Characterization of the distribution of the L_z index of person fit according to the estimated proficiency level

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Gilles Raîche, Université du Québec à Montréal
Jean-Guy Blais, Université de Montréal

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Abstract

The distribution of person fit indices is not easy to describe in tests where the item sample is too small to conform to a theoretical asymptotic statistical distribution, particularly the normal N(0,1). In practice, it is always the fact and, consequently, it is difficult to get the critical percentile value indicating person misfit. First, we investigate if we obtain at least a $N(\bar{Lz} \mid \hat{\theta}, S^2_{Lz} \mid \hat{\theta})$ distribution of L_z conditional on the estimated proficiency level. In a second phase, we verify if it is possible to linearly predict the critical first and fifth percentiles of this same distribution by the estimated proficiency level and his standard error. Results of 2000 simulations of 85 fixed items tests at each of 23 simulated proficiency levels showed that the first percentile of the L_z distribution generally can't be predicted by a $N(\bar{Lz} \mid \hat{\theta}, S^2_{Lz} \mid \hat{\theta})$ distribution according to the related 23 estimated proficiency levels. The proficiency level was estimated with MLE, MAP, WLE, EAP and AEAP methods. More promising, the first and fifth percentiles of the *Lz* distribution were well predicted by a linear regression on the estimated proficiency level and his standard error ($r > 0.90$), particularly with the MLE, WLE and AEAP methods.

1. Introduction

The distribution of parametric person fit indices, would it be L_z, L^*_z, *Zeta*, *Infit*, or *Outfit*, is not easy to describe in tests where the item sample is too small to conform to a theoretical asymptotic statistical distribution, particularly the normal N(0,1). In practice, it is always the fact and, consequently, it is difficult to get the critical percentile values indicating person misfit.

This paper is aimed at easier strategies to obtain these critical percentile values.

2. L_z person fit index

The L_o person fit index was proposed by Levine and Rubin (1979) and applied many times (Drasgow and Guertler, 1987; Drasgow and levine, 1986; Drasgow, Levine and McLaughlin, 1987, 1991; Drasgow, Levine and Williams, 1985; Levine and Drasgow, 1982, 1983). However. this index was difficult to interpret because it was not independant of the proficiency level of the subject. To palliate the problem, Drasgow, Levine et Williams (1985) modified it to give a new standardized index: L_z. The L_z index would follow a N(0,1) probability distribution, so interesting statistical and practical proprieties would be associated to it. For a n items test, if the probability of a correct response at each item of the test, $P(X_g = 1|\theta)$, is obtained according to one of a model from item rersponse theory, L_z is equal to:

$$L_z = \frac{L_o - E(L_o)}{\left[Var(L_o)\right]^{1/2}},$$

(Equation 1)

where

$$L_o = \sum_{g=1}^{n}\left[X_g \ln P(X_g = 1|\theta) + (1 - X_g)\ln P(X_g = 0|\theta)\right],$$

(Equation 2)

$$E(L_o) = \sum_{g=1}^{n}\left[P(X_g = 1|\theta)\ln P(X_g = 1|\theta) + P(X_g = 0|\theta)\ln P(X_g = 0|\theta)\right],$$

(Equation 3)

and

$$Var(L_o) = \sum_{g=1}^{n}\left\{P(X_g = 1|\theta)P(X_g = 0|\theta)\left[\ln\frac{P(X_g = 1|\theta)}{P(X_g = 0|\theta)}\right]\right\}.$$

(Equation 4)

In this paper, only the three parameters logistic model is at study to computed the probability of a response:

$$P(X_g = 1 | \theta) = c_g + \frac{1-c_g}{1+e^{-1,7(\theta-b_g)}} .$$

(Equation 5)

So, the probability of the complete pattern of responses is:

$$P(X = x | \theta) = \prod_{g=1}^{n} \left[c_g + \frac{1-c_g}{1+e^{-1,7a_g(\theta-b_g)}} \right]^{X_g} \left[1-c_g + \frac{1-c_g}{1+e^{-1,7a_g(\theta-b_g)}} \right]^{1-X_g} .$$

(Equation 6)

The L_z person fit index is not specific to a proficiency level estimation method. However, past studies showed that the probability distribution and power of detection of L_z vary with the proficiency level estimation method (Raîche, 2002). For this reason the maximum likelihood (MLE), the maximum a posteriori (MAP), the weigthed maximum likelihood (WLE), the expected a posteriori with a N(0,1) a priori (EAP) and an uniform a priori (EXP), like an adaptive expected a posteriori (AEAP) estimation methods were used.

The adaptive expected a posteriori (AEAP) estimation method was studied by Raîche and Blais (2005; Blais and Raîche, 2005) and presents interesting properties concerning the diminution of the bias and standard error of the profiency level. The computation of the AEAP estimator is done with the replacement of the a priori probability distribution of the proficiency level at each *k* item with the mean of this a priori probability distribution fixed at the value of the proficiency level estimation based on all the preceding *k-1* items of the test.

3. L_z critical score

Previous studies showed that the critical score of the L_z person fit index, would it be conditional on θ or $\hat{\theta}$, does not follow a N(0,1) probability distribution in tests where the item sample is too small to conform to a theoretical asymptotic statistical distribution.

This distribution would be specific to the test, varying with the items parameters (Meijer, Molenaar and Sijtsma, 1994; Meijer and van Krimpen-Stoop, 1998; van Krimpen-Stoop and Meijer, 1999), and also with the proficiency level (Raîche and Blais, 2003). Consequently, it is difficult to get the critical percentile values indicating person misfit.

This paper is aimed at easier strategies to obtain these critical percentile values. First, the focus will be on only one person fit index, L_z. Secondly, it will be verified if the first and fifth percentiles of the L_z distribution can be predicted by a $\mathrm{N}(\overline{Lz} \mid \hat{\theta}, S^2_{Lz} \mid \hat{\theta})$ probability distribution. Finally, a prediction of the first and fifth percentiles of the *Lz* distribution based on a linear regression analysis with the estimated proficiency level and his variance as predictors will be checked.

4. Methodology

4.1 Simulated data

Two thousand administrations of the placement test TCALS II were simulated for proficiency levels, θ, varying between –3.00 and 3.00, by increments of 0.25. Each of the simulations considered the five following strategies for the proficiency level estimation: (1) maximum likelihood (MLE), (2) maximum a posteriori (MAP), (3) weigthed maximum likelihood (WLE), (4) expected a posteriori with a N(0,1) a priori (EAP), (5) expected a posteriori with a an uniform a priori (EXP) , and (5) adaptive expected a posteriori (AEAP).

A normal response pattern was generated following:

$$\text{If } P(X_g = 1 \mid \theta) \geq U(0,1), \text{Then } X_g = 1; \text{Else } X_g = 0. \qquad \textbf{(Equation 7)}$$

Fit 1.0 software was used to simulate the data, and compute the estimated proficiency levels, like the L_o person fit index.

4.2 Simulated test characteristics

The study used the item characteristics of a 85 items large-scale fixed items test used at the placement of french collegial students at different level of english as a second language in Québec Province (Canada): the TCALS II. The items of the TCALS II were calibrated with Bilog 3.0 according to a three parameters logistic model (3PL) and according to the maximum likelihood estimation method (MLE) of the proficiency level.

The TCALS II is most informative at a proficiency level of about –0.40. At proficiency levels higher than 0.50 the test is not very informative, and higher difficulty items would be useful. The test information curve is presented at figure 1.

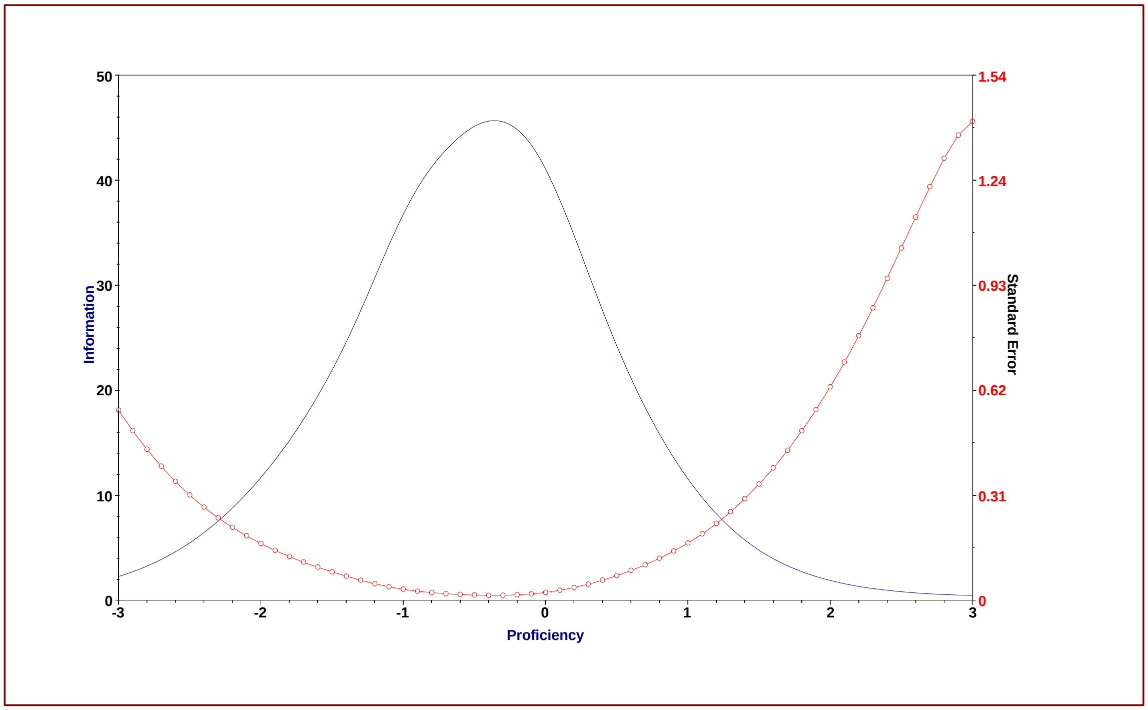

Figure 1. TCALS II Test Information Curve

The difficulty parameters of the TCALS II, *b*, have a mean of –1.11 and a standard deviation of 0.86. The discrimination parameters, *a*, present a mean value of 1.17 and a standard deviation of 0.43, while the pseudo-guessing parameters, *c*, have a mean of 0.20 and a standard deviation of 0.06. Table 1 presents the item parameters for each item of the TCALS II. It can be seen clearly that difficult items are underrepresented.

Table 1. Items Parameters of the TCALS II

a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
1.307	-1.885	0.210	1.413	-0.454	0.076	1.863	-0.663	0.222	1.585	-1.006	0.148
0.690	-2.411	0.210	1.192	-0.008	0.117	1.192	-0.993	0.224	1.858	-0.621	0.256
1.236	-2.439	0.192	1.159	0.805	0.094	0.733	-2.392	0.196	1.387	-0.248	0.273
1.581	-1.282	0.294	0.605	-1.229	0.185	0.731	-1.743	0.209	1.560	-0.086	0.330
0.738	-1.930	0.170	0.478	0.687	0.215	1.540	-1.867	0.161	1.117	-1.107	0.238
0.850	-1.554	0.205	1.040	-1.104	0.192	1.189	-1.696	0.156	0.597	-1.268	0.210
1.144	-1.705	0.246	0.777	-1.441	0.134	1.289	-1.284	0.227	0.583	-1.115	0.203
1.655	-0.551	0.105	1.669	-0.257	0.341	0.434	-2.356	0.213	0.791	-0.612	0.153
1.565	-0.626	0.095	1.357	-0.282	0.239	1.513	-1.444	0.175	0.814	-0.589	0.171
1.891	-0.246	0.078	0.239	-1.103	0.236	1.576	-1.107	0.275	0.738	-0.182	0.196
1.511	0.251	0.147	0.509	-2.224	0.195	1.029	-1.352	0.228	1.693	0.840	0.233
1.358	0.189	0.147	0.536	-3.457	0.194	0.691	-2.014	0.222	0.685	-1.377	0.190
1.039	-1.534	0.185	1.030	-1.658	0.210	0.602	-1.364	0.172	0.859	-1.103	0.283
0.949	-3.088	0.202	1.435	-2.070	0.168	1.230	-1.232	0.243	1.744	0.711	0.089
1.238	-1.139	0.224	1.032	-1.782	0.233	1.509	-0.565	0.166	1.209	-0.265	0.142
0.855	-1.673	0.182	1.015	-1.991	0.214	1.807	-0.265	0.235	1.137	-0.648	0.304
1.012	-1.349	0.195	0.522	-2.815	0.201	1.684	-0.032	0.236	0.940	-1.093	0.384
1.051	-1.481	0.144	1.628	-1.412	0.252	1.535	-0.115	0.092	1.286	-0.664	0.180
2.076	-0.962	0.168	0.863	-1.531	0.218	2.340	0.120	0.063	1.136	-0.952	0.316
0.949	-1.441	0.133	1.020	-1.945	0.253	0.963	-2.139	0.200			
0.691	-0.549	0.223	1.252	-0.810	0.295	1.116	-1.889	0.191			
1.303	-0.814	0.115	2.248	-0.729	0.212	1.047	-1.639	0.187			

4.3 Analysis method

First the the first and fifth empirical percentiles of the L_z statistic are compared to the theoretical percentiles of a probability distribution N($\overline{Lz} \mid \hat{\theta}, S^2_{Lz} \mid \hat{\theta}$) . Since the first and fifth empirical percentiles of the L_z statistic are computed from a sample of 2000 observations, an estimation of the standard error of these percentiles can be computed according to:

$$\sigma_p = \frac{\sigma}{\sqrt{N}} * \frac{\sqrt{pq}}{y}, \qquad \textbf{(Equation 8)}$$

where σ is the empirical estimated standard deviation of L_z in each class of the estimated proficiency levels, N is the class size (number of observations), p is equal to

0.01 or 0.05 respectively for the first and the fifth percentiles, q is equal to $1-p$, and y being the ordinate at the z score associated to the specific percentile.

According to equation 8, with a sample of 2000 observations and the variance of the sampled L_z statistics conservatively at 1.00, the standard error of the first centile is at most equal to 0.06, while the standard error of the fifth centile is at most equal to 0.04.

In a second phase, multiple linear regression on the proficiency level, the square of the proficiency level, and his variance will be used to verify the quality of the prediction of the first and fifth percentiles of the L_z statistic:

$$b_0 + b_1\hat{\theta} + b_2\hat{\theta}^2 + b_3 S_{\hat{\theta}}^2 \qquad \textbf{(Equation 9)}$$

Residuals from these multiple regressions will be analysed.

Finally, the hit rate obtained with each of the predicted first and fifth percentiles will be analysed.

5. Results

Empirical and theoretical probability distribution of Lz

Tables 1 to 6 present the first and fifth empirical and theoretical percentiles of the L_z statistic according to the estimated proficiency level and to the proficiency level estimation method. Figures 1 and 2 present the difference between the empirical and the theoretical fifth percentiles. The mean of the estimated L_z statistic, like its standard deviation and percentiles, varies greatly with the proficiency level.

With all estimation methods, the difference between the empirical and the theoretical fifth percentiles is generally not too important, being usually less than 0.10. The biggest differences are at extreme values of the estimated proficiency level. It’s not the case with the first percentile where the difference between the empirical and theoretical values is generally important.

Also, due to the characteristics of the TCALS II, note that the maximal proficiency level that can be estimated varies with the estimation method. MAP and EAP were the worst methods, not being able to estimate a proficiency level over 1.75 and 2.00 respectively.

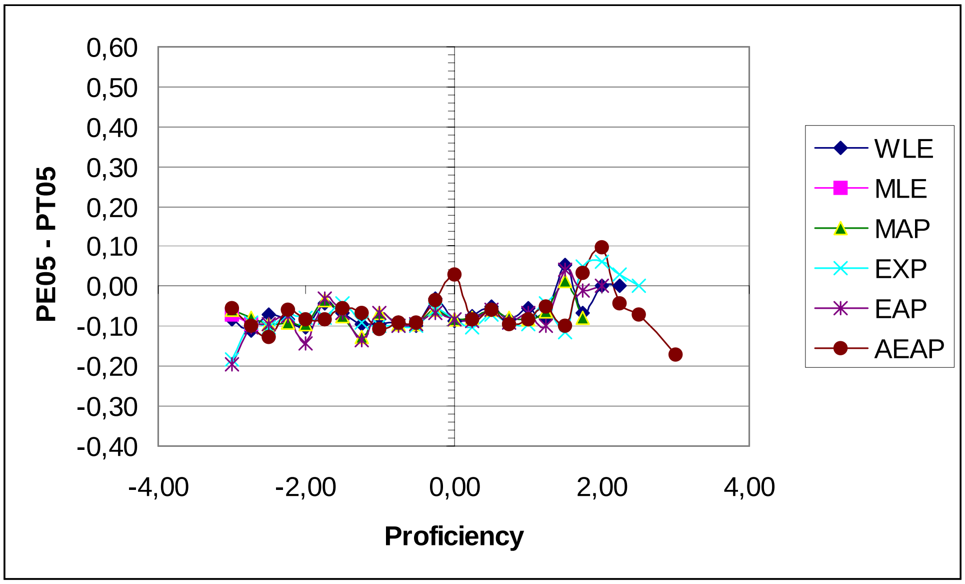

Figure 2. Difference between the empirical and the theoretical L_z fifth percentiles

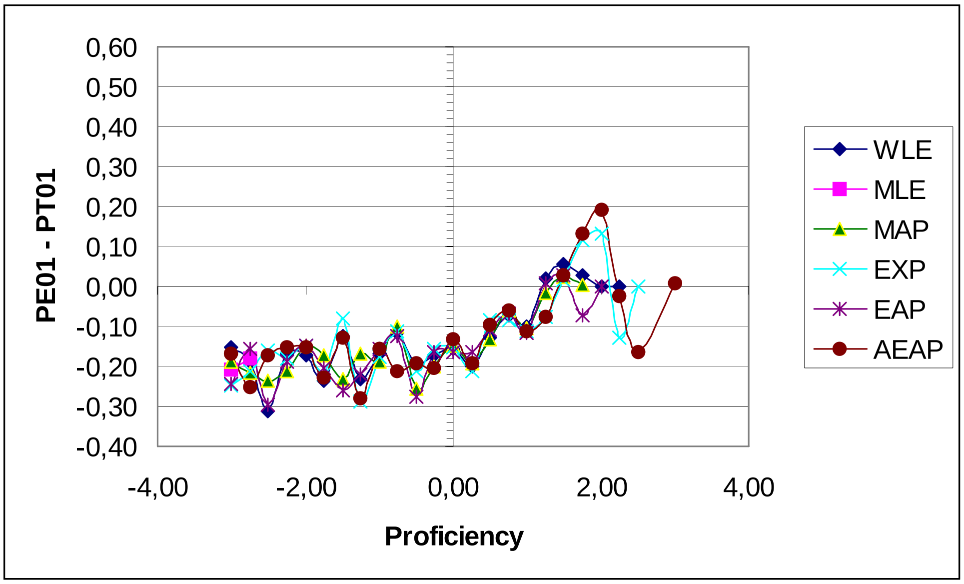

Figure 3. Difference between the empirical and the theoretical L_z first percentiles

Table 2. Empirical and theoretical percentiles - WLE estimation method

OBS	N	$\hat{\theta}$	$s_{\hat{\theta}}$	$\bar{L}z$	$s_{\bar{L}z}$	$P05E_{Lz}$	$P05T_{Lx}$	$\Lambda05_{Lz}$	$P01E_{Lz}$	$P01T_{Lx}$	$\Lambda01_{Lz}$
1	2028	-3.00	0.71640	0.19413	0.89649	-1.36050	-1.27612	-0.08438	-2.0459	-1.89469	-0.15121
2	1563	-2.75	0.40628	0.09695	0.90193	-1.49480	-1.38222	-0.11258	-2.1938	-2.00455	-0.18925
3	1950	-2.50	0.33641	0.10933	0.93646	-1.49680	-1.42646	-0.07034	-2.3832	-2.07261	-0.31059
4	2127	-2.25	0.28343	0.12265	0.95789	-1.53290	-1.44829	-0.08461	-2.2754	-2.10924	-0.16616
5	2076	-2.00	0.24575	0.02893	0.98756	-1.69410	-1.59067	-0.10343	-2.4460	-2.27209	-0.17391
6	2106	-1.75	0.21833	0.02779	0.97393	-1.61340	-1.56945	-0.04395	-2.4777	-2.24146	-0.23624
7	2079	-1.50	0.19800	0.08074	0.96076	-1.56070	-1.49491	-0.06579	-2.2804	-2.15783	-0.12257
8	2057	-1.25	0.18162	0.08823	0.93571	-1.54240	-1.44634	-0.09606	-2.3232	-2.09198	-0.23122
9	1999	-1.00	0.16886	0.07499	0.89338	-1.48410	-1.39015	-0.09395	-2.1763	-2.00659	-0.16971
10	2020	-0.75	0.16100	0.07541	0.85305	-1.41295	-1.32359	-0.08936	-2.0268	-1.91220	-0.11460
11	2043	-0.50	0.15966	0.04582	0.80459	-1.37270	-1.27371	-0.09899	-2.0250	-1.82887	-0.19613
12	2045	-0.25	0.16380	0.05589	0.76036	-1.22310	-1.19110	-0.03200	-1.8892	-1.71574	-0.17346
13	2007	0.00	0.17377	0.10471	0.72914	-1.17830	-1.09108	-0.08722	-1.7497	-1.59418	-0.15552
14	2089	0.25	0.19209	0.10855	0.67941	-1.08000	-1.00569	-0.07431	-1.6744	-1.47448	-0.19992
15	2065	0.50	0.22044	0.15878	0.61593	-0.90380	-0.85135	-0.05245	-1.4057	-1.27634	-0.12936
16	2184	0.75	0.25859	0.17034	0.60839	-0.90850	-0.82742	-0.08108	-1.3209	-1.24721	-0.07369
17	2362	1.00	0.30822	0.20205	0.57208	-0.79150	-0.73615	-0.05535	-1.2314	-1.13089	-0.10051
18	2620	1.25	0.37596	0.26538	0.52120	-0.66785	-0.58938	-0.07847	-0.9272	-0.94901	0.02181
19	2712	1.50	0.46745	0.26318	0.38195	-0.30900	-0.36322	0.05422	-0.5713	-0.62676	0.05546
20	2706	1.75	0.57612	0.43491	0.27524	-0.08170	-0.01648	-0.06522	-0.1765	-0.20639	0.02989
21	1139	2.00	0.70010	0.48520	0.00000	0.48520	0.48520	-0.00000	0.4852	0.48520	-0.00000
22	6023	2.25	0.89210	0.81600	0.00000	0.81600	0.81600	0.00000	0.8160	0.81600	0.00000

Table 3. Empirical and theoretical percentiles - MLE estimation method

OBS	N	$\hat{\theta}$	$S_{\hat{\theta}}$	$\bar{L}z$	$S_{\bar{L}z}$	$P05E_{Lz}$	$P05T_{Lx}$	$\Lambda05_{Lz}$	$P01E_{Lz}$	$P01T_{Lx}$	$\Lambda01_{Lz}$
1	2484	-3.00	0.89487	0.06068	0.90231	-1.4879	-1.4191	-0.0688	-2.2516	-2.0417	-0.2099
2	1504	-2.75	0.40628	0.04205	0.89451	-1.5204	-1.4249	-0.0955	-2.2230	-2.0422	-0.1808
3	1817	-2.50	0.33644	0.06357	0.94269	-1.5607	-1.4824	-0.0783	-2.4082	-2.1329	-0.2753
4	2025	-2.25	0.28346	0.09994	0.95913	-1.5445	-1.4730	-0.0715	-2.2914	-2.1348	-0.1566
5	2007	-2.00	0.24561	0.03792	0.98807	-1.6738	-1.5825	-0.0913	-2.4016	-2.2643	-0.1373
6	2056	-1.75	0.21827	0.01743	0.97357	-1.6474	-1.5792	-0.0682	-2.4758	-2.2510	-0.2248
7	2068	-1.50	0.19797	0.09145	0.95464	-1.5218	-1.4742	-0.0476	-2.2744	-2.1329	-0.1415
8	2037	-1.25	0.18159	0.08941	0.93721	-1.5234	-1.4476	-0.0758	-2.3198	-2.0943	-0.2255
9	1982	-1.00	0.16886	0.07551	0.89366	-1.4832	-1.3901	-0.0931	-2.1749	-2.0067	-0.1682
10	1989	-0.75	0.16100	0.06720	0.85138	-1.4184	-1.3291	-0.0893	-2.0313	-1.9165	-0.1148
11	2011	-0.50	0.15965	0.02271	0.81054	-1.3975	-1.3066	-0.0909	-2.0603	-1.8658	-0.1945
12	2039	-0.25	0.16379	0.02801	0.75873	-1.2586	-1.2163	-0.0423	-1.9186	-1.7398	-0.1788
13	3081	0.00	0.92624	-0.04134	0.59641	-1.0203	-1.0194	-0.0009	-1.6570	-1.4310	-0.2260
14	1992	0.25	0.19191	0.03070	0.68240	-1.1634	-1.0884	-0.0750	-1.7745	-1.5593	-0.2152
15	2001	0.50	0.22041	0.03599	0.62889	-1.0548	-0.9954	-0.0594	-1.5464	-1.4293	-0.1171
16	1978	0.75	0.25846	0.03153	0.62335	-1.0755	-0.9908	-0.0847	-1.4965	-1.4209	-0.0756
17	2010	1.00	0.30728	0.02456	0.59058	-1.0288	-0.9440	-0.0848	-1.4643	-1.3515	-0.1128
18	1990	1.25	0.37472	-0.03413	0.55582	-1.0248	-0.9457	-0.0791	-1.3800	-1.3292	-0.0508
19	1910	1.50	0.46543	-0.03208	0.53906	-0.9374	-0.9161	-0.0213	-1.1696	-1.2881	0.1185
20	1575	1.75	0.57711	-0.09746	0.46804	-0.7537	-0.8650	0.1113	-1.0628	-1.1880	0.1252
21	1674	2.00	0.75057	-0.23367	0.23291	-0.6778	-0.6156	-0.0622	-0.7585	-0.7764	0.0179
22	1746	2.25	0.95520	0.00665	0.33956	-0.5307	-0.5502	0.0195	-0.9115	-0.7845	-0.1270
23	6024	3.00	3.02659	0.27302	9.45997	0.3949	-15.2413	15.6362	0.3949	-21.7687	22.1636

Table 4. Empirical and theoretical percentiles - MAP estimation method

OBS	N	$\hat{\theta}$	$S_{\hat{\theta}}$	$\bar{L}z$	$S_{\bar{L}z}$	$P05E_{Lz}$	$P05T_{Lx}$	$\Lambda05_{Lz}$	$P01E_{Lz}$	$P01T_{Lx}$	$\Lambda01_{Lz}$
1	450	-3.00	0.45085	0.75453	0.87944	-0.7469	-0.68774	-0.05916	-1.4835	-1.29455	-0.18895
2	1247	-2.75	0.37370	0.47655	0.93332	-1.1330	-1.05409	-0.07891	-1.9155	-1.69808	-0.21742
3	2287	-2.50	0.31778	0.28821	0.92489	-1.3245	-1.22860	-0.09590	-2.1035	-1.86677	-0.23673
4	2589	-2.25	0.27304	0.19037	0.96981	-1.4915	-1.40012	-0.09138	-2.2794	-2.06928	-0.21012
5	2442	-2.00	0.23878	0.08424	0.98514	-1.6271	-1.53139	-0.09571	-2.3597	-2.21114	-0.14856
6	2292	-1.75	0.21331	-0.00055	0.96067	-1.6110	-1.57605	-0.03495	-2.4101	-2.23891	-0.17119
7	2216	-1.50	0.19435	-0.00042	0.97932	-1.6807	-1.60650	-0.07420	-2.5158	-2.28223	-0.23357
8	2202	-1.25	0.17878	0.01358	0.93176	-1.6404	-1.51451	$\underline{-0.12589}$	-2.3238	-2.15743	-0.16637
9	2085	-1.00	0.16648	-0.01334	0.90277	-1.5624	-1.49388	-0.06852	-2.3050	-2.11680	-0.18820
10	2020	-0.75	0.15895	0.00453	0.85274	-1.4877	-1.39397	-0.09373	-2.0822	-1.98236	-0.09984
11	2058	-0.50	0.15764	-0.02376	0.80804	-1.4447	-1.34895	-0.09575	-2.1614	-1.90650	$\underline{-0.25490}$
12	2107	-0.25	0.16162	0.00429	0.76631	-1.3123	-1.25246	-0.05984	-1.9804	-1.78121	-0.19919
13	2009	0.00	0.17121	0.05642	0.73208	-1.2277	-1.14418	-0.08352	-1.7992	-1.64931	-0.14989
14	2090	0.25	0.18864	0.06424	0.68323	-1.1345	-1.05626	-0.07824	-1.7197	-1.52769	-0.19201
15	2129	0.50	0.21531	0.12877	0.61726	-0.9422	-0.88354	-0.05866	-1.4406	-1.30945	-0.13115
16	2299	0.75	0.25029	0.18052	0.60492	-0.8886	-0.81156	-0.07704	-1.2909	-1.22895	-0.06195
17	2601	1.00	0.29447	0.24422	0.56371	-0.7620	-0.68025	-0.08175	-1.1750	-1.06921	-0.10579
18	3550	1.25	0.35561	0.42501	0.49765	-0.4537	-0.39114	-0.06256	-0.7509	-0.73452	-0.01638
19	4165	1.50	0.42827	0.51638	0.30056	0.0354	0.02346	0.01194	-0.1584	-0.18392	0.02552
20	7162	1.75	0.50349	1.00470	0.11981	0.7292	0.80821	-0.07901	0.7292	0.72553	0.00367

Table 5. Empirical and theoretical percentiles - EXP estimation method

OBS	N	$\hat{\theta}$	$S_{\hat{\theta}}$	$\bar{L}z$	$S_{\bar{L}z}$	$P05E_{Lz}$	$P05T_{Lx}$	$\Lambda05_{Lz}$	$P01E_{Lz}$	$P01T_{Lx}$	$\Lambda01_{Lz}$
1	992	-3.00	0.16993	0.61696	0.91330	-1.0635	-0.88086	-0.18264	-1.7590	-1.51104	-0.24796
2	3221	-2.75	0.25748	0.11381	0.89954	-1.4512	-1.36144	-0.08976	-2.1956	-1.98212	-0.21348
3	2214	-2.50	0.30364	-0.01934	0.92948	-1.6456	-1.54369	-0.10191	-2.3435	-2.18503	-0.15847
4	1899	-2.25	0.29299	0.00646	0.96350	-1.6450	-1.57369	-0.07131	-2.4166	-2.23851	-0.17809
5	1845	-2.00	0.25892	0.01628	0.97518	-1.6621	-1.58302	-0.07908	-2.4043	-2.25590	-0.14840
6	1933	-1.75	0.22748	0.03155	0.98110	-1.6550	-1.57745	-0.07755	-2.4729	-2.25440	-0.21850
7	2127	-1.50	0.19614	0.12436	0.94810	-1.4733	-1.43052	-0.04278	-2.1647	-2.08471	-0.07999
8	1735	-1.25	0.19928	0.10003	0.95748	-1.5619	-1.47024	-0.09166	-2.4195	-2.13091	-0.28859
9	2261	-1.00	0.15845	0.14944	0.88368	-1.3984	-1.29980	-0.09860	-2.0839	-1.90954	-0.17436
10	1659	-0.75	0.17824	-0.00651	0.84612	-1.4949	-1.39415	-0.10075	-2.0907	-1.97798	-0.11272
11	2019	-0.50	0.15874	0.10730	0.81179	-1.3241	-1.22404	-0.10006	-1.9975	-1.78418	-0.21332
12	2283	-0.25	0.15650	0.00374	0.75538	-1.2931	-1.23508	-0.05802	-1.9143	-1.75630	-0.15800
13	1564	0.00	0.19446	0.03543	0.73073	-1.2464	-1.16296	-0.08344	-1.8203	-1.66716	-0.15314
14	2099	0.25	0.18448	-0.02327	0.69736	-1.2711	-1.16694	-0.10416	-1.8614	-1.64811	-0.21329
15	1844	0.50	0.22684	-0.06188	0.63017	-1.1648	-1.09536	-0.06944	-1.6143	-1.53017	-0.08413
16	1856	0.75	0.25880	-0.08203	0.63276	-1.2121	-1.11976	-0.09234	-1.6384	-1.55637	-0.08203
17	1751	1.00	0.31274	-0.15050	0.61125	-1.2474	-1.15295	-0.09445	-1.6855	-1.57471	-0.11079
18	1668	1.25	0.37987	-0.26134	0.60636	-1.2976	-1.25576	-0.04184	-1.7489	-1.67415	-0.07475
19	1637	1.50	0.46002	-0.32860	0.56437	-1.3675	-1.25417	-0.11333	-1.6260	-1.64358	0.01758
20	1580	1.75	0.52276	-0.46083	0.53734	-1.2919	-1.34207	0.05017	-1.5986	-1.71283	0.11423
21	1945	2.00	0.55485	-0.39014	0.44664	-1.0605	-1.12264	0.06214	-1.2987	-1.43082	0.13212
22	3845	2.25	0.54416	-0.03356	0.32563	-0.5396	-0.56760	0.02800	-0.9220	-0.79228	-0.12972
23	6023	2.50	0.49800	0.71320	0.00000	0.7132	0.71320	-0.00000	0.7132	0.71320	-0.00000

Table 6. Empirical and theoretical percentiles - EAP estimation method

OBS	N	$\hat{\theta}$	$S_{\hat{\theta}}$	$\bar{L}z$	$S_{\bar{L}z}$	$P05E_{Lz}$	$P05T_{Lx}$	$\Lambda05_{Lz}$	$P01E_{Lz}$	$P01T_{Lx}$	$\Lambda01_{Lz}$
1	139	-3.00	0.17804	0.98931	0.90705	-0.6935	-0.49826	-0.19524	-1.3684	-1.12413	-0.24427
2	1861	-2.75	0.25348	0.50006	0.91179	-1.0998	-0.99528	-0.10452	-1.7791	-1.62442	-0.15468
3	2611	-2.50	0.28340	0.20984	0.93231	-1.4145	-1.31914	-0.09536	-2.2568	-1.96244	-0.29436
4	2431	-2.25	0.26785	0.16434	0.96675	-1.4974	-1.42113	-0.07627	-2.2744	-2.08819	-0.18621
5	2270	-2.00	0.24205	0.06540	0.98130	-1.6867	-1.54392	-0.14278	-2.3676	-2.22102	-0.14658
6	2185	-1.75	0.21395	0.02774	0.96711	-1.5899	-1.55832	-0.03158	-2.4285	-2.22563	-0.20287
7	2375	-1.50	0.18878	0.02451	0.97076	-1.6348	-1.56754	-0.06726	-2.4962	-2.23737	-0.25883
8	1785	-1.25	0.19509	0.03322	0.94305	-1.6478	-1.51338	-0.13442	-2.3835	-2.16409	-0.21941
9	2404	-1.00	0.15446	0.06855	0.89285	-1.4642	-1.39572	-0.06848	-2.1665	-2.01179	-0.15471
10	1657	-0.75	0.17740	-0.06378	0.84475	-1.5479	-1.44917	-0.09873	-2.1558	-2.03205	-0.12375
11	2090	-0.50	0.15514	0.06141	0.81032	-1.3624	-1.26751	-0.09489	-2.1029	-1.82663	-0.27627
12	2377	-0.25	0.15375	-0.03043	0.76442	-1.3514	-1.28409	-0.06731	-1.9748	-1.81154	-0.16326
13	1581	0.00	0.19322	0.03412	0.73166	-1.2476	-1.16580	-0.08180	-1.8350	-1.67065	-0.16435
14	2235	0.25	0.18073	0.01176	0.69704	-1.2183	-1.13139	-0.08691	-1.7746	-1.61235	-0.16225
15	1968	0.50	0.22203	0.03206	0.62187	-1.0457	-0.98780	-0.05790	-1.5253	-1.41689	-0.10841
16	2153	0.75	0.25120	0.06996	0.61404	-1.0280	-0.93707	-0.09093	-1.4291	-1.36076	-0.06834
17	2269	1.00	0.29945	0.10802	0.58936	-0.9250	-0.85853	-0.06647	-1.3819	-1.26519	-0.11671
18	2749	1.25	0.35614	0.19538	0.53611	-0.7810	-0.68385	-0.09715	-1.0463	-1.05376	0.00746
19	3051	1.50	0.41878	0.19790	0.39164	-0.4040	-0.44439	0.04039	-0.6860	-0.71462	0.02862
20	3786	1.75	0.46815	0.52260	0.23453	0.1258	0.13798	-0.01218	-0.0958	-0.02385	-0.07195
21	6023	2.00	0.49940	0.97170	0.00000	0.9717	0.97170	-0.00000	0.9717	0.97170	-0.00000

Table 7. Empirical and theoretical percentiles - AEAP estimation method

OBS	N	$\hat{\theta}$	$S_{\hat{\theta}}$	$\bar{L}z$	$S_{\bar{L}z}$	$P05E_{Lz}$	$P05T_{Lx}$	$\Lambda05_{Lz}$	$P01E_{Lz}$	$P01T_{Lx}$	$\Lambda01_{Lz}$
1	3581	-3.00	1.50212	-0.22476	0.89769	-1.75060	-1.69698	-0.05362	-2.4825	-2.31638	-0.16612
2	1389	-2.75	0.43992	-0.08562	0.89336	-1.65000	-1.55074	-0.09926	-2.4194	-2.16716	-0.25224
3	1562	-2.50	0.35704	-0.03704	0.93844	-1.70340	-1.57609	-0.12731	-2.3968	-2.22362	-0.17318
4	1782	-2.25	0.30236	0.02415	0.96241	-1.61230	-1.55420	-0.05810	-2.3717	-2.21827	-0.15343
5	1835	-2.00	0.24985	0.01856	0.97653	-1.66570	-1.58295	-0.08275	-2.4107	-2.25676	-0.15394
6	1973	-1.75	0.22909	0.03431	0.97828	-1.65120	-1.57007	-0.08113	-2.4724	-2.24509	-0.22731
7	1997	-1.50	0.18766	0.11162	0.95911	-1.51570	-1.46133	-0.05437	-2.2506	-2.12312	-0.12748
8	2007	-1.25	0.19863	0.12236	0.94187	-1.48970	-1.42230	-0.06740	-2.3535	-2.07219	-0.28131
9	1932	-1.00	0.15117	0.11552	0.88174	-1.43690	-1.33054	-0.10636	-2.0930	-1.93895	-0.15405
10	1962	-0.75	0.17380	0.09335	0.85187	-1.39380	-1.30371	-0.09009	-2.1017	-1.89150	-0.21020
11	1979	-0.50	0.16573	0.03299	0.80793	-1.38370	-1.29202	-0.09168	-2.0416	-1.84949	-0.19211
12	1999	-0.25	0.15117	0.02750	0.75795	-1.25010	-1.21554	-0.03456	-1.9442	-1.73853	-0.20567
13	3259	0.00	0.70967	-0.06511	0.59565	-1.01060	-1.04199	0.03139	-1.5849	-1.45299	-0.13191
14	1924	0.25	0.17846	-0.02483	0.69271	-1.24440	-1.16087	-0.08353	-1.8316	-1.63884	-0.19276
15	1920	0.50	0.22176	-0.04937	0.63285	-1.14555	-1.08724	-0.05831	-1.6194	-1.52390	-0.09550
16	1839	0.75	0.24671	-0.07507	0.62972	-1.20200	-1.10781	-0.09419	-1.6012	-1.54231	-0.05889
17	1787	1.00	0.29456	-0.14055	0.60630	-1.21690	-1.13489	-0.08201	-1.6662	-1.55323	-0.11297
18	1742	1.25	0.34987	-0.22862	0.60234	-1.26660	-1.21645	-0.05015	-1.7081	-1.63207	-0.07603
19	1573	1.50	0.42456	-0.29333	0.55893	-1.30790	-1.20998	-0.09792	-1.5661	-1.59564	0.02954
20	1476	1.75	0.51622	-0.41543	0.54598	-1.27590	-1.31083	0.03493	-1.5561	-1.68755	0.13145
21	1185	2.00	0.61737	-0.40268	0.54312	-1.19350	-1.29339	0.09989	-1.4742	-1.66815	0.19395
22	1370	2.25	0.79100	-0.63113	0.29444	-1.15620	-1.11401	-0.04219	-1.3412	-1.31717	-0.02403
23	765	2.50	0.93287	-0.80270	0.19444	-1.19250	-1.12158	-0.07092	-1.4212	-1.25575	-0.16545
24	7162	3.00	4.95222	0.05690	0.25711	-0.53430	-0.36476	-0.16954	-0.5343	-0.54217	0.00787

Prediction of the first and fifth percentiles

Tables 3 and 4 let us remark that the prediction of the first and fifth percentiles critical values by a multiple linear regression on the estimated proficiency level, his square, and variance, is fairly good. Except for the prediction of the fifth percentile by the EXP estimation method, the coefficient of determination is always equal or superior to 0.90.

Table 8. Multiple regression for the prediction of the fifth percentile

METHOD	R^2	EQUATION
WLE	0.99	$-1.224 + 0.372\hat{\theta} + 0.070\hat{\theta}^2 + 1 .013\, s^2_{\hat{\theta}}$
MLE	0.98	$-1.265 + 0.209\hat{\theta} + 0.031\hat{\theta}^2 + 0 .085\, s^2_{\hat{\theta}}$
MAP	0.99	$-1.519 + 0.201\hat{\theta} - 0.018\hat{\theta}^2 + 7.467\, s^2_{\hat{\theta}}$
EXP	0.84	$-1.095 + 0.573\hat{\theta} + 0.244\hat{\theta}^2 - 6.788\, s^2_{\hat{\theta}}$
EAP	0.91	$-1.367 + 0.506\hat{\theta} + 0.225\hat{\theta}^2 - \underline{0.333}\, s^2_{\hat{\theta}}$
AEAP	0.92	$-1.277 + 0.093\hat{\theta} - 0.031\hat{\theta}^2 + 0.031\, s^2_{\hat{\theta}}$

Table 9. Multiple regression for the prediction of the first percentile

METHOD	R^2	EQUATION
WLE	0.99	$-1.817 + 0.544\hat{\theta} + 0.110\hat{\theta}^2 + 1.093\, s^2_{\hat{\theta}}$
MLE	0.98	$-1.859 + 0.345\hat{\theta} + 0.057\hat{\theta}^2 + 0.078\, s^2_{\hat{\theta}}$
MAP	0.99	$-2.154 + 0.324\hat{\theta} - 0.019\hat{\theta}^2 + 8.992\, s^2_{\hat{\theta}}$
EXP	0.90	$-1.655 + 0.733\hat{\theta} + 0.284\hat{\theta}^2 - 7.456\, s^2_{\hat{\theta}}$
EAP	0.94	$-1.947 + 0.706\hat{\theta} + 0.283\hat{\theta}^2 - \underline{0.857}\, s^2_{\hat{\theta}}$
AEAP	0.95	$-1.879 + 0.219\hat{\theta} - 0.007\hat{\theta}^2 + 0.031\, s^2_{\hat{\theta}}$

A closer examination at the estimated prediction functions by the residuals inspection (figures 4 and 5) let us see that the prediction is generally acceptable in the middle range of the estimated proficiency level. EXP and EAP give extreme bad values at high estimated proficiency levels. Tables 10 to 15 present with details these residuals.

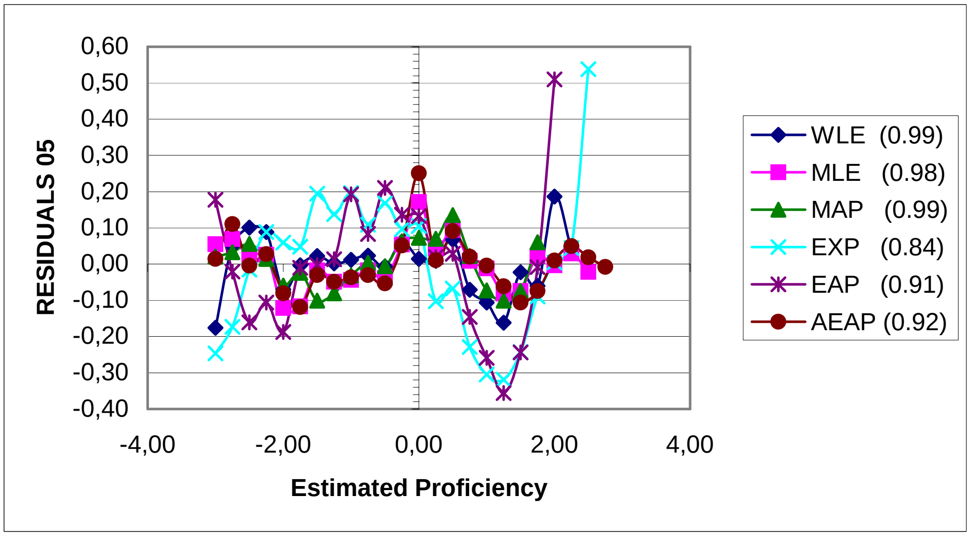

Figure 4. Residuals and coefficients of determination for the fifth percentile

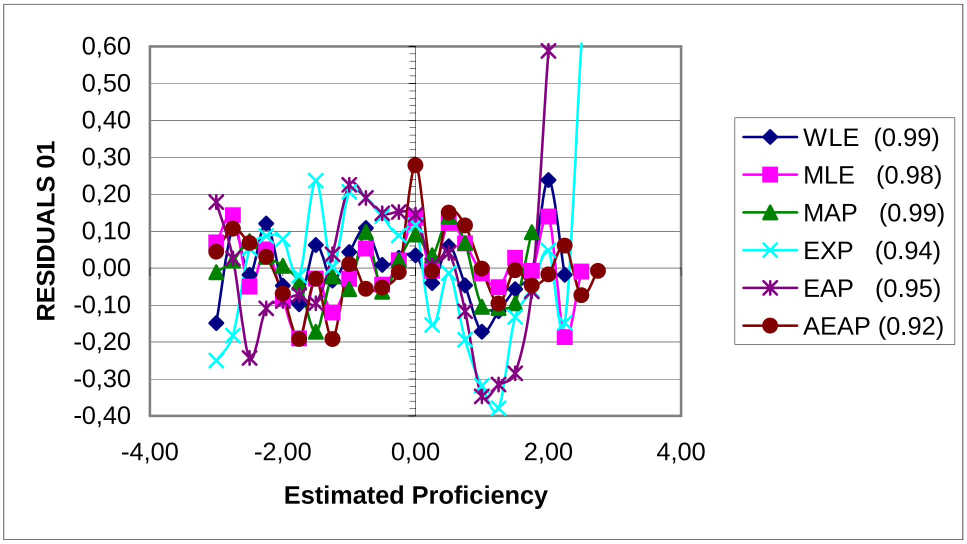

Figure 5. Residuals and coefficients of determination for the first percentile

Table 10. WLE percentile residuals

	$\boldsymbol{P}05$				$\boldsymbol{P}01$		
Obs	$\boldsymbol{P}05\boldsymbol{P}_{Lz}$	$P05E_{Lz}$	Residual	Obs	$\boldsymbol{P}01\boldsymbol{P}_{Lz}$	$\boldsymbol{P}01\boldsymbol{E}_{Lz}$	Residual
1	-1.3605	-1.1845	-0.17600	1	-2.0459	-1.8974	-0.14850
2	-1.4948	-1.5456	0.05080	2	-2.1938	-2.3001	0.10630
3	-1.4968	-1.5978	0.10100	3	-2.3832	-2.3653	-0.01790
4	-1.5329	-1.6218	0.08890	4	-2.2754	-2.3960	0.12060
5	-1.6941	-1.6240	-0.07010	5	-2.4460	-2.3988	-0.04720
6	-1.6134	-1.6100	-0.00341	6	-2.4777	-2.3799	-0.09780
7	-1.5607	-1.5829	0.02220	7	-2.2804	-2.3426	0.06220
8	-1.5424	-1.5447	0.00233	8	-2.3232	-2.2891	-0.03410
9	-1.4841	-1.4960	0.01190	9	-2.1763	-2.2200	0.04370
10	-1.4130	-1.4365	0.02350	10	-2.0268	-2.1349	0.10810
11	-1.3727	-1.3660	-0.00667	11	-2.0250	-2.0338	0.00881
12	-1.2231	-1.2849	0.06180	12	-1.8892	-1.9170	0.02780
13	-1.1783	-1.1930	0.01470	13	-1.7497	-1.7841	0.03440
14	-1.0800	-1.0889	0.00887	14	-1.6744	-1.6339	-0.04050
15	-0.9038	-0.9709	0.06710	15	-1.4057	-1.4644	0.05870
16	-0.9085	-0.8374	-0.07110	16	-1.3209	-1.2740	-0.04690
17	-0.7915	-0.6851	-0.10640	17	-1.2314	-1.0591	-0.17230
18	-0.6679	-0.5056	-0.16230	18	-0.9272	-0.8104	-0.11680
19	-0.3090	-0.2860	-0.02300	19	-0.5713	-0.5144	-0.05690
20	-0.0817	-0.0209	-0.06080	20	-0.1765	-0.1649	-0.01160
21	0.4852	0.2984	0.18680	21	0.4852	0.2472	0.23800
22	0.8160	0.7760	0.04000	22	0.8160	0.8343	-0.01830

Table 11. MLE percentile residuals

	$\boldsymbol{P}05$				$\boldsymbol{P}01$		
Obs	$\boldsymbol{P}05\boldsymbol{P}_{Lz}$	$P05E_{Lz}$	Residual	Obs	$\boldsymbol{P}01\boldsymbol{P}_{Lz}$	$\boldsymbol{P}01\boldsymbol{E}_{Lz}$	Residual
1	-1.4879	-1.5432	0.05530	1	-2.2516	-2.3208	0.06920
2	-1.5204	-1.5895	0.06910	2	-2.2230	-2.3659	0.14290
3	-1.5607	-1.5825	0.02180	3	-2.4082	-2.3582	-0.05000
4	-1.5445	-1.5701	0.02560	4	-2.2914	-2.3419	0.05050
5	-1.6738	-1.5526	-0.12120	5	-2.4016	-2.3175	-0.08410
6	-1.6474	-1.5307	-0.11670	6	-2.4758	-2.2854	-0.19040
7	-1.5218	-1.5045	-0.01730	7	-2.2744	-2.2459	-0.02850
8	-1.5234	-1.4742	-0.04920	8	-2.3198	-2.1991	-0.12070
9	-1.4832	-1.4399	-0.04330	9	-2.1749	-2.1451	-0.02980
10	-1.4184	-1.4015	-0.01690	10	-2.0313	-2.0839	0.05260
11	-1.3975	-1.3591	-0.03840	11	-2.0603	-2.0154	-0.04490
12	-1.2586	-1.3126	0.05400	12	-1.9186	-1.9397	0.02110
13	-1.0203	-1.1920	0.17170	13	-1.6570	-1.7919	0.13490
14	-1.1634	-1.2074	0.04400	14	-1.7745	-1.7664	-0.00810
15	-1.0548	-1.1484	0.09360	15	-1.5464	-1.6686	0.12220
16	-1.0755	-1.0849	0.00939	16	-1.4965	-1.5632	0.06670
17	-1.0288	-1.0167	-0.01210	17	-1.4643	-1.4499	-0.01440
18	-1.0248	-0.9431	-0.08170	18	-1.3800	-1.3282	-0.05180
19	-0.9374	-0.8631	-0.07430	19	-1.1696	-1.1969	0.02730
20	-0.7537	-0.7758	0.02210	20	-1.0628	-1.0554	-0.00736
21	-0.6778	-0.6749	-0.00289	21	-0.7585	-0.8980	0.13950
22	-0.5307	-0.5601	0.02940	22	-0.9115	-0.7241	-0.18740
23	0.3949	0.4169	-0.02200	23	0.3949	0.4043	-0.00942

Table 12. MAP percentile residuals

		$\boldsymbol{P}05$				$\boldsymbol{P}01$	
Obs	$\boldsymbol{P}05\boldsymbol{P}_{Lz}$	$P05E_{Lz}$	Residual	Obs	$\boldsymbol{P}01\boldsymbol{P}_{Lz}$	$\boldsymbol{P}01\boldsymbol{E}_{Lz}$	Residual
1	-0.7469	-0.7672	0.02030	1	-1.4835	-1.4719	-0.01160
2	-1.1330	-1.1660	0.03300	2	-1.9155	-1.9351	0.01960
3	-1.3245	-1.3807	0.05620	3	-2.1035	-2.1763	0.07280
4	-1.4915	-1.5063	0.01480	4	-2.2794	-2.3100	0.03060
5	-1.6271	-1.5677	-0.05940	5	-2.3597	-2.3660	0.00634
6	-1.6110	-1.5865	-0.02450	6	-2.4101	-2.3704	-0.03970
7	-1.6807	-1.5793	-0.10140	7	-2.5158	-2.3432	-0.17260
8	-1.6404	-1.5600	-0.08040	8	-2.3238	-2.3012	-0.02260
9	-1.5624	-1.5313	-0.03110	9	-2.3050	-2.2475	-0.05750
10	-1.4877	-1.4914	0.00373	10	-2.0822	-2.1801	0.09790
11	-1.4447	-1.4387	-0.00604	11	-2.1614	-2.0969	-0.06450
12	-1.3123	-1.3756	0.06330	12	-1.9804	-2.0009	0.02050
13	-1.2277	-1.3005	0.07280	13	-1.7992	-1.8901	0.09090
14	-1.1345	-1.2046	0.07010	14	-1.7197	-1.7541	0.03440
15	-0.9422	-1.0774	0.13520	15	-1.4406	-1.5800	0.13940
16	-0.8886	-0.9112	0.02260	16	-1.2909	-1.3587	0.06780
17	-0.7620	-0.6893	-0.07270	17	-1.1750	-1.0699	-0.10510
18	-0.4537	-0.3525	-0.10120	18	-0.7509	-0.6426	-0.10830
19	0.0354	0.1104	-0.07500	19	-0.1584	-0.0630	-0.09540
20	0.7292	0.6692	0.06000	20	0.7292	0.6323	0.09690

Table 13. EXP percentile residuals

		$\boldsymbol{P}05$				$\boldsymbol{P}01$	
Obs	$\boldsymbol{P}05\boldsymbol{P}_{Lz}$	$P05E_{Lz}$	Residual	Obs	$\boldsymbol{P}01\boldsymbol{P}_{Lz}$	$\boldsymbol{P}01\boldsymbol{E}_{Lz}$	Residual
1	-1.0635	-0.8172	-0.24630	1	-1.7590	-1.5086	-0.25040
2	-1.4512	-1.2782	-0.17300	2	-2.1956	-2.0133	-0.18230
3	-1.6456	-1.6305	-0.01510	3	-2.3435	-2.3965	0.05300
4	-1.6450	-1.7334	0.08840	4	-2.4166	-2.5036	0.08700
5	-1.6621	-1.7214	0.05930	5	-2.4043	-2.4824	0.07810
6	-1.6550	-1.7028	0.04780	6	-2.4729	-2.4518	-0.02110
7	-1.4733	-1.6674	0.19410	7	-2.1647	-2.4007	0.23600
8	-1.5619	-1.7001	0.13820	8	-2.4195	-2.4223	0.00280
9	-1.3984	-1.5948	0.19640	9	-2.0839	-2.2902	0.20630
10	-1.4949	-1.6035	0.10860	10	-2.0907	-2.2811	0.19040
11	-1.3241	-1.4918	0.16770	11	-1.9975	-2.1378	0.14030
12	-1.2931	-1.3895	0.09640	12	-1.9143	-2.0026	0.08830
13	-1.2464	-1.3520	0.10560	13	-1.8203	-1.9365	0.11620
14	-1.2711	-1.1679	-0.10320	14	-1.8614	-1.7074	-0.15400
15	-1.1648	-1.0973	-0.06750	15	-1.6143	-1.6007	-0.01360
16	-1.2121	-0.9834	-0.22870	16	-1.6384	-1.4444	-0.19400
17	-1.2474	-0.9430	-0.30440	17	-1.6855	-1.3666	-0.31890
18	-1.2976	-0.9784	-0.31920	18	-1.7489	-1.3701	-0.37880
19	-1.3675	-1.1248	-0.24270	19	-1.6260	-1.4933	-0.13270
20	-1.2919	-1.2022	-0.08970	20	-1.5986	-1.5388	-0.05980
21	-1.0605	-1.0654	0.00494	21	-1.2987	-1.3467	0.04800
22	-0.5396	-0.5837	0.04410	22	-0.9220	-0.7738	-0.14820
23	0.7132	0.1752	0.53800	23	0.7132	0.1058	0.60740

Table 14. EAP percentile residuals

$\boldsymbol{P}05$				$\boldsymbol{P}01$			
Obs	$\boldsymbol{P}05\boldsymbol{P}_{Lz}$	$P05E_{Lz}$	Residual	Obs	$\boldsymbol{P}01\boldsymbol{P}_{Lz}$	$\boldsymbol{P}01\boldsymbol{E}_{Lz}$	Residual
1	-0.6935	-0.8711	0.17760	1	-1.3684	-1.5469	0.17850
2	-1.0998	-1.0789	-0.02090	2	-1.7791	-1.8050	0.02590
3	-1.4145	-1.2531	-0.16140	3	-2.2568	-2.0136	-0.24320
4	-1.4974	-1.3909	-0.10650	4	-2.2744	-2.1657	-0.10870
5	-1.6867	-1.4991	-0.18760	5	-2.3676	-2.2785	-0.08910
6	-1.5899	-1.5793	-0.01060	6	-2.4285	-2.3562	-0.07230
7	-1.6348	-1.6322	-0.00262	7	-2.4962	-2.4009	-0.09530
8	-1.6478	-1.6612	0.01340	8	-2.3835	-2.4209	0.03740
9	-1.4642	-1.6565	0.19230	9	-2.1665	-2.3913	0.22480
10	-1.5479	-1.6309	0.08300	10	-2.1558	-2.3451	0.18930
11	-1.3624	-1.5722	0.20980	11	-2.1029	-2.2506	0.14770
12	-1.3514	-1.4878	0.13640	12	-1.9748	-2.1267	0.15190
13	-1.2476	-1.3798	0.13220	13	-1.8350	-1.9795	0.14450
14	-1.2183	-1.2377	0.01940	14	-1.7746	-1.7812	0.00660
15	-1.0457	-1.0745	0.02880	15	-1.5253	-1.5658	0.04050
16	-1.0280	-0.8822	-0.14580	16	-1.4291	-1.3126	-0.11650
17	-0.9250	-0.6661	-0.25890	17	-1.3819	-1.0349	-0.34700
18	-0.7810	-0.4253	-0.35570	18	-1.0463	-0.7310	-0.31530
19	-0.4040	-0.1602	-0.24380	19	-0.6860	-0.4014	-0.28460
20	0.1258	0.1346	-0.00876	20	-0.0958	-0.0324	-0.06340
21	0.9717	0.4620	0.50970	21	0.9717	0.3836	0.58810

Table 15. AEAP percentile residuals

	$\boldsymbol{P}05$				$\boldsymbol{P}01$		
Obs	$\boldsymbol{P}05\boldsymbol{P}_{Lz}$	$P05E_{Lz}$	Residual	Obs	$\boldsymbol{P}01\boldsymbol{P}_{Lz}$	$\boldsymbol{P}01\boldsymbol{E}_{Lz}$	Residual
1	-1.7506	-1.7654	0.01480	1	-2.4825	-2.5263	0.04380
2	-1.6500	-1.7607	0.11070	2	-2.4194	-2.5255	0.10610
3	-1.7034	-1.6989	-0.00447	3	-2.3968	-2.4640	0.06720
4	-1.6123	-1.6401	0.02780	4	-2.3717	-2.4023	0.03060
5	-1.6657	-1.5848	-0.08090	5	-2.4107	-2.3413	-0.06940
6	-1.6512	-1.5329	-0.11830	6	-2.4724	-2.2805	-0.19190
7	-1.5157	-1.4851	-0.03060	7	-2.2506	-2.2208	-0.02980
8	-1.4897	-1.4404	-0.04930	8	-2.3535	-2.1613	-0.19220
9	-1.4369	-1.4003	-0.03660	9	-2.0930	-2.1033	0.01030
10	-1.3938	-1.3633	-0.03050	10	-2.1017	-2.0455	-0.05620
11	-1.3837	-1.3305	-0.05320	11	-2.0416	-1.9887	-0.05290
12	-1.2501	-1.3016	0.05150	12	-1.9442	-1.9329	-0.01130
13	-1.0106	-1.2617	0.25110	13	-1.5849	-1.8630	0.27810
14	-1.2444	-1.2549	0.01050	14	-1.8316	-1.8233	-0.00826
15	-1.1456	-1.2369	0.09140	15	-1.6194	-1.7694	0.15000
16	-1.2020	-1.2230	0.02100	16	-1.6012	-1.7165	0.11530
17	-1.2169	-1.2125	-0.00438	17	-1.6662	-1.6641	-0.00213
18	-1.2666	-1.2056	-0.06100	18	-1.7081	-1.6121	-0.09600
19	-1.3079	-1.2019	-0.10600	19	-1.5661	-1.5604	-0.00570
20	-1.2759	-1.2012	-0.07470	20	-1.5561	-1.5086	-0.04750
21	-1.1935	-1.2034	0.00993	21	-1.4742	-1.4568	-0.01740
22	-1.1562	-1.2056	0.04940	22	-1.3412	-1.4019	0.06070
23	-1.1925	-1.2116	0.01910	23	-1.4212	-1.3478	-0.07340
24	-0.5343	-0.5271	-0.00724	24	-0.5343	-0.5261	-0.00817

Hit rate

Finally, the predicted first and fifth percentiles critical values were used to verify the hit rate at the 0.01 and 0.05 error level with each estimation method. According to figures 6 and 7, like table 16, the hit rate, varies with the estimation method. The hit rate varies also according to an interaction between the estimation method and the estimated proficiency level.

For the general case, AEAP seems an interesting estimation method, covering the larger part of the estimated proficiency level. At some values of the estimated proficiency level the AEAP method would have to be replaced with a more efficient estimation method. For example, if the estimated proficiency level is equal to 0.00, would it be to predict the hit rate at the 0.01 or 0.05 level, all the other methods, except MLE, would be more attractive.

In general, the worst estimation methods are the EXP and EAP estimation methods.

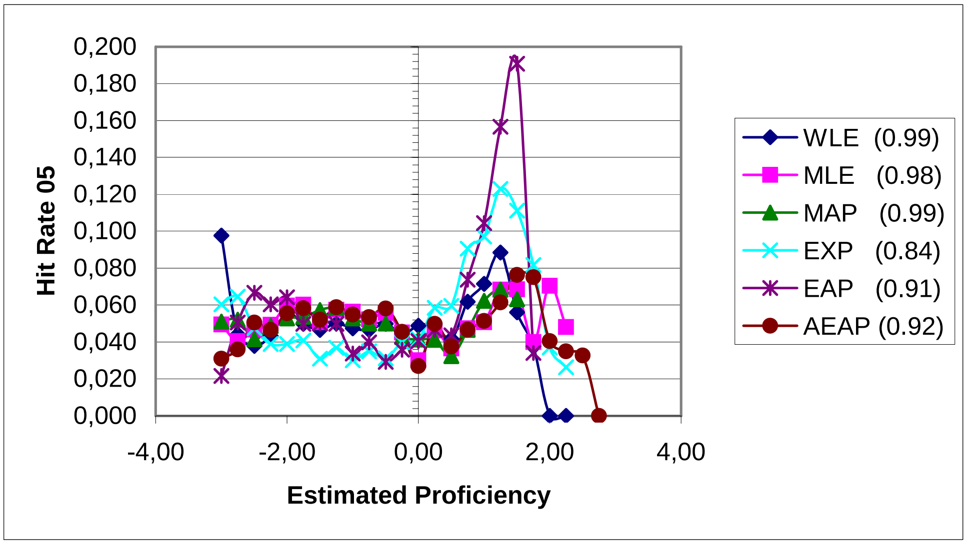

Figure 6. Hit Rate at the 0.05 Predicted Error Level

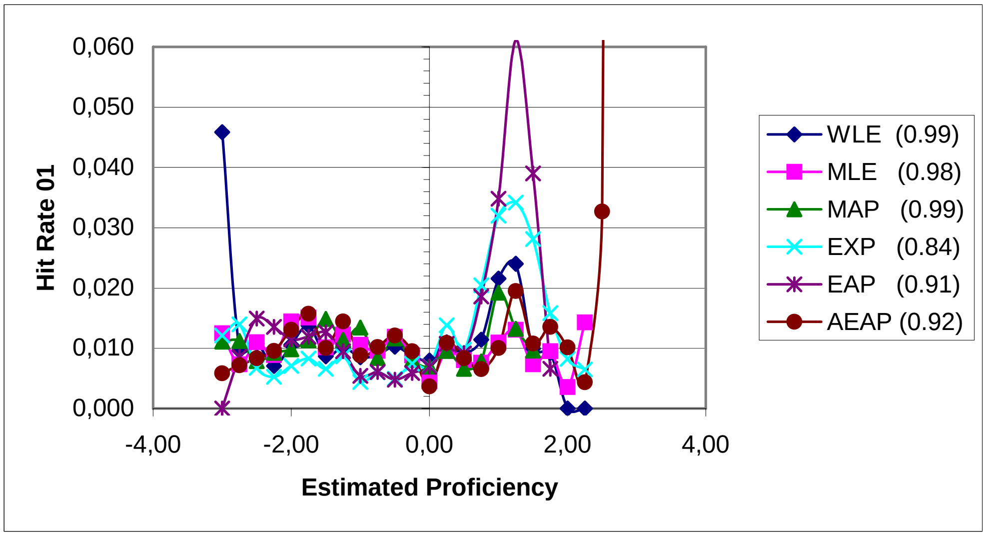

Figure 6. Hit Rate at the 0.01 Predicted Error Level

Table 16. Hit Rate

Obs	$\hat{\theta}$	WLE		MLE		MAP		EXP		EAP		AEAP	
1	-3.00	0.098	0.046	0.050	0.012	0.051	0.011	0.060	0.012	0.022	0.000	0.031	0.006
2	-2.75	0.044	0.010	0.041	0.007	0.052	0.011	0.065	0.014	0.051	0.009	0.036	0.007
3	-2.50	0.038	0.011	0.047	0.011	0.042	0.008	0.047	0.007	0.067	0.015	0.051	0.008
4	-2.25	0.043	0.007	0.049	0.009	0.049	0.009	0.039	0.005	0.060	0.014	0.047	0.010
5	-2.00	0.056	0.011	0.060	0.014	0.053	0.010	0.039	0.007	0.064	0.011	0.056	0.013
6	-1.75	0.050	0.014	0.060	0.015	0.054	0.011	0.041	0.008	0.050	0.012	0.058	0.016
7	-1.50	0.047	0.009	0.051	0.010	0.057	0.015	0.031	0.007	0.050	0.013	0.052	0.010
8	-1.25	0.050	0.010	0.057	0.012	0.059	0.011	0.037	0.009	0.050	0.010	0.059	0.014
9	-1.00	0.048	0.009	0.057	0.011	0.053	0.013	0.030	0.004	0.034	0.005	0.055	0.009
10	-0.75	0.047	0.009	0.052	0.010	0.050	0.008	0.035	0.006	0.040	0.006	0.054	0.010
11	-0.50	0.050	0.010	0.056	0.012	0.050	0.012	0.031	0.005	0.029	0.005	0.058	0.012
12	-0.25	0.044	0.009	0.045	0.009	0.043	0.008	0.040	0.007	0.036	0.006	0.046	0.010
13	0.00	0.049	0.008	0.030	0.005	0.041	0.007	0.042	0.007	0.040	0.007	0.027	0.004
14	0.25	0.049	0.011	0.046	0.010	0.041	0.010	0.059	0.014	0.047	0.011	0.050	0.011
15	0.50	0.042	0.009	0.036	0.008	0.032	0.007	0.060	0.010	0.044	0.009	0.038	0.008
16	0.75	0.062	0.011	0.048	0.008	0.047	0.008	0.091	0.020	0.074	0.019	0.047	0.007
17	1.00	0.072	0.022	0.051	0.011	0.062	0.019	0.097	0.032	0.104	0.035	0.051	0.010
18	1.25	0.089	0.024	0.068	0.013	0.068	0.013	0.123	0.034	0.157	0.061	0.061	0.020
19	1.50	0.056	0.010	0.069	0.007	0.063	0.010	0.111	0.028	0.191	0.039	0.076	0.011
20	1.75	0.037	0.009	0.040	0.010			0.082	0.016	0.034	0.007	0.075	0.014
21	2.00			0.070	0.004			0.037	0.008			0.041	0.010
22	2.25			0.048	0.014			0.026	0.007			0.035	0.004
23	2.50											0.033	0.033

6. Conclusion

Results of 2000 simulations of 85 fixed items tests at each of 23 simulated proficiency levels showed that the first percentile of the $\boldsymbol{L_z}$ distribution generally can't be well predicted by a $\mathrm{N}(\bar{\boldsymbol{Lz}} \mid \hat{\theta}, \boldsymbol{S}^2_{Lz} \mid \hat{\theta})$ distribution according to the related 23 estimated proficiency levels. The prediction of the fifth percentile, on the other side, is acceptable. The proficiency level was estimated with MLE, MAP, WLE, EAP and AEAP methods.

More promising, the first and fifth percentiles of the *Lz* distribution were well predicted by a multiple linear regression on the estimated proficiency level and his standard error ($\boldsymbol{r} > 0.90$), particularly with the MLE, WLE and AEAP methods. Only the EXP estimation method shows low determination coefficients, mostly with the fifth percentile ($R^2 = 0.84$).

All methods, except EXP and EAP, present acceptable predicted hit rate, but the prediction varies with the estimated proficiency level. For this reason, it would be advisable to use different estimation methods according to the estimated proficiency level.

This study considered only one specific fixed item test. The results are not very exportable for the moment. We have to apply this analysis for each specific fixed item test. To elaborate a more general strategy, later, percentiles prediction by multiple regression on any test conditional on the number of items and on the distribution of the item parameters would have to be analysed.

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