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Bus Routing Algorithms: Application to a Rural School District

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Foreword

Foreword

This paper reports the results of an exercise in applied mathematic completed as a class project; but with a unique motive: It addresses a critical issue for rural parents and communities by asking a dangerous, or at least impertinent, question: does off-the-shelf bus-routing take rural place into consideration? What would routing that did take such concerns into consideration look like?

Center faculty understand that such concerns comprise an important motive—some of us argue the principal motive—for studying rural education (including mathematics education). In any case, opening such motive for scholarship in mathematics education is one of the missions of the Center.

The appearance of this paper, particularly in a *mathematics* class, struck members of the Center's management team as strong evidence that students were embracing this motive (i.e., the motive to study rural concerns and issues related to schooling). The present version of the paper is being temporarily posted as Working Paper No. 27. Readers must remember two facts: (1) this is student work; it has limitations associated with its origins as an assignment, and (2) the manuscript is currently undergoing substantial work: It truly is a "work-in-progress."

At the present moment (late August 2005), two derivative manuscripts are being developed. First, an article describing the origins and meaning of this project is now being written for the Fall 2005 issue of the Center's online magazine, the *Rural Mathematics Educator*. That article will link to Working Paper No. 27. Second, a more scholarly revision is being undertaken by members of the study team and their professor, Dr.

Gleason. The aim with the scholarly article will be to describe more fully the connection between this work and the research literature on rural school busing—which was a minor consideration in the case of the class assignment and is only modestly (but quite evidently) considered in the present version. When the journal article manuscript is complete, it will most likely replace this student paper as Working Paper No. 27. In the meantime, because of the intent to produce a manuscript based on this project for scholarly publication, the Center has decided to post the current version in the Working Paper series. Lacking that intent, please note, this paper would have been entirely suitable as one of the Center's Occasional Papers—the occasion in this instance being a mathematics class that elicited work in *mathematics* that takes account of the rural circumstance.

Craig Howley
Athens, OH

Bus Routing Algorithms: Application to a Rural School District

Madison County Schools presently is comprised of six elementary school districts – Brush Creek, Laurel, Hot Springs, Mars Hill, Marshall and Walnut. All students in the system attend Madison Middle School and Madison High School. The high school and middle school are located approximately five miles apart on the Marshall to Walnut Road. Because of unique land characteristics of this county in the Appalachian Mountains, these districts are not very adjustable. The school system now uses double routes on almost all roads as no elementary students are placed on busses with older students. This means almost every road in the county is traveled at least twice – once to pick up the elementary students and again to pick up the high school and middle school students. In talking with parents, administrators and school board members, it was clear that there was not a state law or policy that forbid younger and older children from riding together on the same bus. It was also discovered in our initial survey that there was little opposition to all student-aged children being on the same bus. The reason for this separation seemed to be that the computer system that the state of NC uses to design bus routes includes this in their programming. The people of rural Madison County feel that their children grow up together and saw no reason that the older kids and younger kids could not ride together. All children in a rural community are considered family and it is assumed that older kids will look after younger kids. Parents like the idea of younger kids modeling the behavior of their older siblings.

Inquiring about the NC computer program presently in use to design bus routes resulted in this letter being sent to this project team:

> I'm not a mathematician (sic) and my involvement with routing algorithms is limited to use of the high-level software (Edulog.nt) employed by North Carolina school districts. I'm afraid my help to you will be limited largely to a demonstration of that application, if that interests you. That and a couple of references to other people that might be of help to you.
>
> In the software, routing algorithms can be applied at three levels. First, the software can be requested to select the optimum assignment of stops to runs and their sequence on the runs. Second, you can use the software to group individual runs onto routes that minimize the number of buses required and the amount of unloaded travel time. Third, the software can choose an optimal path to connect the stops on the routes.
>
> The mathematics of the algorithms behind the interface is apparent only through the opportunity for the user to set parameters for the stop-to-run optimization routine. It is in the setting of these parameters that differences in rural and urban networks can be accommodated. To be honest, the setting of those parameters is for most users, if not all, a matter of trial and error. I would be happy to demonstrate the software to you. Edulog also has a training document for run optimization that might be of use to you.

> Regarding referrals, Dr. Dave Hartgen is on the faculty of the Department of Geography and teaches transportation planning courses. I think he's probably as good a resource as any on campus. I've also requested a contact name at Edulog, the company that authors the routing software used by North Carolina school districts.
>
> Regional Transportation Director

It became clear to our Project Team after this letter and a consultation with the Edulog Company in Montana that rural issues were not part of the design of the program. We investigated their parameters and discovered that there was nothing that allowed for rural problems involving time on bus and no reasonable way to turn the bus around at the end of mountain coves. Thus, the need for this study seemed evident.

Additionally, it was decided to continue to operate the after school buses that take students from the high school and middle schools to their local elementary schools. One bus is run to each of the five elementary schools from the middle school and high school to enable rural students to participate in sports or extra-curricular activities such as tutoring, clubs or meetings. No bus is needed to Brush Creek elementary, as it is located behind the middle school. No change in after school buses was needed and the present system is the most efficient as it now operates.

Abstract
This paper proposes a mathematical model that delineates a feasible system of bus transportation for this multiple-school district. The model is composed of six elementary school districts which are part of the overall middle school and high school district. This proposal attempts to show Laurel district busing as a representative sample of what needs to be done in all six elementary districts. Final transportation routes from all six districts to the centrally located county Middle and High Schools are included and explained. Both elementary district and entire county algorithms are included. First, the population distribution was analyzed and bus stops were created for the sample Laurel district such that the average number of children per stop is maximized and each child walks no more than a predetermined distance. Next, the bus stops were grouped geographically into regions of roughly equivalent number of students, and the viable routes for each region were determined. Finally, these routes were sequenced by the implementation of the NCL (No Child Left at Bus Stop) algorithm created for the Madison County Schools. The algorithm has been created so that changes can be made to accommodate adjustments in parent and school board decisions.

Introduction
Each year, thousands of schools nationwide must consider the transportation needs of their students. In many cases, a large number of the students live too far away to be expected to walk to school each day. It is also unreasonable to expect all families to have the resources necessary for the daily transportation of their children. This leads to the

need for school busing. Many districts provide transportation routes that are not optimal; these routes waste time or money for those involved, necessitating an accurate mathematical model to increase efficiency. Such a solution must take into consideration many factors, including economic concerns, time issues, route efficacy, and especially rural concerns unique to Madison County Schools. The rural and mountainous nature of Madison County present some unique concerns that can only be addressed by a group familiar with these issues. Having members of the community on our design team makes this proposal unique and optimal for the realistic nature of the problem. This model attempts to put children first and realizes that rest time, study time and family time are significant contributors to improving learning and test scores. Herein, a realistic model of an effective busing system is presented.

Summary

The main objective of this project is to mathematically represent a feasible system of school bus transportation in a given region, and optimize it with regards to total transit time and net operation costs. Because of unique characteristics of a mountainous rural region and the parent concerns expressed at recent school board meetings, minimizing time on the bus for students will be the primary focus of this project.

Given an arbitrarily distributed population of high school and elementary school students in an area with one or more fixed schools and bus stops, the model can systematically employ algorithms to designate bus stops, divide these bus stops into appropriate regions, and assign bus routes for each region so that all students can be transported to their respective destination within certain time restraints. In the formulation of such algorithms, numerous viable factors are taken into account, ranging widely from gas prices to yearly bus maintenance bills. In addition to the actual formulation of bus routes, a sufficiently accurate and generic model for the net cost is presented as well as relevant investigations into advanced graph theory and adolescent education and psychology. The model is far from perfect and only includes one district as a representative sample of what our company will design when awarded the bus contract for the 2005-06 school year. The unpredictable nature of reality creates an inevitable capacity for error; however, this model is ultimately recommended over any primitive model lacking mathematical consideration, and is in fact efficient and applicable in most real-life rural situations. Past experience in rural bus design has enabled our team of engineers to modify algorithms whenever needed to accommodate issues that only occur in rural environments.

General Model – Definitions and Assumptions

Before discussing the model and its limitations of the method, some definitions and assumptions must be explained. In this county, there is one high school and one middle school. Bus drivers all live on the routes that they drive or are reasonably close to routes. All buses are fueled during the day while they are parked at the high school or the middle school so no time to do this is allocated in the model. Further, all bus drivers who transport kids from their elementary district to the central high school or middle school are employed full time with benefits at one of these two sites. This allows them to be onsite at all times in case of early dismissal due to frequent weather such as flooding and snows. All work as teacher assistants, cafeteria or janitorial staff. Student routes do not change much as new housing is very infrequent in this region. Rural families living nearby or with relatives means that routes are seldom changed within a 5-year cycle of this contract. Numerous roads are dirt or difficult to travel but almost all roads have at least one house with a stop that needs to be reached. Students are very randomly distributed throughout the elementary districts. Educational research supports the idea that minimal amount of mixing of age level students is recommended. However, in the rural area, most younger and older students know each other and their families well if they live in the same school communities. So, for this reason, and because of the excessive mileage involved between houses, students within the same elementary district have been mixed on buses in this model. It is an economic hardship to send separate buses out 26 miles to a single house or group of houses to pick up middle school, high school and elementary school students at the same stop as is presently done.

Route is defined as the path a bus takes from the time it leaves its starting point (bus driver's house) until it reaches its terminal destination. There is a limit to the number of buses that the county has, and their ability to purchase more is limited. N C State Law limits the number of students that are allowed on a bus at any one time. The present situation of providing an after school bus to each elementary district from the high schools and middle schools is maintained in this model. Without this after school situation, students in this rural system with limited means would not have access to sports or extra-curricular activities. NC school law also does not allow students on buses with commercial signs. So, no money for advertising on the buses can be generated. All buses are equipped with cameras and emergency phones although due to mountainous terrain, some areas are inaccessible by phone. Each additional bus needed for this school system is a prohibitive cost. There are restrictions set on amount of time allowed on the bus in a day and the earliest possible pick-up time allowed.

Assumptions about routes:

- all roads used are maintained by the county or state and are passable with no allowances given for construction
- there is no deviations needed due to traffic flow – bus speeds have taken passing situations into account
- students living outside the county will provide there own transportation or will be issued a permit based on space available for the nearest bus stop
- no students from different elementary districts will be placed on routes going to the high school or middle schools as requested by parents
- geographical distances and single middle and high school for all prohibit much variation in school schedules among schools

Graph Theory Concepts

The following mathematical concepts were studied to enable the design of this project: *Discrete Mathematics Concepts* : Graph theory including, critical paths, Euler Circuits/ Paths, Hamiltonian Circuits/ Paths,

Related Mathematics Concepts:
Geometrical concepts including vertices, edges, Sequences, Series.
Problems in the above mentioned application domains are often closely related to (albeit not restricted to the field of) classic problems examined in graph theory:

- (all pairs) shortest path
- critical path
- minimum spanning tree
- travelling salesman
- Chinese postman
- Hamiltonian cycle
- Euler cycle

List of Parent Concerns
Parental concerns are well documented in the small reseach literature on rural school busing: Test Scores, Homework Time, Lack of Sleep and tiredness of children, Family Time (highly important for rural families), Extracurricular Activity Time, Tutoring Time and Chore Time (significant in rural families).

All of these issues seemed to be connected to lack of time in some way. In our evaluation of the current school bus system, we questioned several parents about their concerns about their children on the bus. The overwhelming response was time spent on the school bus in the mornings and evenings. Some of the students ride the bus as much as two hours each way. Parents are concerned that their children aren't getting the rest they need. Some rural children get up at 5:00 AM to be able to catch their bus at 6:00. Then, they ride for two hours and by the time they get to school, it is too late for breakfast. In the afternoons, they get home at 5:00pm, and have chores to do, then eat dinner, and have very little time for homework, because they must go to bed in time to get up at 5:00. Families who lived the farthest from school were obviously the most concerned, and also angry. In many locales, these children are the poorest, and the least likely to have a ride (either because they had no car, or parents needed the car to work.) This longest time on the bus was one more strike against the poorer children in the county. It was decided that if the time on the buss issue could be resolved, other concerns of parents could be eliminated.

Model Constraints
Design of the bus routes took several factors into account that put constraints on the model. These included: Money, Number of Buses allocated by the state of NC and that could be purchased with local funding, Bus Speed – capability and NC Law, Terrain of the area and Rural issues affecting routing, Number of Roads that buses could safely and legally travel, turn around space for buses at end of routes,
Geographical Distribution of Children, Number and Geographical Location of Schools – fixed School Schedule.

Elementary Model Example – Laurel District

The Laurel Elementary district was chosen as a sample elementary district since it is very similar to the others. A design for any of the elementary districts would follow this same model. There are a few more coves with one way paths in and little turn around space for a bus than the other districts. Laurel is probably the remotest of the districts. So, tackling the district of greatest difficulty to prove the soundness of our design seemed fitting.

Mathematical explanation and validity of model chosen

What we found in trying to find the most efficient school bus routes for Madison County was that the algorithms that we learned in Graph Theory do not fit the situation. Any road that school-age children live on that can be driven by a school bus must be traveled. However, many of these roads end in dead ends, which means that there are not very many circuits. The solution that we came up with is this:

1. First we rated the roads by grade and condition of the road. We ranked them as 1, 2 or 3. Roads that were ranked 1 would have an average bus speed of 45 miles per hour. Roads ranked 2, would have an average bus speed of 35 miles per hour; so we multiplied the distance of that section of road by 1.25. Roads ranked 3, would have an average bus speed of 30 miles per hour; so we multiplied the distance by 1.5. We called 1, 1.25 and 1.5 the Condition Factor.

2. We counted the number of bus stops per section of road and the number of children. Children who live within one quarter of a mile of each other get on at the same stop. However, a lot of the homes are fairly isolated with no other houses within one-fourth of a mile. We calculated an average of 1 minute per stop. So the time for each section of road is 60(Actual Distance × Condition Factor)/45 + Number of Stops. The number of stops is only added when the bus is actually picking up children, normally the first time the bus traverses a section of road.

3. Park the school busses as close to the first stop as possible. In this community there are churches down most roads. Most churches will allow a school bus to park at night and the driver's car during the day.

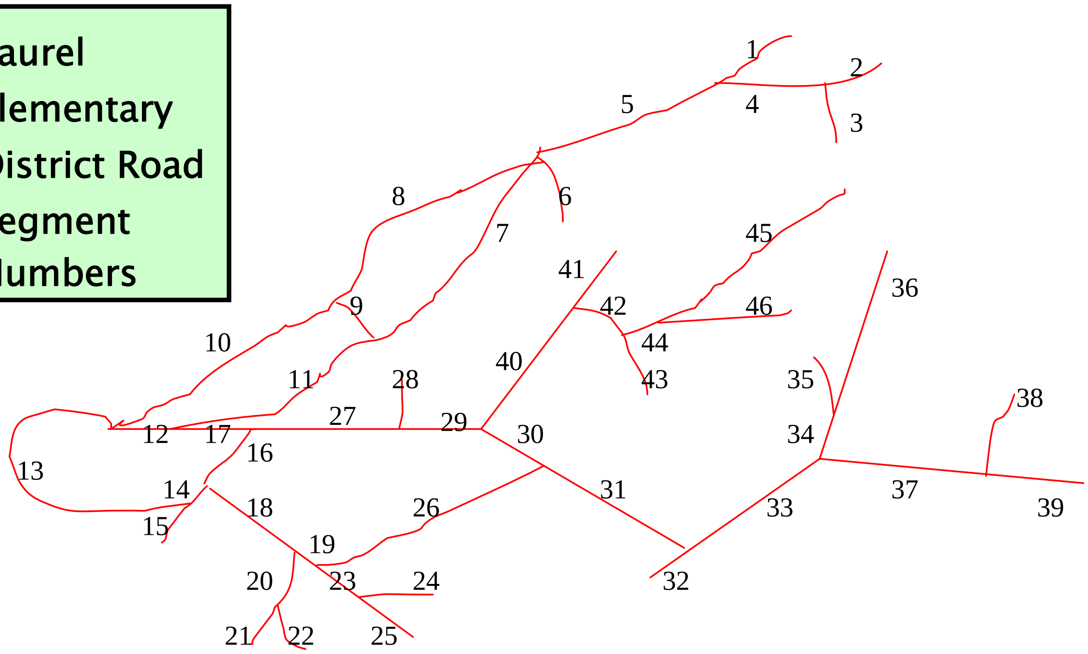
4. In determining a route so that no child rides the bus more than 60 minutes before arriving at Laurel Elementary School, we first establish the shortest time back to the school from the first stop. This can be done using Djikstra's algorithm.

5. Each intersection of roads is a node. At each node calculate the shortest route back to the school. Add the time already on the road to the shortest time back to the school, if this time is less than 60 minutes; find a road out of the node that has children who have not already been picked up. Pick up all the children on this road until you get to the next node.

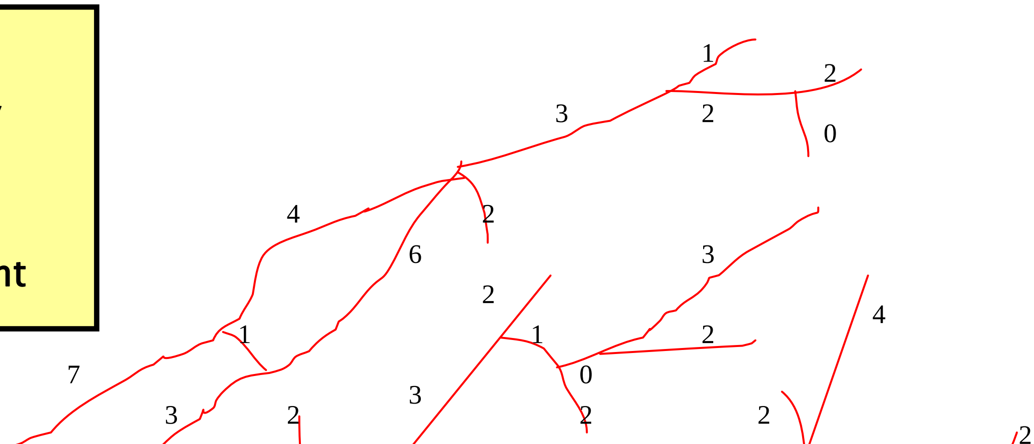
6. When you get to a node and the time already on the road plus the time back to the school is 60 minutes or more, take the shortest route back to the school.

It is somewhat arbitrary how you select the starting points for the buses, and there was a little bit of trial and error getting all the routes to approximately 60 minutes. But if you look at the map (see figure) there are branches fanning out from the school. We first tried putting one bus on each branch, but the routes were longer than 60 minutes; so some branches needed more than one bus, with the second bus starting at the point where the first bus had reached its limit. Using this method, we were able to get all 225 children picked up and taken to school in less than 60 minutes with 7 buses.

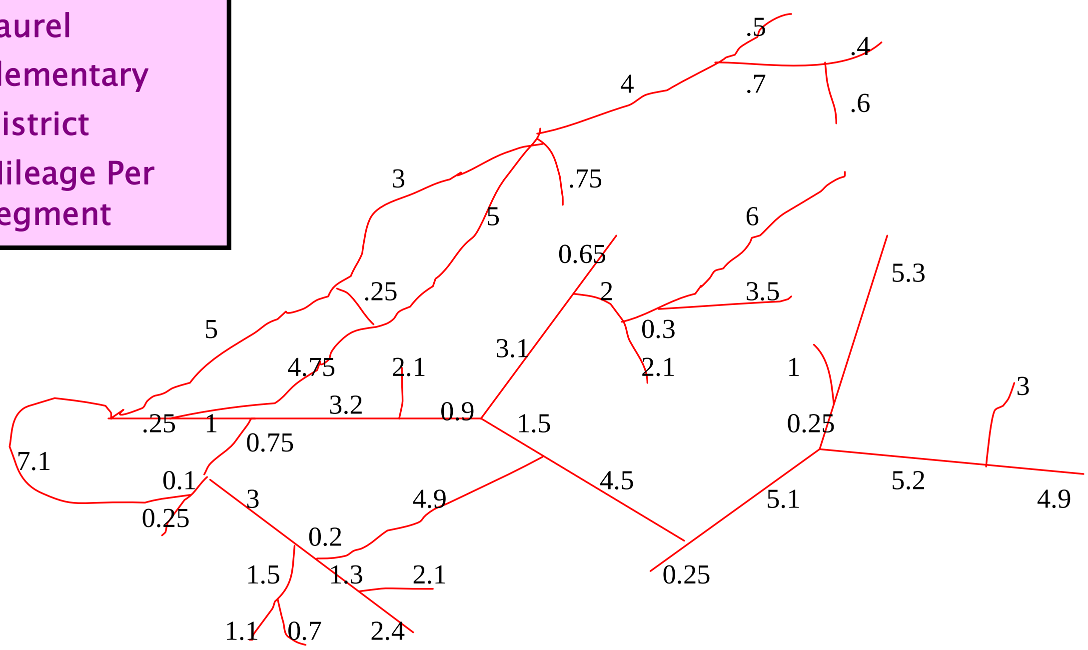
Laurel Elementary District Road Segment Numbers



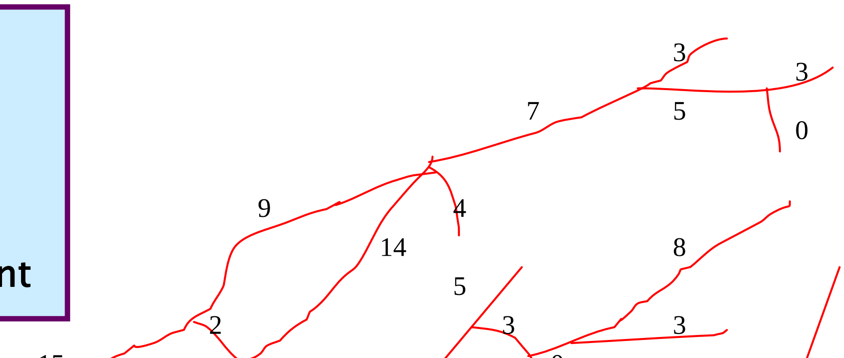
Laurel Elementary District Bus Stops Per Segment



Laurel Elementary District Mileage Per Segment



Laurel Elementary District Children Per Road Segment



Elementary District Generalization – Laurel School Sample

Factors used Laurel District Bus Routes

Route Number	Factor
1	1
2	1.25
3	1
4	1.25
5	1.25
6	1.25
7	1
8	1
9	1.25
10	1.5
11	1.25
12	1.5
13	1.5
14	1.25
15	1.5
16	1.25
17	1.25
18	1.5
19	1.25
20	1.25
21	1.5
22	1.5
23	1.25
24	1.25

25	1.25
26	1.25
27	1.25
28	1
29	1.5
30	1
31	1
32	1.25
33	1.5
34	1.5
35	1.25
36	1.5
37	1.5
38	1.25
39	1.5
40	1
41	1.25
42	1.25
43	1.5
44	1
45	1.25
46	1

Proposed Laurel School Bus Routes

Bus Route 1

Bus Route 1

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
5	3	7	4	1.25	5	
1	1	3	0.5	1	0.5	
1	0	0	0.5	1	0.5	
4	2	5	0.7	1.25	0.875	
2	2	3	0.4	1.25	0.5	
2	0	0	0.4	1.25	0.5	
4	0	0	0.7	1.25	0.875	
5	0	0	4	1.25	5	
7	6	14	5	1	5	
11	0	0	4.75	1.25	5.9375	
17	0	0	1	1.25	1.25	
16	2	3	0.75	1.25	0.9375	
16	0	0	0.75	1.25	0.9375	
27	0	0	3.2	1.25	4	
29	0	0	0.9	1.5	1.35	
30	0	0	1.5	1	1.5	
31	0	0	4.5	1	4.5	

	16	35			39.1625	68.21667

Bus Route 2

Bus 2

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
11	3	7	4.75	1.25	5.9375	
9	1	2	0.25	1	0.25	
8	4	9	3	1	3	
6	2	4	0.75	1.25	0.9375	
6	0	0	0.75	1.25	0.9375	
8	0	0	3	1	3	
10	7	15	5	1.5	7.5	
12	0	0	0.25	1.5	0.375	
17	0	0	1	1.25	1.25	
27	3	6	3.2	1.25	4	
29	0	0	0.9	1.5	1.35	
30	0	0	1.5	1	1.5	
31	0	0	4.5	1	4.5	
	20	43			34.5375	66.05

Bus Route 3

Bus 3

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
13	6	13	7.1	1.5	10.65	
15	1	3	0.25	1.5	0.375	
15	0	0	0.25	1.5	0.375	
14	0	0	0.1	1.25	0.125	
18	4	7	3	1.5	4.5	
20	2	5	1.5	1.25	1.875	
20	0	0	1.5	1.25	1.875	
19	0	0	0.2	1.25	0.25	
23	0	0	1.3	1.25	1.625	
24	2	6	2.1	1.25	2.625	
24	0	0	2.1	1.25	2.625	
23	0	0	1.3	1.25	1.625	
26	0	0	4.9	1.25	6.125	
31	0	0	4.5	1	4.5	
	15	34			39.15	67.2

Bus Route 4

Bus 4

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
42	1	3	2	1.25	2.5	
43	2	6	2.1	1.5	3.15	
43	0	0	2.1	1.5	3.15	
44	0	0	0.3	1	0.3	
45	3	8	6	1.25	7.5	
45	0	0	6	1.25	7.5	
46	2	3	3.5	1	3.5	
46	0	0	3.5	1	3.5	
44	0	0	0.3	1	0.3	
42	0	0	2	1.25	2.5	
40	0	0	3.1	1.25	3.875	
30	0	0	1.5	1	1.5	
31	0	0	4.5	1	4.5	
Total	8	20			43.775	66.36667

Bus Route 5

Bus 5

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
33	4	7	5.1	1.5	7.65	
34	0	0	0.25	1.5	0.375	
35	2	5	1	1.25	1.25	
35	0	0	1	1.25	1.25	
36	4	8	5.2	1.5	7.8	
36	0	0	5.2	1.5	7.8	
34	0	0	0.25	1.5	0.375	
33	0	0	5.1	1.5	7.65	
	10	20			34.15	55.53333

Bus Route 6

Bus 6

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
37	5	9	5.2	1.5	7.8	
38	2	6	3	1.25	3.75	
38	0	0	3	1.25	3.75	
39	3	6	4.9	1.5	7.35	
39	0	0	4.9	1.5	7.35	
37	0	0	5.2	1.5	7.8	
33	0	0	5.1	1.5	7.65	
	10	21			45.45	70.6

Bus Route 7

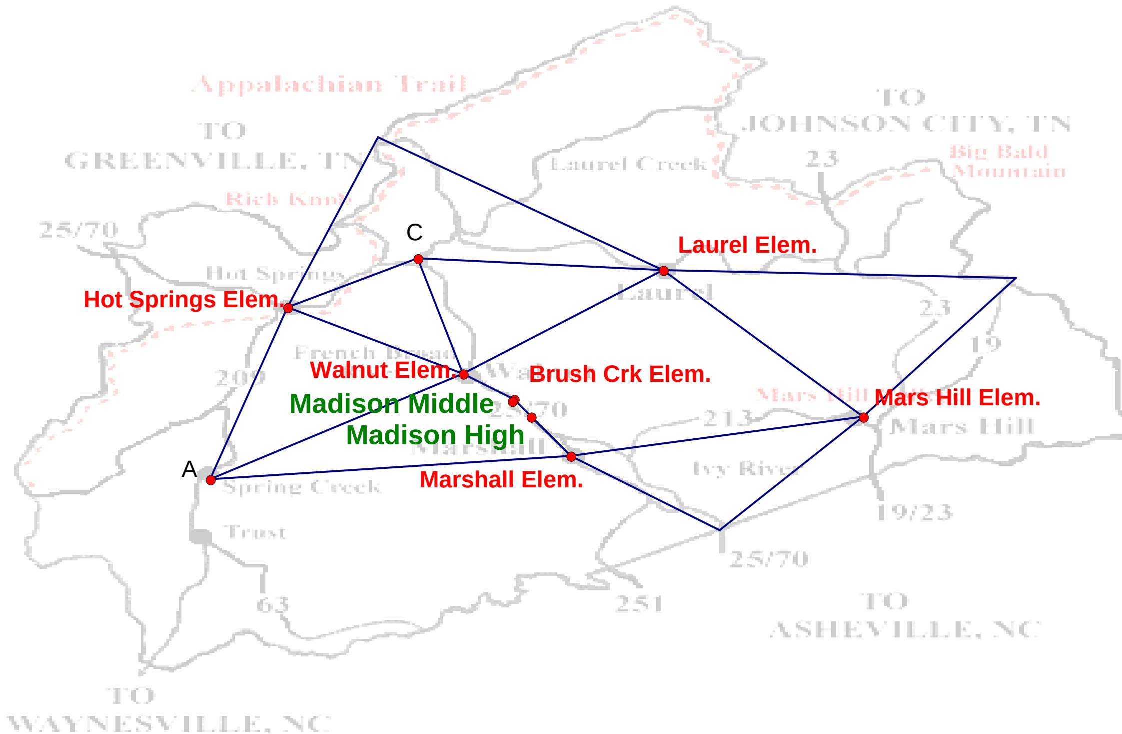
Bus 7

Segment	Stops	Children	Distance	Factor	Total Distance	Total Time
26	7	13	4.9	1.25	6.125	
30	2	6	1.5	1	1.5	
29	1	3	0.9	1.5	1.35	
28	2	5	2.1	1	2.1	
28	0	0	2.1	1	2.1	
29	0	0	0.9	1.5	1.35	
30	0	0	1.5	1	1.5	
40	3	8	0.65	1	0.65	
41	2	5	2	1.25	2.5	
41	0	0	2	1.25	2.5	
40	0	0	0.65	1	0.65	
30	0	0	1.5	1	1.5	
31	4	10	4.5	1	4.5	
	21	50			28.325	58.76667

Consolidated Bus Plan
Madison High School and Madison Middle School

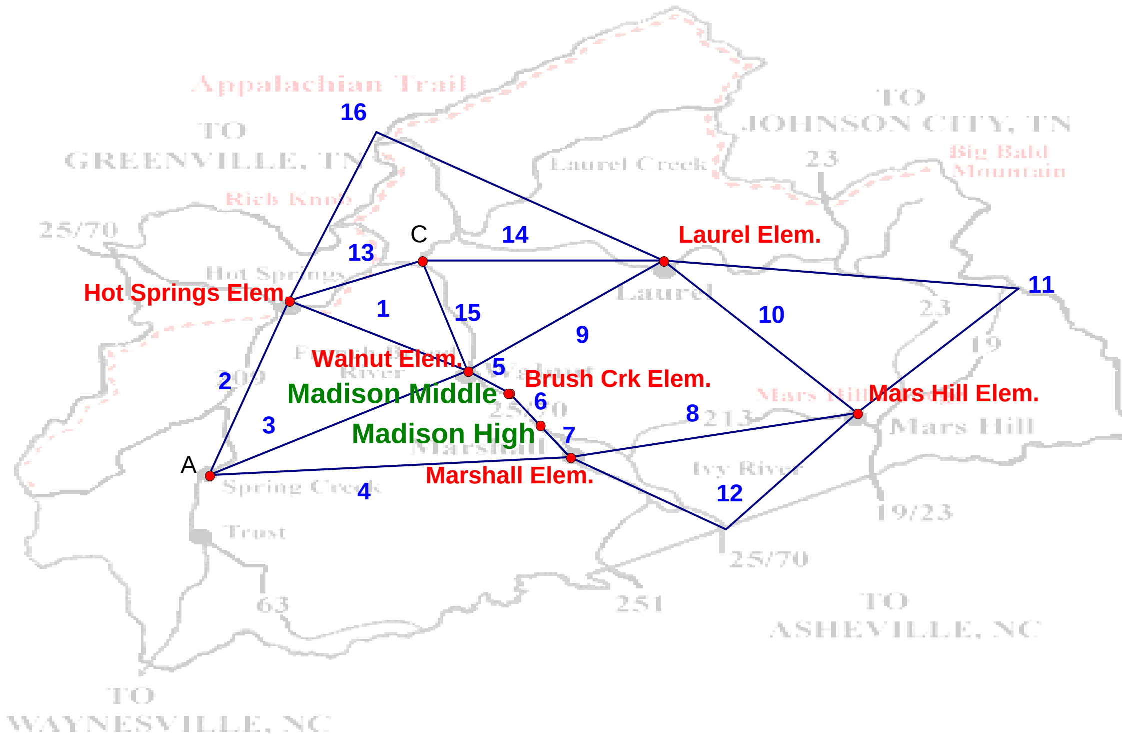
Mathematical Explanation and validity of model

There are six elementary schools that feed into Madison Middle and High Schools. Each of the six elementary schools defines a node of their respective communities. Children are bused from home to the elementary school first and then directly to Madison Middle and High School. The spanning tree that outlines the bus routes for each community joins the graph below at each corresponding community elementary school.



Once each community's routes were configured, the main routes from each elementary school to Madison Middle and High School were to be optimized. There are multiple routes that could be traveled from each elementary school. Our goal is to find the routes that are most time efficient, since students' time on the bus should be a minimum.

We started by numbering sections of road that are similar in characteristics. The defining characteristics were average bus speed attainable, average road grade, and road condition. Average road grade was further defined as a weighting factor of 1 if the average grade was 0-2%, 1.25 if the average grade was 3-6%, and 1.5 if the average grade was greater than 7%. Road condition was weighted according to a weighting factor of 1 if a U.S. highway, 1.25 if a county road, and 1.5 if an unpaved county road. The sections of road are mapped below (numbered 1-16).



Once road sections were identified two additional weighting factors, (distance and number of stops), were included. Road 5,6 and 7 were the only roads with stops as all buses are going from elementary pick-up to high school and middle school. These roads either join, (road 6), or connect to the Middle and High School, (roads 5 and 7). The stops were estimated as 10 min/stop. A formula was developed to aid in weighting the roads by time factors.

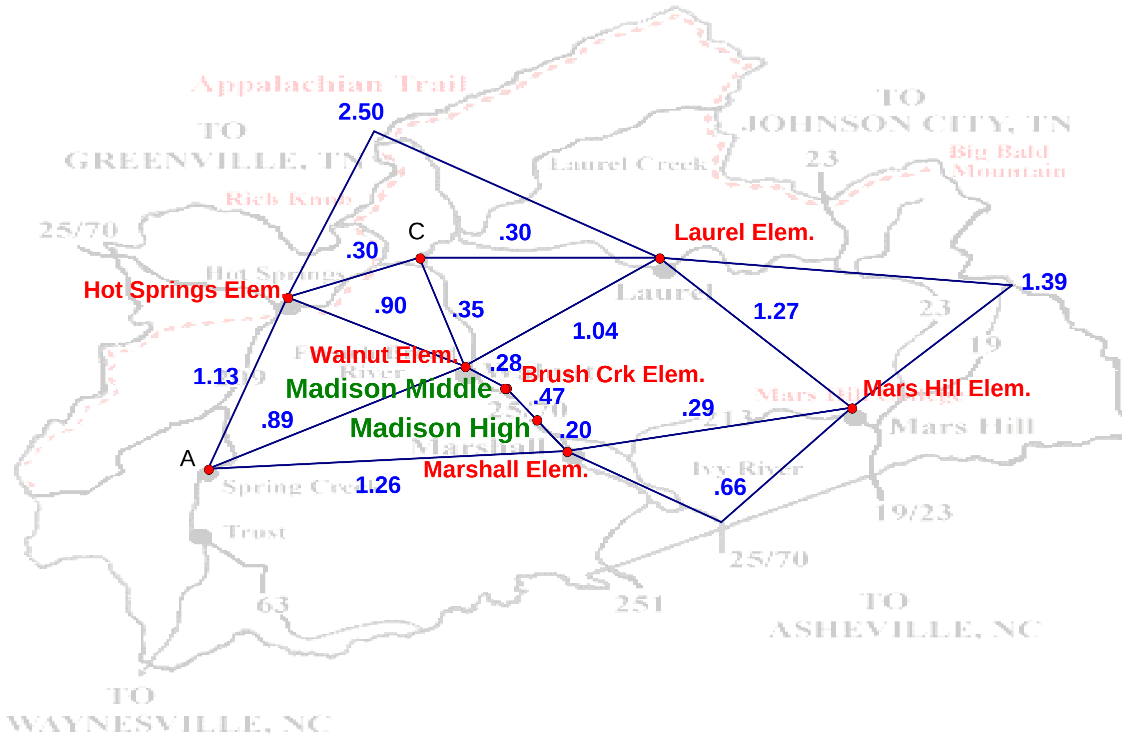
$$\text{TTW} = \frac{DGC}{R} + \frac{S}{6}$$

TTW = total time weight
D = distance in miles
G = average road grade
C = road condition
R = average bus speed in mph
S = number of middle/high school transfer stops, where each stop is estimated as 10 minutes.

Dimensional analysis of $\frac{DGC}{R}$ reveals our intention of emphasis on time. Distance divided by rate equals time in hours where G and C are unit-less weighting factors. Also, $\frac{S}{6}$ can be rewritten as $\frac{10S}{60}$ where $\frac{10}{60}$ changes 10 minutes to hours by dividing by 60 minutes. Now the number of stops multiplied by hrs/stop will leave units of time in hours. The total time weight, (TTW), equation is not being used as an accurate predictor of time on the bus but simply as way to weight each road for comparison. Once an optimal path is found, time trials will be done to reveal an accurate travel time from each elementary to the middle and high school. See the spreadsheet and graph that follows for factors and results for total time weight calculations.

Weighted Map & Table

Time Weight by Road Number						
Roads	Distance (miles)	Avg speed limit (mph)	Avg.Road Grade (1-3)	#Stops	Road Quality (1-3)	Time Weight
1	12	25	1.5	0	1.25	0.90
2	15	25	1.5	0	1.25	1.13
3	20	35	1.25	0	1.25	0.89
4	23.5	35	1.25	0	1.5	1.26
5	4.2	45	1.25	1	1	0.28
6	4.8	45	1.25	2	1	0.47
7	1.6	45	1	1	1	0.20
8	15.8	55	1	0	1	0.29
9	19.4	35	1.5	0	1.25	1.04
10	27	40	1.25	0	1.5	1.27
11	31.2	35	1.25	0	1.25	1.39
12	29	55	1.25	0	1	0.66
13	9.7	50	1.25	0	1.25	0.30
14	8.6	45	1.25	0	1.25	0.30
15	11.1	50	1.25	0	1.25	0.35
16	38.9	35	1.5	0	1.5	2.50



Dijkstra's Algorithm

Once the roads were weighted, we began Dijkstra's shortest path algorithm from each elementary school to the middle and high school. It did not matter if we reached the middle or high school first just that we stopped at both ending our route at the last stop. The non-geometric and geometric methods are commonly used to carry out Dijkstra's shortest path algorithm. What follows is an example of both.

The non-geometric method begins by choosing a starting vertex, (Hot Springs Elementary in our first case), and placing a 0 in the L(U) column that corresponds to that vertex. The distance in weight from each adjacent vertices to the starting vertex is also entered in column L(U) along with the vertex that leads to the adjacent vertex placed in the P(U) column. In our initial step the preceding vertex will of course be our starting vertex HS.

The first iteration of our algorithm will begin with choosing the path with the least weight from the starting vertex to an adjacent vertex, (C in our case). The vertex selected was indicated by bold type as we progressed. The preceding vertex in the shortest path that leads to all adjacent vertices is placed in the P(U) column.

The second iteration begins by choosing a vertex adjacent to a bolded vertex with the least combined weight from the starting vertex. Once the vertex is selected, all vertices adjacent to bolded vertices are labeled with combined weight of the shortest path from the starting vertex. In addition, the preceding vertex leading to the adjacent vertex is placed in column P(U). The rest of the algorithm continues in this way until both the middle and high schools are chosen.

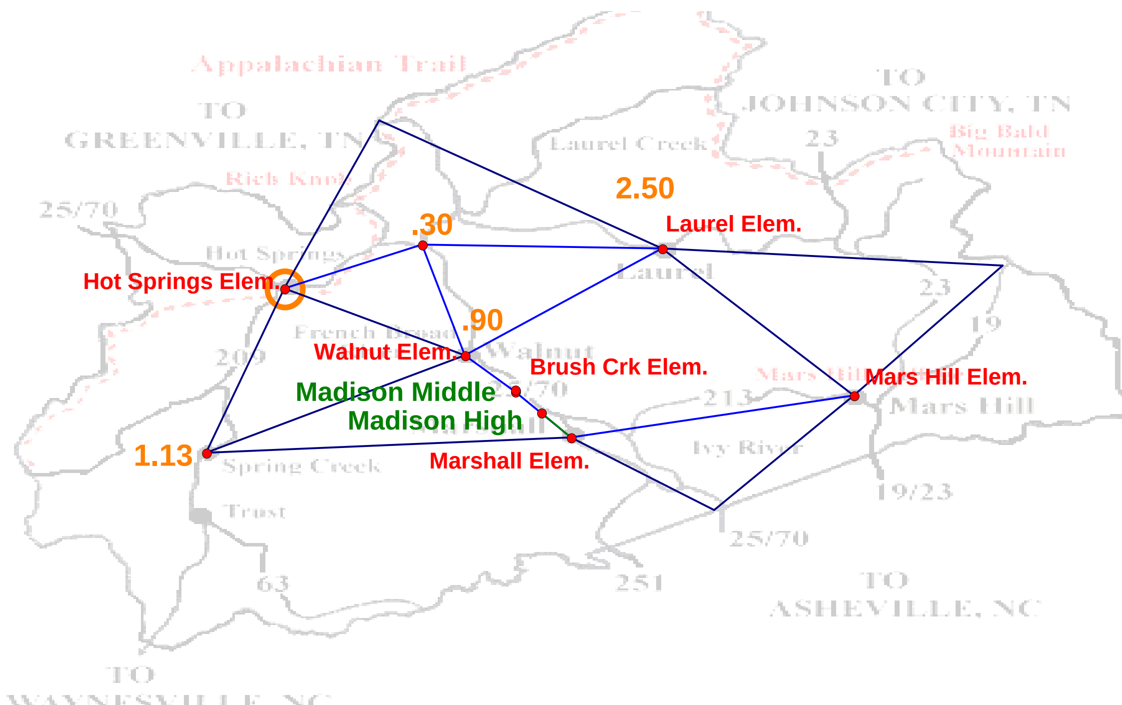
Dijkstra's Non-Geometric Algorithm (Hot Springs)							
Vertex	L(U)(Init.)	L(U)(1st)	L(U)(2nd)	L(U)(3rd)	L(U)(4th)	L(U)(5th)	L(U)(6th)
HS	0	0	0	0	0	0	0
A	1.13	1.13	1.13	1.13	1.13	1.13	1.13
C	0.3	0.3	0.3	0.3	0.3	0.3	0.3
LE	2.5	0.6	0.6	0.6	0.6	0.6	0.6
WE	0.9	0.65	0.65	0.65	0.65	0.65	0.65
MMBE				0.93	0.93	0.93	0.93
MH					1.4	1.4	1.4
ME						2.39	1.6
MHE			1.87	1.87	1.87	1.87	1.87
	P(U)(Init.)	P(U)(1st)	P(U)(2nd)	P(U)(3rd)	P(U)4th	P(U)5th	P(U)6th
HS							
A	HS	HS	HS	HS	HS	HS	HS
C	HS	HS	HS	HS	HS	HS	HS
LE		C	C	C	C	C	C
WE	HS	C	C	C	C	C	C
MMBE				WE	WE	WE	WE
MH					MMBE	MMBE	MMBE
ME						A	MH
MHE			LE	LE	LE	LE	LE
Path Backwards		MH	MMBE	WE	C	HS	

The shortest path can be identified by tracing the path back using the P(U) column in the last iteration. For example the High School (MH) was reached by traveling first to the Middle School (MMBE) and the Middle School was reached by traveling first to Walnut Elementary (WE) and Walnut Elementary was reached by traveling first to road intersection C and C was reached by starting at Hot Springs Elementary (HS).

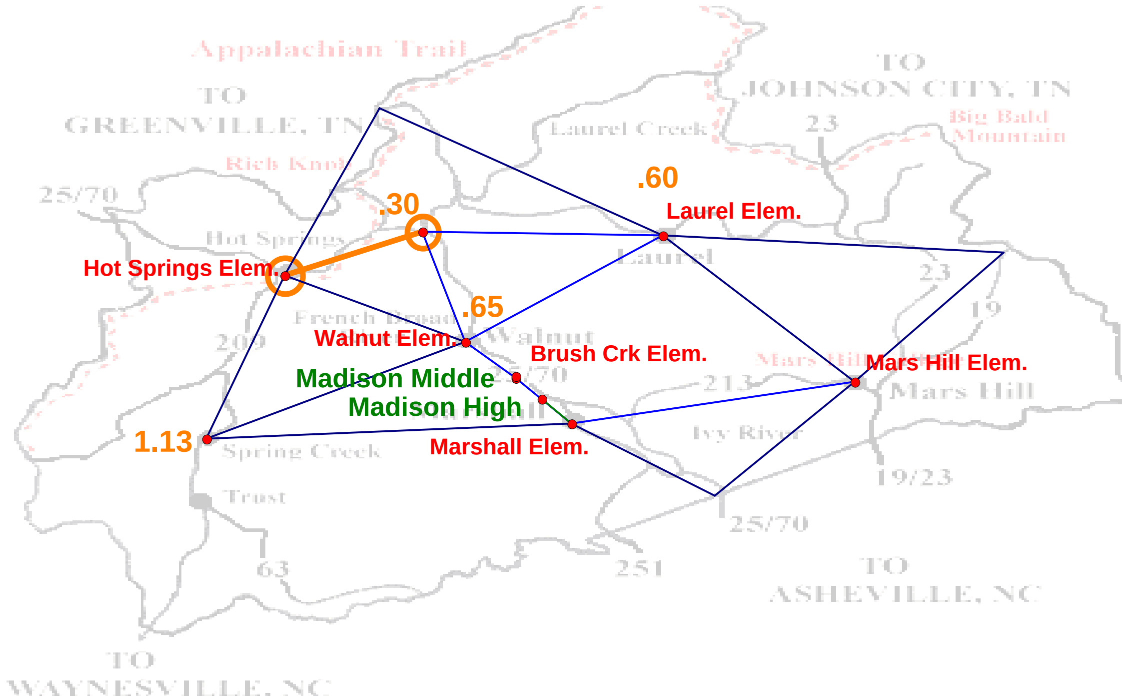
The geometric algorithm is carried out in almost the same way. Selected vertices are circled instead of bolded and the weights of the combined path from the starting vertex are labeled beside each vertex adjacent to circled vertices. The shortest path is traced as we progress. All iterations are shown for the Hot Springs geometric algorithm for illustration with the final iteration shown for the remaining elementary schools.

Dijkstra's Iterations

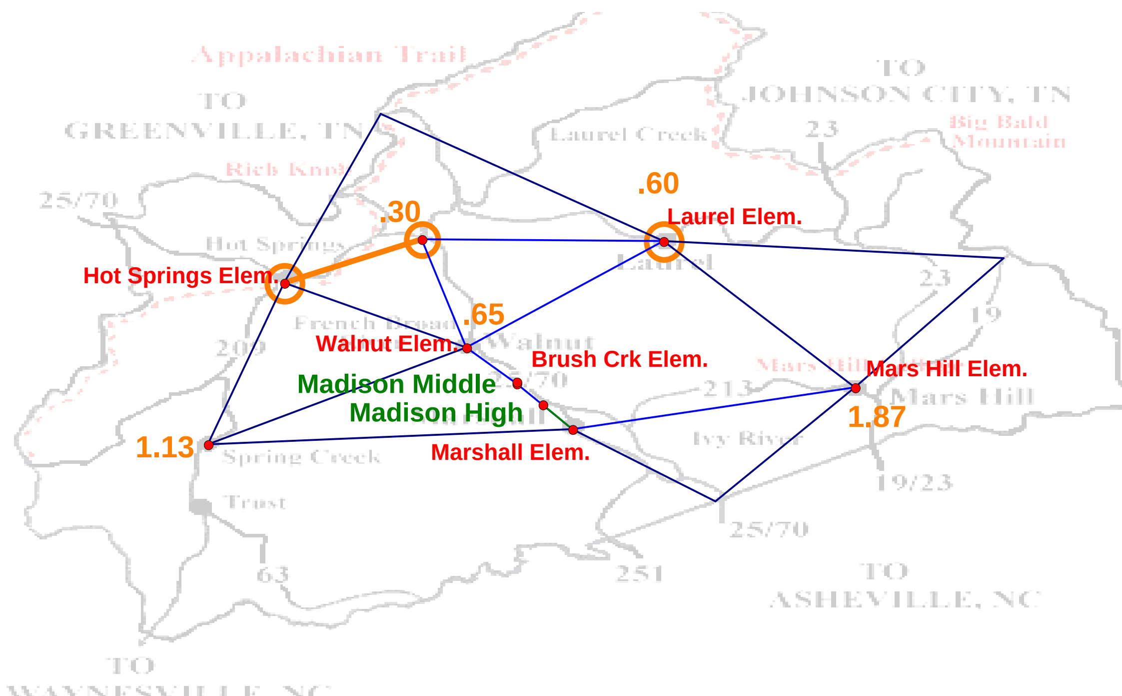
Hot Springs to Madison Middle or High (Initial)



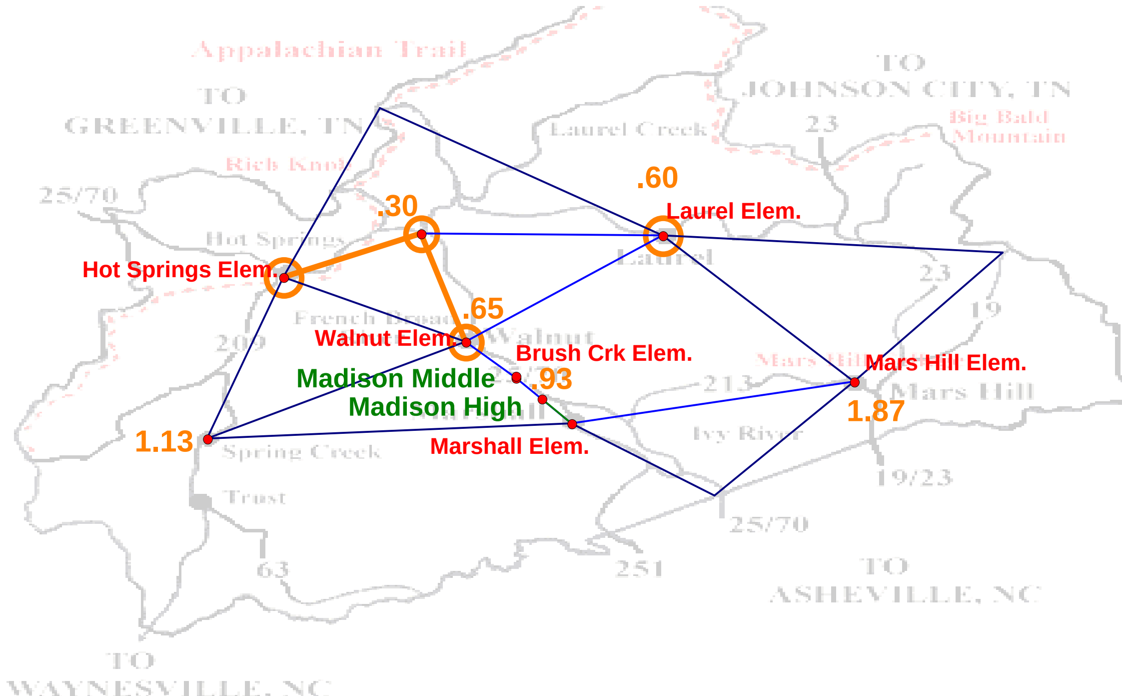
Hot Springs to Madison Middle or High (1st Iteration)



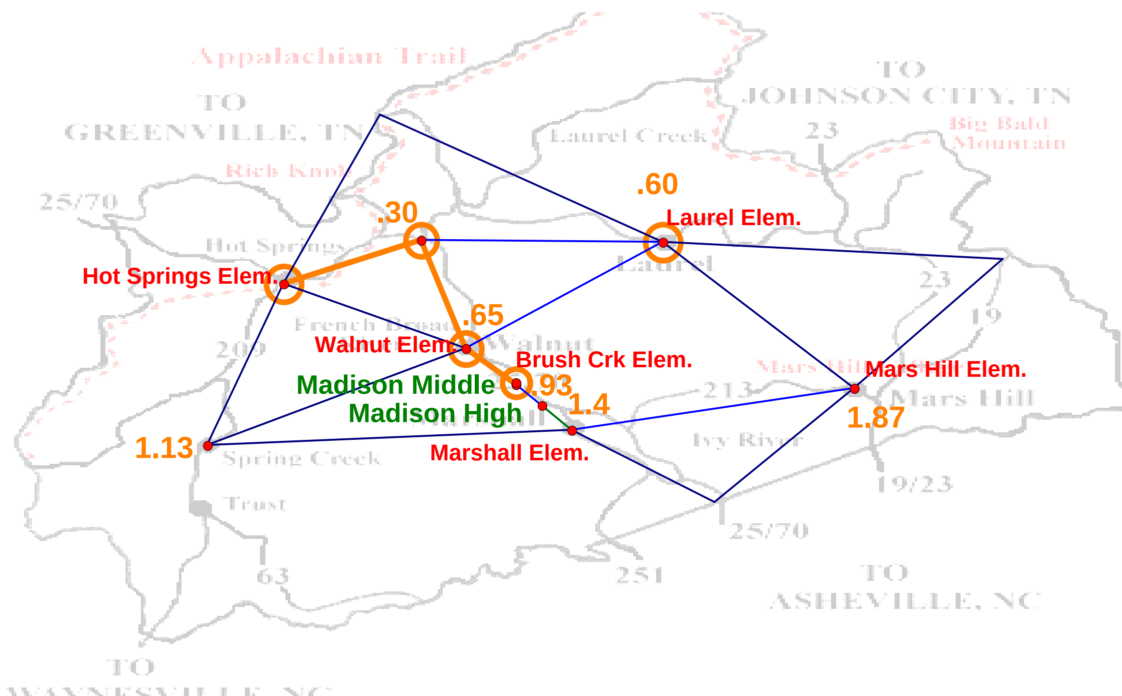
Hot Springs to Madison Middle or High (2nd Iteration)



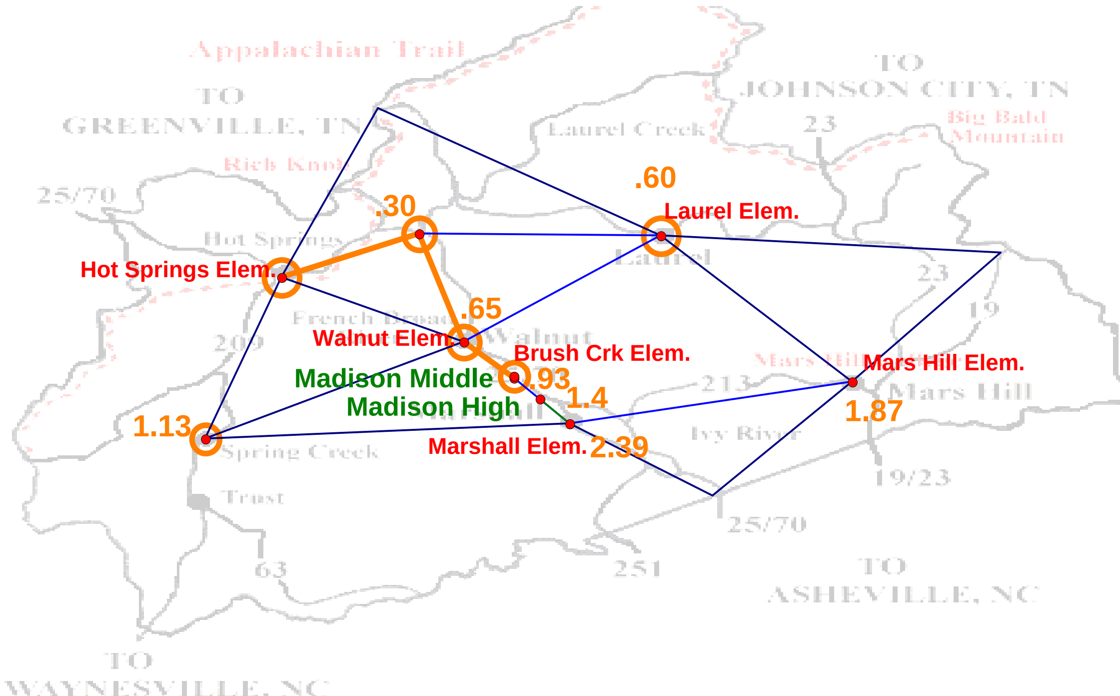
Hot Springs to Madison Middle or High (3rd Iteration)



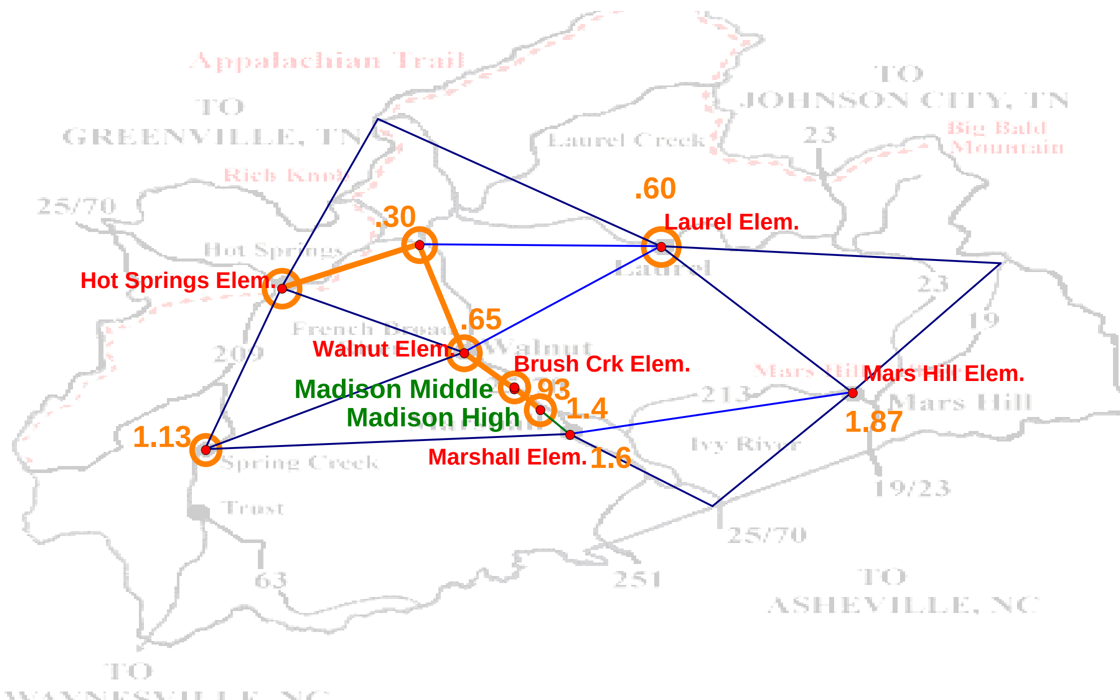
Hot Springs to Madison Middle or High (4th Iteration)



Hot Springs to Madison Middle or High (5th Iteration)

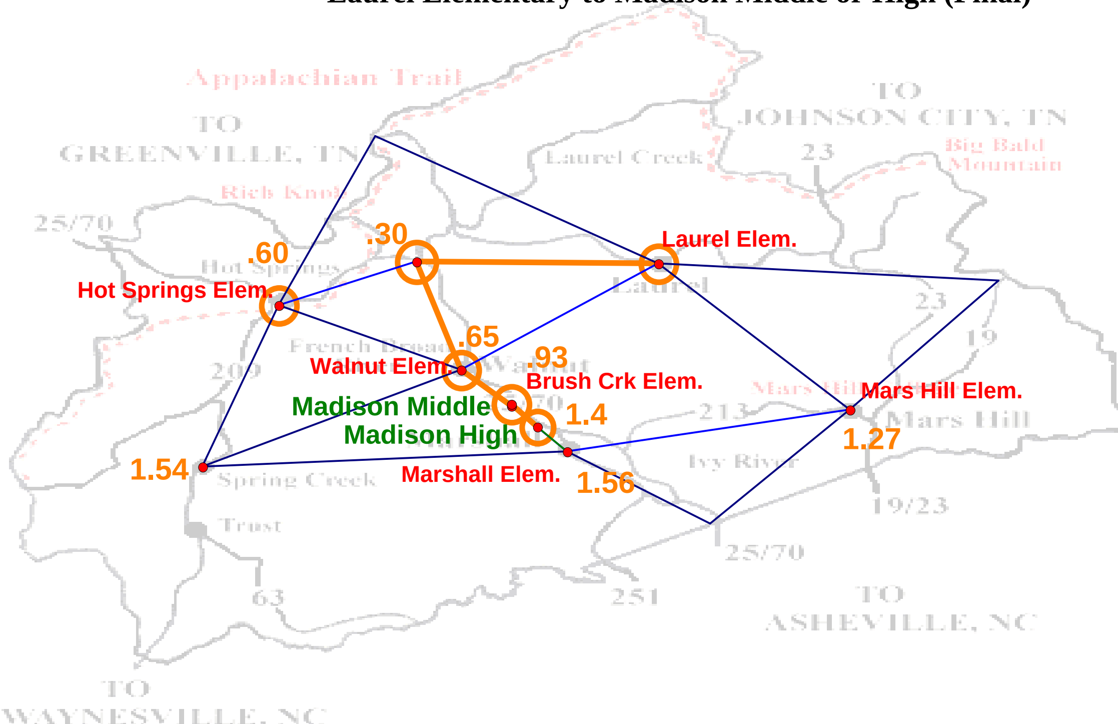


Hot Springs to Madison Middle or High (6th Iteration)



Dijkstra's Non-Geometric Algorithm (Laurel Elementary)						
Vertex	L(U)(Init.)	L(U)(1st)	L(U)(2nd)	L(U)(3rd)	L(U)(4th)	L(U)(5th)
HS		0.6	0.6	0.6	0.6	0.6
A			1.73	1.54	1.54	1.54
C	0.3	0.3	0.3	0.3	0.3	0.3
LE	0	0	0	0	0	0
WE	1.04	0.65	0.65	0.65	0.65	0.65
MMBE				0.93	0.93	0.93
MH					1.4	1.4
ME						1.56
MHE	1.27	1.27	1.27	1.27	1.27	
	P(U)(Init.)	P(U)(1st)	P(U)(2nd)	P(U)(3rd)	P(U)4th	P(U)5th
HS		C	C	C	C	C
A			HS	WE	WE	WE
C	LE	LE	LE	LE	LE	LE
LE						
WE	LE	C	C	C	C	C
MMBE				WE	WE	WE
MH					MMBE	MMBE
ME						MH
MHE	LE	LE	LE	LE	LE	LE
Path Backwards		MH	MMBE	WE	C	LE

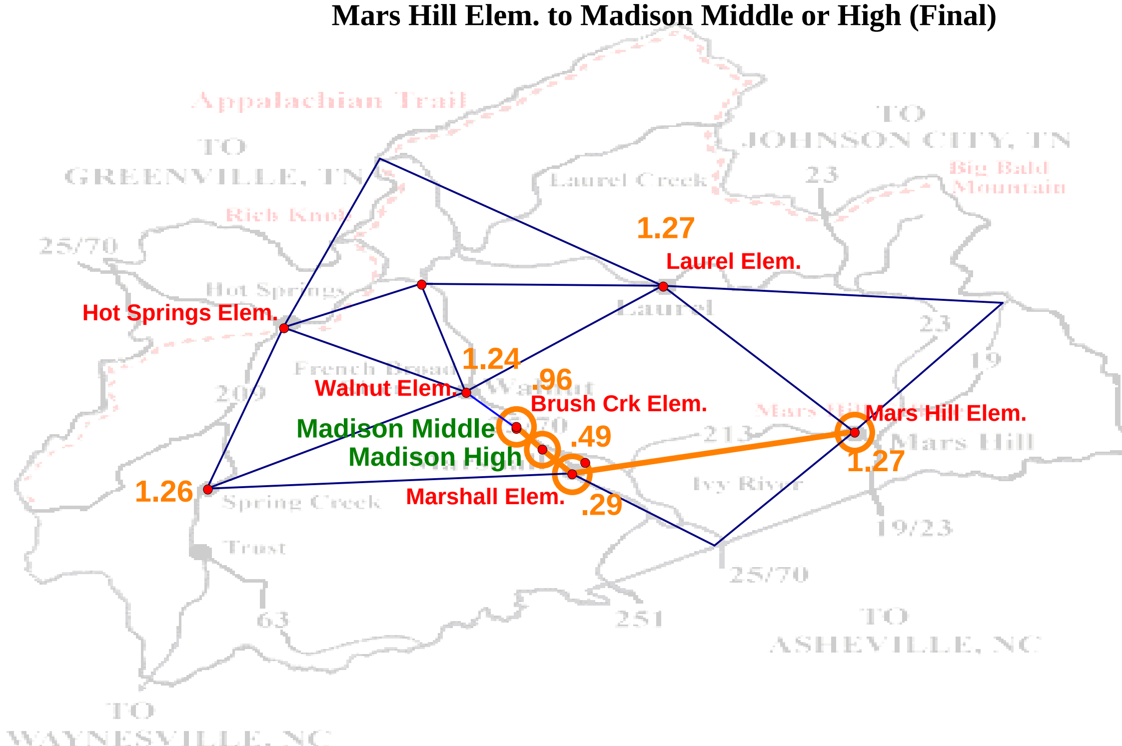
Laurel Elementary to Madison Middle or High (Final)



Dikjstra's Non-Geometric Algorithm (Mars Hill)				
Vertex	L(U)(Init.)	L(U)(1st)	L(U)(2nd)	L(U)(3rd)
HS				
A		1.26	1.26	1.26
C				
LE	1.27	1.27	1.27	1.27
WE				1.24
MMBE			0.96	0.96
MH		0.49	0.49	0.49
ME	0.29	0.29	0.29	0.29
MHE	0	0	0	0
	P(U)(Init.)	P(U)(1st)	P(U)(2nd)	P(U)(3rd)
HS				
A		ME	ME	ME
C				
LE	MHE	MHE	MHE	MHE
WE				MMBE
MMBE			MH	MH
MH		ME	ME	ME
ME	MHE	MHE	MHE	MHE
MHE				

Path Backwards	MMBE	MH	ME	MHE

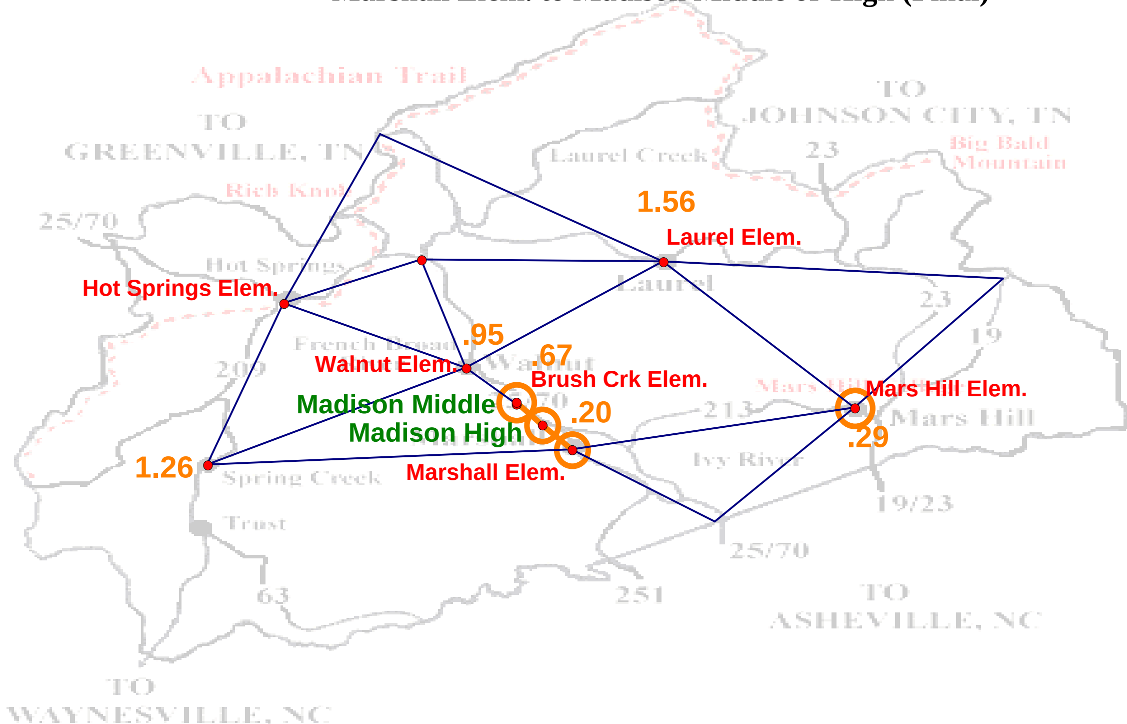
Mars Hill Elem. to Madison Middle or High (Final)



Dijkstra's Non-Geometric Algorithm (Marshall Elementary)				
Vertex	L(U)(Init.)	L(U)(1st)	L(U)(2nd)	L(U)(3rd)
HS				
A	1.26	1.26	1.26	1.26
C				
LE			1.56	1.56
WE				0.95
MMBE		0.67	0.67	0.67
MH	0.2	0.2	0.2	0.2
ME	0	0	0	0
MHE	0.29	0.29	0.29	0.29
	P(U)(Init.)	P(U)(1st)	P(U)(2nd)	P(U)(3rd)
HS				
A	ME	ME	ME	ME

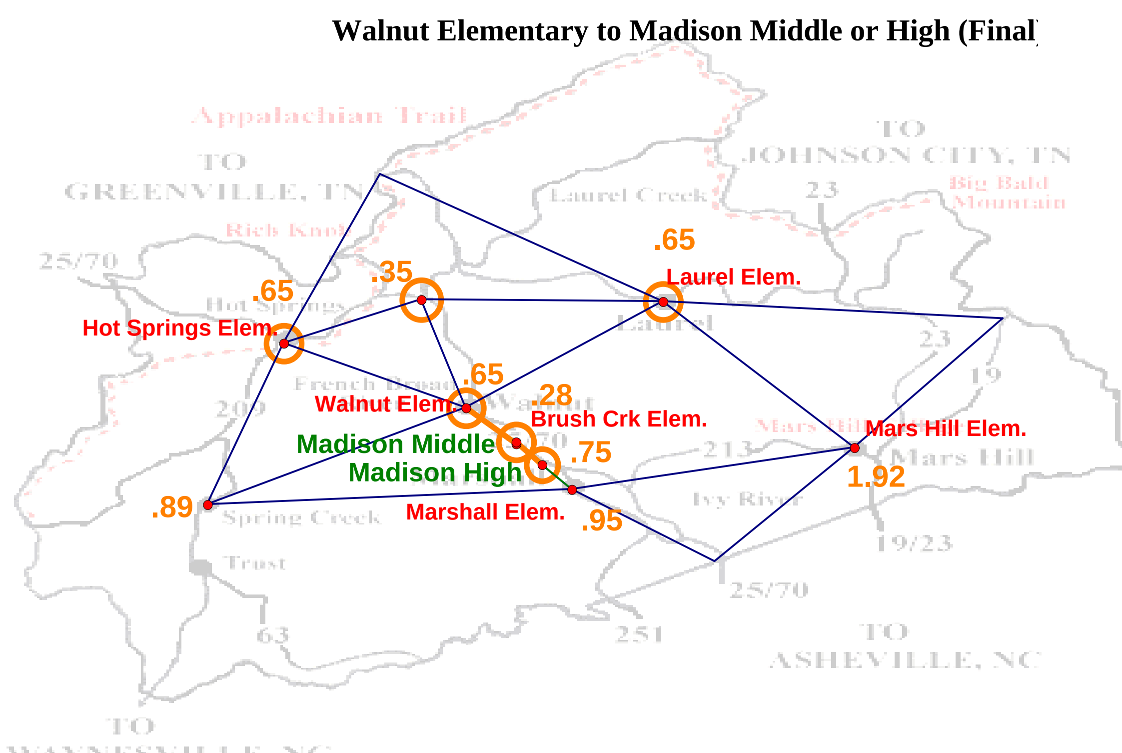
C				
LE			MHE	MHE
WE				MMBE
MMBE		MH	MH	MH
MH	ME	ME	ME	ME
ME				
MHE	ME	ME	ME	ME
Path Backwards		MMBE	MH	ME

Marshall Elem. to Madison Middle or High (Final)



Dikjstra's Non-Geometric Algorithm (Walnut Elementary)						
Vertex	L(U)(Init.)	L(U)(1st)	L(U)(2nd)	L(U)(3rd)	L(U)(4th)	L(U)(5th)
HS	0.9	0.9	0.65	0.65	0.65	0.65
A	0.89	0.89	0.89	0.89	0.89	0.89
C	0.35	0.35	0.35	0.35	0.35	0.35
LE	1.04	1.04	0.65	0.65	0.65	0.65
WE	0	0				
MMBE	0.28	0.28	0.28	0.28	0.28	0.28
MH		0.75	0.75	0.75	0.75	0.75

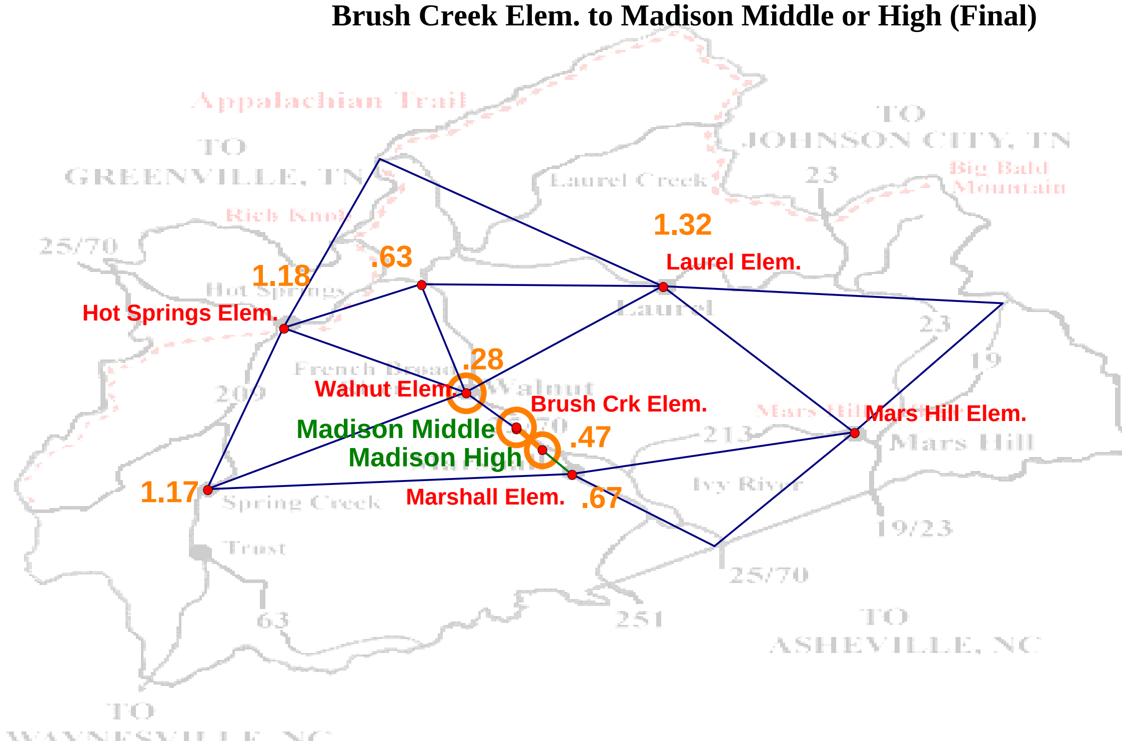
ME						0.95
MHE				1.92	1.92	1.92
	P(U)(Init.)	P(U)(1st)	P(U)(2nd)	P(U)(3rd)	P(U)4th	P(U)5th
HS	WE	WE	C	C	C	C
A	WE	WE	WE	WE	WE	WE
C	WE	WE	WE	WE	WE	WE
LE	WE	WE	C	C	C	C
WE						
MMBE	WE	WE	WE	WE	WE	WE
MH		MMBE	MMBE	MMBE	MMBE	MMBE
ME						MH
MHE				LE	LE	LE
Path Backwards		MH	MMBE	WE		



Dijkstra's Algorithm Brush Creek Elem.			
Vertex	L(U)(Init.)	L(U)(1st)	L(U)(2nd)
HS		1.18	1.18

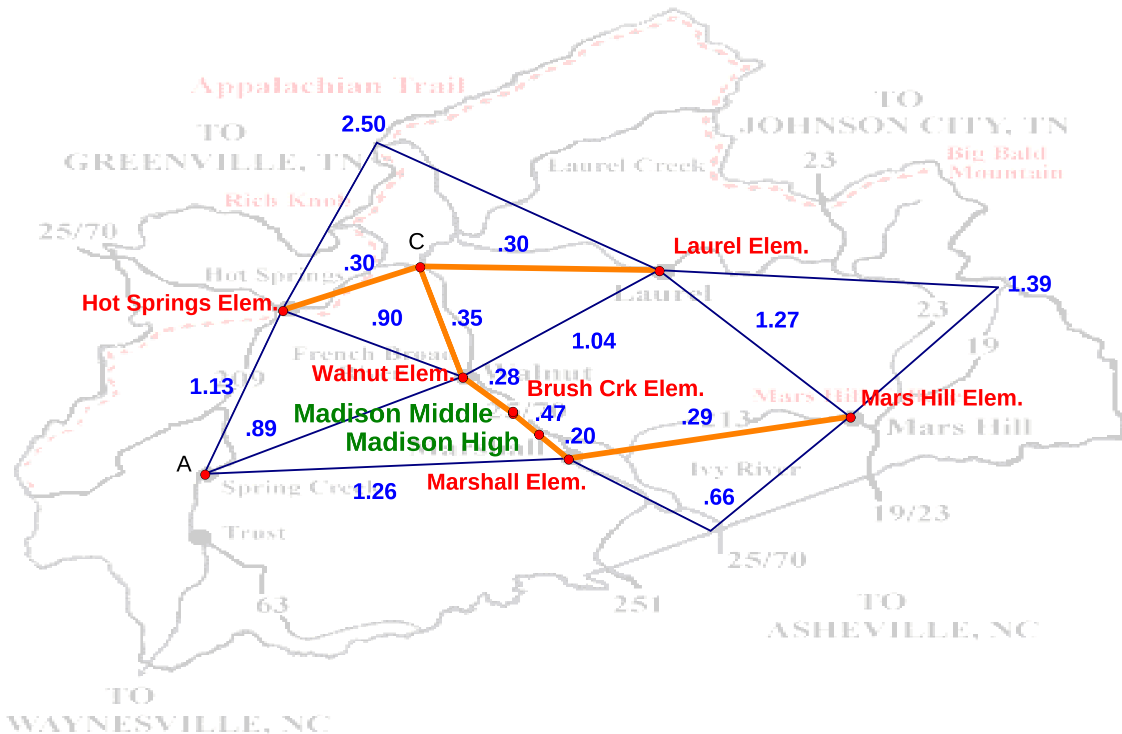
A		1.17	1.17
C		0.63	0.63
LE		1.32	1.32
WE	0.28	0.28	0.28
MMBE	0		
MH	0.47	0.47	0.47
ME			0.67
MHE			
	P(U)(Init.)	P(U)(1st)	P(U)(2nd)
HS		WE	WE
A		WE	WE
C		WE	WE
LE		WE	WE
WE	MMBE	MMBE	MMBE
MMBE			
MH	MMBE	MMBE	MMBE
ME			MH
MHE			
Path Backwards		MH	MMBE

Brush Creek Elem. to Madison Middle or High (Final)



Minimal Spanning Tree

Once all paths are found the combination of these paths are shown below as a minimal spanning tree that makes up the skeleton of our express routes from each elementary school.



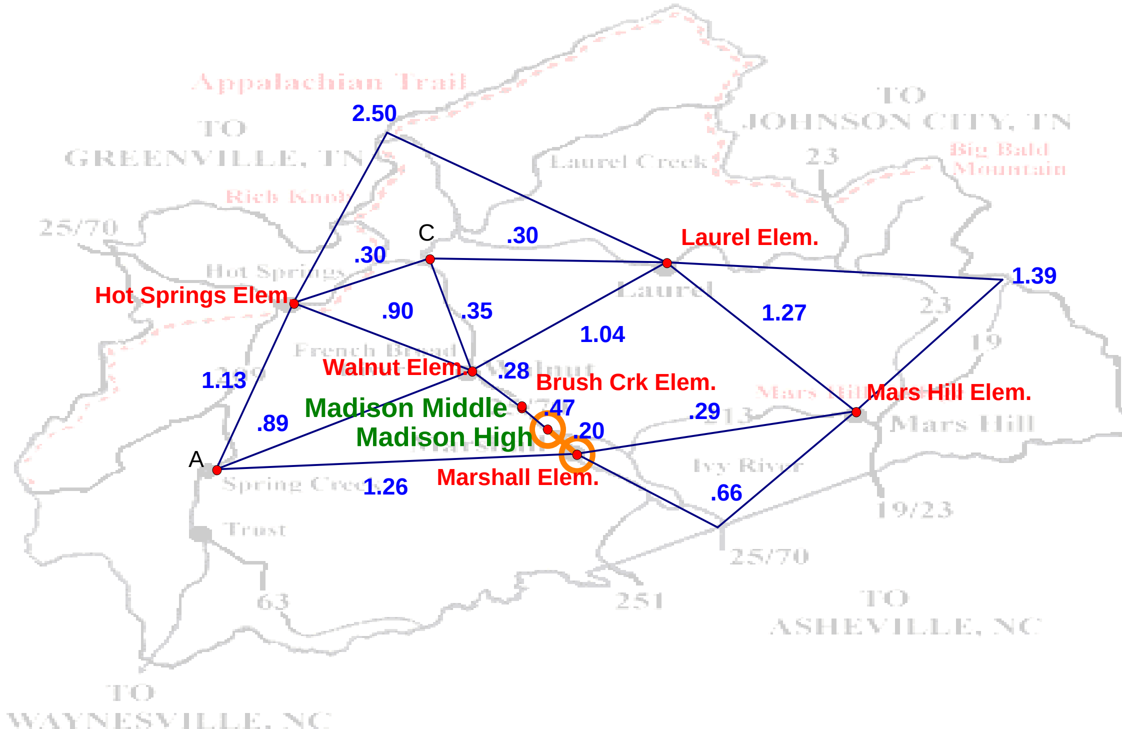
A spanning tree is a connected graph with no cycles with each vertex included. A connected graph means that any two vertices can be picked and there exist a path between them. A cycle is when there are two unique paths between two vertices. The express route skeleton is a spanning tree where the elementary, middle and high schools are vertices of the tree.

Prim defined an algorithm for finding the minimal spanning tree of a connected graph. This means that the combined weight of all the sections of road that make-up the tree is the least of any other combination that could have been chosen in the larger graph. This would mean that buses are running the least weighted roads in combination, which would optimize overall time on road.

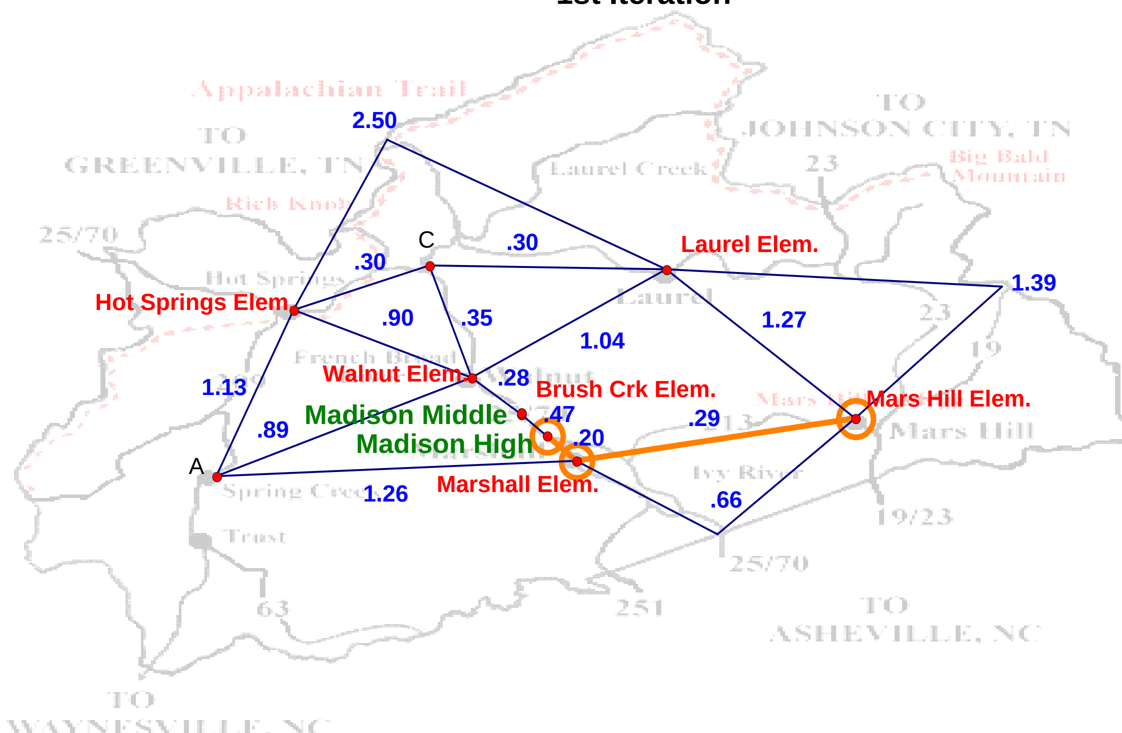
In the initial step we find a section of road that has the least weight and select the road and its vertices. In the first iteration we look for sections of road adjacent to the original section that has the least weight and select it and its vertices. The algorithm is continued until all vertices are selected with n-1 sections of road, where n equals the number of vertices. This method serves as an alternative approach to finding the express route skeleton that contains the least weight of combined routes that still reaches all elementary schools. What follows is a geometric illustration of Prim's model. The result is the same as Dijkstra's algorithm helping to confirm our choice of routes.

Prim's Algorithm

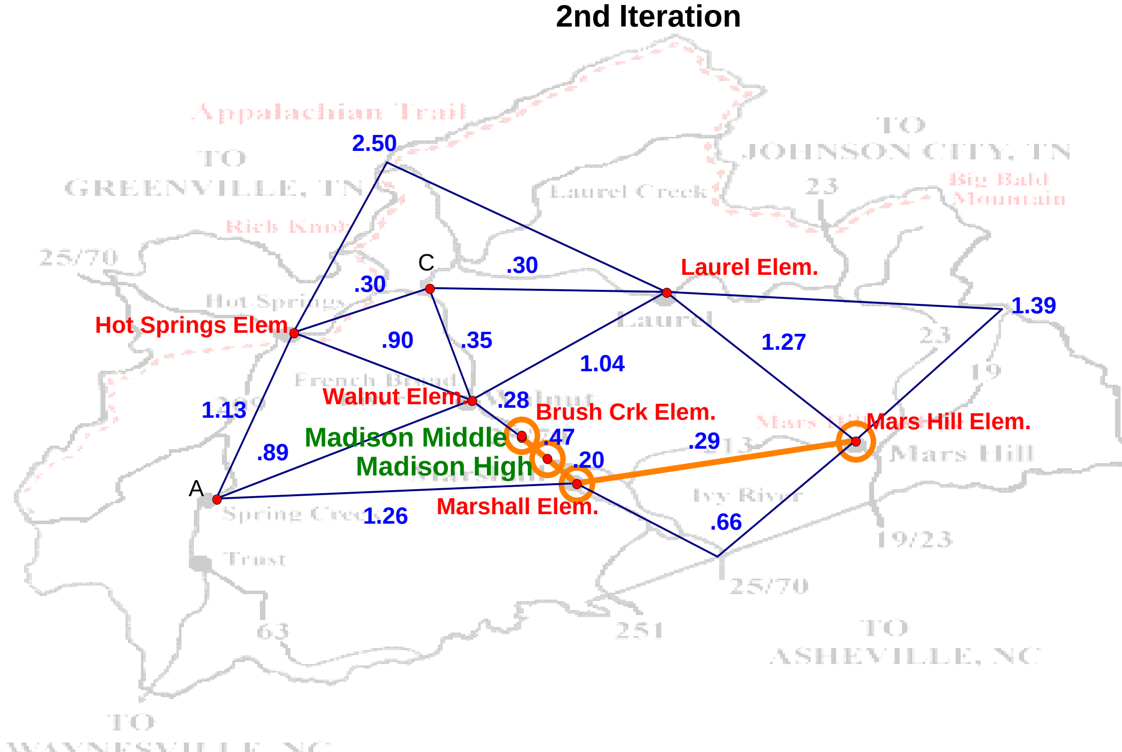
Initial



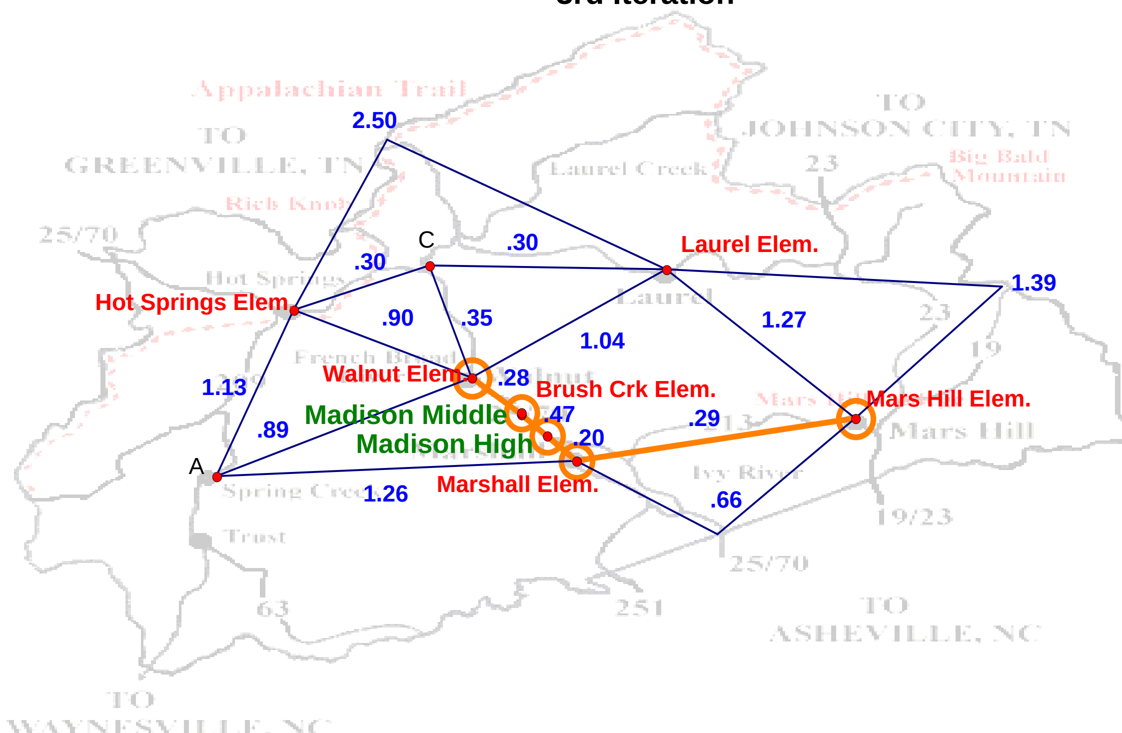
1st Iteration



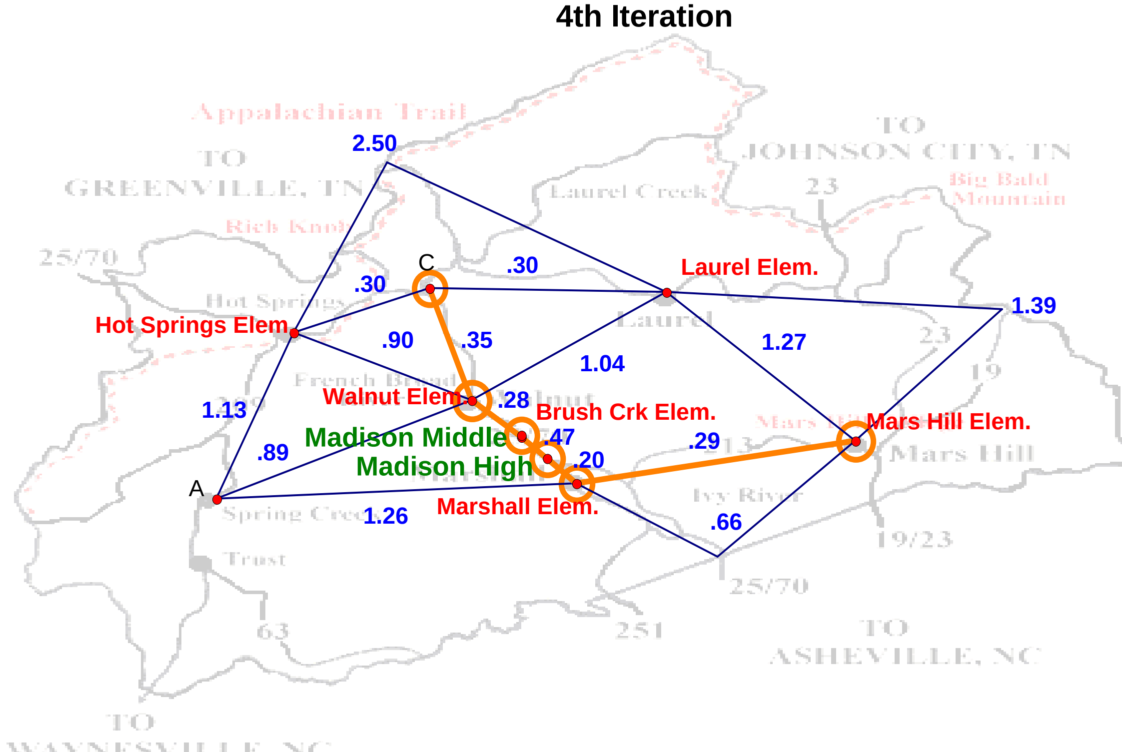
2nd Iteration



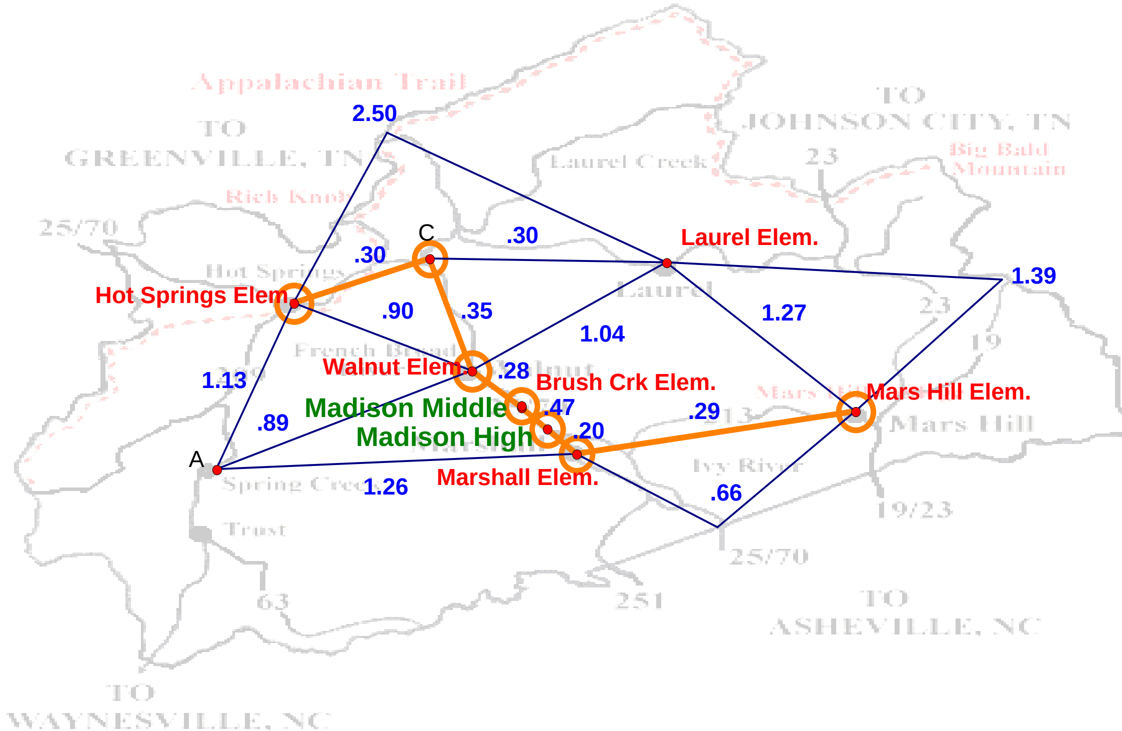
3rd Iteration



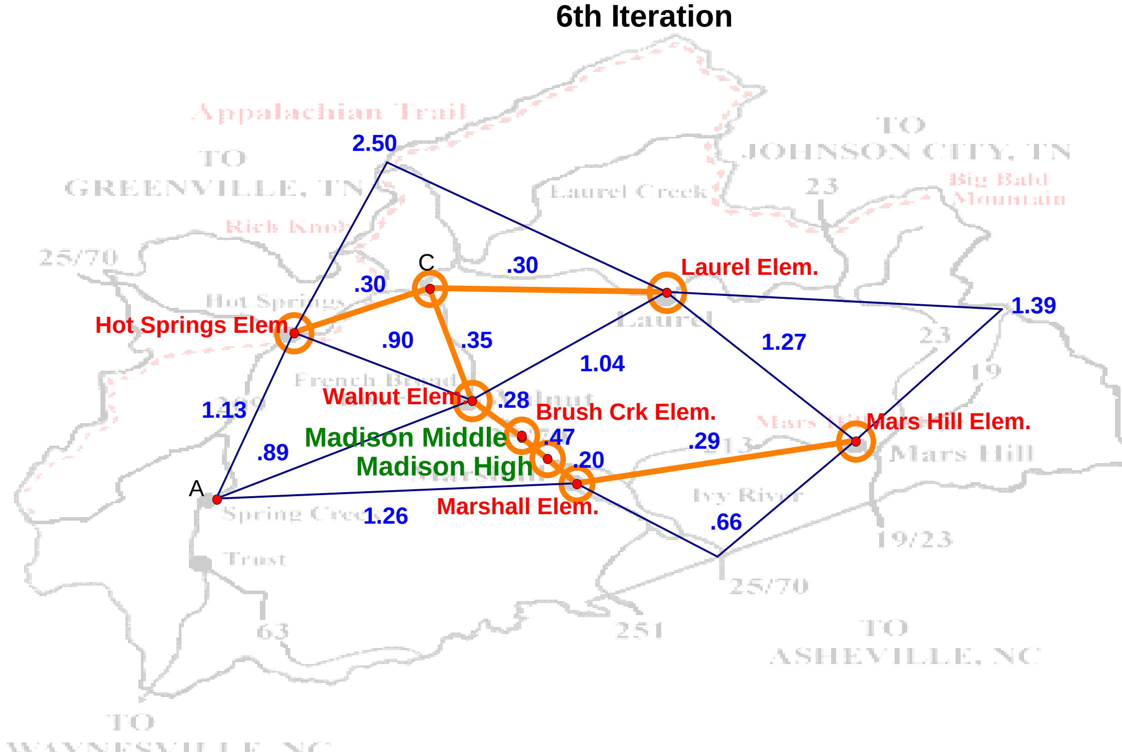
4th Iteration



5th Iteration



6th Iteration



Summary of Project
The main purpose of the model is to guarantee a decent solution to a complex, variable problem regardless of the situation. The algorithms, equations, and procedures discussed yield a solution that is at least as good as and usually better than a random, primitive method derived without mathematical consideration. This model respects the expertise of the creators of the present Madison County bus system by drawing on their previously gained knowledge about the system, its rural nature, terrain and students. This model allows for changes and accommodations for special concerns. These are guidelines to a more comprehensive model. Additional work with the continued assistance of Madison County Schools Transportation Director will result in a district by district model.

As mentioned previously, reality is far more unpredictable than a controlled mathematical model: the difficulty of constructing an accurate model lies herein. The proposed model makes several unrealistic, but indispensable, assumptions, though great efforts were taken to significantly reduce the severity of error these assumptions may create. However, the model nevertheless offers a great deal of flexibility for the variance in situations. For example, given any road and road configuration, functional bus routes are supplied by the model, which is actually quite impressive and applicable in most situations. The optimality of those routes is difficult with rural terrain and small school size but give great versatility and value.

Another important aspect of this model is that it can be computer generated to using graph theory and computer applications. Although it may not be optimal due to money and bus constraints, the model provides a good, workable bus route assignment that can serve as the basis for an even better system by employing personal discretion and common sense. It is hoped that the partnering of the Madison County School System and the NCL (No Child Left at Bus Stop Company) can enable the kids of Madison County to have the best system in place possible for getting students to and from their educational environments.

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