SEQUENCED BENCHMARKS FOR K–12 MATHEMATICS

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PREFACE

This report is one in a series of reference documents designed especially to assist those who are directly involved in the revision and improvement of content standards. It presumes a basic understanding of the purposes for standards and the process of standards review. It is important to note that it is intended to be a desktop reference as opposed to a practical guide. Readers desiring more background and context for the work described here may wish to consult *A Technical Guide for Revising or Developing Standards and Benchmarks* (Kendall, 2001).

This report presents mathematics benchmarks organized into instructional sequences by topic. Each sequence is based upon an analysis of the order of content as it appears among a set of highly rated state standards. This report does not assign content to specific grades or recommend that content be taught at specific grades; however, each item within a sequence includes information on the grades at which that content is found within the state standards documents analyzed. The report is intended to inform and guide state or district curriculum directors or others who, starting from content that is placed within broad grade bands, need to assign specific grades to benchmarks or objectives. Alternatively, those who have benchmarks or objectives already placed at specific grades may wish to compare the sequence of content outlined in this report in order to confirm the commonality of that sequence. Teachers may find the sequence of content useful for organizing instruction.

INTRODUCTION

At the beginning of the standards movement in the early to mid 1990s, most states delineated the content of standards as benchmarks or objectives for a range of grades, such as K–4, 5–8, and 9–12. As states have revised their standards, usually as part of an established review cycle, they have described them in narrower grade ranges, for example, K–2, 3–5, 6–8, and 9–12. Some states have taken this process a step further by describing content for each level from kindergarten through grade 8. A significant number of states, however, still do not provide such grade-by-grade distinctions in standards.

This lack of grade-level benchmarks or objectives provides districts with some freedom to define the local curriculum yet, for many districts, it also presents an immediate problem. In order to implement standards, districts must find a way to translate these grade-range benchmarks into meaningful benchmarks or objectives for day-to-day schooling at specific grades. Unfortunately, there has been little guidance available to districts as they undertake this process. Documents in each subject area produced by national professional organizations might be considered the highest authority in this regard, but none provides grade-by-grade recommendations. It seems likely that these organizations were deterred by the lack of research that guides the placement of content at specific grade levels. In addition, many organizations avoid the assignment of content to a grade because it is likely to be seen as overly prescriptive. Yet, the problem for a school district remains. In most schools, content must be assigned to a specific grade because it must be taught at a specific grade.

Although it seems likely that there will never be adequate research to support the assignment of specific content definitively to a specific grade, we can infer that the sequence of content in state standards documents reflects the authors' beliefs about how the ideas and skills that students learn in different grades and topics depend on and support one another. To assist educators in the development of grade-by-grade benchmarks in mathematics, this report presents information about the sequence in which knowledge and skills appear in a small set of mathematics documents that have been highly rated by a number of national organizations. The method employed here, and discussed in detail below, entails the analysis of exemplary standards documents for patterns of content sequence by three individuals, and the review of the established content sequences by two reviewers experienced in mathematics instruction. This method has been used previously by Mid-continent Research for Education and Learning (McREL) to identify the sequence of content in the English language arts (Kendall, Snyder, & Flynn, 2003) and science (Kendall, DeFrees, & Richardson, 2002). Users of the previous studies have reported that the sequencing information has been useful as a comparison with their state standards and as a help in developing district objectives. Teachers also report consulting the sequenced topics to help them organize instruction.

SOURCE DOCUMENTS

Three evaluation reports were used to help select the state standards documents analyzed in this study. One report was the American Federation of Teachers' (AFT) *Making Standards Matter* (1998), which includes ratings of the state standards in terms of specificity and clarity. Another perspective on state standards was published by the Fordham Foundation in the report *State Mathematics Standards: An Appraisal of Math Standards in 46 States, the District of*

Columbia, and Japan (Raimi & Braden, 1998). Finally, the Council for Basic Education evaluated mathematics documents across the states in *Great Expectations: Defining and Assessing the Rigor in State Standards for Mathematics and English Language Arts* (Berman & Joftus, 1998).

Although a variety of state standards documents have been highly rated for their mathematics standards, five state documents that were highly rated by all three organizations *and* that identify objectives at each grade level from kindergarten through 8th grade were selected as source documents for this analysis:

- *Utah's Core Curriculum Standards: Mathematics* (1994)
- *Mathematics Content Standards for California Public Schools* (1990)
- *Mathematics Standards of Learning for Virginia Public Schools* (1995, June)
- *Ohio's Model Competency-Based Mathematics Program* (1990, November)
- *West Virginia Programs of Study: Instructional Goals and Objectives* (1995, June)

Not since these ratings appeared in the mid 1990s has there been any comparable review of state standards documents by multiple organizations. Thus, in order to select standards documents for this report that are widely endorsed for their quality, we were limited to the documents reviewed in the mid 1990s, which are now some 10 to 15 years old. However, we believe that these documents fairly represent the current state of content standards in mathematics. This view is based on our continued familiarity with state standards over the last 10 years as we have conducted our own studies or reviews for state and district clients. For this sequencing study, we determined that it was preferable to select standards documents that were highly rated by multiple organizations, although somewhat dated, than to select documents that had been highly rated by only a single organization.

METHOD

In order to track mathematics content across these state documents, each of which varies somewhat in the content described, a uniform set of benchmarks was required as a basis for comparison. McREL's online standards database was used for this purpose. The database, the online equivalent of *Content Knowledge: A Compendium of Standards and Benchmarks for K–12 Education* (3rd ed.) (Kendall & Marzano, 2000), provides a synthesis of 137 standards documents representing 14 content areas. By using the benchmarks in this document as a basis for comparison, McREL analysts were able to track the presence or absence of mathematics content in each of the state documents in order to find content that was addressed in common.

It should be noted that benchmarks in the *Compendium* were not used in the actual sequencing of content; they were merely used as a means for tracking and organizing the content of the five standards documents analyzed. On some occasions, in fact, benchmarks that appear in the same grade band within the *Compendium* — and thus indicate no preferred sequence of instruction — were found to have a preferred sequence of instruction based upon evidence from the state documents analyzed in this study.

An example can be found under the topic of "integers." In the *Compendium*, the concept of the characteristics and properties of integers, as well as the concept of the role of positive and negative integers, appear in benchmarks at the 6–8 grade range; at this same level appears a benchmark regarding addition, subtraction, multiplication, and division of integers. In the course of our analysis, we found evidence that three states consistently address the first two topics in a grade prior to addressing the skills of addition, subtraction, multiplication, and division of integers. In order to uncover this sequence and many others like it, it was necessary to deconstruct the more broadly written benchmarks of the *Compendium* into their component parts, then combine those content elements that were found to be shared among the source documents and could be established in a sequence.

In addition to identifying a set of benchmarks for comparison, it also was necessary to use a set of topics to help organize this information. A topic is a level of organization that is more specific than a standard, but more general than a benchmark. A topic names an idea that organizes a small collection of benchmarks or objectives. In McREL's *Compendium*, several topics are commonly found within a standard, and each topic organizes two or more benchmarks (for a description of the process of topic development and samples in mathematics and language arts, see Kendall, 2000). The list of topics used here, organized by sub-discipline, is presented in Exhibit 1.

The articulation of benchmarks under each topic in this report was based entirely on the presence of that content in the state standards source documents specified earlier. Each state document was reviewed for any *sequence information* it provided by topic. Sequence information is defined as the presence of a concept or skill in a grade that is topically related to another concept or skill at a higher or lower grade. That is, in any given document, two or more topic-related concepts must appear separated by at least one grade to be considered informative in the development of articulated content. Simply put, if two benchmarks addressing a given topic appeared in the same grade, it was inferred that the authors of the document did not consider the difference between content to be significant enough that the benchmark or objective should be addressed in separate grades. In such a case, the state document did not contribute information about the sequence of content.

Thus, each articulation under a topic was established by sufficient evidence from the state standards documents. Closely related ideas beneath a topic often appeared at different grade levels in each of the standards documents, but as long as these ideas were presented in the same order in each of *at least three documents*, a sequence was established. For example, the idea that students should understand the defining properties of three dimensional figures was found in three of the standards documents at grades 6 or 7 (see Topic 18, Shapes and Figures). Four of the documents also addressed, at different grades, the concept that students should be able to predict and verify the effects of combining, subdividing, and changing basic shapes, yet each document introduced this concept prior to the grade at which it introduced the idea that students should understand the defining properties of three dimensional figures, thereby establishing a sequence of information.

Exhibit 1. Summary of Mathematics Topics

Addition/subtraction	Multiplication/division
Analytic geometry	Number systems
Basic and linear measures	Patterns
Communicating about mathematics	Perimeter/area/circumference
Data collection and sampling	Perimeter/area/volume
Data distribution	Permutations/combinations
Data organization and display	Precision/accuracy
Decimals	Problem-solving
Equations and inequalities	Proof and empirical verification
Estimation	Rate
Experimental probability	Ratio/proportion/percent
Exponents/logarithms/roots	Reasoning and predicting from data
Expressions	Representing problems
Factors/multiples/primes	Sequences and series
Formulating/testing hypotheses	Sets
Fractions	Shapes and figures
Functions	Similarity and congruence
Graphs and graphing systems	Solution strategies
Integers	Triangles
Likelihood/chance/certainty	Trigonometry
Lines and angles	Units of measurement
Mathematical enterprise	Uses of mathematics
Mathematical reasoning	Variables
Mathematics, science, and technology	Vectors
Matrices	Volume/capacity
Measurement estimation	Weight and mass
Motion geometry/transformations	Whole numbers/place value/numeration

In other words, if the presence of sequenced content was established in any one document, the same relative sequence — that is, the sequence from earlier to later grade — had to be supported by at least two additional documents in order to be considered useful relative to the articulation of content within the topic. One additional requirement was established to ensure that the sequences were meaningful. If a sequence of content was found in the reverse order in any other standards document, it was removed from consideration. For example, one standards document anticipated that students should predict and verify the effect of combining, subdividing, and changing basic shapes in the grade prior to their understanding that shapes can be congruent or similar. Another standards document reversed the grade sequence; that is, students were first expected to understand that shapes could be congruent or similar in the grade before they were expected to be able to predict and verify the effect of operations on shapes. Because of this reversal of grades, no sequence of content was established, despite evidence of content sequence available from other states.

The product of this analysis across the standards documents is presented in 31 topic sequences. Each sequence represents the articulation of that aspect of a topic that was supported by evidence from state standards documents. There are 27 topics addressed in the 31 topic sequences because some topics involve more than one sequence to completely cover the associated content. For example, the topic of Addition/Subtraction has one sequence that addresses the process itself (Topic 8) and another that addresses knowing the properties and order of operations (Topic 9).

It should be understood that each of the topics sequenced in this document has more benchmarks associated with it, both in the *Compendium* and in the state standard documents, than appear in this document. Only those benchmarks whose articulation could be supported by the state standards documents appear here.

Similarly, there are many topics that are not articulated in this study because there was inadequate support from the state standards documents. Of the 54 topics identified for the mathematics standards in McREL's standards database, approximately 57 percent could be presented in some sequence.

The topics that could be sequenced in this report comprise 123 unique benchmarks. Readers should be aware that the content described in 100 benchmarks of McREL's *Compendium* could not be sequenced using the methodology adopted for this study. The fact that this content could not be sequenced of course says nothing about its relative importance. For the benchmark content that could not be sequenced in this report, readers will need to employ other strategies to determine grade level placement (for a discussion of such strategies, see Kendall, 2001, pp. 16–19). This report will not be useful for helping to identify *all* the significant content in mathematics. For such a purpose, readers might consult the highly rated state standard documents listed above, or consult a synthesis of these highly rated documents (see Kendall, Snyder, Schintgen, Wahlquist, & Marzano, 1999).

HOW THIS DOCUMENT CAN BE USED

This collection of content sequenced by topic should prove useful for those districts and schools that seek to assign their state's grade-range content to specific grades for instruction. It is quite likely that the topics in the pages that follow are addressed in nearly every state's mathematics standards. Reviewing each topic in turn, users can compare the content to their own state standards document.

Of first interest during such a review is whether all content identified in this study for a given topic also can be found in the state or district standards being compared. Users may likely determine that any content not found should be added to their own standards because this content is commonly found in highly rated documents. Once the scope of content for a topic has been reviewed, the content should be examined for grade placement. For this work, two kinds of information are available. First, the study indicates where content, in the form of benchmarks, appears in sequence relative to other content in the same topic across three or more states. Second, for each benchmark in a topic, the grade or grades at which that content is found in the state documents is also provided.

Users of this document will want to consider the impact that grade placement might have on ensuring that local standards continue to be aligned with their state standards. For example, if differing source documents indicate that given content could be placed at either grade 4 or 5, and the user's own state standards place very similar content at the K–4 grade band, then grade 4 would be the better choice. Similarly, any information available about the assessment of mathematics content should be part of the decision-making process. If mathematics content is assessed in the fall of grade 6, for example, the appropriate content should be placed in the prior grade so that students will have an opportunity to learn the material before being tested on it.

Once a grade has been assigned to any one benchmark, the user then should consider for grade assignment those benchmarks in the topic sequence that come before or after the benchmark (that is, that are positioned above or below the given benchmark in the charts that follow). The two primary questions for the assignment of a grade to any benchmark, then, are, Does the grade assigned to the benchmark appropriately reflect the grade range identified in the state standards? and, Does the grade assigned to the benchmark make sense in light of the benchmarks that will come before and after it? In similar fashion, the remaining topics should be reviewed for information that is available from the topic sequences in the course of making grade placement decisions.

It is important to keep in mind that the nature of this study is such that concepts and sequences appear as they do only because they appear in some consistent fashion across a number of documents. These documents are otherwise independent of each other and can significantly differ in their grade assignments of content. Our analysis focused only on the sequence of content instruction. We did not anticipate that all topics would fit together as if to reflect a coherent curriculum; the goal was simply to reflect what evidence there is that some concepts and skills are consistently taught earlier or later than others.

While research is not likely to support the idea that specific content should be part of instruction at only one particular grade, there does appear to be some agreement that certain concepts and skills can be articulated in a progression of understanding that helps students learn. This study identifies the articulation of content as it was found expressed in a number of highly rated state standards documents. It also includes information about the grade levels at which that content first appears. It is hoped that educators who seek to make decisions about which content should be covered in which grade will find this information useful.

SEQUENCED BENCHMARKS

For each of the 31 mathematics topic sequences presented in the charts that follow, the placement of the benchmarks indicates the sequence in which content is presented in the state standards. The benchmarks are graphically situated along two axes. The vertical axis, or y-axis, labeled "Content Sequence," indicates the sequence of content established through the analysis of the state documents. If a page is viewed from the bottom to the top, each benchmark should be considered as appearing prior in sequence to any benchmark above it.

Benchmarks also are organized horizontally, along the x-axis. This organization indicates a type of sequence, but not a sequence of the strength established for benchmarks along the vertical axis. The horizontal axis simply indicates the grades at which the benchmarks appear within the standards documents. Thus, a number of benchmarks along a row might even be introduced at the same grade. Simply put, if, according to the rules outlined earlier in this study, a benchmark could be shown to always follow the benchmarks it currently shares a row with, it would be moved to the row above. As it is, benchmarks appear on the same row only because they share a common relationship to the benchmarks in the rows above and below; that is, they have been shown to consistently precede and follow them.

For example, consider Topic 3, Fractions. The two benchmarks on the second row from the bottom refer, respectively, to understanding the relative magnitude of fractions, and the ability to add, subtract, divide, and multiply fractions. From our study, it is clear that this content always follows content related to the concept of a unit and its subdivision into equal parts, the benchmark that appears on the lowest row. These two ideas consistently precede content regarding the characteristics and properties of fractions, as well as the ability to use proportional reasoning to solve problems, content that appears in the benchmarks on the highest row. It was not possible to establish, however, that benchmarks about relative magnitude consistently followed or consistently preceded benchmarks regarding operations on fractions. Hence, these two benchmarks remain together on the same row. Because evidence has established the grades at which these two separate ideas are introduced in state standards documents, they are organized along the same row to reflect this information.

Each box on each chart includes the benchmark that represents the content addressed by the states. Listed below the benchmark are the grades at which the content is first taught among the states, then a list, by each state, of the grade at which the content is first taught. Following this information is a reference to the *Compendium* benchmark upon which the content was based. (Frequently, content of the *Compendium* benchmarks was revised in order to more accurately capture the content found in the state documents). This reference identifies the content by standard number, grade range, and benchmark number in McREL's standards database.

Topic 1: Communicating about Mathematics

12

Content Sequence

K

Benchmark: Uses formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations

Grades content first taught: 7-12

Grades content first taught, by state:
CA (8-12); OH (9); UT (7-12)

****Compendium reference:***
Std 1, Gr 9-12, Bmrk 5

Benchmark: Understands appropriate terminology and notation used to define functions and their properties

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (11); UT (8-12)

****Compendium reference:***
Std 8, Gr 9-12, Bmrk 1

Benchmark: Understands the role of written symbols in representing mathematical ideas and the use of precise language in conjunction with the special symbols of mathematics

Grades content first taught: 3-6

Grades content first taught, by state:
CA (4); OH (4); UT (3); VA (6)

****Compendium reference:***
Std 1, Gr 6-8, Bmrk 8

Benchmark: Understands the common language of spatial sense

Grades content first taught: K-1

Grades content first taught, by state:
CA (1); OH (K); UT (K); VA (1)

****Compendium reference:***
Std 5, Gr K-2, Bmrk 2

Benchmark: Knows the language of basic operations

Grades content first taught: 1-3

Grades content first taught, by state:
OH (3); UT (1); VA (2)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 9

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 2: Exponents/Logarithms/Roots

12

Content Sequence

K

Benchmark: Understands the properties and basic theorems of roots, exponents, and logarithms

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); OH (10); UT (10-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 2, Gr 9-12, Bmrk 2

Benchmark: Uses a variety of operations (e.g., raising to a power, taking a root, taking a logarithm) on expressions containing real numbers

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); UT (9-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 3,Gr 9-12, Bmrk 4

Benchmark: Understands basic concepts (e.g., roots) applications, and solution methods (e.g., factoring) of polynomial equations

Grades content first taught: 8-12

Grades content first taught, by state: A (8-12); OH (11); UT (9-12); WV (9-12)

****Compendium reference:***
Std 8, Gr 9-12, Bmrk 5

Benchmark: Understands the characteristics and uses of exponents and scientific notation

Grades content first taught: 5-6

Grades content first taught, by state: CA (5); OH (6); UT (5); VA (6)

****Compendium reference:***
Std 2, Gr 6-8, Bmrk 5

Benchmark: Understands the exponentiation of rational numbers and root-extraction

Grades content first taught: 5-7

Grades content first taught, by state: CA (7); OH (5); UT (7); VA (6); WV (6)

****Compendium reference:***
Std 3, Gr 6-8, Bmrk 3

Benchmark: Understands exponentials and the advantages and disadvantages of this type of representation

Grades content first taught: 6-8

Grades content first taught, by state: CA (6); OH (8); UT (7); VA (7); WV (7)

****Compendium reference:***
Std 2, Gr 6-8, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 3: Fractions

Content Sequence: 12 ↑ K

Benchmark: Understands the characteristics and properties of fractions

Grades content first taught: 6

Grades content first taught, by state: OH(6); UT (6); VA (6); WV (6)

****Compendium reference:*** Std 2, Gr 6-8, Bmrk 2

Benchmark: Uses proportional reasoning to solve mathematical and real-world problems (e.g., equivalent fractions)

Grades content first taught: 6

Grades content first taught, by state: OH (6); UT (6); VA (6); WV (6)

*Compendium reference: Std 3, Gr 6-8, Bmrk 6

Benchmark: Understands the relative magnitude of fractions

Grades content first taught: 3-4

Grades content first taught, by state: CA (4); OH (3); UT (4); WV (4)

*Compendium reference: Std 2, Gr 3-5, Bmrk 5

Benchmark: Adds and subtracts fractions with unlike denominators; multiplies and divides fractions

Grades content first taught: 4-5

Grades content first taught, by state: CA (5); OH (5); UT (5); VA (4); WV (4)

*Compendium reference: Std 3, Gr 6-8, Bmrk 2

Benchmark: Understands the concept of a unit and its subdivision into equal parts

Grades content first taught: 1-3

Grades content first taught, by state: CA (2); OH(1); UT (3); VA (2)

****Compendium reference:*** Std 2, Gr K-2, Bmrk 5

*Content in the McREL *Compendium* upon which is this benchmark is based

K → Grades where content first taught → 12

Topic 4: Integers

12

Content Sequence

K

Benchmark: Adds, subtracts, multiplies, and divides integers

Grades content first taught: 6-7

Grades content first taught, by state: CA (7); UT (6); VA (7)

****Compendium reference:*** Std 3, Gr 6-8, Bmrk 1

Benchmark: Understands the characteristics and properties of integers

Grades content first taught: 5-6

Grades content first taught, by state: CA (5); UT (5); VA (6)

****Compendium reference:*** Std 2, Gr 6-8, Bmrk 2

Benchmark: Understands the role of positive and negative integers in the number system

Grades content first taught: 5-6

Grades content first taught, by state: CA (5); UT (5); VA (6)

****Compendium reference:*** Std 2, Gr 6-8, Bmrk 3

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 5: Ratio/Proportion/Percent

Content Sequence

12

K

Benchmark: Understands equivalent forms of basic percents, fractions, and decimals

Grades content first taught: 3-5

Grades content first taught, by state:
CA (3); UT (5); VA (4); WV (4)

****Compendium reference:***
Std 2, Gr 3-5, Bmrk 2

Benchmark: Understands the concepts of ratio, proportion, and percent and the relationships among them

Grades content first taught: 6

Grades content first taught, by state:
CA (6); UT (6); VA (6); WV (6)

****Compendium reference:***
Std 2, Gr 6-8, Bmrk 7

Benchmark: Selects and uses an appropriate direct or indirect method of measurement in a given situation

Grades content first taught: 7-8

Grades content first taught, by state:
OH (7); UT (7); VA (8); WV (7)

****Compendium reference:***
Std 4, Gr 9-12, Bmrk 3

Benchmark: Uses proportional reasoning to solve problems involving scale factors

Grades content first taught: 7-12

Grades content first taught, by state:
CA (7); OH (7); UT (7-8); VA (9-12)

****Compendium reference:***
Std 5, Gr 3-5, Bmrk 7

Benchmark: Uses trigonometric ratio methods to solve mathematical and real-world problems

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); UT (8-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 8

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 6: Whole Numbers – Procedures

12

Content Sequence

K

Benchmark: Adds, subtracts, multiplies, and divides integers and rational numbers

Grades content first taught: 5-7

Grades content first taught, by state: CA (7); OH (5); UT (6); VA (6); WV (6)

****Compendium reference:***
Std 3, Gr 6-8, Bmrk 1

Benchmark: Adds and subtracts whole numbers

Grades content first taught: K

Grades content first taught, by state: CA (K); OH (K); UT (K); VA (K)

****Compendium reference:***
Std 3, Gr K-2, Bmrk 1

Benchmark: Solves real-world problems involving addition and subtraction of whole numbers

Grades content first taught: K-3

Grades content first taught, by state: OH (K); UT (3); VA (3); WV (3)

****Compendium reference:***
Std 3, Gr K-2, Bmrk 2

Benchmark: Performs basic mental computations

Grades content first taught: 1-4

Grades content first taught, by state: CA (2); OH (4); VA (1); WV (1)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 5

Benchmark: Multiplies and divides whole numbers

Grades content first taught: 1-3

Grades content first taught, by state: CA (2); OH (1); UT (3); VA (3); WV (3)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 7: Whole Numbers – Relationships

12

Content Sequence

K

Benchmark: Understands the relationships among equivalent number representations and the advantages and disadvantages of each type of representation

Grades content first taught: 5-8

Grades content first taught, by state: OH (5); UT (6); VA (7); WV (8)

****Compendium reference:*** Std 2, Gr 6-8, Bmrk 1

Benchmark: Understands the characteristics and properties of the set of rational numbers and its subsets

Grades content first taught: 6-7

Grades content first taught, by state: CA (7); OH (7); UT (6); VA (6); WV (6)

****Compendium reference***: Std 2, Gr 6-8, Bmrk 2

Benchmark: Understands the basic difference between odd and even numbers

Grades content first taught: 2-3

Grades content first taught, by state: OH (2); UT (3); WV (2)

****Compendium reference:*** Std 2, Gr 3-5, Bmrk 3

Benchmark: Understands the relative magnitude and relationships among whole numbers, fractions, decimals, and mixed numbers

Grades content first taught: 3-4

Grades content first taught, by state: OH (3); UT (3); VA (4); WV (3)

****Compendium reference:*** Std 2, Gr 3-5, Bmrk 6

Benchmark: Uses models to identify, order, and compare numbers

Grades content first taught: 3-4

Grades content first taught, by state: OH (3); UT (3); VA (4)

****Compendium reference:*** Std 2, Gr 3-5, Bmrk 6

Benchmark: Understands basic whole number relationships

Grades content first taught: K

Grades content first taught, by state: OH (K); UT (K); VA (K); WV (K)

****Compendium reference:*** Std 2, Gr K-2, Bmrk 4

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 8: Addition/Subtraction – Processes

12

Content Sequence

K

Benchmark: Adds, subtracts, and simplifies radical expressions containing positive rational numbers

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (8); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:*** Std 3, Gr 9-12, Bmrk 2

Benchmark: Adds, subtracts, and simplifies rational expressions

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (8); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference***: Std 3, Gr 9-12, Bmrk 1

Benchmark: Adds and subtracts integers and rational numbers

Grades content first taught: 6-7

Grades content first taught, by state: OH (6); UT (7); WV (7)

****Compendium reference:*** Std 3, Gr 6-8, Bmrk 1

Benchmark: Adds and subtracts fractions with unlike denominators

Grades content first taught: 4-5

Grades content first taught, by state: CA (5); OH (5); VA (4); WV (4)

****Compendium reference:*** Std 3, Gr 6-8, Bmrk 2

Benchmark: Adds and subtracts simple fractions

Grades content first taught: 3-5

Grades content first taught, by state: CA (3); OH (4); UT (5); VA (3); WV (3)

****Compendium reference:*** Std 3, Gr 3-5, Bmrk 3

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → 12

Topic 9: Addition/Subtraction – Properties and Order of Operations

12

Content Sequence

K

Benchmark: Adds, subtracts, and simplifies radical expressions containing positive rational numbers

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (8); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 3, Gr 9-12, Bmrk 2

Benchmark:Adds, subtracts, and simplifies rational expressions

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (8); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 3, Gr 9-12, Bmrk 1

Benchmark: Understands the correct order of operations for performing arithmetic computations

Grades content first taught: 4-7

Grades content first taught, by state: CA (5); UT (5); VA (7); WV (4)

****Compendium reference***:
Std 3, Gr 6-8, Bmrk 5

Benchmark: Understands the properties of and the relationships between addition and subtraction

Grades content first taught: 1-2

Grades content first taught, by state: CA (1); UT (1); VA (2); WV (1)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 7

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → 12

Topic 10: Multiplication/Division

Content Sequence

12

K

Benchmark: Multiplies, divides, and simplifies rational expressions

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (9); UT (9-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 3, Gr 9-12, Bmrk 1

Benchmark: Multiplies, divides and simplifies radical expressions containing positive rational numbers

Grades content first taught: 8-12

Grades content first taught, by state:
OH (11); UT (8-12); VA (9-12)

****Compendium reference:***
Std 3, Gr 9-12, Bmrk 2

Benchmark: Multiplies and divides decimals

Grades content first taught: 5

Grades content first taught, by state:
CA (5); OH (5); UT (5); WV (5)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 2

Benchmark: Multiplies and divides fractions

Grades content first taught: 5-6

Grades content first taught, by state:
CA (5); OH (5); UT (5); VA (6); WV (5)

****Compendium reference:***
Std 3, Gr 6-8, Bmrk 2

Benchmark: Multiplies and divides integers and rational numbers

Grades content first taught: 5-7

Grades content first taught, by state:
CA (5); OH (6); UT (6); VA (6); WV (7)

****Compendium reference:***
Std 3, Gr 6-8, Bmrk 1

Benchmark: Understands the correct order of operations for performing arithmetic computations

Grades content first taught: 5-7

Grades content first taught, by state:
CA (5); OH (5); UT (5); VA (7)

****Compendium reference:***
Std 3, Gr 6-8, Bmrk 5

Benchmark: Performs basic mental computations

Grades content first taught: 1-3

Grades content first taught, by state:
CA (1); OH (3); UT (3); VA (3); WV (3)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 5

Benchmark: Understands the properties of and the relationships between multiplication and division

Grades content first taught: 1-3

Grades content first taught, by state:
CA (2); OH (1); UT (3); WV (3)

****Compendium reference:***
Std 3, Gr 3-5, Bmrk 7

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 11: Basic and Linear Measures

12

Content Sequence

K

Benchmark: Selects and uses an appropriate direct or indirect method of measurement in a given situation

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (9); UT (7-12); VA (8);WV (9-12)

****Compendium reference:***
Std 4, Gr 9-12, Bmrk 3

Benchmark: Understands relationships between measures (e.g., between length, perimeter, and area)

Grades content first taught: 4-7

Grades content first taught, by state: CA (4); OH (7); UT (7); VA (5); WV (5)

****Compendium reference:***
Std 4, Gr 3-5, Bmrk 4

Benchmark: Understands procedures for basic indirect measurements

Grades content first taught: 5-8

Grades content first taught, by state: CA (5); OH (8); WV (8)

****Compendium reference:***
Std 4, Gr 6-8, Bmrk 9

Benchmark: Understands the basic measures length, width, height, weight, and temperature

Grades content first taught: K

Grades content first taught, by state: CA (K); OH (K); UT (K); VA (K); WV (K)

****Compendium reference:***
Std 4, Gr K-2, Bmrk 1

Benchmark: Makes quantitative estimates of familiar linear dimensions, weights, and time intervals and checks them against measurements

Grades content first taught: K-3

Grades content first taught, by state: CA (3); OH (1); UT (1); VA (2); WV (K)

****Compendium reference:***
Std 4, Gr K-2, Bmrk 4

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → 12

Topic 12: Perimeter/Area/Volume

12

Content Sequence

K

Benchmark: Solves problems involving missing parts of figures, lengths, and distances, using formulas, similar figures, and properties of figures

Grades content first taught: 7-12

Grades content first taught, by state:
CA (8-12); UT (7-12); VA (8); WV (9-12)

****Compendium reference:***
Std 4, Gr 9-12, Bmrk 3

Benchmark: Solves real-world problems involving three-dimensional measures (e.g., volume, surface area) using formulas

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (9); UT (8-12); VA (8); WV (8)

****Compendium reference:***
Std 4, Gr 9-12, Bmrk 4

Benchmark: Understands formulas for finding measures (e.g. volume, area)

Grades content first taught: 4-6

Grades content first taught, by state:
CA (4); OH (5); UT (5); VA (4); WV (6)

****Compendium reference:***
Std 4, Gr 6-8, Bmrk 7

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → 12

Topic 13: Units of Measurement

12

Content Sequence

K

Benchmark: Uses unit analysis to solve problems involving measurement and unit conversion

Grades content first taught: 7-8

Grades content first taught, by state:
CA (7); OH (8); UT (7-8); WV (7)

****Compendium reference:***
Std 4, Gr 9-12, Bmrk 5

Benchmark: Knows approximate size of basic standard units and the relationships between them

Grades content first taught: 1-3

Grades content first taught, by state:
CA (2); OH (2); UT (3); WV (1)

****Compendium reference:***
Std 4, Gr 3-5, Bmrk 3

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 14: Volume and Capacity

12

Content Sequence

K

Benchmark: Understands the relationships among linear dimensions, area, and volume and the corresponding uses of units, square units, and cubic units of measure

Grades content first taught: 5-8

Grades content first taught, by state: CA (5); OH (5); UT (7-8)

****Compendium reference:*** Std 4, Gr 6-8, Bmrk 3

Benchmark: Selects and uses appropriate tools for given measurement situations

Grades content first taught: 1-3

Grades content first taught, by state: CA (3); OH (2); UT (1); WV (1)

****Compendium reference:*** Std 4, Gr 3-5, Bmrk 2

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 15: Weight and Mass

12

Content Sequence

K

Benchmark: Makes quantitative estimates of familiar weights and checks them against measurements

Grades content first taught: 2-3

Grades content first taught, by state: CA (3); UT (2); VA (2)

****Compendium reference:***
Std 4, Gr K-2, Bmrk 4

Benchmark: Understands the basic measure of weight

Grades content first taught: K

Grades content first taught, by state: CA (K); UT (K); VA (K)

****Compendium reference:***
Std 4, Gr K-2, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → 12

Topic 16: Lines and Angles

12

Content Sequence

K

Benchmark: Uses synthetic representations and analytic methods to solve problems involving symmetry and transformations of figures (e.g., problems involving distance, midpoint, and slope)

Grades content first taught: 7-12

Grades content first taught, by state:
OH (8); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 3

Benchmark: Uses inductive and deductive reasoning to make observations about figures and to verify properties of and relationships among figures (e.g., the relationship among interior angles of parallel lines cut by a transversal)

Grades content first taught: 8-12

Grades content first taught, by state:
OH (8); UT (8-12); WV (8)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 10

Benchmark: Uses properties of and relationships among figures to solve mathematical and real-world problems

Grades content first taught: 8-12

Grades content first taught, by state:
CA(8-12); OH (10); UT (8-12); WV (9-12)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 11

Benchmark: Understands the defining properties of three-dimensional figures (e.g., a cube has edges with equal lengths, right angle corners)

Grades content first taught: 5-8

Grades content first taught, by state:
CA (7); VA (5); WV (8)

****Compendium reference:***
Std 5, Gr 6-8, Bmrk 2

Benchmark: Understands the basic measures perimeter, angle

Grades content first taught: 2-3

Grades content first taught, by state:
CA (3); VA (3); WV (2)

****Compendium reference:***
Std 4, Gr 3-5, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 17: Motion Geometry

12

Content Sequence

K

Benchmark: Understands the concept of a function as the correspondences between the elements of two sets (e.g., functions relate sets of points to their images under motions such as flips, slides, and turns)

Grades content first taught: 9-12

Grades content first taught, by state:
UT (9-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 8, Gr 9-12, Bmrk 6

Benchmark: Uses synthetic representations and analytic methods to solve problems involving transformations of figures

Grades content first taught: 7-12

Grades content first taught, by state:
CA (8-12); OH (8); UT (7-12)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 3

Benchmark: Understands geometric transformations of figures

Grades content first taught: 5-7

Grades content first taught, by state:
OH (6); UT (6); VA (7); WV (5)

****Compendium reference:***
Std 5, Gr 6-8, Bmrk 4

Benchmark: Uses motion geometry to understand geometric relationships

Grades content first taught: 2-4

Grades content first taught, by state:
OH (4); UT (3); VA (4);
WV (2)

****Compendium reference:***
Std 5, Gr 3-5, Bmrk 5

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 18: Shapes and Figures

12 ← Content Sequence → K

Benchmark: Uses synthetic representations and analytic methods to solve problems involving symmetry and transformations of figures

Grades content first taught: 7-12

Grades content first taught, by state:
OH (8); UT (7-12); VA (9-12)

****Compendium reference:***

Benchmark: Uses geometric constructions to complete simple proofs, to model, and to solve real-world problems

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (10); VA (9-12)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 5

Benchmark: Uses inductive and deductive reasoning to make observations about and to verify properties of and relationships among figures

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); UT (8-12); VA (9-12)

****Compendium reference:***

Benchmark: Uses properties of and relationships among figures to solve mathematical and real-world problems

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (10); UT (10-12); VA (9-12); WV (9-12)

****Compendium reference:***

Benchmark: Understands the defining properties of three dimensional figures

Grades content first taught: 6-7

Grades content first taught, by state:
CA (7); OH (6); UT (6)

****Compendium reference:***

Benchmark: Predicts and verifies the effects of combining, subdividing, and changing basic shapes

Grades content first taught: 2-5

Grades content first taught, by state:
CA (2); OH (2); UT (3); VA (5)

Benchmark: Understands that geometric shapes are useful for representing and describing real-world situations

Grades content first taught: K-1

Grades content first taught, by state:
CA (K); OH (K); UT (K); VA (1)

****Compendium reference:***

Benchmark: Understands that patterns can be made by putting different shapes together or taking them apart

Grades content first taught: K-1

Grades content first taught, by state:
CA (1); OH (K); UT (K)

****Compendium reference:***

*Content in the McREL *Compendium* upon which is this benchmark is based

K ← Grades where content first taught → 12

Topic 19: Data Organization and Display – Methods

Content Sequence (K → 12)

Benchmark: Collects and represents information about objects or events in simple graphs

Grades content first taught: K

Grades content first taught, by state: CA (K); OH (K); VA (K)

****Compendium reference:**** Std 6, Gr K-2, Bmrk 1

Benchmark: Organizes and displays data in simple bar graphs, pie charts, and line graphs

Grades content first taught: 3

Grades content first taught, by state: CA (3); UT (3); VA (3); WV (3)

****Compendium reference:**** Std 6, Gr 3-5, Bmrk 4

Benchmark: Understands that data come in many different forms and that organizing and displaying data can be done in many ways

Grades content first taught: 3-5

Grades content first taught, by state: CA (5); OH (3); VA (5)

****Compendium reference:**** Std 6, Gr 3-5, Bmrk 6

Benchmark: Organizes and displays data using tables, graphs, frequency distributions, and plots

Grades content first taught: 5-7

Grades content first taught, by state: CA (7); OH (5); UT (6); VA (5); WV (6)

****Compendium reference:**** Std 6, Gr 6-8, Bmrk 6

Benchmark: Understands different methods and applications of curve-fitting

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); VA (9-12); WV (9-12)

****Compendium reference:**** Std 6, Gr 9-12, Bmrk 4

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → **12**

Topic 20: Data Organization and Display – Reading and Interpretation

12

Content Sequence

K

Benchmark: Understands the concept of correlation

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (9); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 6, Gr 9-12, Bmrk 3

Benchmark: Understands that the same set of data can be represented using a variety of tables, graphs, and symbols and that different modes of representations often convey different messages

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); VA (9-12); WV (9-12)

****Compendium reference:***
Std 6, Gr 6-8, Bmrk 8

Benchmark: Understands how the reader's bias, measurement error, and display distortion can affect the interpretation of data

Grades content first taught: 9-12

Grades content first taught, by state: OH (9); UT (10-12); VA (9-12)

****Compendium reference:***
Std 6, Gr 9-12, Bmrk 6

Benchmark: Reads and interprets data in charts, tables, and plots

Grades content first taught: 5-7

Grades content first taught, by state: OH (6); UT (6); VA (7); WV (5)

****Compendium reference:***
Std 6, Gr 6-8, Bmrk 4

Benchmark: Understands faulty arguments, common errors, and misleading presentations of data

Grades content first taught: 6-8

Grades content first taught, by state: OH (7); UT (6); WV (8)

****Compendium reference:***
Std 6, Gr 6-8, Bmrk 7

Benchmark: Reads and interprets simple bar graphs, pie charts, and line graphs

Grades content first taught: 2-3

Grades content first taught, by state: OH (2); UT (3); VA (3); WV (3)

****Compendium reference:***
Std 6, Gr 3-5, Bmrk 5

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught → 12

Topic 21: Data Collection and Sampling

12

Content Sequence

K

Benchmark: Understands the basic concept of a sample

Grades content first taught: 6-8

Grades content first taught, by state: CA (6); OH (8); UT (7)

****Compendium reference:***
Std 6, Gr 3-5, Bmrk 7

Benchmark: Understands the basic concepts about how samples are chosen

Grades content first taught: 6-8

Grades content first taught, by state: CA (6); OH (7); UT (7-8)

****Compendium reference:***
Std 6, Gr 6-8, Bmrk 10

Benchmark: Understands that data represent specific pieces of information about real-world objects or activities

Grades content first taught: 2-4

Grades content first taught, by state: CA (4); OH (2); UT (4)

****Compendium reference:***
Std 6, Gr 3-5, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 22: Data Distribution

12

Content Sequence

K

Benchmark: Understands measures of central tendency and variability and their applications to specific situations

Grades content first taught: 7-12

Grades content first taught, by state:
CA (7); OH (10);
VA (9-12); WV (9-12)

****Compendium reference:***
Std 6, Gr 9-12, Bmrk 2

Benchmark: Understands the concept of correlation

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (11);
VA (9-12); WV (9-12)

****Compendium reference:***
Std 6, Gr 9-12, Bmrk 3

Benchmark: Understands different methods of curve-fitting and various applications

Grades content first taught: 7-12

Grades content first taught, by state:
CA (8-12); OH (9);
UT (7-12); VA (9-12);
WV (9-12)

****Compendium reference:***
Std 6, Gr 9-12, Bmrk 4

Benchmark: Understands sampling distributions, the central limit theorem and confidence intervals

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (9);
VA (9-12)

****Compendium reference:***
Std 6, Gr 9-12, Bmrk 7

Benchmark: Understands the properties of the normal curve and how the it can be used to answer questions about sets of data

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (11);
VA (9-12); WV (9-12)

****Compendium reference:***
Std 7, Gr 9-12, Bmrk 5

Benchmark: Understands basic characteristics of frequency and distribution

Grades content first taught: 5-6

Grades content first taught, by state:
CA (6); OH (6); UT (5)

****Compendium reference:***
Std 6, Gr 6-8, Bmrk 2

Benchmark: Understands that a summary of data should include where the middle is and how much spread there is around it

Grades content first taught: 2-4

Grades content first taught, by state:
CA (2); OH (4); UT (4)

****Compendium reference:***
Std 6, Gr 3-5, Bmrk 3

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 23: Likelihood/Chance/Certainty

Content Sequence

12

K

Benchmark: Knows that a random variable can be represented by a discrete or continuous probability distribution

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); UT (10-12); VA (9-12)

****Compendium reference:*** Std 7, Gr 9-12, Bmrk 1

Benchmark: Determines probability using simulations or experiments

Grades content first taught: 4-6

Grades content first taught, by state: CA (6); OH (5); UT (4); VA (6)

****Compendium reference:*** Std 7, Gr 6-8, Bmrk 2

Benchmark: Determines probability using mathematical/theoretical models

Grades content first taught: 5-8

Grades content first taught, by state: CA (6); OH (5); UT (7-8); VA (7)

****Compendium reference:*** Std 7, Gr 6-8, Bmrk 1

Benchmark: Recognizes events that are sure to happen, events that are sure not to happen, and events that may or may not happen

Grades content first taught: 1-4

Grades content first taught, by state: OH (1); UT (3); VA (4)

****Compendium reference:*** Std 7, Gr 3-5, Bmrk 2

Benchmark: Understands that some events are more likely to happen than others

Grades content first taught: K

Grades content first taught, by state: OH (K); UT (K); VA (K)

****Compendium reference:*** Std 7, Gr K-2, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 24: Reasoning and Predicting from Data

12

Content Sequence

K

Benchmark: Understands the concept of correlation

Grades content first taught: 7-12

Grades content first taught, by state: OH (9); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:*** Std 6, Gr 9-12, Bmrk 3

Benchmark: Understands the properties of the normal curve and how it can be used to answer questions about sets of data

Grades content first taught: 8-12

Grades content first taught, by state: OH (11); UT (8-12); VA (9-12); WV (9-12)

****Compendium reference:*** Std 7, Gr 9-12, Bmrk 5

Benchmark: Understands how the reader's bias, measurement error, and display distortion can affect the interpretation of data

Grades content first taught: 7-12

Grades content first taught, by state: OH (9); UT (7-12); WV (9-12)

****Compendium reference:*** Std 6, Gr 9-12, Bmrk 6

Benchmark: Understands how predictions are based on data and probabilities

Grades content first taught: 5-8

Grades content first taught, by state: OH (5); UT (7-8); VA (7); WV (7)

****Compendium reference:*** Std 7, Gr 6-8, Bmrk 3

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 25: Graphs and Graphing Systems – Correspondence to Algebra

12

Content Sequence

K

Benchmark: Understands that objects and relations in geometry correspond directly to objects and relations in algebra

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (8); UT (7-12); WV (9-12)

****Compendium reference:*** Std 5, Gr 9-12, Bmrk 1

Benchmark: Understands properties of graphs and the relationship between a graph and it's corresponding expression

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (9); UT (7-12); WV (9-12)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 4

Benchmark: Understands the basic concept of inverse function and the corresponding graph

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); OH (9); UT (9-12); WV (9-12)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 10

Benchmark: Uses the rectangular coordinate system to model and to solve problems

Grades content first taught: 5-7

Grades content first taught, by state: CA (7); OH (5); UT (7); WV (7)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 9

Benchmark: Understands special values (e.g., minimum and maximum values, x- and y-intercepts, slope, constant ratio or difference) of patterns, relationships, and functions

Grades content first taught: 7-8

Grades content first taught, by state: CA (7); OH (8); WV (8)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 7

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 26: Graphs and Graphing Systems – Solving Problems

12

Content Sequence

K

Benchmark: Understands that objects and relations in geometry correspond directly to objects and relations in algebra (e.g., a line in geometry corresponds to a set of ordered pairs satisfying an equation of the form of ax + by = c)

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (8); VA (9-12); WV (9-12)

****Compendium reference:***
Std 5, Gr 9-12, Bmrk 1

Benchmark: Understands the properties of graphs and the relationship between a graph and its corresponding expression

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (9); VA (9-12); WV (9-12)

****Compendium reference:***
Std 8, Gr 9-12, Bmrk 4

Benchmark: Uses graphs to represent functions, patterns, and relationships

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (9); VA (9-12); WV (9-12)

****Compendium reference:***
Std 8, Gr 9-12, Bmrk 7

Benchmark: Uses graphs to solve systems of equations and inequalities

Grades content first taught: 8-12

Grades content first taught, by state:
CA (8-12); OH (9); VA (9-12); WV (9-12)

****Compendium reference:***
Std 8, Gr 9-12, Bmrk 11

Benchmark: Uses graphs to solve linear equations

Grades content first taught: 7-12

Grades content first taught, by state:
UT (7-12); VA (8); WV (7)

****Compendium reference:***
Std 8, Gr 6-8, Bmrk 5

Benchmark: Knows basic characteristics and features of the rectangular coordinate system

Grades content first taught: 4-5

Grades content first taught, by state:
UT (5); VA (4); WV (4)

****Compendium reference:***
Std 8, Gr 3-5, Bmrk 6

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 27: Equations and Inequalities – Representation

12

Content Sequence

K

Benchmark: Uses expressions, equations, and inequalities to represent situations and translates among these representations

Grades content first taught: 6-7

Grades content first taught, by state: CA (6); OH (7); UT (7); WV (7)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 2

Benchmark: Uses a variety of models to represent functions, patterns, and relationships

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (7); UT (7-12); VA (8)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 7

Benchmark: Knows that an expression is a mathematical statement using numbers and symbols to represent relationships

Grades content first taught: 2-5

Grades content first taught, by state: OH (2); UT (5); VA (5); WV (3)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 1

Benchmark: Knows that an equation is a number sentence that shows two quantities that are equal

Grades content first taught: 1-2

Grades content first taught, by state: OH (1); UT (1); VA (2); WV (1)

****Compendium reference:*** Std 8, Gr 3-5, Bmrk 4

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 28: Equations and Inequalities – Solving Problems

12

Content Sequence

K

Benchmark: Solves polynomial equations using a variety of methods

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); UT (7-12); VA (9-12)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 5

Benchmark: Solves systems of equations and inequalities using a variety of methods

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (9); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 11

Benchmark: Solves linear equations using concrete, informal, and formal methods

Grades content first taught: 5-6

Grades content first taught, by state: CA (6); OH (6); UT (5); VA (6)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 5

Benchmark: Solves inequalities and non-linear equations using concrete and informal methods

Grades content first taught: 7-8

Grades content first taught, by state: CA (7); OH (8); UT (7); VA (7)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 6

Benchmark: Solves simple open sentences involving whole numbers

Grades content first taught: 1-3

Grades content first taught, by state: CA (3); OH (1); UT (2); VA (2)

****Compendium reference:*** Std 8, Gr 3-5, Bmrk 5

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 29: Expressions

12 ▲ Content Sequence ▼ K

Benchmark: Adds, subtracts, multiplies, divides, and simplifies rational expressions

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); OH (9); VA (9-12); WV (9-12)

****Compendium reference:*** Std 3, Gr 9-12, Bmrk 1

Benchmark: Adds, subtracts, multiplies, divides, and simplifies radical expressions containing positive rational numbers

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); VA (9-12); WV (9-12)

****Compendium reference:*** Std 3, Gr 9-12, Bmrk 2

Benchmark: Knows that an expression is a mathematical statement using numbers and symbols to represent relationships and real-world situations

Grades content first taught: 5-7

Grades content first taught, by state: CA (5); OH (5); VA (7); WV (7)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 1

Benchmark: Understands various representations (e.g., algebraic expressions) of patterns and functions

Grades content first taught: 5-8

Grades content first taught, by state: CA (5); OH (5); VA (7); WV (8)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 3

Benchmark: Understands basic operations (e.g., combining like terms, expanding, substituting for unknowns) on algebraic expressions

Grades content first taught: 5-7

Grades content first taught, by state: CA (5); OH (5); UT (7); VA (7); WV (6)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 8

*Content in the McREL *Compendium* upon which is this benchmark is based

K ——— Grades where content first taught ——— 12

Topic 30: Patterns

12

Content Sequence

K

Benchmark: Identifies a pattern in order to understand new mathematical content and to develop more efficient solution methods or problem extensions

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); OH (9); UT (8-12); WV (9-12)

****Compendium reference:***

Benchmark: Uses a variety of models to represent patterns

Grades content first taught: 7-12

Grades content first taught, by state: CA (8-12); OH (9); UT (7-12); VA (9-12); WV (9-12)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 7

Benchmark: Understands various representations of patterns

Grades content first taught: 6-7

Grades content first taught, by state: CA (7); OH (6); UT (6); VA (7); WV (6)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 3

Benchmark: Understands the properties of arithmetic and geometric sequences

Grades content first taught: 6-7

Grades content first taught, by state: CA (7); OH (7); VA (7); WV (6)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 11

Benchmark: Understands special values of patterns

Grades content first taught: 7-8

Grades content first taught, by state: CA (7); OH (8); WV (7)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 7

Benchmark: Recognizes a wide variety of patterns and the rules that explain them

Grades content first taught: 2-3

Grades content first taught, by state: CA (3); UT (3); VA (3); WV (2)

****Compendium reference:*** Std 8, Gr 3-5, Bmrk 1

Benchmark: Understands the same pattern can be represented in different ways

Grades content first taught: 3-4

Grades content first taught, by state: CA (4); OH (4); UT (4); VA (3); WV (4)

****Compendium reference:*** Std 8, Gr 3-5, Bmrk 2

Benchmark: Recognizes regularities in a variety of contexts

Grades content first taught: K-1

Grades content first taught, by state: CA (1); OH (K); UT (K); VA (K); WV (K)

****Compendium reference:*** Std 8, Gr K-2, Bmrk 1

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

Topic 31: Variables

12

Content Sequence

K

Benchmark: Represents variable quantities using expressions, equations, inequalities, and matrices

Grades content first taught: 6-7

Grades content first taught, by state: CA (6); OH (7); VA (7)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 2

Benchmark: Understands the concept of a function as the relationship between variables that represent numbers

Grades content first taught: 8-12

Grades content first taught, by state: CA (8-12); OH (8); VA (9-12); WV (9-12)

****Compendium reference:*** Std 8, Gr 9-12, Bmrk 6

Benchmark: Understands that a variable can be used in many ways (e.g., as a placeholder; as a representative of a range of values)

Grades content first taught: 5-7

Grades content first taught, by state: OH (5); UT (7); VA (5); WV (7)

****Compendium reference:*** Std 8, Gr 6-8, Bmrk 2

Benchmark: Knows that a variable is a letter or symbol that stands for one or more numbers

Grades content first taught: 1-4

Grades content first taught, by state: CA (4); OH (3); UT (1); VA (2); WV (3)

****Compendium reference:*** Std 8, Gr 3-5, Bmrk 3

*Content in the McREL *Compendium* upon which is this benchmark is based

K — Grades where content first taught — 12

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