

# **LANGUAGE-BASED PRIOR KNOWLEDGE AND TRANSITION TO MATHEMATICS**

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**ABSTRACT:** The paper provides a college mathematics student's concept maps, definitions, and essays to support the thesis that language-based prior knowledge can influence students' cognitive processes of mathematical concepts. A group of intermediate algebra students who displayed terms mainly from the spoken language on the first and the second concept maps and essays included terms reflecting more of an equation/formula-based conceptualization of the function concept on their third and the last concept maps and essays.

## **INTRODUCTION**

Common language meanings students bring into the mathematics classroom may influence or hinder the learning process. We use the term "common language" for the knowledge and meaning of concepts that are used outside the context of mathematics (used in spoken language). As Tobias (1993) states, students struggle to make sense of new information and their common language knowledge of many mathematics concepts. For instance, "multiply", meaning increase in common language, may dominate students' cognitive processes so much so that they may struggle or not understand why multiplying fractions results in a smaller value/quantity. A student struggling to make sense of the mathematical meaning of a concept in the presence of his or her prior knowledge may fail to construct meaningful links between the two meanings, and eventually, out of frustration, turn to rote learning. That is, he or she may choose to focus on fragmented

information leading to the memorization of isolated facts with few links between (Kaput, 1989). However, students may be able to acquire new information meaningfully provided that they make necessary connections between their prior knowledge, whether it is mathematical or common language, and the new information. Therefore, it is crucial that mathematics teachers be aware of students' prior knowledge, and guide students in making meaningful links between the concepts possessed and the new information (Bishop, 1985; Maher, 1996; National Research Council, 2000).

There are studies reporting difficulties with the learning and understanding of the mathematical meaning of function. The abstract nature, point-wise preference, fragmented knowledge structures, and the compartmentalization of new information are some reported in literature as the source of students' difficulties (Bills, 2001; Carlson, 1998; Trigueros and Ursini, 2003; Williams, 1998). The latter two can be associated with the knowledge structures of weaker students and those struggling to make sense of prior knowledge in the context of new information. Ferrini-Mundy and Graham (1994) reported that many students' perception of function is that of an algebraic formula or an equation. They stated that, for many calculus students, the use of the word "function" triggers a search for an algebraic formula that defines the function, and a strong urge to substitute numbers into the formula.

## **METHOD**

Concept maps are considered by many educators to be explicit, overt representations of the concepts and propositions students hold (Kinchin and Hay, 2000; Novak and Gowin, 1984; Williams, 1998). They are graphical representations illustrating connections between concepts and related topics (Novak and Gowin, 1984). In a concept map,

concepts are represented as nodes, and the perceived relationships between concepts are indicated by linking lines (see Figures 2, 3, 4 and 5). One may also require linking-words along the linking lines, and accompanying interpretive essays. Concept maps are widely used as instructional, assessment, and research tools to support, enhance, and document the connectedness of students' knowledge of various science and mathematics concepts (Bolte, 1998, 1999; McGowen and Tall, 1999; Williams, 1998).

In our study, we used concept maps, definitions, and essays to investigate the college mathematics students' knowledge structures of function. Considering that internal representations of knowledge can be organized/connected in a non-linear manner (web-like, hierarchical or combination of both etc...), each student was given the freedom to organize and construct his/her concept maps in a way that makes the most sense to him/her. Data was gathered during the fall 2003 semester from two intermediate algebra sections at a four-year university. These courses were selected based on a few criteria: The course content (whether the function concept is included in the curriculum) and the instructor's willingness to work with the researchers. The difficulty level of the course and the characteristics of the student population (age, race, major etc...) did not play a role in the selection process. The majority (approximately 75%) of students turned out to be Hispanic, majoring in social sciences including education. They all spoke both English and Spanish at varying proficiency levels.

During the first in-class meeting, students were introduced to concept mapping and provided samples of concept maps and accompanying interpretive essays on unrelated topics such as rainbow. Throughout the semester, a total of four concept maps were assigned as take-home activities. Students were given the time from one class

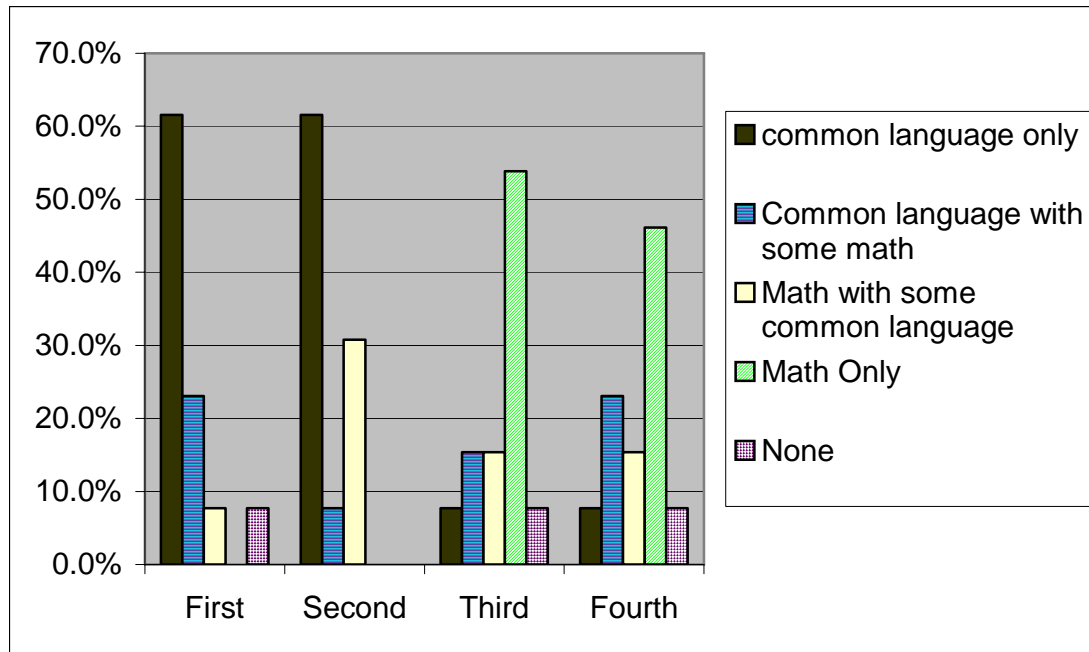
meeting to the next (usually two days) to complete the assignment. The first map, used to document students' prior knowledge of function, was intentionally given during the first week of the semester before students' exposure to any lessons on function. Students were also asked to define function and provide essays stating the reasoning they used for concepts and links displayed on their maps.

## **RESULTS AND DISCUSSION**

There were 13 students out of 35 enrolled in the particular intermediate algebra sections who displayed at least one common language term on their concept maps (see Figures 2, 3, 4 and 5). For instance, car and computer are considered as terms that are used in common language but not in the context of mathematics.

Figure 1 provides the percentage break down of the terms displayed on these thirteen students' concept maps under the five categories that were formed later on during the data analysis phase. About sixty two percent displayed only common language terms on their first and second concept maps. There was one student who continued displaying only common language terms on the third and the last concept maps. That is, throughout the semester he did not seem to be able to make a transition from his dominant prior knowledge (common language knowledge) to mathematical meaning of function. The eleven of the thirteen students' common language terms and links accounted for more than 50% of the terms displayed on their first concept maps. Throughout the semester, the number of students who continued to display terms from common language decreased from 100% on the second to 46.2% on the fourth map. One may wonder whether the students who replaced common language terms with increasingly more mathematical

concepts on their maps, definitions and essays were able to construct meaningful knowledge of function in mathematical context.



**Figure 1.** Common language and mathematical terms displayed on students’ concept maps (N=13). \*

\* “Common language only” refers to the concept maps with only common language terms, no mathematical terms.

\* “Common language with some math” refers to concept maps with 50% or more terms and/or links from common language, and a few mathematical terms.

\* “Math with some common language” refers to concept maps with 50% or more terms and/or links from mathematics and a few common language terms.

\* “Math only” refers to the concept maps with only mathematical concepts, no common language terms.

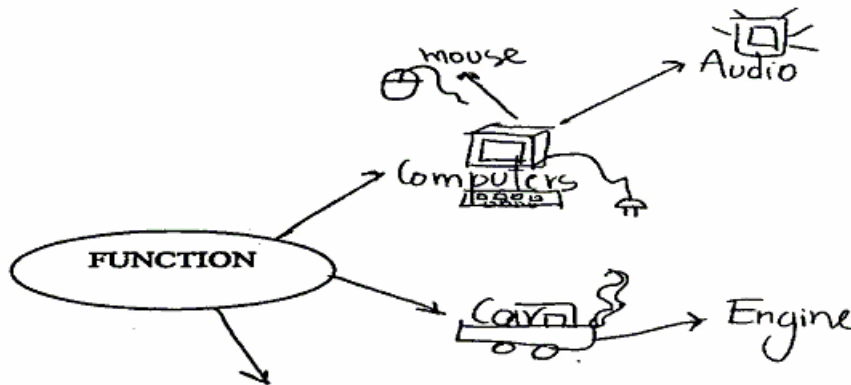
\* “None” refers to students who did not turn in their maps.

A closer look at the mathematical terms displayed, however, indicates that the elimination of common language terms (or the consideration of mathematical terms in isolation from the common language knowledge) did not seem to have helped these students make meaningful transitions to the mathematical meaning of function. The terms they have displayed were mainly fragmented/disconnected facts such as formulas, equations, and examples of specific functions that were introduced around the time the

maps were constructed (Ferrini-Mundy and Graham, 1994). That is, the majority of the students did not include terms and links that would imply a conceptual understanding of basic definitions and the characteristics of function, for instance, the characteristics determining whether a relationship is a function; such as, values of the independent variable paired with unique values of the dependent variable.

### ***STUDENT G***

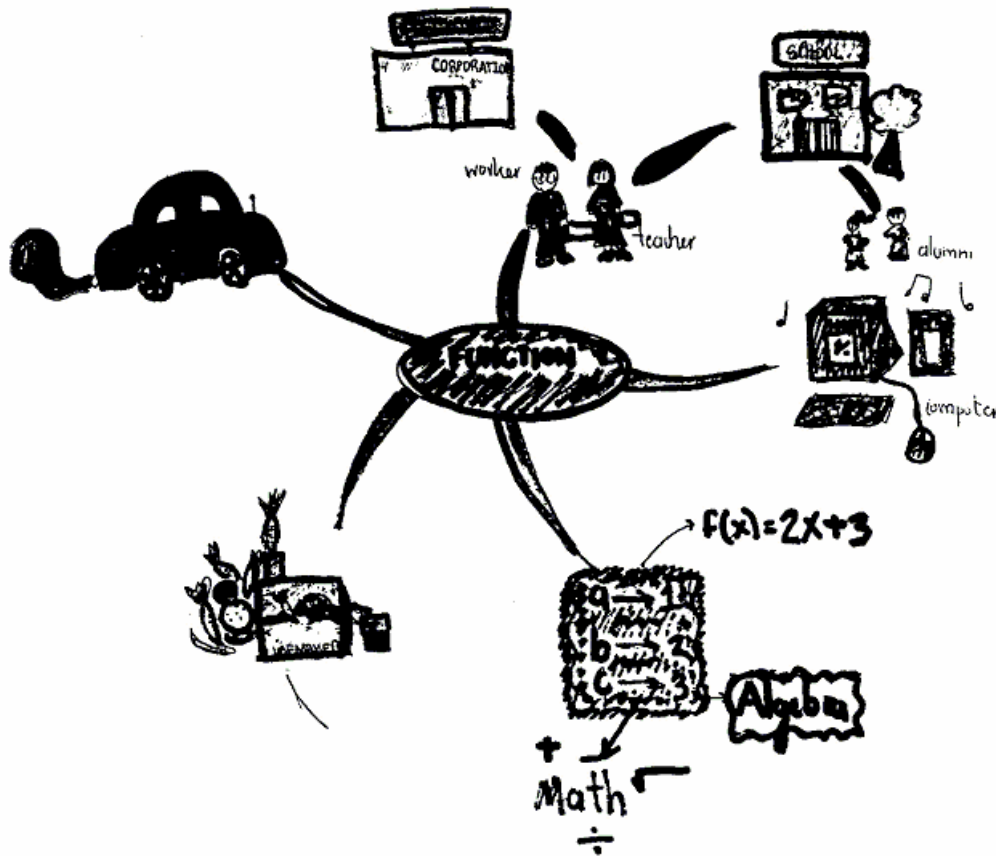
Student G's work represents the nature of the work of those students who appeared to have been unsuccessful in making meaningful transitions from common language-based prior knowledge to mathematical understanding of function. She is a Hispanic, 19-year-old female student majoring in education. Her initial concept map of function elicited only the common language words "computers" and "car." The map's lack of mathematical terms suggests that the student may not have had any understanding of function in the mathematical sense.



**Figure 2 .** Student G's first concept map accounted for her prior knowledge. Assigned on August 29, 2003.

After an initial lesson on function where related definitions, examples, equations, and graphs were explicitly introduced, student G developed another concept map (see Figure 3), this time adding a block with a few mathematical terms, but further elaborating

on common language terms. Words like “algebra”, “math”, an algebraic expression, “ $f(x)=2x+3$ ”, and a finite list of variables connected to numbers with arrows (point-wise perception (Ferrini-Mundy and Graham, 1994; Williams, 1998) appearing on her second map seem to indicate that the student may have picked up a few ideas from the lesson.



**Figure 3.** Student G’s second concept map developed after the first introduction of function. Assigned on September 5, 2003.

She states on her essay:

*“When I started to make this map, first, I think about the function of mathematics. It was very hard to me to think about other thing that relates to function. But then I remember about the function of a computer*

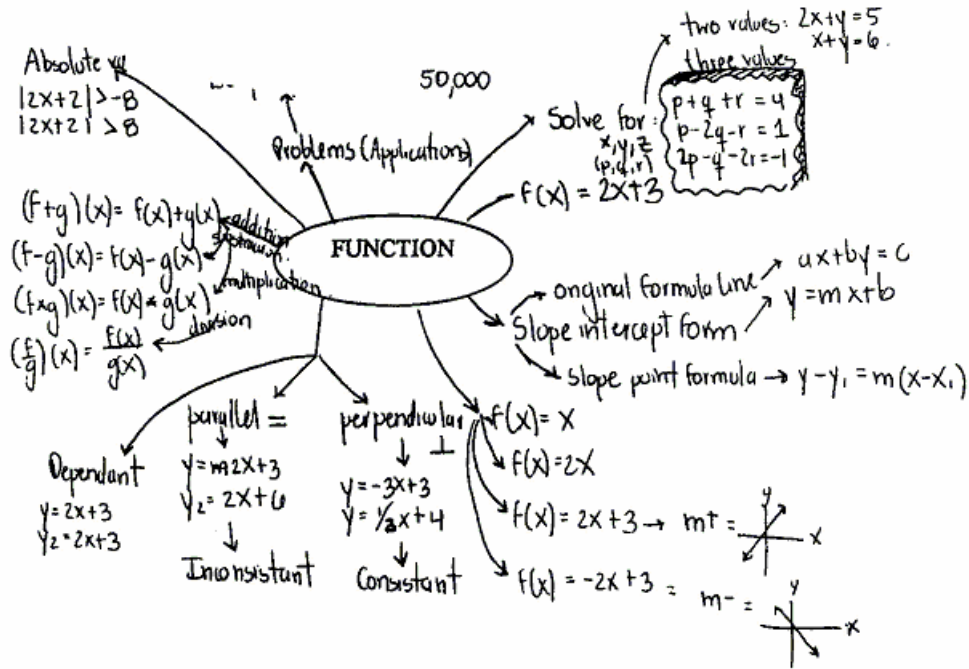
*hardware and the car engine. Then I put a worker and a teacher that work in a corporation and in a school respectively.”*

Student G’s definition of function, “*that works and operates in function of something,*” and her essay indicate that the student’s strongest understanding of function was its definition as the performance of machinery and the role of individuals in society. Note that key words from her definition and essay also appeared on the map. The dominating effect of the student’s prior knowledge seemed to have resulted in low acquisition of mathematical concepts introduced during the initial lesson on function.

The map in Figure 4 created after lessons on various topics such as linear functions, slope and the algebra of functions included words like “addition”, “parallel”, “problems”, and “slope intercept form.” The student’s map reflected the algebraic formulas and equations for linear functions, inequalities, and slope as well as a few graphical examples of increasing and decreasing functions. One should note that these terms appeared in the lessons covered around the time the map was constructed. Notice also that the student’s concept map no longer contained point-wise correspondence between sets of variables and numbers (the most closely related one to the definition of function) nor common language terms that were reflected on her earlier maps. Her definition of function, “*It’s an equation [student’s spelling] which its graph passes the test of the vertical line,*” indicates that the student considered functions as equations/formulas (Carlson, 1998; Ferrini-Mundy and Graham, 1994; Williams, 1998). The student explains her inclusions by writing in her essay:



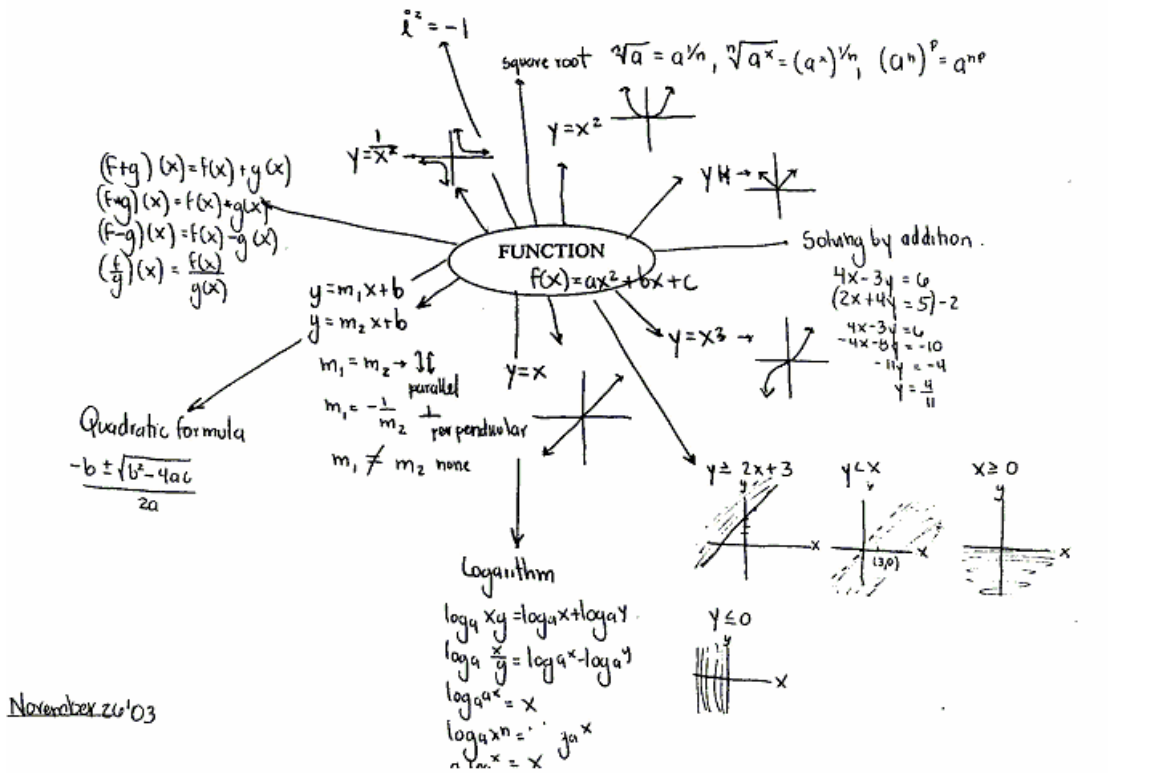
“What I thought when I saw the word function was all the operations that we used to solve a problem. For example, I put the addition, subtraction, multiplication and division of two functions.”



**Figure 4.** Student G’s third concept map developed after lessons on various topics such as slope and algebraic operations on functions. Assigned on October 13, 2003.

It is apparent that what was being discussed in class was now playing a major factor in what the student was writing in her essay and displaying in her concept map. Student G seemed to have begun constructing her understanding of mathematical function just as algebraic operations of functions were being introduced during the 4<sup>th</sup> and the 5<sup>th</sup> weeks of the semester. This appears to have limited her mathematical understanding of function to only those ideas being introduced at the time. This might

also be attributed to the dominating role that the common language knowledge had on her cognitive processes during the first month of the semester.



November 24, 2003

**Figure 5.** Student G’s fourth concept map developed towards the end of the semester. Assigned on November 24, 2003.

Student G appears to have completed the semester with a formula-based conceptualization of function as indicated on her last map (see Figure 5), constructed towards the end of the semester (Carlson, 1998; Ferrini-Mundy and Graham, 1994; Williams, 1998). On the map, she displayed the graphical representations of examples of functions such as “ $y=x^2$ ,” “ $y=x^3$ ,” “ $y=\frac{1}{x}$ ,” and inequalities. Furthermore, she elaborated on equations and formulas of specific functions. She seemed to have recognized a set of examples specific functions and their algebraic and geometrical representations, but

lacked the general and deeper understanding of the mathematical meaning of the function concept.

## **CONCLUSION**

The common language knowledge of function of the thirteen intermediate algebra students displayed on their concept maps, definitions and essays was mainly focused on:

- Things that work. For instance, the washing machine functions (works) properly.
- Role of a person, his/her profession. For instance, the teacher's function (role) is to teach.
- Events, gatherings or meetings. For instance, the family function (gathering) is tomorrow.

Prior knowledge of a subject that is conflicting with class lessons may keep students from making meaningful connections to the new concepts [Bishop, 1985; Maher, 1996; National Research Council, 2000; Tobias, 1993]. It appears from the percentages reported in Figure 1 and student G's concept maps, definitions, and essays that common language knowledge may lead to knowledge acquisition that is more likely a formula/equation based (Carlson, 1998; Ferrini-Mundy and Graham, 1994; Williams, 1998). The thirteen students seemed to have struggled to make sense of mathematical meaning of function in the context of common language knowledge. Many of them chose rote memorization of the formulas and equations introduced in class, which became the basis for their understanding of the concept (Carlson, 1998). This may also explain why the word "function" triggers a search for an algebraic formula, and a strong impulse to substitute certain numbers into the formula for some of the calculus students in Ferrini-

Mundy and Graham's study (1994). Mathematics teachers need to be aware that their students may bring in prior knowledge that conflicts with what is introduced in class. Students may fall behind struggling to make sense of prior knowledge, and as a result, consciously or unconsciously turn to rote memorization of the facts and formulas introduced at the time.

The findings suggest that concept maps along with interpretative essays are a viable addition to traditional assessment in mathematics classes. They can provide substantial insight into the type of prior knowledge and the degree of connectedness of the knowledge to the new information introduced in class (Williams, 1998). Hence, the instructors of mathematics can use concept maps and accompanying essays as alternative assessment tools to document students' prior knowledge, and its degree of connectedness.

The paper shed light on how students display new information on their concept maps and essays in the presence of the prior knowledge that is dominated by common language understanding. Other factors such as instructional styles, and apathetic behaviors/beliefs may also influence knowledge acquisition. Further research focusing on the degree to which the language-based knowledge influences mathematical cognition appears to be in order.

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