DEVELOPMENT OF UNDERSTANDING AND SELF-CONFIDENCE IN MATHEMATICS; GRADES 5–8

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This paper presents some preliminary results of the longitudinal aspect of a research project on self-confidence and understanding in mathematics. We have collected a survey data of 3057 fifth-graders and seventh-graders and a follow-up data of ten classes (191 pupils) one and a half years later. The longitudinal data indicates that the learning of mathematics is influenced by a pupil's mathematics-related beliefs, especially self-confidence. Pupils' level of understanding fractions also influences their developing understanding of infinity. These relationships between different variables depend also on pupils' gender and age.

INTRODUCTION

Pupils' conceptions on themselves as learners are strongly connected with what kind of general attitudes they have toward the discipline in question. Mathematics is a highly valued discipline in school, and therefore, pupils experience success in mathematics important. It has been observed, that pupils' beliefs on mathematics and on themselves as mathematics learners have a central role in their learning and success in mathematics (e.g. Schoenfeld 1992). The importance of beliefs in mathematics education is in concordance with the constructivist understanding of teaching and learning. We understand beliefs as "*an individual's understandings and feelings that shape the ways that the individual conceptualizes and engages in mathematical behavior*" (Schoenfeld 1992, 358). Mathematical beliefs can be divided into four main components: beliefs on mathematics, beliefs on oneself as a mathematics learner/applier, beliefs on teaching mathematics, and beliefs on learning mathematics (e.g. Lester *et al.* 1989).

Mathematics can be described as a combination of calculation skill and competence in mathematical reasoning, but neither of these alone characterizes mathematics. There is much research evidence that many pupils learn mathematics as a symbol manipulation without meaning (e.g. Resnick & Nelson – Le Gall 1987). Mathematical understanding can be distinguished from the neighbourhood concepts 'skill' and 'knowledge', for example as follows: Mathematical knowledge answers the question 'What', and one may remember mathematical facts. Mathematical skill answers the question 'How'; which includes, for example, the traditional calculation skill (procedural knowledge). Only mathematical understanding answers the 'Why' - question; it allows one to reason about mathematical statements. These are intertwined concepts, since understanding contains always knowledge and skill. Another view perceives mathematical understanding as a process that is fixed to a certain person, to a certain mathematical topic and to a special environment (Hiebert & Carpenter 1992).

Several studies have shown that beliefs about oneself have a remarkable connection with success in mathematics (e.g. Hannula & Malmivuori 1996, House 2000). However, to establish a causal relationship between self-concept and achievement is more problematic. In a literature review, Linnanmäki (2002) found out that in some studies no evidence for causality could be found, in other studies evidence was found for the causality from self-concept to achievement, while yet others found evidence for an opposite direction. The seemingly contradictory results indicate a developmental trend, where causality is mainly from achievement to self-concept during the first school years, it changes into a reciprocal linkage for the latter part of the comprehensive school, and in the upper secondary school level the causal direction is from self-concept to achievement (Chapman, Tunmer & Prochnow 2000). In her own study on self-concept and achievement in mathematics, Linnanmäki (2002) found evidence for this developmental trend in mathematics for grade 2 to grade 8 pupils. Looking at a more broadly defined concept, attitude, Ma and Kishor (1997) synthesised 113 survey studies of the relationship between attitude towards mathematics and achievement in mathematics. The causal direction of the relationship was from attitude to the achievement. Although the correlations were weak in the overall sample, they were stronger throughout grades 7 to 12, and in studies that had done separate analysis of male and female subjects.

Gender differences favouring males in confidence in mathematics are well recorded. Differences among teenagers have been reported, for example, by Bohlin (1994), Hannula and Malmivuori (1997), Pehkonen (1997), and Leder (1995). Vanayan *et al.* (1997) reported that already in grade 3 boys estimated themselves to be better in mathematics than girls. In mathematics achievement the results on gender differences are less clear. In IEA's large international studies the gender differences previously favoured boys (Husén 1967). In more recent studies the gender differences have decreased and in many countries disappeared completely (Beaton *et al*. 1997). However, robust gender differences are still found, for example, in some tasks on infinity (Hannula *et al*. 2002) and fractions (Hannula 2003).

The focus of this paper is to reveal the development on pupils' understanding and self-confidence from grade five to grade eight. Furthermore, the most important predictors of results are looked for.

METHOD

The study forms a part of a research project "Understanding and Self-Confidence in Mathematics" financed by the Academy of Finland (project #51019). The project contains a large survey with a statistical sample from the Finnish pupil population of grades 5 and 7 with 150 school classes and 3057 pupils. The survey was implemented fall 2001, and the information gathered was deepened with interviews and observation in 10 classes at convenient locations. In spring 2003 the questionnaire was administered a second time in these 10 classes. Altogether 101 pupils in the younger sample and 90 pupils in the older sample have answered our questionnaire twice. Because the number of classes in the longitudinal sample was small we need to control for possible deviations before making generalizations. The questionnaire was planned especially for the project. It contained, in addition to background variables, 19 mathematics tasks, estimations on success expectation and success confidence as well as a belief scale (25 items); see for more details Nurmi *et al.* (2003).

For the analyses of the longitudinal data we recoded the summary variables into three categories (lowest quartile, middle values, highest quartile) and used general linear model multivariate analyses (GLM Multivariate). It provides regression analysis and analysis of variance for multiple dependent variables by several covariates. The first measures of variables were considered as the covariates and the second measures of the same variables as the dependant variables. For each dependent variable, the overall η^2 (eta-squared) statistic is reported as a measure of the proportion of total variability attributable to the covariates. Statistically significant ($p < .01$) η^2 are also reported separately for each pair of covariate-dependent variable as a measure of the proportion of total variability attributable to the specific covariate.

ON RESULTS

In this paper we will report the results of the longitudinal data. There are many published papers on some specific features of the initial results for the research project: on infinity (Hannula *et al*. 2002), on confidence (Nurmi *et al*. 2003), and on number concept (Hannula 2002, 2003).

Here we shall use three sum variables for success in mathematics test (fractions, infinity, other tasks), and three sum variables for beliefs (self-confidence, success orientation, defence orientation). The mathematics variables are based on the analyses made in Hannula (2002). The two first ones (fractions, infinity) represent our indicator for understanding, and the third variable (other tasks) consists mainly of more computational tasks. Belief variables (self-confidence, success orientation, defence orientation) were constructed with the help of factor analyses from the belief scale (cf. Nurmi *et al.* 2003). The self-confidence factor consists of ten statements that were adopted from the self-confidence subscale of Fennema-Sherman Matehmatics Attitudes scales (Fennema & Sherman 1976). For background variables we shall control the effects of gender and grade.

The longitudinal sample of the fifth-graders did not differ much from the overall sample, if we look at the averages of all sum variables (there is only a small effect[1] favouring focus classes in fractions). However, the seventh grade longitudinal sample had better skills in mathematics (small to medium effects) and a slightly lower defence orientation (small effect) than the full sample. The largest effect was found in infinity, where the large sample of 7th graders had a mean score 4.7 (SD 2.9) while the longitudinal sample had a mean score 5.9. In our interpretations of the results we need to be aware of these deviations from the larger sample.

[1] Here we use the *d*-value = $|mean_1 - mean_2| / SD$ as a measure for difference and a convention established by Jacob Cohen (Cohen, 1988) that sets norms for “small,” (*d* = 20) “medium,” (*d* = 50) or “large” effects (*d* = 80).

Development observed

We noticed a rapid development in all achievement variables from grade 5 to grade 6. The development continued to be rapid in the domain of infinity after grade 7, but it slowed down in other variables, probably due to a ceiling effect[2] (Figure 1). In belief variables, we saw decline in self-confidence and success orientation together with an increase in defence orientation from grade 5 to grade 6 and from grade 7 to grade 8. However, grade 7 measures differed from grade 6 measures to another direction. Here we need to be aware of the differences between the younger and the older sample (Figure 2).

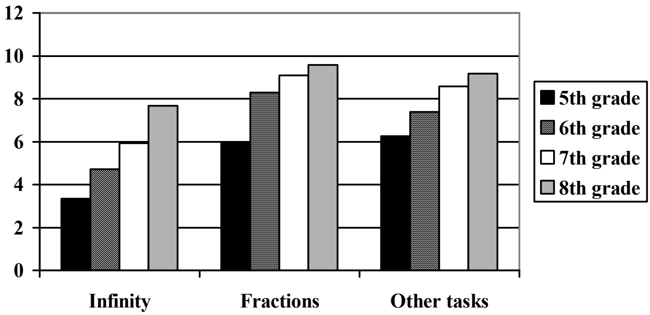

Figure 1. Development in mathematics achievement (infinity, fractions, other tasks).

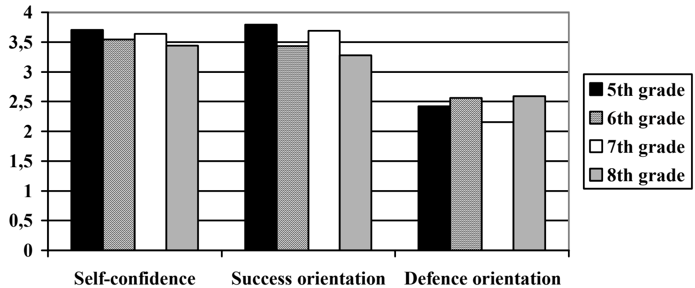

Figure 2. Development in belief variables (self-confidence, success orientation, defence orientation).

Close to half of variation in mathematics achievement was predicted by achievement in the previous test (Figure 3). Only a small part of this variation was attributable to pupil's achievement in the same topic in the previous test. Fractions was an important

[2] The theoretical maximums for variables are 14 (infinity), 13 (fractions), and 12 (other tasks).

predictor for success in infinity and other tasks. For the belief variables we see that self-confidence and success orientation were fairly well predicted by earlier beliefs (Figure 4). Self-confidence was a more important predictor of these two variables. Defence orientation seemed to be relatively unstable variable, and only 8 % of the variation in the later test could be explained.

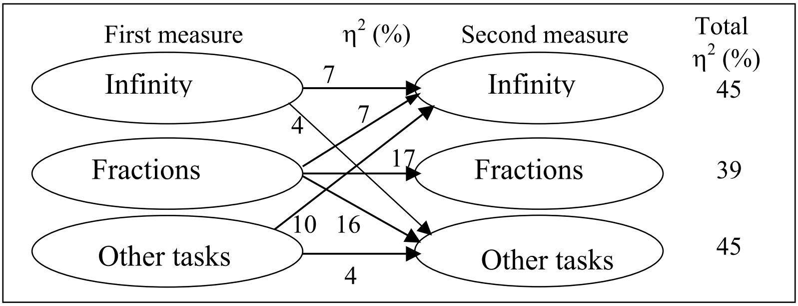

Figure 3. GLM Multivariate analyses of mathematics achievement

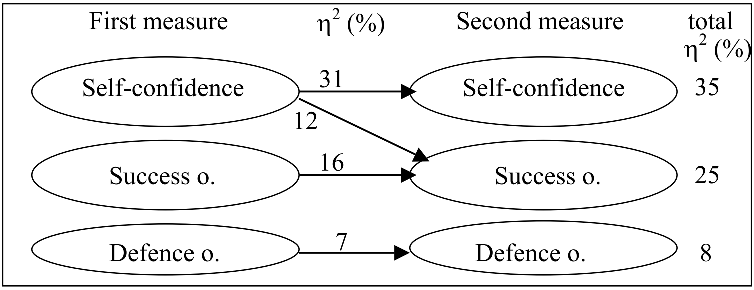

Figure 4. GLM Multivariate analyses of belief variables.

When beliefs and achievement were combined in one model, the explained variance for fractions increased from 39 % to 46 %. Minor increase (1 – 4 %-units) was observed for all other variables except success orientation. A statistically significant new effect was found from self-confidence to fractions (η^2 = 6 %). When analyses were made separately for boys and girls, and for the two age samples, we found some variations in the model:

- Infinity was more strongly predicted by fractions in older samples (η^2 = 13 % (girls), η^2 = 15 % (boys)) and infinity predicted fractions among older girls (η^2 = 16 %).
- Other tasks became a less stable variable in older sample.
- Gender and age had an influence on the stability of beliefs. Self-confidence in the younger sample was more stable among boys (η^2 = 11 % (girls), η^2 = 40 % (boys)), while in the older sample among girls ((η^2 = 37 % (girls), η^2 = 25 % (boys)). Success

orientation was more stable in older samples (η^2 = 35 % (girls), η^2 = 24 % (boys)) and defence orientation was more stable in older boys' sample (η^2 = 21 %).

- Self-confidence was strongly predicted by infinity in older girls' sample (η^2 = 36 %).

We made a more detailed analysis on the connection between fractions and infinity because the connection seemed to be an important one, not easy to explain theoretically, and because the underlying factor constructs were not very strong (Hannula, 2002). We made a GLM multivariate analysis on task level, and found that one fraction task (Figure 5) was by far the most important predictor, and that the effect was on two of the three infinity tasks: "How many numbers are there between numbers 0.8 and 1.1?" and "Which is the largest of numbers still smaller than one? How much does it differ from one?" Looking at the correlations between the two measures of these three variables we found the strongest correlations between the first measure of the fraction task and the later measures of the infinity tasks.

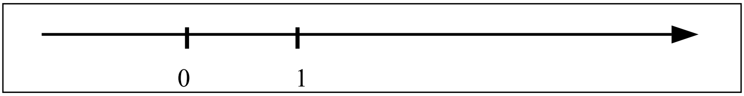

Figure 5. The task 2c: mark 3/4 on the number line.

CONCLUSION

There is significant development during grades 5 to 8 in topics measured in the mathematics test. Most notably the development is rapid both in the topics that are covered in the curriculum (fractions), but also in topics that are not dealt directly with in the curriculum (density of rational numbers). At the same time, there is – somewhat paradoxically – a negative development in beliefs. In both samples self-confidence and success orientation became lower in the second measurement, and defence orientation increased. Confusingly, the beliefs of the older sample at the beginning of the seventh grade were more positive than the results from the younger sample at the sixth grade. Partially the difference can be explained by the differences in the samples (the older sample deviating from the average towards more positive in both achievement and beliefs). This somewhat odd difference in the 6th and 7th graders' beliefs might also be partially due to the effects of time of the measurement, beliefs possibly declining by the end of spring term and increasing again by the beginning of a new term ('a fresh start').

Stability of the measured belief variables seems to be related to pupils' gender and age. Defence orientation is the least stable of the constructed belief variables, and we might even question the validity and usefulness of the variable. However, there seems to be a developmental trend for this orientation to become more stable among older boys. Possibly defensive approach to mathematics is something that develops slowly during school years and more typically for boys.

Mathematics achievement in this specific test can be predicted to a large extent from the pupils' past achievement in the same test. Most notably the pupils' success in fractions becomes an increasingly important predictor for future achievement in the used tasks on number concept, as the pupils grow older. As a specific task, pupils' ability to perceive a fraction as a number on a number line predicts their future understanding of density of number line. This finding highlights the importance of number line as a conceptual tool.

Self-confidence is another variable that seems to be an important predictor for future development. A pupil's self-confidence predicts largely the development of self-confidence in the future, but also the development of success orientation and achievement. A strong connection between self-confidence (and other beliefs on oneself) and mathematical achievement has been found also in earlier research (i.a. Hannula & Malmivuori 1997; Tartre & Fennema 1995).

Regarding the relationship between beliefs and achievement, our analyses suggest that the main causal direction already from grade 5 onwards is from self-concept to achievement. In the older sample we also found achievement in infinity to be a strong predictor for the development of the girls' self-confidence, which supports the hypotheses of a reciprocal linkage. Like the results of Ma and Kishor (1997), also our findings indicate that gender is an important variable in any analyses of causal relationship between affect and achievement.

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