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ABSTRACT

This paper reviews the research on mathematics teacher education and mathematics education reform to provide a possible foundation for educating mathematics teachers in rural Appalachia. The paper takes the position that teacher change has certain characteristics and impediments that are not limited to any particular circumstance. Research on elementary mathematics teachers in various countries reveals teacher misconceptions and lack of knowledge about mathematics, their difficulty in appreciating its connectedness, and their beliefs about mathematics as a frightening subject. Research on secondary mathematics teachers suggests that secondary teachers' beliefs about mathematics and the intensity of their beliefs impede reform efforts. Commonalities found in research into preservice and inservice teacher education focus on difficulties teachers have in knowing mathematics, knowing about mathematics, and knowing about the teaching of mathematics. From these commonalities, three principles are identified for teaching mathematics to preservice teachers: that preservice teachers should experience mathematics as a pluralistic subject, should explicitly study and reflect on school mathematics, and should experience mathematics in ways that support the development of process-oriented teaching styles. Barriers to school improvement in rural areas are discussed, followed by implications of the rural circumstance for improved teaching/learning of mathematics and improved training of rural mathematics teachers. (Contains 60 references.) (SV)

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Working Paper Series

Mathematics Teacher Education in Rural Communities: Developing a Foundation for Action

Working Paper No. 6

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University of Georgia
March 2003

ACCLAIM's mission is the cultivation of *indigenous leadership capacity* for the improvement of school mathematics in rural places. The project aims to (1) understand the rural context as it pertains to learning and teaching mathematics, (2) articulate in scholarly works, including empirical research, the meaning and utility of that learning and teaching among, for, and by rural people, and (3) improve the professional development of mathematics teachers and leaders in and for rural communities.

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Given the necessity of a learned society in an increasingly technologically-oriented world, the education of the young deserves the scrutiny and support of those responsible for education. Education necessitates attention to a variety of principles and mores, some specific to mathematics teacher education, e.g., those developed by the National Council of Teachers of Mathematics (NCTM), and some unique to the particular locale being served, e.g., those involved in the ACCLAIM project. In this paper, attention will be given to research on mathematics teacher education; particular attention will be given to those commonalities that seemingly have no geographic boundaries. Subsequently, a possible foundation for educating mathematics teachers in rural Appalachia will be offered.

The term reform is often bandied about when talking about changes in mathematics education. By definition, "reform" implies moving away from traditional or established practices, whatever they might be. In the United States, reform in mathematics education is best represented by the *Principles and Standards for School Mathematics*, developed and promulgated by the National Council of Teachers of Mathematics (NCTM, 2000), and its three predecessors: *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the *Professional Standards for Teaching Mathematics* (NCTM, 1991), and the *Assessment Standards for School Mathematics* (NCTM, 1995). Collectively these publications (subsequently referred to as the Standards) emphasize the processes of doing mathematics, in particular the processes of problem solving, communicating, reasoning, proving, making connection, and representing.

Reform world wide has usually focused on similar kinds of processes albeit in different contexts. Although reform will be used throughout this paper, it should be realized that reform is not a monolithic entity encased by the same objectives. Thus reform in Appalachia may or may not coincide with reform as defined by the Standards but it does, nevertheless, imply moving away from the current practice of teaching mathematics in Appalachia. Reform by any other definition would imply the perpetuation of the status quo. This paper is less about the form of reform in Appalachia and more about what is involved in promoting reform, whatever shape it may take. The position will be taken that teacher change – that is, reform – has certain characteristics and impediments that are not limited to any particular circumstance. There is much to be learned about reform efforts worldwide that has implications for what reform might look like in Appalachia.

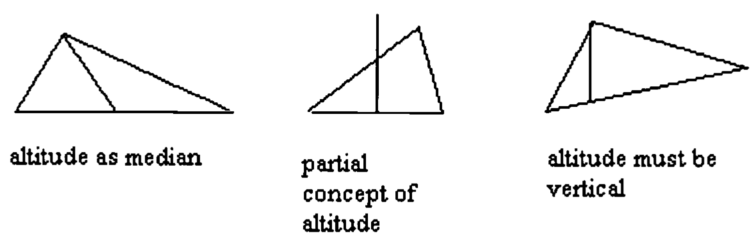
Learning from Research on Mathematics Teacher Education

A considerable body of knowledge is emerging about the processes of mathematics teacher education and what sense teachers make of that enterprise. This section will focus on that research and its implications for educating preservice and inservice mathematics teachers.

Preservice Teacher Education

Although issues regarding the education of preservice elementary teachers have a family resemblance to those of educating preservice secondary mathematics teachers, there is enough difference to warrant separating these two populations in the following analysis.

Elementary Teachers' Conceptions of Mathematics and Its Teaching. Brown, Cooney, and Jones' (1990) review of research in mathematics teacher education pointed out the lack of elementary teachers' knowledge of mathematics and their difficulty in appreciating the connectedness of mathematics. More recently, Ma (1999) observed the difference between Chinese teachers' conceptual orientation and understanding of mathematics and teachers from the United States who held a more procedural understanding of mathematics. Although Ma's study revealed elementary teachers' weaknesses in mathematics, particularly when compared to their Chinese counterparts, elementary teachers' difficulty in understanding mathematics is hardly limited to the United States. For example, Gutiérrez and Jaime (1999) found that Spanish elementary preservice teachers had a poor understanding about the altitudes of various types of triangles as shown below.



The authors claimed that these misconceptions were the product of teachers only encountering triangles with altitudes that were vertical to a horizontal base.

Similarly, Stacey, Helme, Steinle, Baturo, Irwin, and Bana (2001) found that preservice Australian elementary teachers failed to correctly make decimal comparisons such as the following:

Decimals with unequal length – larger decimal is shorter. Compare 0.75 vs. 0.8.
Zero in tenths column of decimal which would otherwise be larger.
Compare 3.72 vs. 3.073.
One decimal is truncation of the other. Compare 8.245 vs. 8.24563.
Compare positive decimal with zero. Compare 0 with 0.6.

The authors reached the following conclusion:

> Only 57% of the preservice teaches reported that students might have difficulty with the items of the type that they got wrong themselves, indicating that quite a sizable proportion of preservice elementary school teachers may not suspect they are making errors. (p. 222)

The upshot of studies such as Gutiérrez and Jaime (1999) and Stacey, et al. (2001) is that teachers lack the sophistication necessary to appreciate children's understanding of mathematics and to promote the connectedness of which the Standards speak.

Ebby (2000) emphasized the importance of developing student teachers' habits of mind in which they can learn to grow professionally by learning from their own teaching. For example, when preservice teachers observed how children made sense of mathematics, the teachers had the potential to envision instructional strategies that promote students' sense-making activities. Similarly, Mewborn (2000) argued that "observation can be a powerful learning technique when it is coupled with discussion and deconstruction of what was observed" (p. 42) especially when preservice teachers work in cohorts. But Grant, Hiebert, and Wearne (1998) demonstrated that what teachers see when observing other teachers is determined to a great extent by what they believe about mathematics and its teaching. For example, teachers who consider mathematics as a collection of procedures are unlikely to appreciate the efforts of colleagues whose lessons promote the processes of doing mathematics (i.e., the approach prescribed by the Standards). If preservice teachers' beliefs blind them to what the observed teacher intends, the observer probably will not learn much from the experience; consequently, allowing preservice teachers to observe reform-oriented teachers without giving the observers any additional grounding in the principles or methods of reform will likely not promote reform.

In an intriguing study of 42 preservice elementary teachers in Germany, Gellert (2000) found that the teachers envisioned their task as that of protecting students from the abstract terrain of mathematics that they had experienced in their learning of mathematics. For example, Gellert found that teachers' use of games and stories was more for the comfort of their students than as a vehicle for helping children learn mathematics. Gellert's evidence supported the "assumption that agreement on the importance of games and stories is only on the surface and not rooted in a didactic conception of mathematics teaching and learning" (p. 263). This is consistent with my observations of preservice elementary teachers in that they like using games with children because of the "fun factor," but they frequently miss opportunities to extract meaningful mathematics from the game. Gellert's study highlights how teachers' beliefs about mathematics as a frightening subject can actually hinder the effective teaching of mathematics.

Cooney and Krainer (1996) emphasized the importance of listening—not only for teachers listening to their students, but also for teacher educators listening to their teachers. Through the process of listening, one comes to understand how individuals are interpreting and making sense of what is being taught thus providing a foundation for meaningful instruction. Similarly, Crespo (2000) studied teachers working with fourth grade students and noted that the preservice teachers had not expected to learn about students' thinking in a methods course. Consequently, the topic was met with both intrigue and excitement. For some teachers, listening was a matter of determining whether students were on the right track. Others, however, viewed listening more broadly: as a means of understanding how students were making sense of the mathematics. Listening enabled those teachers to change how they interpreted students' understanding, and thus to change how they taught.

In an effort to promote reform, it is clear that elementary teachers' knowledge of mathematics needs to be improved. But the tricky part is to address the issue of *what mathematics* should be addressed and by *what means* it should be addressed. This challenge will be the subject of a later section. Suffice it to say here that there is no simple means of addressing this problem.

<u>Secondary Teachers' Conceptions of Mathematics and Its Teaching</u>. There is a dearth of research about preservice secondary teachers' knowledge of mathematics, the assumption apparently being that their undergraduate study of mathematics has given them a solid foundation for understanding school mathematics. This may be a reasonable assumption in that secondary teachers can do the mathematics of the secondary school curriculum in the sense of solving the problems. A study by Cooney, Wilson, Albright, and Chauvot (1998), however, demonstrated that secondary preservice teachers entering their mathematics education sequence of courses displayed a limited view of mathematics despite the fact that they had completed a substantial part of their formal mathematical training. For example, when the preservice teachers were asked, "What is a function?" their responses often focused on equations or a formula involving computations, as illustrated in the following responses:

> A function is a formula that can have various items inserted.
> A function is an equation or graph in which there are not two y's for every x.
> Is an algebraic equation of a line in a plane.

The students' responses failed to capture the essence of what constitutes a function, namely, an explicit stated relationship between two varying quantities.

Although these teachers' views of mathematics broadened as they progressed through their remaining undergraduate program, the fact remains that they needed this added mathematical maturity in order to realize reform in the teaching of mathematics. Brown, Cooney, and Jones (1990) pointed out that secondary preservice teachers often hold limited views of mathematics. It follows that teacher education programs should address the issue of how these views can be broadened.

Kinach (2002) demonstrated that you can affect preservice secondary teachers' knowledge of mathematics by addressing their instructional explanations and raising the question of what constitutes a good explanation. By focusing on explanations, which necessarily involves the teaching of mathematics, students encountered a considerable amount of school mathematics and thereby strengthened their mathematical understanding of what they would eventually be teaching. Langford and Huntley (1999) provided preservice secondary teachers with real world internship experiences that involved using mathematics in the workplace. The researchers concluded that the internships helped to shape and modify the teachers' views of mathematics and its teaching. The interns reported seeing mathematics from a more holistic perspective as a result of their real world experiences.

Bowers and Doerr (2001) found that preservice secondary teachers developed insights into the teaching of mathematics by virtue of working in microworlds and in mini-teaching situations. The microworlds (interactive models of real-world systems) allowed the teachers to study motion and change by considering relationships of position vs. time, velocity vs. time, and acceleration vs. time, all of which were linked to an animated simulation. These "student" experiences provided an effective means for eliciting perturbations among the teachers as they reflected on their beliefs about teaching mathematics. Further, the use of microworlds allowed the teachers to juxtapose their dual roles as students and teachers and consider the implications of those roles for developing mathematical and pedagogical knowledge. In a four-year longitudinal study following preservice teachers through their second year of teaching, Steele (2001) identified several factors that influenced their conceptions and choices of teaching style: teachers' personal commitments, their professional strength, their beliefs and knowledge of what they are teaching, and support from the administration. Steele's message is that teachers cannot be expected to reform their teaching without supporting curricula, collegial support, and support from administrators once in the field.

Frykhom (1999) conducted a three-year study of 6 cohorts of 63 preservice secondary preservice mathematics teachers that focused on their beliefs and knowledge about the Standards contrasted with their teaching practices. Most of the preservice teachers were enthusiastic about the Standards, some suggesting that they constituted "the Bible in mathematics education." Other students, though, felt that the Standards were shoved down their throats by the instructor. In general, the preservice teachers saw the Standards as a set of rules and viewed them as content to be learned rather than a philosophy to be adopted. They felt that it was important to learn the Standards because it gave them a basis for talking about reform in contexts such as a a job interview. But they also felt that it was unrealistic for them to implement the Standards given their lack of experience. Further, they recognized the contrast between the kind of teaching advocated in the Standards and their own (student) teaching, which they felt generally mirrored that of their cooperating teachers. They felt uncomfortable as a result of this recognized contrast.

Although less is known about secondary teachers' knowledge of mathematics, there is a considerable research base about what they believe about mathematics (see Thompson, 1992). Generally, these beliefs are not conducive to reform either because of their substance (e.g., as contradicting the very principles of reform) or because of the intensity (or lack thereof) with which the beliefs are held. In either case, the challenge of such beliefs represents a concern most conveniently addressed in teacher education programs.

There is little research on middle school preservice teachers' conceptions of mathematics. But given the literature on elementary and secondary teachers' conceptions, there is little reason to expect middle school teachers' conceptions of mathematics to differ significantly from those of the secondary teachers. (One would expect their knowledge of mathematics to be stronger than that of their elementary counterparts, however.) Issues identified with educating elementary and secondary preservice teachers are likely to permeate the education of middle school teachers as well.

Inservice Teacher Education

Saxe, Gearhart, and Suad Nasir (2001) demonstrated the value of professional development programs: they found that upper elementary students with teachers who participated in professional development programs that emphasized reform had gain scores significantly greater (topic: fractions) than students whose teachers did not participate in such programs. The professional development programs emphasized reflective discussions among teachers, teachers collectively identifying problems in teaching and sharing successful strategies, and the use of various activities that enhanced teachers' mathematical understanding of the curriculum and how students made sense of that curriculum.

Chapman (1999) provided elementary inservice teachers opportunities to focus on their personal experiences as a way of achieving self-understanding and as a way of reconstructing their personal meanings about problem solving and problem-solving instruction. Through the program's various reflective activities, teachers changed their methods, giving students more opportunities to solve problems and placing a greater emphasis on mathematical processes rather focusing solely on answers. Chapman concluded that the program's explicit recognition of teachers' personal meanings was a significant factor in facilitating their change in practice.

Wilson and Goldenberg (1998) described Mr. Burt, who was willing to reform his teaching, but only to a certain extent. Mr. Burt's teaching moved from a procedural orientation toward a conceptual orientation, but remained teacher centered nonetheless. He was unwilling to allow students to explore open-ended questions or debate possible mathematical interpretations – in general, those contexts in which the outcomes were not predictable. Although he appreciated the more sophisticated mathematics, his beliefs about the teaching of mathematics prevented him from realizing significant reform.

Schifter's (1998) professional development program used student thinking as a basis for middle school teachers' reflective thinking, their reexamination of what it means to do mathematics, and the development of a deeper understanding of the mathematics. She found that teachers can learn mathematics by better understanding their students' mathematics. The apparent success of Schifter's program is grounded in a constructivist approach to teaching, in which teachers adopt a pluralistic view of mathematics and use their students' understanding to guide their teaching rather than following a predetermined curriculum.

Borasi, Fonzi, Smith, and Rose (1999) described a professional development program with middle school teachers in which the intent was to enable the teachers to use an inquiry approach in the teaching of mathematics. The authors concluded that collaboration among teachers who were using supporting curricular materials was essential to realizing reform. Like Schifter (1998) the authors made the case that it was valuable for teachers to experience the content they were to teach. As the authors put it, "...it is important that teachers experience an illustrative unit as learners (emphasis in original) before they implement it as teachers (emphasis in original) as a first field experience." (p. 71). Indeed, the field experiences were a critical addition to the initial summer work in order for the teachers to realize how the curricular materials could be used and how their teaching of the materials influenced students' thinking about mathematics. The authors also found that it was beneficial to have a cadre of 3-4 teachers working in the same school as they could share experiences with one another on what seemed to work and what did not.

Edwards and Hensien's (1999) study of one middle school teacher's struggle to reform her teaching emphasized the need for collaboration and the provision of a context for a teacher's reflective teaching. Although the teacher was in a state of transition when the study began, it was the collaborative and supporting relationship with a university mathematics educator that facilitated the teacher's change in practice. The collaborative context promoted reflection not only on students' sense-making activities but on the teacher's own pedagogical values as well.

Lloyd (1999) studied two high school teachers' conceptions of a reform-oriented mathematics curriculum. Although the teachers appreciated the theory behind the curriculum, their interpretation and implementation of the curriculum differed. While one teacher saw the curriculum's problems as open to student interpretation, perhaps even too open, the other teacher saw those same problems as too structured. Lloyd's study demonstrates how a certain dynamic relationship exists between teachers and particular curricular features.

Knuth (2002) studied 17 experienced secondary teachers' understanding of proof and found that the notion of "proof for all" (i.e., teaching principles of formal mathematical proof to all students) would not be an easy idea for most teachers to implement. The teachers tended to view proof in a pedagogically limited way, that is, as a topic of study rather than as a tool for communicating mathematically. The author concluded that teacher educators should explicitly address the notion and role of proof in classrooms in an effort to enable teachers to extend and enrich their views about the role of proof in teaching secondary school mathematics.

Adler (2000) argued that teacher education programs should enable teachers to be explicitly aware of the resources they have at their disposal and how those resources should be used. Adler pointed out how an obvious but simple resource such as the chalkboard has implications for instruction. Do the students or the teacher primarily control this resource? The controller of this resource speaks volumes as to the kind of teaching that is involved. A second resource Adler discusses is time. Discussions of time raise several questions: What percent of the classroom time is actually devoted to mathematics? How is that time allocated for teacher explanation, student questions, student seatwork, and student homework? Teachers who have been made aware of their use of resources can begin to reflect on whether that usage matches the kind of classroom environment they intend to create.

Nelson (1998) observed that administrators often have well-formed ideas about the nature of mathematics and its teaching and learning. These views shape the type of support provided teachers who are trying to reform their teaching. The author concluded, "if mathematics education reform is to be a permanent feature of school life, the intellectual culture of school as a whole will need to change, as well as the instructional practices of many individual teachers" (p. 212). School administrators bear much of the responsibility for establishing that culture. When administrators hold views of mathematics and teaching that are contrary to reform, change becomes difficult. Krainer's (2001) story of Gisela, a mathematics teacher and a school administrator in Austria, punctuates the importance of involving all significant parties when trying to realize reform.

Inservice programs are diverse and necessarily specific to the teachers and schools being served. Despite that diversity, inservice programs have much in common, consistent with what we know about preservice teacher education. The next section explores these commonalities.

Extracting Commonalities Across Geographic Boundaries

Given their immense diversity, what can we learn from inservice programs preservice teacher education programs? I will focus on three topics: knowing *mathematics*, knowing *about* mathematics, and knowing about the *teaching* of mathematics. Subsequently, three principles for teaching mathematics to teachers that are grounded in these commonalities will be offered.

Knowing Mathematics. What is rather striking about the studies on teachers' knowledge of mathematics is that the mathematics being considered is primarily the mathematics of the schools. Ma's (1999) study addresses the difficulty teachers have in interpreting school mathematics. Similarly, with respect to elementary teachers, the studies of Guitiérrez and Jaime (1999) and of Stacey et al (2001) demonstrate, in two different knowledge domains (geometry and decimals) and in two different contexts, the difficulty teachers have in understanding the mathematics they will be teaching. Although the notion that elementary teachers' knowledge of school mathematics is lacking is legendary, a similar, albeit more muted, concern exists regarding secondary teachers' knowledge of mathematics.. Studies of preservice (Cooney et al, 1998) and inservice (Knuth, 2002) secondary mathematics teachers suggest that teachers' understanding of school curricula needs explicit attention if they are to develop what Ma refers to as a deep and profound understanding of school mathematics.

There is considerable temptation to conclude that the necessity for teachers to understand more mathematics implies that teachers need to study additional mathematics as typically constituted at the collegiate level. Such study may indeed be needed to provide a strong mathematical foundation for understanding school mathematics in depth. But the reality is that such a solution is misleading and ultimately fails to solve the problem, although it might constitute a partial solution. Schifter (1998) pointed us in an alternative direction, showing that teachers can learn the mathematics of teaching by studying their children's mathematics. This novel approach demonstrates one important means of helping teachers understand mathematics by examining and reflecting on the consequences of how children construct mathematical concepts. Bowers and Doerr (2001) demonstrated another means of increasing teachers' knowledge of school mathematics through the use of microworlds. These authors pointed out that teachers' experiences as students can affect not only the teachers' knowledge of relevant mathematics but also can reveal possible pedagogical strategies for teaching mathematics.

It would be foolhardy to suggest that teachers' experiences with school mathematics should replace their study of more advanced mathematics. But it is also obvious that the mathematics of the workplace, that is, the school, requires attention beyond the more formal treatment of mathematics typically associated with studying collegiate level mathematics. Indeed, Owens's (1987) study of preservice secondary mathematics teachers suggests that formal training in mathematics may actually impede a teacher's willingness to embrace a pluralistic view of mathematics – the view that dominates reform-oriented school mathematics. This is not to disparage the study of formal mathematics. But it is to argue that the mathematics of the classroom is not, singularly, a formalized mathematics. The writings of Lakatos (1976) and of Davis and Hersh (1981) remind us of the fallibility of mathematics given its human origins.

School mathematics is no different – a point to be recognized in teaching teachers school mathematics from a pluralistic perspective.

The Issue of Knowing about Mathematics

The literature on how teachers think about mathematics is fairly robust and growing. Literature reviews by Brown, Cooney, and Jones (1990) and Thompson (1992) establish the importance of understanding what teachers believe about mathematics and linkages between those beliefs and the teaching of mathematics, although Skott (2001) reminds us that those connections are not linear. Nevertheless, school mathematics gets defined at least in part by what the teacher believes about mathematics. Lloyd (1999) vividly demonstrated this in her study of two high school teachers' conceptions of a reform-oriented mathematics curriculum. The fact that one teacher saw the materials as too structured while the other teacher saw them as too open-ended speaks to the issue of the influence of teachers' beliefs on interpreting curricula. Further, beliefs can both impede and facilitate teachers' adopting a more process-oriented teaching style. Witness the case of Mr. Burt (Wilson & Goldenberg, 1998), who modified his teaching toward a more conceptual mathematics but maintained a teacher-centered classroom. His beliefs about mathematics may have been influenced, but his beliefs about how that mathematics should be taught remained steadfast.

More generally, the literature on teachers' beliefs indicates that teachers tend to hold rather limited views of mathematics, seeing it as something cut and dried, either right or wrong, and not subject to human fallibility. This is at the heart of Gellert's (2000) analysis of German preservice elementary teachers' views of using activities with young children. Fundamentally, the teachers wanted to protect children from their own tension-provoked mathematical experiences. Wittmann (1992) argues that achieving a more process-oriented teaching style requires the teacher to overcome his or her own formalistic view of mathematics – a view deeply rooted in the teacher's personal mathematical experiences. The ability of the teacher to overcome the "broadcast metaphor" of teaching mathematics is not easy, but must necessarily be grounded in a more pluralistic view of mathematics.

Green's (1971) metaphorical analysis of beliefs suggests that beliefs are grounded in different ways and are held with different levels of intensity. Some beliefs are what Green calls *nonevidential:* they are based on other beliefs essentially grounded in what an authority has proclaimed as truth. Nonevidentially held beliefs are unlikely to change. Other beliefs are held evidentially: they are grounded in evidence and, consequently, subject to change should the evidence change. [For a more detailed analysis of the implications of Green's analysis applied to research on teachers' beliefs, see Cooney , Shealy, and Arvold (1998).] Suffice it to say here that some teachers' beliefs are subject to re-examination when different experiences are provided and reflected upon. Langford and Huntley (1999) demonstrated this in their work with preservice secondary teachers when the teachers participated in internships in which they used mathematics to solve real world problems and subsequently re-examined their beliefs about mathematics. Steele (2001) suggests beliefs are more likely to change in the presence of supporting curricula, collegial support, and the provision of opportunities to explore mathematics from a pluralistic perspective. This kind of support allows teachers to share ideas and compare experiences with their colleagues, allowing them to consider various kinds of evidence about what constitutes effective teaching. This kind of examination promotes evidentially held beliefs that can form a foundation for teacher change.

Knowing about the Teaching of Mathematics

The reformulation of beliefs about the teaching of mathematics parallels that of reformulating beliefs about mathematics. Preservice teachers are not *tabula rasa* when it comes to their beliefs about the teaching of mathematics. The combined studies of Ebby (2000), Mewborn (2000), and Grant, Hiebert, and Wearne (1998) strongly suggest that much can be gained from teachers observing reform-oriented teachers. Observers, however, must be positioned to appreciate what they observe; , should teachers' beliefs about mathematics and the teaching of mathematics run counter to a particular teaching reform, their observations are unlikely to have much impact on them. . Simply put, what gets observed is what one is willing to observe.

There is also substantial evidence that teachers need considerable support in order to successfully reconceptualize their teaching style. Krainer (1999) makes the case that teachers need "critical friends" and a supporting environment to realize change. Nelson's (1998) conclusion that reform requires a change in the intellectual culture of the school implies that inservice programs need to involve those who have responsibility for overlooking the instructional program. Too often, teachers are what Krainer calls "lone fighters" who seldom achieve change in the face of institutional inertia.

The professional development program of Borasi et al (1999) is a case in point. Their program demonstrated that teachers and university personnel needed to collaborate as the teachers struggled to infuse their teaching with inquiry-based curricula. Two additional points should be made. First, the innovative curricular materials dealt with the content taught by the teachers but from an inquiry-based perspective. Second, the collaboration lasted from a summer program throughout the subsequent school year, so that the teachers received continued support during the actual teaching process. The importance of longevity and collaboration was also evident in the study by Edwards and Hensien (1999).

Successful professional development programs engage teachers in reflection, allowing them to reconsider their existing practice without creating an external threat. Jaworski (1998), for example, engaged teachers in a kind of action research in which the teachers, working in conjunction with Jaworski, created a research agenda that set goals for their own teaching and then reflected on how they might best achieve those goals. The teachers were largely responsible for setting the research agenda; Jaworski's role was to encourage them to reflect on practice and consider possible ways of achieving their goals. An essential part of the program of Borasi et al. (1999) was also encouraging teachers to reflect on practice, as was the case for Chapman's (1999) teachers as they reflected on their personal experiences with problem solving.

Several facts become apparent when reviewing research about teachers' efforts to change their practice. First, it takes time. Short summer programs have little hope of having much impact beyond providing information. Second, teachers need to be in a collaborative environment in which they can share their experiences with their colleagues and with their mentors, who act as "critical friends." Third, reflective thinking is essential to the re-examination of beliefs and possible changes in practice. In almost every program that demonstrated changes in teachers' beliefs or practice, reflection constituted a fundamental element.

Three Principles for Teaching Mathematics to Teachers

Cooney and Wiegel (in press) identified the following three principles for teaching mathematics to preservice teachers.

1. Preservice teachers should experience mathematics as a pluralistic subject;
2. Preservice teachers should explicitly study and reflect on school mathematics; and
3. Preservice teachers should experience mathematics in ways that support the development of process-oriented teaching styles.

These principles were grounded in much of the literature previously cited. Although the principles were primarily aimed at preservice teacher education programs, they seem equally applicable to inservice programs.

The first principle is fundamental to any kind of reform movement. Too often, mathematics is seen as a harsh and abstract terrain, devoid of the human circumstance. What does it mean to teach mathematics from a pluralistic perspective that embraces human invention? Basically, it involves seeing mathematics as an empirical science and envisioning connections within mathematics and between mathematics and the real world. This is not to exclude mathematical formalism, but rather to include other kinds of mathematical experiences. It encourages the student/teacher to solve problems using multiple methods and to engage questions to which there is not a single right answer. The provision of these kinds of mathematical experiences need not be limited to advanced mathematics. Young children's mathematics involves counting objects and playing games in which mathematics is part and parcel of the experience. The same approach can be applied at higher levels. Algebra students can solve simple equations using a variety of tools. For example, every algebra student is required to solve equations such as the following:

$$2x - 5 = 7 - 4x \qquad\qquad x^2 + x = -12$$

The task is usually housed in the context of applying certain algebraic procedures aimed at determining the value of x. Imagine how the concept of equation, and of mathematics more generally, might be altered by solving these two equations using a graphing calculator in which the student graphs each member of the equation and searches for the intersection of the graphs or uses a spreadsheet in which values for x are assigned in an effort to determine that value for which the right member is equal to the left member. The point is not that these methods are easier (although they might be, depending on the equation). Rather it promotes a broader conception of what it means to solve equations.

Another aspect of mathematical pluralism involves the posing of open-ended questions that require communication beyond the generation of a single number. Consider the questions below, taken from Cooney, Sanchez, Leatham, and Mewborn (2002).

1. Two bugs start walking at point A on a circle. The first bug takes about 20 seconds to walk around the circle and return to point A. The second bug walks along the diameter of the circle from point A to point B (on the other side of the circle) and then back to point A along the same path. If they walk at the same rate, is it possible to determine how long it takes the second bug to walk its path? Explain why or why not. (figure omitted; see Cooney et al., 2002, p.)

2. Ruanda has made 9 out of 16 free throws so far this basketball season. Her teammate Lisa has shot fewer free throws. Could the percentage of free throws Lisa has made be the same as the percentage of free throws Ruanda has made? Why or why not?

3. Write a rational function whose graph could be the one below. Explain why your equation could have the given graph.

4. Two cylinders have equal surface areas. If both are filled completely, is it possible for them to hold different amounts of liquid? Why or why not?

Students often become accustomed to thinking mathematics questions require the generation of a single number, for example, when solving an equation. The questions above require more than the generation of a single number and, furthermore, have no set procedure on how they should be answered. (For more open-ended questions and students' responses to such questions, check the following website: www.heinemann.com/math.)

The evidence from research is strong that teachers can not only learn mathematics but also generate ideas on how to teach that mathematics from studying the mathematics of their workplace. An often missing link in mathematics teacher education is an emphasis on the mathematics of the classroom. Consequently, the second principle, that preservice teachers should explicitly study and reflect on school mathematics, should be central to teacher education programs.

Lastly, mathematics must be taught in such a way that that the student – the prospective teacher – has a model for teaching mathematics that goes beyond the learning of the content itself. Cooney, Brown, Dossey, Schrage, and Wittmann's (1999) approach to integrating content and pedagogy is one attempt to provide a curricular base for such teaching. The notion of integrating content and pedagogy is essential to any attempt to model and promote reform-oriented teaching styles. It is one thing to teach needed mathematics to teachers, but how that mathematics gets taught is quite another thing. That is, the delivery of the mathematics (the medium) is just as important as the content (the message). The deficiency mode of teacher education, a mode that delivers information to teachers in a way that discourages — or even subverts — reflective analysis, is not likely to provide a foundation for reform. Reflection and the generation of evidentially held beliefs hold the key to teacher change.

The commonalities among the various studies cited combined with the three principles explicated above can provide a foundation for developing a teacher education program that supports reform. But how does or should this reform be defined in the context of rural Appalachia? An attempt to address this question is offered in the next section.

The Rural Circumstance

Scholars of rural education tend to be pessimistic about prospects for promoting reform in rural schools. In a key synthesis, Stern (1994) reports a bleak picture of rural America. In addition to the "significant levels of poverty, with many rural citizens ill-prepared to met the challenge of the modern economy" (p. 3), she reports that meager job opportunities prevail in much of the rural U.S. for skilled and well paying jobs. With respect to schools, the synthesis found rural teachers "to be younger, less experienced and less likely to have completed advanced degrees than those in nonrural schools" (p. 39). Further, curricula in rural schools were found to be more limited, with fewer advanced courses than curricula in nonrural schools (Stern, 1994).

Gruchow (1995), writing in a rather resigned tone, concluded that rural children learn that "their parents were expendable and that their duty is to abandon their dreams and to become cogs in the industrial machine" (p. 98). The implication is that if rural children expect to amount to anything, they had better leave home. Orr (1997) sees this circumstance as particularly tragic in that "those who do comprehend our plight intellectually cannot feel it and hence are not moved to do much about it" (p. 95). This perspective is echoed by Howley (1997) who opined that university types may not be the best sources for approaching matters differently in rural education given that "*they are not usually here, or for very long* (emphasis in original)" (p. 133).

Howley (1997) argues that there exists a certain anti-intellectualism that permeates perceptions of matters rural, a circumstance that denigrates what might otherwise be valued and celebrated in rural America. Reliance on machines and now computers has served to "make the culture stupid" (p. 133). Further, Howley claims that school improvement is moved more by societal circumstances including technology than by research. Howley advocates a certain energizing of the intellectual environment in rural communities by appreciating, among other things, the inventiveness of handiwork and, in general, urges researchers and school leaders to embrace an "*interest in matters and minds rural*" (emphasis in original) (p. 136). Howley questions whether rural schools, as presently constituted, can serve as birthplaces for effective reform.

Gruchow's (1995) apparent resignation about the "colonialization of rural America" would suggest that putting an emphasis on matters and minds rural will not be easy. Experts, according to Gruchow, are of little help as local people unwittingly replace their wisdom with that of the supposedly wiser expert. Whether the rural population is "bloated with conventional wisdom and have forgotten how to question" (p. 74) or whether they are "too tired of the world to think for themselves, they will delegate the responsibility for deciding to those who proclaim themselves authorities" (p. 74).

These perspectives raise the specter of a certain inertia that does not bode well for school reform. But, as one of my professors once said, "Don't do nothing." Let me begin by considering the implications of these circumstances for the teaching and learning of mathematics.

The Relevance of Pedagogy

Because of the abstract nature of mathematics, the subject is often considered culture free, unperturbed by social circumstances. In fact, this is not the case. At the philosophical level, Lakatos (1976) and Davis and Hersh (1981) have argued for the fallibility of mathematics, the implication being that mathematics is a matter of human invention. Unfortunately, the teaching of mathematics is usually marked by formalism, perhaps grounded in the very development of the subject. Indeed, Wittmann (1992) has argued that one of the greatest difficulties associated with overcoming the "broadcast metaphor" of teaching mathematics is the very nature of mathematics itself.

The issue of fallibility in mathematics runs deep, but its implications are not solely limited to scholars concerned with the philosophy of mathematics. When Davis (1999) posed the question, "What is mathematics?" to his preservice secondary teachers, the immediate response was one of the teachers searching for some definition previously learned. It was as if the teachers' personal mathematical experiences were irrelevant; they could not fathom a response grounded in their experiences. Rather, they searched for one proclaimed by some authority. And so it is that mathematics is often taught. Mathematics, so perceived, is not a matter of the mind but of the memory.

Unfortunately, this perspective masks many important issues. First of all, whose mathematics should be the mathematics of the schools? At first glance this may not seem like a reasonable question. Mathematics is determined by the state or the local school system. But a more penetrating analysis suggests that the mathematics that gets taught is determined by the expectations of a local community. Donovan (1990), for example, found that working class parents supported the acquisition of basic skills with a limited pedagogy to match. By contrast, middle class parents valued students being engaged in rich learning experiences that transcended the learning of only basic skills. Wilson and Padron (1994) made a broader point in that the mathematics of the classroom should include mathematics from all origins of development and not simply from the Western world that dominates most textbooks today.

Two points seem particularly relevant. First, there is a tendency for parents to define school mathematics in terms of their own school mathematical experiences. For many, and particularly those less well educated, this puts a severe ceiling on what gets taught and learned. Second, the pedagogy associated with limited outcomes is usually formalistic and driven by an adherence to authority, viz., teacher telling. It is interesting to note that Freire (1970) speaks of the "pedagogy of the oppressed" as being that pedagogy that is grounded in a false sense of paternalism in which the better teacher is believed to be the one who best fills the receptacles of the mind. As he puts it, "knowledge is a gift bestowed by those who consider themselves knowledgeable upon those whom they consider to know nothing" (p. 58). By contrast, the raison d'être of libertarian education lies in its ability to enable the individual to transform society. Freire's point is that the pedagogy of the oppressed aims to transform the consciousness of the oppressed rather than transform the situation that oppresses. This is not to equate the rural circumstance with the societal conditions of which Freire speaks. Nevertheless, there is a certain connection among the view of mathematical infallibility, a pedagogy that oppresses, and an adherence to authority that blunts one's ability to question.

Green (1971) makes a distinction between indoctrination and teaching. The former is a matter of providing students information without rationale. By contrast, the latter is a matter of allowing students to reach conclusions based on evidence that goes beyond the proclamations of authorities. For example, stating the Pythagorean Theorem for students without an accompanying justification constitutes indoctrination; enabling students to see that the Pythagorean Theorem is a reasonable conclusion based on either empirical or axiomatic evidence constitutes the teaching of the theorem. This distinction speaks directly to the kind of pedagogy Freire (1970) is talking about when he differentiates pedagogy for freedom and pedagogy for oppression. Dewey (1916), in perhaps a less dramatic but no less important way, argues that education is the backbone of a democracy in that it is through education that individuals are empowered to control their own destinies, and consequently, the destiny of society. For Dewey, indoctrination as Green defines it presents a moral challenge because it leads to what Rokeach (1960) calls a *closed mind*, a mind incapable of transforming society.

A few years ago, I encountered a movie clip of an Abbott and Costello routine in which the focal point was the division problem 28 ÷ 7. The "discussion" centered on whether the answer was 13. The following "rationale" was presented and "debated" by the two comedians.

Since 7 won't go into 2, we have to take 7 into 8 as shown below.

$$\begin{array}{r} 1 \\ 7\,\overline{)\,28} \\ \underline{7} \\ 1 \end{array}$$

The 2 from 28 is brought down, and 7 is divided into 21 (three times) which then gives an answer of 13.

$$\begin{array}{r} 13 \\ 7\,\overline{)\,28} \\ \underline{7} \\ 21 \\ \underline{21} \\ 0 \end{array}$$

The check consisted of adding seven 13's as follows. First, add the 3's (going up on the right side) and then add the 1's (going down the left side). The result is 28 thus "verifying" that 28 ÷ 7 = 13!

	←		←	
	22	13	21	
	23	13	18	
↓	24	13	15	
	25	13	12	↑
↓	26	13	09	
	27	13	06	↑
	28	13	03	

The "discussion" made for great entertainment as one would expect from such talented comics.

Now suppose a teacher were to teach this "division procedure" to students. Would we be confident that students would reject this approach and show the absurdity of the answer? One is reminded of Brownell's (1956) classic interview with two fourth graders who were asked, "How many times are five times nine?" When one child responded that the product was 45 and Brownell countered with the statement that the answer should be 46, the child indicated that that was not the way she had learned it. The second child, when posed the same question and counterpoint, queried, "Are you kidding?" and then proceeded to show Brownell a column of five 9's with the directions to count them, ensuring that Brownell arrived at 45 – demonstrating again the difference between indoctrination and teaching. .

Addressing the Issue of Reform

It seems safe to assume that scholars who focus on rural education and those who write about education more generally share a certain emphasis on reasoning and making education relevant to the child. Gruchow's (1995) concern that most rural schooling breeds indifference by emphasizing only procedures demonstrated by an authority reflects the position that the Standards and most educational scholars speak against. Education is about empowering individuals to think, to question, not simply to acquire information. Orr (1997) desires a rural education that provides students reasons for wanting to know, an education in contrast to indoctrination. How is it that we can educate our teachers to teach and not to indoctrinate? The three principles cited early provide at least one foundation for doing so. Principles 1 (plurality) and 3 (pedagogy) are consonant with the kind of education that not only reflects the Standards but rural writers as well. Gerdes (1998) has demonstrated how teacher educators can take advantage of teachers' experiences by using those experiences as a basis for exploring mathematical ideas. In particular, he encouraged teachers in Mozambique to explore the (implicit) mathematics behind their skills at basket weaving. In doing so, he was able to engender teachers' appreciation of their own mathematics in ways that the teachers were initially reluctant to do, thinking that they knew little of what they perceived mathematics to be. Similarly, Presmeg (1998) encouraged her students to explore the mathematics they had encountered, often implicitly, in an effort for them to realize not only the relevance of mathematics to their lives but also the pervasiveness of mathematics. For example, one of her students explored the geometry of the Korean flag.

Gerdes' (1998) approach is far-reaching in that his goal is to demonstrate the non-neutrality of mathematics and the ethical nature of mathematics education, implicit as that may be. He concludes:

> Mathematics education may be used, consciously or unconsciously, to discriminate against certain social or cultural groups. In this sense, social and cultural-mathematical awareness relates to the ethical dimension of mathematical education, considering it an aim of mathematics education to be emancipatory and to contribute to the critical consciousness. It is for this reason that mathematics teacher educators should attend to the culture of their students and use that culture as a means of enhancing teacher students' abilities to teach mathematics. (p. 50)

Howley (1997) makes the point that "research may *inspire* or *deflate* (emphasis in original) improvement efforts" (p. 132), a point with which I agree. But it is also important to consider the moral implications of whatever defines school improvement, as suggested by Gerdes (1998). If students acquire more information, perhaps thus elevating standardized test scores, but at the price of developing an ability to question and to invent, is that improvement? School improvement is not just a matter of research nor of quantity but of creating, in Howley's words, an "*interest in matters and minds rural*" (p. 136). For all of its perceived abstractness and certainty and assumed freedom from bias, the teaching of mathematics is ever much a matter of moral decisions as the teaching of any other subject although it is seldom recognized as such.

Re-examining the Education of Mathematics Teachers in Rural Appalachia

In reading the different pieces on rural America, I was struck by the common theme of isolation, or the need to break it down. Gruchow (1995), speaking about prairies, concluded

"The prairie teaches us that our strength is in our neighbors. The way to destroy a prairie is to cut it up into tiny pieces, spaced so that they have no communication." (p. 78). Leo-Nyquist (2001) suggests that "rural teachers can overcome their feelings of isolation by learning that many others have 'been there before,' and have left written accounts of good practice that are both inspirational and useful" (p. 30). The review of the literature provided earlier similarly concludes that communication, often for the purpose of reflection, is essential for teacher change. Jaworksi (1998) provided a context for action research in which researcher and teacher communicated closely about a teacher-defined problem. Krainer (1999) talked about the importance of teachers overcoming the "lone fighter" syndrome. Borasi, et al., (1999) developed a professional development program in which teachers could share their experiences in using inquiry-based materials with their colleagues. For the most part, these domains of communication provided the fodder for reflection, the cornerstone of Schifter's (1998) program. How is it that cohorts of teachers could be formed to promote communication among teachers?

There are other questions that deserve the attention of researchers. How is it that mathematics gets defined in Appalachian schools? What do teachers believe about mathematics and how do they define their roles as teachers of mathematics? To what extent do these beliefs impede or facilitate the use of process-oriented teaching styles? What leverage can be gained so that at least the first and third principles – that preservice teachers should experience mathematics as a pluralistic subject, and in ways that support the development of process-oriented teaching styles – could be realized in collegiate mathematics courses? What kind of support can be generated for beginning teachers to help ensure their professional survival and enable them to develop critical friends, thus avoiding the lone fighter syndrome? A starting point for addressing these questions could be the generation of thick descriptions about preservice teachers' experiences as they move into their initial teaching experience and about experienced teachers who experience the isolation of teaching. Further, descriptions of inservice programs that involve entire mathematics staffs and their administrations could be useful in identifying those circumstances that seem particularly effective in promoting reform.

I believe parallels exist between mathematics teacher education more generally and mathematics teacher education in Appalachia. All teachers need the kind of mathematical insights that allows them to appreciate their students' mathematical understandings. This requires training in the knowledge of the mathematics itself (Principle 2) and in developing frameworks for understanding students' mathematics. Teachers need networks of colleagues so they can share and reflect on their experiences with their critical friends. Teachers need time to try new ideas and to adopt more process-oriented teaching styles. Finally, teachers need the support of those responsible for the school's culture so they can be assured their efforts will be rewarded and appreciated. I do not believe these circumstances are unique to any particular locale.

Education is a human activity. The renown Dutch mathematics educator Hans Freudenthal (1973) long ago argued that reform is not a matter of paper and pencil but of people. What makes a novel interesting is our identification with the characters – our sense that we are a part of the novel. In that sense, the growing number of published stories about mathematics teacher education can speak to all of us in one way or another. Our storytelling should be one of liberation in that we seek to serve humanity rather than to acquire truth in an absolute sense. Just as we should promote mathematical pluralism, our theories and orientations should also be pluralistic and eclectic in the best sense of the word. Eclecticism, if not denied, is an inherent characteristic of humans, rural or otherwise. Feyerabend (1988) claims "Science needs people who are adaptable and inventive, not rigid 'imitators' of 'established' behavioral patterns" (p. 165). Where better to be adaptive and inventive than engaging people in the study of how others learn how to teach mathematics in rural Appalachia?

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