Evaluation of Linking Methods for Placing Three-Parameter Logistic Item Parameter Estimates Onto a One-Parameter Scale

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Abstract

Different item response theory (IRT) models may be employed for item calibration. Change of testing vendors, for example, may result in the adoption of a different model than that previously used with a testing program. To provide scale continuity and preserve cut score integrity, item parameter estimates from the new model must be linked to the item parameter estimates obtained from the previous model. Given that the assumptions of different models vary, it is necessary to identify linking methods that best place item parameters scaled using the new model to item parameters scaled using the old model.

In this study, we explore the results of equating 3PL parameter estimates to 1PL parameter estimates, using Moment, Characteristic Curve, and Theta Regression methods. The data set consists of 31,813 student responses to a 78 item, multiple choice, End-of-Instruction exam. The evaluation criteria include the impact of different linking methods on scale score means and standard deviations, scale score frequency distributions, Test Characteristic Curves and Standard Error Curves, test information, and the classification of students into the different proficiency levels.

The Characteristic Curve linking methods best aligned the 3PL scale to the 1PL scale. From the results, if aligning the mean and SD of the scale score distribution is perceived to be most important, then the Stocking and Lord method is preferable. If the classification of students into different performance categories is deemed most important, then the Haebara method is recommended. In either case, the differences are trivial.

Objective/Purpose

Different item response theory (IRT) models, for example the One-, Two-, or Three-Parameter Logistic models, are available for large-scale educational assessment to calibrate multiple-choice items. Change of testing vendors and/or preferences of state educators and/or Technical Advisory Committee members may result in the adoption of a model different from that previously used with a testing program. To provide a continuity of scale, item parameter estimates obtained from the newly selected model should be linked to the item parameter estimates obtained from the previous model. This requirement is especially important when the State proficiency level standards must be preserved for future administrations.

Each IRT model functions under a unique set of assumptions. For example, the Three-Parameter Logistic (3PL (Lord, 1980)) model assumes that items vary in discrimination and students can correctly answer multiple-choice (MC) items by guessing. The Rasch (1PL (Rasch, 1960)) model assumes that all items discriminate similarly and there is no guessing. The assumptions for the 1PL model are strong and less likely to be strictly met (Divgi, 1986; Traub, 1983).

Several test equating methods and designs are described in the literature for linking different forms and tests (Kolen and Brennan, 1995). However, no formal study (as far as we know) exists on maintaining a scale obtained from the 1PL model when the 3PL model is to be used for future item calibrations. Since linking methods may provide different results, and given that the assumptions of 1PL and 3PL models are dissimilar, it is essential to investigate which linking method provides results best aligned with the original scale. The main objective of this study is to investigate, for a large scale assessment, which linking method places a 3PL scale onto a Rasch scale while best preserving proficiency standards set on the 1PL scale if the cutscores set under the previous model are to be maintained. The evaluation criteria include the impact of different linking methods on scale score means and standard deviations (SDs), scale score frequency distributions, Test Characteristic Curves (TCC) and Standard Error (SE) Curves, test information, and the classification of students into the different proficiency levels.

Theoretical Framework and Perspective

There exists a need to explore the psychometric challenges faced in designing and conducting cross-IRT model linking/equating, and thus, we hope to provide grounds for discussion of evidence and our claim of which method best links 3PL parameter estimates to a Rasch scale. Exploratory analysis is necessary due to the lack of documentation on this specific process and the changing needs of State educators. Our hope is to provide a basis for discussion and further research.

The most commonly employed linking and equating methods are Characteristic Curve methods (Stocking and Lord, 1983 and Haebara, 1980) and Moment methods (Mean/Mean method of Loyd and Hoover (1980) and Mean/Sigma method of Marco (1977)). We plan to use each of the aforementioned methods, as well as linear regression of person ability estimates (mentioned hereafter as Theta Regression) from the two models, to link an administration calibrated in the 3PL model to an anchor scale in the 1PL model, in order to examine the alignment of each technique's 3PL estimates with the desired scale (1PL). Several statistical and graphical criteria were used to compare methods, but it is our perspective that the method that best preserves proficiency level classifications and ability distributions will most effectively link to the 1PL scale.

Methods

Calibration

The three-parameter logistic (3PL) model (Lord, 1980) was used for item parameter estimation in 3PL metric and the two-parameter partial credit (2PPC) model, a special case of Bock's (1972) nominal model, with a single slope was used for item parameter estimation in 1PL metric. The PARDUX (Burket, 2002) microcomputer program was used for calibration. PARDUX constrains the mean and SD of the examinee ability distribution to 0 and 1, respectively, during the item parameter estimation process to obtain model identification using the Marginal Maximum Likelihood Estimation technique for item parameters and Maximum Likelihood Estimation for person ability.

Linking Methods to Transform the 3PL Item Parameters to the 1PL Scale

In order to compare item parameters and equating results between the 1PL and 3PL models, a reparameterization was necessary. Algebraically, the exponential term for a correct response under the 2PPC single slope parameterization is written as: $\exp(f(\Theta - g))$, where f is the slope, Θ is the person parameter, and g is the item difficulty. Under the traditional 3PL parameterization, the exponential term for a correct response is written as exp (1.7A (Θ – B)), where A is item discrimination, and B is item difficulty. The relationships between f and A (f=1.7A) and between g and B (g=1.7AB) are then used to obtain A (A=f/1.7) and B (B=g/1.7A). The A and B values derived from the 1PL model are the item parameters in the 3PL metric. The C-parameter was set to zero. The item parameters on the 1PL scale metric were transformed to the 3PL metric for the purpose of doing the linkings.

Moment methods (Mean/Mean and Mean/Sigma), Characteristic Curve methods (Stocking and Lord (SL) and Haebara), and Theta Regression were utilized to link item parameters from the 3PL model to the 1PL model scale. The application of these equating methods is described comprehensively in Kolen and Brennan (1995). For the Theta Regression method, notice that there is a one to one correspondence between the student's thetas estimated from the 1PL and 3PL models. Let scale I be based on the item parameters from the 1PL model transformed to the 3PL metric. Let scale J be based on the item parameters from the 3PL model. The relationship between the θ-values for the two scales is given by:

$$\theta_{Ji} = M_1 \theta_{Ii} + M_2$$

where M_1 and M_2 are linear scaling constants and θ_{Ji} and θ_{Ii} are values of θ for individual i on scale J and I. For the linear regression procedure, regression coefficients (scaling constants) were determined by considering the examinee ability estimates obtained from the 1PL model as the independent variable and those from the 3PL model as the dependent variable.

The item parameters on the two scales are related as:

aJi=aIj/M1, bJi=M1bIj + M2, and cJj = cIj,

where aJj, bJj, and cJj are the item parameters for item j on scale J and aIj, bIj, and cIj are the item parameters for item j on scale I. The scaling constants used to transform 3PL to the 1PL scale under the Mean/Mean method were obtained from the following relationships:

M1 = Mean (a, 1PL)/Mean (a, 3PL), and M2 = Mean (b, 1PL) – M1*Mean (b, 3PL).

The scaling constants used to transform 3PL to 1PL scale under the Mean/Sigma method were obtained from the following relationships:

M1 = SD (b, 1PL)/SD (b, 3PL), and M2 = Mean (b, 1PL) – M1*Mean (b, 3PL), where SD=standard deviation.

In order to estimate scaling constants for the SL (Stocking and Lord, 1983) procedure, let $\hat{\psi}_j$ be the estimated true score obtained from the 2PPC (single slope) model in the 3PL metric and $\hat{\psi}_j^*$ be the estimated true score obtained from the 3PL model after it has been transformed to the 1PL scale

$$\hat{\psi}_j = \hat{\psi}(\theta_j) = \sum_{i=1}^{n} P_i(\theta_j; a_i, b_i, c_i)$$

$$\hat{\psi}_j^* = \hat{\psi}(\theta_j) = \sum_{i=1}^{n} P_i(\theta_j; \frac{a_i}{M_1}, M_1 b_i + M_2, c_i)$$

The SL procedure, also known as the Test Characteristic Curve method, determines the scaling constants (M1 and M2) in such a way that the average squared difference between true score estimates is as small as possible. That is, M1 and M2 can be found by minimizing the quadratic loss function (F):

$$F = \frac{1}{N}\sum_{a=1}^{N}(\hat{\psi}_j - \hat{\psi}_j^*)^2$$

The Haebara method, also called the Item Characteristic Curve method (Haebara, 1980), minimizes the multivariate function shown below to estimate scaling constants (M1 and M2).

$$F = \frac{1}{N}\sum_{a=1}^{N}[P_i(\theta_j; a_i; b_i; c_i) - P_i(\theta_j; \frac{a_i}{M_1}; M_1 b_i + M_2; c_i)]^2$$

These scaling constants for the Mean/Mean, Mean/Sigma, SL, and Haebara methods were estimated from a micro computer program ST (Hanson and Zeng, 1995). Final scaling constants (M1'=75 and M2'=680) were used to place the item parameters onto the final scale (see Table 1). These item parameters were used to score students, estimate their scale scores and performance level classification (based on previously established cut scores).

Data Sources

Data for this study were obtained from a large-scale English II End-of-Instruction (EOI) test designed for students in grades 8 through 12. The data set consisted of demographic information for 31,813 "Regular" students and their responses to 80 operational items. Two of the items were dropped from the data analyses for poor item characteristics. The "Regular" category excludes students who are English language learners (ELL), those in an individualized education program (IEP), high mobility students, second time testers, and those taking a Braille version of the test.

Results

Scaling Constants

The scaling constants obtained from the transformation methods are presented in Table 1. These scaling constants were used to transform 3PL item parameters onto the 1PL scale. The SL method, followed by the Haebara method, resulted with the largest additive constant (M2) and the Mean/Sigma method resulted with the smallest. The largest multiplicative constant or slope (M1) resulted from the Mean/Mean method and the lowest from the Mean/Sigma method. Since the scaling constants are indicative of the resulting scale score mean and SD, similar results can be expected when the scale is transformed to the final scale score metric.

Test Characteristic Curves, Standard Error, Frequency Distribution, and Test Information Functions

The TCCs of the various methods are shown in Figure 1. The SL and Haebara methods resulted in a better alignment of the TCCs with the original 1PL model from the mid to high range of the ability scale. At the lower end, the methods can be distinguished by the differences between the 3PL and 1PL TCCs' upper asymptotes. The standard error curves (Figure 2) indicated that the Mean/Sigma method resulted in the smallest standard errors (SEs) in the middle range of the score distribution. Although the Mean/Mean method produced the highest standard errors in the center of the distribution, its SEs were nearly constant across the distribution. The smallest SEs near the LOSS and HOSS were associated with the Mean/Mean linking method, which was also closest to the 1PL SEs in the proximity of the LOSS. In the score range from the center of the distribution towards the HOSS, the linking methods that produced SE curves most in line with the 1PL SE curve were the SL, Haebara, and (to some extent) Theta Regression methods. For ease of comparison between the 1PL SE curve and those of each method, please refer to Appendix A.

The frequency distribution curves (Figure 3) indicate that the scale score distributions that resulted from the SL and Haebara methods are very close to that of the 1PL model. In comparison, the frequency distribution from the Mean/Mean method is flatter and the frequency distribution from the Mean/Sigma method is peaked slightly at the lower end of the distribution, indicating that these methods resulted in slightly smaller scale scores than the 1PL, SL, and Haebara methods. Theta Regression produced a frequency distribution that is similar to that of the 1PL model, but shifted slightly towards the low end of the scale.

The test information functions are shown in Figure 4. The Mean/Mean method resulted in a rather flat test information function, indicating that the measurement precision is low and similar across the scale, for this method. The Mean/Sigma method provided higher precision towards the middle of the scale than did the SL and Haebara methods. The 1PL method produced a relatively flat information curve, but also provided the highest precision at the lower end of the scale. The SL and Haebara methods provided the best information at the upper range of the scale. Theta Regression method provided slightly more information than the Characteristic Curve methods at the lower end of the scale.

Scale Score Mean, Standard Deviation, and Cumulative Frequency Distribution

All students were scored using the transformed item parameters on the final scale score metric. The scale score mean and SD are shown in Table 1 and plotted in Figures 5 and 6, respectively. The results showed that the scale score mean from the SL method (676.7) is closer to the 1PL model mean (681.9) than the mean from any of the other methods. The scale score means produced by all other methods are smaller than the SL method, as follows: Haebara (675.7), Theta Regression (652.2), Mean/Mean (635.0), and Mean/Sigma (628.5).

The standard deviation (SD) of the scale score distribution that resulted from the use of the Mean/Sigma method (78.5) is closest to the 1PL model SD (80.3), followed by the SDs of the SL (88.3), Haebara (90.3), and Theta Regression (93.3) methods. The Mean/Mean method produced a comparatively large SD (122.2). Note that both the scale score mean and SD are smallest for the Mean/Sigma method.

Cumulative frequency distributions are plotted in Figure 7. The proportion of students obtaining a given scale score is very similar for the 1PL, SL, and Haebara methods. The cumulative frequency distribution curves for SL and Haebara are virtually indistinguishable. The Mean/Mean, Mean/Sigma, and Theta Regression methods resulted in lower scale scores than the 1PL, SL, and Haebara methods throughout most of the ability range. Note that due to the assumptions in the 1PL model that disallow for guessing to be modeled, the scale score corresponding to the first percentile is higher for the 1PL method, than any of the others.

It is evident from the results above that the SL and Haebara methods provided a more accurate link to the 1PL scale than the Mean/Mean, Mean/Sigma, and Theta Regression methods.

Proficiency Level Classification

The impact of the linking methods on the classification of students into the different performance level categories is shown in Table 2. As is evident from the cumulative distribution function, the SL and Haebara methods classified similar proportions of students into the Unsatisfactory, Limited Knowledge, Satisfactory, and Advanced categories as the 1PL model. Both of the SL and Haebara methods classified 32.1% students at or above the Satisfactory category, which is very similar to the classification by the 1PL model (1PL classified 33.4% students at or above the Satisfactory category). Looking at the Satisfactory and Advanced levels separately, the Characteristic Curve methods' classification percentages (Haebara: Satisfactory = 23.7%, Advanced = 8.4%; SL: Satisfactory = 24.1%, Advanced = 8.0%) are close to those of the 1PL method (Satisfactory = 23.1%, Advanced = 10.3%). The classification percentages of the Haebara method align best with those of the 1PL method. The Moment (Mean/Mean and Mean/Sigma) and Theta Regression methods classified more students in the Unsatisfactory category and fewer students at or above the Satisfactory category. The

Mean/Mean method placed fewer students in the Limited Knowledge category, and the Mean/Sigma method placed more students in the Unsatisfactory category, than any other method.

Summary and Discussion

This study evaluated the use of several equating methods to link 3PL parameter estimates to a 1PL scale in order to identify empirical evidences depicting the effects of different equating/linking techniques when the linking administration was calibrated under a different IRT model than the model for the current administration. The results indicated that, except for test information, the SL and Haebara methods of linking to the 1PL scale showed results most similar to the 1PL model. The difference at the lower asymptote in the SL and Haebara methods' TCCs and the 1PL method TCC is characterized by the difference between the 3PL and 1PL models in modeling guessing. The Moment and Theta Regression methods estimated a higher proportion of students in the lower ability range.

Since the standard error of the scale scores produced by the Mean/Sigma method is smallest in the middle of the scale score distribution, and the SL and Haebara methods resulted in the smallest standard error at slightly above the middle of the distribution (see Figure 2), the information provided by these methods are also higher at the given scale score range (see Figure 4). The standard errors of Moment and Theta Regression methods are comparatively larger in the middle of the scale score range. Some of these methods, for example the Theta Regression method, showed smaller standard error (closer to the 1PL) for the scale scores at the two extremes.

In summary, the Characteristic Curve methods best aligned the 3PL scale to the 1PL scale as evaluated by the TCCs, standard errors, mean and SD of scale scores, and the classification of students into the different proficiency levels. The SL method produced a similar mean and standard deviation of the scale score distribution to the 1PL model. The use of the Haebara method resulted in performance level classification percentages more similar to the 1PL model than the SL method, although the differences were small. From the results above, if aligning the mean and SD of the scale score distribution is perceived to be most important, then the SL method is preferable. If the classification of students into different performance categories is deemed most important, the Haebara method is recommended. However, the differences between the two Characteristic Curve methods are trivial. The results that the characteristic curve methods of transformation are superior to the Moment methods are consistent with the earlier findings (Baker & Al-Karni, 1991; Hung et al., 1991; Way & Tang, 1991).

Finally, it is hoped that the results from this study contributed to our understanding of the inherent differences and commonalities between Moment and Characteristic Curve linking methods. We were privileged to use a large set of data for this study and advise that future comparison of linking methods across models is needed to examine the effects of sampling on linking results. The data set we used was moderately large (>30,000 cases) and the raw score distribution was negatively skewed (see Appendix B). We plan to further investigate the impact of these methods on score distribution when the sample size is smaller and/or with various sample raw score distributions, for example normal, bimodal, etc.

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Table 1. Scaling Constants and Scale Score Descriptive Statistics

	Scaling Constants to Place 3PL Item Parameters onto 1PL scale		Final Scaling Constants (M1'=M1*75, M2'=M1*M2+680)		Scale Score Descriptive Statistics	
Methods	**M1**	**M2**	**M1'**	**M2'**	**Mean**	**SD**
1PL	1.0	0.0	75.0	680.0	681.9	80.3
Mean/Mean	1.4	-0.4	107.5	652.2	635.0	122.2
Mean/Sigma	0.9	-0.5	65.6	640.0	628.5	78.5
Regression	1.1	-0.2	79.1	665.7	652.2	93.3
SL	1.0	0.1	73.8	689.6	676.7	88.3
Haebara	1.0	0.1	75.8	688.9	675.7	90.3

Table 2. Performance Level Classification, N=31,813

Methods	Unsatisfactory	Limited Knowledge	Satisfactory	Advanced
1PL	39.7%	26.9%	23.1%	10.3%
Mean/Mean	55.5%	20.2%	15.3%	9.1%
Mean/Sigma	63.9%	26.1%	8.6%	1.3%
Regression	49.9%	27.5%	17.1%	5.5%
SL	37.7%	30.1%	24.1%	8.0%
Haebara	38.4%	29.5%	23.7%	8.4%

Figure 1. Test Characteristic Curves

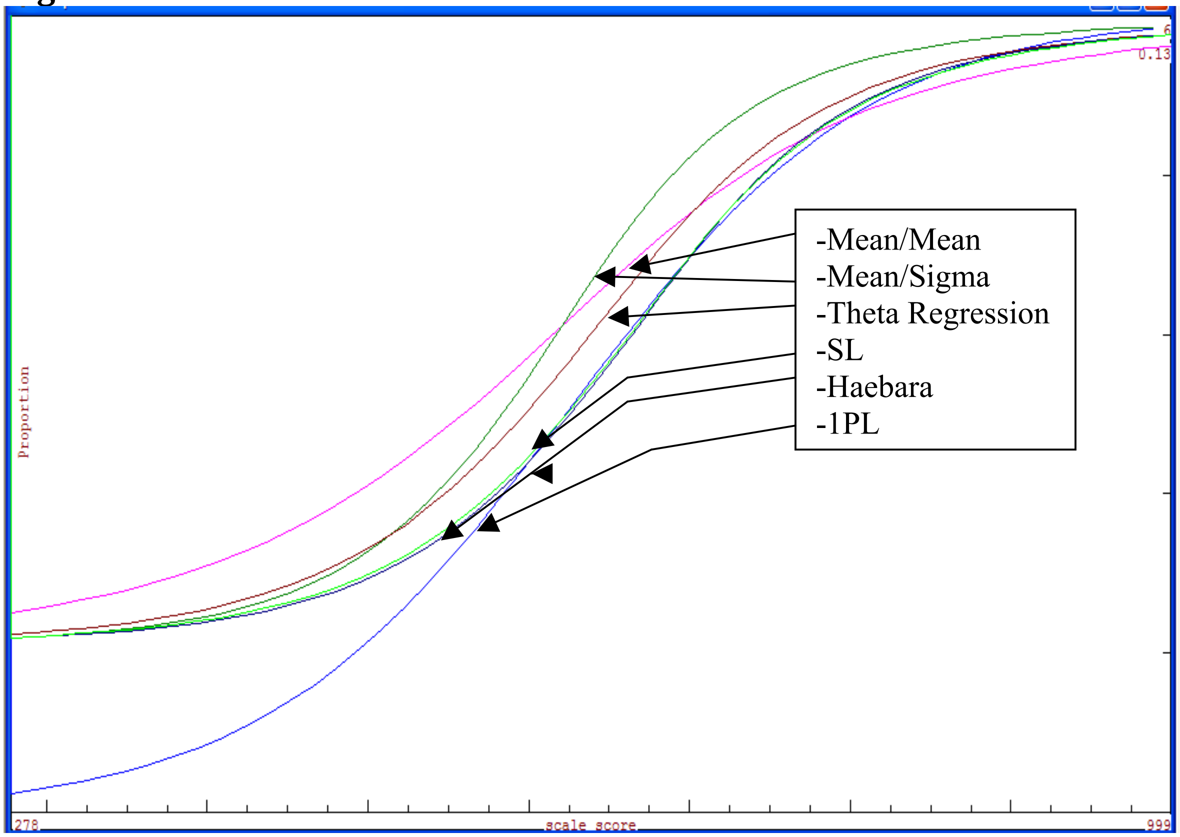


Figure 2. Standard Error Curves

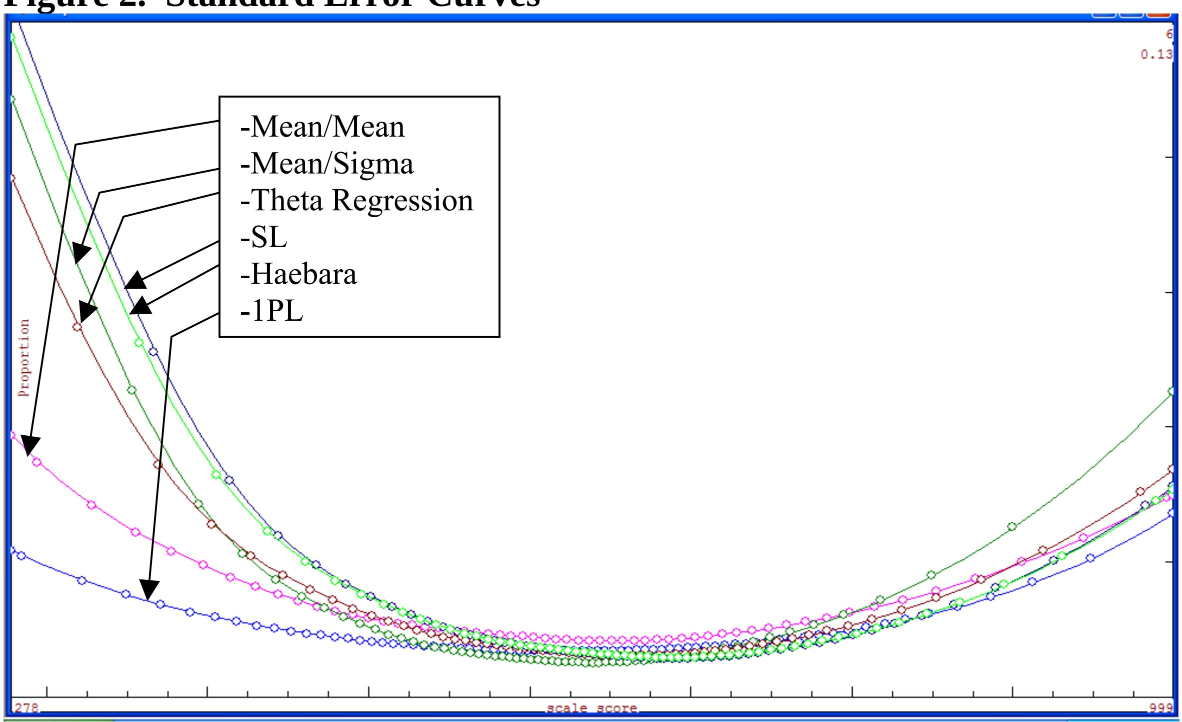


Figure 3. Frequency Distribution Curves

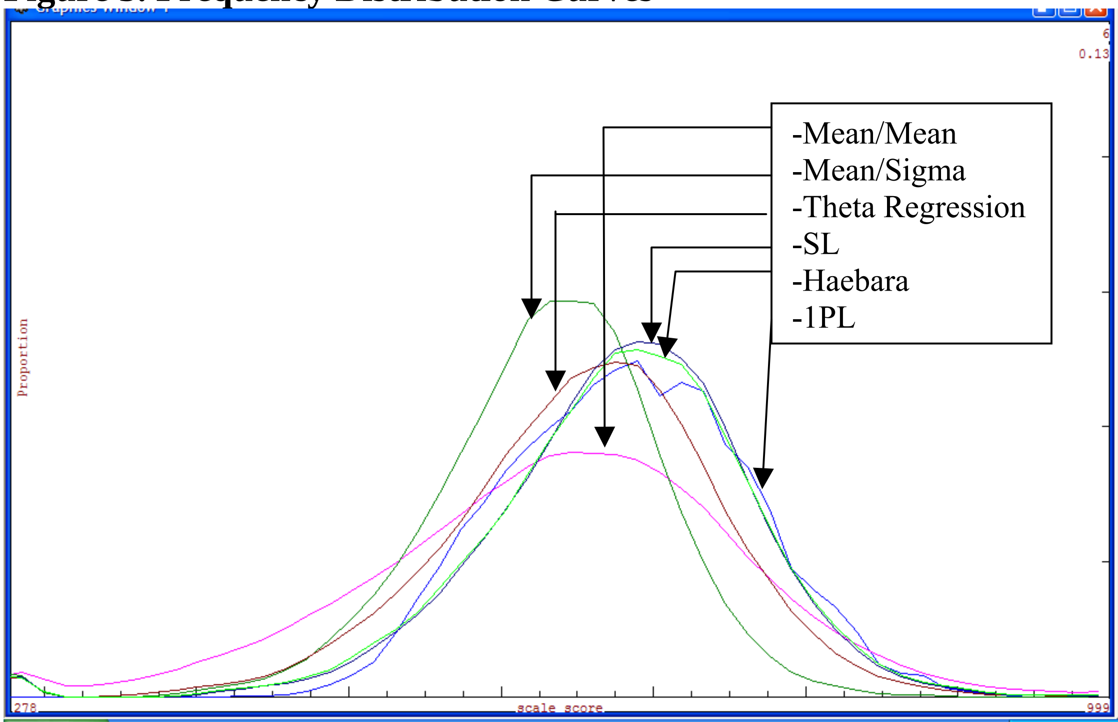


Figure 4. Test Information Functions

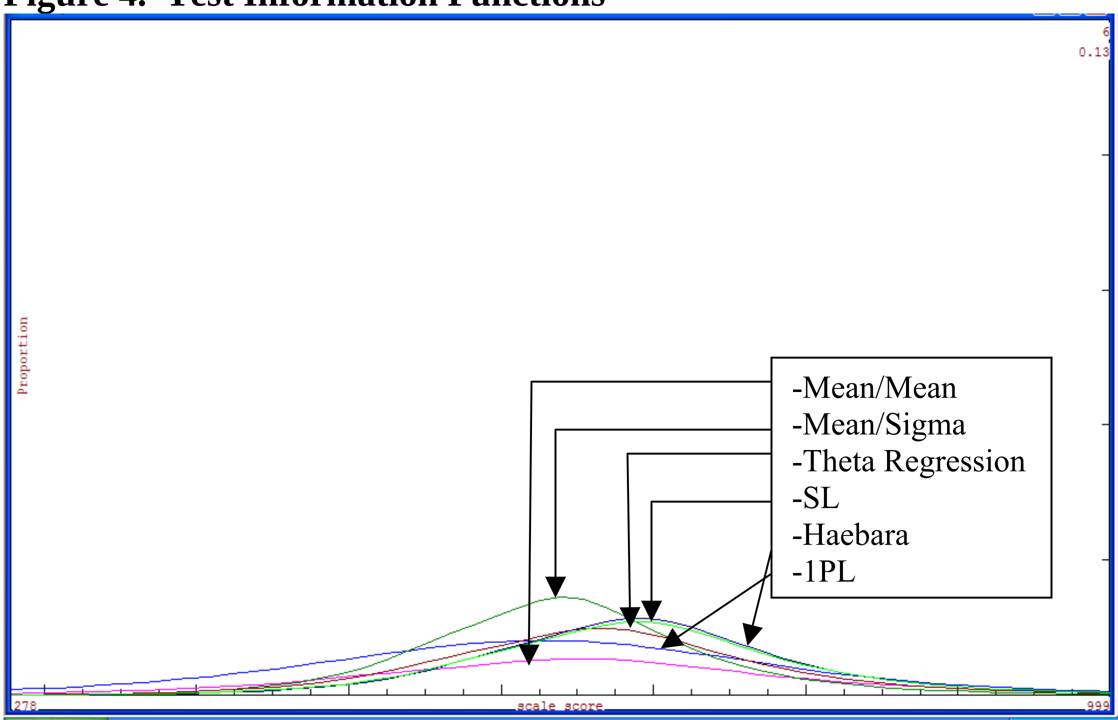


Figure 5. Mean Scale Score Across Methods

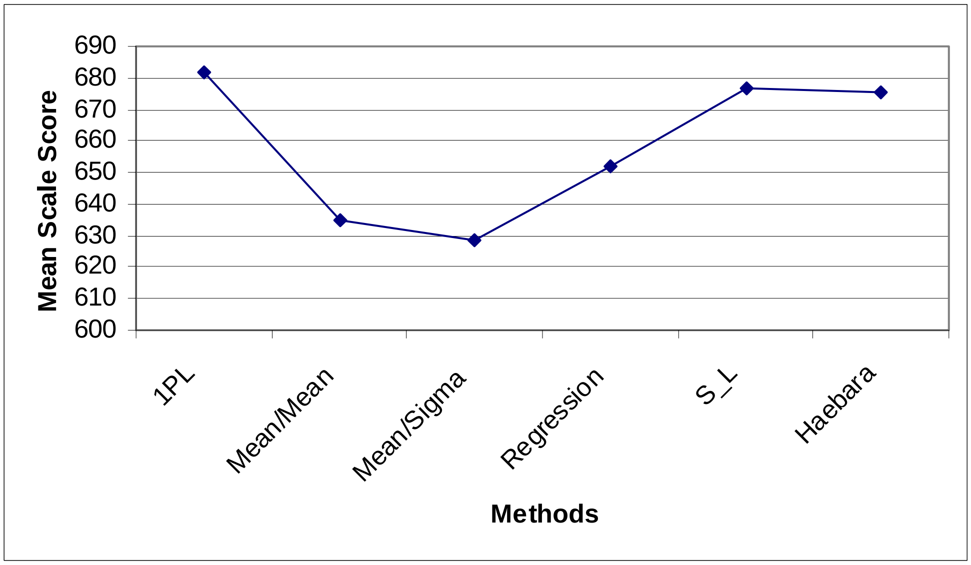


Figure 6. Standard Deviation of Scale Score Across Methods

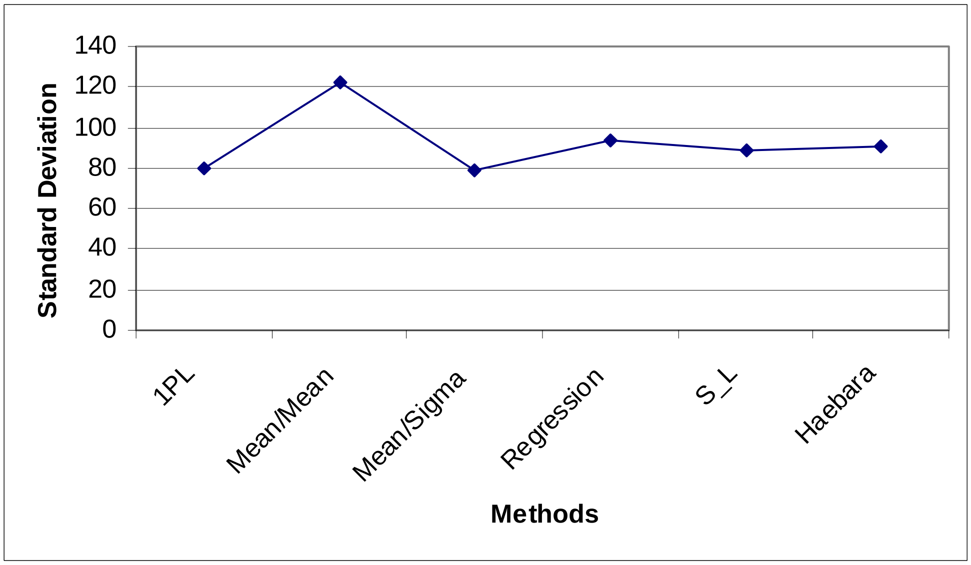
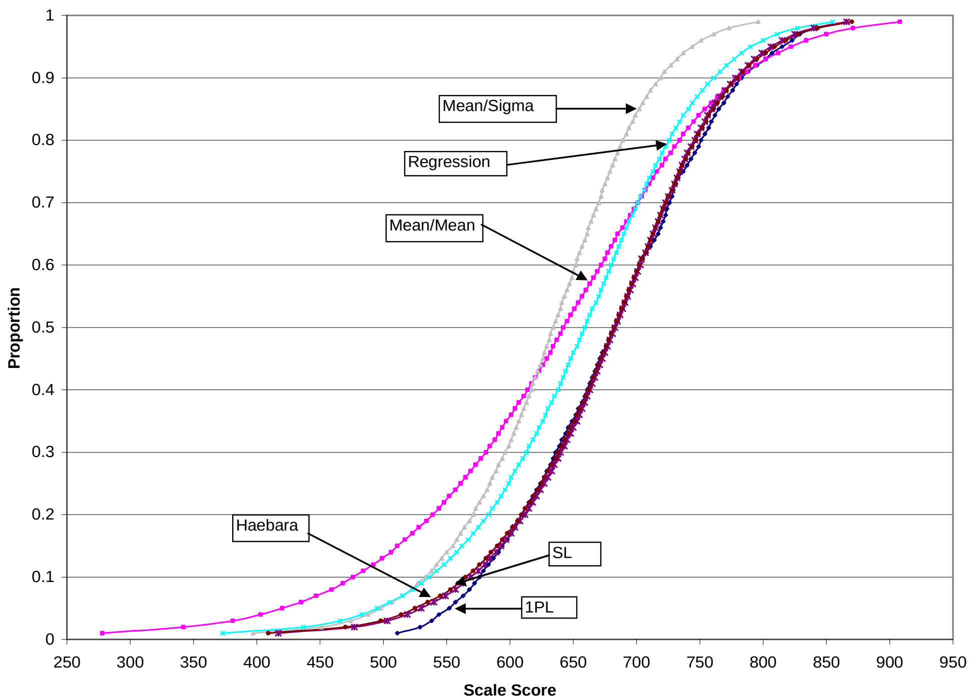
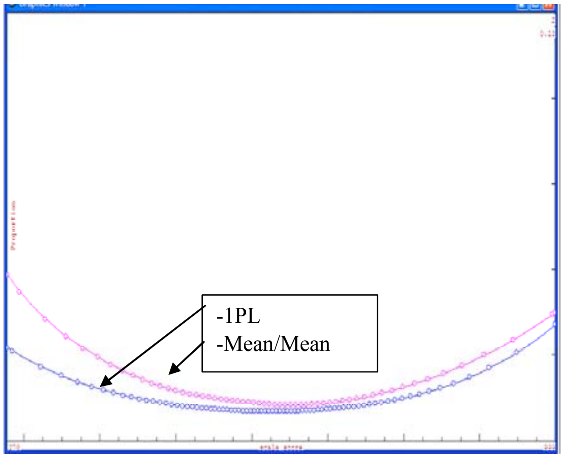
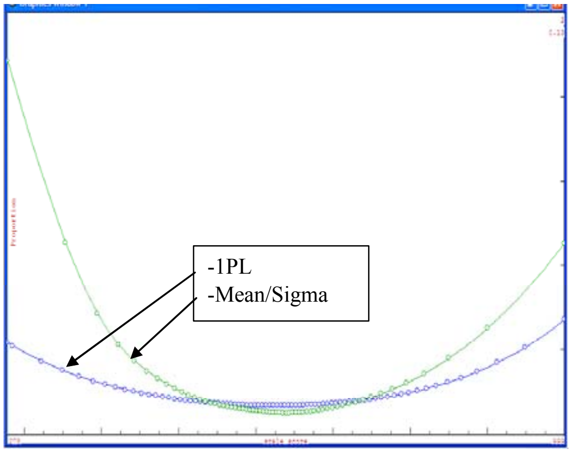


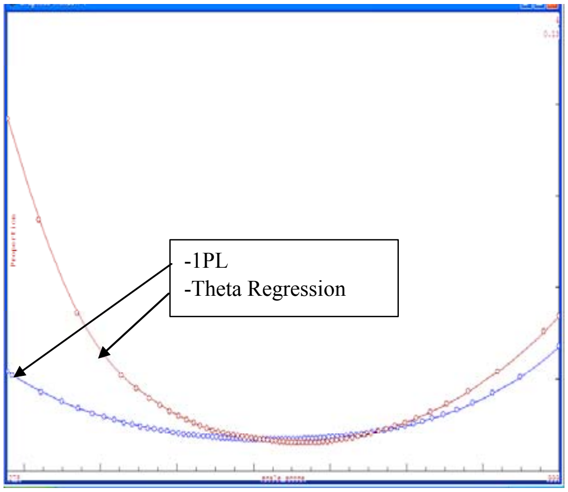
Figure 7. Cumulative Frequency Distributions

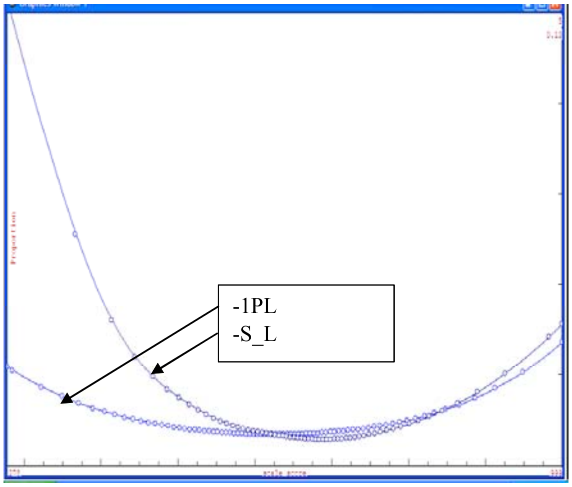


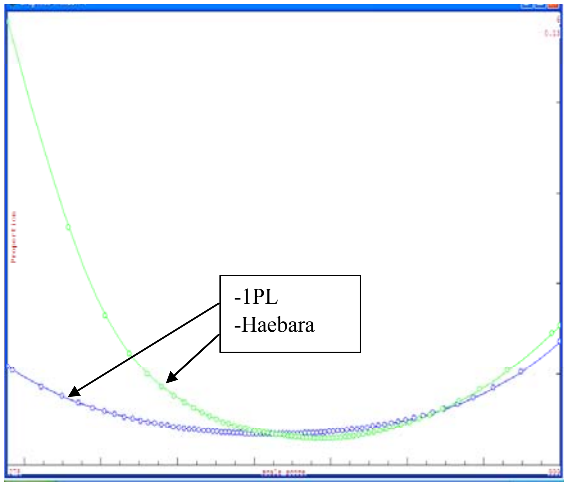
Appendix A. Comparison of Standard Errors











Appendix B. Raw Score Frequency Distribution