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ABSTRACT

An evaluation was conducted to examine the extent to which the District Mathematics Plan (DMP) initiatives of the Los Angeles Unified School District (LAUSD), California, such as adopting standards-based textbooks and professional development opportunities, have led to improvement in classroom practices and student outcomes. The evaluation involved 160 elementary and secondary classrooms in 40 randomly selected LAUSD schools. Data collection consisted of direct classroom observations, interviews of teachers and administrators, and extracts of student data from the Student Information System. Data collectors used standard protocols for the classroom observations, as well as for the teacher and administrator interviews. The key evaluation method involved the combined use of quantitative and qualitative data. Results reveal that although Stanford 9 achievement test mathematics total scores improved at some grade levels following the first year of implementation of DMP, the improvement was limited to items focusing on mathematics procedures, rather than problem solving. These achievement results are consistent with the discourse norm of mathematics teaching and learning that have been observed since spring 2001 (the baseline). The reason only limited improvement has been seen in the way mathematics is being taught has to do with teachers' limited opportunities to learn new ideas about mathematics and about new teaching practices. (Contains 11 tables and 12 references.) (Author/SLD)

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DISTRICT MATHEMATICS PLAN EVALUATION

2001 - 2002 EVALUATION REPORT

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TABLE OF CONTENTS

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ABSTRACT

The focus of the current evaluation is on examining the extent to which the District Mathematics Plan (DMP) initiatives (e.g., adopting standards-based textbooks) and professional development opportunities (e.g., coaching) have led to improvement in classroom practices and student outcomes. The evaluation involved 160 elementary and secondary classrooms in 40 randomly selected LAUSD schools. Data collection consisted of direct classroom observations, interviews of teachers and administrators, and extracts of student data from the Student Information System. Data collectors used standard protocols for the classroom observations, as well as for the teacher and administrator interviews. The key evaluation method consisted of combined use of quantitative and qualitative data. Results revealed that although SAT/9 math total scores improved at some grade levels following the first year of implementation of DMP, the improvement was limited to items focusing on math procedures, rather than problem solving. These achievement results are consistent with the discourse norm of mathematics teaching and learning that we have observed since spring 2001 (i.e., the baseline). The reason why we have seen only limited improvement in the way mathematics is being taught has to do with teachers' limited opportunities to learn new ideas about mathematics and about new teaching practices.

EXECUTIVE SUMMARY

Introduction

The District Mathematics Plan (DMP), initiated in the 2001-2002 school year, aims to improve students' mathematical competencies and give all students access to Algebra. Student improvement in mathematics is dependent upon having teachers who both know the subject matter and know how their students best learn challenging materials. The DMP aims to improve student performance and teaching practice through reform of curriculum, professional development, intervention, on-going support, and assessment. The plan aligns instruction, professional development, textbooks, and assessment with the Mathematics Content Standards for California Public Schools (hereafter referred to as the Standards).

The Standards explicitly state the mathematics content that students need to acquire at each grade level, with an emphasis on computational and procedural skills, conceptual understanding, and problem solving. The Standards, with which DMP is aligned, are intended to assure that mathematics teaching and learning become more thoughtful and demanding (i.e., focus on conceptual understanding and problem solving). Many regard the goals for students to acquire computational and procedural skills as relatively easier to attain than the goals for students to obtain conceptual understanding and problem solving skills. The difficulty derives from the fact that the Standards only describe what to teach, not how to teach it. This pedagogical challenge, that is, moving mathematics teaching and learning toward intellectually rigorous instruction, is one of the central problems that DMP faces.

Such a movement is asking teachers to make ambitious and complex changes, which requires more than being told how to implement effective practices. Rather, teachers must take an active part in changing and have the resources to change. Research on instructional policy and classroom teaching and learning has shown that effective operation of any instructional policy depends in considerable part on professionals' learning. Teachers have to learn new views of mathematics and new approaches to math teaching in order to fully implement the DMP. Teachers' opportunities to learn (OTL) will influence their ability to adopt new beliefs and practices.

Therefore, the focus of the current evaluation report is on examining the extent to which DMP initiatives (e.g., adopting standard-based textbooks) and professional development opportunities (e.g., coaching) have led to changes in classroom practices and student outcomes.

We conducted data collection in Fall 2001 and Spring 2002 from 40 randomly selected schools across the Los Angeles Unified School District. The 40 schools consisted of 20 elementary, 10 middle and 10 high schools. Four teachers, on average, were randomly selected from each school. This yields a total of 160 teachers or classrooms, from which we collected data. Data collection consisted of direct classroom observations, interviews of teachers and administrators, and the collection of student data from the Student Information System. Data collectors used standard protocols for the classroom observations, as well as for the teacher, math coach, and administrator interviews.

Analysis of interview and observation data from all participating schools led to the following findings:

Findings

1. Following the first year of implementation of DMP, SAT/9 math total scores increased between 2001 and 2002 more than they did between 2000 and 2001 at the 2nd and 3rd grade levels. Grades 4-5 and grades 10-11 also saw slight improvement in SAT/9 matched total math gain scores following the first year of implementation of DMP. No improvement in SAT/9 matched total math gain scores was observed at grades 6-8.
2. For year 2002, SAT/9 math scores on procedural items were substantially higher than those on problem solving items at grades 1-3. SAT/9 math scores on procedural items were about the same as those on problem solving items at grades 4-5, 7; whereas the scores on procedural items were moderately lower than those on problem solving items at grades 6 and 8.
3. For year 2002, the percent of students who performed at the proficient or advanced levels on the California Math Standards Test ranged from 10.1% to 34.1% across grades 2-11. The percent of students who performed at the proficient or advanced levels on the California Math Standards Test declined with each increase in schooling level: on a percentage basis, more elementary school students performed at the proficient or advanced levels on the California Math Standards Test than students at middle schools, who in turn, outscored high school students.
4. Younger students outscore their older peers at every math level. In Algebra 1, more 8th grade students achieve proficiency (22%) than 9th (7.9%), 10th (4.7%) or 11th (5.5%) grade students. This pattern holds for Geometry: 8th (48%), 9th

(22.8%), 10th (7.3%), and 11th (4.8%); and for Algebra 2: 9th (28%), 10th (19.7%), and 11th (5.4%).

5. Classroom observations indicated that the discourse norm of mathematics teaching and learning (i.e., the way mathematics knowledge is presented, the roles teacher and students played, the way they interact about mathematical knowledge in classrooms) remained remarkably stable during the first year of implementation of DMP. In other words, mathematics instruction continues to put a heavy emphasis on computational and procedural skills, instead of conceptual understanding and problem solving.
6. Teachers reported most off-site district-sponsored professional development workshops in mathematics focused on how to use the new textbook series. Teachers' descriptions of their experiences indicate that they may have not yet acquired the skills and understandings intended from participation in these activities.
7. Teachers reported only limited involvement with other ongoing professional development opportunities such as on-site staff development workshops, instructional leadership from administrators (e.g., classroom observations that focused on the quality of instruction), and teacher involvement in coaching practices.
8. Teachers predominantly reported positive attitudes towards mathematics and a high level of confidence in their subject knowledge and pedagogy. In contrast, math coaches expressed less confidence in teachers' content knowledge and ability to use different teaching strategies. Administrators' confidence in

teachers' content knowledge and ability to use teaching strategies lay between that of the teachers and of the math coaches.

9. Math coaches reported teachers' resistance to change, teachers' lack of trust, and time constraints as the top three barriers to coaching practice. Teachers, coaches, and administrators expressed concern over the lack of time, particularly for those math coaches who had to work with more than one school.
10. Math coaches' confidence in their coaching skills was not as high as their confidence in their math content knowledge and in their ability to use different teaching strategies.

Conclusions

In summary, although there has been a trend of improvement in students' SAT/9 math total scores at some grade levels following the first year implementation of DMP, there are still many challenges to improving mathematics teaching and learning in the direction of conceptual understanding and problem solving. One of the most difficult challenges will be how to engage and motivate teachers to adopt new beliefs and practices through providing them with ongoing professional development opportunities that will enable them to learn something of lasting value (not merely focusing on textbooks). Unless this challenge is overcome, it will be difficult to accomplish the DMP's goal of improving students' mathematical competencies and providing all students access to Algebra.

Next Steps

Our future data collection will continue to investigate the influence of the initiatives outlined within the District Mathematics Plan on mathematics teaching and learning in the district. Specifically, we will attempt to address the following areas in-depth:

- The quality and content of various ongoing professional development activities
- The impact of these activities on teaching practices and student achievement
- The students' experiences of learning mathematics
- The evidence of change (if any) in teaching practices and student performance and how this is related to DMP's initiatives and teachers' opportunities to learn through various DMP professional development activities

DISTRICT MATHEMATICS PLAN EVALUATION: 2001-2002 REPORT

This report presents the findings of a districtwide evaluation focusing on the first year implementation and impact of the District Mathematics Plan (DMP). Last year's report, the baseline report of this five-year evaluation, provided several important insights into math instruction in the district and suggested problem areas that need immediate attention and sustained effort in order to align instruction with the goals of the Mathematics Content Standards for California Public Schools and the District Mathematics Plan. The present evaluation examines how the District Mathematics Plan and initiatives outlined within DMP influence mathematics teaching and learning in the district in relation to baseline results.

The report consists of four sections. The introductory section provides the background of the evaluation, the evaluation model and its theoretical perspectives, and the research questions. Section II describes the evaluation methodology, including sample section, data collection (e.g., classroom observations and interviews), and data analysis (qualitative and quantitative). Section III presents the findings. Section IV discusses the implications of the findings.

INTRODUCTION

Background

In 2001, Los Angeles Unified School District (LAUSD) adopted a five-year math plan that aims to improve mathematics teaching and learning. The plan aligns instruction, professional development, textbooks, and assessment with the Mathematics Content Standards for California Public Schools. Highlights of the plan include:

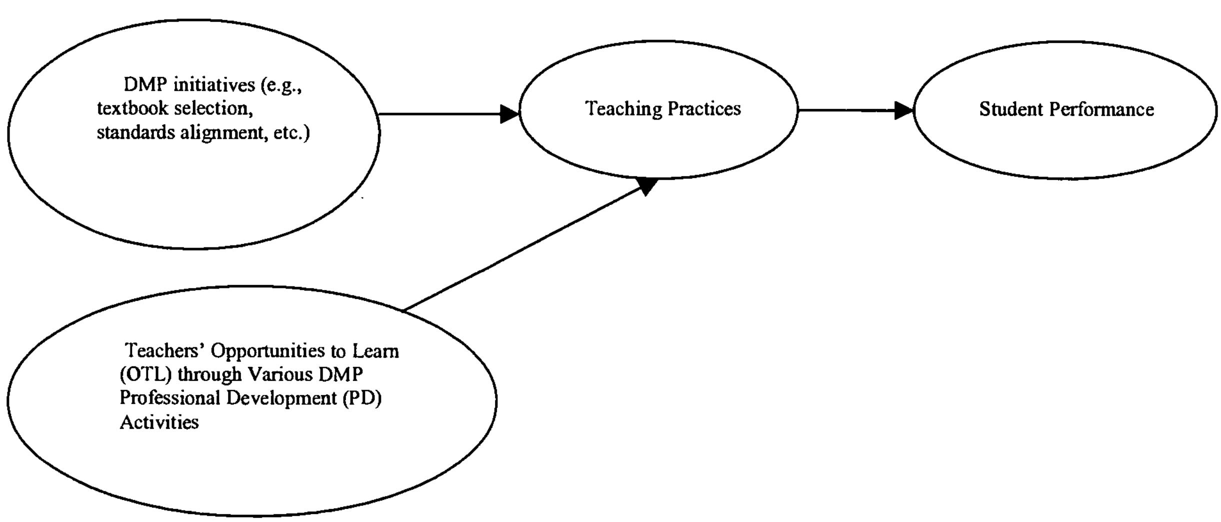
- Aligning new textbooks for grades K through Algebra with the California State Standards.
- Requiring all students to take Algebra and to pass the High School Exit Exam in order to graduate.
- Hiring math coaches to support classroom teachers in math.
- Offering intensive teacher training through the Governor's Institute, Publisher Workshops, and AB1331.

Evaluation Model and Theoretical Perspectives

In the baseline report (Ai, 2002), we pointed out that our central challenge is how to document a clear path of influence that extends from the program and initiatives outlined within the district mathematics plan to student outcomes. We rely on data collected via direct classroom observations to gauge the influence of the district mathematics reform. We focused on the dimensions of teaching practice most likely to result in changes in student learning.

First, direct observation of teaching practice provides us with data that informs us of the types of intellectual tasks teachers ask of students and the methods they use to translate content into student learning. Other data collection methods such as teacher self-report of teaching practice would be less direct and of questionable quality (Aschbacher, 1999; Clare, 2000; Kennedy, 1999; Mayer, 1999). Secondly, teaching is a multidimensional practice. Studies on reform and teaching (e.g., Spillane & Zeuli, 1999) have suggested that some dimensions of this practice appear to be more responsive to reform than others. Specifically, there is evidence that teachers revise dimensions of their practice, including materials used and grouping arrangements, more readily and more dramatically than dimensions such as discourse norms and academic tasks (Spillane & Jennings, 1997). The second consideration in our evaluation scheme, therefore, was to focus on the dimensions that reflect more complex and sophisticated understandings of classroom instruction. These dimensions have been shown to be linked to student learning and ability to engage in rigorous intellectual work within a subject. Furthermore, these dimensions are important concerns for both the Mathematics Content Standards for California Public Schools (hereafter referred to as the Standards) and the DMP.

Such a movement is asking teachers to make ambitious and complex changes, which requires more than being told how to implement effective practices. Rather, teachers must take an active part in changing and have the resources to change. Research on instructional policy and classroom teaching and learning has shown that effective operation of any instructional policy depends in considerable part on professionals' learning. Teachers have to learn new views of mathematics and new approaches to math teaching in order to fully implement the DMP. Teachers' opportunities to learn (OTL) will influence teachers' ability to adopt new beliefs and practices. Our evaluation model can be graphically shown as follows:



In the above model, students' mathematics performance is the ultimate dependent measure of DMP, and teaching practice is both an intermediate dependent measure of DMP and an independent factor that exerts direct influence on students' performance. To a great extent, teachers' opportunities to learn what DMP implies for instruction through ongoing professional development will influence directly their instructional practices, and thus affect students' performance.

Therefore, the focus of the current evaluation report is on examining the extent to which DMP initiatives (e.g., adopting standard-based textbooks) and professional development opportunities (e.g., coaching) have led to improvement in classroom practices and student outcomes.

Research Questions

The guiding questions for this evaluation include:

1. To what extent has the first year implementation of DMP demonstrated improved student outcomes?
2. To what extent does the alignment of textbooks with the State Standards influence teaching practices?
3. To what extent are teachers able to connect what they have learned through various DMP professional development activities (e.g., coaching) to their teaching?
4. Under what conditions is DMP effective in working towards meeting its goals?

METHOD

Sampling Procedure and Sample

The sampling procedure used to select our original sample proceeded in two stages. First stage selection consisted of probabilities proportional to size selection of schools based on school enrollment. A total of 40 schools were selected in this manner from separate strata defined by schooling level: 20 elementary schools, 10 middle schools, and 10 high schools. At stage two, we randomly selected a constant number of teachers at specific grade levels within each school: two 2nd and two 4th grade teachers in each elementary school, four 8th grade teachers in each middle school, and four 10th grade teachers in each high school. A 10th grade math teacher was defined as having more than 50% 10th grade students enrolled. This resulted in 160 teachers (or classrooms). The resultant sample is an equal probability sample of teachers/classrooms within each schooling level, and our findings are generalizable to the district as a whole.

To the extent possible, we observed (and will continue to follow) the same teachers during the 2001-2002 school year. If a teacher left the school, we randomly selected a replacement teacher to observe.

Data Collection Procedures

Observations. As part of the annual data collection for the 5-year evaluation, each teacher was observed three times in fall 2001 and again three times in spring 2002. Observations lasted from 45 minutes to a little more than an hour. Before going to each school, we informed the principal of the timeframe (rather than the exact dates) of our visits. We then contacted the teachers to make sure our timeframe (a range of about three weeks) was convenient for them.

Experienced data collectors wrote detailed field notes describing: (1) teacher activity; (2) student activity; (3) math content (math problems or tasks that students are working on); (4) social organization; (5) number of students, their gender and ethnicity; (6) materials in use (e.g., textbook); (7) interactions between teachers and students, and among students; and (8) where applicable, students' solutions to the problems and/or their thinking process.

Interviews. Prior to each observation, we briefly interviewed the teacher about the focus of the lesson. In spring 2002, we conducted full interviews with each teacher, focusing on the instructional context for the observations (e.g., how was the lesson planned), teaching preparation, professional development, and teaching background.

In addition, we interviewed school administrators (principals or assistant principals in elementary and middle schools, and math department chairs in middle and high schools) and math coaches. These interviews focused on key issues such as coaching practice and professional development opportunities that had been provided to the teachers at the school.

Extraction of student performance data. We collected students' performance records from the district Student Information System. The following student performance information was extracted: (1) SAT/9 matched NCE gains in math total scores between 2000 and 2001, (2) SAT/9 matched NCE gains in math total scores between 2001 and 2002, (3) SAT/9 math NCE scores on problem solving items and on procedural items in 2002, [1] (4) performance levels on the California Math Standards Test in 2002, [2] (5) performance levels on the California content standards tests of Algebra 1, Geometry, and Algebra 2 in 2002. [3]

Data Analyses

We integrated various quantitative (e.g., student achievement) and qualitative (e.g., classroom observations and interviews) data to address important issues related to program implementation and impact during the first year implementation of DMP.

Descriptive statistics were used to explore: (1) Student SAT/9 math achievement over a three-year period, namely, the matched math total gains in the year before (i.e., between 2000 and 2001) and during (i.e., between 2001 and 2002) the first year implementation of DMP, (2) SAT/9 math NCE scores on problem solving and procedures in 2002, (3) performance levels on the California Math Standards Test in 2002, and (4) performance levels on the California content standards tests of Algebra 1, Geometry, and Algebra 2 in 2002. These analyses will help us to understand whether there has been any improvement in student achievement following the first year implementation of DMP and, if so, which area has improved and which has not.

Qualitative analyses of snapshots of classroom examples across grade levels were utilized to describe the discourse norm of teaching and learning mathematics. The theme of these qualitative analyses was related to the quality of teaching practice and its connection to students' opportunities to be engaged in mathematics learning. By

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[1] We used the subscores on problem solving and procedural items for 2002 only, because these subscores were available. Subscores on problem solving and procedural items were available only for the elementary and middle school grade levels.

[2] We presented the performance level data for spring 2002 only, because performance level data for the California Math Standards Tests were given only in spring 2002.

[3] The scale of the California Math Standards Tests performance levels is as follows: 1 – far below basic, 2 – below basic, 3 – basic, 4 – proficient, and 5 – advanced.

examining how teachers present or transfer the content knowledge to students via qualitative analyses of what they normally do about mathematics on daily basis, we are able to add meaning to the numbers (i.e., scores) on the standardized tests and understand at least partly where these numbers (or results) come from. In other words, these analyses will provide some contextual explanations of student math achievement scores.

Descriptive statistics and qualitative analyses of interview data were employed to examine issues in teachers' opportunities to learn through their participation in various professional development activities. The combined use of descriptive statistics and qualitative analyses will help us to understand not only the extent of teachers' participation in various professional development activities, but also the experiences of their participation. Examinations of the extent and experiences of teachers' participation in various professional development activities will help us understand whether or not teachers have benefited from these activities.

Finally, the combined use of descriptive statistics and qualitative analyses was again deployed to address conditions that may facilitate or obstruct DMP's effort to meet its ultimate goal of improving mathematics teaching and learning in the district. These analyses help us to comprehend the complexities and challenges that are imbedded the implementation of DMP, the understanding of which will provide bases for our continued effort in ensuring effective program implementation in the years to come.

RESULTS

The current evaluation examines the extent that DMP initiatives and professional development opportunities have led to improvement in classroom practices and student outcomes. In line with our theoretical evaluation model, the current report attempts to address four key questions, the results of which are presented in the following sections.

To what extent has the first year implementation of DMP demonstrated improved student outcomes?

Following the first year implementation of DMP, SAT/9 math total scores increased between 2001 and 2002 more than they did between 2000 and 2001 at the 2nd and 3rd grade levels. Other grade levels (except for the 7th grade) had about the same NCE gain in math between 2001 and 2002 as they did between 2000 and 2001. SAT/9 math NCE scores on procedural items in 2002 were substantially higher than those on problem solving items at grades 1-3, the grade levels that scored above the national average on SAT/9 math test in 2002. Despite these gains, only small percentages of students performed at the proficient or the advanced level on the California Math Standards Test or on the California Standards Tests in Algebra 1, Geometry, and Algebra 2.

<u>How do SAT/9 matched gains in math compare with prior year's performance?</u> First, we examined students' matched SAT/9 NCE gain scores in math between 2000 and 2001 and the gains between 2001 and 2002 by grade level. Table 1 displays the average SAT/9 matched NCE gains (math total scores) in the year before and after the implementation of DMP by grade level.

Table 1: Students' SAT/9 Matched NCE Gains: Math Total Scores (2000 – 2002)

Grade Level	Matched Gain 2000 – 2001	Matched Gain 2001 – 2002
2	2.24	5.09
3	2.52	3.70
4	-1.44	-1.03
5	2.05	2.26
6	2.60	2.28
7	-1.15	-2.12
8	.55	.15
9	3.70	3.66
10	-3.79	-3.11
11	.09	.62

Following the first year implementation of DMP, SAT/9 math total scores increased between 2001 and 2002 more than they did between 2000 and 2001 at the 2nd and 3rd grade levels (see Table 1). At the 2nd grade level, the SAT/9 matched gain in math between 2000 and 2001 was 2.24 NCEs, whereas the gain between 2001 and 2002 was 5.09 NCEs, an almost 3 NCE improvement. At the 3rd grade level, the SAT/9 matched gain in math between 2000 and 2001 was 2.52 NCEs, whereas the gain between 2001 and 2002 was 3.7 NCEs, a better than 1 NCE improvement.

Except for the 7th grade, other grade levels had about the same gain in math between 2001 and 2002 as they did between 2000 and 2001. At the 7th grade, SAT/9 matched gain in math between 2000 and 2001 was –1.15 NCEs (i.e., a little more than 1 NCE loss), whereas the gain between 2001 and 2002 was –2.12 NCEs (i.e., a little more than 2 NCE loss), roughly a 1 NCE decrease in average SAT/9 gain following the first year implementation of DMP.

Examinations of students' matched gains in SAT/9 total math scores, therefore, suggested a trend of improvement for the 2nd and 3rd grade levels, whereas no improvement in SAT/9 matched gain in math was observed for other grade levels.

What were students' SAT/9 scores in math problem solving and procedures? Next, we examined SAT/9 math scores on problem solving and procedural items in 2002 for grade levels where such information was available. Table 2 summarizes the average SAT/9 math NCE scores on problem solving and procedural items by grade level.

Table 2: Students' Math NCE Scores: Problem Solving and Procedures (2002)

Grade Level	Problem Solving	Procedures
1	48.12	55.82
2	49.74	56.22
3	51.17	53.80
4	48.94	49.38
5	47.29	46.56
6	48.07	44.57
7	43.44	41.64
8	44.13	39.10

At grades 1-3 that scored above the national average on the math test in 2002, SAT/9 math NCE scores on procedural items were much higher than those on problem solving items (see Table 2). At grades 4-5 and 7, SAT/9 math scores on procedural items were about the same as those on problem solving items in 2002; whereas at grades 6 and 8, the scores on procedural items were moderately lower than those on problem solving items.

What were students' performance levels on the California Math Standards Tests? Finally, we examined students' performance levels on the California Math Standards Tests and the California Standards Tests in Algebra 1, Geometry, and Algebra 2. Table 3 displays the percentages of students who reached the proficient or advanced levels by grade level.

Table 3: Students' Performance on the Math Standards Tests and the California Standards Tests in Algebra 1, Geometry, and Algebra 2: Percent Proficient or Above (2002)

Grade	Math	Algebra 1	Geometry	Algebra 2
2	34.1			
3	30.7			
4	30.9			
5	19.8			
6	18.3			
7	16.1			
8	12.6	22.0	48.0	
9	10.1	7.9	22.8	28.0
10	10.5	4.7	7.3	19.7
11	10.6	5.5	4.8	5.4

For year 2002, the percent of students who performed at the proficient or advanced levels on the California Math Standards Test ranged from 10.1% to 34.1% across grades 2-11. The percent of students who performed at the proficient or advanced levels on the California Math Standards Test declined with each increase in schooling level. On a percentage basis, more elementary school students performed at the proficient or advanced levels on the Math Standards Test than students at middle schools, who in turn, outscored high school students.

Younger students outscore their older peers at every math level. In Algebra 1, more 8th grade students achieve proficiency (22%) than 9th (7.9%), 10th (4.7%) or 11th (5.5%) grades students. This pattern holds for Geometry: 8th (48%), 9th (22.8%), 10th (7.3%), and 11th (4.8%), and for Algebra 2: 9th (28%), 10th (19.7%), and 11th (5.4%).

To summarize, results presented to this point suggest that although SAT/9 math total scores went up between 2001 and 2002 more than they did between 2000 and 2001 at the 2nd and 3rd grade levels, the improvement was mostly limited to scores on procedural items rather than scores on problem solving items. Despite these gains, only small percentages of students performed at the proficient or advanced levels on SAT/9 math tests or the California standards tests in Algebra 1, Geometry, and Algebra 2. The majority of the students across all grade levels were performing below the proficient level. These findings are consistent with the discourse norm of teaching and learning we continue to observe in mathematics classrooms; that is, mathematics teaching and learning still put heavy emphasis on procedural skills rather than on problem solving and conceptual understanding. Students are rarely given the opportunities to communicate about mathematics, to reason mathematically, to make mathematical arguments, to see the connection between different mathematical concepts/ideas, or to understand the relevance of mathematics in their daily lives.

To what extent does alignment of curriculum (i.e., textbooks) with the Standards influence teaching practices?

In the baseline report (Ai, 2002), we pointed out that mathematics instruction evidenced a disproportional emphasis on computational and procedural skills rather than on conceptual understanding and problem solving. The Standards, with which DMP is aligned, are intended to assure that mathematics teaching and learning become more thoughtful and demanding (i.e., more focused on conceptual understanding and problem solving). Our observation data described mathematics teaching and learning as basic, lacking intellectual demands. Advancing these aspects of practice continued to be one of the most challenging tasks that DMP faces. From the snapshots of classroom examples across grade levels, we will show that the discourse norm of mathematics teaching and learning (i.e., the way mathematics knowledge is presented, and the roles of teacher and students assumed through the way they interact around this knowledge in classrooms) remains remarkably stable following the first year implementation of DMP.

Our purpose of describing the discourse norm of mathematics teaching and learning in these classrooms is not to criticize current teaching practices. Rather, our intention is to show the limited progress we have made toward fundamentally changing the way teachers and students interact about mathematics in the classroom.

Inside Five Classrooms: Following the teaching of a basic mathematics idea across the mathematics curriculum

The first series of examples were taken from three different elementary classrooms (two 2nd grade and one 4th grade). The focus of instruction in these three classes at the time of our observations was on a basic mathematics idea, namely the rate for composing a higher value unit and its relation to regrouping in subtraction. One of the 2nd grade classes was observed about three weeks ahead of the other 2nd grade class. This allows us to get a sense of the progression in the teaching and learning of the topic at the same grade level (i.e., 2nd). Our observations of the 4th grade class enabled us to see the application of the same basic mathematics idea at a higher level of sophistication than what was observed in the 2nd grade classes.

First 2nd grade class: Tens and ones.

The first setting consisted of a 2nd grade math class engaged in three consecutive lessons designed to elucidate the concept of place value. The goal of the first lesson, as described by the teacher, was for students to understand "what a ten is. That a ten is made up ten ones".

> T: Are we ready for math? We are moving on today with bigger numbers. Tens are bigger numbers. With bigger numbers, it's a lot easier to count by ten.
>
> [T spills a handful of colored, plastic, counting disks on the screen of the overhead projector.]
>
> T: How do I find out how many [counters] I have?
>
> Ss: Don't know.
>
> T: Why?
>
> Ss: They are all stuck together.

T: Would it be easier if I had one group of ten? Each of these is called a one. How many counters in the group?

[T counts out 10.]

T: One group of ten is equal to ten ones.

[T makes another group of 10.]

T: How many groups of ten do I have?

Ss: 2.

T: Here I have ten ones and here I have ten ones. How many groups is that?

Ss: 2.

T: How many are in these two groups?

GH: 20.

T: How many ones are in these two groups?

Ss: 20 ones.

[T makes a third group of ten.]

T: I have ten ones plus ten ones plus ten ones. How many is that?

BH: 30

T: I'm going to make another group. [Counting out the rest] Do I have enough? No. I only have seven. Let's count our tens.

T with Ss: 10,20,30

T: So, I have 30 and 7. My number is 37.

As this snapshot shows, the concept of tens and ones was stated to students as a fact. Once the rule that 1 ten equals 10 ones was established, the teacher led the class to practice repeatedly that 1 ten equals 10 ones, that 10 ones equal 1 ten, that 2 tens equal 20 ones, that 20 ones equal 2 tens, and so on. In fact, for the remainder of the class (58 minutes long), all the class did was to practice the same drill over and over again. At the end of the class, the teacher tried to summarize what they learned by asking:

T: When you look at your math words, which ones did we learn today?

Deiree: 100

Patrick: tens

Anas: tens and ones

T: We talked about tens and groups of ones. At home there's an activity to do with your family. Put the letter under your box to take home today.

Although the teacher's attempt to help the students remember the concept they just learned at the end of the lesson was well intended, the teacher had failed to achieve her stated purpose (i.e., to understand what a ten is), because the teacher never helped the students to understand the relationship between tens and ones, other than stating that 1 ten equals 10 ones and 10 ones make 1 ten. As the students' responses showed, Desiree gave an irrelevant number (100), while Patrick gave part of the number (tens). Although Anas was able to articulate that they learned tens and ones, she did not state the relationship between the two, at least in the form that was taught (i.e., 1 ten equals 10 ones and 10 ones make 1 ten). The teacher's intention was for students to know this fact, therefore, following Ana's response, the teacher could have at least given some feedback such as "We talked about tens and ones and groups of tens. We have learned that 10 ones make 1 ten, whereas 1 ten equals 10 ones."

Of course, teaching students the mere fact that a ten is made up of ten ones, or 10 ones make 1 ten without placing such a relationship in the context of how numbers are composed in the decimal system and its application in basic mathematical operations (e.g., subtraction with regrouping or addition with carrying) certainly does not help students understand why they need to learn 1 ten is made up of 10 ones, or 10 ones make

1 ten. In fact, when we tell students that 1 ten is made up of 10 ones and that 10 ones make 1 ten, we have touched upon a fundamental concept of how numbers are composed in the decimal system.

Ma (1999) in her study of teachers' understanding of fundamental mathematics in China and the United States showed that Chinese mathematics teachers who aim to teach for understanding a simple concept as regrouping would emphasize the importance of packaging all the related mathematical knowledge surrounding regrouping and make this package of knowledge explicit to students. For instance, they would emphasize that before students are exposed to regrouping, they need to learn a basic mathematics idea – the rate for composing a higher value unit. As one teacher pointed out (Ma, 1999):

> What is the rate for composing a higher value unit? The answer is simple: 10. Ask students how many ones there are in a 10, or ask them what the rate for composing a higher value unit is, their answers will be the same: 10. However, the effect of the two questions on their learning is not the same. When you remind students that 1 ten equals 10 ones, you tell them the fact that is used in the procedure. And, this somehow confines them to the fact. When you require them to think about the rate for composing a higher value unit, you lead them to a theory that explains the fact as well as the procedure. Such an understanding is more powerful than a specific fact. It can be applied to more situations. Once they realize that the rate of composing a higher value unit, 10, is the reason why we decompose a ten into 10 ones, they will apply it to other situations. You don't need to remind them again that 1 hundred equals 10 tens when in the future they learn subtraction with three-digit numbers. They will be able to figure it out on their own. [p. 10-11]

Otherwise, students may learn all the mathematical facts and/or procedures, but may never understand the underlying concepts. A simple question such as "what happens to the number as we go from 9 to 10?" could have led the students into a rich discovery process of the meaning of place value rather than the mathematically incomplete notion that 10 equals ten 1's.

During our second observation of the same class, the teacher indicated that the focus of the lesson was to "represent a number with a model. Say a number in three different ways." At the beginning of the lesson, the teacher briefly reviewed with the class that 10 ones equal 1 ten and 1 ten equals 10 ones. The teacher then began the topic of the day.

> T: Help me make a model for the number 25. [T holds up two rods. Students count with the teacher]
>
> T & Ss: 10,20
>
> T: That's 2 tens. That's my model for 20. But I need 5 more. Do I need 5 tens or 5 ones?
>
> Ss: Ones
>
> T & Ss: 1,2,3,4,5 [count out 5 ones]
>
> T: This is my model for 25 using my base 10 blocks. [T repeats the same procedure for the number 43].

After demonstrating how to model 25 and 43, the teacher called a volunteer to make a model for the number 16. Ingrid volunteered but did not know what to do when she got up.

T: Can someone help her?

Kevin: 1 ten [Ingrid put down 1 ten]

T: Right. We need to show 16.

Kevin: 6 more.

T: What are they called?

Ss: Ones. [Ingrid put down 6 ones]

The three modeling activities to this point seemed to be consistent with the teacher's stated goal that students would learn to "represent a number with a model". The purpose for such activities though was unclear. In other words, we do not know what mathematical idea(s) students were to learn from participating in such activities. After Ingrid finished modeling the number 16 with the help of her classmates, the teacher returned to the number 25.

T: We are going to make a model drawing a picture for the number 25. [She draws:]

	Tens	Ones
=	2	5

T: This is my place value chart, ones have a special place, tens have a special place. That means if I put five here, it has a special value. What is it?

Ss: 5.

T: How many tens do I have?

Ss: 10,20,2

T: 2. How many fives[4]?

Ss: 5

At this point, the teacher appeared ready to teach the concept of place value, but the teacher simply mentioned, "This is my place value chart, ones have a special place,

[4] The teacher said 5's, not 1's and never corrected her misstatement or student's misunderstanding.

tens have a special place." In this example (i.e., 25), the teacher never pointed out that 25 is a 2-digit number that consists of 2 at its tens place and 5 at its ones place. And the place (or position) of 2 or 5 determines their values. For instance, 2 is at the tens place, therefore, it represents 2 tens or 20 ones, but 5 is at the ones place, therefore it represents 5 ones. Because of this, we can decompose 25 as 2 tens and 5 ones. This would naturally lead to teacher's introduction of "say a number in three different ways". Instead, after the teacher simply made the statement about the place value chart, the teacher continued with modeling more 2-digit numbers. Then the teacher told the students to take out their math books, missing the opportunity to discuss the utility of place value.

> T: Let's look at the first model. Put your finger on it. How many tens do you see?
>
> G: 3.
>
> T: 3 tens. How many ones?
>
> Kevin: 4.
>
> T: We have 3 tens and 4 ones. What number does that make?
>
> G: 34.
>
> T: This first one looks a little different. 3 tens 4 ones = 34. Another way of saying 34 is 30 + 4. And we can write 34. We can say it in three different ways.

The teacher then led the class through the problems in the math book in the same manner. Although the students could write a number, say, 94, in three different ways as: (1) 9 tens 4 ones = 94, or (2) 90 + 4 = 94, or (3) 94, they will never understand the purpose for such exercises, because the teacher never explicitly taught the students the underlying mathematical concept, namely, the concept of place value and the relationships among different digits within the same number (in this case, 2-digit numbers).

The observations so far indicated that the mathematical ideas introduced to the students in these two lessons were taught in a fragmented way. Naturally some students may not even understand the procedural part of the concepts they had learned, as shown in the beginning of the third lesson. The teacher was reviewing with the students what they had learned in the past two days. The teacher gave several 2-digit numbers and asked the students the numbers of tens and ones in each number. When she gave the number 19 and asked how many tens, a few students yelled out 9, 19, and 10 ones; when she asked how many ones, one student gave 2 while another student said, "I forgot". The review was a nice check for student understanding, however, *nothing was done to correct student misunderstanding.*

The focus of the third lesson, as the teacher stated, was for students to "understand the difference between a digit and a number. The value of a number depends on the place value of the digit." The second part of the teacher's response makes us wonder whether the teacher herself has a clear understanding of the difference between a digit and a number. The statement that "the value of a number depends on the place value of the digit" is mathematically confusing, because comparing the values of different numbers has nothing to do with the place value. It is when we compare the values of different digits within a multi-digit number, the place value of each of the digits matters.

The way the teacher helped students to "understand the difference between a digit and a number" was as a matter of fact, as the following interaction shows:

T: Let's say I want this number. Raise your hand and tell me this number.
[T wrote on board.] 63.

B: 63.

T: And this number? [T wrote] 36.

Desiree: 3.

T: Place Value Chart

	Tens	Ones	
I have a 6 and a 3 here.	6	3	
I have a 3 and 6 here.	3		6

I have 2 digits in both. Is it the same number?

Ss: No.

Ss: Yes.

T: It is? Okay. I owe you $63 but I'm only going to pay you $36.

B: No, that's a lot of money.

T: Each numeral is called a digit. We write digits [writes] 0,1,2,3,4,5.6,7,8,9. All digits can make all kinds of numbers. We call it a 2-digit number. It has a 6 and a 3. We call this a 2-digit number. It has a 3 and a 6. We call the whole thing a number. With a raised hand, show me on your fingers, how many digits in 36?

Ss: 9, 4, 3

[T repeated the digit and number explanation.]

T: [wrote 126] How many digits?

T & Ss: [count together] 1, 2, 3.

T: It makes a difference where we put our digits on our place value chart.

As shown in the above example, the concept of digit and number was not explained well. The teacher simply stated "I have 2 digits in both [63 and 36]". No wonder when the teacher asked immediately "Is it the same number?" some students were confused and said "yes". Possibly they thought the teacher meant, "Do 63 and 36 both have 2 digits?" since the teacher had just said "I have 2 digits in both". The teacher used a money example (i.e., the difference between $63 and $36) to help students understand that the two numbers were not the same. However, instead of explaining why they are different, the teacher made a statement that digits 0 to 9 can make all kinds of numbers without really helping students to understand what this means. The rest of the lesson continued with teacher modeling 2-digit numbers and asking students "How many tens? How many ones?"

The examples from this class showed that the fundamental concept of how numbers are composed in the decimal numeral system was treated in a fragmented and disconnected way. The students may have learned that 1 ten equals 10 ones, or that 10 ones make 1 ten, or that 36 can be represented as 30 + 6 or 3 tens and 6 ones, but were not led to discover or understand the utility of place value. Students left the series of lessons without understanding the concept of place value within the decimal system. This is critical since it not only enables them to see how numbers are composed (i.e., the relationship between different digits of the same number), but also prepares them for a soon-to-be-learned topic, namely regrouping.

Second 2nd grade class: 2-digit number subtraction with regrouping

Three weeks later, we observed a second 2nd grade class for 3 consecutive days. At the time of our observations, this class was learning 2-digit numbers subtraction and regrouping. The first of the 3 lessons introduced the steps of regrouping, whereas the second lesson focused on subtraction of 2-digit numbers with 0 in the ones place. The third lesson reviewed the first two lessons. The focus of the first lesson, as the teacher stated, was to "subtract 2-digit numbers, regrouping when necessary, deciding if they need to regroup, remember the steps of regrouping".

After reviewing with the students that difference in mathematics means the correct answer to a subtraction problem, the teacher writes the word "regroup" on the board.

Ss: Regroup.

T: What do we call it when we're adding?

G: Carry

T: [writes Carry] What about take away?

Ss: Borrowing.

T: [writes: Borrowing]

Carry

Regroup

Borrowing

T: Guys, guess what? Regroup can mean either one.

[T gets cubes/blocks, puts 10 single cubes on the table]

T: My friends, how many do I have here? I'll help you count [picking up the cubes so Ss can see better]

T & Ss: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

T: How many do I have here? [Holds a 10 block]

Ss: 10

T: So which has more?

Ss: Same

T: So I can take this [holds the 10 block], saw it and get 10 little cubes, or

I can take these 10 [holds 10 single cubes], glue them together and get this [holds 10 block]

T: Carry means putting together and borrowing is taking apart. I don't know if Ms. M [the sub] explained that very well, because regrouping can mean either one.

As shown in this interaction, the concept of regrouping as applied in addition and subtraction problems was merely taught as a fact, namely, regrouping is called "carrying" when we are adding and "borrowing" when we take away (i.e., in subtraction). Furthermore, the relationship between the two different applications of the same concept was not explained but merely stated as "Guys, guess what? Regroup can mean either one." In fact, helping students to understand the concept and application of regrouping is closely related to the fundamental idea of how numbers are composed in the decimal system. Helping students to understand the connection between two different applications of the same mathematical concept (i.e., regrouping) in addition (i.e., carrying – composing) and subtraction (i.e., borrowing – decomposing) certainly would enable them to learn something of lasting value, rather than a mere fact that regrouping is called carrying in addition but borrowing in subtraction.

Although the teacher tried to use the manipulative to convey the idea that regrouping "can mean either one", she had failed to facilitate students' understanding, because the way the manipulative was used bears no connection between the visual symbols (i.e., breaking a 10 block into 10 pieces or gluing 10 pieces of cubes together) and the mathematical idea of carrying (composing) in addition or borrowing (decomposing) in subtraction. Therefore, without a conceptual understanding of how regrouping works in different situations, students have learned the concept in a procedural way. The danger of failing to achieve conceptual understanding was apparent when the teacher tried to progress, only to discover that her students were not ready to move on. She assessed student understanding, but only at the level of recitation. Indeed, some had memorized her script, though few had learned what it meant.

12:44

T: We worked on some problems like this [wrote]

$$\begin{array}{rr} 4 & 5 \\ \underline{-2} & \underline{7} \end{array}$$

T: If you don't remember then you need to pay really good attention.

T: If it helps you, you can draw a line. [Drew dotted line down between tens and ones column in the above problem]

T: What is the first thing we do in a subtraction problem? [1/2 of class raise hands] It starts with the word "look". Taylor

Taylor: Look at the ones.

T: Look at the ones. You can't even think about take away until you think about the ones. Always start on the right. Reading we go from left to right, but this is backwards from reading. We go right to left. Ste 1, what is it?

Ss: Look at the ones.

T: We are looking at subtraction. Step 1, say it.

Ss: Look at the ones.

T: [wrote and said] Look at the ones.

T: Marvin, why are we looking at the ones? What are we looking for?

Marvin: [quiet]

T: Marvin, would you like to pick someone who has their hand up? [1/2 have hands up].

Marvin: Samantha.

Samantha: To see if you need to regroup.

T: [wrote, adds to #1, Look at the ones: to see if you need to regroup]
Now what's step 1? Say it.

Ss: Look at the ones to see if you need to regroup.

T: What's step 1? Vanessa.

Vanessa: Look at the ones to see if you need to regroup.

T: What's step 1? [Pointing to another girl]

G: Look at the ones to see if you need to regroup.

T: What's step 1? Jonathan.

Jonathan: Look at the ones to see if you need to regroup.

T: How do I know? [1/2 raised hands] Keano?

Keano: [mumbles]

T: Can some one tell me in a different way? Taylor?

Taylor: Top number take away bottom number.

T: Top number take away bottom number. Say it.

Ss: Top number take away bottom number.

T: That's why we look at the ones.

Once the students firmly memorized step 1, the class moved on.

12:54

T: Everybody, let's talk to Mr. 4.

Ss: Mr. 4.

T: Ask him if you can have one of his tens.

Ss: Mr. 4, can I have one of your tens?

T: Yes, you may, but how many will you leave me? Paul?

Paul: 3.

T: [wrote 3 in the box above 4] Am I done? Marvin?

Marvin: No.

T: What do I put up here? [Pointed to the box above 5]

Ss: 15

T: [wrote 15] Why did I put 15 in this box? Fernando?

Fernando: [quiet]

T: I used to have [drew 5 circles]... Then I borrowed [drew 10 more circles]

Now I have 15. You can do it quicker by putting a 1 in front of the 5. [demonstrated].

T: We have to do all these steps just to get ready to subtract. Now we're ready to subtract.

T: Now, what am I going to subtract? Fernando, what do I subtract?

Fernando: 7

T: What am I going to take 7 away from? Don't tell me the answer, I will be very upset. Wyneil, am I taking away from 5?

Wyneil: No.

T: Daniel, what am I taking away from?

Daniel: [quiet]

T: Taylor, what am I taking away from?

Taylor: 15.

T: What's 15 take away 7?

Ss: 8.

T: [Put down 8] Am I done?

Ss: No.

T: Marvin, what do I do? [Class quiet] Everybody say it out loud.

Ss: 3 take away 2 is 1.

T [put down 1 in the tens place].

The regrouping (i.e., borrowing, decomposing) step, as shown in this example, was not taught for understanding at all, because the two digits (4 and 5) in the number 45 were treated as if they were independent neighbors. If we do not have enough ones in 5 to take away 7, we can simply go to our neighbor, Mr. 4, to borrow one of his tens. If so, what are we going to return later to Mr. 4? We have emphasized that 4 and 5 in 45 are two inter-related components of one number due to the way numbers are composed in the decimal system. In addition, when we have too many ones in the ones place, we compose them into units of 10 and put them in the tens place. Consequently, when we do not have enough ones to carry out the subtraction, we can decompose the tens back into ones. On the surface, this procedure seems similar to what was used in this class, yet the underlying concept is completely different. Composing/decomposing explains the concept underlying the algorithm, whereas borrowing a ten from Mr. 4 does not. In language that 2nd graders should be able to understand, Mr. 40 needs to change his name to Mr. 30 plus 10. This representation, which can certainly be improved upon, at least captures correctly the concept of decomposition. The merely procedural approach to conducting 2-digit subtractions with regrouping failed to help this group of students to understand the basic concept and may have reinforced their misunderstanding. After practicing the same procedure with different 2-digit subtractions, the teacher asked, "Do we always have to regroup?" The students replied, "Yes." They were so used to the procedures taught that they thought they needed to follow the same procedures in all kinds of 2-digit subtractions.

Since the concept underlying the algorithm (i.e., regrouping) was never explained to the students, 2-digit subtractions with zero in the ones place was treated as a separate topic, even though conceptually nothing new is involved in such applications. The focus of the second lesson, as the teacher stated, was "subtracting with zero in the ones place, deciding if they need to regroup, and remember the steps of regrouping." The steps that were taught in the previous lesson were used.

T: [wrote]

70

$\underline{\quad - 24}$

T: Nothing changes. What's step 1?

Ss: Look at the ones to see if you need to regroup.

T: Since …

Ss: The bigger number is on the bottom…

T: Is the bigger one on the bottom?

Ss: Yes.

T: Do you need to regroup?

Ss: Yes.

T: Now here's the tricky part, but it's not that tricky. If you have no cookies can you take 4 away?

Ss: No.

T: Let's talk.

Ss: Mr. 7 can I borrow one of your tens?

T: Yes, you can, but how many will you leave me?

Ss: 6.

T: [crossed out 7, replaced with 6]

B1: Put 10 in the box.

T: [Turned 0 into 10]

T: The only time you can put 10 is if there is a 0. Let's go, talk with me.

Ss: 10 take away 4 is 6, 6 take away 2 is 4.

T: Ariana, I think you will learn more if you pay attention. Say it.

Ariana: 10 take away 4 is 6, 6 take away 2 is 4.

T: [wrote 46]

T: That's not so hard. Now I have a question. Is there a time when there is a zero and I don't have to regroup? Daniel, what do you think? Is there a time when there is a zero and I don't have to regroup? When would that be? Can you give me an example?

Daniel: 10.

T: [wrote]

70

-10

T: Paul, what do you think? Right or wrong?

Paul: Wrong.

T: Let's pretend I looked at that and said, "Do I need to regroup?"

Ss: Mr. 7, can I borrow one of your tens?

T: Yes, you can, but how many will you leave me?

Ss: 6.

T: [Crossed out 7, replaced with 6. Crossed out 0, replaced with 10].

6 10

~~7~~ | ~~0~~

-1 | 0

10

T: When you see a number like that, [pointed to the bottom 10], there's a problem because the biggest number you can have is a 9. The rule didn't change. Is the bottom number bigger?

G: Same.

T: Can I have zero and take away none of them?

Ss: Yes

As shown in this example, the teacher decomposed 1 ten the number 70 into 10 ones and said, "The only time you can put 10 is if there is a 0", which is not true. Although the standard procedure in most 2-digit subtractions with regrouping would normally combine the decomposed 10 ones with other ones (for instance, in 53 – 17, we normally would decompose 53 into 40 + 13), we do not have to. In fact, we could regroup 53 as 40 + 10 + 3 and conduct the subtraction in this way: In the tens place, 40 minus 10 equals 30, whereas in the ones place, 10 minus 7 is 3, plus 3 that is originally in the ones place, we would get 6. So the final answer would be 36. Of course, the second approach of regrouping 53 (i.e., 40 + 10 + 3) is unconventional, but perfectly o.k. In fact, unconventional approaches such as this are widely used in daily mental arithmetic calculations. It is unacceptable for the teacher to lead students to believe what is typically done is the only correct way to solve the problem. Another example is that the teacher told the students that the big number is always on top (as in 72 – 54). When she asked what if she switched 72 with 54, the students yelled out, " Math jail!" Even though at 2nd grade, students are not taught how to subtract a bigger number from a smaller one, they should not be confused by the false idea that the big number is always on top and therefore 54 minus 72 is wrong.

The second point worth mentioning is that when the teacher asked the class, "Is there a time when there is a zero and I don't have to regroup?" Daniel gave an example of 10 (in 70 – 10). The teacher asked Paul why the example was right and Paul replied, "Wrong". Without inviting Paul to explain why he thought the example as wrong (i.e., Paul thinks that one needs to regroup in 70 – 10), the teacher led the class through the same procedures. Now the class found out that they would get 10 ones in the ones place when they regrouped 70 as 60 plus 10. At this point, the teacher simply stated, "When you see a number like that, there's a problem because the biggest number you can have is a 9. The rule didn't change." The teacher never even attempted to help students understand why we do not leave 10 units in ones place, which related directly to the fundamental idea of how numbers are composed in the decimal system. Perhaps Paul was thinking to decompose 70 as 60 plus 10, therefore, "70 – 10" becomes "60 + 10 –10" which gives 60. This would be an alternative approach to solving the problem that is perfectly correct.

As we have seen so far the concept of regrouping was never taught for conceptual understanding, but as a set of fixed computational steps. The third math lesson just reinforced students of these steps. In the end, students may or may not memorize these steps, or they may just automatically apply these steps regardless of whether it is necessary or not. For instance, some students automatically applied the regrouping procedures in 36 – 24.

The two 2nd grade classes described to this point give us a picture of how little the students understood the mathematical ideas of tens and ones, and regrouping in 2-digit subtractions, this happened because the teaching was fragmented and procedural-driven and did not emphasize conceptual understanding. The teacher never explained, nor led students to discover the fundamental ideas of: how numbers are composed in the decimal system; regrouping in addition and subtraction; the relationship between how numbers are composed in the decimal system and its implication for regrouping in addition or subtraction; and, the relationship between the application of the same concept (regrouping) in two reverse mathematical operations (addition and subtraction). Naturally, students taught in this way will most likely unable to extend their learned knowledge to new situations. Another teacher shared the same view (Ma, 1999):

> To discuss the rate for composing a higher value unit here is not only helpful for them to deal with subtraction of multidigit numbers, but also other more complicated versions of problems. To decompose a ten into 10 ones or to decompose a hundred into 10 tens is to decompose 1 unit into 10 units of the next lower value. But sometimes we need to decompose one unit into one 100, one 1,000 or even more units of lower value. For example, to compute 302 – 17, we need to decompose one hundred into

100 ones. Again, conducting the subtraction 10,005 – 206, we need to decompose one unit into ten-thousand lower-valued units. If our students are limited to the fact that 1 ten equals 10 ones, they may feel confused when facing these problems. But if at the beginning of learning, they are exposed to the rate for composing a higher value unit, they may be able to deduce the solutions of these new problems. OR at least they have a key to solving the problems. [p. 11]

As these two Chinese teachers pointed out, teaching students a key to solving problems will enable students to go a long way. When we confine students with all kinds of mathematical statements, rules, or facts, they will not be able to be on their own when confronted with more complicated mathematical problems. As our examples showed, 2-digit subtractions with 0 on the ones place were treated as a separate topic.

Unfortunately, we saw evidence that these misunderstandings are repeated, not replaced, as students get older. The same procedures we observed in those two 2nd grade classes were used by fourth grade teachers teaching 4-digit subtractions.

4th grade class: 4-digit addition and subtractions with regrouping

At the time of our observations, this 4th grade class was learning 4-digit addition and subtraction that involved regrouping. The first of the 3 lessons introduced the steps of carrying out 4-digit addition and subtraction. The second lesson reviewed the topics of the first lesson and the last lesson was about subtracting numbers with zeros.

The following example, taken from the first lesson, showed how 4-digit addition and subtraction was introduced to the students.

On board: Add and subtract 4-digit numbers using regrouping.

9910	9910	6899	9674	8902	9201
+ 7340	-7340	+2267	-1406	-5730	+1321

12:35

T: Can I start anywhere? (1)

Christian: You have to start in ones column. (2)

T: [Wrote 0, 5] Can I squeeze 12 in there? (3)

9910
+ 7340
$_{12}50$

Ss: No. (4)

T: Why? (5)

Ss: You have to regroup. (6)

T: [Uses manipulatives] I have 12 hundreds. I can't put it all in the hundreds place, so I put 1 in the thousands place, and have 2 left over so I put the 2 down. (7)

9910
+ 7340
17250

T: Can we have less talking? It takes too much time. Now we are regrouping in the thousands. Now we'll do it with subtraction. (8)

9910
- 7340

T: Where do I start? (9)

Ss: Ones. (10)

T: Tamara, can we take away 4 from 1 in tens column? No. We regroup in next column and change the 9 to 8 and this becomes 11. 11 take away 4? (11)

Ss: 8. (12)

Chase: 7. (13)

T: Good. Check before you call out. [Class moved on.] (14)

Erica: 5 (15)

Jeremy: 2. (16)

T: Did we get closer? Yes, because this number rounded would be 3,000. (17)

The addition problem the class worked on (i.e., 9910 + 7340) involves two places where one would need to regroup. The first regrouping, as shown in the interaction, was composing 10 hundreds in the hundreds place into 1 thousand and put it in the thousands place. The teacher asked the class, "Can I squeeze 12 in there?" and "why?" The students knew the answer (i.e., they cannot squeeze 12 in the hundreds place), but could not explain why, because they were never taught why. The teacher did not evidence understanding in her comments (line 7) that described the procedure of regrouping, but not the reason why we need to regroup. As explained earlier, the reason why we regroup has to do with how numbers are formed in the decimal system. This fundamental idea was never made explicit to the students in any of the three classes where the focus of the lessons was on regrouping.

Another point worth mentioning is that although the teacher demonstrated the procedures of 4-digit subtraction and addition using the same numbers, the connection between how regrouping was used in these two inverse mathematical operations (i.e., addition and subtraction) was never discussed. The students were never given the opportunity to learn that regrouping in addition involves composing 10 units of lower place value into 1 unit of the immediate next higher place value, whereas regrouping in subtraction involves decomposing 1 unit of higher place value into 10 units of the immediate next lower place value. The teacher herself did not demonstrate her understanding of the relationship, because at the beginning of the lesson, the teacher said, "We will also be practicing addition and subtraction of 4-digit numbers using regrouping. Regrouping is to borrow from the next place value. Does any one remember what sum means?" The definition of regrouping that the teacher described here applies only to subtractions, but she immediately asked the students if they remembered what sum means. We do not know with certainty whether or not the teacher thinks regrouping means the same thing in both applications (i.e., in addition and subtraction).

Because the students were never taught to understand conceptually how regrouping works in either mathematical operation (i.e., addition and subtraction), these students could only be expected to recall memorized procedures and carry out the operations. Some students, however, would have difficulties, particularly in conducting subtractions with regrouping. As our observations showed, at the end of the first lesson, quite a few students did not know how to subtract 4-digit numbers that involve regrouping. What they did was simply switching the digits to make an easier problem that did not involve regrouping. For instance, one student did not know how to compute 3204 – 2413, so what this student did was simply changing the problem into 3414 – 2203 (i.e., switch 1 and 4 in 2413 with 0 and 2 in 3204 respectively).

$$\begin{array}{r} 3204 \\ -\underline{2413} \\ 791 \end{array} \qquad \begin{array}{r} 3414 \\ -\underline{2203} \\ 1211 \end{array}$$

The teacher noticed this common practice among some students and decided to a "real-life" example to show why they should not do that.

T: Everyone get your seats and wait for corrections. Tonight's homework will be more practice of 4-digit adding and subtracting. A common error is inverting the numbers. Maybe you think regrouping takes too much work. Don't flip the problem around. It's going to be wrong. If I go to a department store and the cashier says, "That's 54.35." I give her a $100 bill and cashier gives me a $5 dollar bill. The cashier says, "I don't feel like regrouping."

100.00
-54.35
4.35

Ss: that's rip off.

T: Right. You have to learn this so you don't have to get ripped off.

First of all, the teacher did not even conduct the calculation right, if she were to demonstrate the same switching practice that some students were doing. Using these students' practice, $100 subtract $54.35 would give us $154.35 instead of $4.35. So the cashier would give back more change than necessary (which should be $45.65). Of course, if the teacher followed the exact procedure that these students used, she would not be able to convey the message that they were being ripped off. Second, instead of showing and helping the students understand the correct procedure, the teacher simply concluded that they had to learn this so they would not get ripped off. The way the teacher help those who had trouble regrouping was simply, "I will give you homework that will help you to regroup. Please see me and I'll give it to you."

During the second lesson, the teacher reviewed 4-digit addition and subtraction with the students, going over the rules of regrouping:

12:55

T: We're going to regroup 10 ones into one while 10. We'll put the zero down here and the 1 in the box. This doesn't mean you're not smart, it just means you need practice.

Step 2: [T read] Put the 2 down and carry the 1 into the hundreds place. [T continued Step 3 and Step 4]

If the same approach of how regrouping was taught to the students did not work in the first place, it certainly would not work when it was used again, because the approach focused on the procedure, not the understanding. The teacher, however, thought that understanding would arise simply from more practice, because the activities of the second lesson were merely applying the 4-steps in different 3-4-digit addition and subtraction problems. The teacher, therefore, failed to explore the reasons for student mistakes, cutting off an opportunity to understand and correct the error at the source.

As shown in the remarks of the two elementary Chinese math teachers, teaching for understanding involves equipping students with a key to problem solving, that is, to discover the mathematical concepts underlying any particular algorithm and to see the connections among different mathematical ideas. In the context of learning regrouping, both these Chinese teachers emphasized the importance of helping students to understand how numbers are composed in the decimal system (i.e., the rate for composing a higher value unit) and how this idea is related to regrouping in subtraction.

This emphasis is in sharp contrast with how regrouping was taught in different classrooms and across grade levels in our three examples. Earlier we have showed that 2-digit subtraction with zero in the ones place was treated as a special topic and was the focus of one whole lesson in the 2nd grade class. Similarly, subtraction involving multi-digit numbers with zeros was treated as a separate topic and took one whole lesson to learn in this 4th grade class. These teachers, of course, were mostly following the steps and procedures in the textbook. The treatment of these topics (place value, regrouping, and 2-digit subtraction with regrouping) in the teachers' manual revealed that the textbook (particularly Scott Foresman) at least has made an attempt to emphasize the connection among these mathematical concepts and the importance of helping students understand the concept of place value. However, such information may be useless if teachers do not recognize its significance or do not have time and energy for careful study of manuals (Ma, 1999). This issue will be further discussed in later sections.

The focus of the third lesson, as the teacher stated, was "subtracting numbers with zeros (regrouping zeros)".

12:40

On board – Subtracting numbers with zeros using regrouping.

300	900	600	100	300	1000	3000	7100	1000	5000	3000
-<u>12</u>	-<u>374</u>	-<u>211</u>	-<u>29</u>	-<u>287</u>	-<u>384</u>	-<u>2241</u>	-<u>3291</u>	-<u>328</u>	-<u>3024</u>	-<u>280</u>
51000	6000	5000	8000							
-<u>3906</u>	-<u>2900</u>	-<u>2630</u>	-<u>7887</u>							

T: Over the past few days we've been practicing regrouping in addition and subtraction. I noticed a part that's little more difficult for everyone. When I was in fourth grade, anytime we had a problem with zeros my teacher said cross it out and put it a 9. I had no idea why I did it. When I became a teacher I learned in the teaching manual, why. I never understood it. I don't want you to be like me. I'm going to attempt to show you two different ways. The first problem is:

300
- <u>12</u>

T: Subtract what's on bottom from top row. Can we take it from tens column? No. Can we borrow from hundreds column? Yes. How much is it worth?

Some: 10.

Some: 4.

T: It's always 10, change the 1 to 10 ones. I traded one ten for ten ones.

$$\begin{array}{l} \ {}^{2}\ {}^{9}{}^{1} \\ \cancel{3}\cancel{0}\,0 \\ -\ \underline{1\ 2} \\ \ 2\ 8\ 8 \end{array}$$

T: Here's where the 9 comes from.

Although the teacher told the students that she was going to show them two different ways, she actually only demonstrated one way as described in this interaction. She repeated the same approach using a different example (900 – 374), if that was what she meant. The teacher intended to help the students understand how regrouping was typically conducted when both the ones and the tens places are zero in 3-digit subtraction, because she never understood this as a student. The way the teacher explained things to the students, however, might not be helpful for their understanding. First, the rate of change from one place value to the next immediate place value was not made clear to the students. The teacher simply stated, "It's always 10." Second, how was 300 decomposed as 2 in the hundreds place, 9 and 1 in the tens place was not explained to the students. The teacher simply said, "change the 1 to 10 ones. I traded one ten for ten ones." Given that students in this class were still struggling with regrouping, the teacher could have at least pointed out that once she decomposed 1 unit in the hundreds place into 10 units in the tens place, she then regrouped 10 ten as 9 tens and 1 ten. This 1 ten was further decomposed into 10 units into ones place so that we can subtract 2 from 10 in the ones place, and 1 from 9 in the tens place. The teacher never explained clearly where 9 and 1 came from and why.

The teacher's inability to do this might be due to the fact that she did not really understand the fundamental idea of how numbers are composed in the decimal system, because for the reminder of the lesson the teacher kept saying, "I'll check my neighbor and if I get some I'll lend it to you. What do you always give? Magic number?" So 10 became a magic number instead of being explained as the rate of change from one place value to the immediate next place value in the decimal system. Furthermore, whenever 1 unit was "borrowed" (using the teacher's term), it was automatically rewritten as 9 and 1 without explaining clearly to the students why. No wonder when a student was called to solve 1000-384, the student automatically wrote:

$$\begin{array}{cccc} & 1 & 1 & 1 \\ \cancel{1} & 0 & 0 & 0 \end{array}$$

It was obvious that the student did not understand how regroup works in this context. In fact, a lot of students in this class probably did not understand either, since the teacher observed, " I still see a lot of errors. We're still not ready to go on our own yet." The teacher led the class to practice more exercises of the same nature. As mentioned earlier, students will never be able to be on their own if they are not equipped with the key to problem solving. As we have seen through these examples from 3 different elementary classrooms that showed how a basic mathematical idea was taught. From this, we surmise that mathematics is being treated as set of isolated facts, rules, and steps. To these teachers, learning mathematics simply means mastering these disconnected facts, rules, and steps. Acquiring these facts, rules, and steps with accuracy and fluency equals understanding. Therefore, mathematics understanding is regarded as arising automatically from repeated practice.

Students taught in this way will rarely achieve the deep understanding called for by the Standards. To make meaning, it requires a systematic presentation of concepts that makes transparent the many interconnections. When students move on from grade to grade with poor foundational skills, math mastery becomes increasingly difficult. It becomes the proverbial house built on sand. The foundation is insufficient to build understanding. Moreover, it is difficult for students subject to this type of mathematics teaching to develop confidence in themselves as mathematicians. Students lack mathematical reasoning skills because they are not taught these skills – classroom conversations about mathematics where students take an active role are rare.

Our next two examples came from an 8th grade Algebra 1 class and a high school Honors Algebra 2 class.

Beyond *x*'s and *y*'s: Dynamics of the middle and high school Algebra classes. The first series of examples were taken from an eighth grade Algebra 1 class. Students in this class were on the two-year Algebra 1 pathway, which means that these students were not as advanced as those who complete the same course within one year. The Honors Algebra 2 class, on the contrary, consisted of mostly advanced students. Despite perhaps big differences in student population in terms of academic ability, the underlying classroom discourse was similar in many aspects.

An 8th grade class: Algebra 1

As the class started, the teacher gave the students a warm-up activity:

7:26

Warm up

1. 12 + 3x when x=0

2. 12 + 3x when x=1
3. 12 + 3x when x=2
4. 12 + 3x when x=3

Ss opened their notebooks and began working on the warm up.

T: I am going to do number 3 for you as an example. [wrote on OHP]

12 + 3x when x = 2.

T: [Wrote and said] 12 + 3 times 2. This gives us [wrote 12 + 6 = 18].

Several Ss visited the pencil sharpen to sharpen their pencils. This disrupted the instruction.

T: Please pay attention!

Ss continued working on the warm up.

7:45

Ss became extremely noisy.

T: Stop taking! You are 8th graders. [T had a confrontation with a boy for leaving his book at home.] The exercise you are doing is worth 10 points. O.k. three more minutes.

7:48

T: Look at number 1. It's 12 + 3 times 0. This gives us [wrote 12 + 0 = 12]. 12 + 3 times 1. [T went through all warm up exercises in the same manner].

T: Open your books to page 59. This is an open book test. [T put slide on OHP]

Test problems:

Evaluate the variable expression when y=3 and x=5.

1. $5y + x^2$	3. $2y + 9x - 7$
2. 24/(y-x)	4. (5y + x)/4

In exercises 5-7, write the expression in exponential form.

5. 5y.5y.5y.5y

6. Nine cubed

7. six to the nth power

8. Insert grouping symbols in 5 . 4+6 so that the value of the expression is 50.

Express as a variable, an equation or an inequality expression.

8. 7 times a n

9. 9 is less than t

10. 8 minus s is 4

11. y decreases by 3.

T: You have 15 minutes on this test.

As shown in this example, to help students work through the warm-up exercise, the teacher first demonstrated the calculation steps using 12 + 3x when x=2 as an example. The students were then on their own. Once time was up, the teacher gave out the calculations and answers to each problem. There was no interaction between the teacher and the students about mathematics. All interactions pertained to disciplinary problems. Furthermore, when the warm-up activity was over, the students were given an open book test that consisted of dry and meaningless drill exercises. After the test, the teacher spent the rest of the class time reviewing with the students "how to arrange numbers according to the ascending and descending order of a number".

9:00

T: You must copy your new words.

<u>New words</u>

- Real number
- Real number line
- Positive number 5
- Negative number 5
- Integer + or – number 5
- Whole number
- Graph of a number

9:05

T: [Wrote on board] 3, 4,5,1. I draw a number line and locate these numbers on it.

0_____1_____2_____3_____4_____5. This is all you are supposed to do. I gave you a series of numbers to arrange in an ascending order. Now I am going to give you numbers to arrange in descending order [Wrote on board] –4, 5, -3, 6, -1, 2, -2. Put these on a number line.

[Observer note: After a minute or so, I walked around. One girl counted her money rather than working on the assignment. Another boy, balled paper and used the trashcan for a basketball basket. Three boys on the back row played instead of working on assignment. I estimated that about 10% students tried to do the assignment]

9:13

T: [Wrote on board]

_/__/__/__/__/__/__/__/__.__/__/__/__/__/__/__/__

-7 -6 -5 -4 -3 -2 –1 0 1 2 3 4 5 6 7

T: [pointed to the right of 0] If you move this way you increase and decrease if you move this way [pointed to the left of 0]. What are numbers on the left?

Girl: Negative.

T: Right. The only time that's not negative or positive is 0.

Again in this example, we saw that the teacher first demonstrated how to arrange numbers on a number line using an example that consisted of 3, 4, 5, 1. The students then were simply told, "This is all you are supposed to do" and were on their own. After giving students some time to work on their own (as we can see from the description, the majority of the students were off task), the teacher showed them the answer. There was one interaction here where the teacher asked, "What are the numbers on the left?" When a student gave the correct answer (i.e., negative), the teacher said, "Right."

In terms of discourse of teaching and learning mathematics, math instruction in this Algebra class is remarkably similar to that in elementary classes. Mathematics concepts and ideas (in this case, x's and y's or numbers) were thrown at the students through meaningless drills. There was no connection among different activities. The teacher was the center of instruction in each activity, demonstrating how to solve a problem and assigning problems of the same kind for the students to practice. The students' role in learning was to execute correctly the same procedures or steps that the teacher had shown. In this 8th grade class, there were almost no conversations about mathematics.

As mentioned earlier, students in this class were not advanced. Indeed, our observations showed that these 8th graders possessed very poor foundational mathematics (or arithmetic) skills. For instance, when the teacher called on several students to give the answer to $15 - 12 / 3 + 17$, only one student was able to give the correct answer. Poor foundational skills of students in combination with their lack of interest in class work (there were constant disruptive behaviors in this class) make the task of teaching Algebra for understanding even more daunting. Student motivation is an important issue that will be addressed in later sections. Now let us go into a high school Honors Algebra 2 class to see what mathematics teaching and learning looks like. Since it was an Honors Algebra 2 class, one would expect to see a quite different picture from the one we saw in the 8th grade Algebra 1 class.

A high school class: Honors Algebra 2

The teacher began by stating the purpose of the lesson as, "graphing linear equations using only the x and y intercepts, and graphing absolute value functions". After telling several students that they were going to demonstrate the next day how to solve one problem in the previous day's homework, the teacher reviewed the signs of *x*'s and *y*'s (i.e., positive or negative) in the four quadrants of the Cartesian Coordinate system. Then the teacher told the students to look at page 108 of the text:

> T: It says in Example 2, find and graph 5 solutions of $3x + 2y = 4$. I am going to ask you to graph $Ax + By = C$. And I'm pretending that A, B, and C are positive. [T wrote]
>
> $Ax + By = C \quad A>0\ B>0\ C>0$
>
> Let $x = 0$
>
> $By = C$
>
> $Y = C/B$. [Repeat for let y=0]
>
> Y – intercept (0, C/B) x – intercept (C/A)
>
> T: If I graph this I get more or less something like this. [Drew]
>
> [insert graph here]
>
> T: If I ask you to graph this one, $2x + 3y = 5$. It looks like this, right? [Drew]
>
> [insert graph here]
>
> T: So you can graph anything with this system.

Once again, we see a similar approach of math instruction, namely, teacher demonstrating to students how to do something (in this context, how to graph a linear equation). Using the same approach, the teacher went over different examples in the text (e.g., how to graph $y=x$, $y=|x|$, $y=-|x|$, $y=|x-a|$, $y=|x+a|$, $y=|x-a| + b$, and $y=-|x-a|+b$). During these demonstrations, the teacher was the center of instruction, occasionally asking the students questions such as "What is the definition of absolute value?" or "If

|y|=5, then what would y equal?" After going through different examples in the text, the teacher finally came to the concept of "the slope of a line".

T: Turn to page 112, 3.3, the slope of a line. What is a slope?

Girl: The rise over run.

T: The rise over run. When do you feel a slope?

[No response from the students.]

T: Is it when you go up a 100 stories?

Ss: Yes.

T: If I take all of you outside and make you run up a hill, will you feel it?

Ss: Yes.

T: If I make you go up that hill 100 times, sprinting all the way, would you feel it?

Ss: Yes.

T: Yes, your heart would be pounding, right?

Ss: Yes.

T: So the slope, or the rise over run is what you would feel. If I picked two points, and called them P and Q, and Q was at (1,1), and P was at (2,2), they would look like this:

[insert graph]

The subscripts just help differentiate the numbers.

The slope will be: slope=(y2-y1)/(x2-x1).

T: And I'm going to test you on the formula. So make sure you understand it. All of you know how to find the slope. So if I give you a slope like this:

Slope=(1/2 – 3/2)/(5-2)   (2,3/2) (5,1/2)

What would your slope be?

Ss: -1/3.

The concept of "slope" was merely treated as "the rise over run", a correct definition yet hardly bears any meaning. The teacher tried to help the students "feel" a slope by giving examples such as "go up 100 stories" and "run up a hill". These examples do not express the concept of slope. The formula for calculating the slope was given to the students who were told that they would be tested. Surely the expectation on the students was to memorize the formula and execute it with accuracy. But what does the formula mean? Why would the formula give the slope of a line? These questions were never discussed, because the concept of slope was narrowly defined as the rise over run instead of being conceptualized as a rate that describes the linear relationship between variables x's and y's (i.e., amount of change in y for every unit change in x as the formula shows).

Obviously, the roles that the teacher and the students played in this lesson were essentially the same as the ones that we observed in the 8th grade Algebra 1 class. Of course, students in this Honors Algebra 2 class, by definition, were far more advanced than those 8th graders who possessed very poor arithmetical skills. However, we have seen that even these advanced students typically were not experiencing the type of mathematics teaching and learning that reform asks for. Naturally, these students' opportunities to learn and to develop confidence in their own mathematical reasoning powers were limited, which can be seen shortly.

Earlier we mentioned that the teacher nominated several students who were going to demonstrate their solutions to a homework problem the next day. The following examples described these students' demonstrations and explanations of their solutions. When these students were selected the day before the presentation, the teacher had indicated to them that, "You will be graded on the presentation and the correctness of your answer. To get the full 10 points for your answer, you will have to include all the steps it took to get that answer, and I will have to like the way you did it." The problem read like this:

> In Exercises 9 – 12, the digits of a positive two-digit integer N are interchanged to form an integer K. Find all possibilities for N under the conditions described:
> Prob. 9: N is odd and exceeds K by more than 18.

Kyle was the first one to present his solution, which is as follows:

Kyle's solution:

$N > K + 18$		
N = AB, K=BA	A	
A< B	$A > 4$	{4,5,6,7,8,9}
B = odd	B = 1,3,5,7,9	41, 51, 61, 71, 81, 91
B		63,73,83,93,85,95}
$B < 5$		
B odd {1, 3, 5}		

Kyle: It says that N is odd, the second digit is odd too. B is less than or equal to 5 or else it doesn't work. Because the number goes too high. So B has to be 1,3 or 5.

T: You lost me on that last bit. Can you go back and explain it?

Kyle: The next number is 7 because it's going to be odd. Like 71 would be a number. If you flip it around, it would be 17. Then 71 + 18 would be more than the 17. So there's no number greater than 5 that would work. A is greater or equal to 4 or else it doesn't work in the problem. So 4, 5,6, 7,8, and 9. So 41 is going to be the first number. And 4-1 is 3 because this is A and this is B. A – B has to be greater than or equal to B in the problem, so 4-1 has to be greater than or equal to 3. So 51, 61, 71, 81, 91. So the next number 43 does not work because 4

– 3 is not greater or equal to 3. So 63, 73, 83, and 93. and then 75 doesn't work because 7 – 5 is not greater than or equal to 3. So 85, and 95 work.

T: Go over how you got A to be 4, 5, 6, 7, 8, and 9. And go over how you got B to be 1 through 5.

Kyle: O.K.

T: Take it from the top.

Kyle: O.K. B is only the odd numbers and it said that in the problem, so 1, 3, 5, 7, and 9. But it has to be less than or equal to 3 if you reverse it. So like if you have 75 and you reversed it you get 57 + 18 is not less than 75. [T let Kyle go at this point.]

Although Kyle got the correct answer to the problem, it was not clear how he arrived at the answer based upon his explanations. If we examine his explanations carefully, we can find ambiguities, inconsistencies, and even wrong logics. Kyle was right to begin with the condition for the solution since N is an odd number, the ones digit represented by B must be odd. But without making a mathematical argument, he immediately jumped to the conclusion that B "is less than or equal to 5 or else it doesn't work". When told to go back and explain his logic again, Kyle used 7 as an example (judging from the context, what Kyle intended to say was that the next higher number than 5 B could be is 7) for B, and used 71 as an example for K. Doing so, Kyle was showing that if K was 71, then N would be 17, then 17 would not be greater than 71 + 18 (Of course, one has to guess from his words to infer that he meant this).

The way Kyle proved his statement that B could not be greater than 5 was not convincing enough, because he used only one example to demonstrate that when B was 7 and A was 1 (i.e., N=17) it did not work. What about other possibilities? Unless Kyle could show that all other possibilities (i.e., 2-digit numbers) when B was greater than 5 did not work, his argument was prone to questioning. The teacher let Kyle go at this point. Kyle then made another statement about the tens digit without supporting argument, that A is greater than or equal to 4 so that A was 4,5,6,7,8, and 9. Not only didn't Kyle make an argument why A was greater than or equal to 4, but he made a false statement about the relationship between A and B. Kyle stated that A – B has to be greater than or equal to B in the problem. Putting aside the problem of lacking an argument to support his statement, we can see that Kyle did not even notice the inconsistency between his statement about the relationship between A and B and the fact that in numbers 85 and 95 (two of the possible numbers for N), this relationship obviously did not hold and yet 85 and 95 still worked. Even worse, after Kyle finished his explanations, the teacher asked him to start all over again, Kyle gave very confusing explanations, "B is only the odd numbers and it said that in the problem, so 1,3,5,7, and 9. But it has to be less than or equal to 3 if you reverse it. So like if you have 75 and you reversed it you get 57 + 18 is not less than 75." The last two sentences do not make any sense. Besides, it is ambiguous what has to be less than or equal to 3. Therefore, even though Kyle got the correct answer to the problem, we do not know how he arrived at the answer judging from his reasoning.

After Kyle finished, the teacher called Mike to present his solution, which read like this:

Mike's solution:

$N>K+18$

$53 = 5(10) +3$

$10t + u > 10u + t + 18$

$9t>9u + 18$

t>u+2

u = 1,2,3,4,5,6,7,8,9,

2> nothing will work

3> nothing will work

4>1+2

t = 4,5,6,7,8,9

4>3

5>3+2 no

5>1+2

5>3

6>1+2

6>1+3

7>1+2

7>3+2

7>5+2

8>1+2

8>3+2

8>5+2

9>1+2

9>3+2

9>5+2

These would give you 41,51,61,63,71,73,81,83,85.

Mike: Well, I separated it into the ten's and units and then like for 53, you can write 53, or you can write 5 times the 10's + the units. Because when you're flipping around the numbers, to get the 2 digit number, 10 times the 10's digit + the unit is greater than switching this around. So you just multiply the units by 10 and the 10's is going to switch places. So like 9t>9u+18, so t>u+2. So it connects to the tens, so 1,2,3,4,5,6,7,8,9. Bu N is odd, so it's not 2,4,6,8. You can't start out with 1,2,or3. So let's say t is 2, the only way that this would work, it won't work. If you go to 3, it won't work. Because like if you had 30 and you switch the digits around to 03, it won't work. So 4 is greater than 1+2. So you start the tens with 4,5,6,7,8, and 9. So you plug in all these numbers. So then is 4 greater than 3? Yes, so the first number is 41. Is 4 greater than 3+2? No. So none of the other 4's work. So then we go to 5. Is 5 greater than 1+2? Yes. So 51 works. But that's the only 5 that works. So then go to 6. Is 6 greater than 1+2? Yes, so 61 works. So then is 6 greater than 3+2? Yes, so 63 works. So no other 6's will work. So 7, is it greater than 1+2? Yes, so 71 works, so 7 is greater than 3+2. that's all the 7's that work. So is 9 greater than 1+2? Yes, so 91 works. And 9 is greater than 3+2. so 9 is greater than 5+2. And that's it.

Boy: What about the 8's?

Mike: Oh, yeah. So is 8 greater than 1+2? Yes, so 81 works. So 8 is greater than 3+2, and 8 is greater than 5+2.

Compared to Kyle, Mike made an important advancement in his reasoning, using the idea of how two digit numbers are formed and the condition given in the problem to establish an important relationship between the tens and ones digit (Mike used units instead of ones digit), namely the tens digit must be greater than the ones digit plus 2 (i.e., $t>u+2$). Mike also inferred that the ones unit (u) could not be 2,4,6, or 8 because N was an odd number. Instead of combining these pieces of information to make a mathematical argument of all the possibilities of N, Mike stated that, "You can't start out with 1,2,or 3". Mike did not really make a rigorous case why one can't start out with 1,2 or 3 (In fact, using 30 was inconsistent with the condition that N was an odd number) and concluded that, "So you start the tens with 4,5,6,7,8,and 9". After this, Mike used plug in method to test different possibilities and found the answer. A third student, Chris also presented his solution, which was the same as Mike's, however, Chris was also unable to explain clearly why the tens unit has to be greater than 4. The teacher asked him why 4, Chris simply used 3 as an example to show that it did not work. After that, Chris' logic became confusing and inconsistent:

> T: Why 4?
>
> Chris: Because when you put 3 in, it does not go, so it has to be greater than 4. So when you plug in 1, 3 isn't greater than 3, so 2 isn't greater than 3, so it has to be 4-10, and u has to be 1,3,5,7,or9.
>
> T: What is 4 – 10?
>
> Chris: 4 –10 is t. No, it's 1-9 is t.
>
> T: O.K.

As can be seen, when Chris said, "1-9 is t", he was contradicting his earlier statement that t has to be greater than 4 (which is incorrect, since t can be 4). Moreover, Chris' conclusion that "u has to be 1,3,5,7, or 9" not only was incorrect but also inconsistent with his final answer.

The three examples presented here showed that even though these students could give the correct answer to the problem, none of them were able to make a clear, consistent, and logical mathematical argument how they arrived at the correct answer. Moreover, *the teacher did not capitalize on the errors to generate understanding with the group*. Instead, *the correct solution took precedence over the mathematical understanding*. Although Mike and Chris were able to use the idea of how numbers are formed in the decimal system to establish the relationship between the tens and the ones digit in N (i.e., tens digit must be greater than the ones digit plus 2, or alternatively, the tens digit minus the ones digit must be greater than 2), neither one exhibited an analytical ability (other than using the ambiguous plug-in method described by Mike and Chris) to connect this piece of information with the other information given in the problem (i.e., N is an odd number) to make an argument that the ones digit in N can only be 1,3, or 5, whereas the corresponding tens digit would be {4,5,6,7,8,9}, {6,7,8,9,}, and {8,9} respectively. This would give us all the possible 2-digit numbers that N could be.

During these students' presentations, the teacher could have played a more active role than that we had observed, even though he asked them to clarify their reasoning at several places. When the students were unable to better explain their logic than what was already given, however, the teacher did not explore those areas where understanding was not evident. Moreover, once the presentations were over, the class was not given an opportunity to discuss and compare the solution methods presented to them. Therefore, we do not know if any of the ambiguous, inconsistent, or incorrect reasoning was taken as truth by the rest of the class. Importantly, an opportunity was lost for furthering the understanding of those not yet clear.

These classroom snapshots across grades and across levels indicate mathematics teaching and learning remains at the descriptive level. Teaching practices appear procedure-driven with scant attention to conceptual understanding and problem solving. This is so, regardless of whether the mathematics topics were 2-digit or multi-digit subtractions with regrouping, or Algebra. All topics were essentially reduced to basic arithmetic manipulation.

Nine Hundred Observations – No Teaching for Understanding

One point worth mentioning is that these snapshots came from over 900 classroom observations. We searched for examples of teaching that do not appear procedure-driven, but rather emphasize conceptual understanding. We were not able to find a single example that meets our expectations. In last year's evaluation report (Ai, 2002), we described one elementary teacher who was reviewing and teaching her 2nd graders the concept of probability through experiment. She first asked one student to read the definition of probability and then did the experiment to show what it meant.

T Yesterday, we used cubes. Today we are going to use something different. Here is a card, one side is red and one side is blue. I'm going to drop the card on the floor 10 times. Sam, read the definition of probability.

Sam [read the definition]

T What is the probability that it will end up more red or more blue?

Sam [no response]

T We are going to flip it 10 times to see what comes up. I need a helper, Samantha, to check off on the chart whether it comes up blue or red, the probability or how likely something will happen. Jessica, I'd like you to call it when it drops on the floor.

The teacher dropped the card 10 times and the results were as follows:

	1	2	3	4	5	6	7	8	9	10
blue	x	x	x	x	x	x				
red							x	x	x	x

T How many times did it land on blue?

Ss [counted] 6

T How many times did it land on red?

Ss [counted] 4

T Did it end up in a tie?

Ss No.

T Close to a tie?

Ss Yes.

T It's pretty close to being even.

In fact, this teacher was following the textbook. Although the textbook's idea of teaching the concept of probability through experiment was a good one, whether students were able to understand the concept or not ultimately depends on the teacher's understanding of the concept, particularly when students were confused by the seemingly inconsistent experiment result and what they had learned about the concept of probability, as the following clip shows:

T Let's try a different object. The next object is a penny. Joseph, how many sides to a coin?

Joseph 2

T How many sides are heads?

Joseph 1

T How many sides are tails?

Joseph 1

T Is there a greater chance for it to be heads, tails or equal?

Sam Equal, because there are 2 sides.

T What about this? [She refers to the experiment results of the card chart. See page 61] Why?

Derek Because there are 2 sides, 1 heads and 1 tails

T Does anyone think any differently?

Brazil Heads more.

T Why? Raise hands if you agree.

[Some children did]

T Raise your hands if you think it will land on tails.

[Some children did]

In this example, most of the interactions proceeded well — the teacher asked a few scaffolding questions before turning to the first main question, that is, "Is there a greater chance for it to be heads, tails or equal" if she were to drop the coin. Sam answered the question correctly by saying that it would be "equal", but he did not just stop by offering only an answer. Sam also explained that the reason why he thought the answer was equal was "because there are two sides [to a coin]." The teacher challenged Sam by asking "what about this?" and "why?" referring to the result from the experiment they did earlier in the lesson. The teacher did an excellent job here by pushing Sam to think further. In the experiment they did earlier, a card with two sides (blue and red) was dropped 10 times and the result for the card to be blue and red turned out to be 6 and 4, respectively. So if a card with two sides, just as a coin has heads and tails, did not come out even when dropped 10 times, could it be possible that the chance for it to be heads and tails not be equal when the coin is dropped a few times? Sam did not follow up the teacher's challenge with an answer, but Derek did, although his answer was irrelevant – "Because there are 2 sides, 1 heads and 1 tails." The teacher did not point out the irrelevance of Derek's answer; instead she asked, "Does anyone think any differently?" Brazil realized that the chance for the card to be equally blue and red was not supported by the result of their experiment. She then answered, "Heads more". It was unclear why she chose heads as if it were the blue side of the card used in the earlier experiment, but the important thing is that she probably had made the connection that if the blue and red did not turn out to be equal, heads and tails would probably not turn out to be equal. The teacher asked, "why?" but then forced a yes/no choice from the students by asking them to raise hands. This was where the teacher failed to move the interaction along the direction of discovering reasoning instead of pursuing a correct answer. If the teacher had probed further using the why questions instead of asking the students to raise hands if they agreed or disagreed, they would have had an opportunity to discuss, if not discover, why an object with 2 sides (e.g., the card with blue and red, a coin with heads and tails) did not come out equal when dropped a few times.

The students probably had learned that the chance should be equal, given that this was their third lesson on the concept of probability. The teacher could be of great help in this situation. For instance, after further probing, if the students still could not explain why, the teacher could have helped the students by pointing out that the card was dropped only 10 times. If they were to drop the card many times to infinity, the result for the card to be blue and red would be exactly the same. And when we talk about the probability of something being equal, we are referring to infinity or in the long run. 10 times is not infinity. This rationale would have helped the students to explain the discrepancy between what they learned and what they saw from the experiment and therefore understand the concept of probability better. However, the discussion was very brief and the teacher did not pursue it further in the direction of helping students to conceptually understand the concept of probability. In doing so, the teacher has failed to demonstrate her fundamental understanding of the concept of probability, which, in turn, has limited her ability to clarify the confusion on the part of the students.

Relying on textbook alone, therefore, will not bring out teaching for understanding, which requires a deep understanding of: (1) the mathematical topics and ideas in the textbook; (2) the connection between different mathematical topics and ideas; and (3) the pedagogies that are helpful for student understanding. Such an understanding can arise only from careful studying of and thinking about the materials in the textbook. Simply going through the procedures or steps in the textbook will not solve the problem of students' lack of understanding of materials presented to them. As pointed out earlier, the treatment of topics such as place value, regrouping, and 2-digit subtraction with regrouping in the teachers' manual revealed that the textbook (particularly Scott Foresman) at least has made an attempt to emphasize the connection among these mathematical concepts and the importance of helping students understand the concept of place value. However, such information may be useless if teachers do not recognize its significance or do not have time and energy for careful study of manuals.

Our purpose of describing the discourse norm of mathematics teaching and learning in these classrooms is not to criticize current teaching practices. Rather, our intention is to show the limited progress we have made toward fundamentally changing the way teachers and students interact about mathematics in the classroom. These cases highlight the stable discourse norm of teaching and learning mathematics that is so resistant to change. We appreciate the challenges and complexities involved in transforming mathematics teaching and learning from its depleted current state towards greater intellectual demands.

This would require teachers to make ambitious and complex changes. For this to occur, teachers will require considerable assistance. Research on instructional policy and classroom teaching and learning has shown that effective operation of any instructional policy greatly depends on professionals' learning. To fully implement the DMP, teachers must develop a deep conceptual understanding of mathematics and new pedagogies that emphasize the development of student understanding. Teachers' opportunities to learn (OTL) will influence teachers' ability to adopt new beliefs and practices. These opportunities to learn will come mainly through their participation in various professional development activities.

To what extent are teachers able to connect what they have learned through various DMP professional development (PD) activities (e.g., coaching) to their teaching?

Our current investigation of the extent to which teachers are able to connect what they have learned through various DMP professional development activities focused on: (1) teachers' participation in professional development workshops (off-site and on-site); (2) support for math instruction teachers have received; and (3) teachers' involvement in coaching practice.

The majority of the trainings that teachers had received during the first year implementation of DMP were offered by the district. These trainings mostly focused on how to use the new textbook series, which teachers considered as neither useful nor helpful. It remained questionable, therefore, whether teachers had learned anything from these training workshops. Teachers reported limited involvement in other ongoing professional development opportunities such as on-site staff development workshops, instructional leadership from administrators (e.g., classroom observations that focused on the quality of instruction), or teacher involvement with math coaches.

<u>In what types of professional development activities did teachers participate?</u> Teachers' participation in various off-site professional development workshops mainly consisted of textbook publisher workshops, other district-sponsored workshops, 5-day Governor's Math Institute for elementary teachers, textbook publisher workshops, other

vendor workshops, and workshops at professional conferences for secondary math teachers (see Table 4).

Table 4: Teachers' Participation in Professional Development Activities (Off-Site)

School Level	Types of Activities	Percent (%)
Elementary	Textbook publisher workshops	59.4
	Other district-sponsored workshops	21.9
	5-day governor's institute	17.2
	None	19.0
Secondary	Textbook publisher workshops	40.8
	Other vendor (non-district) workshops	28.6
	Conferences	28.6
	None	36.4

Nineteen percent of the elementary teachers did not participate in any off-site professional development workshops. Of those who did, 59.4% of them had participated in workshops sponsored by textbook publishers, which topped the list. Close to 22% of the elementary teachers had participated in other types of workshops sponsored by either the central or local districts. About 17.2 % of the elementary teachers reported attending the 5-day Governor's Math Institute.

A little over 36% of the secondary math teachers did not participate in any off-site professional development workshops. Of those who did, 40.8% of them attended workshops sponsored by textbook publishers. A little less than 29% of the secondary math teachers attended workshops offered by other vendors. Similarly, 28.6% of the secondary math teachers had attended workshops at professional conferences.

Overall, the type of professional development activities that most elementary and secondary teachers had participated in was textbook publisher workshops that focused on how to use the new textbook series. In addition, secondary math teachers' participation in professional development activities in general was not as high as elementary teachers, since almost twice as many secondary math teachers had not involved in any types of professional development activities as elementary teachers (i.e., 36.4% vs. 19.0%).

Examinations of teachers' responses raised questions about the extent to which teachers had benefited from their participation in these activities. Teachers had difficulties recalling the content, the titles, and sponsors of these trainings. These responses call into question whether teachers' practice had benefited from their participation in these workshops. As one teacher commented:

> I did a five day thing, the acronym is LUCI, L-U-C-I, and it's in conjunction with UCLA or the University of California and I quite frankly thought it was useless, a waste of the taxpayers' money, as did most of the other people in the class think too.

Another teacher remembered the $500 that was offered:

> I was up to Lake Arrowhead and got and--- whatever the thing was for LA Unified, I got my five hundred dollars.

Some teachers also commented on the textbook professional development. As one teacher said:

> Well I went to one during the summer for teaching this program by the company that makes the math program itself, Harcourt Brace. A sales representative tried to teach us about some math and was giving us a

program that we had already brought. So it was kind of worthless. **[Interviewer interjection: How long was that workshop?]** One day.

Another teacher shared similar experience:

> Well we just had a workshop in the use of our textbooks by McDougall Littell. I don't know the exact name. But we just did that. **[Interviewer interjection: OK, any others?]** No, I don't remember right now.

Next, we examined the types of on-site professional development activities that teachers reported having engaged in (see Table 5).

Table 5: Teachers' Participation in Professional Development Activities (On-Site)

School Level	Types of Activities	Percent (%)
Elementary	Math-coach sponsored workshops	52.1
	Bank time PD not by math coaches	47.9
	None	39.2
Secondary	Math-coach sponsored workshops	11.4
	Bank time PD not by math coaches	88.6
	None	42.1

Roughly 40% of the elementary teachers had not participated in any on-site professional development workshops. For those who did, 52.1% of them had attended on-site workshops sponsored by their math coaches, whereas 47.9% of them participated in professional development activities held as part of banked-time Tuesdays.

Approximately 42% of the secondary math teachers had not attended any on-site professional development workshops. For those who did, 11.4% had participated in workshops sponsored by their math coaches, whereas 88.6% attended school-based professional development activities held as part of banked-time Tuesdays.

Overall, math coaches played a key role in sponsoring school based professional development workshops in elementary schools, whereas banked-time Tuesdays functioned as the key channel for on-site professional development opportunities for secondary math teachers.

Examinations of teachers' descriptions of their participation in these on-site professional development activities showed, however, that these activities might have had little impact upon teachers. Most teachers were not able to provide specific descriptions of activities they had participated in. As one teacher put it, "I can't remember, because we have it every Tuesday." Or as another teacher said, "I can't give you the name of the title. They were staff development and several teachers gave math workshops, so I can't give you the exact title what they did. One had to do with an abacus, using an abacus and the other one I don't remember the title that the teacher gave us." One teacher did describe one thing that she had learned and how she tried it with her students:

> We had someone who had gone to a math training, I'm not sure who it was, who brought back some of their hands-on ideas, one of which is the multiplication card game. I don't know if you've been here when we've used it. But if you're at school another time when I'm doing it, come in. It's great. The kids each get a card and they have answer and problem. And the child calls out the solution and that person has to tell the problem. They have the card with the problem on it. It's very fun and very fast paced and they love it.

This teacher's description, however, indicated that the hands-on idea of using a multiplication card game was merely for fun. There was no indication as to how the hands-on idea this teacher had learned from the workshop had helped or improved her teaching of multiplication. In contrast to this teacher who had at least tried to use an idea she learned in her class, another teacher was antagonistic to the professional development activities provided:

> Professional development workshops, we just had one the day before yesterday about computer software which I didn't agree with. But there's really been very few professional developments that are of any use other than pushing someone else's agenda such as the program or collaborative learning type lessons, which I disagree with. So they've all been a waste of time so I don't really remember, I choose not to remember very many of them.

To summarize, teachers reported participating in professional development workshops sponsored by the district. The focus of these trainings was mainly on how to use the new textbook series. On-site professional development activities in elementary schools were mainly sponsored by the math coaches, while those in secondary schools were primarily staff development on banked-time Tuesdays. Teachers' descriptions of these workshops in general suggested that they probably had little impact.

<u>What types of support for math instruction had teachers received at school?</u> We asked teachers what kinds of support for math instruction they had received at school.

Table 6: Types of Support for Math Instruction Teachers Had Received at School

School Level	Types of Activities	Percent (%)
Elementary	Math coach support	71.1
	Peer support	17.8
	None	43.0
Secondary	Math coach support	35.8
	Administrator support	22.6
	Peer support	49.1
	None	31.2

Math coaches were the primarily provider of support for math instruction for teachers in elementary schools (see Table 6). Roughly 71% of the elementary teachers reported receiving support from their math coaches. Peer support, albeit not very common (18%), was another main source of support for elementary teachers. In contrast, peer support functioned as a main type of support for secondary math teachers (49.1%). Support from math coaches was the second most frequently mentioned type of support for secondary math teachers (35.8%). Additionally, secondary math teachers (22.6%) reported receiving support from their administrators (principals, assistant principals, or math department chairs). Finally, a considerable number of teachers in both elementary (43.0%) and secondary schools (31.2%) had not received any support for math instruction at their schools.

The next section will discuss the nature of teachers' involvement in coaching practices in detail. With respect to peer support, examinations of teachers' descriptions showed that this type of peer support was mostly informal and unsystematic, occurring mainly at staff meetings. As one teacher put it:

> We have only our own staff development when we sit as a department and as a group for each level and we discuss what we should do to improve the success rate for our students, which we have within our department.

Another teacher described similar experience:

> The mathematics teachers will get together every once in a while and we discuss everything from students to textbooks to mathematical subjects and, you know, we act as our own resource people, we help each other.

Administrators' support normally took the form of providing instructional materials to teachers (e.g., the department chair is always giving us new materials for us to go through). From teachers' perspective, therefore, the kinds of support for math instruction at schools primarily came from math coaches (particularly in elementary schools) and peers (particularly in secondary schools).

Table 7: Administrators' Observations of Math Instruction

School Level	How Often	Percent (%)
Elementary	Never	5.6
	Three to five times per year	5.6
	Once a month	33.3
	Two or three times a month	44.4
	Each week	11.1
Secondary	Never	48.4
	Once per year	6.5
	Twice per year	16.1
	Three to five times per year	9.7
	Two or three times a month	3.2
	Each week	16.1

The majority of administrators in elementary schools (primarily principals) conducted classroom visits during math instruction at least monthly. About 33.3% of them observed math instruction once a month, whereas 44.4% observed twice or three times a month. In secondary schools, however, about half (48.4%) of the administrators (i.e., principals and math department chairs) had never conducted non-evaluative observations of math instruction. Secondary administrators, when they did report observing math instruction, did so much less frequently than elementary administrators.

Examinations of administrators' descriptions suggest that more than 75% of these classroom observations did not focus on math teaching practices per se, but on rather broad areas. Additionally these classroom visits usually occur in an informal way (e.g., drop-in), particularly in secondary schools. For instance, one principal described the observation focus as:

> I'm looking at a classroom environment. I'm looking to see how the class is set up, what kind of stations they have. If they have a math word wall. I'm looking to see if the curriculum are there, supplementary materials that they are looking at the proximity of the teacher where the teachers actually instructing the classroom. I'm looking to see if the other kids are being engaged. I'm looking to see if they are actually teaching from our standards and their standards are posted. I'm looking to see if kids' work is available and visible on the board – on the walls of the both the board. I'm looking for samples of those work and seeing that the standards are there as well as rubrics and criteria charts to see if the work has been actually scored using the rubrics and seeing that the kids are given some type of feedback, written feedback on their work. I look for things that they were learning for the most part. If there's an opportunity to talk to a kid and if the kid understands the lesson that they're doing, and why

> they're doing it. And if they understand, and if they understand how to actually score their work based on what the rubric is. I'm looking to see if there's any teaching plan, and also looking to see if the sequence of the lesson is following the first page of the plan, given to the staff by the math coach.

In secondary schools, it is not usual for math department chairs to conduct classroom observations. This kind of activity is often regarded as the role of administrators who will undertake this activity during formal evaluations of teachers. For instance, when asked how often the chair visits math teachers (even though we emphasized that we meant non-evaluative observations), one chair replied:

> One thing you need to understand. I'm not a supervisor. So I have no involvement with Stull evaluations. In fact, it would be evaluations. Other than that just informally, sometimes when I go in to talk to people about other things like, you know, look and see what's going on occasionally. But like I say, it's not any systematic thing where I'm looking for something particular. It's not systematic observation in that sense.

Another chair described his observations as:

> No, I don't have a procedure that I can put my hands on right now. I just keep my mouth shout and observe. I try not to disturb what they're doing. And I certainly wouldn't want to undermine what they're doing.

It seems that classroom observations in secondary schools by someone other than an administrator who plays the role of teacher evaluations occur only in the context of a mentor-mentee relationship. Otherwise, having another teacher in the classroom is considered as having an authority present, as one chair described:

> I try to make myself as inconspicuous as possible, so as not to disrupt the class. Because if there's another teacher there, that they think may be an authority, even if they're wrong, in this case they're wrong, but the children may view it that way. And they may quite often see themselves as getting the teacher in trouble. So, I try to be very inconspicuous and not stay too long at a time. I don't write anything. I just kind of walk through and do things very briefly and get out. I have stayed in there a while, the classroom was perfectly quiet, the moment I leave out the door, they go crazy, 'cause they think they have to be quiet for me and then they revert back to the way they were before. So I work with the teacher afterwards on how they can do things as far as classroom management and not put them on the spot, 'cause I want the students to feel that they are in complete control. They are the authority, not anyone else, not an administrator, not another teacher. No one that walks into the room has more authority than the teacher in the room.

Therefore, the support that teachers received (particularly secondary math teachers) in the form of classroom observations was limited, even though such observations can be very beneficial. As one math chair described:

> The only tenured math teacher I've observed was when we did our Learning Walk. And that was enlightening to me. I was interested to watch him work. But that was the only time. I was interested to see how differently he does things than I do. I was watching how he was doing things.

In addition to examining administrators' non-evaluative observations of math instruction, we looked at the kinds of professional development workshops and other support related to math that administrators reported were available to teachers at their schools.

Table 8: Math-Related Opportunities and Support That Were Available to Teachers

	Categories	Elementary	Secondary
Workshops	District sponsored	72.2%	71.0%
	Coach sponsored	38.9%	9.7%
	Outside consultant	11.1%	3.2%
Other Support	Resources (manipulatives, and so on)	16.7%	22.6%
	Peer support	27.8%	67.7%
	Coach support	11.1%	19.4%

The majority of the math-related professional development workshops, according to school administrators, were sponsored by the district (central and/or local) (see Table 8). 72.2% of the elementary and 71.0% of the secondary administrators reported that teachers at their schools had been to these workshops. In addition, 38.9% of the elementary administrators indicated that math coaches had provided on-site workshops to the teachers, whereas 9.7% of the secondary administrators said the same thing. Schools occasionally also invited outside consultants to provide workshops to their teachers (11.1% elementary and 3.2% secondary). Other resources that were available to teachers included instructional materials, and peer and coach support (excluding receiving trainings from coaches at the workshops). Peer support, consisting of mostly informal conversations or formal discussions at grade level/departmental meetings, was the primary source of support for math teachers (27.8% elementary and 67.7% secondary).

Other types of support such as providing instructional materials or using coach help were less common than peer support (16.7% and 11.1% elementary, 22.6% and 19.4% secondary).

Teachers' involvement in coaching practice. "Coaching" can be defined as a non-evaluative relationship in which a teacher provides another teacher with opportunities for demonstration, practice, feedback, reflection, and/or collaborative problem solving. We first examined the coaching involvement from the teachers' perspective (see Table 9).

Table 9: Key Coaching Practices and Teacher Involvement

	Categories	Teachers
Elementary	Classroom observations	33.3
	Model lessons	58.3
	Answer questions	27.8
	Help with lesson plans	33.3
	Attending staff meetings	13.9
	None	35.7
Secondary	Classroom observations	41.2
	Model lessons	29.4
	Answer questions	29.4
	Help with lesson plans	35.3
	Attending staff meetings	11.8
	None	58.5

A considerable portion of teachers (35.7% elementary and 58.5% secondary) had not been involved in any of the key coaching practices during the first year implementation of DMP. For those who did, the most frequently reported coaching activity that teachers had engaged in was model lessons (58.3%) for elementary teachers and classroom observations (41.2%) for secondary math teachers. Classroom observations (33.3%) and help with lesson plans (33.3%) were also popular coaching activities that elementary teachers had participated in, whereas help with lesson plans (35.3%), model lessons (29.4%), and answer questions (29.4%) were the popular coach services that secondary math teachers had used. Teachers (13.9% elementary and 11.8% secondary) also reported that math coaches had attended school staff meetings.

Except for a few cases, teachers reported positive experiences with their math coaches. For instance, one teacher described the help she had received from her math coach as:

> My math coach spends a lot of time with me helping me. If I have any questions she's available. She will come in and she demo'd maybe three lessons for me. She's also observed me. She has helped me do some plans as well.

Another teacher commented on how wonderful a resource her math coach was:

> I just want to say that, I was thinking for a minute, as far as our math coach is concerned I really love having a person, a go-to person at this school, especially someone like [our coach] who has this depth of understanding and materials for us to go to, because when you're, sometimes you're just hitting your head against the wall with some kids, you don't know what strategies, and I've got five years of teaching but there is so much that I don't know about different ways of teaching, materials, things like that and I have to say whenever I go to my coach with a question or need something she's there and, you know, gives me a different way of looking at something or a different approach, and, I think that it's wonderful to have somebody like that as well as our Open Court coach, because these people have so much experience and I'm so glad that

the coaching program is pulling in these experienced people that have so much to share and, I wish there was more of that. When I was a newer teacher it was – oh it felt that you got the teacher guide handed to you and it's like go to it and there was no support, everybody is so darn busy, nobody has time to sit down and I wish that, we could have more time set aside for regular meetings with these people, but it seems like they're here or they have to be at other meetings or [***interviewer interjection: Other schools***] Yeah, other schools, they are just all over the place so.

According to their self-report, despite the potential benefits of math coaches, the majority of the teachers had not had the one-on-one opportunity to work with their coaches.

Math coaches, however, reported frequent activities (on a daily or weekly basis) in each of the main coaching activities (see Table 10). Except for lesson planning, more than 70% of the math coaches had reported participating in each activity on a daily or weekly basis. This seems somewhat inconsistent with teachers' reported involvement in coaching practices, because the majority of the teachers reported having not participated at all in coaching activities. It is possible that coaches were involved frequently with only a limited number of teachers, because only 22.4% of the school administrators reported widespread use of coaching for math instruction at their schools.

Table 10: Coach Activities

Activities	How often	Percentages
Demonstrations of math lessons	Daily	20.8
	Weekly	50.0
	Monthly	16.7
	Every few months	8.3
	Never	4.2
Classroom observations of math lessons	Daily	58.3
	Weekly	37.5
	Monthly	0
	Every few months	4.2
	Never	0
Lesson planning with teachers	Daily	4.2
	Weekly	54.2
	Monthly	33.3
	Every few months	0
	Never	8.3
Giving feedback based on classroom observations	Daily	33.3
	Weekly	58.3
	Monthly	4.2
	Every few months	4.2
	Never	0
Reflective conversations with teachers	Daily	25.0
	Weekly	58.3
	Monthly	12.5
	Every few months	4.2
	Never	0

Of course, providing teachers with resources to change is a necessary, but not sufficient condition for moving mathematics teaching and learning toward intellectually rigorous instruction. As pointed out earlier, such a movement requires ambitious and complex changes, and cannot be accomplished by simply telling teachers to implement effective practices. Rather teachers must take an active part in changing. Our next section will explore several complex, yet important issues in implementing DMP, conditions that might work for or against DMP in working towards meeting its goals.

Conditions under which DMP is effective in working towards meeting its goals

This section addresses four of the most important issues that are directly related to DMP's ability to work towards meeting its ultimate goals of improving mathematics teaching and learning in the district in order to improve students' mathematical competencies and give all students access to Algebra. These four issues are: (1) teachers' reported attitude towards and confidence in mathematics and in using different teaching strategies; (2) teachers' resistance to accepting, let alone trying out new ideas (e.g., coaching); (3) challenges encountered during the first year implementation of DMP in using coaching as one of the main support for math teachers; and (4) cultural beliefs about the nature of mathematics knowledge and its relationship to educational practices.

<u>Teachers' self-perceived attitude and confidence.</u> Teachers have positive attitudes towards math and high confidence in their mathematical ability (see Table 11). The majority of the teachers reported that mathematics is useful for problem solving (99.4%), that there is more than one way to solve a math problem (99.4%), and that math is not simply memorizing facts (76.9%). In addition, teachers felt that they were well prepared

or very well prepared for different main areas of mathematics. Teachers who report they are well (or very well) prepared in number sense (96.2%), measurement (87.8%), Geometry (88.4%), data analysis (83.3%), and Algebra (91.0%). Finally, teachers reported extensive or almost complete confidence in their knowledge of math (89.1%) and ability to using teaching strategies for math (86.5%).

Table 11: Teachers' Self-Perceived Attitude and Confidence

	Statements or Areas	Choice	Percentage
Attitude	Math is useful for solving problems.	Agree	99.4
	Math is memorizing facts	Disagree	76.9
	There is only one way to solve a problem.	Disagree	99.4
Preparedness	Number sense	Well prepared+	96.2
	Measurement	Well prepared+	87.8
	Geometry	Well prepared+	88.4
	Data analysis	Well prepared+	83.3
	Algebra	Well prepared+	91.0
Confidence	Math knowledge (in general)	Extensive/complete	89.1
	Teaching strategies	Extensive/complete	86.5

Administrators were less sanguine concerning teachers' content knowledge and ability to use teaching strategies. Fewer administrators than teachers reported having extensive or almost completed confidence in teachers' content knowledge (63.3%) and teachers' ability to use teaching strategies (48.9%). Similarly, even fewer math coaches than administrators reported having extensive or almost complete confidence in teachers' content knowledge (33.3%) and teachers' ability to use teaching strategies (33.3%).

Therefore, from teachers' perspective, it may sound strange if they are told that they need further learning and improvement. However, from administrators' and math coaches' perspectives, this may not be an unreasonable expectation. Furthermore, our classroom observations have shown that the discourse norm of mathematics teaching and learning has not changed to any measurable extent. As the snapshots of those classroom interactions indicate, mathematics teaching and learning are disconnected from teachers' reported beliefs that mathematics is useful for problem solving, that there is more than one way to solve a math problem, and that math is not simply memorizing facts (or steps). What teachers said was not what they did, for their stated beliefs were far more progressive than their teaching practice would have indicated.

A sharp contrast, therefore, exists between what we have observed in classrooms and teachers' self-perceived attitudes or confidence. Less extreme, but still evident, the contrast exists between teachers' self-perceptions and those of school administrators and math coaches. These contrasts suggest that teachers may believe that what they have been doing works and therefore needs no improvement. Such beliefs could explain teachers' resistance to trying out new ideas (e.g., coaching).

<u>Teachers' resistance to change.</u> Teachers' reliance on textbooks, worksheets, and homework was standard practice since the early 20^{th} century (Cuban, 1988). Learning a new mathematics, therefore, is much more formidable for teachers than students (Cohen, 1991), because teachers must un-learn the mathematics and teaching practices that they have used for decades. As one teacher put it: "It's sort of hard to get teachers to start something new". Another experienced teacher said, "Standards are pretty much the same. It's just the wording changes slightly."

The existence of such a mentality among teachers, particularly those who have been teaching for many years, has made new practices such as studying curricular materials and preparing a quality lesson plan – prerequisite for a quality lesson – a difficult one to implement.

This may explain why, while the DMP has successfully aligned math textbooks to the Standards, it has had little impact upon how teachers practice teaching. In fact, almost all teachers had a hard time answering the question of "how was the lesson planned?" A few secondary math teachers reported that the lesson was off the top of their head, because they have been teaching for many years. As one teacher put it, "I have taught it many times so it's off the top of my head at this point, but I know what specific questions I'm trying to bring out." While another teacher shared the same experience, "I've done it a long time and not much planning's necessary anymore." Several secondary math teachers frankly reported that they looked through the teacher's guide the night before (less than 30 minutes) and that was how the lesson was planned. As one teacher mentioned, "Usually I look at the book the night before and look at the topic and follow the examples in the book and I just make up real world situations of my own or drawings. Maybe a half hour's worth of preparation." The majority of the teachers gave vague responses even though they indicated that they planned according to the book and/or standards. As one teacher said, "I planned it – I made lesson plans and I made sure that I had all my equipment, which was the overhead, I use the overhead, I use the math manipulative kit. I did that and I made sure that the standards were posted."

Simply going through and following the procedures in the textbook will not change in any fundamental way what teachers and students do about mathematics when they are together in classrooms (Cuban, 1988; Elmore, 1996). The snapshots of classroom observations presented earlier clearly showed that the discourse norm of mathematics teaching and learning still conforms to the portrait of standard practice (Chazan, 2000):

> That mathematics classroom interaction, most often consists of teacher exposition, teachers' evaluative questioning of student, and student request for clarification. That the teacher is the sole authority for right answers and students memorize procedures and mechanically find answers. That most classrooms are collections of individuals in which mathematics is portrayed as a body of isolated concepts and procedures. That students are passive and that conversation about mathematics is rare. [p. 112]

Therefore, "the core of educational practice" (practice pertaining to mathematics in this context), using Elmore's term (1996), has yet to take the first step on its journey to change and improvement. Relying on the textbook alone will not bring out teaching for understanding, because understanding how different mathematical topics and ideas in the textbook are connected and how to present these topics to students to help them understand the concepts underlying these topics can arise only from careful studying of and thinking about what is presented in the textbook. Simply going through the procedures in the textbook will not solve the problem of students' lack of understanding of materials presented to them. As mentioned earlier, information described in the teachers' manuals may be useless if teachers do not recognize its significance or do not have time and energy for careful study of manuals (Ma, 1999). As one teacher described her use of the teacher manual:

> I like this new math plan because I don't have to do any prep work it's done for me. I don't have to worry about covering the standards; I mean

it's done. Somebody got paid to do all this, so my work is done. It takes a lot of the pressure off.

Barriers to coaching practice. As indicated earlier, "coaching" can be defined as a non-evaluative relationship in which a teacher provides another teacher with opportunities for demonstration, practice, feedback, reflection, and/or collaborative problem solving. Coaching practice, therefore, is intended as an instructional support for teachers. Teachers, however, may think or feel otherwise, which could be a barrier to coaching practice. As one teacher described, "If you ask for help you're criticized. So people don't ask for help anymore." In fact, close to half (i.e., 45.8%) of the math coaches reported "lack of teacher trust" and "teacher resistance to change" as two of the main barriers to coaching practice.

With respect to "lack of teacher trust", math coaches reported that teachers felt "threatened/scared", treated math coaches as "an administrator and evaluator", or felt that math coaches were there "to provide more work rather than to be used as a resource". How to build trust with teachers and establish productive rapport with them, therefore, seem to be of paramount importance in order to "open" teachers up to the idea of coaching practice. Only 50% of the math coaches reported "extensive" or "almost complete" confidence in their coaching skills, whereas 95.8% and 100% of these coaches reported "extensive" or "almost complete" confidence in math content and in using different strategies for math instruction, respectively.

"Teacher resistance to change" was another main barrier to coaching practice, according to math coaches. Some veteran/experienced teachers "think they know it all". Or teachers were reluctant to "try new things". As one math coach put it: "The only thing that I've seen is, a little reluctance to change to new ways of doing things." Another coach shared similar experience of this barrier: "Trying to get some of the veteran teachers convinced that I'm not coming in to tell them how to teach, but to have them feel comfortable in seeing alternatives for teaching strategies." In fact, if teachers were open to the idea of having someone come in to observe and/or demonstrate a lesson, they might learn alternative ways of doing things. Based upon the majority of the math coaches' experiences (54.2%), lesson demonstration was the most useful in their role as a math coach during the first year implementation of DMP. As one coach described:

> Just to show a teacher something new cause, you can make – you can see a lesson written in writing, you know, on a piece of paper and, but how do you do it, you know? There's a teacher there, she's never used Algebra tiles. And she said, you know, I'd like to use this, but I don't know how to do this. Could you come and show me? And I think, you know, we learn by seeing what people do.

Another coach shared her successful experience of getting some veteran teachers to open to her presence because of the opportunity for her to do lesson demonstrations in their classrooms. This was what she said:

> Some difficulties have come up especially with the veteran teachers, they are very limited about coaching, you know, and so mostly what I do is with the veteran teachers I volunteer to demonstrate in their classrooms and they like that and I think they learn from it too because since not all of them but most of them would say "You are welcome in my room any time", to teach. But I know that they did like me to get in there, which is good.

Besides teachers' resistance or lack of trust in math coaches, lack of time was another frequently reported barrier to coaching. According to math coaches, 33.3% of them had experienced difficulty in providing effective coaching due to lack of time, because they were assigned to work with two schools, which had made them "spread too thin". One consequence of this limited availability of math coaches who had to work with two schools was that teachers who would like to use them as resources for help did not get what they needed in time, which in turn, had caused certain degree of resentment (albeit uncommon) among teachers. As one teacher indicated:

> She [the math coach] seems to be unavailable or not understand what the definition of her position is. She's a very nice person but she usually comes in my room, kind of glances at what a child is doing. She's in there for ten minutes and leaves. She's handed me a paper one time on how to use the review at the end of the book that was so confusing I'm just doing it page by page. She just seems unavailable as if she's always around doing other things, but she's never really in working with the teachers.

Another teacher shared similar viewpoint:

> I'm very disappointed in the fact that we're supposed to have someone onsite that is supposed to assist us with strategies or anything else that we might be needing besides professional development. Yes. We do have one that we pay. And I've yet to see this person. I've yet to see the benefit from this person's presence on campus. As far as I can tell, the person that spends the majority of time doing other things, being elsewhere than going into a classroom, seeing what's going on, working with us, working with our students, improving their situation. Asking us what resources we might be needing that we have, that we may not have. Asking us anything that has to do with improving our math schools here. I don't want to get

into a personal issue. I don't want to say what I see this person doing when this person is supposed to be getting paid for what she does. But I'm very disappointed that LA Unified is supplying someone with us that isn't doing what they're supposed to be doing during working hours.

Being aware of this, the math coach of these two teachers expressed her concern of having to work with two schools:

> I don't think coaches should be at two schools. It's too inconsistent. You know if someone asks for something on fractions and by the time you get it it's three days before you come back, so they've either forgotten or found it or improvised themselves.

These teachers' sentiment had made this math coach to react that:

> At this school I would prefer I just work really with small groups for remediation. A few new teachers have been receptive for me to come in but this school has never had a coach before so they don't know what to do with a coach and they don't want a coach, and they have made that quite clear.

The lack of communication between the math coach and the teachers at the school due to the coach's limited availability having to work with two schools had been a severe hurdle for her to play an active and effective role. On top of this, the school administrator who was well aware of all this termed it as "a delicate situation". While emphasizing the fact that "it doesn't mean she's not a good coach", the principal nonetheless indicated that:

> The coach is a half-time coach. From what I understand and this is based on people coming in and telling me this is that. I don't think it's been a good experience. I think it's been a very negative experience. They're not too happy with the coach. They don't think, I don't know how to say it, but they just don't feel they're getting what they should be getting, you know, the support and the help. They feel that she's giving them too many dittos, she's not really spending the time to explain concept. It's more superficial. Most of the time she's not there, so it's not reliable because she has a lot of meetings. She's only there half the time and many times during that period she's not here because she's meeting. It's not a good attitude and they're not really utilizing her. So I don't know. I have talked to her about this, she is aware of it. And I told her to make herself available. She has her own personal feelings also about this school, it's not really positive. So it's not such a great thing. I think they're more or less – whatever they're doing, they're doing it on their own, you know. And next year, she will not be here any longer. We will not have a math coach and they're happy about that.

The school therefore had lost their math coach. This extreme case suggests that lack of time can really work against the effectiveness of coaching practice, as one coach put it:

> I think that it's a wonderful thing that the District has instituted and it could work very, very well. But I truly feel – I'm having a rather large success in being split into three schools, but I truly feel that the job was not designed for the coach to be split that way. The more time that the coach can spend at one school, I think the more positive changes can be made at that school.

In some cases, math coaches reported having difficulties scheduling activities with teachers (e.g., post-conference following a classroom observation where one-on-one reflective conversations took place), because teachers did not want to use their break time or stay after school. As one math coach described:

> Some of the problems that I find is finding time to talk to the teacher I observe right away because sometimes they're not ready to talk or they're going somewhere after that and we can't talk or some teachers just don't want to talk and I don't know how to go about that. Some teachers are waiting to talk but some of them just don't have time and I don't know how I can give them the right feedback or I don't know how I can help them better if I don't talk to them. Even if I give them written feedback I don't know if they agree with what I've written there and how I can defend whatever I wrote there or why they think that my suggestion is wrong. And in that way if we talk I could also learn from them or what are their weaknesses or why they're shy about talking to me. So there's just time constraints and coaching that is especially this school there are so many new teachers this year and I don't have time, sometimes I'm out of here going to training or we have meetings...

A third barrier to effective coaching practice is the inconsistency between the key role that math coaches are supposed to play and the role that they were actually playing at schools. Some math coaches were acting as tutors, sub-teachers, or administrative assistants, rather than being instructional or intellectual resources to teachers. The district had invested a lot in hiring and training math coaches. This has been a good thing, as one math coach described:

> It is a wonderful experience, it is giving me more insight into teaching and I am also learning for my personal expertise and knowledge. I have developed professionally a lot just during the year I started coaching.

But if coaches were spending their time doing all this but not working with teachers to support their math instruction, the benefits of all the trainings that math coaches have been receiving cannot even reach classrooms, let alone having any impact upon improving teacher practice and student achievement. As one math coach put it:

> This year was really rough because it's this math coach position's completely new in our District and overall in many Districts so they are still trying to organize how we were going to do it.

Another coach shared the same view:

> [S]ometimes they give me other responsibilities that I'm not supposed to be doing. So then I don't do what I'm supposed to do and those are the problems. But I enjoy helping teachers and I know that I have been successful with most of them. I love math and I love teaching math and I would like to share my knowledge of my strategies and my expertise because I think it's necessary for the kids to learn how to love math. Kids are scared of math because sometimes it's the way you teach it I think and I show them a fun way of teaching math, that's like my goal so that the children will love math and will not fear math. And I studied just what I told you, the math puzzles monthly and the principal loved that. At first nobody was participating, they don't even know that it exists but it's been since October so now a lot are participating and now I have a problem. But it's a nice problem.

To summarize, the three issues that we have addressed to this point (i.e., teachers' self-perceived attitude towards and confidence in mathematics, teachers' resistance to change, and barriers to coaching practice) are some of the main reasons why the typical practice of mathematics teaching and learning is so stable and resistant to outside influences. These barriers are reinforced by western cultural beliefs about the nature of knowledge, which in turn, has shaped standard educational practice in the United States the way it has been for more than a century. It is this issue that we now turn to address.

Cultural beliefs about the nature of mathematical knowledge and its relationship to educational practice. Building on Cuban's (1993) view of the role of cultural beliefs, Chazan (2000) has suggested that the widespread and deeply rooted cultural beliefs about the nature of knowledge, how teaching should occur, and how children should learn has a special flavor when we focus on mathematics instruction. According to Chazan, in western views of knowledge, mathematics is often described as the most certain branch of human knowledge. In mathematics, it is easy to distinguish "right" from "wrong". The notions central to this set of beliefs about mathematics instruction are that:

- All statements of school mathematics can be judged unequivocally right or wrong.
- A central role of the teacher is to exercise this judgment.
- These judgments can be used effectively to label students' "ability" or aptitude in mathematics. [p. 115]

Because of these beliefs, Chazan argued that teachers have difficulties in creating authentic conversations with students about mathematics. Naturally, if teachers know what is right or wrong, what is there to discuss? Therefore, in typical teacher-centered classrooms, a large chunk of instructional time is devoted to teacher lecture. Students ask clarifying questions. If there is any confusion on the student part, this is regarded as problematic and therefore further explanation or practice is required without a diagnosis of why students do not understand the materials. Besides a heavy emphasis on truth and correctness, it is a common practice in the United States to give much weight to ability, which unlike other societies that may emphasize effort rather than ability (Stevenson & Stigler, 1992; Stigler, Lee, & Stevenson, 1990). On top of all this, it is widely accepted in the United States that elementary mathematics is basic, superficial, commonly understood, and repetitive (Ma, 1999). Therefore, starting from the stage of building foundational skills, students are not taught or given the opportunities to reason mathematically, to communicate about mathematical ideas, and make connections among mathematical ideas and between mathematics and their own daily lives. However, when they do not do well on tests, they are labeled as "low ability" and are therefore held back and considered as problematic (e.g., lack of maturity to learn the topics, lack of motivation, disciplinary behaviors, and so on).

These problems came up among several secondary math teachers during our interviews. As one teacher described that Algebra 1 should be for 9th rather than for 8th graders, because the latter group was not "mature" enough:

> I would like to let the District know, especially my first year of teaching, that the pacing plan that they gave us is not planned according to the students – I mean I know they want – like for my Honors class I know they want the whole book covered, but it's impossible to get the whole book covered because of what they understand. At this grade, at 8th grade I know they want to make it mandatory that our 8th graders, you know,

> have Algebra. But you don't start taking Algebra till 9th grade. But I guess nowadays they want them to take Algebra in 8th grade, but the maturity level for the 8th graders is very difficult. The math will be really difficult because of the maturity level and that's a concern I have. If they want me to cover the whole book I'm gonna say I'm gonna go by what my students will learn. I will not cover – I will cover real quickly so that I can go all the way to chapter 12 and then they do not learn anything. You know, I want them to at least learn something I mean, so that they can carry it onto high school instead of just like OK you don't learn it, fine, whatever, just move on 'cause we need to get to chapter 12. I don't teach like that. I want my kids to know. If the majority understand, I will move on. If the majority do not understand, I'll go back and re-teach because I mean it's me. So my concern for the district is that they need to know, whoever makes this lesson plan up, please go into the classroom and observe and see what the students can do and what they can't do. But I have a very high expectations of my students, but at the same time the maturity level is not that high so I have to go with what they-they can pick up for that day. You know, I can't just zoom on and just keep on going. So they just need to kind of like is this gonna work in reality or is this not gonna work in reality. That's the main thing I wanted them to see. Yeah.

Another teacher expressed concerns with students' lack of work ethics and motivation and therefore they should be held accountable:

> I wish that the Board of Education would make the 8th graders pass their classes in order to go to the high school. The District's excuse is that they have to pass kids because they can't retain them, they don't have the room. Frankly I don't care if the kids have to sit on the floor, they should be held accountable, they should have to pass their classes, because they go to the high school and they're not prepared, they don't have the foundation, the

> kids that I have right now, a lot of them, not all of them, but a lot of them, don't do their homework, don't pay attention because they know they don't have to pass math to go to the high school, but when they go up there the new rule says everybody has to take Algebra and they're not going to have the foundation, they're not going to have the work ethic to say "Hey I've got to get my work done", they're not prepared emotionally, mentally, psychologically, and this is why a lot of them are failing, this is why the math scores are so dismal and low. We have got to toughen up, we have got to draw the line and say "Hey this is it, no more horse-playing, no more fooling around, we're holding you accountable, you don't pass you come back here next year", or else at least make Summer School mandatory, make these kids go to Summer School and take math, 'cause a lot of them aren't doing well in math or English, you know, make them go. They've got to be held accountable, at some point we've got to say "No, no you're not going to pass, no you're not going to make it and you better get your act together now because when you go out into the world it's going to be any easier and you're going to need every ounce of education that you can get". We've got to start doing something because it's not good the way it is and I blame the parents for a lot of it, parents need to be more involved, frankly they need to do a better job of raising their kids, and many of my kids don't respect me, too many of my kids laugh at me, too many of my kids don't turn in their homework, and when I say you're not doing well they look at me and shrug their shoulders, okay, so what. Got to draw that line somewhere.

However, Chazan showed through his own teaching of mathematics (Algebra) to lower-track secondary students that the notion of ability was problematic, because: (1) when students accept this label (i.e., low ability), they have a ready-made explanation for expecting that tasks posed for them are too difficult to attempt; whereas if teachers accept this label, their energies and efforts at understanding the students are undercut; and (2) the impact of the notion of ability on classroom dynamics in less advanced track classrooms can be two-fold. If students are so used to being judged as "low ability", they are afraid to share their thinking because they are concerned that once again they may be evaluated to show how little they know. On the other hand, students may not listen to one another, because if they have all done poorly in mathematics, why should they listen to each other?

These two dynamics in classrooms consisting of primarily less advanced students make it particularly challenging for teachers who desire to make mathematics teaching and learning to be meaningful. As shown previously, some teachers expressed their frustration over the students' lack of motivation, work ethics, or readiness, and so on. However, as Chazan pointed out, it is problematic for both teachers and students to accept the notion of "low ability" (and its associated terms such as effort, motivation, and so on). It would be interesting to hear students' experiences on these issues from their perspectives. This will be one of the topics for next year's evaluation report.

SUMMARY AND DISCUSSION

The 5-year district mathematics plan (DMP) aims to improve students' mathematical competencies and give all students access to Algebra. DMP intends to achieve this goal through alignment of math curriculum with the Standards and provision of on-going professional development to teachers. The focus of the current evaluation report, therefore, is on examining the extent to which DMP initiatives (e.g., adopting standard-based textbooks) and professional development opportunities (e.g., coaching) have led to changes in classroom practices and student outcomes.

The results of this evaluation revealed that although there was a trend of improvement in students' SAT/9 math total scores at some grade levels following the first year implementation of DMP, the improvement was in procedures scores rather than in problem solving scores. Moreover, only modest percentages of students reached the proficient or advanced level on the California Math Standards Test and the California Standards Tests in Algebra 1, Geometry, and Algebra 2.

These achievement results are consistent with the discourse norm of mathematics teaching and learning that we have observed since spring 2001 (i.e., the baseline). Mathematics instruction continues to put a heavy emphasis on computational and procedural skills, instead of conceptual understanding and problem solving.

The reason why we have seen only limited improvement in the way mathematics is being taught has to do with teachers' limited opportunities to learn new ideas about mathematics and about new teaching practices. Examinations of teachers' opportunities to learn through their participation in various professional development activities (e.g., workshops, instructional support, coaching practice) suggest that teachers may have not acquired the skills and understandings intended from participation in these activities.

There were several issues that might explain why there has not been more evident improvement in mathematics teaching than what we have been seeing. These issues are related directly to DMP's ability to work towards meeting its goals, which include teachers' self-perceived attitudes towards and confidence in mathematics and in using different teaching strategies, teachers' resistance to new ideas and change, barriers to coaching practice, and cultural beliefs about the nature of mathematics knowledge and its relationship to educational practices.

Specifically, this evaluation yielded the following findings:

1. Following the first year implementation of DMP, SAT/9 math total scores increased between 2001 and 2002 more than they did between 2000 and 2001 at the 2nd and 3rd grade levels. Grades 4-5 and grades 10-11 also saw slight improvement in SAT/9 matched total math gain scores following the first year implementation of DMP. No improvement in SAT/9 matched total math gain scores was observed at grades 6-8.
2. For year 2002, SAT/9 math scores on procedural items were substantially higher than those on problem solving items at grades 1-3. SAT/9 math scores on procedural items were about the same as those on problem solving items at grades 4-5, 7; whereas the scores on procedural items were moderately lower than those on problem solving items at grades 6 and 8.
3. For year 2002, the percent of students who performed at the proficient or advanced levels on the California Math Standards Test ranged from 10.1% to 34.1% across grades 2-11. The percent of students who performed at the proficient or advanced levels on the California Math Standards Test declined with each increase in schooling level: on a percentage basis, more elementary school students performed at the proficient or advanced levels on the California Math Standards Test than students at middle schools, who in turn, outscored high school students.
4. Younger students outscore their older peers at every math level. In Algebra 1, more 8th grade students achieve proficiency (22%) than 9th (7.9%), 10th (4.7%) or 11th (5.5%) grade students. This pattern holds for Geometry: 8th (48%), 9th

(22.8%), 10th (7.3%), and 11th (4.8%); and for Algebra 2: 9th (28%), 10th (19.7%), and 11th (5.4%).

5. Classroom observations indicated that the discourse norm of mathematics teaching and learning (i.e., the way mathematics knowledge is presented, the roles teacher and students played, the way they interact about mathematical knowledge in classrooms) remained remarkably stable during the first year of implementation of DMP. In other words, mathematics instruction continues to put a heavy emphasis on computational and procedural skills, instead of conceptual understanding and problem solving.
6. Teachers reported most off-site district-sponsored professional development workshops in mathematics focused on how to use the new textbook series. Teachers' descriptions of their experiences indicate that they may have not yet acquired the skills and understandings intended from participation in these activities.
7. Teachers reported only limited involvement with other ongoing professional development opportunities such as on-site staff development workshops, instructional leadership from administrators (e.g., classroom observations that focused on the quality of instruction), and teacher involvement in coaching practices.
8. Teachers predominantly reported positive attitudes towards mathematics and a high level of confidence in their subject knowledge and pedagogy. In contrast, math coaches expressed less confidence in teachers' content knowledge and ability to use different teaching strategies. Administrators' confidence in

teachers' content knowledge and ability to use teaching strategies lay between that of the teachers and of the math coaches.

9. Math coaches reported teachers' resistance to change, teachers' lack of trust, and time constraints as the top three barriers to coaching practice. Teachers, coaches, and administrators expressed concern over the lack of time, particularly for those math coaches who had to work with more than one school.
10. Math coaches' confidence in their coaching skills was not as high as their confidence in their math content knowledge and in their ability to use different teaching strategies.

In summary, although there has been a trend of improvement in students' SAT/9 math total scores at some grade levels following the first year implementation of DMP, there are still many challenges to improving mathematics teaching and learning in the direction of conceptual understanding and problem solving. One of the most difficult challenges will be how to engage and motivate teachers to adopt new beliefs and practices through providing them with ongoing professional development opportunities that will enable them to learn something of lasting value (not merely focusing on textbooks). Unless this challenge is overcome, it will be difficult to accomplish the DMP's goal of improving students' mathematical competencies and providing all students access to Algebra.

FUTURE DATA COLLECTION AND OUR EVALUATION

Our future data collection will continue to investigate the influence of the initiatives outlined within the District Mathematics Plan on mathematics teaching and learning in the district. Specifically, we will attempt to address the following areas in-depth:

- The quality and content of various on-going professional development activities
- The impact of these activities upon teaching practices and student achievement
- The students' experiences of learning mathematics
- The evidence of change (if any) in teaching practices and student performance and how this is related to DMP's initiatives and teachers' opportunities to learn through various DMP professional development activities

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APPENDIX

AUDIENCE FOR THE REPORT

The Program Evaluation and Research Branch provides the evaluative and research component that is required of all organizations delivering educational programs in LAUSD. The intended primary audience for the report includes:

- Board members
- Senior district leadership (e.g., Superintendent and his chief staff)
- Program managers
- Teachers

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