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MATH AND MIND MAPPING: ORIGAMI CONSTRUCTION

Abstract Students with or without disabilities often experience difficulties with abstract math concepts. This paper is intended to help solve the mystery of math concepts through origami construction, a hands-on activity. Students are involved in constructing and deconstructing concepts by folding and unfolding a piece of paper which eventually leads to a three dimensional product. Good teaching practice affords students the opportunity to learn within a constructivist framework. Origami, as an ancient paper art form, activates prior knowledge as well as encompassing hands-on learning, step-by-step instruction, schema building, spatial reasoning and logical concept mapping (Gardner, 1993).

FRAMEWORK

Through origami construction, students have the opportunity to discover both individual difference and universal commonalities through their unique learning styles within western and eastern cultures (National Council of Teachers of Mathematics, 1975). They are able to explore differing perspectives, examine stereotypes, develop global awareness, and hopefully, celebrate the diversity in their own classrooms through a multiple intelligence approach (Reigeluth, 1987). Origami combines different intelligences. It exemplifies the spirit of rural schools: creative, authentic, and coping skills. Origami stimulates more parts of the brain than does the antiquated teacher-lecture format (Stein, & Bovalino, 2004). It is an educational approach involving joint intellectual effort by students, or students and teachers together (Dewey, 1902). As students practice origami, they begin to understand and experience some of the complexities and interconnections of life laws and perseverance. Hands-on learning such as origami proves to be effective among the different learning styles as well as students from diverse population. Teachers can apply the knowledge to involve a student in a total learning experience which enhances the student's ability to think critically, to create a class dialogue, and to pose questions to encourage higher level of thinking (Bruner, 1966). More than just folding paper, origami can help teach students learn important math skills. As students use their fine motor skills to fold and crease paper into fun shapes and structures, they build skills involving spatial reasoning, following precise directions in sequence, fractions, geometry, and more (Olson, 1975). One of the advantages of origami construction is its ability to be applied at all grade levels and to similarly represent appropriate mathematical concepts (Kasahara, 1988). Using paper as a manipulative, teachers can teach basic folding that will be useful for teaching geometry concepts.

INTRODUCTION

Origami does not require fancy equipment. Ordinary paper, even used paper, works well, especially for the early stages of learning. However, when a student is helping other students to fold, it might be more advantageous to make use of computer technology to make diagrams and other teaching aids (Sze, Murphy, & Smith, 2004). Such computer use often serves to make the activities even more appealing to students and to raise the standard for student work (Hull, 2004). Origami can be an answer to the demand from educators and others for activities and entry points to mathematical discourse and applications called for by the National Council of Teachers of Mathematics and other reform efforts (Simon, Arnstein, and Gurkewitz, 1999). One of the most frequently stated goals of such educational reform efforts is to change the role of teacher from lecturer to guide, from the teacher being the only source of knowledge in the classroom to the condition of the classroom being a community in which everyone is an enthusiastic, responsible, and contributing teacher and learner (Prestia, 2003). In the origami community, teachers have appreciated the pleasure and value of being teachers and learners as they bring these experiences to the classroom. With its open-ended nature, communication, and inter-connectiveness, origami, as an activity, is reflective of the epistemology of mathematics that teachers want their students to experience.

OPERATIONAL DEFINITIONS OF ORIGAMI CONSTRUCTION

Origami construction: origami construction is defined as those geometric operations that can be formed by folding a piece of paper, using the raw edges and points of the paper, as well as any subsequent crease lines and points created while folding.

MATH TERMINOLOGY

Origami and mathematics each have their own jargon and are highly useful in describing folding. Sometime students may develop their own jargon. Having students develop their own terms and reconciling those with the traditional terms illustrates both the arbitrariness and usefulness of conventions (Alperin, 2000). Teachers can also discuss the viewing angle of the audience and the use of gesture. Origami activity motivates the explicit use of geometric terms. However, it is important to keep in mind that mathematical learning can be taking place even in the absence of mathematics terminology.

CREASES AND FOLDS

Studying origami crease patterns can help us learn about networks such as subways and phone networks, and how to make them faster and more efficient (Andersen, 2003). In the geometry of paper folding, a straight line becomes a crease or a fold. Instead of drawing straight lines, an individual folds a piece of paper and flattens the crease. Folding paper is analogous to mirroring one half of a plane in a crease, thus folding means both drawing a crease and mapping one-half of a plane onto another. As in the usual geometry, the distinction is being made between experimentation with the physical paper and the abstract theory of "paper folding" (Row, 1966).

IMPLICATION IN MATH EDUCATION

As a natural follow-up to (oral) instruction, students are given the task of preparing directions using their own writing and diagrams. However, it is possible to make acceptable diagrams using a variety of methods (Johnson, 1957). Computer based systems can be used and this can serve as an opportunity to encourage students to refine and polish their work (Demaine, Demaine, and Mitchell, 1999). Many people use specialized tools to produce drawings, but even the basic draw tools can be beneficial. Other options are to scan in hand drawn diagrams, scan actual models in development, or use a digital camera to produce images (O'Rourke, 1999). These images can be marked up using a computer drawing program (Sze, Murphy, & Smith, 2004). Finally, students can produce hypermedia text, images, animation, and sound linked together, with the navigation under the partial control of the user (Zamiatina, 2004). Several different techniques for producing diagrams can be implemented in a math lesson.

FACILITATE HIGHER ORDER THINKING

Some suggestions to teachers as to facilitate higher order thinking (Vygotsky, 1978) through a folding exercise are as follows. Ask students to consider beforehand what will be the results of making a fold. Ask them to visualize it 'in their minds'. Encourage them to pose generalizations on the effects of folds. For example, folding an edge to a parallel edge divides an area in half. Encourage students to compare models to models and folds to folds. Certain models (or partial models) are called bases in origami. Similar folds can be applied to different models in a way analogous to functions or transformations applied to different objects in mathematics. Ask students to describe and keep track of symmetries in models as the folding proceeds. Encourage students to ask themselves why a particular model works. Teachers can ask students how there can be movement in action models. Teachers can also ask students to compute a specific measurement of the final model in terms of the original dimensions. Moving from linear to two and three dimensions, ask students to determine what regions of the paper end up as specific regions of the final model.

TIPS ON TEACHING ORIGAMI MATH LESSON

1. A teacher should be familiar with the sequence of the correct folding steps. As a general guideline for good teaching practice, always practice the activity ahead of time to anticipate any areas of difficulty that students may encounter.
2. Students must be able to see the end product- a completed object in order to be successful. The completed object is served as a visual aid, motivator and roadmap for the students.
3. A teacher can make a list of math concepts, vocabulary, and laws involved in the origami construction.
4. A teacher can generate specific questions to elicit higher order thinking.
5. A teacher can use thin square paper for better result. Types of papers can be extended to include gift wrap catalogs, magazines, menus, and calendars.
6. A teacher should demonstrate the folds with a larger piece of paper and to make sure the paper faces the way the students' paper is facing them.
7. A teacher should support students who need more help with following directions or with manipulating spatial relationships by marking landmarks on the paper with a pencil as he or she goes around the classroom.
8. A teacher can make a dot at the point where two corners should meet.
9. A teacher can arrange the class in clusters and let students who have completed one fold assist other students. This will foster cooperative learning and help the teacher address all students' questions.

FOLDING FUNDAMENTALS: THE POWER OF ACCURACY AND PRECISION

It is important to be accurate when constructing an origami model. There are certain fundamentals students must observe in order to make the end result authentic. According to Sze (2004), neatness truly does count and there is power in accuracy and precision. In contrast to many traditional academic topics, there is less risk of over emphasizing neatness to the detriment of critical thinking or of failing to distinguish between mistakes of accuracy and mistakes in fundamental concepts (Swanson, 1999). The distinction between failings in surface features and failings in deep concepts is clear, even explicit, in origami. Furthermore, the importance of neatness is pragmatic enough that most students will learn this lesson on their own. The following are the fundamentals to build a successful model:

1. Make sure students fold on a smooth, hard, clean surface.
2. Encourage students to make a soft fold and check that the edges line up properly to avoid overlapping. After they make adjustments, they can make a sharper crease using their fingernails.
3. Have students unfold their origami projects to look at the interesting patterns and geometric figures they have created through their series of creases.
4. Challenge them to create their own variations--and make their own diagrams showing how they did it.

COOPERATING TEACHING MODEL

There are general techniques that can be applied to any model and fit very well in the culture of origami, in which every student is a potential teacher. The most basic use of origami is to have students teach models to other students. Various strategies exist for this. Teachers can divide the class into groups and teach each group one model in a set of models of similar complexity (Marton, Hounsell, & Entwistle, 1984). Later, their anchoring task is to teach the model to the individuals in another group. Students can work together, trying different approaches, and then agree on one presentation, or they can each find a partner in the other group (Scandura, 1977). Notice: even if there is one designated teacher, he or she may be using ideas from many students in the final lesson. Of course, teachers must manage the process over time so that the same students are not doing all the talking. Teachers can listen and give feedback to rehearsals or teachers can just let them do it (Bandura, 1977). During rehearsals or after the 'real' lesson, discuss explicitly the uses of language. Point out that specialized language, "jargon", serves a definite purpose.

CONCLUSION

In summary, teachers are required to follow a certain core curriculum to be taught at each level within a fixed time frame. There is very little room to do enrichment activities (Phibbs, 1991). Teachers often lack time to fully implement more in their normal classroom lecture. Origami can be an extremely useful tool in education and mathematics in particular. In this time of budgetary constraints, it is an inexpensive way to brighten up and offer an alternative (Sze & Yu, 2004) to the pure blackboard theory note taking approach to teaching and learning.

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