CHALLENGING STANDARDS IN MATHEMATICS: WHAT ARE HIGH SCHOOLS MISSING?

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PREFACE

This report is one in a series of reference documents designed especially to assist educators who are directly involved in the revision and improvement of content standards. It presumes a basic understanding of the purposes for standards and the process of standards review and evaluation. This document is intended to be a desktop reference volume that curriculum specialists might use to focus and revise existing mathematics content. This report also may be useful to educators and policymakers who want to ensure that the mathematics standards they hold for high school graduates are adequate to satisfy the expectations of post-secondary institutions and employers. Readers who desire more background and context for the work described here should consult *A Technical Guide for Revising or Developing Standards and Benchmarks* (Kendall, 2001).

This report focuses attention on student knowledge and skills in mathematics that recently have been identified as important for success beyond high school but that are not found even among state standards that have been highly rated. Identification of this important but missing mathematics content resulted from a comparison of national and highly rated state documents that describe what students should know and be able to do by the end of 12^{th} grade against documents that describe the knowledge and skills that incoming college students should acquire to be successful in their first year of college.

INTRODUCTION:
RISING EXPECTATIONS FOR HIGH SCHOOL SENIORS

In the last decade, public discourse on college-readiness has undergone a subtle but significant shift. Discussions that were once about what the college-bound student should know now focus on whether *all* high school students are adequately prepared for college-level work. As an example, the first standards document issued by the National Council of Teachers of Mathematics (NCTM, 1989), a report widely credited with inspiring similar standards development efforts from other subject-area disciplines, identified a separate set of standards for "college-intending" students. The 2000 revision of the NCTM standards, however, does not include a distinct set of standards for college-bound students. The closest reference to the place of mathematics beyond high school is the statement that those

> who understand and can do mathematics will have significantly enhanced opportunities and options for shaping their futures. . . . All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. (p. 5)

Growing concerns about the expectations commonly held for all high school graduates have been voiced by a variety of groups. Economists, parents and their children, and educators have somewhat differing points of view, but interests converge on the question of whether high schools currently expect from students all the knowledge and skills they will need for future success. Economists and others argue that the knowledge and skills students acquire in high school will not be sufficient for their continued success; they must go on to higher education. As researchers Carnevale and Desrochers (2003) demonstrate, those who do not graduate from college face an ever-growing loss of opportunity and potential income over the course of a lifetime:

> Since the 1980s, the real inflation-adjusted earnings of male high school graduates and dropouts have declined precipitously.…Overall, the earnings of college-educated workers, compared with high school educated workers, have increased from about 43 to 62 percent since 1979, in spite of the fact that the supply of college-educated workers has doubled over the same period. (p. 3)

Even if students do not graduate from college, Carnevale and Desrochers contend that attending some college has a significant impact on future success:

> The fastest-growing and best-paying jobs have been those that require at least some college. Currently, six in ten jobs are held by workers with at least some postsecondary education or training, compared with two in ten in 1959. (p. 3)

Factory jobs, which were once a haven for high school dropouts, are now increasingly taken by those with some college experience. Between 1973 and 2000, the proportion of factory jobs held by individuals with at least some college education tripled and their wages held nearly steady, while those with a high school diploma or less saw their wages decline (Barth, 2003). It is not surprising, then, that in the public's mind a college education has replaced the high school diploma as the gateway to the middle class (Immerwahr, 2000). From a broader perspective, it is

not simply that individual students lose a brighter economic future if they are unprepared to continue their education. Carnevale and Desrochers present the case that continued international competitiveness and the economic well-being of the U.S. relies upon a system that prepares college-educated workers.

Students, and their parents, share a view that all graduating high school students should be ready for education beyond high school. Parents have higher educational aspirations for their children than ever before: 86 percent of parents want their children to pursue some postsecondary education (U.S. Census, 2003). Hispanics and African Americans have lower participation rates in higher education than the population as a whole, but, in fact, the parents of Hispanic and African American students are significantly more likely than whites to emphasize the value of higher education, not less (Immerwahr, 2000). Students' expectations are likewise high. For example, a study conducted by the U.S. Department of Education's National Center for Education Statistics found that 82 percent of high school sophomores expect to continue their education beyond high school (Ingels & Scott, 2004).

Clearly, these students and their parents need to understand what must be accomplished to meet college entrance requirements. Yet, according to a recent report from Stanford University's Bridge Project (Venezia, Kirst, & Antonio, 2003), one of most common student misconceptions about college readiness is that meeting their high school graduation requirements prepares them for college. It is plausible that such a misconception has its roots in the commonly held belief that readiness for college is the rightful expectation of every high school student who has earned graduation.

Interest in a system of education that encompasses kindergarten through grade 16 has been growing rapidly. The National Commission on the High School Senior Year (2001) urges a system in which "standards, curriculum, and assessment efforts are integrated" and postsecondary education and K–12 are more closely linked (p. 5). A strong linkage and seamless transition between high school graduation and college entry must form part of any such unified system of education. More than a dozen organizations are currently working to advance the move from a K–12 to a K–16 system.

America's commitment to universal public education now appears to embrace the proposition that all students deserve an education that will ready them for success in their post-secondary years, even if their paths should take them directly to the world of work. Under such a view, students might well opt out of college, but their high school diploma should indicate that they are fully prepared to attend. Such a perspective has a tradition, having been articulated late in the 19th century when the Committee of Ten, a National Education Association task force charged with bringing coherence to the high school curriculum and establishing uniform college entrance requirements, issued the following recommendation:

> [The Committee of Ten] unanimously declare that every subject which is taught at all in a secondary school should be taught in the same way and to the same extent to every pupil so long as he pursues it, no matter what the probable destination of the pupil may be, or at what point his education is to cease. (National Education Association, 1893, p. 17)

It is not surprising that one lasting impact of the Committee of Ten's report was the development of the Carnegie unit, which early on was defined as a course of five periods each week for one academic year (Ravitch, 1995). Thus, the belief that all students should be prepared to enter college is not new. What is new is that the view appears within the context of a standards-based system. A standards-based system — one in which the knowledge and skills that students should acquire are clearly articulated — provides a means for communicating specific expectations for students. This level of detail is not available in the Carnegie unit, the commonly used metric for describing high school accomplishment. Although the Carnegie unit is said to include a description of what students should learn, it has come under mounting criticism, most notably for its frequent use as a meaningless label of course content, for equating a high school diploma with earned seat time, and for creating an inflexible course structure (Maeroff, 1993). The underlying idea that led to the Carnegie unit gains focus and greater likelihood for success in a standards-based system: in order to integrate high school expectations with college admissions, clear statements of what students should be learning must be shared by both educational institutions.

WHAT'S NEEDED FOR SUCCESS BEYOND HIGH SCHOOL?

Post-secondary institutions and other organizations have recognized a gap between what is expected from high school students and the requirements for post-high school learning. Post-secondary institutions have found that high school graduates arrive less adequately prepared than they have in the past. To address this problem, many of these institutions offer remedial courses in reading, writing, and mathematics. The National Center for Education Statistics (NCES) defines postsecondary remedial education as "courses in reading, writing, or mathematics for college-level students lacking those skills necessary to perform college-level work at the level required by the institution" (Parsad & Lewis, 2003, p. iii). A recent NCES study of remedial education (Parsad & Lewis, 2003) reports:

> Between 1995 and 2000, the proportion of institutions that reported an average of 1 year of remediation for [incoming] students increased from 28 percent to 35 percent, while the proportion indicating an average of less than 1 year of remediation for students decreased from 67 percent to 60 percent. (p. iv)

Clearly, post-secondary institutions have a strong interest in whether high school graduation standards adequately describe the knowledge and skills that college-bound students need to be fully prepared. It seems logical to suppose that post-secondary remediation rates would decrease if there were a well-defined and agreed-upon set of standards at the high school level that accurately reflect the knowledge and skills students need to succeed at the post-secondary level.

Two organizations in particular have undertaken the task of communicating what post-secondary institutions and the world of work expect of high school graduates. One notable effort to meet the need for a clearer connection between high school courses and university expectations is Standards for Success, a project sponsored by the Association of American Universities in partnership with The Pew Charitable Trusts. One of the project's primary goals was to identify what graduating high school students need to know and be able to do in order to succeed in entry-level university courses. These student expectations, termed Knowledge and Skills for University Success, are presented in *Understanding University Success* (Conley, 2003),

the product of a two-year study in which more than 400 faculty and staff members from 20 research universities participated in extensive meetings and reviews. The disciplines covered included English, mathematics, natural sciences, social sciences, second languages, and the arts.

The American Diploma Project (2004) shares a similar purpose — connecting secondary and postsecondary expectations for success — but its focus is on "what it takes for graduates to compete successfully beyond high school — either in the classroom or in the workplace" (p. 1). The Project, a partnership of Achieve, Inc., The Education Trust, and the Thomas B. Fordham Foundation, spent nearly two years working with two- and four-year postsecondary faculty and front-line managers in high-growth, high-skill occupations to define the core knowledge and skills that high school graduates need in order to be ready to succeed in their organizations. The results of the study are presented in *Ready or Not: Creating a High School Diploma that Counts* (ADP, 2004), which describes student expectations, termed *benchmarks*, for English and mathematics. The report also includes sample workplace tasks and post-secondary assignments, which illustrate in real terms how the knowledge and skills captured in the benchmarks might be applied beyond high school, whether in the workplace or in the college classroom.

WHAT CAN WE LEARN?

In light of these two recent project reports, which outline what high school graduates should know and be able to do, it should be of interest to determine whether the K–12 community currently shares these expectations for students, or if these reports appear to raise the bar for high school graduates. This information will be useful to policymakers, state and district curriculum directors, and others who are interested in determining whether the standards they hold for graduating high school seniors meet the expectations of post-secondary institutions for incoming students or those organizations that employ students with some college education. In order to address this question, McREL analysts compared the expectations described for students in these two reports with current national and state mathematics standards documents considered to be high quality. For standards at the national level, analysts used the mathematics content in the following nationally recognized documents:

- *Benchmarks for Science Literacy* (Project 2061, 1993)
- *Mathematics Framework for 2005* (National Assessment of Educational Progress, 2001)
- *Performance Standards: English Language Arts, Mathematics, Science, Applied Learning, Volume 3, High School.* (New Standards, 1997)
- *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000)
- *Standards for Excellence in Education* (Council for Basic Education, 1998)

The mathematics content of the American Diploma Project and Standards for Success reports also was compared to the high school mathematics standards in documents from states highly rated by national organizations for the quality of their mathematics standards (Kendall, Snyder, Schintgen, Wahlquist, & Marzano, 1999). Three evaluation reports were used to help select these state documents. One report was the American Federation of Teachers' *Making*

Standards Matter (1998), which includes ratings of the state standards in terms of specificity and clarity. Another report on state standards, *State Mathematics Standards: An Appraisal of Math Standards in 46 States, the District of Columbia, and Japan* (Raimi & Braden, 1998), was published by the Fordham Foundation in the report. Finally, the Council for Basic Education evaluated mathematics documents across the states in *Great Expectations: Defining and Assessing the Rigor in State Standards for Mathematics and English Language Arts* (Berman & Joftus, 1998).

Although a variety of state standards documents have been highly rated for their mathematics standards, five state documents that were highly rated by all three organizations were selected:

- *Academic Content Standards: K–12 Mathematics* (Ohio Department of Education, 2001)
- *K–12 Core Curriculum: Secondary Mathematics* (Utah State Office of Education, 2002)
- *Mathematics Content Standards and Objectives for West Virginia Public Schools* (West Virginia Department of Education, 2003)
- *Mathematics Framework for California Public Schools: Kindergarten through Grade Twelve* (California Department of Education, 2000)
- *Mathematics Standards of Learning for Virginia Public Schools* (Commonwealth of Virginia Board of Education, 2001)

Not since these ratings appeared in the mid 1990s has there been any comparable review of state standards documents by multiple organizations. Thus, in order to select standards documents that are widely endorsed for their quality, McREL analysts had to turn to the documents last reviewed in the mid 1990s. We selected the most current versions of the documents for this analysis, but found no significant difference between the more recent versions and those that had been evaluated previously. We believe that these documents fairly represent the current state of content standards in mathematics. This view is based on McREL's continued familiarity with state standards over the last 10 years as we have conducted our own studies and reviews for state and district clients. For this study, we determined that it was preferable to select somewhat older standards documents that were highly rated by multiple organizations, than to select more recent documents that had been highly rated by only a single organization.

Analysts examined each statement of mathematics knowledge or skill described in *Understanding University Success*, published by the Standards for Success project (Conley, 2003), and the mathematics benchmarks identified in *Ready or Not: Creating a High School Diploma that Counts*, published by the American Diploma Project (2004), and compared them to national and state standards. Nearly all of the mathematics content identified in these two documents also was found in the national standards documents as well as within those highly rated state standards documents that were chosen for comparison. However, a number of post-secondary expectations for high school graduates were not present in any of the highly rated state standards documents; some of these statements also were not found in the national documents selected for review. Content identified as important by Standards for Success, the American Diploma Project, or both, but not found in any of the state documents reviewed is listed in Table 1.

Table 1. What Is Missing from State and National High School Mathematics Standards?

	Mathematics Expectations for Post-secondary Incoming Students		State and National Standards for 9–12 Mathematics	
	American Diploma Project	**Standards For Success**	**Present in national standards documents**	**Present in highly rated state standards documents**
Identify, explain the necessity of and give examples of definitions, axioms and theorems[1]	yes	no	no	no
Know the terminology of complex numbers and understand their use[1]	yes	**yes	yes	no
Know the characteristics of the Gaussian normal distribution (bell-shaped curve) [1]	yes	no	yes	no
Understand the different roles played by axioms, definitions and theorems in the logical structure of mathematics, especially in geometry[1]	yes	no	no	no
Recognize that there are geometries, other than Euclidean geometry, in which the parallel postulate is not true[1]	yes	no	no	no
Know how to compose and decompose functions and how to find inverses of basic functions. [2]	*yes	yes	yes (inverse operation only)	no
Use calculators appropriately and make estimations without a calculator regularly to detect potential errors[2]	yes	yes	yes	no
Understand periodicity and recognize graphs of periodic functions, especially the trigonometric functions[2]	*yes	yes	yes	no
Perform appropriate basic operations on sets (e.g., union, intersection, elements of, subsets and complement) [2]	no	yes	yes	no

[1] *Source:* American Diploma Project

2 *Source:* Standards for Success

* "content that is recommended for all students but is required for those students who plan to take calculus in college" (ADP, 2004, p. 56)

** "expected of students who plan to major in these fields of study (mathematics, computer science, statistics)" (Conley, 2003, p. 31)

Of the 87 statements of knowledge and skill from the American Diploma Project that summarize important mathematics for graduating high school students, 9 percent could not be found within any standards from highly rated states and 3 percent were not found in the national

standards. Of the 84 statements of knowledge and skill identified as important by Standards for Success, 6 percent were not addressed in highly rated state standards, but all appeared in the national standards.

Educators and policymakers who want to ensure that the mathematics standards they hold for high school graduates are adequate to satisfy the expectations of post-secondary institutions and employers will want to include, as part of their review, the items identified in Table 1. In every case, the content identified in this table was absent even from those states whose standards have been highly rated. Of particular interest for reviewers should be those expectations identified as important in the national standards in addition to the American Diploma Project, Standards for Success, or both. It also should be noted that the American Diploma Project identifies three expectations not identified by Standards for Success or found in the national standards.

SUMMARY AND IMPLICATIONS

In order to determine if the expectations that post-secondary institutions hold for incoming students are shared by the authors of highly rated state standards and national standards in mathematics, McREL compared expectations for high school graduates with post-secondary expectations. The analysis showed that most of the content identified as important for post-secondary schooling does indeed appear in state and national standards documents; however, a small number of concepts and skills important for incoming college students are not. In other words, although any of the state standards documents used in this analysis may be useful for defining most high school mathematics content, the reports published by the American Diploma Project and the Standards for Success project suggest ways to augment these with additional content likely to improve a student's chances of success in a post-secondary setting. Educators and policymakers will likely want to review the standards they hold for graduating high school students in order to determine whether this content is present.

It is of interest that some content identified as important by the American Diploma Project is not found among the content identified by Standards for Success, and vice versa. Similarly, some content identified as important for the advanced student by one organization is not recognized as such by the other. It would likely further the cause of both organizations were such differences reconciled.

There are a number of reasons that the transition from high school to post-secondary work can be difficult for students. This study has focused on one of the more straightforward questions that can be addressed in order to make this transition easier. This approach takes advantage of the clarity and specificity standards afford as a means for communicating expectations about what students should know and be able to do by the time they graduate from high school. It also serves to illustrate that standards can provide a more effective method for describing student expectations than does the Carnegie unit, which for the most part has become simply a measure for indicating seat time, not student knowledge and ability. Secondary and post-secondary institutions might communicate more directly and thus bring the reality of a K–16 education system a little closer were they to adopt standards as a formal means of identifying their shared expectations for students.

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