DESIGNING FOR LOCAL AND GLOBAL MEANINGS OF RANDOMNESS

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This research aims to study the ways in which 'local' events of randomness, based on experiencing the outcome of individual events, can be developed into 'global' understandings that focus on an aggregated view of probability (e.g. probability of an event). The findings reported in the paper are part of a broader study that adopted a strategy of iterative design, in which a computer-game was developed alongside the gathering of evidence for children's use of the game. In response to a range of tasks, 5 and 8 year-old children manipulated the sample space in ways that generated corresponding outcomes in the game. The findings illustrated how the game provided new tools of experience that afforded the construction of novel meanings for randomness.

INTRODUCTION

Research within the constructionist framework has shown how technology can empower children in the use of stochastics. Wilensky (1997) based his work on the conjecture that both the learner's own sense-making and the cognitive researchers' investigations of this sense-making are best advanced by having the learner build computational models of probabilistic phenomena. He shows that through building computational models, learners can come to make sense of core concepts in probability such as normal distribution. Learners are supported in building and developing their mathematical intuitions and, through this construction process, mathematical objects are seen to be more concrete as learning progresses. Pratt's (2000) study also showed that by using a computational system, children managed to make sense of local and global probabilistic meanings, referring to *local* meanings of randomness as those based on experiencing the outcome of individual events, and *global* meanings that focused on an aggregated view of probability. Local meanings in probability are characterised by the fact that the next outcome is unpredictable and there is irregularity. Global meanings have the following characteristics: the proportion of outcomes for each possibility is predictable, the proportion will stabilise with an increasing number of results (large numbers) and there is control through manipulation of the sample space. Pratt describes the development of a computer-based domain within which children (aged between 10 and 11 years) manipulate stochastic 'gadgets', representing everyday objects such as a die, a coin, a lottery and a set of playing cards. Individual learners were put in situations where they could express their beliefs in symbolic (programming) form and articulate the beliefs that they held, and construct and reconstruct them in the light of their experiences.

The aim of this paper is to describe the design of a game constructed simultaneously to afford expressive power to children aged 5 to 8 in the domain of probability, and to provide a *window* (Noss & Hoyles, 1996) on that thinking as it is developed. It presents the ways in which local meanings of randomness, based on experiencing the outcome of individual events, can be developed into global understandings that focus on an aggregated view of probability (e.g. probability of an event). The broader study also assesses whether and how the explicit linking of local and global meanings via a rule-based system, assists in effecting this evolution. However, in this paper, we will focus on design and on ways in which our design-criteria facilitated children's expressions of their ideas of randomness.

THE MAJOR DESIGN PRINCIPLES OF THE GAME

There were three major principles that informed the design of the game, as are described in the following paragraphs.

The manipulable sample space (and distribution): A 'lottery machine' represented an "executable sample space" or distribution in the game. The 'lottery machine' was a visible manipulable engine for the generation of random events and with it the children could directly manipulate the outcome of the game. The direct manipulation and linked connections provided by the software allowed children to set in motion the mechanism to trigger an event, and be able to link the execution of that event with an outcome on the screen.

The spatial representation of sample space: The presentation of the lottery machine in the game was geometrical/spatial, whereas in previous work (for example Konold, 1989) the sample space was either hidden (i.e. not available for inspection or manipulation) or represented only in quantitative form (by using only numerical quantities). The lottery machine contained balls of different colours, which made it possible for children to carry out as many events as they liked without being obliged to think about numbers (as they would have to using dice, coins etc.). Moreover, in the final iteration, the children also had the opportunity to change the probability of an event by changing the size of the balls and their arrangement.

The existence of local and global events in the game and a visible link between them: The game gave the children the opportunity simultaneously to see on screen the local and global representation of an event in their sample space. A *local* event refers to the trial-by-trial variation and the *global* to the aggregate view of each single trial[1]. In practice, local events might be used by children to make sense of short-term behaviour of random phenomena, while global events are associated with long-term behaviour. The game made visible the link between the short-term and long-term behaviour. Thus, whilst individual outcome could be seen as a single trial in a

[1] Many studies (for example Pratt, 2000; Ben-Zvi and Arcavi, 2001; Konold and Pollatsek, 2002; Rubin, 2002) have shown the importance of linking local and global understandings with an aggregate view, and how complex students may find this process.

stochastic experiment, the totality of these outcomes gave an aggregated view of the long-term probability of the total events.

A DESCRIPTION OF THE GAME AND ITS DIFFERENT PARTS

An example of a lottery machine can be seen in Figure 1.

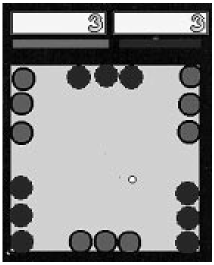

Figure 1: A lottery machine with its scorers showing the local events of the game

The lottery machine here is represented by the large square. In it, a small white ball bounces and collides continually with a set of static *blue* and *red* balls. Children could change and manipulate a number of aspects in order to construct their own sample space: the number, the size, and the position of the balls in the lottery machine, and they could also create new objects with their own rules. As programmed initially, collisions with the *blue* balls (light grey balls in the figures) added one point to the blue score and moved the 'space kid' one step down the screen. In this way the lottery machine controlled the movement of the space kid (see Figure 2).

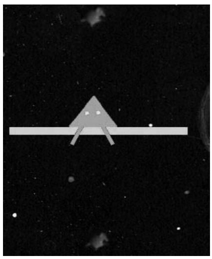

Figure 2: The space kid and the planets that represent the result of the game

An important design criterion was to choose a tool that worked with rules to afford children the opportunity to understand how the elements of the game are connected and moreover, to link local and global events. Figure 3 shows how the features of Pathways[2] provided the link between the lottery machine and the space kid.

[2] This research was funded by the European Union as part of the *Playground Project*, Grant No. 29329. See http://www.ioe.ac.uk/playground/

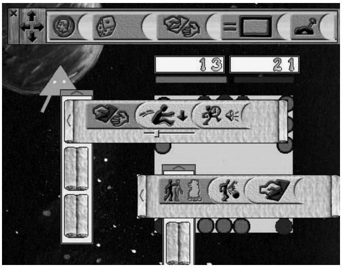

Figure 3: An example of linking rules in Pathways

In Pathways, the library of stones enables the user to make rules that will connect one object to another. So when, for example, the bouncing ball collides with a coloured, *blue* or *red* ball, this ball can instruct another object (by posting a message), to react. For example, the rule in Figure 3 can be read as 'when I touch anything, I bounce off it and I show a *blue* message' and the reaction of the other object is 'when I see a *blue* message, I move a step downwards and I play a sound'. The small ball in the lottery machine moves continuously in 2 dimensions, a design decision made in order that children might visualise the global outcomes of the game. The final structure of the game as the child plays it, is illustrated in Figure 4.

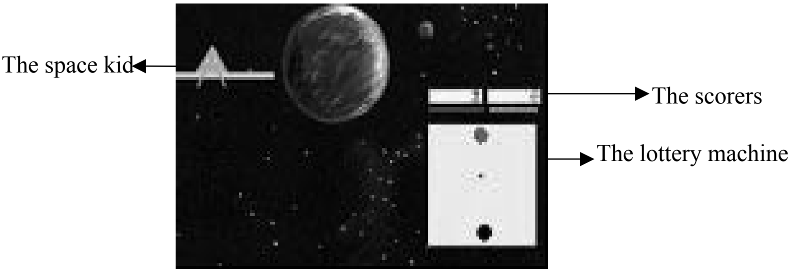

Figure 4: A screenshot of the game

Whilst individual collisions can be seen as single trials in a stochastic experiment, the totality of these movements gives an aggregate view of the long-term probability of the total events. In order to explore the connections children make between fairness and randomness, we began with a situation in which the children had to try to make the space kid move around a centre line in order to construct a 'fair sample space'.

THE 'CONCEPT' OF THE GAME

The game gave children the opportunity, by manipulating the sample space and distribution, to identify intuitively whether an event is possible (whether it is impossible, certain or somewhere in between). The game consisted of two key pieces: a 'lottery machine' and a link between the local and global events. Diagram 1 gives the 'concept' of the game.

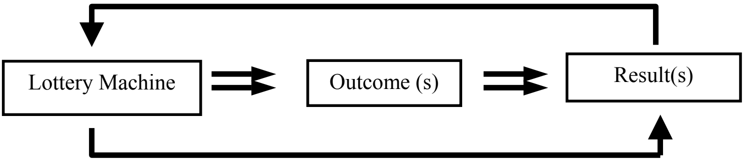

Diagram 1:

The concept of the game: the connection between the lottery machine, the outcomes and the results

Diagram 1 shows how the ingredients of the game are connected. The lottery machine generated an outcome and this affected the result of the game. The short arrows illustrate how children, by manipulating the lottery machine, were intended to experience the outcome of an individual event in the machine (i.e. a collision between two balls) and how this was connected to a single result in the game (i.e. effecting a movement of the space kid). The single outcome from the lottery machine provides an idea of a local event. The totality of the outcomes of the game gave a more aggregate view of the results and of the lottery machine's construction. The manipulations made via the lottery machine could have short-term and long-term outcomes within the game. For example, children could make decisions about their next change in the lottery machine based on the long-term results of their previous constructions.

THE CHOICE OF PATHWAYS SOFTWARE FOR DESIGNING THE GAME

Pathways was designed as a medium where children can build and modify games using the formalisation of rules as tools in a constructive process. This enabled the construction of the game, which afforded a simple means for programming the direct manipulation of objects, with which children could express meanings from actions and build new meanings of probabilistic ideas.

Hence, there were three reasons for the choice of Pathways: 1. evident, iconic rules that could be understood by children of this age 2. easily manipulated objects and 3. a clear message-passing mechanism providing a means of linking local and global events (see Figure 3). Thus, rules could be created specifying how each object of the game works. This afforded the children the opportunity to understand how the objects of the game were interrelated. It also allowed them to manipulate and link local and global events. Diagram 2 shows how the mathematical ideas and programming criteria are interrelated.

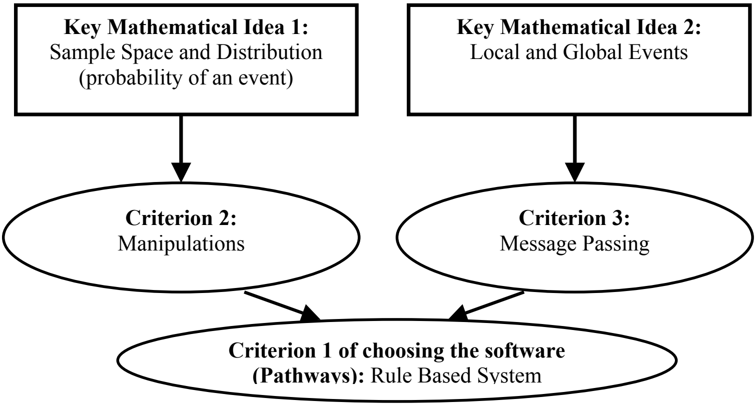

Diagram 2:

The interrelations between the mathematical ideas developed in the game and the criteria for Pathways choice

The key contribution of Pathways was that it allowed manipulations of the sample space and distribution, while its message-passing feature gave the opportunity to link local and global game events. Both criteria were related to the rule-based system on which Pathways (and therefore the game) was built.

Our approach followed that of iterative design (diSessa, 1989), which facilitates, through the gradual refinement of computational tools, an increasingly fine focus on the primary issues involved in the restructuring process. In fact, the iteration involved three components: while each version of the game was informed by children's activities with the previous version, Pathways too was being refined and debugged, so that it could serve as a suitable medium in which to construct the game. This hardly made the iterative design process less complex, but it ensured a better match between the platform, the game and children's emergent activities.

METHODOLOGY FOR CHILDREN'S USE OF THE GAME

In keeping with the iterative design approach, the game was developed alongside the gathering of evidence for children's use of the game. All interviews were videotaped and transcribed. Twenty-three children were involved in the final iteration of the main study. The children worked with the software individually for a period lasting between 2 and 3 hours. The role of the researcher was that of participant observer, interacting with the children in order to probe the reasons behind their answers and actions.

A BRIEF GLIMPSE OF CHILDREN'S USE OF THE GAME

The early iterations of the design process gave the researcher an opportunity to observe the characteristics of children's ideas about randomness, and how they expressed these ideas. The dominant tendency was the use of patterns, as other related research has shown (Konold, 1989; Pratt, 2000). Piaget and Inhelder (1975) also claim that children under the age of 8 years old are most interested in the point of view that considers the pattern of the total number of balls and the 'effect' of each experiment on the next.

An example below demonstrates how the design of the game helped a child make the link between short-term movement of the white ball and long-term movement of other elements. In the following episode, Victoria (girl, aged 6 years) tries to construct a fair lottery machine that will keep the space kid near the yellow line.

Researcher: How did you arrange the balls?

Victoria: If it (the white ball) goes like this it might get the red (ball) and then the blue (ball) and then to move down like this or it may move like this and move up and get this one and then that one or to move in a different place and get this and this.

She starts the game.

V: Come on! Look! It (the white ball) gets red again! Now it (the space kid) moves down and then up and…Whatever I say it happens! Come on…It (the space kid) moves up and then down. Oh…no, we have more points for the red colour. I wanted to get one red and one blue. Go up, now move down…

R: Does it listen to you?

V: No! I will place somewhere else the white ball. Here! Let me start the game again.

V: Come on… (*she knocks on the table*), come on…Oh, not again. I can't control this white ball….

She stops the game.

Here, Victoria based her ideas on the previous movement of the white ball, thinking that it would follow the same path. As the game progressed, she began to realise that even if she was able to predict where the white ball ended up, she could not predict how it might get there (i.e. its path of movement), and she could not control or predict exactly its next move. Finally, Victoria focused on the movement of the space kid instead of the white ball.

At the beginning of the game, all the children participating in the main study tried to find patterns to 'explain' the movement of the white ball. These patterns were based on the path traced by the white ball in order to 'achieve' a hit. However, because of the continuous movement of the ball it quickly became impossible for them to predict a particular path. The characteristic of the game of connecting the short-term and long-term behaviours of the system, provided children with corresponding local and global events. The research findings (see also Paparistodemou, Noss & Pratt, 2002) show that the continuous movement of the ball links short-term and long-term behaviours, thus discouraging children from simply looking for patterns in randomness. It is conjectured that the components of the game, and the connections between them, helped children to connect "pieces" of the system mentally, and to introduce corresponding structure into their thinking about randomness.

It might have been expected that children this age would find it extremely difficult to construct a rich set of meanings for the challenging idea of randomness. One reason for this might be the noted tendency of young children to focus on controlling the 'thing' that delivers randomness. However, the episodes of the main study indicate that in the special dynamic medium of expression provided by the game, children did construct meanings for randomness, in the sense that they realised the need to control the outcome without controlling the random movement.

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