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ABSTRACT

As a part of a long term district-wide study, we report on achievement test results involving students in a third-grade classroom taught by a teacher in a professional development program led by the authors and directed towards the integration of algebraic reasoning with elementary school mathematics. The teacher's practice, reported upon elsewhere, embodied many of the features targeted in the professional development program, and her students' performance on a state-wide standardized test for fourth graders exceeded those of a comparable control class from the same school and those of her district's fourth graders, and approximated the performance of fourth graders state-wide. (Author)

STUDENT ACHIEVEMENT IN ALGEBRAIC THINKING: A COMPARISON OF 3RD GRADERS' PERFORMANCE ON A STATE 4TH GRADE ASSESSMENT

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Abstract: As part of a long term district-wide study, we report on achievement test results involving students in a 3rd-grade classroom taught by a teacher in a professional development program led by the authors and directed towards the integration of algebraic reasoning with elementary school mathematics. The teacher's practice, reported upon elsewhere, embodied many of the features targeted in the professional development program, and her students' performance on a state-wide standardized test for 4th graders exceeded those of a comparable control class from the same school and those of her district's 4th graders, and approximated the performance of 4th graders state-wide.

Perspective and Purpose of the Study

We are currently engaged in a 5-year district-based project integrating the development of algebraic thinking into elementary school mathematics (Kaput, 1999) in an educationally underachieving school district. While our primary focus is on K–5 teachers and administrators, their professional development is ultimately validated through evidence of student achievement as measured by independent standards—in this case, students' capacity for problem solving and generalization reflecting the Patterns, Functions and Algebra Strand of the NCTM *Principles and Standards for School Mathematics* (2000). Other analyses of the 3rd grade classroom in which this study occurred have shown that the teacher, a participant in the project, supported practices of algebraic thinking and that her students showed an emerging capacity for this as expressed through their activities of generalizing (Blanton & Kaput, 2000a; Blanton & Kaput, 2000b).

Methods/Data Source

We administered a set of fourteen test items (see Appendix) selected from the 4th grade Massachusetts Comprehensive Assessment System (MCAS), a state-wide mandatory exam, to Jan's class of fourteen 3rd grade students, a class with low socio-economic status (SES) by standard measures. The same items were administered to a second 3rd grade control class with comparable SES from the same school. Analysis of students' responses to the MCAS items, in both individual and partner settings, offers evidence to support the strategy we have adopted with elementary teachers as a means to build classrooms that prioritize understanding mathematics, in particular as it relates to students' abilities to form generalizations in their mathematical thinking and express those generalizations in increasingly formal ways. The test enables us to compare these students' achievement with students across the district and state.

For the experimental class, the test was administered through a 3-day process. Students first completed the exam individually under standard MCAS conditions (first day), then with a partner (second day), and finally, through whole-class discussion (third day). Students were also asked to provide written justifications for their responses to multiple choice items (10 out of 14 items were multiple choice; the remainder were open-response items). The controls took the exam individually only and were not asked to provide explanations of their answers, but used approximately the same amount of time on the test. Their instructor, an experienced teacher, had not participated in our professional development seminars and employed methods similar to those common across the district. We regard the comparison with another 3rd grade class as important due to the literacy demands of the 4th-grade test and the fact that well over half of the experimental class' students were from homes where English was a second language.

Results and Conclusions

Item-Based Comparison with Control Class: State-wide levels of achievement for the MCAS (from which these items were selected) were determined in the Spring 1999 assessment to be (a) advanced—81%; (b) proficient—67%; and (c) needs improvement—41%. Based on these cut-off levels, it follows that Jan's 3rd grade students performed significantly above the 'proficient' level on 33% of the items. They were at or above the 'needs improvement' level for 74% of the items.

An item analysis comparing the results of the experimental group (Jan's classroom) with the control group shows that students in Jan's classroom outperformed the control group on 11 of the 14 test items (see Figure 1). Moreover, Jan's students performed significantly better (at level *alpha* = 0.05) than the control group on 4 of these 11 items. We note that the items on which the control group outperformed the experimental group (although not at a statistically significant level) were multiple choice (items # 3, 9, and 13), and we don't have information from the control group (i.e., written justifications) to determine if their performance was due to chance. Of these 3 items, Jan's students' scored 22% on item 13, much lower than the 41% cut-off level and the control group's score of 50%. In their written justifications for this item, we found that the most common error was that students added numbers given in the problem or the list of possible responses to determine their answer (e.g., 9 + 4 = 13 and 3 + 4 = 7 produced responses of 9 and 7, respectively). Of those 4 students who answered the problem correctly, one described that he had counted and one drew a model of divided oranges. Of the remaining 2 students, one did not give a written justification and the other gave a response that was unintelligible. Only 50% of the students provided a written justification.

Of the 14 items on the assessment, we identified seven as being deeply algebraic in nature (items #2, 3, 6, 7, 8, 11, and 14), requiring students to find patterns generated numerically and geometrically, understand whole number properties (e.g., commutativity), and identify unknown quantities in number sentences. We find it significant that the experimental class outperformed the control class on 6 out of 7 of these items.

We also note that the experimental class performed at the 'proficient' or 'needs improvement' level for 79% of the items according to the state standards for those items. Moreover, they scored at these levels on 72% of the algebra items, which tended to be harder for all groups.

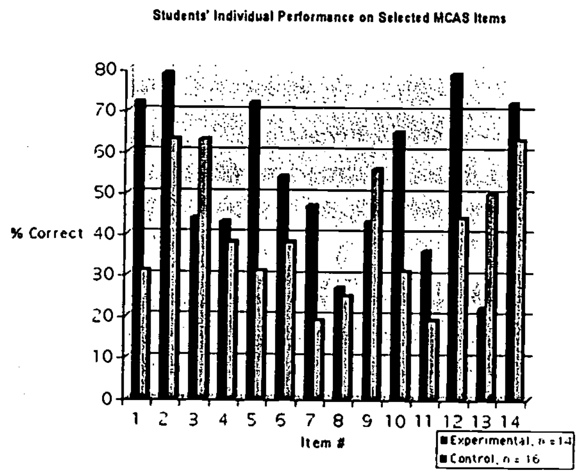

Figure 1. Item comparison with control class.

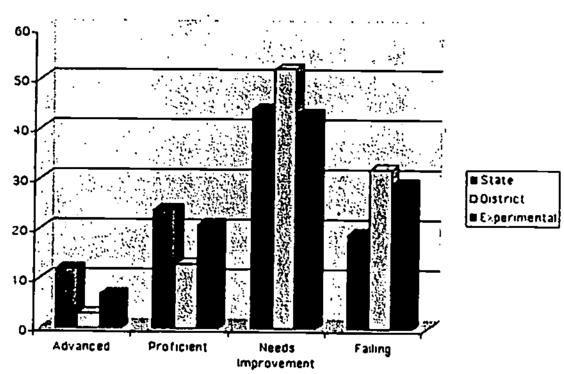

Figure 2. State/District MCAS scores compared to experimental scores.

Overall Comparison with State and District Performance: Jan's students' individual performances, while varied, also support the promising results indicated by the item analysis comparison with the control group (Figure 1) and when compared with State and District results (see Figure 2). The respective State/District achievement levels on the *entire* test are compared with the experimental group's scores by percent per category in Figure 2, with the experimental group's score in **boldface**. (That is, the **boldface** score represents the percentage of students in Jan's class who scored at a particular level.)

Advanced:	12/3/**7**
Proficient:	24/13/**21**
Needs Improvement:	44/52/**43**
Failing:	19/32/**29**

These results indicate that the experimental 3rd-grade class performed approximately as well as the 4th graders state-wide, and significantly better than the district 4th graders (whose performance had improved considerably over the prior year). In our view, given the significant advantage of an additional year's instruction of the 4th grade comparison groups at the state and district levels, and especially given the significant development in the necessary verbal skills during the intervening year relevant to several of the items as well as the low SES factors, especially language, for the experimental class, we regard these as strong results.

Item Analyses for Individuals: In conjunction with Figure 2, Figure 3 provides an item analysis of those MCAS items used in our assessment that were selected from the 1999 MCAS (items 1, 2, 3, 4, 5, 6, 9, 10, 12, and 14). In particular, it shows a comparison of performances by the experimental and control groups with performances at the state, district, and school levels (state, district, and school data for the other items used here were not available). Note that the 14 assessment items we selected came from a variety of MCAS resources, including the 1999 MCAS items, and from multiple strands represented on the MCAS. Again, we see that Jan's students' performance was comparable to that at the state, district, and school levels for most of the items (i.e., # 1, 2, 5, 6, 10, 12, 14), and in some cases her students outperformed 4th-grade results overall (i.e., #10, 12).

Analyses for Students Working in Pairs: Finally, we include in Figure 4 results of an item analysis of Jan's students working in pairs to complete the MCAS items. As expected, students' performed even better when working with a partner. In particular, the analysis showed that students performed at the 'advanced' level on 57%

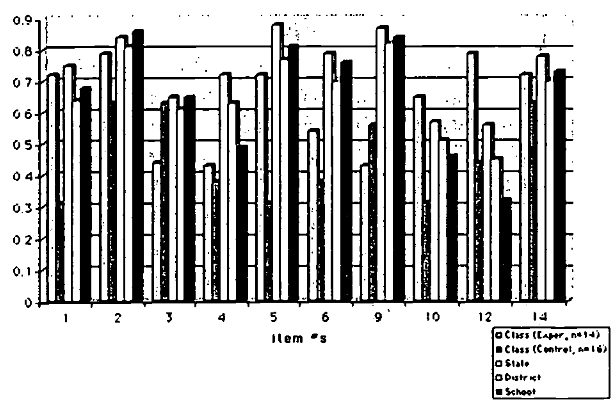

Figure 3. Percentage of correct responses for each comparison group by item.

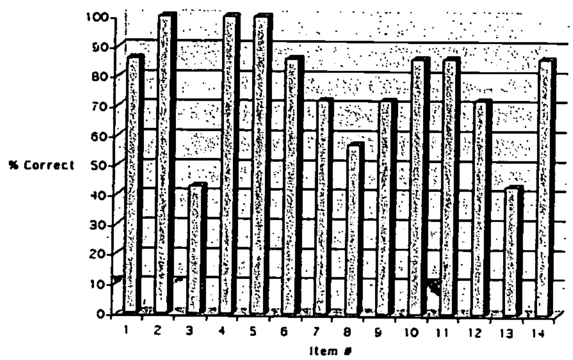

Figure 4. Item analysis of students' performance when working in pairs.

of the items and were 'proficient' or above on 79% of the items. Also, students performed at least at the level of 'needs improvement' on all of the items. Additionally, of the problems for which students scored at the 'proficient' or 'advanced' levels (items # 1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 14), five were problems we identified as strongly algebraic. Thus, working in pairs, students scored at or above the 'proficient' level on 72% (5 out of 7) of the algebra items—scores at this level were achieved in but a few schools state-wide. They scored at least at the level of 'needs improvement' on all of the items we characterized as strongly algebraic.

Overall scores for partner-pairs were quite strong. We believe that, beyond practice effects, this reflects the factoring out of verbal skills as the students were able jointly to interpret the text of problem statements. Out of 7 partner groups, 57% (4 out of 7) performed at the 'advanced' level, 29% (2 out of 7) performed at the 'proficient' level, and one group performed at the level 'needs improvement'. All partner groups ranked under this scheme, scoring at least at the level 'needs improvement', with a maximum score of 90% and a minimum score of 47%. Anecdotally, we note that for the most part students took the assessment quite seriously and, as we observed, were deeply engaged in a process of argumentation and justification with each other during the partner exam and the whole-class discussion. We take this as additional confirmation of the type of socio-mathematical norms that had evolved in Jan's classroom throughout the year, norms that we take to be critical for the development of students' algebraic thinking (Cobb & Bauersfeld, 1995).

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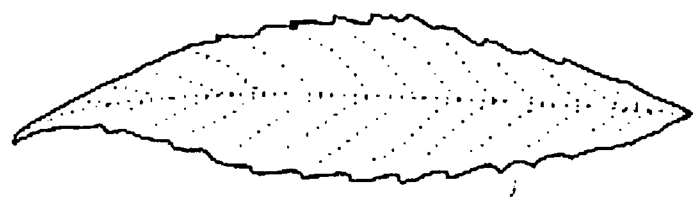
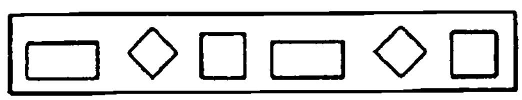
Appendix

1. How many CENTIMETERS long is the leaf?



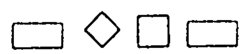
Use the pattern in the box below to answer the next question.



2. What are the next four figures in the pattern above?

A. 

B. 

A. 

B. 

3. What number does *n* stand for in the sentence below?

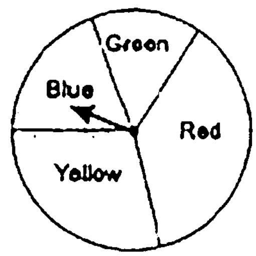
$(8 + 2) + 6 = 8 + (n + 6)$

A. 2 B. 6 C. 8 D. 16

4. Your lunch time begins at 12:40 P.M. If your lunch time is 35 minutes long, what time does it end?

A. 12:05 P.M. B. 1:10 P.M. C. 1:15 P.M. D. 1:30 P.M.

5. This is a spinner for a game. Which color are you most likely to spin?

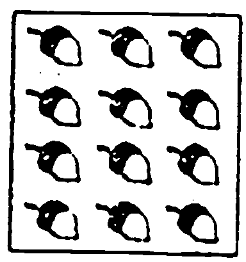


A. blue B. green

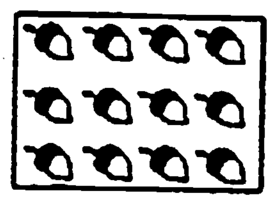
C. yellow D. red

6. Melvin collected acorns from the yard.

First he placed them like this:



Then he placed them like this:

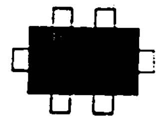


Which number sentence shows the TWO ways Melvin placed his acorns?

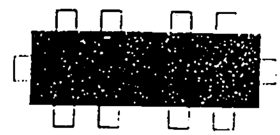
A. $3 \times 4 = 4 \times 3$ B. 3×4 C. $3 \times 4 > 4 \times 3$ D. $3 \times 4 \geq 4 \times 3$

7. Andrew is setting up tables for a birthday party.

He knows that six people can sit about this table:



When he puts two of these tables together end to end, he can seat ten people.



How many people can Andrew seat if he puts three tables together end to end?

8. Write the RULE to find the next number in this pattern.
87, 81, 75, 69, ____

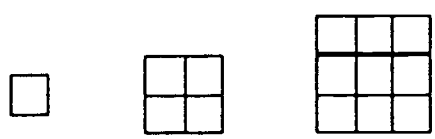
9. There are 60 pieces of art paper and 42 children. If each child gets one piece of art paper, how many pieces will be left for another project?

A. 9 B. 18 C. 27 D. 42

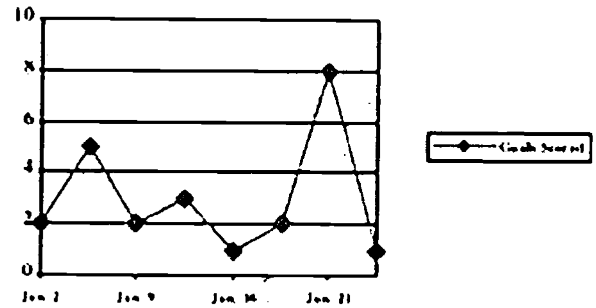
10. What is the GREATEST number of different outfits you can make with 2 pairs of pants and 5 shirts? (Each outfit must have exactly one pair of pants and one shirt.)

A. 5 B. 7 C. 10 D. 25

11. How many of the smallest squares will be in Figure 5 if this pattern continues?



12. How many goals did the Boston Bruins score in their game on January 16, 1999?

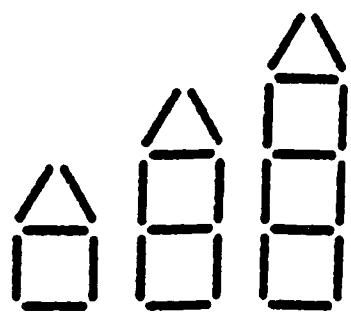


(a) 1 goal (b) 2 goals (c) 4 goals (d) 3 goals

13. Mr. Gillman wants to give apple slices to his 13 soccer players during their game. Each player will receive 3 slices. He plans to cut each apple into 4 slices. How many apples will Mr. Gillman need?

A. 8 B. 10 C. 7 D. 9

14. Donna made this pattern using sticks. Draw the next figure in the pattern.



Explain how you got your answer.

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