# THE ALBERT SHANKER INSTITUTE

**From Best Research to What Works:
Improving the Teaching and Learning of Mathematics**

National Press Club
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**Greetings:**

**Eugenia Kemble**
Executive Director
Albert Shanker Institute

**Moderator:**

**Toni Cortese**
Executive Vice President
American Federation of Teachers

**Featured Speakers:**

**James Hiebert**
Robert J. Barkley Professor of Education
University of Delaware

**Deborah Loewenberg Ball**
William H. Payne Collegiate Professor of Education
and Director of Teacher Education, University of Michigan

EUGENIA KEMBLE:  (In progress) – continue to have your lunch.  I'm Eugenia Kemble.  I'm executive director of the Albert Shanker Institute, which, as most of you probably know, is named after Al Shanker, former president of the American Federation of Teachers.  I was just talking to Charlotte Frank over here, who goes back with the union and with Al Shanker, and it occurred to me I really must tell you all that he was a math teacher.  And when you look at our logo here for the Shanker Institute, that is a variation on the infinity symbol, which we thought was appropriate logo to identify the Shanker Institute.

What we're trying to do with these forums – and I think this is probably about the sixth one that we've had – is take an area that is very important for public policy in education and where we think significant work has been done, but where policy and practice lag significantly, and try to bring together some of the best researchers with those who can influence the use of that research.  And the way we select the people that we bring to these forums is through what I call a kind of informal peer review process.  We – and Burnie Bond here to my left did all the work on this for the Shanker Institute – talk to the people whom we believe (from what we can surface) know a lot about the research in a particular area.  And we make the rounds with them, asking them who they think the best researchers are.  And those names that rise to the top are the ones that we select for these forums.  Now that may not be a review process that would pass in a methodology test, but we have found it works pretty well.  So the people that you have here today are the result of that process.  And then we also invite by selection.  These are not wide open, public forums.  We try to invite people who know something about the field and who are in a position to influence it, and we've found that the result is very good discussions about important policy areas in education.

I want to introduce Toni Cortese, who is going to moderate this session.  She is the new executive vice president of the American Federation of Teachers elected this past July.  She comes from New York State United Teachers, where she was actually instrumental in the creation of that organization from a merger in that state between the NEA and the American Federation of Teachers, and has carried the education policy portfolio for that organization and now will carry it for the AFT.  She has served in many capacities that give her this expertise and where she's been able to bring leadership to the field, including the Learning First Alliance, representing the AFT there, the Partnership for 21st Century Skills, the National Board for Professional Teaching Standards when she was in New York State.  And she is also past chair of the AFT's pre-K through 12 Program and Policy Council, where policies that ultimately are passed by the organization get hatched.  So Toni is in a real leadership role at the AFT now, and she's taken a lot of interest in our forums.  And we're glad that she's here.

Before she starts this, we thought it might be useful if we could just go around the table and get your name and identification, not a speech about all the things you do, so that we can know who you are.  And we would ask you, when this begins, to please identify yourself.  We put the transcripts of these sessions up on our website, and we've found that they attract considerable interest, which shouldn't intimidate you.  But we do want to know who you are.  Okay, can we start with you over here?

**KRISTAN VAN HOOK**:  Sure.  I'm Kristen van Hook for the Teacher Advancement Program Foundation, which is a new initiative out of the Milken Family Foundation.

**RUTH WATTENBERG**:  Ruth Wattenberg, editor of the American Educator, AFT's educational issues magazine.

**AMY WOOTEN**:  Hi, I'm Amy Wooten.  I am at the National Board of Professional Teaching Standards.

**RANDY GARTON**:  Randy Garton, Shanker Institute.

**LISA HANSEL**:  Lisa Hansel, also with American Educator.

**JUDY DAVID**:  Judy David with Pal-tech.  I'm working on the Head Start contract.

**LINDA JAGIELO**:  Linda Jagielo with Head Start.

**CHARLOTTE FRANK**:  Charlotte Frank, McGraw-Hill Companies.

**JANIS SOMERVILLE**:  Jan Somerville of the National Associate of University Assistant Heads and the Education Trust.

**VANCE ABLOTT**:  Vance Ablott, the executive director of Triangle Coalition.

**JUDY WURTZEL**:  Judy Wurtzel at the Aspen Institute.

**PATRICIA ROBINSON**:  Patricia Robinson.  I'm a math supervisor at Arlington Public Schools.

**ALICE GILL**:  Alice Gill, AFT Education Issues Department.

**GERI ANDERSEN-NIELSON**:  I'm Geri Andersen-Nielson.  I'm a retired teacher and a math education consultant.

**KATHLEEN MORRIS**:  Kathleen Morris, the American Association for the Advancement of Sciences.

**DENISE MCKEON**:  Denise McKeon, NEA researcher.

**GERALD SROUFE**:  Gerry Sroufe at the American Educational Research Association.

**STEW MAGNUSON**:  Stew Magnuson, Education Daily.

**JANE LEIBBRAND**:  Jane Leibbrand with the National Council for Accreditation of Teacher Education.

**TANYA SHUY**:  Tanya Shuy, the National Institute for Literacy.

**KIYUK KUK**:  Kiyuk Kuk, Korean Daily.

**ARDEN BEMENT**:  Arden Bement, director of the National Science Foundation.  I think I was invited not because of my experience in math education, but for my experience investing.

(Laughter.)

**KEN KREHBIEL**:  I'm Ken Krehbiel.  I'm director of communications for the National Council of Teachers of Mathematics.

**BURNIE BOND**: Burnie Bond, Albert Shanker Institute.

**TONI CORTESE**:  And I'm Toni Cortese.  I'm the executive vice president of the American Federation of Teachers, and I'm also a board member of the Albert Shanker Institute.  I've known Charlotte Frank for quite a while, and when this meeting started, she and I were saying that there were vague memories of having discussed some of the very same things 20 years ago about how to improve math education and how to improve science education.  And except for one little thing being launched into space, which created a lot of fury for a few years, there really hasn't been a lot of the focus and intensity around math and, I would also add in to that certainly, around science.  So every time we have an international comparison, as we did with PISA and the latest TIMSS, there's a flurry of activity about what we ought to do.  And again, we look at where the United States ranks.  And even though in the TIMSS study our eighth graders improved a little, we're not in the top 10 percent of the countries that took it, and our students are not scoring there.

So if we look at kind of the mediocre performance of our students, we have to ask ourselves why that is.  And I think our two speakers today hopefully will help us determine what the reason is.  We do know one thing, however, that we can't have good math instruction going on in our schools, and it can't be taught well unless our teachers know more about math than they do and more deeply about mathematics.  And even when we look at the elementary scores, we know there's nothing really very elementary about what K through 6 teachers have to teach in the schools.  In fact, it's very fundamental in terms of understanding concepts to probably how well they'll do in their future endeavors in the whole mathematics area and whether they'll be even interested in future endeavors.

The second part of it, I think, has to do with how we define what it is that teachers should know in order to be able to teach math.  We have generally across this country had

it defined by almost every teacher education institution, but not in terms of the knowledge base, but more, you shall have nine credits in this and 12 credits in that and 15 credits in something else.  And because there's not a centralized type of curriculum, that could mean very different things depending on which teacher preparation program you happen to have gone through.  So I think that's another side of it that we want to look at today and what we can do in terms of teacher preparation and also what we can do to help our students achieve better in terms of instructional practices and things that they might need in our schools.  When we just look at the NCLB results, we know that a lot of our students are not reaching the proficient level.  But again, that proficient level depends on what state you happen to be talking about and what we don't seem to have is any kind of national standard to judge that by.  And I guess that's why we go back to the TIMSS and the PISA studies.  So that certainly is lacking.

What we need to do today is to address some of those issues, but to actually ask ourselves, what can we do?  What kind of action plan, what will it take to actually move some change in the way that we teach math in our schools?  Certainly professional development is an important part of that.  Teacher education is an important part of that.  But there's also another piece that would provide the will and the resources to be able to really get serious about a problem that's plagued us for decades.  So what we're going to do here today is we have two very distinguished speakers, and we're going to give them each about 20 minutes to talk about their particular facet on this subject.  And then we'll open it up for questions for all of you.

Our first speaker is James Hiebert, and he's the Robert J. Barkley professor of education at the University of Delaware.  He teaches actually in programs of teacher preparation – so maybe he'll shed some light on what it is that teachers should know – and professional development and doctoral studies.  His research and professional interests focus on mathematics teaching and learning in the classroom.  I think what's even more interesting is that he directed the mathematics portion of the TIMSS 1999 video study of math and science teaching in seven countries.  So we're very glad to welcome you here today, and we look forward to your comments.

**JAMES HIEBERT**:  Thanks, Toni, and thanks to the Albert Shanker Institute and Burnie for organizing this nice forum.  It's a pleasure to be here and to share some thoughts with you about the problems that Toni identified.  The sort of bottom-line problem that we're all concerned about is student achievement in mathematics.  And Deborah and I are going to be talking about some of the possible explanations for the kind of achievement American students have shown.  And as Toni mentioned, this isn't a new problem.  People have been addressing this problem for a considerable length of time.  So I would like to talk mostly about teaching, and Deborah's comments will focus mostly on teachers and teacher knowledge.  What I'd like to do is draw on research conducted in the United States and also and especially international comparisons of mathematics teaching in different countries.

I'd like to begin by just reminding us of a principle that often gets lost in discussions of teaching and what counts as good teaching and so on.  And that is that

teachers teach in order to help students learn something, and this means in general that teaching isn't universally effective or ineffective. Teaching is more or less effective for particular kinds of learning goals. So when we talk about what counts as good teaching, we also need to include in that statement good teaching for what. What are we trying to accomplish? And over the last few years, there have been debates in the United States and in other countries actually as well about what sort of learning goals should be valued most highly. And those of you that have been reading or involved in these debates know that they can get quite heated at times. And they often are – if you probe – about some differences in the kind of goals that we value for students.

I'm going to build on a report that some of you or maybe most of you are familiar with that came out in 2001, sponsored by the National Research Council called "Adding It Up," a report that was compiled by stakeholders in this national debate about mathematics learning and teaching and is, in a sense, a consensus document about what goals we should be headed for in this country. Here's just a quick summary that I think will be sufficient for the comments I'm going to make. The goal that is laid out in this book is much more ambitious than just the skills and understanding distinctions that usually head up the headlines when people discuss what we should be doing in mathematics. The report suggests that although both of those are important, there are additional proficiencies that students should acquire, many of them surround issues of mathematical reasoning and problem solving skills. The rather unique and, I think, significant part of this document and its description of this learning goal is the fact that these five strands that have been identified are strengthened by tight integration, by intertwining learning of skills, concepts, problem solving. It's not the case that we do skills on Monday, understanding on Tuesday, reasoning on Wednesday, and so on, but that we try and understand how to design the curriculum and teaching so that in every lesson, there is some attention to more than one of these strands.

With that said, I'd like to offer some overview comments about what we know from research with respect to the kinds of teaching that support this learning goal. First of all, we don't know – at least yet – what the best teaching method or methods might be to help students accomplish this learning goal of mathematical proficiency. And there are a number of reasons that we don't yet know this. One, of course, is that this goal itself has been stated rather recently so there have not been a huge number of studies conducted with this particular goal in mind. But there are many additional problems, which make it difficult to isolate particular methods of teaching that are going to be the most effective for helping students accomplish this. One problem is that there are huge numbers of factors which affect students' achievement. And I hear about many of these as I talk about the international comparisons because most people are quick to point out that different cultures and different countries support education in different ways and are probably partly responsible for the level of mathematics achievement in these different countries. So there are a huge number of factors that go into explaining and accounting for the level of achievement of students.

Isolating particular features of teaching that are especially effective for this particular learning goal is a very difficult task. I don't think it's inherently impossible,

but I think it's difficult.  One of the reasons I think it's not impossible is I think we're on the way to beginning to identify some features that look like they will be important in any emerging methods that look to be effective for accomplishing this goal.  So one thing we're finding is that not all teaching methods are equal.  So saying that we don't yet know what the best methods are is not the same as saying anything goes.  Different teaching methods are differentially effective.  Not all of them are equally effective, and there are two key features that I want to identify as being features that I can claim with some level of confidence will be part of methods of teaching that help students achieve well, especially if we have this ambitious learning goal in mind.

One of these features is explicit attention to relationships – relationships between different facts students are learning, between the procedures they're learning, and the underlying concepts.  The important thing is seeing how things are connected in mathematics.  And this explicit attention might come from the teacher.  It might come from the students.  There are a lot of ways in which students' attention might be drawn to relationships.  The second is an opportunity during mathematics for students to wrestle with, grapple with, struggle, become sort of frustrated about temporarily, get into the sort of topic that they're studying.  This is different than receiving small bits of information at a time and ingesting each one without recognizing any particular trouble.  Asking students to solve challenging problems is one way to get them engaged in this kind of thinking.

Okay, what I'd like to do now is to ask about whether the international comparisons between the kinds of teaching we see in the United States and the kinds of teaching in other countries have anything to say about these particular features because, you know, high-achieving – scoring high on the international achievement test doesn't mean students have this mathematical proficiency that I mentioned earlier.  They might, but those tests aren't direct measures of this.  So one of the questions is: could we learn anything about these kinds of features if we look at how teachers teach in other countries?  And the answer, I think, is yes.  But I'm going to show you an example of a finding that I think is illuminating in terms of how we teach compared to other countries.

So I'm going to talk a little about his video study that was initiated in 1999, completed in 2002, and had some very simple goals.  One was just to describe eighth grade math and science teaching in different countries.  The science results are almost ready to be released, but I'll just be talking about the math results.  And the second was to create a digital library of public use videos of classroom lessons that can be used to study teaching.  These aren't videos meant to be imitated.  They're not exemplars of excellent teaching in each of these countries.  These videos represent average teaching in the countries that were included in the study, and they're intended for use by teachers to analyze and examine and ask themselves questions about, not to assume that they're supposed to imitate what they see from teachers in other countries.

Okay, the countries that were included in the study are shown on this slide.  All of them are higher achieving than the United States.  These scores are based on the 1995 achievement results, which were the results used to select the countries for the study,

which was begun in 1999.  An overview of the results, and then I'd like to show an example of a result.  One thing we found very quickly is that each of these countries has their own method of teaching mathematics.  It's not as if all the higher achieving countries went off somewhere and learned how to teach and then we were left out and didn't get that memo, and now we – oh, that's how you teach.  That's not the case.  Each country has developed their own method of teaching, and it's distinctive in some ways from methods in other countries.  One thing this means is that there isn't one way of teaching that's associated with high achievement.  So that opens the sort of playing field in terms of wondering about what kind of methods might be appropriate to use in the United States.

But there were a few features that all of the high achieving countries shared, and these are the ones that I think are the most interesting to examine.  And so I'd like to pull out one of these results and to just tell you what I think it means in terms of the sort of one possible explanation for the level of achievement we see in the United States.  Okay, here's the sample result, and it was generated by looking at all the kind of math problems students solved in lessons.  By the way, students in all countries during math class, solve problems.  That's what they do 80 percent of the time.  They're busy solving a math problem.  Sometimes the problems are hard.  Sometimes they're real easy, but eighth graders don't hear long lectures from teachers.  They basically work at solving math problems, sometime during exchanges with teachers.  But we decided to understand what was going on in these lessons.  We needed to understand the kind of problems that students were solving.

We could classify every problem into one of three types.  Stating concepts are not very frequent, but they're the kind where a teacher will say, can you please tell me what the two important properties are of an equilateral triangle.  So students have memorized something that they'll repeat back to the teacher.  So it's the stating part of this that's the key to this particular category.  The next two are the most interesting, and I'd like to focus on those.  Using procedures is the most common kind of problem in almost all countries.  If you remember to any math course you took, you did a lot of this.  Teachers would give you 10 problems.  You'd kind of know how to do them.  You would practice doing the procedure to solve the problem – not much creativity, but getting good at doing something.

And then a third kind of problem, which are problems that could be hypothesized to support this mathematical proficiency goal.  They're problems where students are asked to look for connections, to develop relationships, to look for patterns, to make conjectures, a whole variety of interesting mathematical reasoning activities wrapped up with making connections.  Because these are somewhat different, let me just show you quick examples, two of them drawn from the videotapes.  The first example says, solve these two equations, and describe what is different about their solutions.  What makes this a making connections problem, using our term, is that students were asked to describe what was different about those two solutions.  Just solving each one independently would put it into the other category, but it was asking students to look back and forth, figure out what's the same or different.  The second problem finding a patter

for the sum of the interior angles of a polygon – it's the finding a pattern part that makes this a making connections problem.  So the importance of these kinds of problems is in the kind of reasoning that's required to complete them.

Now, here is the data on the kind of problems presented to students.  These are – you could infer that the problem or the teacher intended students to engage in either using procedures or making connections by the way in which the problem was presented.  And there are two interesting results on this slide that I want to point out.  One is that Hong Kong and Japan are on the opposite ends of the spectrum here, and these were the two highest achieving countries.  The second interesting thing is that the U.S. is right in the middle.  So the U.S., on this measure, doesn't look substantially different that any of the other countries. So we asked ourselves, how this could be?  How could be Japan and Hong Kong differ so dramatically on a feature which we thought would correlate with level of achievement?  And what's so different about the U.S.?  It looks like teachers in the U.S. are doing pretty much the same thing as teachers in other countries.

To find out, we looked again at the problems, in particular at how the problems were worked on during the lesson. So we coded each problem a second time, and this is where teaching really matters – because, although problems are presented a certain way in the curriculum, teachers have a lot of control over how the problems are implemented. They can be transformed.  So here's an example.  Here's a problem that I mentioned earlier – finding a pattern for the sum of the interior angles of a polygon. If a teacher was going to work on this as a making connections problem with students, they would ask students, for example, to measure the sum of the angles of three-, four-, and five-sided polygons, and then maybe predict what's going to happen with six sides.  If students are in eighth grade, they could say, can you figure out a way to find the sum if all I give you is the number of sides?  Can you do that quickly?  Is there a formula maybe that you can develop, and so on?  Alternatively, you could work on this problem by saying, hey kids, there is an easy way to find the answer.  It's this.  Go ahead and work on 12 problems like this. Use the formula. And this turns into arithmetic problem when students are presented with the problem.

So now, what do we find in countries around the world when we look at how problems are worked on?  So again, this is the slide we just saw, the percentage of problems as they're presented.  And what I'd like you to focus on now are the blue – the blue data.  So let's look at what happens to the making connections problems when they're implemented in the classroom.  How do teachers work on these with the students?  This is very, very telling from my point of view. What you see now is a very similar pattern across the highest achieving countries. The four countries that were highest achieving in our sample look very much the same on this. But, in the U.S., there are essentially no problems that end up getting worked on in this making connections way. Teachers often step in and do the interesting mathematical work for the students, probably because they sense students are confused or frustrated or getting antsy.  And so they think it's their job to step in. It's not that they're trying to subvert students' learning; at least I'm convinced they're not after talking with lots of teachers about this. So this is

an important finding for us, I think, in trying to understand what kind of teaching is going on in the U.S. and how it differs from high achieving countries.

So, in conclusion, what I'd like to do is to look just briefly again at the comparison between Japan and Hong Kong because, remember, the problems that were presented – the percentage of different types of problems that were presented were very different. But when you look at the data on whether they were working on them as making connections problems, they look very much the same. So there are some similarities across these countries. They attend explicitly to connections among facts, procedures, and skills; and they provide some opportunities for students to wrestle with challenging problems. So these two features seem to be present in both of these countries even though their style of teaching is dramatically different. There are differences in the relative emphasis that they give to skills versus understanding versus reasoning. In Hong Kong, a much higher percentage of time is devoted to practicing skills. In that way, they look more like the U.S. But then there are times during the lesson where they do something very different, and kids work very hard at solving problems independently, creatively, analytically, and so on.

And finally there are – if you watch videotapes of Japan and Hong Kong, there are just striking differences in the roles that teachers play. In many of the lessons we have from Japan, the teacher presents a problem to the student and then steps back and lets the student struggle with it, work on it. Many of you might have seen those tapes. In Hong Kong, the teacher plays a much more active, directive role. The teacher is up in front of the room, directing students, asking particular questions. But if you sort of probe beneath those surface features, you see lots of interesting, challenging mathematics going on.

So, before I turn this over to Deborah, let me mention that there are several things we need to understand much more about. One is the sort of exact nature and configuration of these features that promote mathematical proficiency. Something I would like to do is to understand better these differences between Japan and Hong Kong because they're striking at the surface level – but, underneath, there's something that's similar about them. I think we would make some headway in understanding the range of possible teaching methods we could use by understanding those differences and similarities. And a second question is, what do teachers need to know to be able to teach like this, either like the Hong Kong teachers do or the teachers in any of these other high performing countries? And let me pause there because that's sort of the story that Deborah is going to pick up on.

**MS. CORTESE**: Thank you very much. Our next speaker is Deborah Loewenberg Ball. She's the William H. Payne collegiate professor of mathematics education and teacher education and director of teacher education at the University of Michigan. She is co-director of the Study of Instructional Improvement, which is a large, longitudinal study of the effort to improve instruction and learning in reading, language arts, and mathematics in high poverty schools. She also directs the Mathematics Teaching and Learning to Teach project. She has published in many articles on teacher

learning and teacher education as well as the role of subject matter in teaching and learning to teach. So we're very pleased to have her here to give the perspective of the teacher in the classroom.

**DEBORAH LOEWENBERG BALL**: Thank you. As Jim said, we're both very pleased to be here. And we had a chance to talk in advance with Burnie, and we did try to design this so that Jim sets me up. And what I'm going to take up now is the question that he ended with, and that is, what is it that we know at this point about what teachers need to know in order to teach mathematics? So I'm going to take us into that space. And I want to start by clarifying the question. I think it will fit well with what we've just been discussing.

On one hand, we hear this issue spoken about as though it's a problem of teacher quality. In the NCLB [No Child Left Behind] legislation, we see a mandate that each child must have a highly qualified teacher, and it's hard to argue with that. The interesting point is that we still lack agreement about what it might mean to be highly qualified. So what I want to try to talk about today is an approach to trying to attack that question, beginning with practice and attempting to answer the question by linking a practiced-based approach to studying what teachers might need to what their students would learn. In order to do that, it's important to keep in mind that the real problem, as Jim said, that we're trying to solve is not how to have a teaching force that just knows mathematics better or is somehow of greater quality, but a teaching force that is able to work to create high quality mathematics teaching for each student.

Because of that, you might say, then why are we talking about teacher knowledge? Let's keep talking about teaching. The reason that we talk about teacher knowledge is that there's substantial evidence that how teachers know mathematics and, I would add here, the ability they have to use that mathematical knowledge in the service of the work of teaching makes a difference for the quality of what they can do with students. And that's the frame with which I'd like to begin. So, to be clear, although you said that I would be speaking about teachers, in fact, in talking about teachers, I'd like us to keep our collective eye on teaching – that is, to ask the question, to do the work of teaching students well, what do teachers have to have at their disposal?

So let's just take a moment to think about how the framing is usually done around the question of teachers' knowledge or teacher quality, and there are a couple of basic points to be made that about which I think we can agree. One is what Toni already said. I think there is substantial evidence that teachers in the United States don't know mathematics well enough, but I'm not going to say much on this are right now. There is a range from pretty good evidence about what teachers have not had the opportunity to learn (and I want to emphasize that this is an important way to understand the problem) to anecdotes that people like to tell about some teacher somewhere who misspoke or taught things the wrong thing. But there is a substantial body of evidence that teachers lack the knowledge that they should have. I do want to say that whenever we talk about this, it's important to acknowledge two things. One I've already mentioned, and that is that it's a question about what teachers have had opportunities to learn, not anything about the basic

level of intelligence of teachers. And second, if you were to ask non-teachers in our population the sorts of questions I'm about to show you, they would understand them much less well than teachers do. So this is in the context of a country that does not educate any of its population well in mathematics. Teachers have been among the best educated mathematically. I want us to understand that when we say teachers don't know mathematics well; it's because they're being asked to do something with mathematics that most well educated people are never asked to do. And that is to help other people learn mathematics. Another problem that is worth mentioning alongside this one is that we have a particular problem in schools of the greatest need. It's most likely that students in high poverty or urban schools will have a teacher who is entirely unprepared in mathematics and has had no opportunity to study mathematics for teaching, may not him- or herself be oriented toward mathematics knowledge for teaching. And this adds to the intensity of the problem we see.

Now these are problems that it's hard to argue about, although we may discuss them differently. But what's important is to notice how we tend to approach the solution to these problems. And there are two basic ways we tend to think about this currently. One option, which is logical, is to say let's increase the number of courses required to become certified or let's increase the rigor somehow of the mathematics that teachers study, which sounds reasonable. And yet there is no evidence that just by adding course work or by saying, oh, let's make it tougher, that this will in fact do anything to increase someone's ability to hear a student's mathematical point and understand whether it's valid or not or be able to come up with a suitable representation to making a mathematical point clear to a student. It is not clear that taking more mathematics courses automatically guarantees that kind of mathematical facility, and most of what I'll do today will focus on that question. What is it that one has to do mathematically? I'm not saying that this option isn't part of the solution. But, by itself, a quantitative solution – for example, just increasing teachers' math coursework requirements – doesn't address the question of what exactly teachers need to be able to do with mathematics.

The second popular policy solution, which also makes sense on the surface, is to say, look, there are a lot of people out there who have managed to get through our system and have learned mathematics. Let's create incentives for these people to enter the work of teaching and therefore increase the intensity of mathematical training among the teaching force. There are two points that are worth noticing here. First of all, that is similar to my first point, it's not obvious that someone who's trained as an economist or who has spent life as an accountant or has been working as an engineer at Ford knows mathematics in ways that will equip that person to teach mathematics to students. Those are different kinds of mathematical work. And when we say mathematically trained, the lack of distinction about what we mean by training in mathematics is worth noticing. Someone could be extremely good at using mathematics as a physicist and be nowhere close to being able to use mathematics as a teacher. Second, there is a sheer scale problem, and I know my colleagues in this room are more able than I to speak to this. But we have a teaching force in this country of roughly 3.6 million teachers, a very large fraction of whom teach mathematics. I don't care how far you want to look at the recruitment question. It's a good idea to recruit well educated people into teaching, but

what we have is a much different problem. And that is how to figure out what perfectly ordinary people need to know to teach children well and to develop a system of professional education such that we can have a teaching force full of people who are equipped to teach mathematics. So while this is a perfectly good strategy, it's a tiny drop in the bucket given the scale issue. Teaching is the largest occupation in this country. Recruitment will not solve the problem. You will not be able completely to replace the current teaching force even if you wanted to argue that that an alternative teaching force would know mathematics for teaching, which I doubt it would.

So my colleagues and I have taken an approach which is a bit different. We are trying to understand exactly what is it that someone would have to know to teach mathematics well. And rather than looking at standards or looking at the curriculum, which are reasonable things to do, we started by asking, in some ways like Jim and his colleagues, what is it that someone does when they teach mathematics – to begin to answer the teacher knowledge question by looking at the work. So we study instruction, and as we study teachers at work teaching mathematics, we try to do what we call identify the mathematical work in which teachers engage. We try to notice the things one does routinely in the course of classroom work that requires mathematical skill, for example, hearing a student's response and judging its validity, being able to give the next example to a student when a student gives an answer, being able to select a particular representation out of the textbook, or being able to tell that the textbook uses an inaccurate one. Those are all things teachers do. Then we try to identify the mathematical knowledge one would need to carry out these tasks that teachers do routinely. So you can see we're working kind of bottom-up, from the work toward a theory of what teachers have to know.

We have also been engaged in developing measures of teachers' mathematical knowledge – and I'll talk more about that in a moment – as a means to test our own ideas from the work we've been doing. We began qualitatively, with looking at teaching, and ended by trying to create measures and correlating those with what a larger population of teachers does with students.

So let me just clarify a couple of terms. When I say mathematical knowledge for teaching, there are a couple things worth saying here. First of all, I mean not just knowledge – although that's the word I'm using here. I mean things like skills, habits of mind, orientation to mathematics, as well as knowledge, that are entailed; that is, used and required in order to do the work. So then you might say, well, what do you mean by the work of teaching? And although I just gave you some examples, I'd like to broaden your sense of what I mean by saying I don't only mean what teachers do inside their classrooms with students. I also mean the tasks they engage in as they carry out the professional work for which they're responsible. So that would include speaking to parents about a student's progress, talking with community members about the curriculum, defending what one is doing to one's principal, talking with colleagues. Teachers do many things, and many of these tasks involve mathematical skill.

So now I'm going to make you do a little work so that you understand more concretely what I mean. I'm going to use multiplication of decimals to help you see the difference between knowing mathematics as any well-educated adult, we would hope, might know it, and what someone who is going to teach mathematics would have to know. And I'm hoping to convince you that – before even talking about how to teach or what to know about students –the kind of mathematics that someone needs to teach is considerably more than would be required for someone who wouldn't, which I think echoes something Toni said.

So first I'd like you to just multiply these two numbers and get an answer: 3.5 x 2.5  =  ____.   (Pause.)

What did you get? (Off-mike responses from audience.) Excuse me? Eight-point – what did you get – 8.75?

So we could stop here and have a discussion of possibly different answers you got or what method you used.  I'm not going to do that right now, but I'm going to refer to this from here on out as the kind of knowledge we would expect someone who finished public school in this country to have; that is, to be able to calculate this. And there are lots of ways you might have done that, and I'm not going to dwell on it.  I'm going to call this for the moment just common knowledge of mathematics. And a teacher who is going to teach let's say fifth grade or sixth grade where this might get taught needs to know what you just did. It would be ludicrous to imagine someone teaching who couldn't get the answer to that problem.

But teachers do much more than solve problems; in fact, they rarely are doing problems themselves. They're doing other things. So let's imagine other things teachers do. One thing that teachers do is that their students don't get 8.75 quite a reasonable amount of the time, so they're in the position of looking at other answers that students get and trying to figure out what mathematical steps might have produced these other answers. So I want you to try that out.

I'm going to show you two other answers that students might get, and I'm not asking anything about student thinking; I'm only asking you to see if you can figure out what mathematical steps were taken that produced these two answers.  (Pause.)

(a)      3.5                              (b)      3.5
        x  2.5                                   x  2.5
         255                                      62.5
          80_
        10.55

Anyone want to try one of these?  To explain it?  (Pause.)

**MS. ?**:  I'll do one.

**MS. BALL**:  You'll do one – okay, which one?

**MS.  ?**:  Well, they just took '5 x 5" and put down the 25 with the decimal point in between, and then '2 x 3' is 6 and they put the 6 on the front.

**MS. BALL**:  Why might they put the decimal point in between?

**MS. ?**:  It's because it's there in two places.  Sure.

**MS. BALL**:  So you could just carry it down possibly. Okay, so they multiplied down. Do you see that?

Anyone see A?  (Pause.) Is there anyone in the room who sees it? So let me pause and make a point about that.

A teacher needs to see that rather quickly, needs to have a kind of fluency. It's not just that you're puzzling about something. A teacher actually can't afford the time to stand there and look at that and puzzle it out. Maybe if the teacher is sitting at home grading papers, which is something that teachers do, they might have the time to think about it. But, in fact, they need a kind of speed for sizing up student responses, a kind of fluency – it's a kind of mathematical analysis that teachers do all of the time and that someone who doesn't teach doesn't have to do. They could just ask the student what he or she did, but I caution you before you conclude that that's what a teacher has the luxury of doing. In classes of 30 and 35, teachers are not going to interview students at every moment. I certainly don't do that when I teach. It also equips me a lot better to figure out what question to ask a student next if I have a guess about what might he or she might have been doing. Furthermore, when I'm grading papers at night and trying to figure out what to do the next day, it's quite helpful if I'm not in the dark about what might have produced the answer.

So do you want me to tell you what it was or do you want to – for me to leave it with you? So if you multiply 5 x 5 – on the right – and get 25, you would carry the 2.  If you add the 2 to the 3, that gives you 5 – 5 x 5 is 25, and if you repeat that in the next row – 5 x 2 is 10, carry the 1, 1 + 3 is 4 x 2 is 8. The student has added the carry, so to speak, before multiplying.

Now you know that you don't do that. You know that what you should do is first multiply then add the carry, so we can go down a little further here and say a teacher not only has to remember that that's the procedure, but actually would have to be able to explain, why do we multiply in this procedure before we add? Why does that matter? It doesn't matter in addition– you can add the carry before or after. It doesn't make any difference.

So we won't go there right now, but that's another aspect that a teacher has to know that the average person wouldn't. And being able to tell quickly that it was a carrying error would help equip a teacher to figure out what to ask or what to do to reach

that student. And similarly here, it would be helpful to know why do you have, quote, as many decimal places in the answer; like there should be – you know, the decimal – the number of decimal places should be two, among the other errors there, so why is that? We won't take that up.

Now students don't only get things wrong; they also get things right. So let's take another stab at what teachers have to do with this very same problem. Students might multiply 3.5 x 2.5 and get 8.75 but do it using an approach or a method that you've never seen before. And you might really have to worry about whether it's a fluke or it's kind of interesting. Is it worth discussing? Is it better to say don't do that? Whatever you're going to do pedagogically, it's useful to know what mathematical steps were involved.

So the questions here are whether there a method at all, and whether what the student has done here something that is a method would work for all decimal numbers or is there something peculiar about these numbers that made that work. So take just a moment to see if you can figure out what's going on here. (Pause.)

(a)  $$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline .25 \\ 1.5 \\ 1 \\ \underline{6\ \ \ } \\ 8.75 \end{array}$$

(b)  $$\begin{array}{r} 3.5 \\ \times\ 2.5 \\ \hline 1.25 \\ \underline{7.5\ \ } \\ 8.75 \end{array}$$

Anyone want to try one? Alice, you look like you want to –

**ALICE GILL**: The student has actually said what's happened. 0.5 x 0.5 is .25, and then what's half of three? That's 1-1/2. And then again, 2 x ½ is 1, and 2 x 3 is 6, so that works.

**MS. BALL:** Okay, so Alice has a particular way of narrating it, and you might have narrated it differently, but each part of that problem is being multiplied separately, basically like the way you might multiply in algebra. So that would be one way you could establish that it works. In general, it's essentially using the distributive property to multiply – but it's important to be able to figure out what's going on and also whether it would work in general.

And we won't take up the other right now. I'll just leave it with you with the point that when teachers see unusual methods, they need to figure out if these methods are indeed methods, and if they work in general, even to make a decision about what to do next. So we're still not up to teaching; we're just up to the point of being able to make choices as a teacher that are valid.

Another thing that teachers do is use representations, and we won't actually get to do this, but I just want to signal this as another extremely common part of teachers' work.

Teachers make up representations; that is, diagrams, models, metaphors to help students understand content – they make them up. But even if they don't make them up, they're in the textbooks, and you know what? The textbooks are wrong some of the time, and it's useful to know that that model that the authors put in the textbook is dead wrong and you'd better not use it, or there's something about it that you have to understand how it works in order for students to understand what's being highlighted. And if you don't understand what's being demonstrated with a diagram, students and the teacher can be quite confused. So no matter how the teacher is interacting with the text, this is a huge part of the work.

So here are two possible diagrams that could be used to represent 3.5 x 2.5. If you were a teacher, you would have to decide if they are in fact good representations of the multiplication problem. And you might want to ask yourself the question – since it's not visible in both drawings – where's the 3.5, for example, on the left? The problem is supposed to be 3.5 x 2.5. Where is the 3.5? Where is the 2.5 in the problem on the right? Where is the 8.75 in either of them, and you might want to be clear about that before you started to use it with students or ask the students what was going on. I'm not going to actually explain this one to you, but this work is part of what I mean when I say teachers do mathematical work that's well beyond where we started. All I asked you to do was find the answer. Being able to find the answer is nowhere close to enough mathematical knowledge.

(a)

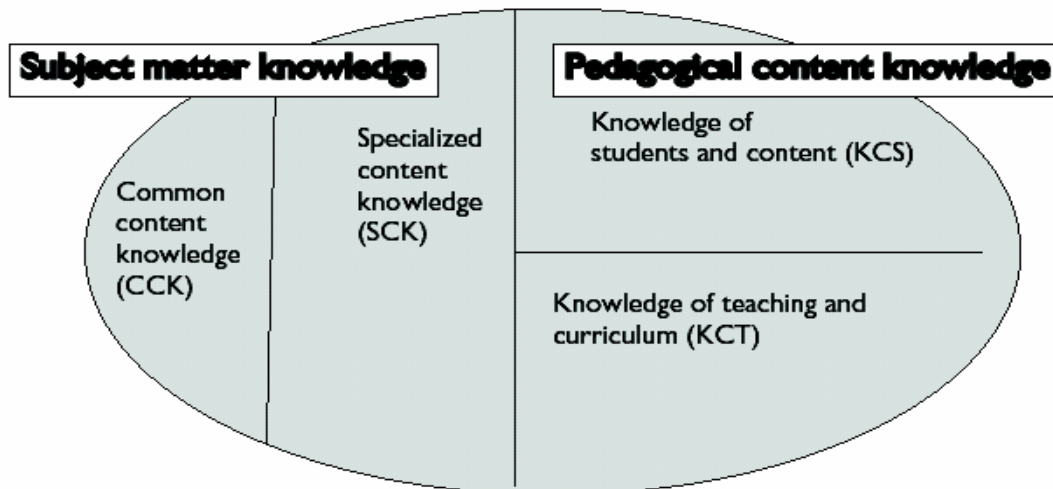6.0   1.5

1.0   .25

(b)

3.5   3.5   3.5
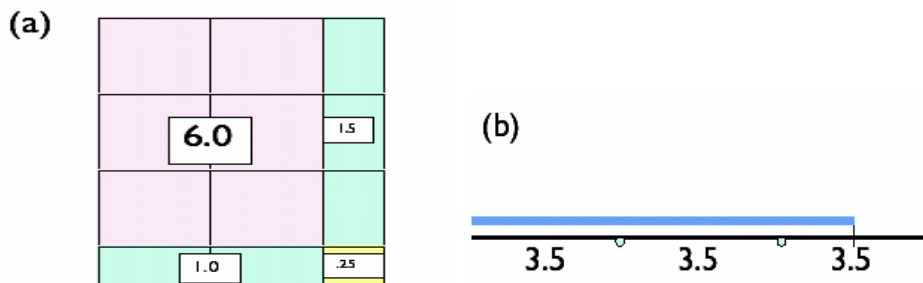
I haven't, however – I want to be clear – said anything about knowing about students or how they learn this content. I've not said anything about how to teach it well. All I'm trying to show you is the kind of mathematical knowledge that we're learning teachers would need in order to carry out the tasks they do as teachers. So let's talk for a moment about what hypotheses there might be about what knowledge is needed for teaching, and I'll again contrast what is sort of prevalent out there with the way my colleagues and I are beginning to think about this.

One way – I think we already heard this one today – is to say that teachers need to know whatever they're going to teach, whatever level they're going to teach, plus "n" years beyond that. So if a teacher is going to teach $4^{th}$ grade, that teacher should know $4^{th}$ grade math plus four more years. It sounds a little silly, but some policy discussions are seriously about what beyond the level at which one teaches ought one to know, and it's a reasonable question. It just, again, doesn't really touch what it is someone will do with that knowledge. And then there are terms that float around that we've become fond of using – pedagogical content knowledge or curricular knowledge – which are poorly

defined, and we use them thinking we know what one another means. But in fact we may not be using these terms in the same way. So I want to offer what our own current hypotheses are about that and then just say how that fits with some of the other language.

In our work these days, from the analyses we've been doing, we divide knowledge of subject matter and its usefulness for teaching into four domains: one we call common, which is what I illustrated for you by saying, for example, just get the answer to 2.5 x 3.5. Another we would call specialized, and that's what I've just been illustrating for you; that is, a way of knowing the content that a teacher would need based on the work that teachers would do, but it's not about students or about teaching; it's just about the math. A third category would be a mixture of knowledge of students and how they learn a particular topic as a topic, such as knowing what errors might be typical for students when learning multiplication, or knowing some of the ideas students might have about what multiplication means. Those would be in this third category. And a fourth category would be knowledge of content and teaching, and also the curriculum. So this might be knowing what are good models for trying to teach multiplication of decimals and how might they be sequenced, and how do base ten blocks compare with using other kinds of models – knowing the kinds of ideas that would get one closer to how to teach that particular content.



**Shulman's Original Category Scheme (1985) Compared with Ours**

Subject matter knowledge

Pedagogical content knowledge

Common content knowledge (CCK)

Specialized content knowledge (SCK)

Knowledge of students and content (KCS)

Knowledge of teaching and curriculum (KCT)

Now you might ask, well, what does that have to do with this famous term, "pedagogical content knowledge," and I want to take a moment to clarify how it fits with that. In the original work on pedagogical content knowledge, Lee Shulman and his colleagues differentiated – among the things they mentioned were subject matter knowledge and pedagogical content knowledge. And pedagogical content knowledge, we understand from the literature, to be an amalgam of knowledge of content together with knowledge of the students they're teaching, so in our lexicon, we would divide into these two categories: knowledge of content together with students; that is, for example, errors kids might typically make when studying a topic; and knowledge of how to teach that

particular topic. On that right-hand side of the diagram, therefore, we would be dividing pedagogical content knowledge into two parts. In my comments with you today, though, I've focused only on the left side, really, and that is to distinguish within the area we might just call pure subject matter knowledge – that knowledge you'd expect educated people to have – from the specialized knowledge required for the work of teaching.

I want to close by saying a little bit about how we've been testing these ideas empirically rather than simply analyzing teaching and developing ideas about it. We had an opportunity – and Toni mentioned when she introduced me – in a very large study we've been conducting at the University of Michigan called the Study of Instructional Improvement, in which we studied three comprehensive school reform models, and tried to understand how, in high-poverty elementary schools, these reform models played out over a period of several years. The models were Success for All, America's Choice, and Accelerated Schools.

I won't talk about that study. If you'd like to know about that some other time I will tell you. The reason I bring it up here is because it created a context in which we could try to take these ideas about teacher content knowledge and take them to scale. We've hypothesized that, in teachers' interactions with these reform models, their knowledge of content might be one of the variables that would interact with the way they interpreted the curriculum in the reform model they were working with, the professional development they might use or need as they worked with the reform, and how their students learned across time as the school made this shift to the reform. But I'm not talking about these specific reforms.

So we had these instrument development goals in order to conduct the study that constrained and created opportunities. One was that the scale of the study meant that we needed to study teacher knowledge at scale. No longer could we afford to interview individual teachers or go into their classrooms to see the way they used math because it's a very large study. So we had to create ways of measuring content knowledge that could be used with 5,000 teachers, which led us in the direction of creating multiple choice survey measures of teachers' content knowledge, but in the genre of what I just illustrated for that kind of mathematical knowledge. That meant that we wanted to try to build survey measures of the content that teachers actually use in their teaching and the way they use it, not just what they happen to know as adults. And we wanted to be able to differentiate among teachers such that we could distinguish a teacher who might know less of this kind of knowledge from one who knew more, because that might be one of the variables that would predict how the reform played out in the classroom. And finally – and this is important – we sought to build measures that weren't partisan to a particular view of good teaching because, in fact, these models differed in what they considered to be good teaching, and we wanted measures that could be used irrespective of the particular view given to teaching.

I'll give you an example of an item just so you can picture it. It's like the kinds of things I've just shown you, very similar to the problem you've just seen. We would present a person taking one of our questions with three different methods of multiplying –

in this case 35 x 25, two whole numbers – and ask the question, "Which of these shows a method that could be used to multiply any two whole numbers?" And a person would choose: "Student A is using a method – yes/no," "Student B – yes/no," "Student C – yes/no." So there would be three questions asked about this. Again, it's not about students or teaching in this case, but only about the mathematics. So an item would look something like that. It is situated in the work of teaching, but is really a mathematics knowledge item. We also measured other kinds of knowledge, but I'm not talking about that today.

| Student A | Student B | Student C |
|---|---|---|
| 35 | 35 | 35 |
| x25 | x25 | x25 |
| 125 | 175 | 25 |
| +75 | +700 | 150 |
| 875 | 875 | 100 |
| | | +600 |
| | | 875 |

So let's say a little bit about what we were able to use these data for, and then I'll wrap up. We did three kinds of tests to understand whether we were on the right track in the way we were hypothesizing what teacher knowledge might be. We conducted factor analyses, we analyzed the validity of what we were up to, and then we tried to use the measures to see if they told us anything interesting at all about the relationship of what teachers knew and what their students learned, or what teachers might learn from their own opportunities for professional development.

I'm not going to actually talk about these; I'll just give you the headlines and skip to the last one about student achievement because I think it's the most important. A factor analysis permits you to understand whether you are in fact measuring different sorts of things or just one thing. For example, you might just think, well, you have all these nice questions, they look great on their face, but what you've really got is just a measure of intelligence. And it would be worth knowing that that's not all you are measuring. So we conducted analyses to understand whether, in these kinds of questions we'd written, we were in fact differentiating what I'm calling specialized content knowledge from, say, content knowledge and students, or common content knowledge. And in fact, in the data what we're learning is that these are different constructs; that common content knowledge is not the same thing as specialized knowledge or that knowledge of content and students is not the same thing as knowledge that's specialized for the work. And I – there are things that I could offer you if we had more time to talk about it, but I'm going to go on.

A second thing we had to do is to ask, well, this all sounds convincing to us, but how would we want to interpret a teacher who scored well on this. There are things we

might want to make of it. We might want at least to claim that the questions do in fact measure what the teacher knows. You wouldn't want a situation where someone who didn't know the content got the question right and – (audio break, tape change) – math effectively when she was teaching. And to do that, we conducted a videotape validation study in which we taped teachers at work in their classrooms, and attempted to code the mathematics that they used in the course of their teaching and correlate that with their scores. So, in other words, if a teacher scored well on our measure, we would want the result to be reflected when we observed that teacher teaching mathematics – to see that the teacher was agile with mathematics in the course of the lesson, didn't make errors, spoke clearly about mathematics, was able to interpret students' thinking. We weren't coding the quality of the instruction, just the quality of the mathematics in use, because that's what we were trying to measure. Actually we're not done with this and we're blind to the teachers' scores as we code their teaching. But it's been a very instructive thing to learn to code teaching for its mathematical quality.

Obviously, the bottom line question would be whether teachers who score better have students who gain more knowledge in the course of the year, and I'm going to talk a little more about that. And finally, we wanted to know whether the scores differentiated different kinds of content knowledge, and to do that, we compared the responses of teachers with people who weren't teachers, for example mathematicians, accountants, or other non-teachers. Again, I won't dwell on that. I say this to you right now primarily to show that the demands of building measures of this kind are considerable and include substantial attention to whether the validity of the measures is what you hoped it would be.

So let me conclude by talking about the student achievement results, because they're among the things I'm most fascinated with at this point. We asked the question – do students learn more if teachers score better on these measures? Or another way to put it would be, would teachers who score better have students who learn more in the course of a year? So we constructed a measure consisting of thirty of these items that we've been writing. Interestingly, even with only thirty items, the scale of reliability was .88, which is pretty good reliability for a measure that has so few items on it. And then we asked ourselves the question, what student gains would you expect to produce in the course of a school year? We used the Terra Nova test administered to get scores at two points in time: the beginning and the end of the school year. In one case, it was spring-spring; in one case it was fall-spring. And we built models that included the variables that you would think might predict student achievement – for example, student background and family background variables, how often this kid had been absent from school, the teacher absence rate. You could name others; I'm just giving you some examples. And we also put teacher characteristics that we would expect might predict student achievement into the models, including our own content knowledge measure, but it was side-by-side with other things that one would expect might be a predictor. And really, the question one is asking then deals with the variability in students' gain in scores, specifically is any of that variability significantly predicted by teachers' content knowledge scores.

And what we found was that the teacher content knowledge variable, as we measure it, was significantly associated with gains in student achievement. And I want to say a couple of things about this. First of all, the effect size is very small. The way best to explain it is in terms of two teachers who score one standard deviation apart on our measure – the students of the higher-scoring teacher would have a gain score in a course of a year of less than one-tenth of a standard deviation higher than the students of the lower-scoring teacher, which sounds pretty little. But if you translate it into more meaningful terms, that's equivalent to about two to three weeks more instruction for the students of the higher-scoring teacher. So it's not trivial; although the effect size sounds small, it's not trivial.

I think this is even more interesting – the student's family background variable predicts achievement at about the same effect size as the teacher content knowledge measure. In other words, family background accounted for as much of the variation or, said differently, the teacher content knowledge score accounted for about as much of the variation – predicted about as much of the achievement gain – as did the family background variable. That to me seems incredibly important. These results were obtained through analyses of student achievement data from the first and third grades. We're currently working on the second grade and fourth grade achievement data, and an article reporting these results is in your packet.

So in conclusion, one point I've tried to make here is that, to answer the teacher quality question, we must know that the mathematical knowledge needed for teaching is specialized. It's different from the mathematical knowledge required for other mathematically intensive work like physics or accounting or engineering. The mathematical demands of the work are specialized. Figuring out what teachers know is more than getting experts in the room and asking them what they think or looking at the curriculum; it requires multiple kinds of work and different kinds of research-based evidence in order to establish answers to that question. I think I'm here to say that it's possible to survive the effort of trying to write multiple-choice items that can both reliably and validly measure something that seems to be meaningfully related to quality in instruction, but that it takes a lot of work – and I'd be happy to talk more about that.

One interesting sort of by-product is that questions of this type appear to be more credible to teachers than a conventional math test. We routinely get comments back from teachers who take these measures, saying that this reminds them of the sorts of questions that come up in the course of their work and that they appreciate this sort of a test. They see this as beginning to articulate a kind of professional knowledge of the content. In fact, we're in the middle of a national probability sample of middle school teachers and, on the questionnaires, we're getting back many comments of this kind saying they enjoyed taking the items, that they were reminded of their own work.

We do think that work on questions of this kind is central to the improvement of math education. The kind of problem that Jim described – the work that it takes to keep a connections problem up and alive in the classroom as a connections problem – requires the kind of sensitivity to the math that I've been talking about. So if what we want is to

have that zero in Jim's last slide be a very different kind of number for the U.S., I would hypothesize that attention to the kind of mathematical knowledge it takes is one piece of that answer.

MS. CORTESE:  Thank you very much.  Well, I'm only going to speak for myself, but now that you have blown apart most of the assumptions I've made for the last few years, I just wanted to ask you a question – have you been able to apply any of your research knowledge in terms of a teacher preparation program or a professional development program?

MS. BALL:  I have two pieces to that. One thing is we have used these measures to study different professional development programs. So, although we didn't design those programs, we've been comparing – measures of this kind could be used to compare what teachers learn in different professional development programs. And I thought about talking about that today, but didn't. And we find that programs differ – it's a little like Jim's findings – programs differ in whether teachers learn mathematical knowledge of this type in the program, and what differentiates the programs doesn't appear to be some of the surface features we often associate with high-quality professional development. So, for example, some short institutes produce big gains in teachers' mathematical knowledge – even though they were short, and we like to say professional development needs to be long. So it's kind of like your question, looking more deeply at what might characterize high-quality professional development, that question is peaked by those results, I think.

A second thing is that I, as a teacher educator, and my colleagues – and Jim can certainly speak to this too – are engaged in trying to develop approaches to teacher preparation that make use of this work. So I'm humbled constantly by what it might mean to use the space we're accorded in teacher education to think about how best to make use of this. But we are trying to develop the ability to work with teachers in this way and then, ultimately, we'll try to study the effects on their capacity to teach mathematics and on their students' learning. But at this point, it's in my teaching that I'm working on it and less as deliberate research yet. So it's both research on professional development and active practice as a teacher educator.

MS. CORTESE:  Are there any questions of either presenter? Okay, I'm going to start on this side of the table. I'd ask you to identify yourself because this is being transcribed and recorded.

MR. BEMENT:  Arden Bement, National Science Foundation. Of course, we're in the business of sponsoring research in this area, and I'm curious to know what regimens of research might be beneficial in coupling teacher educators with content area specialists and what would that research be?

MS. BALL:  Research coupling, is that what you said?

**MR. BEMENT**: In other words, mathematicians working with educators in mathematics – first of all, is the premise correct, is there some benefit in carrying on research where both contribute as a team, and secondly, what would be the goals and objectives of the research, if the first answer is yes?

**MS. BALL**: Certainly, in our research, we found the involvement of research mathematicians to be vital. So that's the simple answer to your question. We have in fact begun to do some analysis of the ways in which mathematicians' contributions to our work have played out. So one way in which mathematicians have played a significant role is in helping to develop these measures. We've had mathematicians both helping to write questions and evaluating and critiquing and reviewing them, and we presented a paper last month at the American Educational Research Association, which was our first ever to analyze how mathematicians worked on the items, and we learned a couple of interesting things.

One was that, in our experience of trying to do this over the last five years, in general, research mathematicians did not produce useable items for our question bank. And we tried to analyze why that was, and it actually takes me back to some of what we were talking about. The nature of the problems that mathematicians found to be good problems were in fact good mathematics problems in the discipline, they were really interesting problems. But they weren't the kind of problems that teachers work on. That is, they might be problems a teacher would teach, so for that purpose they would have been useful, but they weren't the kinds of mathematical problems teachers face in their work. And so they were rather remote from what we were trying to write.

There were other features of their problems that made them difficult to convert to problems that measured mathematical knowledge for teaching. For example, sometimes the grain size was much too large – not at the level at which we would want to measure a teacher's ability to distinguish two mathematical ideas. However, mathematicians were extraordinarily good at being able to review and critique items very, very finely. We took as a standard in our work that we wanted any item that we eventually had out in the field and finished to be of mathematical quality, such that anybody in the mathematics profession who read them would not find any gross mathematical errors, but also imprecisions in the language or distortions – subtle distortions – in the math. And there were many items that would never have made it in the field when we piloted them had mathematicians not been among those who were reviewing them. They caught things that would have caused these items to fail psychometrically. And I think that that was a very important part of the work.

There are other things to say about that, but in the item development, I would not approach this without involving research mathematicians. I wouldn't begin by assuming that they would be the best developers of the initial drafts of items, but they were crucial collaborators in the production of those items. There are other things too but that's one context to which I could speak.

**MR. BEMENT**: Thank you.

**MS. CORTESE**: Next around the – yes, sir.

**MR. SROUFE**:  Well, good afternoon, I am Gerry Sroufe, with AERA. I thought the ratio of insights to time was exceptionally high and positive here, so thank you both for your presentations. These studies strike me as being incredibly expensive, and perhaps more desirable for that reason. I'm just wondering if you would say a word about sources of support for the work that you've accomplished.

**MR. HIEBERT:**  I'll just mention the support for the TIMSS video studies and then make an additional comment. The video studies are incredibly expensive and were supported primarily by the U.S. Department of Education through the National Center for Education Statistics, with additional support from the National Science Foundation, and some countries invited themselves into the study by paying their own way. So not all of the studies were supported by money from the U.S., but five out of the seven were I believe. There is currently a program supported through the National Science Foundation that is a network of centers around the United States pursuing different research agendas in this general area, and a number of the centers, which have five-year life cycles, are working on these issues. Deborah and I are both members of different centers, which share sort of a common research agenda. But a lot of the current support for the work I'm doing at the University of Delaware comes from the National Science Foundation through that center activity.

**MS. BALL**:  Our work also has been generously supported by the National Science Foundation – all of the work on the measures development that I reported and the initial analyses and the ongoing analyses of teaching practice to derive the initial ideas about what teachers need to know – all of it was supported by National Science Foundation funding. The large study of instructional improvement, which creates a data set for us to conduct these achievement analyses, was funded not only by the National Science Foundation but also the U.S. Department of Education. And very generous support from the Atlantic Philanthropic Society sponsored that research as well.

And you're quite right; I think that one thing to say about expensive research, however, is that we tend, in our field, to do lots of duplication of effort. We considered ourselves to have funding to develop measures that we thought from the beginning were measures we hoped to develop and methods of developing those that could be shared by other projects. So in the study of instructional improvement, a huge body of instrumentation has been developed that could either be directly used by other projects or, with guidance, adapted. And the field has suffered from the lack of common measures, so I think I want to say something about the importance of investment in measures that – in which one has the time and the money to develop them rigorously with the data about their psychometric qualities and their validity that often really have not been done in our field. And we won't make progress we need on these sorts of research questions without this kind of investment and instrumentation.

**MS. CORTESE**:  Yes, please.

**MS. LEIBBRAND**:  Jane Leibbrand, with the National Council for Accreditation of Teacher Education. It seems that teachers teach the way that they were taught, and it seems like we're in this cycle of repeating teaching the procedures – when the gentleman showed us the slide where everybody in the United States was just focusing on the procedures rather than teaching the relationship between the concepts, facts, and procedures where the other countries were doing that. And I was just thinking, you know, how can we break this logjam? How can we impact the way that teachers are taught so that they teach differently, so that their students will teach differently – again, because we teach the way we were taught? And we accredit 602 colleges of education in the United States that produce two-thirds of the nation's new teachers annually. So we do have a huge impact on the teaching force.

We used national professional standards in our accreditation system, and one of the sets of standards we use is that of the National Council for Teachers of Mathematics. And Ken is here and I – Ken, I don't remember chapter and verse every one of the 19 sets of specialized standards that we use, but it seems that we do include language in the standards about the fact that teachers should integrate this knowledge and should be going into the deep conceptual knowledge. But when we see the slide, it seems not to be happening in practice. And so what can we do on a national level, and with the standards, and with the way that we're approaching this to help initiate more teaching of the relationship between the facts, procedures, and concepts.

**MS. CORTESE**:  Who would like to answer that?  (Laughter.)  I could answer, but it would be totally non-factual, so I – (laughter).

**MR. HIEBERT**:  Mine probably won't be factual either. Okay, let me say a few comments while I'm thinking. The introductory comments to this are that you're absolutely right in the fact that we've been teaching this way for a long time. In fact, the earliest descriptions of mathematics classrooms in the late 1800s sounded strikingly like the classrooms you see today. Not in surface features, but in the underlying interaction between a teacher and a student; it hasn't changed a lot in a very long time. We also have good evidence that teacher preparation programs that try to do something dramatically different often have effects in the first year maybe, when teachers go into the field, but when teachers hit the first real high-pressure points of tests, and et cetera, et cetera, they'll fall back on the way their favorite fourth grade teacher taught. So the system is over-determined. There are huge numbers of factors that keep it in place. Changing one factor probably won't affect the system, because many other factors will rush in and take its place. So it's not a question, I think, of finding one thing that we could do to affect teachers' teaching. It will only happen, I think, by the reinforcing effect of multiple things in the system, each of which sort of gradually changes toward a common goal.

So an example of this is that, if teachers move through a pre-service program where they have to struggle with these ideas in the same way that kids need to struggle with mathematical ideas – and if teachers entered the profession understanding that they were novices and that they could get to be good teachers if they studied their practice for

another ten years, if they had that orientation – I think they would have a shot. If they then entered a school setting where there was encouragement for teachers to work together to study their own practices, I think that would be a powerful shaping of the way teachers began teaching.

There are some – sort of – levers or triggers that seem to make a big difference for teachers. One, for example, is listening to students. It's quite remarkable, working in the pre-service program at Delaware, and also with teachers in the field, how much focus there is on observing the teachers when you watch a classroom, and not looking at the students or listening to the students. It takes a lot for us as teacher educators at Delaware to shift our pre-service teachers' attention away from the teacher and toward looking at the students while they're watching a videotape, for example, of a classroom interaction.

Listening to the students and being really interested in what they have to say, seems to – but this is anecdotal so I'm in thin air here – but it seems to have an impact on teachers unlike being told by anyone that they should be teaching in a different way. Understanding that their students understand as little as they do, when they thought they had given a perfectly clear explanation, can be a sort of powerful wake-up call for a lot of teachers. And it doesn't take a lot to set up the situation where teachers encounter that circumstance, but it takes some organization, something in the system, that values finding out what your students are thinking while you are teaching. And I was going to say one more thing, but I forgot, so I'll let Deborah pick up.

**MS. BALL**:  Do you want to follow up?

**MS. CORTESE**:  Why don't we see if we can get some other people around the table? I'm going to start back around. Alice, and then Judy.

**MS. GILL**: Deborah, you said that looking at the kinds of knowledge in the way teachers have to know it is one piece of what we really need to get a handle on in order to improve teaching and learning. Would you comment on the other pieces that we also need to focus on?

**MS. BALL**:  Well first of all, I chose not to talk about the other aspects of content knowledge today. Like I do think that it's important to get a much clearer about what this kind of knowledge and reasoning is around using mathematics to interpret and listen to students. One piece of it will be knowing mathematics in the ways I tried to show you, but there are other things, I think, that will be important to understand. Some of it is knowledge. You can learn that there are certain buggy algorithms or errors or things that help students to learn, but teachers also – there's some fraction of teaching that requires a kind of reasoning about students and about the content and we need to understand that kind of dynamic better. I think that's also a kind of teacher knowledge.

And I think we're finding in our data where we try to study knowledge of content and students, we get both mixed together. Sometimes teachers see situations they've seen before and they quickly say, oh I've seen that before, that's a typical error, kids all make

that error. And sometimes they're able to reason about what's likely to have occurred and you see skillful teachers able to use their own knowledge of math to make good hypotheses – even though they haven't seen it before, it's actually not a very common error – and you need both. So we can elaborate this content knowledge and its use, but then there are whole domains other than that. I think the question that was just asked that was just asked that Jim was describing – you want to go that far, right? What we would need to have a different quality of instruction, is that what you're asking?

I mean, then we need a whole system of professional education that is entirely different than what we have, I mean vastly different. I don't think we're talking about moving deck chairs around. It just needs to be completely different than what we do and the scale problem that I mentioned earlier is a reality with which we have to deal. We're not talking about a small numbers of teachers. We're talking both about a very large teaching force for whom we don't have a system of reliable, decent professional education, and we produce a very large number of teachers every year who are not well prepared to begin. And those problems are with us and we all have to break off pieces to work on them.

So one thing I think I'd add to the mix that we've talked about today goes beyond the teacher knowledge question, and that is a much more performance-oriented approach to professional education. Teaching is about doing things with students. For some reason that I don't fully understand, teacher education has shifted increasingly over the last twenty years at least, maybe longer, toward a highly cognitive view of teaching, as though teaching were mostly about reasoning and thinking and knowledge. As a person who has spent a lot of my career as an elementary school teacher, I recognize how much I think and reason and make decisions. But I'm also doing a lot, and there's a lot to learn about that doing. There are many things that teachers do every day that we do nothing to prepare them for.

And, in fact, it almost appears as though we inadvertently communicate that we have nothing to teach. We can routinely communicate to people that they'll learn this through experience, that this is something that will develop over time. And, while I agree with Jim that we need to equip people to learn from their practice, there are a lot of things one can learn to do well and other professions do in fact isolate those practices that professionals need and teach them to beginners, and they make good decisions about which things beginners have to do reliably.

Let's take one example: all teachers, I think – all elementary school teachers at least – must interact with students' parents or whomever the caregiver is for the child in ways that are pretty constant. There are conferences with parents; there are phone calls to parents. Depending on the community, the communication patterns are very different. Very few teacher education programs prepare people for that work. It's work that imbeds many things we like to talk about, like equity considerations, culture, language, content knowledge, knowledge of students, knowledge of communication. I'm sure there are programs that do it, but it's not a common part of teacher education. You could imagine practicing and learning to get very good at that before you first have to do it. And yet

right now – and probably you've all been in parent conferences with someone who hasn't ever done one before – that very vital link between the out-of-school experience of the kid and school isn't something we teach anything about. So try to imagine a system of professional education that not only equipped people with the kind of usable content knowledge that we've been describing, but also focused on key performances of the work of teaching, and helped people learn to do those. I think we'd have a very different system and, I would add to that and not elaborate, that implies that we would need a system of assessment of those performance – from the beginning of a program and throughout the program that would allow us to help beginning teachers learn whether they were in fact developing those capacities and, when they weren't, the instructional system would help them to improve those.

We're so far from that it's slightly shocking. The way that most teachers get recommended for certification from their programs is because they've finished the program and completed the assignments and gotten grades. But if one scrutinized those, I think it would be hard to defend these as estimates of a teacher's ability to do things with students. Now that may be all way too radical but, as a confirmed member of the teacher education profession, I know that if we don't do it, someone else will tell us how to do it or do it instead of us, and I don't know that that's better.

**MS. CORTESE**: Judy, and then I'll take the questions here, and then Genie had a question; then I think just about at that time, our time will be up.

**MS. WURTZEL**: Thanks, Antonia. I'm Judy Wurtzel with the Aspen Institute. Deborah, I'm trying to think about how to connect your comments to the point you made at the very beginning about the policy conversation about highly qualified teachers today, in which we're looking at a range of measures – the number of courses taken, process scores, certification, length of time on the job, mathematical background in your prior profession. Have you started looking at correlations between scoring high on your measures of mathematical content knowledge and any of those other kinds of measures that are being discussed – and if you haven't done it yet – what are your hunches about how we should start thinking about those connections?

**MS. BALL**: We try to do some of that. For example, we've tried to look at comparisons between these measures and certification or these measures and some of the other things you've mentioned, though I don't have that with me. But we have begun to do that because it's an important piece of the question. I mean, on the point of legislation requiring highly-qualified teachers, I think that what I'd like to reiterate is that we have a professional responsibility to provide useful answers that are valid about what it means to be highly qualified. We wouldn't want to be in the position of appearing to say that we shouldn't have highly qualified teachers. If we don't step up to articulating more clearly what that would mean, with evidence, I think we will inherit rather useless measures of teacher quality, which won't in fact accomplish anything for students. So I think this work is slow but requires more collective – sort of – collaboration within the profession than we've seen.  I can give you those data, but I just don't have them at my fingertips. And very many of them aren't well correlated, which suggests that we have work to do.

That is – scores on these measures aren't well correlated necessarily with some of the things we commonly like to use.

MS. CORTESE:  Charlotte?

MS. FRANK:  Just some experience that had been mentioned, I used to be the head of curriculum and instruction for New York City – the whole city – and I was once a math teacher, so I find this interesting. I'm thinking I'm going back – I have to go on my next step – what would you recommend would be the next step that we take because you're asking for change to take place, and we know the only one who wants to be changed is a baby – (chuckling) – and we have been trying for years. No, but these are very practical issues. We have to try and change the system. You're part of the system. We're part of the system. We have coaches in schools now. We have coaches with new teachers, trying to do all sorts of stuff. What would you recommend – one, two things – whatever you think we should go back and say, hey, we've really got to change this, because we've been changing the system forever.

MR. HIEBERT:  I'll give a quick response that won't be very satisfying and then let Deborah give – (laugher).

MS. FRANK:  Is that an example we should take now when we answer questions?  (Chuckling.)

MR. HIEBERT:  I think the most sort of systemic obstacle to change in the United States is the conflicting and mixed messages we give to teachers about what's important to do in the classroom. In other words, we might all sit around this table thinking this learning goal of mathematical proficiency – which is much more ambitious than we usually ask teachers to teach for – is what's important, but teachers are asked to do an enormous number of things throughout the year that don't align with that at all.

Many of the tests that they have to prepare students for, many of the expectations of the parents, and much of their background doesn't align with that. So my comment would be if we endorse this learning goal in mathematics – and I think we have to decide first on what goals we're shooting for – what do we want students to learn? If we decide on this one, and I think it's a good one, the best thing we can do is figure out what about the system is in conflict with that and blow it up – somehow change some part of the system, which prevents teachers from spending their energy on helping students get there. And there are huge numbers of these.

MS. CORTESE:  Did you want to – Deborah – did you want to comment?

MS. BALL:  It's a really good question and a hard one to answer in a couple of minutes – but I think it's a good one to ask. One thing that I would say is that organizing – beginning to try to organize a more systemic approach to professional learning for practicing teachers would be a step that we could take. And that includes attending to the very many people who have to do professional education who randomly end up in that

role with virtually no preparation to do it – attending to their development would be one thing.

But more important, I guess I would say, is that the thing we're beginning to see that might matter for the quality of teachers' ongoing learning would be to organize it around the two obvious axes of their instruction – and one is students' learning and the other is the curriculum. So professional development would be much more systematically organized around the curriculum that teachers actually teach, which astonishingly is rarely done – that would be one support. And as I said earlier (and I wish I had a better phrase for it), we need a system that prepares ordinary people to teach. All I mean by that, respectfully, is that perfectly regular smart people need to be able to learn to teach well. We can't do this by populating the teaching profession with heroines and heroes. It is heroic work, but regular people have to be able to do it. And curriculum is one of the tools. No curriculum materials are perfect; they all have flaws. Organizing the work around systematic use of the curriculum – learning how to judge it, learning what to skip, being in conversations about it, which in some sense borrows from what I think we see in other countries – curriculum is one major axis of the work.

And I agree with what Jim said, so I won't pick that up again. That work would include making judgments about how this intertwined goal of proficiency would be used in a curriculum. I think that teachers in the schools where I go find themselves separating out, to the extent that they're working on this, they are doing what Jim is saying. They're feeling worried about covering everything. Help in learning how to intertwine those in the context of the curriculum would help.

The second issue – and we see some of this – is a more systematic orientation of professional learning around whether kids are learning. They need to learn how to look at student outcomes in all different kinds of outcome measures – and try to understand what that means in relation to what they tried to do. So students didn't learn; let's look back and see what happened – doing that at more frequent intervals, having that be much more part of the professional work. I think those two things would be parts of the pillar because they're inside the work, rather than external kinds of things. I think having coaches, all of that is probably fine, but it's not necessarily in the substance of the work. Coaches could be one tool in that, but so could other structures. If the substance of professional learning isn't about the curriculum and about student learning, I doubt we're going to make real changes. And the nice thing about that is that those are already in the system if we could start using them as resources for professional work. Now that may sound abstract, but you didn't give me a long time to talk.

The other thing is that I would reorient professional pre-service education to make much tougher choices about what the high leverage, early performances are that people need to be able to do and stop either trying to do so much or trying to say that nothing can be done. Lots can be done. You know there are other professions that prepare people to begin high-quality professional work in the same amount of time that we do, and it's not clear to me why we can't do better than we do. I think we don't make the hard choices about what the essential work of beginning teaching is and then determine that we'll

construct a curriculum at the pre-service level that gets people started well. I'm really not interested in extending the length of pre-service teacher education until we use the time we have better. And, from my estimates of looking at what I and my colleagues do, I don't think we use it well; there's a lot of waste in that time. And I don't say that so that others who criticize us can now say we don't use our time well.

I think there are some tough choices to be made about what will give beginning teachers high leverage over the challenges of the work. I don't see that conversation and it would be a good one to be having. Teachers could contribute to it, researchers could, lots of people could help with that question where we'd be forced to say we're only going to do some things, but we're going to do them very, very well – and ensure that people start knowing how to do some of the things that are critical to the work. Then people would feel more successful from the beginning and could enter a system that was better designed for ongoing learning. We equip people for failure in a sense.

**MS. FRANK**: That sounds like the next conference. (Chuckles.) What are the beginning steps if we're going to make this change?

**MS. CORTESE**: One more question, Genie, and then – I think that we're out of time, unfortunately.

**MS. KEMBLE**: I think I heard you say that the family background variable predicted as much as the teacher performance on your instrument in terms of –

**MS. BALL**: Roughly similar statistically –

**MS. KEMBLE**: Yes, which I found stunning because we've always – I mean I think it's kind of been conventional wisdom that math gives you a truer read on school and teacher effects than, say, reading and language and so on, because presumably the impact of family background would be less. And in literacy and language development, we do have some of these early studies like the Hart-Risley studies that give you some guidance about the kinds of things that very young kids need, even though we haven't been able to do that at the family level. And I'm just wondering if you could say a little bit about what this is about, in terms of family background and math. And are there studies being done about this that would be comparable to the Risley studies? Can you just talk about that a little bit, because I found that very surprising and a little bit depressing.

**MS. BALL**: It's depressing; it's interesting that you find it depressing. I'm probably thinking about it in a different way, which was given what I said at the beginning, that students in high-poverty schools are most likely to have an under-prepared teacher, an unqualified teacher by any standard of qualification. This suggested to me that investments in teachers' usable content knowledge in those environments would get us a real payoff. Despite the fact that there are many things that create impediments for kids to learn in our culture, this suggested to me that investments in the

quality of their teachers' knowledge would make it more likely that they would be able to make those gains in school. That was how I was thinking about it.

On the question of how family background interacts with mathematic school achievement, I think we're beginning to get into a new era on that. Traditionally, mathematics was seen as somewhat different from reading in that it demanded less cultural knowledge. You see these interesting studies of upward social mobility for students from poverty backgrounds because somehow school provided more difference for those students – that's some of what you're talking about I think. Here I think we don't know, and maybe there is some research I'm not aware of – I had a graduate student who was working on this, but it still seemed to be at a very early stage in the field – is whether the mathematics – as the goals in this country are shifting towards mathematics that involves more problem-solving and reasoning – now whether that's changing is another whole question – but if it were, and if mathematics were becoming more discursive and more linguistically dependant than it had been in the past, it may be that that difference between math and literacy won't be as pronounced as it's been. That actually is a little worrisome. It may be that doing well in math in school will require more cultural linguistic confidence than it has in the past and that supports for that are going to be even more crucial. So that seems more where your question was. But the reason I wasn't discouraged by those results is because they suggested to me that, while the problems of poverty seem so overwhelming, investments in some manageable domains might pay off.

**MS. CORTESE**:  Well, thank you both very much.  It was a conversation that probably could have gone on a while longer, but we are at our appointed ending time. I guess Charlotte said it out loud, but this probably would be an excellent topic to follow up on with a little bit more in-depth discussion. So we certainly appreciate you bringing this kind of information to us and also getting people around the table – whether they are involved in teacher preparation or whether they're teachers or in foundations – to look at the way we teach math and what teachers need to know, using very different kinds of measurement than we've been using in the past.  So thank you both very much. (Applause.)

(END)