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

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PRE-CALCULUS

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks**:**

NUMBER AND OPERATIONS

1. Illustrate and explore the characteristics and operations connecting sequences and series.

a. Express sequences and series using recursive and explicit formulas.
b. Evaluate and apply formulas for arithmetic and geometric sequences and series.
c. Evaluate and apply infinite geometric series.
d. Use mathematical induction in proofs.

ALGEBRA

2. Analyze, manipulate, and solve equations and inequalities.

a. Determine characteristics of the graphs of parent functions (domain/range, increasing/decreasing intervals, intercepts, symmetry, end behavior, and asymptotic behavior).
b. Determine horizontal, vertical, and slant asymptotes and holes of rational functions.
c. Determine the domain and range of piece-wise functions.
d. Determine the end behavior of polynomial functions.
e. Decompose composite functions into component functions.
f. Solve exponential and logarithmic equations to include real world applications.
g. Find the possible number of rational roots using the Rational Root Theorem.
h. Find the zeros of polynomial functions by synthetic division and the Factor Theorem.
i. Graph and solve quadratic inequalities.
j. Factor using advanced techniques to include rational and negative exponents and non-standard difference of squares.
k. Decompose a fraction into partial fractions.

GEOMETRY

3. Recognize, sketch, and transform graphs of functions.

a. Recognize the graphs and the general equations of the parent functions (linear, quadratic, cubic, absolute value, rational, exponential, logarithmic, square root, cube root, and greatest integer).
b. Perform translations, reflections, and dilations on parent functions.
c. Graph exponential, logarithmic, rational, and piece-wise functions with and without technology.

MEASUREMENT

4. Use formulas to determine characteristics of geometric figures.

a. Apply area and volume formulas to various shapes.

DATA ANALYSIS AND PROBABILITY

5. Adapt curves to data.

a. Fit exponential and logarithmic regressions to data using technology.
b. Fit cubic regressions to data using technology.

PRE-CALCULUS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	On the first day of January, Bob ate one candy bar. Each day thereafter, he ate one more candy bar than the previous day. Determine the number of candy bars he ate during the month of January.	Class discussion
1	a	Find the number of calories in a candy bar of your choice. Calculate the caloric intake for that month. Estimate the possible weight gain by the end of the month.	Class discussion
1	a	Count the spirals (left and right) on a pinecone, pineapple, or artichoke. Illustrate that the number of spirals are numbers in the Fibonacci Sequence. Write the first ten terms of the sequence, and have the students write recursive formulas for each term.	Teacher observation
1	b	Tear a square piece of paper with an area of one, in half. Tear it in half again. Predict the area of one of the resulting rectangles after six tears.	Demonstration
1	c	Divide the class into groups and provide each group with a ball. As the ball is thrown or dropped, use technology (Computer Based Learning) to record the path of the ball.	Teacher observation
1	d	Use mathematical induction to prove that a formula is valid for all positive integral values of n.	Teacher evaluation
		Journal Write: In your own words, explain what is meant by a proof by mathematical induction. Give an example.	Teacher evaluation
2	a	Given a function, predict the domain and range. Enter the function on the graphing calculator. Use a piece of spaghetti to find the domain and range. Compare the prediction to the spaghetti results.	Teacher evaluation
2	a	Using the graphing calculator, determine the end behavior of a function and write a paragraph explaining the results. Discuss how the degree of the function affected the end behaviors. Create a spreadsheet of values to determine the end behavior of a graph.	Teacher observation
2	a	In small groups, determine if families of functions have the same symmetry as the parent function $f(x) = 1/x$ and justify the answer.	Discussion
2	a	Using a graphing calculator, determine the intercepts of a function. Discuss why the intercepts of a function should be found prior to sketching the function. Discuss what zeros of the function are.	Teacher observation

PRE-CALCULUS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	b	Using a graphing calculator, determine asymptotes of a function and write a paragraph explaining the results. Discuss how the vertical and horizontal asymptotes are determined.	Teacher evaluation
2	b	Using a graphing calculator, graph a rational function like $f(x) = (2x^2 + x - 1)/(x^2 - 1)$. Label the vertical asymptote and the hole. Analyze the function algebraically by factoring the numerator and denominator. Identify the vertical asymptote and hole, and compare the results to those on the graphics calculator.	Discussion
2	c	Using a graphing calculator, determine domain and range of a piece-wise function. Discuss how to determine domain and range of a piece-wise function, and how to use tables, graphs, etc of the graphing calculator to do so.	Teacher evaluation; Discussion
2	d	Using a graphing calculator, determine end behavior for several different polynomial functions. Give students different polynomial functions and ask them to describe the end behavior of that function and the equation of the function.	Teacher evaluation; Discussion
2	d	Using the graphing calculator, determine the end behavior of a function and write a paragraph explaining the results. Discuss how the degree of the function affected the end behaviors. Create a spreadsheet of values to determine the end behavior of a graph.	Rubric; Teacher evaluation
2	e	Discuss what composite functions are and write a paragraph describing your conclusions. Using a graphing calculator, graph composite functions and their inverses. Describe in your paragraph any similarities/dissimilarities you may see.	Discussion
2	e	Fold graph paper about the line $y = x$. Draw the graph of any function. Trace the graph on the other side of the fold to reveal the inverse.	Teacher evaluation
2	e	Using the composition $h(x) = f(g(x))$ where $h(x) = \sqrt{x^2 - 4}$, decompose $h(x)$ into the two components $f(x)$ and $g(x)$. Have students find more than one correct answer.	Teacher evaluation

PRE-CALCULUS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	f	In small groups, create, explain, and verify specific examples for each of the properties of exponents.	Student work
2	f	Compare the relationships of a logarithmic function and the inverse of an exponential function. Write an equation in one form, exchange papers, and write the inverse form.	Teacher evaluation
2	f	Solve growth and decay problems involving half-life using logarithms.	Teacher evaluation
2	f	The population of Jackson, MS (in thousands) is given by the exponential formula $P = 220e^{kt}$, where $t = 0$ is the year 2000. In 1980 the population was 169,500. Find the value of k, and use the results to predict the population in 2020.	Teacher evaluation
2	g	In small groups, discuss what the rational root theorem is and when to use it. Write a paragraph about the group discussion. Give each group a polynomial function and ask them to use the rational roots theorem, and each member of the group must perform a step in this process. They may report their findings to the class in a variety of ways.	Teacher evaluation; Student work
2	g	Use the table features of a graphing calculator to determine which of the possible roots are zeros of the polynomial $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$ or other given polynomials. Set the table feature on "ASK" mode and enter each of the possible roots to find the zeros.	Teacher observation
2	h	In small groups, write a paragraph containing the rules of synthetic division and the Factor Theorem. Using these rules, give each group polynomial functions and require them to use both synthetic division and the Factor Theorem to solve for zeros of the polynomial functions. Also have the students compare the synthetic division to long division, and compare the two methods.	Student work; Teacher evaluation
2	h	Create and graph an equation of a polynomial function. Write a paragraph explaining the zeros of a function and how to determine where they are located on the graph.	Teacher evaluation
2	i	In small groups, create, solve, and graph rational and polynomial inequalities. Extend to graphing systems of inequalities by shading solutions with colored pencils. Given a function, find the maximum and minimum of the shaded region.	Teacher evaluation
2	i	Given a quadratic inequality, find and verify the values of x and express the solution in inequality notation, interval notation, and graphically.	Teacher observation

PRE-CALCULUS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	j	In small groups, factor using rational and negative exponents, and then factor with non-standard difference of squares. Write a paragraph explaining each process.	Teacher evaluation
2	k	In small groups, give students index cards that contain an integral with rational functions. Decompose the fraction into partial fractions. Write each step omitting parts of that particular step, and leave the final answer blank. Ask each group to fill in the missing pieces of information. Give the groups several of these, with different pieces missing for each integral. Each student should write a paragraph explaining how the group answered each step.	Student evaluation
2	k	Write the partial fraction decomposition of the rational expression $1 / (a^2 - x^2)$. Check your results algebraically. Then assign a value to the constant a, and check the result graphically.	Teacher observation
3	c	Using a graphing calculator, graph exponential, logarithmic, rational, and piece-wise functions and then sketch on graph paper. Discuss what characteristics of the graph in your calculator you used to begin sketching on paper. Would you use those same characteristics if you had not seen the graph in the calculator?	Teacher evaluation
4	a	In small groups, give students various shapes and require that the area and volume of each shape be to be found. Require students to show process of solving for each. Show groups (and pass around) area and volume representation with real life examples. (boxes, cylinders, cones, etc).	Teacher evaluation
4	a	Find the surface area and volume of a right circular cylinder. Have the students make a vertical cut of a toilet paper roll or paper towel roll, and with the resulting rectangle and circles compose their own formulas for the surface area and volume.	Teacher observation
5	a,b	Give students data from application problems. Use the graphing calculator STAT PLOT feature to fit the data to exponential, logarithmic, and cubic regressions.	Student evaluation

DISCRETE MATHEMATICS

Discrete Mathematics is the study of processes that involve sequences of individual or countable steps as opposed to the study of continuously changing processes addressed in Calculus. Topics of study include number systems, logic of compound statements, mathematical induction and recursion, graph theory and set theory. Discrete Mathematics, a one-half credit course, is designed to provide students who have completed Geometry and Algebra II with an overview of concepts needed for computer science, electrical engineering, or fields requiring networking.

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DISCRETE MATHEMATICS

CONTENT STRANDS:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

NUMBER AND OPERATIONS

1. Explore the relationships among number systems and use matrix operations to analyze graphs.

a. Convert decimal numbers to binary numbers and vice versa.
b. Convert decimal numbers to hexadecimal and vice versa.
c. Convert between decimal, binary and hexadecimal numbers.
d. Add and subtract binary numbers.
e. Determine the number of walks in an undirected graph of length *n*.

ALGEBRA

2. Use algebraic methods to represent simple and complex relationships among statements and use models to represent patterns and operations.

a. Define sentence (proposition), true and false in relation to logic.
b. Define the simple compound statements: negation, conjunction, disjunction, contradiction, and tautology using truth tables.
c. Define a conditional statement using truth tables.
d. Define the inverse, converse, and contrapositive of a conditional statement.
e. Apply the principles of logic to determine the validity of arguments.
f. Define a sequence recursively and explicitly.
g. Find the explicit formula for a recursively-defined sequence using iteration.
h. Use mathematical induction to verify explicit formulas for arithmetic, geometric, and other sequences and/or series.
i. Define terminology and symbols associated with sets.
j. Perform basic operations, unions, intersections, differences, and complements.

GEOMETRY

3. Use geometric models to describe and analyze mathematical relationships, establish the validity of conjectures, and determine solutions to real applications.

a. Construct a logic circuit from a Boolean expression to determine output.
b. Construct a Boolean expression given a logic circuit.
c. Construct a logic circuit and Boolean expression given an input/output table.
d. Use Venn diagrams to represent basic operations on sets.

e. Define basic terminology associated with graphs.
f. Determine the number of vertices and edges in a graph.
g. Determine walks, paths, and circuits in a graph.
h. Construct walks, paths, and circuits given an edge/vertex string.
i. Determine whether Euler and Hamiltonial circuits exist in a given graph.
j. Construct a graph given the adjacency matrix of the graph and vice versa.
k. Determine connectivity of a graph using an adjacency matrix.
l. Determine the number of walks between two vertices using powers of the adjacency matrix.
m. Determine whether a graph is a tree.
n. Determine the level, parent, siblings, ancestors, descendants and height of a rooted tree.
o. Determine the shortest route in a spanning tree.

DISCRETE MATHEMATICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a,b,c	Write numbers in different bases in terms of a polynomial representation for quick recognition. $a_nx^n + a_{n-1}x^{n-1} + \ldots\ldots + a_2x^2 + a_1x^1 + a_0x^0$ this polynomial can be used to represent any number where x is equal to the base in question such as two, eight or ten, and the a_n's are numerals used in said base. Example: Write 2743 in polynomial form. $2\ (10)^3 + 7(10)^2 + 4(10)^1 + 3(10)^0$ Where $a_0 = 3$, $a_1 = 4$, $a_2 = 7$, and $a_3 = 2$	Teacher observation; Student response
2	e	Provide students with sample arguments, and have the students decide if these arguments are valid using the rules of logic.	Teacher observation; Student response
2	g	Have students use a recursive definition of multiplication such as f (1) = 3, and f(n) = 3 + f(n-1), to see that multiplication can be defined as repeated addition. Use a similar idea to see that powers can be defined as repeated multiplications and in turn repeated additions.	Teacher observation; Student response
2	j	Use different identifying properties to create sets of students found in the class, such as color of clothing, wearing earrings, wearing glasses, etc. from these sets have students determine the elements of unions, intersections, differences, and complements of sets.	Teacher observation; Student response
3	a,c	Use a circuit board to build simple circuit from a Boolean expression and note the output.	Teacher observation; Student response
3	b,c	Provide the students with a simple circuit on a circuit board and have the students write down a Boolean algebra statement for the circuit.	Teacher observation; Student response
3	f	Provide students with examples of platonic solids, and the students count the number of edges and vertices. From several examples have the students write a general statement about the number of edges and vertices: Euler's Formula.	Teacher observation; Student work sample

CALCULUS

Calculus is the study of the mathematics of change. The major focus is on differential and integral calculus. The use of graphing calculators and other technologies are integral parts of the course. This one-credit course is designed for the student who has a thorough knowledge of college preparatory mathematics.

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CALCULUS

CONTENT STRANDS:

Number and Operations
Algebra
Geometry
Measurement
Data Analysis & Probability

Competencies and Objectives/Benchmarks:

1. Demonstrate basic knowledge of functions, their behavior and characteristics.

a. Predict and explain the characteristics and behavior of functions and their graphs.
b. Investigate, describe, and determine asymptotic behavior.
c. Discuss and determine continuity and discontinuity of functions.
d. Analyze parametric, polar, and vector functions.

2. Evaluate limits and communicate an understanding of the limiting process.

a. State and apply properties of limits.
b. Calculate limits using algebra.
c. Estimate limits from graphs or tables of data.
d. Verify the behavior and direction of non-determinable limits.
e. Use L'Hopital's Rule to evaluate simple indeterminate forms.
f. Apply L'Hopital's Rule to determine convergence of improper integrals and series.

3. Use the definition and formal rules of differentiation to compute derivatives.

a. State and apply the formal definition of a derivative.
b. Apply differentiation rules to sums, products, quotients, and powers of functions.
c. Discuss and demonstrate the differences between average and instantaneous rates of change.
d. Use the chain rule and implicit differentiation.
e. Extend knowledge of derivatives to include exponential, logarithmic, trigonometric and inverse trigonometric functions.
f. Calculate derivatives of parametric, polar, and vector functions.

4. Apply derivatives to find solutions in a variety of situations.

a. Interpret and communicate the purposes of the derivatives.
b. Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed and acceleration.
c. Apply the derivative to find tangent lines and normal lines to given curves at given points.

d. Apply Rolle's Theorem and the Mean Value Theorem and their geometric consequences.
e. Apply differentiation techniques to curve sketching.
f. Explain and predict the relationships between functions and their derivatives.
g. Model rates of change to solve related rate problems.
h. Solve optimization problems.
i. Determine an understanding of Newton's Method to approximate roots.
j. Investigate local linear approximations.
k. Interpret differential equations using slope fields.
l. Solve differential equations by Euler's Method.
m. Analyze planar curves given in parametric, polar and vector form including velocity and acceleration vectors.

5. Employ various integration properties and techniques to evaluate integrals.

a. Demonstrate the concept of the integral as an accumulator.
b. Use Reimann's Sum and the Trapezoidal Rule to approximate definite integrals.
c. State and apply the First and Second Fundamental Theorem of Calculus.
d. Evaluate the average value of a function on an interval.
e. Apply the power rule and u-substitution to evaluate indefinite integrals.
f. Extend techniques of integration to include integration by parts and simple partial fractions.

6. Adapt integration methods to model solutions to problems.

a. Investigate and apply integration to solve problems including area, volume, and cross sections.
b. Employ integration to compute distance traveled by a particle along a line.
c. Solve differential equations using integration and separation of variables.
d. Utilize integrals to model solutions to real-world problems.
e. Solve logistic differential equations and use them in modeling.
f. Apply integration to find length of a curve.

7. Explore the concepts affecting relationships among different kinds of series.

a. Identify different types of series and their characteristics.
b. Apply different types of tests to create valid arguments to determine convergence or divergence of series.
c. Use Lagrange's Method for computing errors of Taylor polynomials.
d. Formulate new series from known series to include Maclaurin and Taylor series.

Calculus

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a, b, c	Distribute examples of graphed functions. For each example: a. Use the graph to identify intervals where the function is continuous. b. Discuss and identify the values of the function where failure occurs for each of the three tests of continuity.	Short answer
1	c	Explore Layman's version of continuity: A function is continuous if you can draw it without ever lifting your pencil.	Teacher observation
1	d	Use technology to model parametrics by revisiting an old algebra problem of two trains traveling on the same track.	Peer evaluation
2	a,b,c	Divide the class into groups. Each group will investigate the function: $f(x)=\frac{x^3-1}{x-1}$ Group assignments: 1) Have one group create a table of 10 to 20 function values for $[1, 2]$ 2) Create table values for $[0, 1]$. 3) Graph function using a decimal (friendly) calculator window. List five (5) observations about what happens to *y* values as *x* gets closer to 1. 4) Predict what graph will look like and list at least five characteristics. 5) Algebraically explore the function: "Can it be factored? "	Group work; Class discussion
2	c,d	Compare the graphs of several rational functions to table values for behavior at points near where the denominator is undefined.	Rubric
2	d	Compare a list of indeterminate forms and discuss why they are indeterminant.	Student work sample
2	d	Use $\lim_{x\to\infty}(1+\frac{1}{x})^x$ to show/explore why 1^∞ is an indeterminant form.	Short answer

Calculus

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	a	Using an overhead-graphing calculator to create overheads of different functions, create two bugs (from hole punched dots) to travel along the overhead functions. Get students to predict what will happen as both bugs walk along the curve toward each other and a string connects the two bugs—one bug stays still and the other approaches the first bug.	Presentation
3	b	**Quotient Rule** Hi = Numerator Lo = Denominator Lo de Hi – Hi de Lo and down below the denominator squared must go.	Demonstration
3	c	Provide students with a table of values of time and speed. Have them calculate the average speed. What method(s) were used? Compare to instantaneous rates.	Discussion
3	b, d	After basic differentiation rules have been introduced, provide memory tools. For example, PI **(P**ower then do the **I**nside), and PTA (**P**ower, **T**rig, **A**ngle).	Test
3	f	Make a set of match cards with derivatives, graphs, and different forms (parametrics, polar, and vector) and have groups match and sort.	Free response
4	a, b	Given the graph of a function draw the tangent line at a variety of points on the function. Estimate the slope and analyze in terms of rate of change.	Student work sample
4	c	Determine the tangents to the curve $4x^2+9y^2=36$ at the ends of each axis. Describe the relationship between the two sets of tangents.	Short answer
4	d	Explain the similarities and differences between Rolle's Theorem and the Mean Value Theorem.	Essay
4	e,f	Give students a function like $f(x) = x^5 + 3x^4 – 4x^3 – 12x^2$ a) Where are the zeros for f'(x)? b) Identify intervals where graph is increasing/decreasing. c) Have students compute derivative and graph the derivative. Where is f(x) above the x-axis; Below the x-axis? d) State x coordinates of max/min points for f(x).	Group work
4	e,f	Make a set of match-cards to include $f(x)$, f', (x), f" f '(x), f" (x) for each group of students. (Extend: Critical number cards) Have groups match all the parts, then present one complete solution to the class.	Group investigation

Calculus

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	h	Find examples of real-world situations that involve solving optimization problems. Follow-up with a class discussion.	Project
4	h	Investigate why a soda can is the shape and size it is.	Project
4	i	Use technology to demonstrate finding roots using Newton's Method.	Demonstration
4	j	Justify how linear approximations are used to model local linearity of different functions.	Short answer
4	k	Create an overhead with families of curves that are solutions to a particular differential equation. Give each group a copy of an extra transparency. Have groups draw tangent lines at given points for different curves. Bring all group transparencies and place on overhead. Discuss the meaning of the slope field.	Small groups
4	l	Get a copy of an Euler method program or use a spreadsheet. Investigate what happens for different functions and different step sizes when using Euler's method.	Discussion
4	m	Model tossing a baseball to a person sitting on a ferris wheel using parameter equations and/or vectors.	Self assessment using graphing calculator
5	a	Provide a data set where an over-estimate and an under-estimate of an integral could be computed. Relate to an example of velocity data and estimate distance traveled.	Constructed response
5	b	Use technology to investigate numerical methods such as the Trapezoidal Rule.	Teacher observation
5	c	Use the Fundamental Theorem of Calculus to explain the difference between definite and indefinite integrals.	Constructed response
5	d	Create a graph that would model the average value formula.	Short answer

Calculus

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	e, f	Divide the class into two teams. Use a football field to score points. Team 1 has four chances to move +0 yards (correct answer = 10 yards). The team quarterback will designate a player to answer a question. All class members will work on the problem. If the designated player misses the question, the side of the room that has the most correct answers either wins the play or blocks the play.	Constructed response
6	a	Compute the area between a curve and the x-axis using geometric shapes and rectangular areas (from grid).	Group investigation
6	a	Use play dough to create solids formed by revolving a region about an axis. Slice into discs to demonstrate where the disc formula for volumes is derived.	Teacher observation
6	b	Use a graph to explain how an integral would model distance traveled.	Class discussion
6	c	Explain the process for solving differential equations by separation of variables.	Essay
6	d, e	Investigate exponential decay and/or logistic functions as they apply to integrals.	Test
6	f	Derive the formula for arc length.	Demonstration
7	a, b	Create a set of matching cards with all the tests for convergence, sample series, and blank index cards. Students will match tests with examples, then use index cards to write an appropriate argument proving convergence or divergence.	Student work sample
7	c	Discuss how to find a value for c on a specific interval as it relates to errors of Taylor polynomials.	Class discussion
7	d	Obtain either a computer or calculator program that will compute the Taylor polynomial. Explain the computer/calculator results for the examples given.	Constructed response

STATISTICS

Statistics introduces students to the major concepts and tools for collecting, analyzing, and drawing conclusions from data. Four major areas of concentration include data explorations, design of experiments, production of models using probability and simulation and statistical inference. The use of technology will be an integral part of the course. This course is designed for students who have successfully completed Algebra II. This is a one-credit course.

The competencies are printed in bold face type and are required to be taught. The competencies combine the content strands: **number and operations, algebra, geometry, measurement, and data analysis & probability** and the process strands **problem solving, reasoning & proof, communication, connections and representation.** The competencies are not listed in order of importance; rather the sequence of competencies relates to the broader K-12 framework. Competencies provide a general guideline of on-going instruction, not isolated units, activities, or skills.

Objectives/benchmarks indicate concepts that enable fulfillment of competencies, describe competencies in further detail, or show the progression of concepts throughout the grades. School districts may adopt the objectives/benchmarks, modify them, and are encouraged to write their own objectives/benchmarks to meet the needs of students in their school district.

STATISTICS

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

Competencies and Objectives/Benchmarks**:**

1. Use graphical and numerical techniques to study patterns and to explore, describe, and interpret data.

a. Interpret graphical displays of distributions of univariate data (dot plots, stem plots, histograms, box plots).
b. Summarize distribution of univariate data and correctly find and use measures of center (mean, median, mode); measures of spread (range, interquartile range, standard deviation); and measures of position (quartiles, percentiles, standardized scores).
c. Explore bivariate data by analyzing patterns in scatter plots and residual plots, performing logarithmic and power transformations to achieve linearity, finding least squares regression lines, and finding correlation coefficients.
d. Explore categorical data, construct, and interpret frequency tables.

2. Plan a study by clarifying a question and deciding upon a method of data collection and analysis.

a. Know the characteristics of a well-designed and well-conducted study and be able to distinguish between observational studies, surveys, and experiments.
b. Design a method for obtaining a simple random sample for a population of interest and for obtaining a stratified sample when appropriate.
c. Identify sources of bias and discuss the concept of sampling error in studies.
d. Design experiments, to include the concepts of confounding variables, control groups, placebo effects, blinding, randomization, replication, blocking, and generalizability of results.

3. Use probability to predict what the distribution of data should look like under a given method.

a. Use concepts of independent and mutually exclusive events, and apply the addition, multiplication, and conditional probability rules to find the probability of events.
b. Produce models using probability and simulation, and explain the “law of large numbers.”
c. Find the mean and standard deviation of a random variable and the mean and standard deviation for the sums and differences of independent random variables.
d. Know properties of the normal distribution, use normal distribution tables, and make inferences from these tables.

e. Simulate sampling distributions (distribution of a sample proportion, distribution of a sample mean, distribution of a difference between two independent sampling proportions, distribution of a difference between two independent sample means).
f. Discuss and illustrate the Central Limit Theorem.

4. Use statistical inference to analyze data, draw appropriate conclusions, and effectively communicate those conclusions.

a. Find and interpret large sample confidence intervals for a proportion, a mean, a difference between two proportions, and a difference between two means.
b. Appropriately use the following tests of significance: large sample tests for a proportion, a mean, a difference between two proportions, and a difference between two means (unpaired and paired); Chi-square test for goodness of fit, homogeneity of proportions, and independence; single sample and two sample t-procedures; and inference for slope of least squares line.
c. Write null and alternate hypotheses for studies, distinguish between one and two-sided tests, calculate appropriate test statistics, find p-values, arrive at appropriate conclusions, and communicate those conclusions effectively.

STATISTICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Open a magazine arbitrarily and record the lengths of all words in the first complete paragraph on the page. Create a dot plot of the lengths (number of letters) of words that were recorded. Write a few sentences describing this distribution of word lengths. (Students may choose various magazines and compare results.)	Check students' graphs and written description
1	a, b	Reconsider the data collected with word lengths. Calculate the five number summary of this distribution and draw a box plot. Comment on what the box plot reveals about the distribution of word lengths. Are there outliers?	Check students' graph and related comments
1	a	Consult the Farmer's Almanac or U. S. Census Report (http://www.census.gov/) to find a data set of interest. The Internet is also a source for interesting data sets. Choose a one-variable data set such as percentage of residents 65 years of age or older in each of the fifty states. Draw a histogram for the data. Make a stem plot for this data. Describe the main features of the distribution. Is it symmetric, right skewed, or left skewed? Single or double peaked? Are there gaps or outliers?	Check students' graphs and analysis of graphs
1	b	Write down all scores on a fictious test. Each student will standardize his/her score. Discuss measures of center for the test scores and also measures of spread.	Class discussion; Teacher observation
1	c	Obtain from a favorite fast food restaurant nutritional information about their sandwiches. List all sandwiches, serving size (in ounces) of each sandwich, and calories for each sandwich. Draw a scatter plot and reveal an association between a sandwich's serving size and its calories. Determine the least squares regression line for predicting calories from serving size. Find the correlation coefficient. Sketch a plot of residuals. How well does the least-squares regression line fit the data?	Check scatter plot, regression line, residual plot, and analysis
1	c	A courtier was offered a reward by an ancient king of Persia. He asked for a grain of rice on the first square of a chessboard, two grains on the second square, then 4, 8, 16, etc. Plot the number of grains on each square against the number of the square for squares 1 to 10 and connect the points with a smooth curve (exponential curve). Take the logarithm of each of the numbers of grains. Plot these logarithms against the numbers of squares from 1 to 10. (straight line) Find the least squares regression line for the logarithms of the number of grains versus the number of squares. Use this equation to predict the number of grains for the 64th square.	Self-check

STATISTICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	d	Classify each member of Congress according to his/her gender and political party. Construct a frequency table with row headings of Republican, Democrat or other. Use column heading of male or female. Interpret the frequency table.	Whole class assessment; Peer assessment
2	a, c	Consult a scientific journal. Find an example of an observational study, a survey, and an experiment. Critique each study to determine if it is a well-designed and well-conducted study. Identify any sources of bias.	Teacher observation and critique
2	b	Design a method of obtaining a simple random sample to determine the typical number of hours studied each weeknight by students in grades 11 and 12 at your school.	Teacher critique
2	b	Design a method for obtaining a stratified sample to determine who among three hypothetical candidates will be elected Homecoming Queen at your school.	Teacher and peer critique of experimental design
2	d	Divide class into groups of three. Each group will design an experiment, keeping in mind the concepts of confounding variables, control groups, placebo effects, blinding, randomization, and replication.	Class activity and discussion
3	a	Using M&Ms, obtain probabilities for various colors. Apply the addition principle to compute the probability of choosing a red or blue M&M, when selecting one at random. Go to www.mms.com for exact data.	Class activity and discussion
3	a	Use the multiplication and conditional probability rules to find the probability of selecting at random two male members of the class. (Assuming all names of class members were put in a hat and two names were drawn without replacement.) Find the conditional probability of selecting a male member of the class, given the student chosen has blonde hair.	Class activity and discussion
3	b,c	Repeatedly toss four coins and record the number of heads obtained on each trial. Find the mean number of heads in 5, 10, 25, 50, and 100 trials of the experiment. (The mean number of heads $\bar{x}$ observed when four coins are tossed many times approaches the population mean of the probability distribution.) The mean of the random variable = 2.) An illustration of the "Law of Large Numbers" follows. $\bar{x}$ will approach $\mu \times = 2$ more closely as the number of trials grow.	Class activity and discussion

STATISTICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
3	d	Each student should calculate the ratio of his height and his arm span (e.g., height divided by arm span). Produce a dot plot of the distribution of these ratios (for all students in class). Does the distribution appear to be roughly normal? Calculate the mean and standard deviation of these ratios. Suppose that these ratios in the population of all college students do in fact follow a normal distribution with mean and standard deviation equal to those found in your classroom sample. Under this assumption, calculate the proportion of all students who have a ratio greater than one (height greater than arm span).	Student and whole class activity; Teacher critique of work
3	e	Consider the population of the M&M®'s candies made by Hershey. Suppose you want to learn about the distribution of colors of these candies but you can only afford to take a sample of 25 candies. Record the number and proportion of each color in your sample. Each student should calculate the proportion of orange candies obtained by the students in the class. If every student estimated the population proportion of orange candies by the proportion of orange candies in his sample, would everyone arrive at the same conclusion? Observing the sample results from the entire class, estimate the population proportion of orange candies. Observe the variation of the sample proportions from sample to sample—the sampling distribution of the sample proportion.	Individual and whole class activity
3	f	Suppose a population consists of five employees for a firm. The number of years of employment are 5, 3, 6, 2, 4. Compute the mean length of employment for the population $(\mathrm{M}\mu = 4)$. Select all possible samples of size two from the population. Compute the mean of each sample. Does the mean of the sample means equal the population mean? Give the sampling distribution of the means. Plot the probability distribution of the sample means and the population. Is the population normally or non-normally distributed? Does the sampling distribution tend to approximate a normal distribution? (Central Limit Theorem)	Teacher critique of answers
4	a	Have students think of a real situation in which they would be interested in producing a confidence interval to estimate a population proportion. Have them describe how they would compute a 95% confidence interval.	Teacher critique

STATISTICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
4	a, b, c	Select one page from the white pages of a telephone book. Disregard all listing of businesses, which provide only initials, and listing with first names that are not gender-specific (like Pat or Chris). For the listings, which can be identified as male or female, count how many are male and how many are female. What is the sample proportion of females in the sample? Use the sample data to form a 95% confidence interval for the actual proportion of all humans who are female. Does the confidence interval provide a reasonable estimate of the actual proportion of all humans who are female? (No) Explain. Using your sample data, perform a test of significance to address whether the sample data support the theory that less than half of all of the telephone books' individual listings carry female names. Write null and alternate hypotheses. Calculate appropriate test statistics, find p-value, and write a paragraph describing your findings and explain how conclusions follow from the test results.	Teacher grades project

SURVEY OF MATHEMATICAL TOPICS

Survey of Mathematical Topics is designed to provide students with the skills necessary in making wise financial decisions. The basic concepts of algebra will be reviewed and extended as students solve real-life problems which affect them and their families. This course will provide skills in probability and statistics, logic, linear programming, and regression analysis. Students are encouraged to use a variety of techniques and appropriate technology (calculators and/or computers) to solve problems. This course is designed for students who have successfully completed Algebra I, Geometry, and/or Algebra II. This is a one-credit course.

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SURVEY OF MATHEMATICAL TOPICS

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

COMPETENCIES and Objectives/Benchmarks:

1. Demonstrate the skills necessary to manage personal finance.

a. Develop a household budget.
b. Maintain and balance a checkbook.
c. Investigate terminology and the process of filing personal income tax.
d. Investigate and explore all the components necessary to own and operate a car.
e. Analyze the options of housing alternatives.
f. Connect and apply appropriate algebraic formulas to personal finance situations.

2. Compute, analyze, and develop a variety of personal and business investments.

a. Analyze information to make wise decisions regarding personal savings.
b. Investigate life and health insurance.
c. Study and investigate the economics of the stock market.
d. Connect and apply appropriate algebraic formulas to personal and business investments.

3. Analyze and illustrate the practices that affect employer and employee decision-making.

a. Compute and compare various forms of earnings and calculate gross pay, deductions, and net pay.
b. Analyze the relationships among cost, revenue, and profit.
c. Apply linear programming to business decisions.
d. Connect and apply appropriate algebraic formulas to employer and employee practices.

4. Demonstrate an understanding of the impact of consumer credit.

a. Compare and contrast the finances of credit cards.
b. Explore the pros and cons of installment loans.
c. Connect and apply appropriate algebraic formulas to consumer credit.

5. Collect and apply information in planning a trip.

a. Investigate and evaluate modes of transportation.
b. Create a travel budget.
c. Make travel plans based upon airline schedules.
d. Utilize map-reading skills.
e. Connect and apply appropriate algebraic formulas to planning a trip.

SURVEY OF MATHEMATICAL TOPICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a	Create a budget for a family of four with a given yearly income.	Student work sample
1	b	Use simulated checks, checks register, and reconciliation forms to maintain a checking account and to reconcile the checkbook with the bank statement.	Portfolio
1	c	Obtain copies of 1040EZ and 1040A forms and instruction booklets from the IRS or local library. In groups, discuss the forms and provide sample information for students to complete both forms.	Discussion
1	d	Create a poster with the following headings for six used cars cut out from newspaper advertisements: • Sticker price • Down payment (use 10%) • Loan amount • Monthly payments (use current interest rate and three years for loan) • Total payments • Total amount including down payment Use a calculator and the monthly payment formula to complete the poster. Justify which car would be the best buy after verifying the condition of the car by visiting the dealership offering the car.	Project; Rubric
1	e	Investigate the following for each of ten local apartments for rent" • Square footage • Monthly rent • Number of bathrooms • Number of bedrooms Using a graphing calculator, calculate linear regression and find the line of best fit to compare any two apartments. Use this information to make predictions.	Representation
1	f	Use a calculator and the appropriate formula to compute monthly payments when buying a car or house.	Student work sample
2	a	Visit local banks to gather information on savings accounts. Prepare a poster, which compares the data.	Student work sample
2	b	Invite an actuary or local insurance agent to speak to the class concerning life and health insurance policies.	Teacher observation
2	c	Contact the Mississippi Economic Council (MEC) for information on participating in the state Stock Market Game.	Portfolio

SURVEY OF MATHEMATICAL TOPICS

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
2	d	Suppose the ancestors deposited $1 in a savings account 20o years ago. Using simple interest of 3%, calculate the value of that account today. Repeat using compound interest. Discuss the results. (Extend: Vary the amount of originally deposited and/or the interest rate.)	Project
2	d	Use the Rule of 72 to estimate how long it would take to become a millionaire with an initial deposit of $1000 with an interest rate of 10%. Repeat varying interest rates and initial deposit.	Teacher observation
3	a	Find gross pay based on commission sales and hourly rate. Use federal and/or state tax tables and FICA percentage rate to calculate deductions and net pay.	Short answer questions
3	b	Find the break-even point given cost and revenue equations. Analyze the regions between the two curves when graphed.	Constructed response
3	c	Use the method of linear programming to maximize or minimize certain factors in a business situation.	Student work sample
3	d	Research different types and financial amounts of fringe benefits offered by local employers. Using this data, compute additional costs associated with employment.	Checklist
4	a	Collect several credit card applications. Compare terms, finance charges, APR, etc. Determine which application is the most advantageous to the consumer.	Discussion
4	b	Create an amortization schedule to illustrate the concept of installment loans.	Student work sample
4	b	Investigate car-buying options involving rebates versus the offer of an extremely low interest rate. Discuss the advantages/disadvantages of each option for the dealer, loan institution, and buyer.	Discussion
4	c	Use the Rule of 78 to estimate the savings when a loan of $1000 for 12 months at 7% is paid off after four months.	Short answer question
5	a,b,c	Plan a trip to a far away city within the 48 contiguous United States. Decide on destination and length of trip. Call a travel agent (or use the Internet) to compare various modes of transportation for cost and time constraints. Prepare a budget of anticipated expenses.	Presentation; Project; Rubric
5	a,b,d,e	Obtain state maps for each student. Given two locations on the map, discuss the best route to travel from one location to another. Calculate the costs of driving a car to this destination. Discuss the pros and cons of driving versus other modes of transportation.	Teacher observation; Discussion

INTRODUCTION TO ENGINEERING

The Introduction to Engineering course is designed to provide students with an introduction to the problem-solving methods that engineers use when applying scientific principles to the "engineering" of realistic solutions. These problem-solving skills should be useful to a student regardless of the academic or vocational direction that the student desires to pursue. The course includes several learning modules that focus on the development of solutions to specific problems. This one-half credit course is designed for students who have successfully completed Algebra I and Geometry.

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INTRODUCTION TO ENGINEERING

CONTENT STRANDS:

Number and Operations *Algebra*
Geometry *Measurement*
Data Analysis & Probability

COMPETENCIES and Objectives/Benchmarks:

1. Explore general problem solving strategies.

a. Analyze and evaluate general problem solving strategies.
b. Develop and use critical thinking skills in solving problems.
c. Identify and utilize various aspects of "best" engineering solutions.
d. Apply Newton's Laws of Physics as they relate to the problem solving strategies.
    1. Calculate mass of objects on earth, moon, etc.
    2. Convert units of English system of measurement to Standard International units.
e. Convert units extensively (i.e., meters/sec to mi/hr, cubic inches/day to liters/min).

2. Solve general civil engineering/surveying problems.

a. Calculate angles of a triangle given lengths of sides.
b. Calculate distance of given angle measurements.
c. Calculate decimal degree form to degrees, minutes, and seconds.
d. Apply Pythagorean Theorem of a right triangle.
e. Calculate the area of a road to be paved over a certain distance using road grade (slope).
f. Calculate difference in percent slope and degree slope.

3. Solve general electrical engineering problems.

a. Calculate work, power, voltage, current, and resistance using given and derived formulas.
b. Apply Ohm's Law to calculate unknown information in a simple series and parallel circuits.
c. Apply Kirchoff's Voltage and Current Laws to derive equations for calculating equivalent resistances in series and parallel circuits.
d. Calculate the number of light bulbs (and other resistors) that could be placed in series or parallel circuitry before a circuit breaker would trip.

4. Solve general chemical engineering problems.

a. Write mass balance equations and solve for some variable in the mass balance equations. Typical problem would be a system having different inflow and outflow rates. Determine drain or overflow rate and calculate exact time for the system to drain or overflow.

b. Write energy balance equations and solve for some variable in the energy balance equations. Typical problem would be an individual consuming a certain diet and exercising a certain amount. Determine an individual's weight gain or loss and his new weight after a specified period of time.
c. Calculate volumes of odd sized containers or systems, which must be incorporated into a mass or energy balance equation in order to solve.
d. Write equations of mass balance and energy balance, which are dependent upon one another. Solve the system of equations using a variety of methods such as substitution method, elimination method, matrices, etc.

5. Identify, study, and solve problems involving the control of room acoustics.

a. Determine and use the root mean square of a signal in further calculations.
b. Use and apply given formulas to solve for sound power, reverberation time, decibel level, and total noise exposure. Students will apply geometry to calculate volume and area in order to use architectural acoustics formulas.
c. Use formulas involving logarithms to calculate decibel levels from sound pressure.
d. Use formulas involving logarithms to calculate total decibel level from multiple sources of sound.
e. Calculate total noise exposure limits per OSHA standards by using a computer spreadsheet.

6. Design and build a coffee-making system.

a. Apply and use mathematical concepts related to basic physics and chemistry including heat exchange, fluid flow, condensation, leaching, mass transfer, and system energy requirements.
b. Collect, apply, and use data to determine actual costs of a morning shower as part of thermodynamics discussion.
c. Calculate BTU requirements and utility costs for heating a specified home.
d. Design and build a system from various hardware materials capable of heating water, transferring water to coffee grounds, and producing hot coffee.

7. Following chemical engineering methods, create calibration curves from laboratory data to establish standard concentrations of various poisons involving a fictitious murder mystery.

a. Use and apply methods of unit conversion.
b. Determine units of concentration (weight percent, volume percent, parts per million, and molarity).
c. Using a periodic table, determine molecular weight of a substance.
d. Use and apply linear relationships and the slope-intercept equation.
e. Determine and use the "line of best fit." Calculate the linear regression of a line by hand, by spreadsheet, and by computer graphing.

8. Design and build a bridge meeting certain specifications.

a. Calculate internal and external forces of members of a structure and loads applied.
b. Use and apply geometry to calculate bending angle formed from various forces acting on a figure.
c. Calculate load paths, functions, and force types of capable, arch, and truss systems.
d. Use and apply equilibrium analysis and strength checks of design.

INTRODUCTION TO ENGINEERING

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
1	a,b,c	Compile list of considerations (critical thinking) needed for designing a useful, safe, reliable, inexpensive coffee-making system and Problem Set 3-5: page 80	Teacher observation
1 2 3 4 5 6 7 8	a,b,c,d,e a,b,c,d,e,f a,b,c,d a,b,c,d a,b,c,d,e a,b,c,d a,b,c,d,e a,b,c,d	Create a detailed logbook with table of contents of examples of all types of problems worked in the course complete with formulas, unit conversions, various calculations and computations.	Teacher observation and student use of logbook during exams
1	d,e	Solve unit conversion problems: HW Set 1-1; page 7	Teacher Observation; Rubric grading
2	a,b,c,d,e,f	Solve survey problems: HW Set 1-2; page 13	Teacher observation; Rubric grading
2	a,b,c,d	Design a in-ground swimming pool to certain qualifications: Project 1-1; page 14	Teacher observation; Rubric grading
2	a,b,c,d	Design a piping system for swimming pool; Project 1-2; page 15	Teacher observation; Rubric grading
3	a,b,c,d	Solve electrical circuit problems; HW Set 1-3; page 23	Teacher observation; Rubric grading
3	a,b,c,d	Project 1-3; page 24-25	Teacher observation; Rubric grading
4	a,c	Mass balance problems; HW Set 1-4; page 29	Teacher observation; Rubric grading
4	b	Energy balance problem; HW Set 1-5; page 33	Teacher observation; Rubric grading
4	d	Have students study and present the steps needed to solve the combined mass and energy balance problem in groups (solution provided): page 34	Teacher observation; Rubric grading

**References to page number are from "Introduction to Engineering: A High School Workbook." Copyright 2001 by Marni R. Kendricks and Michael K. Ponton. ISBN no. 1-58692-323-4*

INTRODUCTION TO ENGINEERING

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
5	b	Acoustics problems: HW Set 2-2; page 48	Teacher observation; Rubric grading
5	c	Architectural acoustics project: Page 49	Teacher observation; Rubric grading
5	d	Have students study and present the steps needed to solve the industrial acoustics problem in groups (solution provided): page 57	Student and Teacher observation
5	e	Computer spreadsheet calculations of OSHA legal noise exposure limits: HW Set 2-3; page 59	Teacher observation; Rubric grading
6	a	Lab activity for determining BTU requirements: page 65 Lab activity for heat transfer: page 71 Lab activity for fluid flow: page 75 Lab activity for fluid flow: page 76	Teacher observation; Rubric grading
6	b	Obtain utility bills and determine exact cost for energy using natural gas (CF) and electricity (KW-HR). Apply energy formula to determine costs for morning shower, heating pot of coffee, heating hot water tank, etc. HW Set 3-1: page 66	Teacher observation; Rubric grading
6	c	Calculate utility costs for heating a specified home with varying numbers of windows using a heat transfer formula and utility costs. Plot the results on coordinate plane (square footage of windows vs. utility costs). Write equation of the line. Predict utility costs with other quantities of windows based on equation. HW Set 3-3; page 72	Teacher observation; Rubric grading
6	d	Have students design and build an "automated" working coffee making system from parts obtained in the local hardware store.	Teacher observation; Rubric grading
7	a,b,c	Chemistry problems: Problem Set 4-1; page 88	Teacher observation; Rubric grading
7	d	Following sample lab results (p.99), have students plot results on coordinate plane (concentration vs. absorbance) and calculate the slope of the line using various methods and write equation of the line. Measured Rise/Run Calculated Delta Y / Delta X Linear regression: Sum of xy / Sum x^2	Teacher observation; Rubric grading

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INTRODUCTION TO ENGINEERING

Comp.	Obj.	Suggested Teaching Strategies	Suggested Assessment
7	e	Murder mystery project: page 1-4-112	Teacher observation; Rubric grading

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LITERATURE CONNECTIONS

The *Literature Connections* is designed to serve as a guide and first-stop source for making cross-curriculum connections. The titles are not extensive and other books should be used in the classroom.

Kindergarten through Second Grade

Adams, Pam	THERE WERE TEN IN THE BED. Child's Play International, 1979. Every time the little one says, "Roll over," a child falls out of bed. The reader manipulates a dial which moves a figure at each turn.
Allen, Pamela	MR. ARCHIMEDES BATH. New York: Lothrop, Lee, and Shepard Books, 1980. A good introduction to volume and water displacement of objects.
Anno, Mitsumasa	ANNO'S MAGIC SEEDS. Philmel Books, 1995. A golden seed grows by one's, two's, and so on. The arithmetic puzzles presented help develop numeration and problem-solving skills.
Anno, Mitsumasa and Masaichiro	ANNO'S MYSTERIOUS MULTIPLYING JAR. Putham Publishing, 1983. This story introduces factorials in a tale about a porcelain jar with a sea inside.
Axelrod, Amy	PIGS WILL BE PIGS. Arladin Paperbacks, 1997. A counting book about a family of pigs. This story contains a menu with prices, money problems, and a bonus problem.
Barret, J.	CLOUDY WITH A CHANCE OF MEATBALLS. MacMillian Publishing, 1978. A story about a town where it rains soup and juice, snows mashed potatoes, and blows storms of hamburgers.
Berenstain, Jean and Stan	THE BERENTAIN BEARS' TROUBLE WITH MONEY. Random House, 1983. To earn coins for the Astro Bear video game, Brother and Sister Bear find ways to work for money. How they find the middle ground Bears with money – between being spendthrifts and little misers makes for a funny, realistic story.
Brisson, Pat	BENNY'S PENNIES. Yearling Books, 1995. This story introduces money and counting by exploring what Benny can buy with five pennies.
Burns, Marilyn	THE GREEDY TRIANGLE. Scholastic Trade, 1995. This story introduces the many different ways shapes appear in the world. A dissatisfied triangle transforms into different shapes, trying to find the best one.

Carle, Eric	THE VERY HUNGRY CATERPILLAR. New York: Philomel Books, 1987. A hungry caterpillar is born on Sunday and on Monday begins to eat his way through fruit and stair-step fashion up to ten pieces on Saturday.
Carle, Eric	1,2,3 TO THE ZOO. Penguin Putnam Books for Young Readers, 1998. This story offers youngsters an introduction to numbers and number sets while learning about different animals.
Carter, David A.	STICKER BUGS NUMBERS. New York: Simon and Schuster, 1996. This book contains a set of stickers for numbers 1 to 10.
Cassidy, John	THE TIME BOOK. Palo Alto, California: Klutz Press, 1991. This book introduces estimates with time along with time intervals.
Clement, Rod	COUNTING ON FRANK. Gareth Steven Children's Books, 1991. A dog named Frank and his young owner explore estimation of measurements and numbers.
Crews, Donald	TEN BLACK DOTS. Mulberry Books, 1995. This book uses rhymes and everyday objects to demonstrate counting up to ten black dots.
Edens, Cooper	THE WONDERFUL COUNTING CLOCK. New York: Simon and Schuster Books for Young Readers. A counting book that presents the numbers 1 to12.
Esbensen, Barbara Juster	ECHOES FOR THE EYES: POEMS TO CELEBRATE PATTERNS IN NATURE. Pennsylvania: Harper Collins Publishers, 1996. This book contains poems and paintings that create images of repeating patterns in nature.
Feelings, Muriel	MOJA MEANS ONE: A SWAHILI COUNTING BOOK. New York: Dial Books for Young Readers, 1971. This book reinforces counting skills by counting objects, one through ten. Swahili names for the numbers are presented as well.
Freeman, Don	CORDUROY. Viking Penguin, 1968. Corduroy is a bear that once lived in the toy department of a big store. Day after day, he waited with all the other animals and dolls for somebody to come along and take him home.
Friedman, Aileen	THE KING'S COMMISSIONERS. Scholastic Press, 1994. This story introduces the concept of people counting by grouping.
Friedman, Aileen	A CLOAK FOR THE DREAMER. Scholastic Press, 1995. This story is about three sons' father, who is a tailor. Each son is asked to sew a cloak to keep out the wind and rain. Each son uses different shapes to make their cloak.

Gerstein, Mordicai	THE SUN'S DAY. New York: Harper & Row Publishers, 1989. This story presents the progression of a day from sunrise to sunset.
Giganti, Paul Jr.	HOW MANY SNAILS? Harper Trophy, 1994. This story is about a child who takes a walk to different places and wonders about the amount and variety of things seen on the way.
Giganti, Paul Jr.	EACH ORANGE HAD EIGHT SLICES: A COUNTING BOOK. Greenwillow, 1992. This story uses familiar objects to introduce familiar math concepts.
Hamm, Diane Johnston	HOW MANY FEET IN THE BED?. Simon & Schuster, 1994. A counting book that has a family of five tumbling in and out of bed while adding and subtracting feet.
Harshman, Marc	ONLY ONE. Dutton Books, 1993. This book uses a county fair to introduce the concepts of parts of a whole.
Haskins, Jim	COUNT YOUR WAY THROUGH CHINA. Minneapolis, Minnesota: Carolrhoda Books, 1987. This story shows how to write and pronounce the numbers one through ten in Chinese. Each number leads to the exploration of Chinese history and culture.
Hindley, Judy	TEN BRIGHT EYES. Peachtree Publishers, 1998. This story is about a mother bird searching for breakfast for her young. Introduces patterns, shapes, and numbers.
Hong, Lily Toy	TWO OF EVERYTHING. Whitman and Company, 1993. A Chinese folktale that helps develop number sense and numeration concepts.
Hughes, Shirley	LUCY AND TOM'S 1, 2, 3. Marking, Ontario: Viking Kestrel, 1987. This book introduces a variety of ways that mathematics is used on an everyday basis. One-to-one correspondence, addition, division, even and odd numbers, and measurement are some of the concepts covered.
Hutchins, Pat	1 HUNTER. Harper Trophy, 1986. 1 Hunter walks through the jungle. He does not see 2 elephants or 3 giraffes, but they see him.
Keats, Ezra Jack	OVER IN THE MEADOW: A COUNTING-OUT RHYTHM. Penguin Putnam, 1999. This story introduces animals and their young using the numbers one through ten.

Leedy, Loreen — FRACTION ACTION. Holiday House, 1996. This story explores fractions by using examples they find in the world around them.

Lewis, Paul Owen — P. BEAR'S NEW YEAR'S PARTY. Tricycle Press, 1999. This book features a large analog clock for exploring time.

Linn, Charles — ESTIMATION. New York: Thomas Y. Crowell Publishers, 1972. This book consists of activities and experiments to help improve skills in estimating countable quantities, volumes, and lengths.

Lionni, Leo — INCH BY INCH. Harper Trophy, 1995. An inchworm demonstrates how he can be used as a measurement tool.

Long, Lynette — DOMINO ADDITION. Massachusetts; Charlesbridge Publishing, 1996. This book has pictures of dominoes to show basic sums for the numbers 0 to 12.

Maccarone, Grace — MONSTER MATH. Scholastic, Inc., 1995. This story follows the activities of twelve monsters that diminish one by one. Includes a section of counting activities.

McGrath, Barbara — THE M&M'S BRAND COUNTING BOOK. Charlesbridge Publishing, 1994. This book uses M&M's to introduce counting, additions, subtraction, sets, colors, and shapes.

McMillan, Bruce — EATING FRACTIONS. New York: Scholastic, 1991. This book introduces wholes, halves, thirds, and fourths through food examples. Recipes are included at the end to extend the discussion to measurement.

Merriam, Eve — 12 WAYS TO GET 11. Aladdin Paperbacks, 1996. This book provides strategies for counting and numeration with collections of objects.

Morozumi, Atsuko — ONE GORILLA. Farrar Strauss & Firoux, 1990. This book is about a gorilla that counts hidden creatures in the jungle.

Murphy, Stuart — SUPER SAND CASTLE SATURDAY. HarperCollins, 1998. This book introduces the concept of nonstandard measurement as three friends compete in a sand castle contest.

Myllar, Rolf — HOW BIG IS A FOOT? New York: Dell Publishing, 1991. This is a tale about nonstandard measures and how they were used to make a bed for a queen.

Neasi, Barbara — A MINUTE IS A MINUTE. Chicago: Children's Press, 1988. This book explores the meaning of a minute from a child's perspective.

Paul, Anne Whitford — EIGHT HANDS ROUND. Harper Collins Juvenile Books, 1991. This story tells the history behind the art of quilting and introduces patterns and symmetry.

Pinczes, Elinor — REMAINDER OF ONE. Houghton Mifflin, 2002. This story applies numerical division to a practical problem in a fun and exciting way.

Pluckrose, Henry — WEIGHT. New York: Franklin Watts, 1988. The need to know is presented text along with applications of the metric system.

Reid, Margarette — THE BUTTON BOX. Puffin, 1995. This book introduces counting concepts with a grandmother's box of buttons.

Resier, Lynn — BEACH FEET. New York; Greenwillow Books, 1996. The beach displays human feet which squish, splash, or rest, as well as animal feet which may number five, six or even nine and which have many uses.

Richardson, John — TEN BEARS IN A BED. Sadie Fields Productions, Inc., 1992. Nine bears fall out of bed one by one when the littlest bear says "roll over."

Rocklin, Joanne — HELLO, MATH READER: HOW MUCH IS THAT GUINEA PIG IN THE WINDOW? New York: Scholastic, 1995. This book introduces money concepts.

Rogers, Paul — THE SHAPES GAME. Holth, 1989. This book introduces geometric shapes with riddles and illustrations.

Russo, Marisabina — ONLY SIX MORE DAYS LEFT. New York: Greenwillow Book, 1988. This story helps generate discussion of countdowns to certain events.

Schwerin, Doris — THE TOMORROW BOOK. New York: Pantheon Books, 1984. This book contributes to a discussion of what tomorrow means.

Scieszka, Jon and Lane Smith — MATH CURSE. Penguin Group, 1995. An amusing look at numbers in everyday life.

Sendack, Maurice — CHICKEN SOUP WITH RICE: A BOOK OF MONTHS. New York: Scholastic, 1986. This story takes the reader through the months of the year by portraying activities for each month.

Shapp, Martha and Charles Shapp — LET'S FIND OUT ABOUT WHAT'S LIGHT AND WHAT'S HEAVY. New York: Franklin Watts, 1975. Two suggested experiments explore the comparative lightness and heaviness of different objects.

Spurr, Elizabeth — THE BIGGEST BIRTHDAY CAKE IN THE WORLD. Harcourt Brace Jovanovich, 1991. The richest fattest man in the world falls into his birthday cake – the biggest in the world.

Srivastava, Jane Jonas — SPACES, SHAPES, AND SIZES. New York: Thomas Y. Crowell Publishers, 1980. The experiments of five mischievous animals help students investigate the concept of volume.

Srivastava, Jane Jonas — WEIGHING AND BALANCING. New York: Thomas Y. Crowell Publishers, 1970. This book provides directions for making a simple balance along with investigations to use this balance with nonstandard and standard units of measurement.

Tucker, Sian — 1-2-3 COUNT WITH ME. New York: Simon and Schuster, 1996. This counting book involves lifting a flap on each page to count the objects hidden beneath.

Viorst, Judith — ALEXANDER WHO USED TO BE RICH LAST SUNDAY. New York: Atheneum Publishers, 1979. Alexander receives $1.00 from his grandparents. Through the course of a week, Alexander's money disappears through a series of events. It is left to the reader to determine whether he has lost all of his money by accounting for all his expenses.

Wahl, John & Stacy — I CAN COUNT THE PETALS OF A FLOWER. NCTM, 1985. This is a counting book.

Ward, Cindy — COOKIE'S WEEK. New York: G. P. Putnam's Sons,. 1988. This book promotes discussion of the sequential days of the week.

Weiss, Malcolm E. — SOLOMON GRUNDY, BORN ON ONE DAY: A FINITE ARITHMETIC PUZZLE. New York: Thomas Y. Crowell Publishers, 1977. This story is about Solomon Grundy and asks if he was really buried a week after his birth and explores the cyclic nature of the days of the week. Patterns, arithmetic, and time can be discussed within the context of this book.

Williams, Vera — A CHAIR FOR MY MOTHER. Greenville, 1984. This book, about a family who saves dimes in a jar to buy a chair after losing everything in a fire, introduces counting and money.

Williams, Vera — CHERRIES AND CHERRY PITS. Mulberry Books, 1991. This book provides story problems related to a bag of ripe red cherries.

Wise, William — TEN SLY PIRANHAS: A COUNTING STORY IN REVERSE (A TALE OF WICKEDNESS—AND WORSE). Dial Books for Young Readers, 1993. This story is about piranhas that eat from ten fish

down to one.

Zimelman, Nathan — HOW THE SECOND GRADE GOT $8,205.50 TO VISIT THE STATUE OF LIBERTY. Albert Whitman and Company, 1992. A story about a group of children trying to raise money to go on a trip.

Third through Fifth Grade

Adams, Barbara Johnson — THE GO-AROUND DOLLAR. Simon and Schuster, 1992. This picture book is about the travels of a single dollar.

Adams, Pam — TEN BEDS TALL. Child's Play, International Ltd., 1988. A washable book that has an attached set of beads and a series of stories and exercises that help students learn about measurement.

Anno, Mitsumas — ANNO'S SUNDIAL. New York: Philomel Books, 1987. These three-dimensional pop-ups illustrate how the motions of the sun and the Earth led people to discover ways to tell the time of the day.

Anno, Mitsumasa and Masaichiro — ANNO'S MYSTERIOUS MULTIPLYING JAR. Putnam Publishing Group, 1983. This book introduces factorials in a story about a porcelain jar with a sea inside.

Burns, Marilyn — SPAGHETTI AND MEATBALLS FOR ALL. Scholastic Trade, 1997. This book introduces concepts of area and perimeter as a family tries to make room for everyone who attends a family reunion.

Burns, Marilyn — THE GREEDY TRIANGLE. Scholastic Trade, 1995. This book introduces the many ways shapes appear in the world.

Burns, Marilyn and Joanne Rocklin — ONE HUNGRY CAT. Sagebrush Education Resources, 1997. Tom the cat tries to evenly divide the snacks he has baked for himself and two friends. Includes division activities.

Clement, Rod — COUNTING ON FRANK. Gareth Sterens, 1991. This book is about a boy who uses fact, figures, and a wild imagination to make counting fun.

Dahl, Ronald — ESIO TROT. Puffin, 1992. This book is about Mr. Hoppy, who devises a plan to win Mrs. Silver's heart that leads to thinking about measurement and division.

Ernst, Lisa Campbell	SAM JOHNSON AND THE BLUE RIBBON QUILT. Mulbery Books, 1992 This story is about quilt-making to introduce patterns and symmetry.
Falwell, Cathryn	FEAST FOR 10. Clarion Books, 1993. This book shows what it takes to make a meal for ten people.
Friedman, Aileen	THE KING'S COMMISSIONERS. Scholastic Trade, 1995. This book introduces the concept of counting by grouping.
Friedman, Aileen	A CLOAK FOR THE DREAMER. Scholastic Trade, 1995. This story is about three sons father, who is a tailor. Each son is asked to sew a cloak to keep out the wind and rain. Each son uses different shapes to make their cloak.
Hutchings, Pat	THE DOORBELL RANG. Mulberry Books, 1989. This is about each time the doorbell rings, someone else shows up to share some cookies.
Leedy, Loreen	MEASURING PENNY. Henry Holt & Company, 2000. This book uses mathematics by Lisa measuring her dog, Penny, in several different ways.
Matthews, Louise	BUNCHES AND BUNCHES OF BUNNIES. Dodd, Mead, 1978. This book teaches the multiplication facts one through twelve using rhymes.
Pittman, Helena Glare	A GRAIN OF RICE. Skylark, 1996. This story is about a farmer teaching an emperor a math lesson. As a reward, the farmer asks for a single grain of rice, doubled every day for 100 days.
Schwartz, David M.	HOW MUCH IS A MILLION? Mulberry Books, 1993. This book gives meaning to one million, one billion, and one trillion.
Schwartz, David M.	IF YOU MADE A MILLION. Lothrop, Lee, and Shepand, 1989. This book explores ways to earn and spend a penny, a nickel, and a million dollars.
Tompert, Ann	GRANDFATHER TANG'S STORY: A TALE TOLD WITH TANGRAMS. Demco Media, 1997. A grandfather tells a story to his granddaughter with arranged tangrams to show the shape of each animal in the tale.

Viorst, Judith — ALEXANDER WHO USED TO BE RICH LAST SUNDAY. New York Atheneum Publishers 1978. This book is about Alexander who receives $1.00 from his grandparents. Through the course of a week, Alexander's money disappears through a series of events. It is left to the reader to determine whether he has lost all his money by accounting for all his expenses.

Sixth through Eighth Grade

Anno, Masaichiro and Mitsumasa — ANNO'S MYSTERIOUS MULTIPLYING JAR. Penguin Putnam, 1999. This story demonstrates the concept of factorials in mathematics.

Apfel, Necia H. — CALENDARS. New York: Franklin Watts, 1985. A book about calendars that integrates social studies, science, and measurement.

Ardley, Neil — MAKING METRIC MEASUREMENTS. New York: Franklin Watts, 1983. Hands-on activities from the Action Science series.

Berg, Olvie S. — I'VE GOT YOUR NUMBER, JOHN. New York: Holt, Rhinehart, and Winston, 1965. This book explores how numbers are used in license plates and area codes. It also explores the binary number system and how it is used in the electronic phone system.

Branley, Franklyn M. — THINK METRIC. New York; Thomas Y. Crowell Publishers, 1972. Explores the origins of measurement units and comparisons between the English and metric systems.

Briers, Audrey — MONEY. New York: Franklin Watts, 1987. Traces the development of currency from bartering to credit cards.

Brindze, Ruth — STORY OF OUR CALENDAR. New York: Vangaurd Press, 1949. A history of the calendar.

Burns, Marilyn — THE I HATE MATHEMATICS BOOK. Boston: Little, Brown and Company, 1975. A book that shows how much fun math can be.

Burns, Marilyn — MATH FOR SMARTY PANTS. Boston: Little, Brown and Company, 1982. A collection of problems and facts to help captivate middle school students.

Burns, Marilyn — THIS BOOK IS ABOUT TIME. Boston: Little, Brown and Company, 1978. This book contains information and activities about time.

Butrick, Lyn McClure — LOGIC FOR SPACE AGE KIDS. Athens, Ohio: University Classics, 1984. Zeno, from the planet Zircon, ponders problems involving logical deduction.

Cantwell, Lois — MONEY AND BANKING. New York: Franklin Watts, 1984. An account of money as it developed to the Middle Ages and the Renaissance. This book also discusses the development of coins and paper money in the United States.

Cushman, Jean — DO YOU WANNA BET? YOUR CHANCE TO FIND OUT ABOUT PROBABILITY. New York: Clarion Books, 1991. Two boys become involved in everyday situations that involve probability.

Dilson, Jesse — THE ABACUS: A POCKET COMPUTER. New York: St. Martin's Press, 1968. Explores numeration and operations through the Chinese abacus.

Fehr, Howard — NUMBER PATTERNS MAKE SENSE: A WISE OWL BOOK. New York: Holt, Rhinehart, and Winston, 1965. Explores number patterns through creative problem-solving.

Fisher, Leonard E. — CALENDAR ART: THIRTEEN DAYS, WEEKS, MONTHS, YEARS FROM AROUND THE WORLD. New York: Four Winds Press, 1987. This book presents various ways cultures throughout history respond to the need for accurate recording of time.

Fisher, Leonard E. — NUMBER ART: 1-2-3s FROM AROUND THE WORLD. New York: Four Winds Press, 1982. The history and development of thirteen number systems are presented.

Hayes, Cyril and Dympna — NUMBER MYSTERIES. Milwaukee, Wisconsin: Penworthy Publishing, 1987. Readers use strategies such as number lines, diagrams, models, graphs, tables, and patterns to solve challenging problems.

Holland, Penny — LOOKING AT COMPUTER PROGRAMMING. New York; Franklin Watts, 1984. This book focuses on using logical thinking to understand how computer programs work.

James, Elizabeth and Carol Barkin — MANAGING YOUR MONEY. Chicago: Children's Press, 1977. This book explores what money is, why we have it, how you earn it, how checking and savings accounts work, what credit cards are, and how to manage money.

Juster, Norton — THE PHANTOM TOLLBOOTH. Random House, 1971. This book brings to life infinity, distance measurement, and averaging while Milo journeys to an unknown land.

Kirst, Werner — TIME. Woodstock, New York: Beekman Publishers, 1977. Short articles related to time measurement.

Kyte, Kathy S. — THE KIDS COMPLETE GUIDE TO MONEY. New York: Alfred A. Knopf, 1984. Demonstrates how kids can manage money and make intelligent decisions about money.

Laithwaite, Eric — SIZE: THE MEASURE OF THINGS. New York: Franklin Watts, 1988. This book presents size in terms of relationships such as scaling, size and time, length and frequency, biggest and smallest, size in numbers, etc.

Laithwaite, Eric — SHAPE: THE PURPOSE OF FORMS. Franklin Watts, 1986. This book is part of a series that connect science with real-world applications.

Lamm, Joyce — LET'S TALK ABOUT THE METRIC SYSTEM. Middle Village, New York: Jonathan David Publishers, 1974. This book discusses the need for measurement, the historical development of measurement, and the confusion of different systems.

Leighton, Ralph and Carl Feynman — HOW TO COUNT SHEEP WITHOUT FALLING ASLEEP. New Jersey: Prentice Hall, 1976. A fictitious account of the development of our number system.

Luce, Marnie — ONE IS UNIQUE. Minneapolis, Minnesota: Lerner Publications, 1969. This book discusses concepts such as cardinality, multiplicative identity, reciprocal, division of fractions, set notation, and infinity.

Luce, Marnie — ZERO IS SOMETHING. Minneapolis, Minnesota: Lerner Publishers, 1969. This book discusses the historical development of our current numeration system.

Parker, Pat and Teresa Kennedy — LOGO FUN. New York: Scholastic, 1985. This book uses computer usage concepts along with problem-solving and geometric concepts.

Petty, Kate	NUMBERS. New York: Gloucester Press, 1985. This book presents increasing complex computer programs and computer games.
Phillips, Louis	263 BRAIN BUSTERS: JUST HOW SMART ARE YOU, ANYWAY? New York: Penguin Books, 1985. A collection of brainteasers.
Pluckrose, Henry	TIME. New York: Franklin Watts, 1988. This book explores units of time measurement along with the importance of timekeeping.
Ritchie, Alan	ERIN MCEWAN YOUR DAYS ARE NUMBERED. Random House, 1991. In danger of flunking sixth grade because of math, Erin takes a job in a deli where she is told to help with the bookkeeping.
Sarasas, Claude	ABC'S OF ORIGAMI. Rutland, Vermont: Charles E. Tuttle Company, 1964. Origami patterns of 26 objects are illustrated with sequenced diagrams.
Saunders, Kenneth	HEXAGRAMS. Stradbroke, England: Tarquin, 1983. A collection of puzzles that can be solved by using a hexagon cut into eight pieces.
Sharp, Richard and Seymour Metzner	THE SNEAKY SQUARE AND 113 OTHER MATH ACTIVITIES FOR KIDS. TAB Books, 1990. A book of problems and puzzles.
Sleater, William	THE BOY WHO REVERSED HIMSELF. Penguin Putnam, 1998. This story takes the mathematical complexity and uncertainty of the fourth dimension and translates it into an entertaining novel.
Stwertka, Eve and Albert	MAKE IT GRAPHIC! DRAWING GRAPHS FOR SCIENCE AND SOCIAL STUDIES PROJECTS. New York: Julian Messener, 1985. This book covers the production, use, and value of many types of graphs.
Simon, Seymour, and Steven D. Schindler	EINSTEIN ANDERSON. Avon Books, 1998. Einstein Anderson is a whiz at science and he investigates the mysteries of science.
Tahan, Malba	THE MAN WHO COUNTED: A COLLECTION OF MATHEMATICAL ADVENTURES. Norton, W.W. & Company, 1992. The adventures of Beremiz Samir takes the reader on a journey where mathematical powers are used to settle disputes, give advice, overcome enemies, and win fame and fortune.
(Unknown Author)	MILLIONS OF PEOPLE. New York: Holt, Rhinehart, and Winston, 1971. Graphs present population data gathered about people from the classroom, the United States, and the world.

Van Note, Peter — TANGRAMS: PICTURE-MAKING PUZZLE GAME. Rutland, Vermont: Charles E. Tuttle Company, 1966. A collection of tangram puzzles.

Zaslavsky, Claudia — TIC-TAC-TOE. New York: Thomas Y. Crowell Publishers, 1982. A book of strategy games.

Ninth Grade through Twelfth

Belton, John and Joella Cramblitt — DOMINO GAMES. Milwaukee, Wisconsin: Raintree Publications, 1976. Domino games that require strategies as well as chance to win.

Diggins, Julia E. — STRING, STRAIGHTEDGE, AND SHADOW: THE STORY OF GEOMETRY. New York: Viking Press, 1965. This book explores geometry in nature along with a history of some famous geometers and how geometry was used in ancient times.

Flannery, Sarah and David Flannery — IN CODE: A MATHEMATICAL JOURNEY. Algonquin Books, 2002. This story tells how the girl next door moved from the simple math puzzles that were the staple of dinnertime conversation to number theory and her creative algorithm breakthroughs.

Haney, Jan P. — CALCULATORS. Milwaukee, Wisconsin: Raintree Publications, 1985. This book contains a history of calculating machines and illustrations of the parts of a calculator. Also, includes calculator games and activities.

Paraquin, Charles H. — WORLD'S BEST OPTICAL ILLUSIONS. New York: Sterling Publishing Company, 1987. This book of illusions, with a question posed about each illusion reinforces spatial perception.

Sakade, Florence — ORIGAMI, JAPANESE PAPER FOLDING. Rutland, Vermont: Charles E. Tuttle Company, 1957. A background on origami.

Singh, Simon — CODE BOOK: HOW TO MAKE IT, BREAK IT, HACK IT, OR CRACK IT. Bantam Doubleday, 2002. This book covers actual instances of code-breaking from its role in the plan to execute Mary, Queen of Scots, to the Navajo code talkers in World War II.

Singh, Simon — FERMAT'S ENIGMA: THE EPIC QUEST TO SOLVE THE WORLD'S GREATEST MATHEMATICAL PROBLEM. Knopf Publishing, 1998. This book brings to life a riveting story of a mathematical problem that sprang from the study of the Pythagorean theorem.

Technology Resource Guide

The Technology Resource Guide is designed as a companion to the 2007 Mississippi Mathematics Framework to provide the mathematics teacher options to include technology in their instructional practices in order to enhance particular strategies. It gives examples of suggested software programs and online resources that maybe helpful.

The mathematics teacher should use this guide to find extra information about technology that can enhance student achievement in mathematics. Due to rapid changes in technology, the resources included here are as current as possible. (September 2005)

Online Resources

Search Tools

Yahoo	http://www.yahoo.com
Excite	http://www.excite.com
Altavista	http://www.altavista.com
Lycos	http://www.lycos.com
Hotsheet	http://www.hotsheet.com
Infoseek	http://www.infoseek.com
Dogpile	http://www.dogpile.com
Metacrawler	http://www.metacrawler.com
Ask Jeeves	http://www.askjeeves.com

Search Engines Especially for Kids

Yahooligans	http://www.yahooligans.com
Surfnet for Kids	http://www.surfnetkids.com
Cyber Kids	http://www.cyberkids.com

Mathematics Websites

Math in Daily Life
http://www.learner.org/exhibits/dailymath/

Algebra Buster
http://www.algebra-online.com/

Geometry Basic
http://www.mathleague.com/help/geometry/geometry.htm

Web Math
http://www.webmath.com/

Hamilton's Math to Build On
http://mathforum.org/~sarah/hamilton/ham.contents.html

Gallery of Interactive Geometry
http://www.geom.uiuc.edu/apps/gallery.html

Math2
http://www.math2.org/

Ask Dr. Math
http://mathforum.org/dr.math/abt.drmath.html

Math Archives
http://archives.math.utk.edu/

Click on Bricks Multiplication from 1 to 4
http://kathyschrock.net/clickonbricks/index2.htm

Discovery School Mathematics
http://school.discovery.com/lessonplans/math.html

Frank Potter's Science Gems in Mathematics
http://www.sciencegems.com/math.html

Math Goodies
http://www.mathgoodies.com/

Math Teacher Link
http://mtl.math.uiuc.edu/classroom_resources.htm

Math World
http://mathworld.wolfram.com/

Mega Mathematics
http://www.c3.lanl.gov/mega-math/

Purple Math
http://www.purplemath.com/internet.htm

Super Kids Math Worksheet Creator
http://www.superkids.com/aweb/tools/math/index.shtml

Algebra Help
http://www.algebrahelp.com/index.jsp

AAA Math
http://www.aaamath.com/index.html

Academic Info - Mathematics
http://www.academicinfo.net/math.html

The Educator's Reference Desk - Mathematics
http://www.eduref.org/cgi-bin/lessons.cgi/Mathematics

Cool Math
http://www.coolmath.com/

Math Magic
http://mathforum.org/mathmagic/

Fun Brain
http://www.funbrain.com/numbers.html

Biographies of Women Mathematicians
www.agnesscott.edu/lriddle/women/women.htm

Math Stories
www.mathstories.com

A+ Math
www.aplusmath.com/

Math Activities
www.activitiesforkids.com/teacher.htm

Hershey Fractions
http://mathforum.org/paths/fractions/hershey.frac.html

Mathematical Interactive
http://mathematics.hellam.net/

Money Factory
http://www.moneyfactory.com/

The Mint Part of Money
www.usmint.gov

Education 4 Kids
http://edu4kids.com/

Place Value: K-3
http://mathcentral.uregina.ca/RR/database/RR.09.96/mcleod1.html

GLOSSARY

This glossary is designed to help teachers understand mathematics terminology. The following terms cover the major terms associated with assessment and the curriculum guide.

absolute value - the distance of a number from zero on the number line.
$|4| = 4; |-4| = 4$

acute angle - an angle with a measure greater than 0° and less than 90°.

addend - any number being added; 4 + 3 = 7 where 4 and 3 are addends.

additive inverse – the opposite of a number. When a number is added to its additive inverse, the sum is zero. 15 and –15 are additive inverses because 15 + (-15) = 0.

algebraic expressions - a group of numbers, symbols, and variables that express an operation or series of operations. For example, $5x^2 + 6$ is an expression.

algebraic thinking - thinking skills which are developed by working with problems which require students to describe, extend, analyze, and create a variety of oral, visual, and physical patterns such as ones based on color, shape, number, sounds from real life and other subjects such as literature and music.

algorithm - a step-by –step procedure for performing a given type of calculation or solving a given type of problem.

alternative assessment - any type of assessment in which students create a response to a question, as opposed to assessments in which students choose a response from a given list, such as multiple-choice, true/false, or matching. Alternative assessment can include short answer questions, essays, performance assessments, oral presentations, demonstrations, exhibitions, and portfolio.

angle - two rays that share an endpoint.

area - the measure in square units, of the interior region of a two-dimensional figure or the surface of a three-dimensional figure.

Associative Property of Addition - the sum stays the same when the grouping of addends is changed. (a + b) + c = a + (b + c), where a, b, and c stand for any real numbers.

Associative Property of Multiplication - the product remains the same when grouping of factors is changed. (a □ b) □ c = a □ (b □ c), where a, b, and c stand for any real numbers.

attribute - quality or characteristic.

authentic assessment -assessment tasks that elicit demonstrations of knowledge and skills in ways that resemble "real-life" as closely as possible, engage students in the activity, and reflect sound instructional practices.

bar graph - a representation of data in which the length of a rectangle or bar is used to represent a numerical amount; a bar graph typically has spaces between the bars.

Calculus - the mathematics of change and motion. The main concepts of calculus are limits, derivatives, and areas under curves.

capacity - the amount that a given container can hold expressed in unit such as milliliters, cups, liters, quarts.

circumference - the perimeter of a circle; ($C=2\pi r$ where r is the radius).

Commutative Property of Addition - the sum remains the same when the order of the addends is changed. a + b = b + a, where a and b are any real numbers.

Commutative Property of Multiplication - the product remains the same when the order of the factors is changed. a □ b = b □ a, where a and b are any real numbers.

conditional statements - a logical statement consisting of two parts, a hypothesis and a conclusion.

congruent (≅) - having the same size and shape.

conjecture - an unproven statement based on observations.

deductive reasoning – the process of reasoning where specific cases are tested against a specific rule.

dilation – a proportional shrinking or enlargement of a figure.

Discrete mathematics the study of mathematical properties of sets and systems that have a countable number of elements.

Distributive property – the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products. a □(b + □ c) = (a □ b) = (a □ c) and a □ (b – c) = (a □ b) – (a □ c), where a, b, and c stand for any real number.

domain – in a function, f(x), the possible values of x in the given situation.

equation – a mathematical sentence with an equal sign. The amount on one side of the equal sign has the same value as the amount on the other side.

estimation - an approximation of a measure or calculation based on a strategy such as rounding, front-end estimation, compatible numbers (see also reasonableness of results).

Euclidean Algorithm – a algorithm to determine the greatest common divisor of two integers.

even number – a whole number that has 2 as a factor. All even numbers end with 0, 2, 4, 6 or 8 and are divisible by 2.

expanded form – a way to write numbers that shows the place value of each digit.

experimental probability - the actual number of occurrences of a particular (favorable) outcome divided by the total number of trials or outcomes for a particular experiment.

exponent -The number that tells how many equal factors there are.

exponential function – a function in which the independent variable is an exponent. Exponential functions have the general form $f(x) = a^n$, where $a > 0$ and $a \neq 1$.

exponential form - a way of writing a number using exponents.

expression - a variable or combination of variables, numbers, and symbols that represent a mathematical relationship.

factor - an integer that divides another with no remainder.

function - a relation in which each element of one set (the domain) is paired with exactly one element of a second set (range).

hypothesis - the "if" part of a conditional statement.

indirect measurement - measurement in which measures cannot be calculated directly but must be determined by finding other direct measures; for example: rates such as miles per hour.

Identity Property of Addition and Multiplication – for any real number, a: a + 0 = a and 0 + a = a; a □ 1 = 1 and 1 □ a = a.

Inductive reasoning - making a generalization based on observation of specific cases and consideration of a pattern.

inequality - a mathematical sentence that compares two unequal expression using one of the symbols $\neq$, <, >, $\leq$, or $\geq$.

inference - a generalization based on statistical data; a prediction based on sample or experimental data.

integer - the set of whole numbers and their opposites
{... -4, -3, -2, -1, 0, 1, 2, 3, 4, ...}

irrational number - a real number that cannot be written as a ratio of two integers.

line plot - a diagram showing frequency of data on a number line.

linear equation - an equation in one or two variables with no exponents other than one and with no products of the variables.

mass - the quantity of matter in an object.

mean - the sum of a set of numerical data divided by the number of items in the set.

measures of central tendency - numbers used to describe sets of data. The mean, median, and mode are measures of central tendency.

median - the middle number of a set of numbers when the numbers are arranged from least to greatest or the mean of two middle numbers when the set has two middle numbers.

mode - the most frequent occurring number in a set of data. There may be one, more than one, or no mode.

model - a representation that uses physical objects or drawings and their actual manipulations to illuminate concepts or problems.

monomial - an expression that is a number, a variable, or the product of a number and one or more variables.

multiple - the product of a whole number and any other whole number.

multiplicative inverse – reciprocal of a number. When a number is multiplied by its multiplicative inverse, the product is always one. Zero is the only number with no multiplicative inverse. $a\frac{1}{a}=1$, where $a \neq 0$.

natural numbers - the counting numbers: 1, 2, 3, 4, 5, ……

net - a two-dimensional shape that can be folded into a three-dimensional figure

non-linear equation - an equation whose graph is not a line such as $\frac{1}{x}=7$, $y=3xy+4$ or $x^3=7.$

non-standard measurement - measuring using materials such as paper clips, different sized scoops of ice, thumbprints, footsteps, etc.

obtuse angle – an angle with a measure greater than 90° and less than 180°.

odd number - a whole number represented by the expression, 2n + 1 where n is a whole number. The ones digit of any odd number is 1, 3, 5, 7, or 9.

open-response tasks - the kind of performance required of students when they are required to generate an answer, rather than select it from among several possible answers, but there is still a single, correct response.

ordinal number - a whole number that names the position of an object in a sequence.

parallel -in Euclidean geometry, always the same distance apart. Parallel lines lie in the same plane and do not intersect. Parallel planes never intersect.

percent - a special ratio that compares a number to 100 and using the symbol % sign.

perimeter - the distance around an object or geometric figure.

perpendicular - intersecting to form right angles.

polygon - a closed, plane figure composed of line segments that meet at their endpoints or vertices.

polynomial - the sum of monomials.

portfolio - a purposeful, integrated collection of student work showing effort, progress, or degree of proficiency.

postulate - a mathematical statement that is accepted as true without proof. Also called an axiom.

prime number - a number that has exactly two positive factors, itself and one.

problem solving - applying or putting together knowledge and skills already learned in new situations or to derive new knowledge and/or solutions.

protractor - a tool or device for determining the measurement of angles.

Pythagorean Theorem - for any right triangle, the sum of the squares of the lengths of the two legs equals the square of the length of the hypotenuse. $a^2 + b^2 = c^2$.

quadratic function – a function described by an equation of the form $f(x) = ax^2 + bx + c$ where $a \neq 0$.

range – the difference of the lowest and highest value within a set of numbers.

rational number - a number that can be expressed as a ratio of two integers.

ratio - a comparison of two numbers using division.

real graph – a graph created with actual objects.

reasonableness of results - acceptability of an estimated or approximate measurement or calculation based on its making sense.

reflection (flip) - a geometric transformation creating a mirror image of a figure on the opposite side of a line.

right angle - an angle that measures 90°.

rotation (turn) - a geometric transformation which a figure is turned a given angle and direction around a point.

rubric - an established and written-down set of criteria for scoring or rating students' performance on tests, portfolios, writing samples, or other performance tasks.

sample space - a list of all possible outcomes of an experiment.

scientific notation - a form of writing numbers as the product of a power of 10 and a decimal number greater than or equal to 1 and less than 10.

simulation - a model of an experiment that might be impractical to carry out.

translation (slide) - a geometric transformation, which involves moving the figure up, down, right or left without changing its orientation.

statistical data - qualitative or quantitative information collected through experimentation and/or observation for the purpose of analysis, presentation (such as graphic) interpretation, or inference.

straight angle - an angle with a measure of 180°.

straight edge - a tool similar to a ruler, but without markings.

theoretical probability - finding the probability of an event without doing an experiment or analyzing data.

variable - a letter or symbol that represents a number value in an expression.

volume - the number of cube units it takes to fill a three-dimensional space.

weight - a measure of the heaviness or force of gravity on an object.

whole number- the set of numbers that includes zero and the natural numbers {0, 1, 2, 3, 4 …}

Zero Product Property - the product of any number and zero is zero. a □0 = 0 and 0 □ a = 0, where a is any real number.

Explanation of Norman Webb's Depth of Knowledge

Depth of Knowledge

Assessments must be demanding cognitively as what students are expected to know and do as stated in the curriculum frameworks. This is one aspect of the alignment between assessments and curricula.

Each test question does not have to have precisely the same depth-of knowledge level as the corresponding competency or objective. However, the majority of test items on an assessment must correspond to the most common depth-of-knowledge level for competency and related objectives.

Two factors:

Sophistication and complexity: Sophistication will depend on the abstractness of the activity, the degree to which simple knowledge and skills have to be recalled or drawn upon, the amount of cognitive processing required, the complexity of the content concepts used, the amount of content that has to be recalled or drawn upon, the lack of routine, and the need to extend knowledge meaningfully or produce novel findings.
Prior instruction: Test items that address complex knowledge can still have a low depth of knowledge level, if the required knowledge is commonly known and students with normal instruction at a grade level should have had the opportunity to learn how to routinely (habitually) perform what is being asked.

Level 1 (Recall) includes the recall of information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula. That is, in mathematics a one-step, well-defined, and straight algorithmic procedure should be included at this lowest level. Other key words that signify a Level 1 include "identify," "recall," "recognize," "use," and "measure." Verbs such as "describe" and "explain" could be classified at different levels depending on what is to be described and explained.

Level 2 (Skill/Concept) includes the engagement of some mental processing beyond a habitual response. A level 2 assessment item requires students to make some decisions as to how approach the problem or activity, whereas Level 1 requires students demonstrate a rote response, perform a well-known algorithm, follow a set procedure (like a recipe), or perform a clearly defined series of steps. Keywords that generally distinguish a Level 2 item include "classify," "organize," "estimate," "make observations," "collect and display data," and "compare data." These actions imply more than step. For example, to compare data requires first identifying characteristics of the objects or phenomenon and then grouping or ordering the objects. Some action verbs, such as "explain," "describe," or "Interpret" could be classified at different levels depending on the object of the action. For example, if an item required students to explain how light affects mass by indicating there is a relationship between light and heat, this is considered a Level 2. Interpreting information from a simple graph, requiring reading information from the graph, also is a Level 2. Interpreting information from a complex

graph that requires some decisions on what features of the graph need to be considered and how information from the graph can be aggregated is a level 3. Caution is warranted in interpreting Level 2 as only skills because some reviewers will interpret sills very narrowly, as primarily numerical skills, and such interpretation excludes from this level other skills such as visualization skills and probability skills, which may be more complex simply because they are less common. Other Level 2 activities include explaining the purpose and use of experimental procedures; carrying out experimental procedures; making observations and collecting data; classifying, organizing, and comparing data; and organizing and displaying data in tables, graphs, and charts.

Level 3 (Strategic Thinking) requires reasoning, planning, using evidence, and a higher level of thinking than the previous two levels. In most instances, requiring students to explain her thinking is a Level 3. Activities that require students to make conjectures are also at this level. The cognitive demands at Level 3 are complex and abstract. The complexity does not result from the fact that there are multiple answers, a possibility for both levels 1 and 2, but because the task requires more demanding reasoning. An activity, however, that has more than one possible answer and requires students to justify the response they give would most likely be a Level 3. Other Level 3 activities include drawing conclusions from observations; citing evidence and developing a logical argument for concepts; explaining phenomena in terms of concepts; using concepts to solve problems.

Level 4 (Extended Thinking) requires complex reasoning, planning, developing, and thinking most likely over an extended period of time. The extended time period is not a distinguishing factor if the required work is only repetitive and does not require applying significant conceptual understanding and high-order thinking. For example, if a student has to take the water temperature from a river each day for a month and then construct a graph, this would be classified as a Level 2. However, if the student is to conduct a river study that requires taking into consideration a number of variables, this would be a Level 4. At Level 4, the cognitive demands of the task should be high and the work should be very complex. Students should be required to make several connections – relate ideas within the content area or among content areas – and have to select one approach among many alternatives on how the situation should be solved, in order to be at this highest level. Level 4 activities include designing and

conducting experiments; making connections between a finding and related concepts and phenomena; combining and synthesizing ideas into new concepts; and critiquing experimental designs.

Mathematics Resources

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 22091

American Mathematical Association
201 Charles Street
Providence, RI 02904-2294
http://www.ams.org

Mathematical Association of America
1529 18th Street, N.W., Suite 600
Washington, DC 20036
http://www.maa.org

American Statistical Association
1429 Duke Street
Alexandria, VA 22314-3415
http://www.amstat.org

Association for Computing Machinery
ACM, 1515 Broadway
New York, NY, 10036
http://www.acm.org

Society of Actuaries
475 North Martingale Rd, Suite 800
Schaumburg, IL 60173-2226
www.BeAnActurary.org

Association for Women in Mathematics
4114 Computer and Space Sciences Bldg.
University of Maryland
College Park, MD 20742-2461
http://www.awm-math.org