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ABSTRACT

This paper describes a models and modeling framework that has been applied to various areas in teaching, learning and problem solving. It examines the implications of that framework on metacognition and higher order thinking during everyday problem-solving situations that required teams of students to produce complex solutions in approximately 1-2 hours. In a team of students, there may be multiple interpretations of a complex problem situation and communication of interpretations effects problem solving. This changes the role of metacognition. (Author)

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A Modeling Perspective on Metacognition in Everyday Problem-Solving Situations

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Abstract: This paper describes a models and modeling framework that has been applied to various areas in teaching, learning and problem solving (Doerr & Lesh, in press). It examines the implications of that framework on metacognition and higher-order thinking during everyday problem-solving situations that required teams of students to produce complex solutions in approximately 1-2 hours. In a team of students, there may be multiple interpretations of a complex problem situation and communication of interpretations effects problem solving. This changes the role of metacognition.

Metacognition has been investigated often over the last 25 years and has remained an unclear, vague, and broad concept in part because of unclear definitions and a large number of other related problem-solving issues (Schoenfeld, 1992). Recent standards documents for science and mathematics list metacognition and higher-order thinking skills as important educational goals for students (National Council for Teachers of Mathematics, 2000; American Association for the Advancement of Science, 1993). However, the questions "What is meta? What is cognitive?" (Brown, 1987, p. 66). can be difficult to answer when considering everyday problems requiring complex products. The framework proposed here attempts to clarify some aspects of metacognition and its application during real-world problem solving. New questions include: what kinds of everyday problems provide opportunities for students to use metacognitive functions? How do new types of problems affect perspectives on problem solving? If problem solving is defined in a new way, then what changes for metacognition?

Model-Eliciting Activities

Model-eliciting activities represent one type of problem that provides opportunities for students to use metacognitive functions. The students are placed in small groups of about three and given a task from a real-world client with specific criteria for successful completion of the task. Their solution takes the form of a procedure, explanation, justification, plan or method for solving the client's problem. Their solution should serve not only the client's immediate needs, but also be generalizable and extendable to other problem situations. Part of this requirement often includes writing a letter to the client describing their solution. Their procedure should be described in enough detail that the client could apply it to a set of data and produce the same results as the students. Two examples of model-eliciting activities used as illustrations in this paper are the Paper Airplane Problem and the Amusement Park Problem. For

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the Paper Airplane Problem, the goal is to devise a set of rules that can be used to find the winners in four categories (Most Accurate, Best Floater, Best Boomerang and Best Overall) at a paper airplane contest. Each team throws their plane three times for three different paths (a straight path, a path including a right turn and a boomerang path so the plane comes back to the thrower). The data provided includes the time in air, length of throw, and distance from a target for each of attempt. The set of rules should work not only for the available data, but other data as well. The types of data analysis procedures the students use (e.g., averaging, ranking) are applicable to other situations where different types of data must be combined and analyzed. The Amusement Park Problem asks students to group a class going on a field trip to an amusement park into chaperone groups and devise a schedule for each group's day at the park. A table listing each student's five favorite rides, a line waiting time chart and a park map are included as data. In the letter to the client (in this case a teacher), the students have to outline their grouping and scheduling procedures so the teacher can use it for other classes going on field trips. The students have to coordinate different types of data to define their process. When they begin writing their schedule, they often use both the park map and the line waiting time chart. They may make a new table listing rides according to wait time (short, medium, long) and notes their location in the park. This information may then be combined with data from the ride preference chart in order to form groups.

The students must judge the usefulness of their solution by criteria explicitly defined by the client. These assessments happen throughout the solution process, but particularly at the end when they are writing their letter and reflecting about what they did. As they write their letter, they have to think carefully about their procedure and how to describe it so someone else could carry it out. Often, during the letter writing, the group may discover a detail they left out or a flaw in their procedure. In the Paper Airplane Problem, the rules have to be fair and realistic. Students will often use their experience with paper airplanes or other contests to evaluate their procedure. The students self-assess their work as they compare their results with the needs and specifications of the client. They also compare their solution with realistic constraints in the problem situation.

Placing the students in small groups means they have to discuss the problem with each other and monitor the work of their group members. Since different students have different interpretations of the problem or may focus on different pieces of information, they have to communicate effectively with each other. Sometimes they listen to each other and sometimes they don't. The small group aids self-assessment since they frequently ask each other for justifications and explanations. During the letter writing process, they have to come to agreement about their procedure. In some problems (e.g., Amusement Park), the group may have divided up tasks among members and they need to assemble their results. They have to communicate their way of thinking about the problem to their group thereby providing a window to the observer about the metacognitive functions they employ.

A Modeling View of Problem Solving

In order to describe the relationship between problem solving and metacognitive functions, there are some distinctions between model-eliciting activities and other more traditional problems that need to be made. In model-eliciting activities, the primary purpose of problem solving for students is to provide an explanation, justification, plan or method as a solution to a complex problem rather than a one word or one number answer. The solutions take the form of complex models (Lesh et al., 2000). The model may include equations, graphical and/or pictorial representations, systems and other mathematical constructs. The model can be generalized and applied to other mathematical situations. In model-eliciting activities, students are often asked to develop a process for solving a problem so "correctness" is often judged by the usefulness of the process. For example, the product of the Paper Airplane Problem is a method for finding a winner of the contest. The method is judged by its fairness and usefulness for the judges of the contest. "Process as product" is a different type of goal than a problem asking the students to find the winner of the contest using established procedures. Students have to clearly define their assumptions as part of describing their process. Defining assumptions helps the client understand why certain decisions were made as part of the procedure. For example, when awarding a prize for the most accurate plane, some students assume this means which plane came closest to the target most frequently. Others take "most accurate" to mean which plane on average came closest to the target. A third interpretation is that the "most accurate" plane came closest to the target at least once even though other planes may have been more consistently close to the target. These different interpretations result from different assumptions about the problem situation and different experiences students may have had with the problem context.

Many model-eliciting activities (and other real world problems) contain too much information or not enough information. So, sorting out the relevant information for solving the problem is important. Sometimes the information needs to be reorganized or reformatted (as in the Paper Airplane Problem or the Amusement Park Problem) in order to be useful. How the students sort the data can depend on their assumptions and interpretations about the situation. In the Amusement Park Problem, students coordinate pieces of data (e.g., the park map and the line waiting time chart) in different ways. They may reorganize information in the charts. They may give different weight in their process to different sources. Some groups start work by looking at the park map. Other groups ignore the park map until later stages of the session. In the end, most groups coordinate information from the different sources because ignoring sources can give an incomplete solution. For example, their schedule wouldn't be complete if they didn't consider the park map. In word problems found in textbooks,

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givens and goals are often clearly outlined and defined. The problem for the student is to find the correct path from the givens to goals. In model-eliciting problems, there is a greater need to define the givens, goals and relevant assumptions of the problem in order to develop a solution. The data provided often needs to be modified to be useful for a solution. (Zawojewski & Lesh, in press)

In addition, everyday problems are often solved in teams of people. This means that compromises must be made between interpretations and communication between team members is critical to successful problem solving. (Zawojewski & Lesh, in press; Developing a process may take a team 1 to 2 hours. This is in contrast to finding a one word or one number answer in only a few minutes. A 1-2 hour process requires more planning and monitoring. Tasks may be divided (e.g., data analysis, different parts of the problem) and students may reconvene at various points. This provides more opportunities for metacognitive functions such as planning, monitoring and reflecting. Planning a one to two-hour process for a group is different than planning a 2-minute process by an individual. Reflecting about what happened over a one to two hour problem solving session is different than reflecting about a 2-minute problem solving process. Metacognitive functions such as planning, monitoring, reflection, control, etc. play a significant role in the development and application of complex models by students working in teams in everyday problem solving situations.

Modeling Views of Metacognition

In general, a modeling definition of metacognitive behaviors includes that students use them whenever they move from thinking with a construct to thinking about the construct. Thinking about the model occurs while they are developing the model and in subsequent problems. For example, students frequently begin using a procedure to process data. Often, while using the procedure, they consider the uses of their results or steps they need to take to solve the problem after they have finished the present procedure. They may also monitor the accuracy of their results as they are developed. Group members may check their own results and the results of others. So, they are not only carrying out a procedure, but also simultaneously thinking about the procedure. For example, instead of planning and then executing a procedure, students may plan the next procedure as they are executing the first. They may evaluate the usefulness or the implications of a procedure as they are carrying it out. For example, the girls in the following excerpt are solving the Amusement Park Problem. At this point, they have just started dividing the class into chaperone groups. They decided they needed to know how many students wanted to ride each ride before they could group them. Three girls in the group are counting the number of students who want to ride each ride. The fourth girl (Emily) is writing down the names of the students who want to ride each ride. At this point (Figure 1), Rochelle has begun to notice patterns as she is counting. Rochelle makes a number of comments throughout the session about how they should make the groups when they are done counting, but is largely ignored until they are ready to start grouping. The two girls discuss their different interpretations

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Rochelle:	I think those guys should definitely be together. These three. Or ... like um. Cause there's only 3 rockets they should be together you know cause...
Emily:	We could also divide it if we look up on here how long take it'll take each of 'em to get through the line if they're like right by each other.
Rochelle:	These should definitely be together because there's not that many people and if they like... They both like ... Emily.
Emily:	Oh wait. Yeah.
Rochelle:	All of 'em both like... the three people who like the rockets also both like the snake so they ... {Emily starts to interrupt} let me finish

Figure 1. Amusement park excerpt.

here, but they have to compromise about what to do later. In this instance, Emily had already formulated a plan for what to do next, and she wasn't ready to hear alternatives. Later in the session, they use a combination of grouping by ride preference and using the line waiting time chart to form their chaperone groups.

There are three characteristics of metacognitive functions related to the shift from thinking with a construct to metacognitively thinking about a construct: (i) connection to lower-order functions, (ii) interactions with lower-order functions, (iii) cyclic development. These characteristics have been observed in students solving model-eliciting activities and examples from sessions will be used to illustrate these characteristics (Lesh, Lester & Hjalmarson, in press).

The first characteristic of metacognition functions is that they are connected in a context-dependent fashion not only to the logical and mathematical aspects of the problem but also to lower-order functions related to the problem context. The connection to lower-order functions results because modeling focuses on interpretations, and interpretations are not restricted by the mathematics of problem solving. Interpretations of a context can depend on prior experience with the context. For example, the girls solving Amusement Park reference their experience with rides at other amusement parks when grouping. They interpret a rides description from its name (e.g., the Monster ride is thrilling, the Boat ride is for little kids). The students' interpretations of the problem are affected not only by their mathematical knowledge, but also by interactions in the group, beliefs associated with the problem context, and other issues. These issues affect mathematical decisions made during everyday problem solving. For example, beliefs and previous experiences about a problem context affect assumptions students may make about the problem and decisions they make to solve the problem. Hence, students are able to examine their beliefs about a problem and the effects of those beliefs on their problem-solving process (Middleton, Lesh, & Heger, in press). The connection to lower-order functions is context-dependent because of students varying capabilities and experiences in different contexts. For example, in the Paper Airplane Problem, students who have had experience with paper airplane contests, flying paper airplanes or other contests will think differently about the problem than students with different types of experiences. Their experiences in the situation affect the types of assumptions they make about the context and procedures they use.

The second characteristic is that metacognitive skills occur in parallel to and interactively with lower-order skills and cognitive functions. The interactive and parallel connection is directly related to the context-dependent nature of metacognitive functions. If metacognitive functions are context-dependent, then the context affects their application. More experience in a particular context may provide more opportunity to use metacognitive functions. One implication of this is that if more thinking is required to carry out the procedure, less thinking will be used for metacognitively thinking about the procedure. When the girls solving the Amusement Park problem started counting how many students wanted to ride each ride, they were very focused on the process of counting. As this process progressed, they began to notice more patterns and consider what they would do once they were done counting. Also, different types of metacognitive procedures may be used when a student first learns about a procedure than later in their development. For example, checking that fractions have been added correctly may be more important or take more time when first learning to add fractions. Planning when to add fractions may be more important later. As procedures become more refined, there is more opportunity and need for students to think about them metacognitively. So, as students are developing mathematical models, they are also developing metacognitive functions that help them understand when, how and why to use those models effectively Figure 2 shows the difference in the relationships between metacognitive and cognitive thinking (Lesh, Lester & Hjalmarson, in press).

This represents an important shift from thinking of lower-order functions as prerequisite to higher-order functions or as a separate goal of instruction. When students solve model-eliciting activities, they develop lower-order skills at the same time as they are using higher-order skills to further the development of lower-order skills. Principally, since model-eliciting activities require more time and more complex solutions, there is more opportunity for students to use and need to use metacognitive functions in a meaningful way. They are also developing an understanding of when, how and why to use metacognitive functions. For example, a group of boys solving a model-eliciting activity called Paper Airplanes spends a significant amount of time working on the problem without monitoring what they are doing. At the beginning of the problem, they read the data table without regard to which numbers represent times and which are distances. Later, they carefully question each other about procedures to make sure they all understand what is happening. In the excerpt in Figure 3, Bob is explaining his averaging technique to Al.

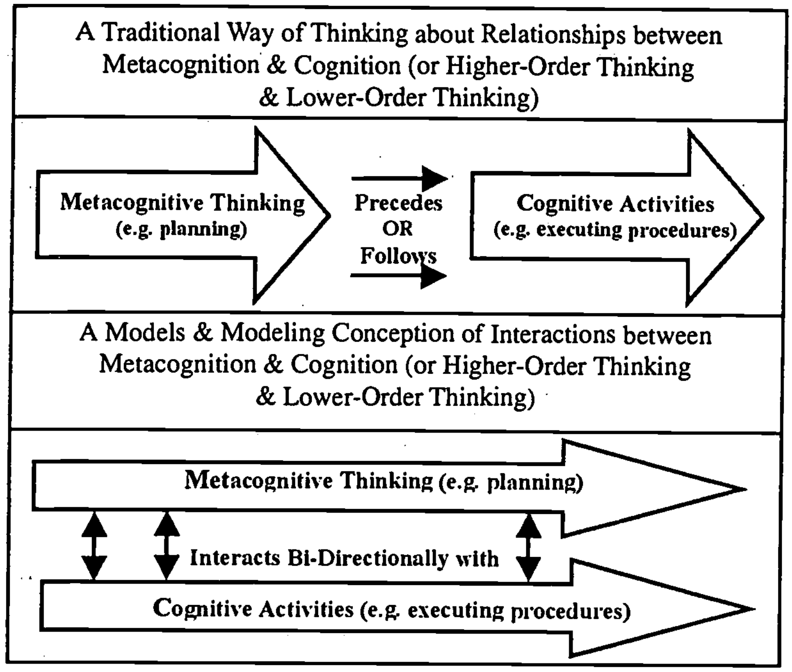

Figure 2. Relationship between metacognition and cognition.

Al: Why we gotta find the average? How we gonna do that?...Let's get a calculator.

Bob: The average is, like, in baseball. ... It's the one in the middle. ... It's like you got three stacks and you gotta make 'em all the same. You move 'em up or down a little – if they're too high or low. ... Here, I'll show you. You gotta even 'em up. ... Take these. [Refers to 1.8, 8.7, and 4.5]. ... Now. Move... uh... move 7 from the big one [Refers to 8.7] to the little one [i.e. 1.8]... No, just move two.

Al: What are you doing? What are you doing? I don't get it.

Figure 3. Paper airplane excerpt.

Bob proceeds to explain the procedure until Al understands how he's finding the average. Carl (their third group member) then gets a calculator from their teacher and asks her how to calculate averages. They use averages in combination with other procedures to evaluate the rest of their data. The developmental view also means that students need the opportunity to develop metacognitive skills in the form of problems requiring such skills. The opportunity may also be provided by problems that require a group to solve. The complex nature of the problems in model-eliciting activities means that in order for students to successfully solve the problem they must plan, monitor, reflect, etc. and work effectively in a group. There is often too much data for one student to process alone in the time period they are allowed. Effective and useful solutions also require input from multiple sources and people.

The final characteristic of a models and modeling perspective is that models are developed in cycles. For example, the boys solving the Paper Airplane Problem spent some time initially examining the data table without much monitoring of each other's interpretations. They didn't pay attention to the different types of values in the table (some distances, some times). Later, they realize that how far a plane flies may be a different size number than how long it was in the air (e.g., 14.9 meters is a reasonable distance, but 14.9 seconds is not a reasonable time.) Later in the session, they carefully monitor each other's interpretations of the problem and procedures used to solve it. In one problem-solving session, there may be multiple cycles or phases of model development. From cycle to cycle, the models move along continuums from unstable to stable, incomplete to complete, and concrete to abstract. For the girls solving Amusement Park, at first they used only one piece of information (the ride preference table). This led them to find only the number of students for each ride. Later cycles of the solution process, they coordinated information between sources (e.g. the park map and the line waiting time chart) because the coordination led to better chaperone group formation. Just as students' models develop in cycles, metacognitive functions also occur and develop in cycles. As students improve their way of thinking about the problem situation, their solution and their model should improve. Hence, it is also important to note when metacognitive functions serve to help students change their way of thinking about the problem as they move from cycle to cycle. Often, at the beginning of the session, the students have not clearly defined the givens and goals and they may have an incomplete notion of the givens and goals in the problem (As Bob, Al, and Carl did for the paper airplane problem above). It is important for them to work through a few cycles of understanding so they can have a clearer, more complete understanding of the problem and how to solve it. One implication of this characteristic and the interactive nature of metacognition with cognitive activities is that it becomes clear that not all metacognitive functions are helpful all of the time in all situations. A metacognitive function that may be helpful at the beginning of the problem solving session may not be helpful at the end of the solution process. For example, reflection may be counter productive during initial brainstorming of ideas about the problem, but reflection is very helpful when finishing the solution to the problem in order to make sure the solution is appropriate and complete. Some metacognitive functions may be helpful when a model is still primitive and incomplete, and different metacognitive functions may be helpful once a model is more stable and complete.

The three characteristics discussed above (connection to lower-order functions, parallel interaction with lower-order functions, and cyclic development) have been accounted for in the development of model-eliciting activities that embody the types of everyday problem-solving activities requiring model development. For cyclic development to occur, it is necessary to have a problem situation requiring more than one cycle of interpretation. Outlining the client's specific criteria for a useful solution helps students self-assess their ways of thinking and move from cycle to cycle. This is aided by having problems where multiple interpretations of data and information are possible. It is also aided by problems asking for a process rather than a one number answer. A problem asking for a complex artifact means they have an opportunity to monitor and reflect on multiple interpretations.

Conclusions

The models and modeling framework as applied to metacognition attempts to clarify issues that have been unclear and examine metacognition from a perspective that is more easily observed and documented in students. Using model-eliciting problems, it examines new characteristics of metacognition for longer more complex problem solving situations asking for the development of a process. In particular, it addresses exactly what students are thinking about when they are thinking metacognitively. The model-eliciting activities provide problem-solving opportunities complex enough for students to have a need for metacognition and evaluate their interpretations of the problem. It also attempts to define how metacognitive functions develop and are employed in everyday problem-solving situations requiring the development of complex models and descriptions of systems. Metacognition remains a complex issue particularly when the interactions between metacognition and other aspects of problem solving are examined. However, this does not mean that it is any less important or that it cannot be examined and developed in a meaningful way.

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