# STUDENTS PREFERENCE OF NON-ALGEBRAIC REPRESENTATIONS IN MATHEMATICAL COMMUNICATION

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*This research study deals with the modes of representation that ninth-graders choose in order to communicate their problem solving paths and justifications, and the relation between these modes of representations and achievement level. The findings are based on analysis of 350 answers to problems that demanded communication of reasoning, explanations, and justifications. The results indicate that only a few students, who are very high achievers, choose to communicate via algebraic representations, even after two extensive years of learning algebra. These results might be related to difficulties students have with the abstraction of algebra and the way algebra is taught in school – an issue that should be considered by curriculum developers and teachers.*

## INTRODUCTION AND THEORETICAL BACKGROUND

Over the past decade the emphasis in the math classroom is shifting from routine procedures towards developing mathematical thinking, reasoning, and communicating. The more the students' mathematical language develops, the better they can reason (NCTM, 1989, 2000). The importance of mathematical communication, using mathematical symbols - in writing, and in 'ordinary language', lies in bridging between articulation and reflection, and serves as a means to participating in the mathematical community (Fried & Amit, 2003; Morgan, 1994).

Mathematical communication is strongly related to problem solving and reasoning. The process of successful problem solving is dependant on the following: *problem representation skills* which include constructing and using mathematical representations in words, graphs, tables and equations, *solving* and *symbol manipulation* (Brenner et al., 1997). Students can communicate their explanations for a mathematical strategy or solution in a variety of ways: symbolically (numerical and/or algebraic symbols), verbally, diagrammatically, graphically, or by tables of data (Shield & Galbraith, 1998). The significance of presentation in problem solving is that it reveals the ways in which students process the problem, and as a matter of fact, the modes of representations are the external reflection of the thinking and solution processes (Cai, Magone, Wang & Lane, 1996). In this study we observe the ways students communicate their justification for solution paths, and how do they choose to represent them.

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## METHODOLOGY

The aims of this study are to examine the modes of representation that ninth-graders choose in order to communicate their problem solving paths and justifications, and to investigate the relation between the modes of representations and achievement level.

### Settings and instruments

The population of this study comprised 164 ninth-grade students (83 male and 81 female) who participated in a regional test. Five multi-ability classes, each from a different school were selected to compose the sample (The number of students from each class: 37, 37, 27, 34, 29). They came from similar socio-economic backgrounds; most of them low to middle class. All students had a similar mathematical background, because their studies followed the national curriculum.

The research instrument was compromised of three problems taken from a regional test in mathematics. The test was consisted of 46 problems referring to varying areas within the ninth grade curriculum. About half of the items were multiple-choice, and the others were constructed response, short answer and a few were open-ended problems (to different degrees of "openness").

Three problems, in which students had to communicate their explanations and justifications, were chosen as the research instrument.

The first problem dealt with optimization, and required that the students choose between two telephone companies and justify their preference. Data on the cost of monthly charges and per-minute costs were provided. Such a problem had never appeared in their math textbooks and therefore was considered to be a non-routine problem.

The second problem dealt with rate of change: water drained from a pool at a given constant rate. The students were asked whether the pool would be completely empty after one hour had elapsed. This type of problem appears frequently in textbooks but in the test they had to justify their conclusions.

The subject of the third problem was the relation between the area and circumference of a rectangle. The problem was formulated in two parts: a multiple-choice question with three choices, and then justification of their choice. This is a non-routine problem and does not appear in textbooks.

### Data Collection and Analysis

Data was collected from students' test booklets. The data were analyzed qualitatively to identify the mode of representation, and then the qualitative results were quantified. (Note: neither the teachers nor the students knew at the time the test was taken that some of the tests would be researched later on. This fact increased the authenticity of the answers.)

## Level of Achievement

The whole test score, excluding the three instrument items, defines the level of achievement in this research. Scores are on a scale of 0-100. The tests were scored by the researchers, according to strict guidelines set by the Ministry of Education.

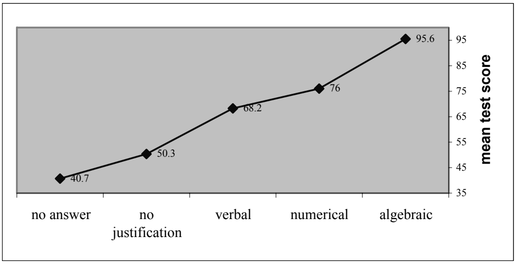
## Modes of representation

In order to find how students communicate, a qualitative analysis was implemented. Students' explanations and justifications were sorted according to the following representation modes: *algebraic* - explanation was represented by an equation or a function; *numerical* – explanation was represented arithmetically, including computations and manipulation; *verbal* - justification was written in words, or *diagrammatically and graphically* - explanation was represented by diagrams, graphs, or other pictorial illustration. This analysis was based on Shield & Galbraith (1998) modes of representations, but modified after a pilot, in such a way that symbolic representation was separated into two categories: numerical and algebraic. For each problem the mean test scores of the students who had chosen to communicate in a specific representation was calculated. This obtained a mean score per representation per problem.

## Findings

Observation of how students chose to communicate indicated that the vast majority preferred verbal and numerical modes, and a minority preferred an algebraic mode. From the total of 350 justified answers, 153 answers (44%) were represented in a verbal mode, 131 (37%) answers in a numerical mode, and 39 (11%) answers were represented in an algebraic mode. In the area-perimeter problem, 26 answers were in a diagrammatical mode. Since this problem was perceived as a geometry problem – the use of a diagrammatical mode was natural. In the other problems, only in one case was there use of a graphical mode.

Examination of the relations between representation and achievement indicates that the students who choose algebraic representations were high-achievers, and their mean test scores were 95%-97% (pending each problem, see below). Those who chose numerical representations had mean scores of 76%-85%, and those who chose verbal representations had mean scores of 68%-75%. Students who did not attempt to justify their answers or did not answer the problems at all were low-achievers in the entire test (scored less then 45%). These results were valid for all three problems. The following graphs represent the relation between achievement level (test scores) and the choice of representation mode for each problem. Enclosed is also the distribution of representation mode within each problem.




Number of Answers:

Algebraic: 18

Numerical: 76

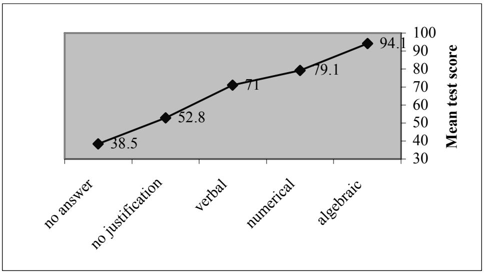
Verbal: 25

Graphical: 1 (excluded from the diagram)

No justification: 7

No Answer: 37

**Fig 1: Optimization problem: Achievement related to representation modes**




Number of Answers:

Algebraic: 15

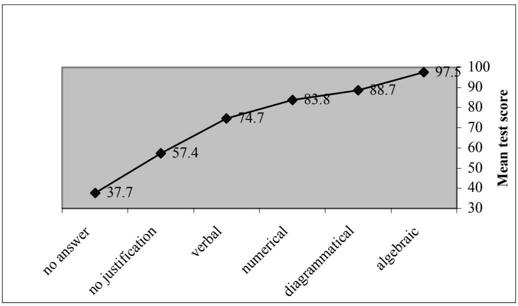
Numerical: 50

Verbal: 57

No justification: 10

No Answer: 32

**Fig 2: Rate of change problem: Achievement related to representation modes**




Number of Answers:

Algebraic: 6

Numerical: 5

Verbal: 71

Diagrammatical: 26

No justification: 22

No Answer: 34

**Fig 3: Area-perimeter problem: Achievement related to representation modes**

## DISCUSSION

Reform in mathematics education emphasizes written communication as a tool to develop and deepen mathematical thinking and reasoning. In this paper, we report on the modes of representation students choose in the process of communicating their problem solving paths and justifications, as appeared to three mathematical items in a regional test. Such communication can be represented in a variety of ways: mathematical symbols (numerical or algebraic), graphs, verbal, diagrams, and sketches (Shield & Galbraith, 1998).

A major finding in this research is that only a few students choose to communicate their solution paths in an algebraic mode. There are other studies reporting students' preference to justify and explain mathematical solutions in a verbal mode (Cai et al., 1996), or to solve problems presented verbally in non-algebraic methods (Nathan & Koedinger, 2000). However, this outcome is quite surprising, bearing in mind that the students in the current study have experienced almost 2 years of algebra studies. Focusing on the results, it is evident that the students, who choose algebraic representations, are students who achieved high scores in the test.

Why is this so? One explanation is that students just find that communicating in an algebraic mode is difficult for them (even after two years of studying algebra!). The use of numbers generalized by letter designations is an abstraction that raises difficulties for students (Hembree, 1992). Some students even fail to construct a meaning for the ideas of algebra or to connect them with pre-algebraic ideas. In order to use the language of algebra, students need to get used to a more different and more abstract mode of thinking than that used in arithmetic, and students tend to retreat to a more "solid ground" such as numbers or words (Herscovics & Linchevski, 1994; Lee & Wheeler, 1989). The same phenomenon was found by Hazzan (1999). She found with undergraduate students, that coping mentally and cognitively with new mathematical styles of representation and new concepts, leads the students to adopt mental strategies that involve reducing the level of abstraction. This type of regression has also been reported for students that had been introduced to algebra but who had the tendency to stay in the arithmetic modes (Lee & Wheeler, 1989), or with calculus students (Amit & Vinner, 1990.) When dealing specifically with argumentation, it was found that algebraic arguments are even harder to follow, and students prefer arguments presented in words (Healy & Hoyles, 2000). All the above indicates that using algebraic means in order to justify and explain problem solving procedures is really hard for students. Therefore it makes sense that only high achievers dare or are willing to choose this mode representation.

Another explanation is related to the way algebra is taught in middle school. According to the national syllabus, the first two years of algebra are mainly devoted to solving equations and systems of equations, plotting graphs, and solving problems with one or two unknowns. When students have (seldomly) to justify their outcomes, they do it by substituting a number in an equation or by using everyday language.

Shield & Galbraith (1998) found that the writing products of students are constrained by the mathematical presentations to which they have become accustomed. If students do not experience whatsoever the use of algebra for argumentation, then only the "talented and brave" dare to do so, as the results of this research indicates.

## CONCLUSIONS and IMPLICATIONS

We do not underestimate the importance of the use of verbal or numeric representation in mathematical communication, especially in local one-time situations. However, we cannot overestimate the importance of an algebraic representation as a powerful, general, global, and comprehensive communication mean. Such means should not be the estate of the high achievers only; rather it should be accessible to every student. The current research indicates that this is not the case. In order to achieve this goal, the use of algebraic representation should be integrated into the teaching of algebra from the first stage, and students should gain experience in using algebra for argumentation and justification. If such a feat is implemented in schools, communication will be a real service to mathematics education rather then a lip service.

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