DOCUMENT RESUME

ED 476 704 SE 067 866

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TITLE	Linking Algebraic Concepts and Contexts: Every Picture Tells a Story.
SPONS AGENCY	National Science Foundation, Arlington, VA.
PUB DATE	2003-00-00
NOTE	12p.; Some diagrams may not photograph well.
CONTRACT	9619102
PUB TYPE	Reports - Descriptive (141) -- Reports - Research (143)
EDRS PRICE	EDRS Price MF01/PC01 Plus Postage.
DESCRIPTORS	*Algebra; *Computer Simulation; *Computer Uses in Education; Curriculum Development; Grade 7; Graphs; Instructional Materials; Learning Strategies; Mathematics Education; Middle Schools; *Motion; Teaching Methods

ABSTRACT

This paper reports on efforts to design a computer-based instructional unit for 7th grade students that would not only address state standards but would also provide some groundwork for the students' subsequent formal study of algebra. SimCalc Mathworlds software was used to explore the same motion from different perspectives. The overall objective is to build on what students know and to provide learning experience on which they build. Instructional sequences implemented and students' comments regarding what they thought they had experienced during the instructional sequences are included. (KHR)

Linking Algebraic Concepts and Contexts:

Every Picture Tells a Story[1]

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[1] This material is based upon work supported by the National Science Foundation under Grant No. 9619102. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

Current trends in middle school curricula highlight the importance of integrating algebraic reasoning throughout the middle grades. The use of innovative technologies offers one exciting way to engage students in algebraic problem solving activities. In this article we report on our efforts to design a computer-based instructional unit for seventh graders that would not only address state standards but, we hoped, would also provide some groundwork for the students' subsequent formal study of algebra. The software we chose, SimCalc Mathworlds, is available free of charge via the world wide web (http://tango.mth.umassd.edu/simcalc/). With this program, students can animate characters on the screen by creating graphs of position and velocity. Our experiences with this software revealed several ways in which the use of technology can affect how algebraic topics can be taught. In particular:

- Animated software provides a compelling context to which students can relate thus promoting algebraic discussions.
- The use of computer-based graphs linked to animations provides a new means for exploring these content areas as integrated rather than isolated topics.
- Once learned, broad-range tools such as SimCalc can provide familiar contexts within which concepts can be introduced and integrated throughout the middle school grades.

The instructional sequence we describe in this article was implemented over a three-week period in an urban setting in Southern California. In an effort to assess what students thought they had experienced during our instructional sequence, we asked them to write a letter to the principal describing what they had learned. The following excerpts include exact spellings to preserve the spirit of the students' comments (approximately 65% of the class was classified as ESL):

• *"I would tell her [the principal] that I learned how graphs work. And I also learned that it does matter where you start the graph, because it could if you start in the rong place it will change the hole graph."*

• *"At first I did not know what n means but now it means a paturn like n•5 means anything times 5. I like working with a partner and it kind of help me."*

• *"We are learning about graphs and velocity which is how far you go per second."*

These three examples were chosen to highlight the diversity of interpretations that the students formed during the three weeks. Taken separately, they reflect the three major themes that we addressed: 1) graphing in a coordinate-plane system, 2) writing and evaluating simple algebraic expressions, and 3) understanding rate of change. Students engaged in activities that integrated topics where graphs derived their meaning from context. This approach places the animation as the focus of activity rather than as an application to be studied after formal notation is mastered. In other words, we feel it is essential that students believe that every picture tells a story.

Our overall objective was to build on what students knew and to provide learning experiences on which they could build. Although these seventh-grade students had not formally encountered line graphs, algebraic notation, or the concept of speed as a rate, we hoped to build on their real-world experiences of speed to introduce conventional notation such as equations of lines and Cartesian graphs of position and velocity. Ultimately we hoped that they would begin to develop an understanding of rate on which they could build in subsequent mathematics courses such as high school algebra and calculus. We chose SimCalc because its design is consistent with these goals.

The SimCalc software was developed by a team of researchers at the University of Massachusetts at Dartmouth (Kaput & Roschelle, 1997). This computer-based environment enables students to explore one-dimensional motion in which any combination of three graphs (position v. time, velocity v. time, and acceleration v. time) can then be linked to a character's motion and to each other (See Figure 1)[2]. Unlike most graphing technologies that include multiple linked representations of the same data set or function, the central focus of the SimCalc software is the exploration of the same motion from different perspectives. For example, when using a graphing calculator students can view a graph linked to a table of coordinates linked to an algebraic expression, all of which refer to the same set of data. In contrast, students using SimCalc Mathworlds can focus on the same *motion* from different perspectives (i.e., from the perspective of a character's velocity over time, or a character's position over time). Because they are linked, a student using the program can create or modify a position (or velocity) graph and view the character's corresponding movement.

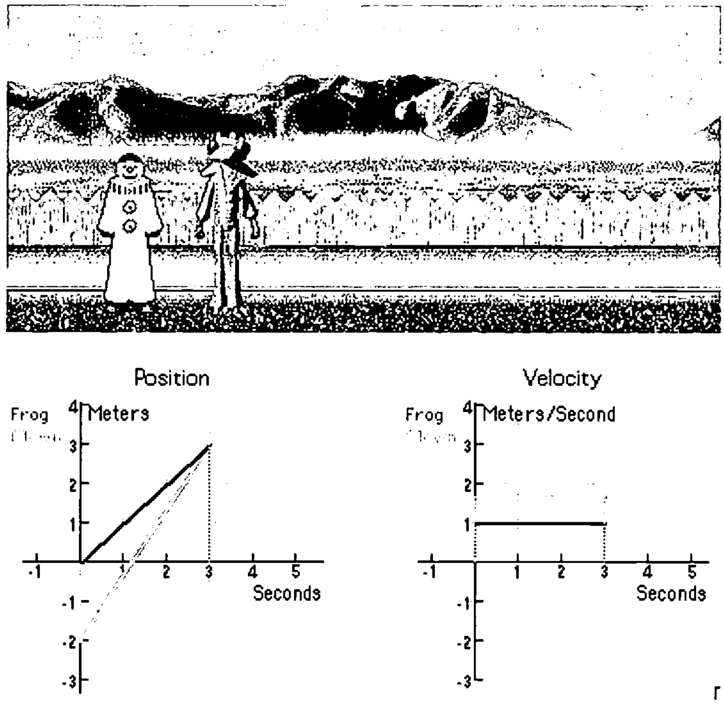


Figure 1. Interface for SimCalc Mathworlds.

One advantage of this software's focus on graphing is that it does not require that students describe motion with algebraic equations. This is advantageous because it provides an opportunity for students to connect the character's motion with the graphical representation without first having to master algebraic equations. In fact, this environment and the activities we created provided a context where variables and algebraic expressions could arise as helpful tools for describing motion.

The Curriculum

As mentioned earlier, our instructional sequence had three major integrated themes: 1) graphing in the coordinate plane, 2) writing and evaluating algebraic expressions, and 3) understanding rate of change. Each of these can be placed in rough correspondence to sets of activities in the three-week unit. During week 1, we focused on line graphs, specifically graphs of

[2] The software we used was developed for the Macintosh computer. A version has since been developed in JAVA that is cross platform.

position. Then our emphasis shifted to using algebraic notation to describe patterns in tables of position data. Lastly, the class conversations focused on rate, and activities focused on graphs of velocity. In the following paragraphs, we highlight activities from each of these three themes. [3]

Graphing in the Coordinate Plane

1. **Experiencing motion with a motion detector**. In the first activity we created worksheets containing target graphs and asked the students to model each graph using a motion detector connected to a computer[4] (See Figure 2). The students enjoyed this activity and were challenged to revise their motion based on the graphical feedback that appeared on the screen. We had two reasons for beginning with a motion detector activity. First, we wanted students to become familiar with graphical representations of position over time. Second, we have found it important that they experience the kinesthetic activity involved in revising and anticipating the graphs in order to recognize that the quantity being measured was a distance between themselves and the motion detector.

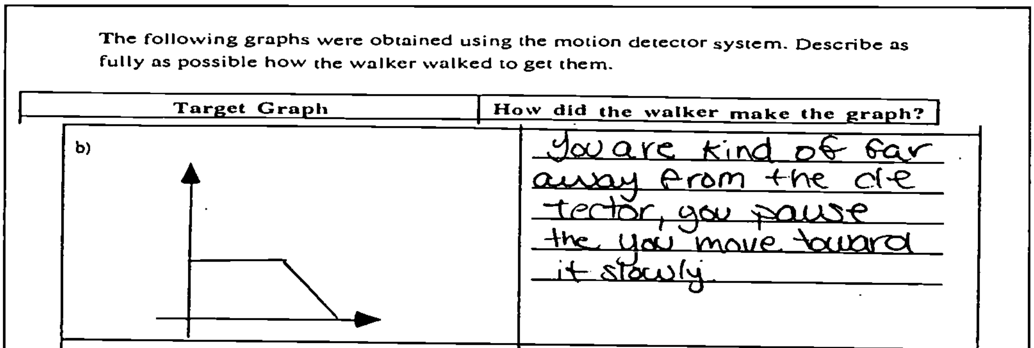


Figure 2. Worksheet for reading the graphs to create a story with the motion detector.

2. **Describing motion with position graphs.** In the second activity, we built on the students' kinesthetic experiences by asking them to predict how two characters in the SimCalc program, Frog and Clown, would move based on given position graphs. After students' predictions were collected, the teacher ran the simulation. The teacher's intention was to focus on qualitative descriptions of graphs. After several initial conversations, we were pleased that some students began to introduce numbers into their descriptions as a way to describe the

[3] The curriculum in its entirety is available via the Internet at http://:www.mathed.com.

motion more precisely. For example, during a whole class discussion one student described the graph in Figure 3 by stating, “He started far away. Took four steps backward. Stood still for two seconds. Took seven steps forward.” This was refined by another student to “Frog started at 3 meters. Walked away at normal speed. Stood for two seconds. Walked toward at normal speed. Total time is eight seconds.” Although some students referred to “steps”, which may have been related to meters as in Figure 3, many students were developing ways to describe motion with numbers. More importantly, they were developing strategies to coordinate distance and time.

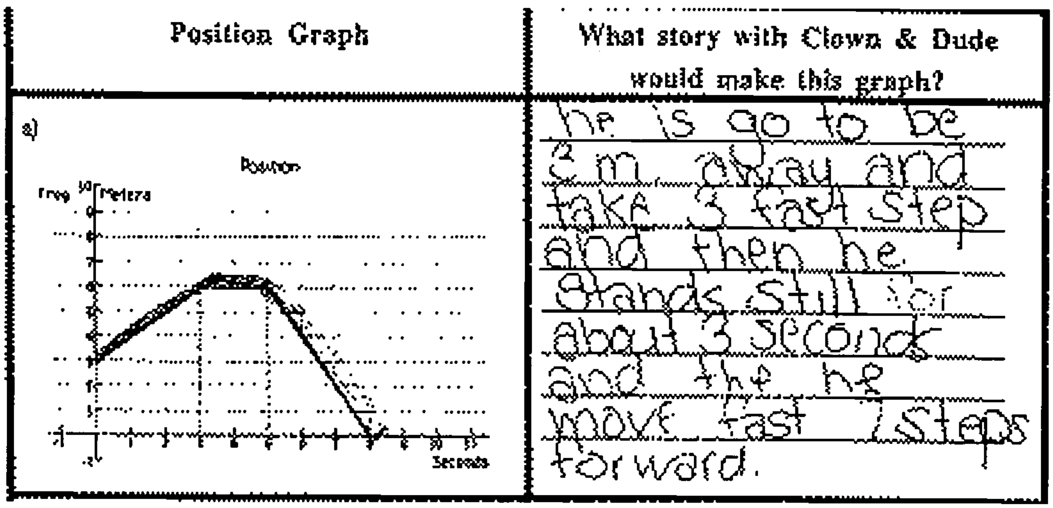


Figure 3. One student’s work describing the story that the position graph depicts.

Once students became familiar with telling stories from a picture (i.e., a graph), we reversed the process by engaging them in lab-based activities in which they viewed an animated story (such as Clown and Dude passing after 3 seconds) and asked students to choose which position graph (from a multiple choice list) best represented the story. This activity turned out to be one of the richest sessions; as the groups discussed the different choices, the students developed compelling mathematical arguments to support their particular preferences.

Writing Algebraic Expressions

[4] In our situation, there were several motion detectors and computers available. Students worked in groups in a lab situation. It is possible to use these activities with little modification if you have no motion detector and one

1. **Describing motion with a table of ordered pairs.** The SimCalc software provides a means of viewing the simulation in second-by-second steps. After drawing position graphs based on beginning locations and time, students used this step tool to record in a table a character's position at each second. Students used this table of values to plot points in a coordinate plane, and, eventually, to sketch linear graphs.
2. **Describing motion with algebraic expressions.** Our goal for introducing algebraic expressions was to have students come to appreciate the power of variables in predicting a character's position at any given time. This goal is consistent with the NCTM Curriculum Standards (1989) which suggest that, at all grade levels, students engage in modeling phenomenon by describing and representing relationships with tables, graphs, and rules and exploring the inter-relationships. As shown in Figure 4, activities focused on students identifying a pattern in the table of ordered pairs and describing it with an algebraic expression.

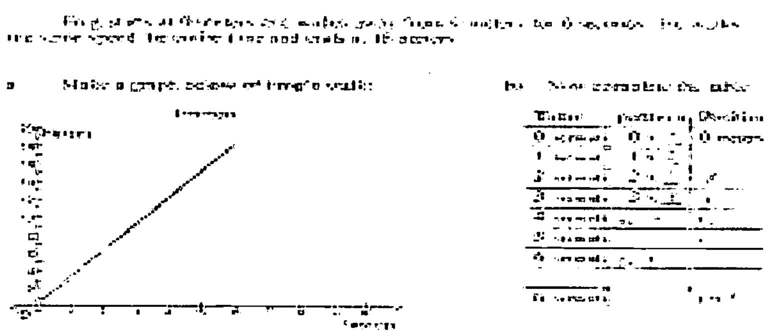


Figure 4. Developing algebraic expressions to describe position from tables of data.

Understanding Rate of Change

1. **Describing velocity.** A third goal of the instructional sequence, which is also a goal of the NCTM's Standards for Grades 5-8 was to "...develop the concept of rates and other derived and indirect measurements" (NCTM, 1989, p. 116). During our final week of instruction, speed became an explicit topic of discussion. The teacher began by asking students for justification as to why a particular character in the simulation would win a race. Arguments

computer with projection capabilities.

that a particular vehicle (say, a sports car) would win initially took the following form: "Both are going the same meters, but one is going in less time. The sports car is going 60 meters in 4 seconds and the van is going 60 meters in 6 seconds. The van took longer." (See Figure 5). Eventually some students realized that these descriptions were inadequate for describing individual rates when the distance (or time) was not the same in both cases. This realization led students to describe speed based on how far the character traveled in the first second (which was accurate because all speeds were constant). For example one student said: "The clown is traveling 2 meters per second because in 1 second he's gone 2 meters and in the second second, he's at 4 meters. Therefore, he has gone 2 meters every second.")

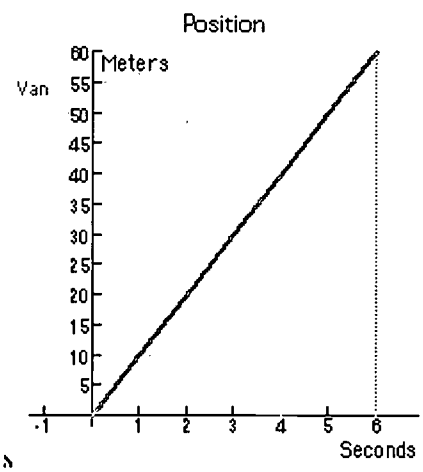
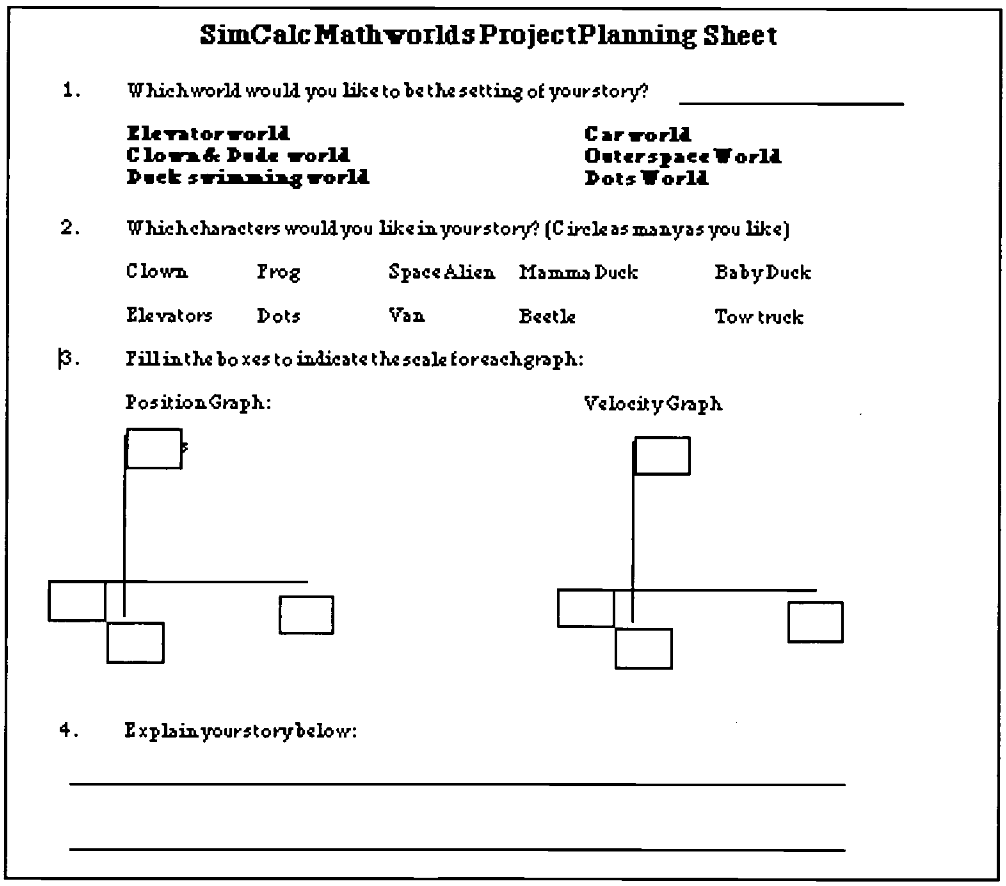


Figure 5. Position graph of a race between a sports car and a van.

2. **Describing motion with velocity graphs.** Students built on this conception of rate to graph constant velocity. Lab-based activities in this final section focused on the relationship between velocity and position graphs. In this activity, students ran a given simulation and made a graph of the character's position, and then chose a velocity graph which best told the story.

As a concluding activity, students planned, created, and presented stories of their own design. Examples included characters racing or meeting to tell secrets, aliens hovering and snatching baby ducks, and sports cars breaking out of traffic jams. Two critical elements of this assignment were: 1) students were required to write their stories before going to the lab, and 2) their stories needed to include more than one character that changed rate and direction. The purpose of asking students to plan their activity prior to going to the lab was so that they would have a goal in mind, rather than just playing around with the graphs and then claiming that the end product was their original goal. To this end, we created a "project planning sheet" as shown in Figure 6. As the students presented their simulations to the class, it became evident that their final projects were quite similar to their original plans, which indicated a strong effort on their part to use graphs to tell a particular story. The students had been writing stories described by graphs but now had a chance to tell *their* story. To more fully appreciate the students' efforts, we invited all class members to ask presenters questions about how the graphical representations and simulation were inter-related. They exhibited tremendous pride in their work as they made their presentations. Many were anxious to show their parents and took their projects home.



SimCalc Mathworlds Project Planning Sheet

1. Which world would you like to be the setting of your story? ______________

   Elevator world | **Car world**
   Clown & Dude world | **Outerspace World**
   Duck swimming world | **Dots World**

2. Which characters would you like in your story? (Circle as many as you like)

   Clown | Frog | Space Alien | Mamma Duck | Baby Duck
   Elevators | Dots | Van | Beetle | Tow truck

3. Fill in the boxes to indicate the scale for each graph:

   Position Graph: | Velocity Graph:

4. Explain your story below:

   ____

   ____

Figure 6. Project planning sheet.

Reflections

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We acknowledge that the use of technology was considerably demanding for the teacher. As with any technology-based lesson, there were several pragmatic difficulties. For example, it was difficult for one teacher to attend to a class of more than thirty students in a computer lab. In our case, this was addressed by having two other adults to help, a situation that we hope other teachers would be able to arrange. Students worked in pairs selected by the teacher to maximize the potential of the group. In addition, the teacher often used an overhead LCD projector to run whole-class discussions rather than holding all classes in the computer lab. A second difficulty was the daunting task of creating an entire instructional sequence. Fortunately, many curricula and instructional sequences for this software are currently being developed by teachers around the country and being shared via the Internet at the website given earlier.

In retrospect, we believe the benefits of this experience greatly outweighed the pragmatic difficulties. Many of the students' letters to the principal mentioned how much they enjoyed the experience, and the "difficult math" they came to understand. From the students' point of view, our data suggests that the use of technology enhanced the appeal of doing mathematics *for a reason*. From the teacher's point of view, it provided a context in which to ground abstract algebraic concepts such as y-intercept, slope, and speed. Lab-based activities and projects enabled students to integrate a number of algebraic notions in a natural way. Results from a post-test indicated that students gained in understanding a number of pre-algebra concepts. In addition, the teacher has noticed a few things as the school year has progressed: The students in this class, more than others in sections that did not engage in the sequence, were able to construct context-based understandings for signed integers, made sense of how to plot points in all quadrants of the coordinate plane and demonstrated stronger results on district assessment items involving graphing. In sum, we believe that technology-based activities in which students link algebraic representations with motion provide opportunities for students to develop a sense that "every picture tells a story."

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