

# TEACHER'S PRACTICES AND DYNAMIC GEOMETRY

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*In this paper we analyze instructional materials supported by a Dynamic Geometry software, which were produced by teachers during an in-service training program. We discuss illustrative examples, as well as the outcomes of the critical discussions that took place during the presentations of these materials by the teachers. In order to analyze these materials, our conception of geometry involved a full spectrum of activities, from concrete exploration and experimentation, through conjecturing, problem solving, and on to formal proof. We also took into consideration results from research on the didactical potential of Dynamic Geometry (DG). It is argued that these programs can help us address fundamental difficulties in developing geometrical thought, as they can provide new representations of geometrical objects. Our aims are to evaluate how close from fulfilling this potential seem to be the instructional materials produced by the teachers themselves and to discuss the main factors influencing teachers' conceptions while developing of these materials.*

## INTRODUCTION

As discussed in Belfort & Guimarães (2002), the need to create opportunities for regular in-service training courses for secondary (that is: year groups from 11 to 17) Mathematics teachers in our geographical area led us to create a two-year in-service training course. We adopt as basic principles that solid mathematical formation and pedagogical and didactical knowledge are essential in training teachers, but they cannot be considered enough. The course should also include activities especially planned to allow teachers to reflect upon their classroom practices and to establish connections between their own learning experiences in Mathematics and their practice as secondary teachers.

During the first semester of the in-service training course, secondary Mathematics teachers revisit basic contents in geometry and functions, in disciplines supported by computer laboratory lessons. As reported in Guimarães et al. (2002), there are indications that most of teachers working in Brazilian secondary schools have “*a less than adequate grounding in geometry*” (p. 213), and it was decided that they should have several other opportunities to revisit geometry in the following semesters of the course.

They acquire some experience in using computers as tools for teaching Mathematics by attending the 'Computers for Mathematics Teaching' (CMT) discipline, which was designed to provide a counterpoint to the previous disciplines, when computers were extensively used as a tool by the instructors. With their newly acquired experience as learners to rely upon, they are asked in CMT to discuss, as well as exercise, the possibilities of the computer as a teaching tool for Mathematics. As part of the

assessment process of the discipline, the teachers are asked to write an essay, in which they must present their own computational instructional materials to other secondary mathematics teachers and justify its use for classroom work. They were encouraged to use Dynamic Geometry (D.G.) software (Barbastefano et al., 2000; Jackiw, 1996; Laborde & Bellemain, 1994) as a way to help overcome their own difficulties in geometry.

A first report of the outcomes of a research project design to evaluate the CMT discipline was presented in Belfort et al. (2001). In this article, under the light of a broader theoretical perspective, we deepen that previous analysis and add new data from other editions of the discipline.

## **THEORETICAL BACKGROUND**

There are several factors to consider if we endeavor to understand teacher's practices. Cooney (1988) described teaching as "an interactive process", in which conscious decision making is needed not only during the planning of the lessons but also "on the stage". Models to explain the role of the teacher usually consider the interaction between teacher's knowledge and beliefs as the basis for their decisions (Fennema et al., 1989). The complexity of the influence of textbooks and other written materials on teachers' practices is also to be considered (Belfort da Silva Moren, 2000).

There are also several indicators from research that a solid subject content knowledge may be essential for a successful teacher (see Grossman et al., 1989; Ball, 1988, 1991; Leinhardt et al., 1991). In particular, Ma (1999) discusses the importance of the "profound understanding of fundamental mathematics (PUFM)" demonstrated by some elementary teachers. According to her, the work of these teachers displays the following characteristics: connectedness, multiple perspectives, awareness of the basic ideas, and longitudinal coherence (p. 122).

If we attempt to transpose these research ideas to the situation of the study of Mathematics at secondary level, it seems that, if the teaching of secondary Mathematics is to display a similar set of characteristics as the ones observed by Ma (1999), the teacher would need to be able to ponder the connections of his/her mathematical knowledge with the mathematical contents of secondary school. In the case of Geometry, it seems that the secondary school curriculum oscillates between more figure exploration/less formal geometry teaching and less figures/more proof elaboration, reflecting the dialectic process between exploratory work with figures and proof elaboration, which can be seen in the historical evolution of geometry.

Concrete exploration, experimentation, conjecturing, problem solving and proof formulation seems to be central points of the set of skills we want the student to acquire in his/her knowledge construction process (Guimarães et al, 2002). But this set of skills, which seems so natural to the scientifically trained, does not come so naturally to the students. The concrete object does not have the same signification and is not explored in the same way by the mathematician and by the student: the way the concrete object is used strongly depends on the previous knowledge of who

is using it. Even more important is that teaching based on the exploration of the concrete object makes the none evident assumption that the interaction with the concrete will effectively produce the construction of the desired knowledge (Balacheff, 1999).

Again in the case of geometry, the concrete object is often a diagram, and to understand the differences in the exploration of this, researchers consider two different objects (Parzys, 1988; Arsac, 1989; Laborde et Capponi, 1994; Balacheff, 1999): one that is concrete, called *the drawing*, and the other formal, corresponding to the class of drawings representing the same set of specifications (called *the figure*). From this point of view, a dynamic geometry software can have a specific contribution: it can provide controlled representations of geometrical objects, which, in some ways, concretize the formal figure.

We take as one of our assumptions that these software can provide new ways to learn geometry, and by way of consequence, new ways to teach geometry. Their use in in-service programs for teachers of Mathematics provides us also with the opportunity to discuss with them how to integrate mathematical software in their teaching toolkit.

## **ANALYSIS OF TEACHERS' INSTRUCTIONAL MATERIALS**

In Belfort et al. (2001), we analyzed the instructional materials developed by the teachers during the first edition of CMT discipline from three different perspectives:

- usage of the computational software as a didactical resource (Software);
- subject matter knowledge and consistent mathematical reasoning (Subject); and
- appropriateness of the didactical proposal, considering the targeted year group (Instruction).

We discussed that, although we were expecting a balance among these three factors, this was achieved only in roughly one third of the essays. The majority was mostly oriented towards a single perspective, ignoring the others. During the following editions of the CMT discipline, the same tendencies were observed. We provide here some examples of the developed materials and briefly describe the outcomes of the discussions that took place during the presentations of these essays to the group.

### **Instructional Materials Oriented Towards the “Software” Perspective:**

The instructional materials in this group (about 10% of the essays) reflect their authors' focus of interest as being the process of mastering the use of the software. The material developed by Marcos<sup>1</sup>, using The Geometer's Sketchpad to draw geometric loci is an extreme example. Figure 1 illustrates two of his sketches.

The typical initial reaction by the teachers to this sort of presentation is admiring the sophisticated use of the package resources that characterizes these materials. Nevertheless, once the question “*what have you learned from this activity?*” is posed

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<sup>1</sup> Here and in the following cases we use fictitious names.

to them, this reaction begins to change. In their final evaluation of the typical material in this group, teachers realize that they allow virtually no room for students' exploration and that mathematical concepts are neither explained nor justified. Teachers become aware that students are, at best, treated as spectators. For instance: in the case of Marcos' materials, all they have to do is to 'click the mouse' over the animation button, and watch curves being 'magically' traced, the proprieties of which they cannot investigate. It was also observed by the teachers the lack of definition of the year group(s) for which the material was intended.

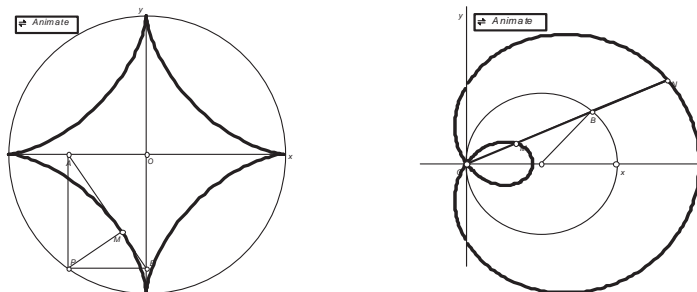


Figure 1: Marcos' instructional material in DG environment.

### Instructional Materials Oriented Towards the “Subject” Perspective:

The instructional materials in this group (about 10% of the essays) usually reflect their authors' focus of interest as being the opportunity to revisit subject contents that can be represented in a simple way using DG. The work developed by José exemplifies this set of materials. As shown in figure 2, he used only the basic tools available in a Dynamic Geometry Software (Tabulæ) to study the points of intersection of the cevians of a triangle. The written essay was mathematically correct, presenting definitions, theorems and well-organized proofs.

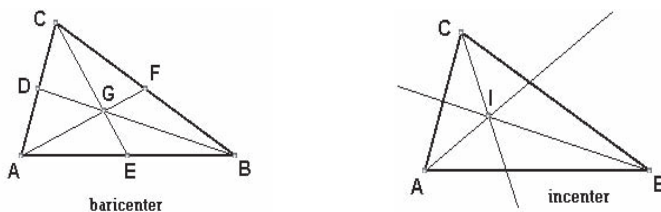


Figure 2: José's instructional materials in a DG environment

During the presentation of the works in this group, it soon became clear to the other teachers that very little is left for students' interactions, as these material are usually ready visualizations of mathematical results, not to be discussed nor justified in the laboratory environment. In the case of José's materials, for instance, the students' job

is reduced to move the vertices of the triangles to verify that the intersection points remain coincident. Since José could not state clearly the objectives of the proposed tasks, the group of teachers searched for didactical alternatives, and some interesting suggestions were presented. Teachers in the group were also unanimous in considering that students should be encouraged to construct the geometrical objects themselves, the more so given that only the simplest resources made available by the DG software were applied on the development of this type of sketches.

### **Instructional Materials Oriented Towards the “Instruction” Perspective:**

Almost half of the materials developed by the teachers can be classed in this group. They reflect their authors’ focus of interest in creating computer assisted learning activities for their pupils. Designed to provide experiences in area measurement, the instructional material presented by Mariana is a typical example. We present a model of the interaction proposed in her first sketch in figure 3: pupils are asked how many square area units are needed to fill the rectangle. The following sketches repeat the experience, but the rectangles get larger. Mariana explained to her colleagues that she expected the students to make the effort to (inductively) conclude the formula for the area of the rectangle, as a means of avoiding the repetitive job.

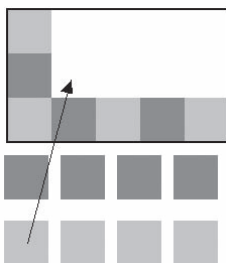


Figure 3: Mariana’s instructional material in DG environment

Teachers’ spontaneous remarks, such as “*this is so boring!*”, “*after that, kids will hate computers!*” and “*this remind me of my textbook!*”, show they acknowledged not only that the typical interaction proposed by these materials underestimates DG’s potential but also the strong influence in these materials of textbook’s approaches to Mathematics contents. That the proposed learning experiences in this group usually display a fragmented vision of the topic and a clear hurry in getting the “formula” is probably a consequence of this very influence.

Although these materials are often designed exploring the resources of DG software at an intermediate level, it was argued by the colleagues that they fail to encourage a meaningful mathematical development, lacking connections and multiple perspectives. For instance, in the particular case of Mariana’s proposed activities, it was observed by one of the teachers that all rectangles in the sketches have integer side measures, even though the year group targeted by her is the very one studying operations with fractions in our schools. It is to be noted that this doesn’t seem

relevant to many Brazilian textbook authors either, as a quick check on their own examples for area measurement indicates.

### Well Balanced Instructional Materials:

An instructional material was considered as “well balanced” whenever all three perspectives were integrated by its author. The materials presented by Helena are a good example. Directed at students in their final years of secondary school, they exploit the characterization of the ellipse as the locus of the points for which the sum of the distances to two given fixed points of the plane is constant. By means of a sequence of activities supported by written guidelines, students are led to the construction of this locus using the so-called director circle. Figure 4 illustrates the initial stage of the third sketch given to the students and also its final stage, in which new constructions were added by the pupils. We have also observed that teachers who produce well balanced materials do not usually rely solely on the content of a single textbook, but seem to seek support for their work by researching a more comprehensive bibliography.

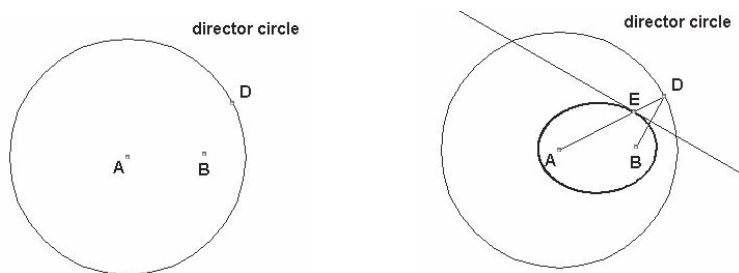


Figure 4: Helena's instructional material in DG environment

Teachers who produced well balanced materials typically chose to simulate a laboratory lesson when presenting their work to their colleagues. During the presentation of these materials, it was observed that teachers, working in pairs, got really involved in solving the proposed tasks. The discussion engaged in by the group after these presentations acknowledged that examples in this category typically provide the students with a sequence of computer activities aiming at developing a well defined mathematical concept. Teachers in the group also commented that these materials display an evident concern with proper definitions and justifications for the geometric constructions and results.

### FINAL REMARKS

The substantial differences in the outcomes of teachers' work suggest that they have different views on Mathematics teaching and learning processes. Some of instructional materials seemed to reinforce the role of the teacher as the knowledge keeper (and teller), while others provided experiences that seemed to be designed to

keep students “busy”, with no clear (minimum) objectives to be achieved. Yet other materials seemed to reflect a problem identified by several researchers (Ball, 1988, Ma, 1999, etc.): the lack of deep, broad and thorough subject matter knowledge on the part of the teachers resulted in fragmented materials. In them, formulas were overestimated and there was strong reliance on the simplified approaches found in some of the poorest textbooks available in the Brazilian market. On the other hand, there were teachers who produced well balanced materials, in which mathematical concepts were treated as connected parts of a body of knowledge.

Our results also suggest that to provide their students with worthwhile learning experiences using computers is an idea that may have a strong appeal for the Mathematics teachers. We contend that this motivation can be explored to help them to overcome content knowledge difficulties, and to develop a critical awareness of the materials available for classroom work.

Although we are well aware that developing instructional materials is not at all a simple task (see Belfort da Silva Moren, 2000), we feel it is worthwhile to give teachers the opportunity to make an attempt at it. The debate that took place in classroom exploring the didactical characteristics of the materials made these teachers more conscious of some critical educational issues related to Mathematics teaching and learning processes, and, in most cases, made them willing to make the effort to overcome their perceived difficulties. It is to be expected that these experiences will reflect positively on their future work.

Finally, we strongly believe that a Dynamic Geometry software is a powerful tool for teaching and learning Mathematics. Nevertheless, as it happens with any other tool, it is the way it is used that determines the final outcomes. If we expect teachers to fully understand the potential of these packages, we’d better start to provide them with rich learning experiences supported by DG environments.

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### **REFERENCES**

- Arsac G., (1989). La Construction du Concept de Figure chez les Élèves de 12 Ans. In *Proceedings of the 13<sup>th</sup> Conference of the International Group for Psychology of Mathematics Education*, pp. 85-92.
- Balacheff N., (1999). Apprendre la Preuve. In J. Sallantin et J.J. Szczeciniarz (eds), *La Preuve à la Lumière de l’Intelligence Artificielle*. Paris: PUF.
- Ball, D. L. (1988). *The Subject Matter Preparation of Prospective Teachers: Challenging the Myths* (Research Report). East Lansing, MI: National Center for Research on Teacher Education.

- Ball, D. L. (1991). Research on teaching mathematics: Making subject matter knowledge part of the equation. In J. Brophy (ed.) *Advances in Research on Teaching* (vol. 2, pp. 1-48). Greenwich, CT: JAI Press.
- Barbastefano, R., Guimarães, L. C. et al. (designers) (2000) - *Tabulae - Educational Software* © by the authors, Rio de Janeiro: UFRJ.
- Belfort S. Moren, E. (2000). *Categorising Different Ways Secondary Mathematics Teachers Use Written Materials for Classroom Work* - Unpublished PhD. Thesis. London: School of Education, King's College London.
- Belfort, E & Guimarães, L. C. (2002) The Influence of Subject Matter on Teachers' Practices: Two Case Studies. In *Proceedings of the 26<sup>th</sup> conference of the International Group for Psychology of Mathematics Education*, v. 2, pp. 23-32.
- Belfort, E; Guimarães, L. C. & Barbastefano, R. (2001) Using Computers in Mathematics Teacher Training Programs: a Reflection upon an Experiment. In: *Annals of The 5th International Conference on Technology in Mathematics Teaching*.. Klagenfurt, electronic volume.
- Cooney, T. J. (1988). Teacher's Decision Making. In D. Pimm (ed.) *Mathematics, Teachers and Children*, pp. 273-286. London: The Open University.
- Fennema, E.; Carpenter, T. & Peterson, P. (1989). Learning Mathematics with Understanding: Cognitively Guided Instruction. In J. Brophy (ed.) *Advances in Research on Teaching*, pp 195-221. Greenwich, CT: JAI Press.
- Guimarães, L.C.; Belfort, E. & Bellemain, F. (2002). Geometry: Back to the Future? In: *Proceedings of the Second International Conference on the Teaching of Mathematics*, Crete, J. Wiley & Sons Inc.
- Grossman, P., Wilson, S. & Shulman, L. (1989). Teachers of substance: Subject matter knowledge for teaching. In M. Reynolds (ed.), *Knowledge Base for the Beginning Teacher*, pp. 23-36. New York: Pergamon Press.
- Jackiw, N. (designer) (1996). *The Geometer's Sketchpad* - Educational Software © Key Curriculum Press, Berkeley, CA: Key Curriculum Press.
- Laborde C. & Capponi B., (1994). Cabri-géomètre Constituant d'un Milieu pour l'Apprentissage de la Notion de Figure. *Géométrie. Recherches en Didactique des Mathématiques* vol 14, n°1.2 pp165-210.
- Laborde, J. M. & Bellemain, F. (designers) (1994). Cabri Géomètre II. Educational Software © Université Joseph Fourier, Grenoble. Dallas: Texas Instruments.
- Leinhardt, G., Putnam, R. Stein, M. & Baxter, J. (1991). Where subject matter knowledge matters. In J. Brophy (ed.) *Advances in Research on Teaching* (vol. 2, pp. 87-113). Greenwich, CT: JAI Press.
- Ma, L. (1999). *Knowing and Teaching Elementary Mathematics: Teacher's Understanding of Fundamental Mathematics in China and the United States*. Mahwah: LEA.
- Parzys B., (1988). Knowing vs Seeing, Problems for the Plane Representation of Space Geometry Figures. *Educational Studies in Mathematics*, n°19.1, pp 79-92.