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Student Difficulties in Learning Elementary Mathematics. ERIC Digest.

Learning mathematics with understanding is the vision of school mathematics recommended by the National Council of Teachers of School Mathematics (2000). In order to design and develop learning environments that promote understanding efficiently, teachers need to be aware of students' difficulties in learning mathematics. Drawing from the research in mathematics education, this Digest focuses on students' difficulties in learning written symbols, concepts, and procedures in elementary mathematics as well as the sources of these difficulties.

DIFFICULTIES IN LEARNING WRITTEN SYMBOLS

Standard written symbols play an important role in student learning of mathematics, but students may experience difficulties in constructing mathematical meanings of symbols. Students derive meaning for the symbols from either connecting with other forms of representations (e.g. physical objects, pictures and spoken language) or establishing connections within the symbol systems (Hiebert & Carpenter, 1992). The meaning of numerical and operational symbols-such as 2, -4, 3/4, 2.4, and +-are constructed by connecting with concrete materials, everyday experiences or language. For example, the symbol "+" takes meaning if it is connected with the joining idea in situations like "I have four marbles. My mother gave me five more marbles. How many marbles do I have altogether?" (Hiebert & Lefevre, 1986). Similarly, students frequently refer to 3/4 as three pieces of a pizza or cake that is cut into four pieces (Mack, 1990).
Although these representations facilitate learning written symbols, the potential for them to create understanding of written symbols is limited, since they are representations themselves. Students might have difficulty in understanding the meaning of a written symbol if the referents do not well represent the mathematical meaning or if the connection between the referent and the written symbol is not appropriate (Hiebert & Carpenter, 1992). For example, geometric regions are the models most commonly used to represent fractions. These models represent the part-whole interpretation of rational numbers. However, the symbol a/b also refers to a relationship between two quantities in terms of the ratio interpretation of rational numbers. Similarly, it is used as a way of writing a/b to refer to an operation. For this reason, teachers need to use other types of representations such as sets of discrete objects and the number line to promote conceptual understanding of the symbol a/b.

Most of the difficulty in understanding symbols comes from the fact that in their standard form, written symbols might take on different meanings in different settings. For instance, in solving the equation 2x+3=4, x is regarded as an unknown, which does not vary, whereas it varies depending on y in the equation 2x+3=y (Janvier, Girardon, Morand, 1993). In order to understand mathematical symbols, students need to learn multiple meanings of the symbols depending on the given problem context. Therefore, they should be provided with a variety of appropriate materials that represent the written mathematical symbols, and they should also be aware of the meaning of mathematical symbols in different problem contexts.

Students also build understanding for written symbols by making connections within the system. For example, a numeral such as 3254 can express the number of the units of any power of ten. In other words, it represents three thousands, two hundreds, fifty-four units as well as three hundred twenty-five tens; thirty-two hundreds; and three thousands. Although these patterns are evident for adults, students might not easily construct these relationships by themselves (Hiebert & Carpenter, 1992). Therefore, teachers should be aware of these difficulties and provide students with opportunities to recognize the patterns and make connections within symbol system.

DIFFICULTIES IN LEARNING ELEMENTARY MATHEMATICAL CONCEPTS AND PROCEDURES

Students learn new mathematical concepts and procedures by building on what they already know. In other words, learning with understanding can be viewed as making connections or establishing relationships either within existing knowledge or between existing knowledge and new information (Hiebert & Carpenter, 1992).
When students attend school, they have intuitive understanding of many concepts in mathematics including numbers, measurements and probability. For example, kindergarten and first grade students intuitively solve a variety of problems involving joining, separating, or comparing quantities by acting out the problems with collections of objects (Carpenter & Lehrer, 1999). Extensions of these strategies can be used as the basis for developing the concepts of addition, subtraction, multiplication and division (Carpenter, Fennema, Fuson, Hiebert, Human, Murray, Oliver, & Wearne, 1999). Despite their mastery of certain mathematical concepts, however, students have difficulty learning elementary mathematics because students are often discouraged from using their informal knowledge (Hiebert & Lefevre, 1986; Romberg & Kaput, 1999). Mathematics instruction, which does not help students build their formal knowledge on their informal knowledge, may cause students to develop two separate systems of mathematical knowledge.

It is interesting to note that students who obtain incorrect answers for their written calculation are often able to find the correct answer by using concrete materials. However, when they are confronted with their written work, about half of these students kept their incorrect answer for written work. This discrepancy between the results obtained from working in two different settings reveals that students often cannot make connections between formal and informal mathematics (Lesh, Landau, & Hamilton, 1983). In another study, fourth graders who had connected decimal fraction numerals with physical representations of decimal quantities were more successful in dealing with problems that they had not seen before-such as ordering decimals by size and changing between decimal and common fraction forms-than students who had not made the same connections (Wearne & Hiebert, 1988). For these reasons, teachers should provide context to help students bring about their intuitive mathematical concepts and procedures, encourage them to argue whether they are reasonable, and guide them to make connections between their intuitive and formal mathematical concepts and procedures (Lampert, 1986).

Student errors are often systematic and rule-based rather than random (Ben-Zeev, 1996). In addition to student inventiveness, these errors may be caused by instruction that focuses on rote memorization. Students abstract or generalize procedures from following the steps in worked-out examples, but when their knowledge is rote or insufficient, they might overgeneralize or overspecialize the rules and procedures (Ben-Zeev, 1996; VanLehn, 1986). For instance, students might overgeneralize the rule for subtracting smaller from larger on single-digit subtractions to multidigit subtraction problems if they are only taught to subtract the smaller from the larger number. Similarly, if students are exposed only to borrowing two digit subtractions, they may overspecialize borrowing from the units-digit to multi-digit subtractions (VanLehn, 1986).

One way to reduce such difficulties is to help students make connections between conceptual and procedural knowledge. The construction of conceptual knowledge requires identifying the characteristics of concepts, recognizing the similarities and differences among concepts according to these characteristics, and constructing the relations among them. On the other hand, procedural knowledge requires constructing skills, strategies or algorithms that are means to an end (Byrnes & Wasik, 1991). For example, students who do not align decimal points while adding or subtracting decimal fractions probably follow the algorithm without making connections between position values of decimals and lining up the decimal points (Hiebert & Lefevre, 1986). More advanced connections, such as adding up values that are alike, require generalizing and reflecting on pieces of information such as lining up decimal points in order to add decimal fractions or looking for common denominators while adding common fractions. Although such connections might be obvious for adults, constructing them might be difficult for the students. Teachers need to design instruction that helps students construct these big ideas.

With regard to classroom instruction, student difficulties can also be attributed to using inappropriate representations. For example, students having difficulty in adding fractions may extrapolate erroneous algorithms from instruction on the representation of fractions. Students who are often presented fractions by using pie graphs perform "1/2 + 1/3 = 2/5" and justify the solution as "adding one piece of a two piece pie and one piece of a three piece pie will result in two pieces out of five pieces altogether" (Silver, 1986; Ben-Zeev, 1996). As discussed earlier, using appropriate representations will help students construct different characteristics of concepts.

CONCLUSION

Developing understanding in mathematics is an important but difficult goal. Being aware of student difficulties and the sources of the difficulties, and designing instruction to diminish them, are important steps in achieving this goal. Student difficulties in learning written symbols, concepts and procedures can be reduced by creating learning environments that help students build connections between their formal and informal mathematical knowledge; using appropriate representations depending on the given problem context; and helping them connect procedural and conceptual knowledge.

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