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ABSTRACT

This paper addresses components of a 3-year longitudinal study in which 9th and 10th grade students in Australia, Canada, and Zambia participated in data-handling programs through networked learning communities. Of interest here are the students' responses to a selection of "ends-in-view" problems, which formed the major part of the data-handling programs. The nature and role of these ends-in-view problems in promoting students' mathematical learning are addressed first. In the second part of the paper, the cognitive and social developments of groups of 9th and 10th grade students as they worked some of the ends-in-view problems are examined. (Author)

STUDENTS' DEVELOPMENTS IN SOLVING DATA-HANDLING ENDS-IN-VIEW PROBLEMS

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Overview

This paper addresses components of a 3-year longitudinal study in which 9th and 10th grade students in Brisbane (Australia), Canada, and Zambia participated in data-handling programs through networked learning communities. Of interest here are the students' responses to a selection of "ends-in-view" problems, which formed the major part of the data-handling programs. The nature and role of these ends-in-view problems in promoting students' mathematical learning are addressed first. In the second part of the paper, the cognitive and social developments of groups of 9th and 10th grade students as they worked some of the ends-in-view problems are examined.

Data Handling

Data handling is recognized as an important topic within school mathematics and has gained an increasingly visible role in the K-12 curriculum (English, Charles, & Cudmore, 2000; Greer, 2000). As Shaughnessy, Garfield, and Greer (1996) noted, data handling is not just dealing with a body of statistical content, rather, it is "an approach to dealing with data, a frame of mind, an environment within which one can explore data" (p. 205). In addition to helping students cope with the increasing use of data and data-based arguments in society, data-handling activities provide realistic contexts for the application of basic mathematical ideas, and can assist students in developing an appreciation of mathematics as a way of interpreting their world (Hancock, Kaput, & Goldsmith, 1992).

Accompanying the calls for students' statistical development is a focus on student-driven classroom projects where students are given opportunities to engage fully in the practices and processes of meaningful statistics (Derry, Levin, Osana, & Jones, 1998; Lehrer & Romberg, 1996; Hancock et al., 1992; Moore, 1998). In the present study, these projects adopted the form of "ends-in-view" problems (English & Lesh, in press). Ends-in-view problems that involve data handling not only comprise meaningful contexts but also address the kinds of mathematical knowledge and processes that are fundamental to dealing with the increasingly sophisticated systems of our society. As indicated later, these problems take students beyond just computing with numbers to making sense of large volumes of data, quantifying qualitative information, identifying patterns and trends, producing convincing arguments supported by appropriate data, and assessing the products generated by their peers.

Ends-in-View Problems

Tasks that present students with particular criteria for generating purposeful, complex, and multifaceted products that extend well beyond the given information are referred to as "ends-in-view" problems (English & Lesh, in press). John Dewey (Archambault, 1964) initially coined the term when he highlighted the importance of inquiries that involve anticipating the consequences of any proposed courses of actions, as well as assessing the means by which these actions can be implemented to achieve a desired end product.

Ends-in-view problems differ in several ways from the classroom problems that students typically meet. With standard textbook tasks, the "givens," the "goals," and the "acceptable" solution steps are usually specified clearly such that the interpretation process for the learner is reduced or removed completely. In contrast, ends-in-view problems not only require students to work out how to get from the given state to the goal state but also require them to interpret both the goal and the given information, as well as the permissible solution steps. This interpretation process presents a challenge in itself: there could be incomplete, ambiguous or undefined information; there might be too much data, or there might be visual representations that are difficult to interpret. When presented with information of this nature, students might make unwarranted assumptions or might impose inappropriate constraints on the products they are to develop.

Another challenging feature of ends-in-view problems is that students are presented with criteria for generating various mathematical products, which could involve constructing a new problem, developing a persuasive case, or designing a data-gathering tool. Students do not know the nature of the products they are to develop; they only know the criteria that have to be satisfied. For example, in constructing new problems, students could be required to make their problems challenging but manageable, free from cultural bias, and interesting and appealing to the solver. There is more than one way of satisfying the criteria of ends-in-view problems, which means that multiple approaches and products are possible. Students at all achievement levels are thus able to tackle problems of this type.

Ends-in-view problems are also designed with group problem-solving processes in mind. That is, the problems require the combined abilities and experiences of a diverse group of students to develop products that can be shared with others (Zawojewski, Lesh, & English, in press). In contrast to other problems that might be worked in group situations, ends-in-view problems specifically require students to develop sharable, mathematical products. This means that the products must hold up under the scrutiny of others. When students don't have to produce something sharable, they can frequently "settle for second best."

Although ends-in-view problems share the above features, they do differ in the type of end products they request. These products may be classified under three broad categories: *tools*, *constructions*, and *problems*, each of which comprises many different examples (English & Lesh, in press). *Tools* as products include models, mathematical descriptions, explanations, designs, plans, and assessment instruments. In general terms, tools are products that fulfil a functional or operational role. A *construction* normally requires students to use given criteria to develop a mathematical item, which can take many forms including spatial constructions, complex artefacts, persuasive cases, and assessments (i.e., the products of applying an assessment instrument). *Problems* as products encompass not only criteria for their construction but also criteria for judging the appropriateness and effectiveness of one problem over another. Examples of these types of products are presented in the next section.

The Data-Handling Program

Participants

Students from the 9th and 10th grades (aged from 13 to 15 years), together with their teachers, participated in the present program. The Australian students were from a private, co-educational school in a middle socioeconomic area of Brisbane. Students from Canada and Zambia also participated in the programs, via the website we established. The Brisbane students commenced the program in their 9th grade and continued with it in the following grade (N=26). The Brisbane teacher remained with the class of students for both years.

Program

A program of data-handling experiences was implemented during the third and fourth terms of the students' 9th grade year and a second, related program was implemented during the first and second terms of their 10th grade year. We worked collaboratively with the classroom teacher in developing and implementing the programs. The students worked in small groups (3 to 4 members per group) to complete the activities. It was important that we established a classroom environment where the students could work as a community of learners (Cobb & Bowers, 1999). In doing so, we encouraged the students to work collaboratively and to express and justify their ideas and sentiments in an open and constructive manner. We also placed an emphasis on analytical, critical, and philosophical thinking.

Appropriate technological support was essential to the program. The students were provided with computer workstations that contained the required software (including Excel) and that were linked to the Internet. We established a website that housed the students' data, their responses to various questions, and their products. The website also provided a forum for the students to communicate with their international peers, and a forum for researcher/teacher collaboration. Each of the 9th and 10th grade programs comprised 16 learning sessions of approximately 70 minutes each.

The ends-in-view problems that were implemented during the programs included the following:

A Tool. In the 9th grade program, the students in Brisbane and Canada were to develop an international survey for completion by all the participating students. After preliminary discussion, the students worked in their groups to develop some questions for inclusion in their survey. This involved the students in extensive posing and refining of questions, taking into account the nature of the data that their questions would yield. The students were presented with the following directions (including criteria that had to be met):

> *Think of 20 questions that you would like to include in the international survey we are constructing. Include a mix of questions that will produce nominal data, ordinal data, and interval data. Write your questions on the back of this sheet as well as creating a Word document. Remember that your questions must be suitable for the students in the other countries.*

Problems. In both the 9th and 10th grade programs, the students were to generate their own statistical problems and related issues for investigation (drawing upon the data generated from their survey). The students had previously developed their own criteria for determining what constitutes a mathematical problem, a "good" mathematical problem, a challenging problem, and a problem that appeals to the solver (including their overseas peers). These criteria not only guided the students in initial problem creation but also in critical assessment of their completed problems. The latter included testing whether one problem creation was better than another (which helped them to refine their problems) and critically analyzing and assessing one another's problems.

Constructions. The students undertook several constructions, including the development of persuasive cases (e.g., using the data from the survey, students were to present a persuasive case that argued for a particular point of view, such as the need for more computers in their school). Another construction involved the writing of a newspaper article that reported on interesting and controversial findings from their data (including comparisons of data gathered across the three countries). The students were to support their constructions with appropriate statistical representations (e.g., tables, graphs). The students also completed critical assessments of their own and their peers' constructions (with provision of constructive feedback).

Data Sources and Analysis

Each session was videotaped and audiotaped, together with fieldnotes being made. We used two video cameras, one focusing on the teacher and the whole class, and the other on groups of students. We rotated the videotaping of the student groups across the sessions, but we audiotaped all groups during each session. Transcripts were made of all the audio and videotapes. We also collected copies of students' artefacts, which included the products they developed for each of the ends-in-view problems and print-outs of their website entries.

"Iterative videotape analyses" (Lesh & Lehrer, 2000), along with analyses of the students' artefacts and the classroom fieldnotes, were used to produce a description of the students' cognitive and social progress during and across the sessions. Given that several complex, dynamic systems were involved in the study (e.g., the classroom community, the student group, the individual student, and the students' evolving products), iterative analyses were deemed essential. As is typical of such analyses, initial interpretations of the data were rather disjointed and barren. However, with each successive analysis, clearer patterns of the students' mathematical and social development emerged. Some of these developments are addressed in the remainder of this paper.

Some Findings

Interpreting the Problem

In the early stages of working the ends-in-view problems, the students' initial responses were quite unstable and disjointed. The students spent considerable time trying to interpret the nature of the end product they were to develop (i.e., interpreting the criteria that had to be met) and tended to focus on certain features of the problem and ignore others. For example, in constructing questions for their international survey, some 9th graders were content with just creating questions -- any questions (and frequently got off task in doing so). Other 9th graders were cognizant of creating questions that would yield different data types, irrespective of whether the questions were appropriate or not. Only a few students in the initial stages attempted to address all of the criteria simultaneously. However, over the course of the survey construction each student group developed the ability to coordinate all the criteria in their product generation. In the two excerpts below, Kate's group is primarily concerned with getting some questions recorded, although they do show some awareness of the appropriateness of their questions. In contrast, Laurel's group spends time interpreting the criteria first, recalling an earlier learning experience to help them.

Kate's group

Kate: What are we supposed to do?
Greer: I don't know. Make some questions up.
Kate: Are we supposed to ask questions like 'What's your name' and 'What school do you go to?'
Greer: These go at the beginning, so don't bother doing it.
Kate: OK. Let's ask a question about 'What do you think about wearing a school uniform?' But don't write that. We have to have 'yes' or 'no.' 'Do you think you should wear a school uniform, um, at your school?'
Greer: 'Do you think school uniforms should be compulsory?'

Kate: Yeah.
Greer: 'Do you think the legal age for driving should be...'
Kate: But it's not a legal age for everywhere.
Bill: Do you want another question? (Kate and Greer welcome his input.)
Bill: 'How old are you?'
Bill (After the girls explain this has been covered): What about like, 'How much TV do you watch?'
Kate: 'Do you think the school hours are too long?'
Greer: 'Do you go to church?'

Laurel's group

Laurel took the problem sheet and read the directions to the group.
Mary: Right. I think it says, like, you know how we did all that data on our three questions ---
Laurel: Yes. I know what it means.
Mary: --- and I think it says, like, what type of question we can ask about those data.
Cindy: That can be subjective or objective response? It can be, like, subjective- ---
Laurel: Tested. We did that in science: testable or untestable. Someone's opinion or someone's actual research.
Cindy: Yeah, whether it's opinion or whether it's fact.
Mary: It should be fact because we had data for them to use.
Laurel: Yeah.
Mary: Like, they're supposed to use that data.
Laurel: Yeah. An answer shouldn't be opinionated, because therefore you wouldn't get a proper answer... All right. So we have to think of what kind of data we expect in the things---

Reconciling Individual Interpretations with Those of Others

A noticeable feature of the students' interactions in working the ends-in-view problems was the reconciling of individual interpretations with those of others. That is, individual students brought their own understandings and experiences to the group situation, at times, ignoring the viewpoints of others. However, they gradually tapped into one another's ideas and subsequently reconciled their viewpoints and interpretations with those of their peers. This reconciliation process was observed to occur in a cyclic fashion during the working of the problems (not surprisingly, students who failed to reconcile their views faced considerable difficulties). In the following excerpt, Laurel, Mary, and Cindy (10^{th} graders) are coming to a consensus on what they consider a problem to be (this was part of their discussion on the criteria they would use to help them create and assess their mathematical problems).

Mary: So, what is a mathematical problem?
Laurel: Something that has the answer in the question.
Cindy: A question involving numbers that has an answer.
Laurel: No, because you can have words ----
Mary: Involving calculations or....
Cindy: It has to involve numbers or formulae. Formulae!
Mary: It doesn't have to have formula.
Laurel: No, it doesn't have to.... you know those things in the maths book where you tick the different things?....Um..there's one down the edge. There's all words and not numbers.
Mary: What do you mean "all words"?
Laurel: You don't always have to have numbers.
Mary: Yeah, but you need some sort of calculation like even if you have just words you need to gather information.
Laurel: Something with a question and an answer has to have a solution, doesn't it?
Mary: That's just any problem. I could ask you "Laura, what is your name?" and the answer is "Laura".
Cindy: That's the most pointless question I've ever heard.
Laurel: Yes, one characteristic....no, but things like - philosophers ask questions that can never be answered. It's the same – and you have to go "I am what I am". It doesn't have to have an answer.
Mary: OK. So it's a problem. It's a question that needs calculation to get the solution. How's that?
Laurel: OK (as she records) A question......that.....involves....calculations... ....to reach an appropriate answer? To reach.....
Cindy: To reach an appropriate conclusion or solution.
Mary: Yes, that's good.

Repeated Cycling of Refinements to Intermediate Products

An important development in the students' responses, particularly when they constructed their own problems and cases, was their attention to refining their intermediate products. This was evident at both grade levels, but there was more commitment to this process in the 10th grade than in the 9th grade. As the students made repeated refinements to their products, they also demonstrated internalization of what was once external to the group (cf. Vygotsky, 1978). The students' cycles of refinement involved improving their contextual language, refining their mathematical language (the structure of the mathematical questions or issues posed), coordinating their contextual and mathematical language, and improving any open-ended questions they wished to include (the open-ended question asked the solver to think in a critical or philosophical manner). The students frequently became bogged down in contextual concerns (e.g., which characters they should include in their problem setting), until one of the group members alerted them to the need to return to the task directions. These refinement cycles were also evident when the students acted on feedback from their peer's critiques of their finished products. In the following discussion, Laurel's group is trying to improve the mathematical questions they wish to pose in their problem. Notice how Mary alerts the group to the inaccuracy of Laurel's suggested question. Mary had internalized the importance of using the appropriate statistical procedures for given data types (the students had explored this at the beginning of each program).

Laurel: All right. We have to find the mean, median, and mode. Or which one do you think? What do you think gives a more accurate answer to the most preferred musical artist: mean, median or mode? We could ask them that........ (No response from others.) That's a question we could ask.
Cindy: (to Laurel): I wouldn't want to be asked that.
Laurel: You don't want to be asked, but we're going to ask it. They're going to give us an answer because we don't have to think of it ourselves.
Cindy: But we do. We have to.
Mary: Well, we'd better get through this, you know.
Laurel: How about 'Find the mean of the most popular musical artist---'
Mary: You can't find the mean. (Laughs)
Laurel: Can we find the mode or....
Mary: Cardinal. Okay, for the cardinal question ---
Laurel: So it's cardinal data?
Cindy: You could ask them to find the mean.
Mary: Slash mode...median.
Laurel: ...and median.
Mary: And you might ask---
Laurel: And how about – can we ask them their opinion on which do you think give the most accurate----
Mary: Yes that's a good one.
Laurel: 'Which of these do you think give the most accurate....'
Mary: 'View of the general time spent on the---'
Cindy: 'Shows the trend in the data.'
Mary: Yeah, 'trend.'
Laurel: 'Which of these......(writing).....do you think...give a true reflection---'
Cindy: 'Gives the most accurate.'
Laurel: 'Gives the most accurate.....description of the trends?'

Following this, the group had considerable discussion on the wording of their problem context, and then returned to the nature of the questions they would pose in their problem. Laurel posed an open-ended question that asked the solver to suggest reasons for the findings: "We can ask, 'Is there . . . what factors do you think might influence the people's choice of listening and music.' The group then reverted to a mathematical question they had posed earlier and tried to incorporate it within their chosen story context, which they found somewhat difficult to do:

Laurel: ..look 'Is there any correlation between the people who are groovy and the people, what music they listen to.' ... 'Please help Groovy Greg discover....find out. . .'
Mary: Kid's language....are you trying to insult us?
Laurel: Find out whether . . . . 'Is there is a link between the people who are groovy and what music they listen to?'

They subsequently revisited their construction of an open-ended question to be included in their problem, and spent time refining it:

Laurel: What was the question?
Mary: Like, think about the reasons why there's this difference between the----
Laurel: Why is this happening?
Mary: Yes, why is this happening.
Cindy: Why are there differences in the ....
Laurel: Just say...I just have to say 'And why?' for that question and they can point out the reason why. Not much to that. And...now, why what?
Mary: Why there are differences between the data of the different countries. Does that make sense?
Cindy: 'Why are there differences between each set of data?'
Mary: Yes, each set of data. That's it.

Concluding Points

This paper has highlighted the importance of ends-in-view problems in the mathematics curriculum and has illustrated how such problems can facilitate learning in the domain of data handling. Ends-in-view problems bring many new features to classroom problem solving. In contrast to the usual tasks that students meet, these challenging authentic problems require students to interpret both the goal and the given information, as well as permissible solution steps. Criteria are presented for generating the end product (i.e., the solver develops an end-in-view of what is to be constructed) but information on the exact nature of the product is missing. However, even though this information is missing, students know when they have developed an appropriate product. This is because the criteria serve not only as a guide for product development but also as a means of product assessment. That is, the criteria enable learners to judge the suitability of their final products as well as enabling them to assess their intermediate products.

In the study addressed here, students displayed repeated cycling of refinement processes as they worked on perfecting their products. In constructing their own problems, for example, the students cycled through the processes of refining their contextual language, refining their mathematical language, and coordinating the two. The students had internalized the key features of effective problems, which they had developed earlier in the program. The fact that the students worked in groups and had to create a product that was to be shared with others strengthened their commitment to a high-quality product.

Students' school mathematical experiences could be made more meaningful, more powerful, and more enjoyable if some of their "standard" tasks were replaced by ends-in-view problems. Further research is needed, however, on students' learning through ends-in-view problems. We need to explore how students, both individually and as a group, develop the important mathematical understandings that these problems foster. At the same time, we need to analyze students' social developments as they work as team members in generating their products.

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