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ABSTRACT

This report contains four papers reflecting the richness and energy of the issues surrounding national standards in mathematics and science. Papers include: (1) "Are the NCTM Standards Suitable for Systemic Adoption?" (Deborah Tepper Harino); (2) "A Mathematician Looks at National Standards" (Judy Roitman); (3) "Comments about NCTM's Curriculum Standards" (Thomas A. Romberg); and (4) "A Commentary on the Profound Changes Expected by the National Science Standards" (John C. Wright and Carol S. Wright). The dominant issue discussed and debated in the papers is the shift in emphasis from memorizing procedures to problem solving and understanding. The notable common theme across the papers is sympathy with the twin goals of the standards to reach more students and make mathematics and science more interesting and meaningful to teach and learn. Within this broad umbrella, the papers differ in the degree of skepticism about how well the standards achieve the objectives. (KHR)

National Institute for Science Education

University of Wisconsin-Madison . National Center for *Improving* Science Education

Occasional Paper No. 3

Commentaries on Mathematics and Science Standards

William H. Clune, Deborah Tepper Haimo, Judy Roitman,
Thomas Romberg, and John C. Wright and Carol S. Wright

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National Institute for Science Education
University of Wisconsin-Madison

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INTRODUCTION

The National Standards in Mathematics and Science: Developing Consensus, Unresolved Issues, and Unfinished Business

William H. Clune

William H. Clune is Voss-Bascom Professor of Law at the University of Wisconsin Law School, Director of the Policy Group of the National Institute for Science Education, and a senior researcher of the Consortium for Policy Research in Education (CPRE). His past research has included school finance, school law, implementation, special education, public employee interest arbitration, school site autonomy, effects of high school graduation requirements, upgrading of the high school curriculum in mathematics and science, and systemic educational policy. His present research includes "program adequacy" (the cost and implementation structure needed to reach high minimum levels of student achievement in low-income schools) and systemic policy in mathematics and science education.

The four papers in this set of commentaries reflect the richness and energy of the issues surrounding national standards in mathematics and science as they begin to reach a wider audience and take shape in implementation. The dominant issue discussed and debated in the papers is the shift in emphasis from memorizing procedures (calculations) to problem solving and understanding. The authors themselves make an interesting group in this respect, comprising three university professors of mathematics or science with longstanding interest in educational reform (Haimo/mathematics, Roitman/mathematics, John Wright/chemistry), a professor of education and the Chair of the standards committee of the standard-setting National Council of Teachers of Mathematics (NCTM standards, Romberg),, and a teacher/ administrator with hands-on experience implementing the new standards (Carol Wright).

A notable common theme across the papers is sympathy with the twin goals of the standards to (a) reach more students and (b) make mathematics and science more interesting and meaningful to teach and learn. Within this broad umbrella the papers differ in the degree of skepticism about how well the standards achieve the objectives. Most skeptical is D. T. Haimo who sees the potential for major distortion, confusion, and lowering of the quality of mathematics content and instruction. Judy Roitman, the other mathematics professor, gives a more favorable but still mixed review, seeing many of the same problems but also many strong points. With his long involvement in the development of the mathematics standards, Tom Romberg tends to view the criticisms as refinements if not quibbles and sees the reforms as a quantum improvement for students who historically received rudimentary and inferior instruction. But Romberg does concede the force of many of the criticisms in pointing to a new round of revisions of the mathematics standards that emphasize the importance of traditional procedures (calculations), the integration of problem solving with content, and the details of curriculum and instruction. The Wrights are unqualifiedly enthusiastic about the science standards (calling them "brilliant") but skeptical about the capacity of our current teaching force and social culture to implement and accept such lofty and ambitious goals-the education of an entire population capable of independent inquiry and critical reflection.

The papers are fascinating in themselves, and I will not attempt a summary or synthesis. Instead, I look across the papers for areas of agreement, disagreement, and a shared sense of the incomplete agenda. I conclude that many of the issues appropriately joined at the conceptual level over the standards can only be resolved in the context of real curricula and instruction, where specific tradeoffs between competing goals and risks are made and can be evaluated.

The area of consensus over goals-more than the "right answer"

An interesting place to start a discussion of the goals of the standards is Romberg's description of the origins and impetus behind the mathematics standards-the highly stratified but overall low level of traditional mathematics instruction among the nation's students. A decade ago, Romberg tells us, about 40% of American students stopped at eighth-grade mathematics, another 30% at the second high school course, another 20% with enough to qualify for selective colleges, and 10% with enough to prepare for scientific training in college.

> Our long-term objective was to change the percentages—40%, 30%, 20%, and 10%—by focusing our work on the needed changes for the 90% of American students who took the least mathematics. (p. 44)

In light of this comment, we might ask ourselves why reform should not consist of simple upgrading and acceleration-more students taking traditional courses. In fact, that is one common strand of reform, sometimes called "intensification," and one that is clearly responsible for some of the gains in student achievement that occurred during the 1980s. Let us be clear about this. Despite the protestations of constructivists, substitution of Algebra for General Mathematics and getting more students to reach Calculus are legitimate goals of reform.

But there is another side of reform captured in Romberg's disparagement of eighth-grade mathematics as "shopkeeper mathematics":

> These students were expected to learn only paper-and-pencil calculations and routines for whole numbers, common fractions, decimals, and percents. (p. 43)
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> Today no one makes a living doing paper-and-pencil calculations. Calculators and computers have replaced shopkeeper calculations in business and industry. . . . The fields that use new technologies are growing rapidly and often require a deep understanding of traditional mathematical topics as well as some topics not in current school courses. (p. 44)

This dislike of mindless calculation has broad support among reformers and the other authors of our papers. If the nightmare of traditionalists is kids who can't get the right answer, the nightmare of other reformers is kids who don't know what a right answer means. Roitman begins her essay with the example of students who, when asked how many buses at 23 children each would be required to carry 121 students, answered 5 6/23, or rounded down to 5. "Our kids could calculate, but they couldn't make sense" (p. 23). D. T. Haimo says that,

> when teachers themselves have little feel for what they are doing, cannot really explain why they perform certain operations or where the subject is heading, they teach mechanically, stressing only formalisms. . . . [T]heir students suffer the consequences. (p. 7)

And for the Wrights, thinking, not calculation, is the essence of science: inquiry, self-confident discovery, disciplined criticism, cooperative problem-solving. This "making sense" of mathematics and science, sometimes called "teaching for understanding" has a special connection with equity. Disadvantaged students often have been taught the lowest level, most mindless version of basic skills and may have a special need for instruction in complex problem solving. For these kinds of reasons, the NCTM standards for mathematics included four "process standards": problem solving, communication, reasoning, and connections.

But the consensus on mathematical and scientific reasoning is deeper yet. Disapproval of "mindless calculations" is not the same as disapproving all calculations or precision. Approval of intuitive "making sense" does not imply disapproval of abstraction. Indeed, all of the authors agree that some kind of powerful, exact conceptual framework or approach-reification as the articles call it-is the essential outcome of the whole learning process. For example, the Wrights say:

> Technology such as computer animations can provide assistance in helping the students to form mental pictures that interrelate physical quantities, but they cannot substitute for the mental pictures that must form [in students' minds] if reification is to occur. As one progresses, mathematics enters and the level of abstraction increases. (p. 57)

From the other side Roitman tells us, "My own work, for example, is in the very abstract world of set-theoretic applications to Boolean algebra and general topology, but I cannot think clearly without using things like dots, lines, and circles" (p. 26, note 2).

The area of disagreement-applications, amateur problem-solving, and the risk of junk mathematics and science

While the approval of making sense and teaching for understanding is almost universal, critics of the mathematics standards see the emphasis on applications, intuitive problem solving, and active learning by students as prone to serious errors. (The science standards have not been so heavily criticized, perhaps because they are much newer.) In the language of investment, the new elements have a greater downside risk because of vagueness, ideology, self-delusion, and unrealistic demands on the nation's teachers.

Haimo articulates the risks of wholesale reform in a way reminiscent of Edmund Burke's critique of the excesses of the French Revolution:

> The drastic abandonment of every aspect of the "traditional" ways might sound good and be appealing theoretically, but it has not yet been shown that it can produce students with greater understanding of mathematics concepts. The "old ways" at least have withstood the test of time so that their strengths as well as their flaws have become clear. Further, over time, some trouble spots have been eliminated and some changes have been made. On the other hand, using the proposed reform to correct all the ills of the past by replacing everything on all fronts with untried proposals is a dangerous route to follow. (p. 18)

The papers cite examples of serious problems in the standards and real classroom practice, for example:

- problems and applications that are vague, overly complex, technically incorrect, and (surprisingly) needlessly technical

- teachers who obviously do not understand the underlying mathematical or scientific principles and who completely overlook both gross errors and powerful insights of their students

- an emphasis on applications and cross-disciplinary problem solving to the exclusion of core subject-matter content.

Such criticisms appear to be having an impact and may produce a new level of consensus. Romberg mentions that the following revisions are planned for the next set of NCTM curriculum standards: the integration of process standards like problem solving with mathematical content, the addition of a fifth process standard on "procedures or routines," increased emphasis on content strands across grade levels (e.g., number, algebra, geometry, statistics), and a careful review of all examples and applications.

Conclusion: Getting specific, encouraging alternatives, and assessing the outcomes

The dominant impression I get from these papers is the need for developing and implementing specific curricula and teaching methods so that the debate can move from the general to the concrete. Ideological broadsides quickly lose their usefulness and become empty slogans: "applications vs. calculations," "problem-solving vs. traditional content," "basics vs. higher order thinking." Sometimes battles between rival camps like the "constructivists" and "mathematically correct" seem aimed more at winning symbolic political victories than real change in the teaching and learning for real children. Most of the ideological dichotomies are misleading or downright false. For example, traditional formal mathematics eventually lends itself to numerous applications in engineering. And constructivist curricula usually build around the framework of traditional mathematics. Development of real curricula forces the proponents of an educational philosophy to become specific about the tradeoffs of different educational goals and risks that are necessary in the limited time available for instruction. As the Wrights conclude, "The standards need to be transformed into workable teaching plans" (p. 65).

So, we desperately need development, implementation,. and evaluation of alternative curricula, each involving an explicitly stated mix of alternative goals. Pieces of new curricula now exist, often developed under sponsorship of the National Science Foundation, including in the sites of the Systemic Initiatives (states, districts, schools). These curricula and methods of instruction put different emphasis on different social goals (preparation for advanced training vs. general vocational competence), the blend of student discovery and control by the teacher, the mix of intuitive applications and formal disciplinary content, and even the mix of enjoyment and hard work. Complete integrated curricula spanning multiple grades are harder to find, raising the disturbing possibility that multiple, disjointed curriculum reform may produce its own kind of "splintered curriculum."

In principle all curricula and programs of instruction can be evaluated in terms of some common set of criteria, such as the four questions recommended by Roitman for evaluating technology in the classroom (and adapted from the assessment standards of the NCTM). As paraphrased by me in terms of curriculum, the four questions are (1) What mathematical or scientific content is reflected? (2) What efforts are made to ensure that the content is significant and correct? (3) Does the curriculum engage the students in realistic and worthwhile mathematical and scientific activities? (4) Does the curriculum produce a deep understanding of aspects of the subject matter that are important to know and be able to do? To these, we might add the acid test for equity suggested by Haimo and Roitman: Regardless of rationale, does the curriculum lower expectations and constrict opportunities for students at any range of performance and achievement? Put positively, the goal of equity must always be twofold: increased access together with higher standards.

Under any set of fair criteria, it seems likely that different kinds of curricula can, in principle, be judged to possess high quality. Unfortunately, here we encounter a possible tension between curriculum diversity and systemic reform. All four papers agree that student achievement-not ideology or rhetoric-is the ultimate test for reform. Since systemic reform is aimed at change in the "whole system," there is a powerful tendency to adopt a single measure of student achievement implying a single authorized curriculum.

Consider, for example, the much publicized case of Michigan students who performed at the top levels of high school grades and the SAT and ACT tests but less well on a new state test emphasizing different kinds of knowledge and skills. My guess is that students who do well on either set of assessments will make good mathematicians and scientists. If that is not true, we certainly need to know. But if the truth is that alternative curricula produce alternatively accomplished students, why should systemic reform be used to champion one and subdue the other? The answer cannot be that all students must be rated on the same test to facilitate selective admission to higher education. American universities already admit students from many different states with various kinds of curricula, grading practices,, and standardized tests.

Any new course or test must establish its credibility and quality with external audiences, such as the labor market and professionals in higher education; otherwise, high performance would not serve as a useful credential. But once quality has been established, why not let states, districts, schools, and even students select from among alternative curricula and tests? A 90th percentile on a test of mathematically correct mathematics ought to be as good as the 90th percentile on a test of deep-inquiry mathematics if they are both of high quality. Systemic reform means higher standards for all, not exactly the same standards or curriculum. In New York, the Regents program coexists with Advanced Placement, each with its own set of examinations. ChemCom is taught in the same schools as traditional Chemistry. Montana is developing a new mathematics curriculum for grades 9-10 that puts more emphasis on probability and statistics than algebra and geometry (as well as on technology); but students can take more traditional courses later in high school.

Somehow the legislated world of entrance requirements and tests must be made to conform to the real world of quality alternatives rather than destroying quality and diversity out of a divisive quest for uniformity. I'm not sure that there is any great harm done if a state does succeed in establishing a single high quality test as the sole measure of achievement in its schools, and perhaps states are the logical unit for experimentation and diversity. Most standardized tests do seem to change gradually in response to reform, by incorporating new items and testing formats (e.g., complex problem solving, actual speaking of foreign languages). Unfortunately, experience has taught us that, in the battle for total victory or defeat, the potential for divisive politics is also quite high. A quest for uniformity could turn ideological wars into real ones.

In conclusion, the debates and issues identified by these: four papers tell us much about the national standards in mathematics and science and also about the process of implementation where the same issues reappear. The positive vision is of rigorous, meaningful, and useful content and achievement for all students realized through different instructional programs. The risks include no change, lower standards, and a repressive uniformity. The only way that we can be faithful to the vision and avoid the risks is to stay with the process and guide it in productive directions.

Are the NCTM Standards Suitable for Systemic Adoption?

Deborah Tepper Haimo

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Introduction

As we consider whether the NCTM *Standards* are suitable for systemic adoption, we need to examine answers to some basic questions. For example, we need to know what all students are expected to learn; how they are to obtain this knowledge; and how we can ascertain whether they actually have done so.

Many of us in academe, whether in classroom teaching or in administrative positions, have had a long and abiding interest in every phase of the educational system. At whatever level one teaches, there is always the previous background of students to consider. In dealing with that aspect in mathematics, substantial information about the learning effectiveness in earlier courses is essential, since much basic material is vital to absorb before any significant progress can be made. Thus, those teaching at the college or university level must be sure that the overall mathematical education of all their students is sound, including that of those who expect to become school teachers.

When teachers themselves have little feel for what they are doing, cannot really explain why they perform certain operations or where the subject is heading, they teach mechanically, stressing only formalisms. As a result, they fail to help their students to understand and appreciate the nature and power of what they are learning, and their students suffer the consequences. For example, a high school teacher once asked one of my colleagues, "Tell me, I know you get two answers when you solve a quadratic equation, but which is really the correct one?" Clearly, this teacher had no sense of his subject, nor could he be expected to find the solution to that question by turning to a calculator or computer. He obviously had been puzzled by this problem for a long time, but never felt he could ask anyone about it before. It is thus imperative that we create an atmosphere where any such difficulty can be identified readily and resolved immediately. Those who go into teaching must be so well grounded in the essence of their subject that they can provide their students with a solid knowledge and understanding of mathematics.

Even more than ever before, it is important that students gain a strong background in mathematics throughout their school years. Those students who intend to continue their formal education at a college or university can then explore the possibilities of the many fields that will be available to them, whereas those who join the work force immediately on graduation from high school will be in a position to take advantage of openings that provide potentially challenging opportunities. It is thus vital that school teachers, at every grade level, help all of their students to gain a mathematical understanding commensurate with the utmost of their abilities. In this way, students can be assured that, whatever course they choose later, they will not be hampered by poor preparation in a fundamental discipline; the results of their school experience will endure well beyond high school graduation.

The Essence of Mathematics

In other days, mathematics was described, on the one hand, as the Queen of the Sciences and, on the other, as the Handmaiden of the Sciences. It would thus appear that there is no question about the gender of mathematics, merely uncertainty about her social status.

Actually, these designations attempt to encapsulate a broad, fundamental discipline and to describe it in a terse, dramatic way. As a consequence, it may not be surprising if we are inclined to dismiss the essence of these titles altogether as not worthy of attention. They may seem rather outmoded and politically unacceptable anyway. Despite any doubts we may harbor, however, let us examine what these views are attempting to connote, and let us determine whether there might be some justification for these characterizations.

Are the titles really descriptive of the discipline? Are there, indeed, two quite different roles that mathematics has, as these titles imply? If we reflect on the nature of mathematics, it seems to me that we cannot but conclude that mathematics does play both roles, although it is hard to regard them to be of equal standing.

In its abstractions and in its theoretical conclusions, mathematics reigns magnificently, perhaps even majestically like a Queen. It is a vital, dynamic, exciting discipline, where new results are continually determined as our knowledge expands. Periodically, long standing problems that have challenged the most brilliant minds at some time finally give way to the persistent and constant attempts to find solutions by generations of determined mathematicians.

A recent example, for instance, is Fermat's last theorem that brought much new mathematics into being before finally succumbing to the untiring efforts of Andrew Wiles of Princeton University. His work so ignited the imaginations even of ordinary citizens that, aside from being featured in major daily newspapers, this famous problem was presented to the public in a large San Francisco auditorium where attendance required the purchase of tickets. The demand was so great that not only were all seats quickly taken, but scalpers even entered the scene selling tickets to the event at their usual highly inflated prices, a truly amazing development for an advanced mathematics program!

In its abstraction, not only does mathematics play the all important role as Queen, but its idealizations have been found to give such accurate descriptions of phenomena in the real world that, through its applications, mathematics is endowed with incredible power to serve as Handmaiden to a broad range of diverse areas. The same mathematical tools, differently interpreted, can be applied to seemingly unrelated subjects to solve problems in a great variety of disciplines.

This is a major contribution of the field that frequently is not fully recognized. Indeed, when those who are mathematically educated leave academe for the commercial world, their titles usually do not reflect their mathematical background. Consequently, the final results of those in areas applying the mathematical theories may be given great prominence, while the fundamental tools that have led to these breakthroughs are generally ignored. The result is that the importance of the role played by mathematics fails to be noted, let alone appreciated.

We must succeed in conveying to our students, to other scientists, and to the general public the notion that mathematics is an intrinsically beautiful and exciting discipline in its own right, in addition to being such a basic subject that it can be applied in many ways to many other areas. In general, most scientists view mathematics merely as a tool in their own profession. While the public is in awe of the discipline, it has no conception of its significant value, nor does it understand its need for public support. We must correct such perceptions.

Mathematics is a unique discipline based, as it is, on abstraction. In this respect, it is distinct from virtually every other subject that appeals to the scientific method and is generally centered on the concrete. Indeed, it is from this abstraction that its own great power for applications is derived. Thus, we have the paradigm of the abstraction leading to applications, or conversely. This allows for substantial flexibility for teaching in either direction, to the great enrichment of students. When, however, the teaching is restricted, say, to the applications, and the abstraction is never reached, students are deprived of the opportunity to step back and see the richness of the subject, learn of its great overall power, and appreciate the position of their particular problem within the discipline.

As we note from Fermat's last theorem, some mathematical work appears to have no relation to anything except to the field itself. Sometimes, however, often unexpectedly and many, many years later, someone ingenious recognizes that mathematics holds the clue to some need in real life, and an important application of mathematics is born.

It was not so long ago, for instance, that number theory was considered as totally "pure." How wrong most of us were to hold that perception! It took centuries, but today, results of number theory affect us all in many ways in our daily lives. From the great strides in cryptography since World War II, to the issue of bank security, we have ample examples to appreciate the important role of a "real-life" application of what had earlier been viewed as a totally abstract area of mathematics.

As the rush for seats in San Francisco indicated, the public is not always concerned just with the practical. Number theory has many problems that are relatively easy to understand, though difficult and far too deep to prove, and yet, the nonprofessional public craves to know more and will seek it out even when the solution cannot be fully followed nor totally appreciated.

As abstraction defines the nature of mathematics, there is the innate requirement of precision. A question has an answer dependent on the hypotheses. That answer can only change as the assumptions do, and it cannot be accepted as valid until it has been firmly established. The solution may be found along many paths, but as long as assumptions remain unchanged, there is no variation in the answer!

Another feature of mathematics is conciseness. The assumptions on which a result depends are best if they are minimal, without any superfluous information provided. Also, conditions given must be consistent. and not contradict one another.

Further, over the years, a compact notation has been developed to express significant ideas as clearly and concisely as possible. For example, something that is now taken for granted and assumed always to have existed, and which indeed has been adopted by many other fields, is the simple introduction of subscripts and superscripts. This symbolism took a long time to develop. It resulted in our ability to obtain many new results and. to express old ones more simply and compactly.

Oral and written communication in mathematics can now occur by using, not only the prevailing ordinary language, but mathematics symbolism as well, Students can thus take advantage of years of developments in the field. By familiarizing themselves sufficiently with the subject matter, they are in a position to learn to express themselves clearly, both in their ordinary language and in the currently available mathematical symbolism.

A rather subjective characteristic that may often be sought in mathematics is elegance. This is what, for some, designates the discipline as an art. An answer to a problem may sometimes be obtained by applying brute force, but if the same result can be derived instead by some technique that establishes it more simply and elegantly, that solution is generally more highly appreciated.

Unlike experiments in other disciplines, those in mathematics cannot be considered as anything but a means of leading to conjectures, and conjectures cannot replace sound theories. Before a result can become a part of mathematical theory, it must be proved completely and convincingly. Ambiguities are not acceptable. Although experimentation may show that it holds for a large number of special cases, perhaps billions or more, until the result has been established conclusively for all cases, it cannot be considered as valid. It must remain merely a conjecture, as was the case with Fermat's result.

Much mathematical theory emanates from conjectures. There are some who are very adept at guessing a possible outcome. Yet, it is extremely important not to be swayed by a seeming pattern. It is folly to draw a conclusion that may be invalid, although a counterexample is not apparent immediately. At the school level, of course, the validity of most results has already been confirmed. This fact must be conveyed to students. They must be made aware of the existence of such a gap when they make generalizations but don't yet have the mathematical maturity to provide ironclad justification.

Thus, to understand and appreciate the nature of the subject, we must recognize abstraction as the fundamental characteristic of mathematics. We must take note of its emphasis on proofs, precision, and conciseness and its striving for elegance. Further, we must realize that from this abstraction the discipline acquires the powerful tool of specific application to a highly varied array of problems in very many different fields. Any educational program must address these two important features of mathematics in a balanced way.

Mathematics Education Reform

Throughout my professional career, there have been many attempts by various groups to reform the teaching of mathematics formally. These have usually occurred after the publication of a serious report that decried the poor status of our educational system, or following the release of some international comparisons in which our students failed to show up well in the rankings. While far-reaching changes were generally proposed in the various reform programs, strong criticism always ensued for one reason or another. For example, a change might come under attack for its failure to reach all students, or for its omission of some pertinent group from initial direct involvement. In every case, the experiments were limited in their reach and invariably were short-lived.

Although many factors, not only inadequate instruction,, are responsible for students' unsatisfactory performance, our concern is with teaching. There is little doubt that poor teaching exists, in the schools as well as in the colleges and universities. Just as teachers who demand mindless calculations in the traditional format fail to convey any semblance of the nature of the subject to their students, so do the ones fall short who currently instruct students in the use of the new technology by indicating what buttons to push on their calculators without providing any explanations whatsoever for the choice.

Despite whatever problems exist in our mathematics educational system, and certainly change is indicated as we strive to reach all students, individual teachers have never ceased to look for effective approaches. They continually seek to improve their impact regardless of formal recommendations. These exceptionally dedicated teachers have greatly inspired and influenced some of their students to become leaders in the field. We must recognize these teachers as being largely responsible for our nation's still holding its place at the forefront of world mathematics.

To maintain that position, the mathematical community must address the current problems where it can improve matters within its area of expertise and influence. In making recommendations for change, however, it is vital that it be cognizant of the history of mathematical achievement and instruction. In the interest of bringing about rapid change, it must not modify those aspects that have been most successful unless there is clear evidence that their replacements will result in significant improvement. Any reform efforts that have as their goal the education in mathematics of all students must be so constructed that the intellectual aspects of the field are not totally overshadowed. In a pursuit of relevance, we must be careful to make sure that the very essence of the field is preserved. In trying to make the subject accessible to everyone, we must not allow our expectations to be so diminished as to be worthless and totally unchallenging to many.

NCTM's Curriculum and Evaluation Standards for School Mathematics

The NCTM *Standards* (1989), as they have come to be called, are different from other attempts at revision in that they have been far more widely disseminated and are being seriously considered by many for systemic adoption. Indeed, they are the closest we have come to the introduction of national standards. They thus merit most careful review so that we are as sure as can be that their provisions are sound. We need to ascertain that they will not only improve the situation for the very many who have not been well served by past practices, but will not destroy what has proved to be successful for some.

We are convinced that it has become vitally essential for us to provide an education resulting in a citizenry better able to cope with the technological society we have created. We anticipate that we will need a more highly skilled and competent work force than ever in the past. In addition to the stars who have led this nation to preeminence in the world of mathematics, we need the many others, at all levels of achievement, to enable us to maintain our preeminent position. We can no longer afford to waste the talent we have in our midst, nor can we accept the unproven conventional wisdom that claims that the bright will emerge anyway and don't need any guidance!

In the current climate of equity, practices of exclusion can no longer be tolerated. We must make sure that everyone is afforded the opportunity to learn, limited only by interest and ability. How much further will we grow as a nation if we take advantage of all the talent within our midst and have a well educated citizenry!

Seeking such an end, the NCTM produced the three standards: the ***Curriculum and Evaluation Standards for School Mathematics*** (1989), the ***Professional Standards for Teaching Mathematics*** (199 1), and the ***Assessment Standards for School Mathematics*** (1995). One of their laudable goals is reaching out to all students. To achieve their objective, they propose a curriculum that they believe will be attractive to a larger number of studlents and that they expect will give these students a deeper mathematical understanding than has been given in the past. They seek to instill in students enthusiasm for a subject that has not been generally popular and that students often seek to avoid as soon as they can. Further, they want to generate in students a feeling of confidence in their own abilities. All are lofty objectives with which one could hardly quarrel.

The problem with the ***Standards*** is that they are so lengthy and vague in many parts that they provide far too much opportunity for misrepresentation and misinterpretation. Even when a statement seems reasonable, it can be, and has been, misconstrued. The result is that, in practice, the entire thrust of the educational experience has been on relevance and examination of "everyday" problems with a total loss of reasonable balance between abstraction and application. Further, there has been a serious downgrading of content and expectations of student achievement, with the ablest students ignored altogether.

For any progress to be made, all students must have basic mathematical tools that must be absorbed by everyone studying any part of mathematics,. Introductory definitions and postulates, as well as computational skills, must be developed to a high degree. At every level, it is important to convey to students an accurate description of the essence of the subject in a comprehensible way. We must impart to them our own excitement and delight with mathematics as a fascinating, intriguing, and enticing area of study. Above all, we must have a solid balance between theory and application. In the "social constructivist" direction of the ***Standards,*** however, there is a clear attempt to reshape the discipline by emphasizing the utilitarian part and blurring any possible distinction in the two roles of mathematics.

We would certainly like to insure that all students have an equal opportunity to learn and to become mathematically literate. Learning any subject thoroughly enough to understand and appreciate its concepts fully, though, requires work. No one can seriously object to keeping students absorbed and interested, and even entertained if this can be achieved in the process. It is not constructive, however, merely to succeed in getting students to acquire a superficial and distorted glimmer of an important discipline while they obtain an unwarranted feeling of competence. Although this undoubtedly is not intended, the impression nonetheless is strong since confidence in one's mathematical ability is highly stressed, and teachers are basically encouraged not to correct any errors. If one accepts these statements literally, the results can be serious indeed.

One of the major problems that strikes one in reviewing the NCTM ***Standards*** and in examining the: current reform movement in mathematics education is that there is a great emphasis on the relevance of mathematics and its applications to everyday events, while the innate beauty and elegance of the subject are rarely brought up. Their mention seems parenthetical. As a result, the entire subject becomes unrecognizable as it is distorted in order to have concrete problems that are considered readily understandable.

In some current educational circles, any reference to one's having to exert some effort to learn is almost never raised. It is thus refreshing to note that, in indicating the purpose of having standards for any subject, Diane Ravitch states unequivocally that it is "to raise the academic achievement of all or nearly all children, to signal students and teachers about the kind of achievement that is possible with hard work" (1995, p. 5). There is no substitute for the requirement of diligence. The ancient admonition, attributed to Euclid, that there is "no royal road to geometry" (or any other subject in whatever area) still holds. Where in the ***Standards*** is the indication that a student would have to work in order to learn? A difficult, basic subject like mathematics requires that nearly all of us work hard to understand it.

Teachers cannot do more than seek to arouse interest and to inspire, but all too many students have little conception of their need to exert some serious attempt to learn. This has generally never been a requirement in their current educational experience, nor is it proposed anywhere in the ***Standards.*** Quite the opposite is implied. Indeed, the entire emphasis is on having teachers in the background. They are charged with creating an atmosphere for learning in which there are no mistakes and in which everyone's contribution is recognized, however wrong! Teachers are to entice students into the subject by making it accessible and by effectively bringing the level of comprehension required down to the poorest student, basically eliminating any challenge for most. The eloquent disclaimers are simply invalid in actual practice.

Let me introduce a not uncommon analogy, which is important as a means of bringing out a comprehensible comparison with mathematics. In the United States, there is a far too prevalent belief that people either are born with mathematical talent or are not. Hard work and application to try to excel are not in general considered essential, since facility with mathematical ideas is all perceived as a function of some inborn ability.

While on the one hand, we consider any effort expended to learn mathematics rather futile, our attitude toward athletics is quite different. Indeed, we accept the fact that in sports there are superstars. We judge their performance most critically, and we expect them to excel at a very high level, not only by virtue of talent, but also by a rigorous and demanding regimen of dedicated hard work. We value also the professionals who are outstanding, but we insist that they must work hard to achieve a high degree of competence. Further, there are the coaches whom we require to be familiar with every nuance of their particular activity and to have learned it well enough to inspire and teach others. In addition, we have the amateurs, who take their roles seriously, also work hard, and enjoy the level they can reasonably attain. Finally, there are the fans who have learned enough and have absorbed the essence of the sport so that they understand what transpires and can appreciate the talent of the best.

Why do many view mathematics in an entirely different light? Why don't we expect our youth to expend the same, or indeed greater, energy and effort on their studies as we demand on the athletic field? In our concern for the previously unrepresented groups, are we ignoring even the most talented among them by reducing our expectations for all in their study of serious academic subjects? Why do we hold teachers accountable for their teaching performance, but ask so little of students who are not supposed to struggle to learn, even when the material they must absorb is difficult to understand? Is there no realization that everyone meets obstacles and difficulties in mathematics? Only hard work enables one to reach one's level of ability. Clearly some have more talent than others, but all have to expend the effort to progress.

While seeking to eliminate difficulties and make the subject more attractive to some, the *Standards* misrepresent the essence of mathematics entirely by portraying it merely as the "foundation discipline for other disciplines (which) grows in direct proportion to its utility" (NCTM, 1989, p. 7). It is distressing to find such an unbalanced view being promulgated to our teachers and students. Despite the glowing descriptions and the references to the "value" of mathematics that all students are to learn (p. 5), the final result is to regard as important only the real-life applications, and to engage in problem solving to the extent that the broad picture and abstract concepts are glossed over as having but a minor role in understanding the nature of the subject.

"Problem solving is to be the central focus of the mathematics curriculum" (NCTM, 1989, p. 23) for those in the early years of K-4. Mathematics is described as "useful" for those in grades 5–8 (p. 65). By high school, a core curriculum is proposed for all students, with three years of mathematics study required of all, but the "curriculum (is to be) differentiated by the depth and breadth of the treatment of topics and by the nature of the applications" (p. 125). Those students who intend to go to college are to be required to study mathematics a full four years. In' general, however, the curriculum described does not offer much to ensure that students will have gained substantially in their real understanding of mathematical ideas at any level.

In attempting to be sure that "everybody counts" in mathematics, the NCTM ***Standards*** highlight topics that are felt to be important enough to be emphasized; others are designated as needing to be de-emphasized. The result, of course, is that, in practice, poorly prepared teachers, as well as many others, take advantage of the situation to interpret what is recommended for de-emphasis as material that is considered less important, and not necessary to bother with. Since the aim is to empower students by having them construct for themselves even well-known mathematical results, and such activities can be extremely time consuming, it has become routine in many classes to omit these items entirely. As a consequence, virtually all students now are required to learn merely an unacceptable minimum. The low floor has become the norm, and there is no challenge at all for any but the mathematically weak student. Our expectations of all our students' performance has been substantially lowered so that we are educating them far more poorly than their abilities would allow.

No one would support the premise that all students need to be able to give careful proofs of every mathematical statement they encounter. Proofs are generally quite difficult to provide and take substantial mathematical maturity to give accurately. Indeed, the ***Standards*** limit the construction of proofs to "college-intending students." Are they suddenly to be exposed to this for the first time? It takes preparation and substantial time to be able to absorb, appreciate, and understand fully what is involved in providing a sound proof. All students in grades 9-12, however, are expected to "make and test conjectures; formulate counterexamples; follow logical arguments; judge the validity of arguments; and construct simple valid arguments" (p. 143). What is the justification for confusing the issue and creating such fuzziness, at all levels, by the introduction and sprinkling throughout of such words as "confirm," "justify," "argue," or "judge" when it is not made clear what is intended and why? How is a conjecture confirmed? Isn't a counterexample a proof? Who determines whether an argument is logical or valid?

In the ***Standards*** (NCTM, 1989), for example, there is a discussion of logarithmic properties, and then high schoolers are asked to "confirm a generalization on their computers" by testing "several numerical values" (p. 44); three are suggested. Is one to conclude that generalizations are valid despite the fact that only a very small finite number of examples is even tried? Instead of being given the opportunity to understand the difference between conjecture and proof or experimentation and ultimate validity, concepts that are very important in mathematics, students are: asked to generalize results without any indication that these generalizations are mere conjectures that may hold even for millions of specific cases and yet not be valid in general.

Although there is some mention of mathematical theory and structure, the ***Standards*** generally omit difficulties without even addressing the need to fill the resulting gaps. There is emphasis on solving of real-life problems, but failure to provide even a minimally balanced curriculum. The need to focus some greater attention on the intellectual phase of mathematics is disregarded. With so much stress on applications and on problem solving, when will the student be given an opportunity to see the broad picture? Where are the theoretical aspects of mathematics that form its core? In this respect, the needs of potentially able and interested students are completely ignored, while the general student is deprived of any real understanding of the nature of an important basic discipline.

Although controversial among psychologists, Anderson, Reder, and Simon (in press) provide supportive evidence that describes a 1987 demonstration by I. Biederman and M. Shiffrar supporting the premise that abstract instruction may at times be even more effective than the concrete. It often enables the learner to transfer the abstract knowledge derived to another context far more readily. A reasonable balance is what is needed.

NCTM's Professional Standards for Teaching Mathematics

In the *Professional Standards for Teaching Mathematics* (NCTM, 1991), the fact is stressed that "each student's knowledge of mathematics is uniquely personal" (p. 2). That observation may be correct, but, however each student learns, the final mathematical result must either agree with the prevailing structure or form the beginning of some extended new theory. If the hypotheses are sound, the conclusion is never in doubt unless it's an unproved conjecture. Ambiguity is foreign to mathematics!

Despite the current trend to regard mathematics as "social," it is not a democratic discipline in the sense of having the majority rule when an incorrect result is involved. Even if each student and the instructor agree on the validity of some conclusion, they could all be wrong. It is thus incumbent on teachers not only to understand thoroughly much more than the material being taught and where it is headed, but to recognize that they may not always know just what the correct answer might be. If they encourage students to "brainstorm," they must be prepared to direct the discussion when it gets off the track. They must also accept the fact that there will be times when they are unfamiliar with the correctness of a statement and must admit as much, while they and their students seek a sound resolution of whatever problem is in question.

"Logic and mathematical evidence" (p. 3) are certainly essential in confirming the validity of a result, but it is the teacher who should be the ultimate judge of what constitutes these characteristics, and the teacher who should have the background to know, or find out, whether the reasoning is sound or merely seems so. In that respect, the teacher must be regarded as the authority on the subject.

The proposed alternative to having a stronger teaching staff is argued by T. Romberg (1992) who states that "to improve teachers, one must improve teaching" (p. 799). It is hard to understand how this can be effected when, in the proposed teaching approach, the teacher is asked primarily to be an expert at asking for explanations and establishing a discourse that allows everyone to feel confident, however misguided.

The *Professional Standards* (NCTM, 199 1) indicate that asking students to explain each statement "consistently, irrespective of the correctness of students' statements, is an important part of establishing a discourse centered on mathematical reasoning" (p. 35). A skillful teacher, who can recognize when an incorrect statement has been made, may be able to get some student to reverse an answer in trying to explain it. Unless such a statement, however, is clearly corrected, and a teacher must be proficient mathematically to effect this, the damage done can be extremely serious. Never, never, never should a student be left with a wrong answer beyond the class time without being made very much aware that a correction is needed. Otherwise, a teacher is failing in a most critical way.

Just as learning is very much an individual's unique approach that must be recognized, so is teaching. The interactions between student and teacher also come into play, as some students react and learn very differently depending on who is teaching and how. While various forms of teaching are mentioned, and teachers are encouraged to use a variety of styles, like much else in the *Standards*, the discussion of style is misinterpreted so that one form seems to emerge as being primarily advocated. Teachers who may by far prefer to use a more traditional method much of the time, having found it more effective in their experience, quickly come to realize that this mode of teaching is regarded as having no value and appears to be relegated to being ipso facto bad.

There is also a strong emphasis on group learning. While mention is made of individual study as well, the thrust of the *Standards* is that mathematics is a "social" activity, and students need to cooperate in groups where they can learn best communally. With the ambiguities inherent in the document, the *Professional Standards* have been interpreted as not allowing for any flexibility in deciding whether that procedure is going to be the best choice in a particular class. Some of the serious problems 'with cooperative learning are merely glossed over. It may be good for some students to develop the ability to work with others on problems, but it is not clear that all will benefit from that format. Indeed, there are those who would prefer by far to work alone, except when they themselves choose to discuss some items with others. Should their wishes be ignored?

It's distressing to note that a teacher thinks that a problem in mathematics yields more than one solution if it has a set of pairs of numbers satisfying the hypothesis. Such an example occurs in a vignette where a teacher provides a problem to contradict "the image of mathematics as a domain of single right answers." As indicated earlier, the solution of a problem depends on the hypothesis. If the hypothesis leads to a single linear equation in two variables, as is the case in the *Professional Standards,* the vignette described in 3.1, of course there will be multiple pairs that will constitute the single solution.

Again, the details of a real-life problem are obscuring what should be opportunities to emphasize significant mathematical properties rather than to use them to disseminate misinformation about mathematics. It is important to differentiate between a complete and an incomplete answer. The image of mathematics that the teacher sought to contradict is, in this case, entirely correct.

How would that teacher deal with, say, a quadratic equation that is taught just a few grades later? Is not the fundamental theorem of algebra important for teachers to know, not only if they are at the high school level, but even if they only teach the sixth-grade classes or those before?

Returning to the vignette described in *3.1* of the *Professional Standards,* we note that attention must be directed toward another aspect of the problem of the Wolverines, a basketball team that has achieved a given score in a game. If all students are to be reached, is the expectation that everyone knows how basketball games are scored? How could that problem have been solved by a student unfamiliar with the scoring unless the information was given that only two- or three-point shots were to be counted?

NCTM's Assessment Standards for School Mathematics

Throughout the commentary in the *Assessment Standards* (1995) is the implication that the glowing description of the proposed changes will result in "(our having) high expectations for all students, envisioning a mathematics education that develops each student's mathematical power to the fullest." We certainly aspire to reach that goal, but is there any valid evidence to substantiate the premise that such will be the effect if we radically reform the curriculum as proposed?

There seems to be great opposition to ability tracking of any kind. Further, low-achieving high school students are no longer to repeat earlier mathematical studies with which they had difficulty, but instead, like their classmates, are to be given a new course of study in keeping with the goals of the core curriculum. Yet, in the ***Curriculum Standards,*** one reads, "It is important to understand that this statement does not imply that students of all performance levels must be taught in the same classroom, and it does not imply that: the content presentation for all students must be the same" (p. 30).

It is not at all clear here what is meant. Is a form of "tracking" proposed with the term "college-intending" used instead? Are these students to be separated, or are expectations to be lowered for all? The drastic abandonment of every aspect of the "traditional" ways might sound good and be appealing theoretically, but it has not yet been shown that it can produce students with greater understanding of mathematics concepts. The "old ways" at least have withstood the test of time so that their strengths as well as their flaws have become clear. Further, over time, some trouble spots have been eliminated and some changes have been made. On the other hand, using the proposed reform to correct all the ills of the past by replacing everything on all fronts with untried proposals is a dangerous route to follow.

It is unreasonable to dismiss the traditional curriculum entirely and fail to recognize that it has served to make our nation foremost in mathematics in the world. While it can be criticized for some of its failings, such as, for example, not taking advantage of the potential talent of those who did not fit a given mold, there is little evidence to support the claims that are being presented as absolute facts. Indeed, until we have a sound means of determining whether, overall, students exposed to the reform curriculum will outperform those of earlier times, there is no valid indication to substantiate systematic adoption anyway.

It is hard to envision that teachers will have time for all the detailed assessments that are mentioned, even though they are supposed to be part of the learning process of students. Is there evidence that performance assessment or portfolio assessment provides valid, reliable information that justifies the time involved? Unless teachers have a real grasp of their subject matter, however, they may not be able to deal adequately with students who, as is suggested in the *Assessment Standards,* "respond in unanticipated ways" (p. 15).

The next sentence on the same page seems to miss the whole point of mathematics when it mentions that "students may need to specify the assumptions they are making when they communicate the results of their work." Are not assumptions the heart of any result, and shouldn't a student know that results without clear assumptions are meaningless?

At one time, the philosophy among some educators was that, if one knew how to teach, it was not necessary to know the subject matter. Fortunately, however, that is no longer the case. In the NCTM *Assessment Standards,* on the other hand, there is too great a requirement that teachers carry out what appear to be unnecessarily demanding assessments. One cannot help but feel that their students would benefit far more from teachers whose background in subject content was greater. Indeed, as has been the case in past experiments, those whose knowledge of the subject is strong will circumvent the flaws in the standards and will thus improve the mathematical education of their students.

In one of the vignettes presented to teachers as examples of good assessment reactions, a teacher appears to miss entirely the trend in the right direction that a seventh grader follows. For example, the teacher's written comments in the *Assessment Standards* (p. 35) fail to recognize that the seventh grader's picture is far more than an indication of his awareness that $1 + 3 + 5 + 7 = 16$, but also that $1 + 3 = 4$, $1 + 3 + 5 = 9$, and that $1 + 3 + 5 + 7 + 9 = 25$. Indeed, he was on the way to a reasonable guess of a generalization, not only by his verbal explanation, but also by his visual one as well. Her example of $11 + 5$ misses the major fact that he brings out in his illustration as well as in his clarification of the addition of the next odd number each time to get the square of the number of terms for the sum. Her questions should have been along the lines he was pursuing to get his conjecture of what he might expect "always." He would then have had the opportunity to perfect his answer and state it more precisely so that others would understand. In addition, she should have taken this opportunity to point out that, at this stage, all he could claim is the belief that his result would always hold. In his case, the generalization is known to be true, as it already had been proven valid.

Memorization of any kind is derided, although no serious mathematician would propose mindless memorization nor consider it productive. We all would favor having students who understand the concepts and appreciate the value of learning. Some simple arithmetic facts, however, must be memorized, at least for efficiency. It is unreasonable, for example, to have no recourse if a computer is down and a clerk cannot carry out primary-grade calculations without its use. Unfortunately, this is not altogether a rare occurrence. It illustrates the justifiable fear of many, based on their empirical and anecdotal evidence, that the availability of calculators and computers does indeed make students dependent on them for the simplest of calculations. It is regrettable that, at an early age, basic principles essential for understanding mathematics are being overshadowed by the new technology before young children have fully mastered important arithmetic facts.

Mathematics cannot be forced to be like other disciplines, nor should it be. It is unfortunate that, in seeking to demystify the subject and make it more accessible, the reform movement is introducing inaccuracies that seriously affect the character of this exciting, unique, and important area.

While emphasizing solving of real-life problems, those seeking to revise totally the assessment procedures and to eliminate comparisons among students, even at the high school stage, are ignoring the fact that such comparisons are a daily occurrence for everyone in the real world. There is a need for an evaluation of students' knowledge that is useful and reliable and simple to administer. Teachers must have an impartial way to assign grades to upper level students that has been found useful in predicting future performance. Just as the NCTM ***Standards*** propose different approaches to teaching that have unpleasant features with which one must contend, so must we introduce assessment procedures that may not be ideal. Without being able to ascertain the effectiveness of the proposed NCTM ***Standards*** in actual practice, we can but consider them an untried experiment that will suffer the fate of previous reforms if they are adopted and left unchanged.

Conclusion

Perhaps the greatest achievement of the standards is the extensive dialogue, generated by their adoption by some schools and consideration for adoption by others, that has ensued within many groups. These groups include parents who have not understood that "a return to basics" is not the answer; business and community leaders who observed at first hand that their employees' knowledge of mathematics needs substantial improvement; mathematicians who have been aroused to give increased attention and time to this very important educational issue where their vision and their knowledge of where the subject is heading can serve as an invaluable resource; mathematics educators who can provide the important pedagogical criteria that might be implemented and who must thus be careful to ensure that they reflect tested current theories about how students learn mathematics; teachers who can best determine whether proposed changes will actually be workable in classrooms; and politicians of all sorts who must satisfy their constituents.

It is not surprising, with such a diverse group, with different views in each, that these reforms, as predicted in the ***Curriculum Standards*** (p. 255), are meeting substantial opposition. That would be expected in any case, as what seems reasonable and straightforward in theory is a far cry from what happens in its actual implementation in the classroom. The concern of some mathematicians is that the subject is being so distorted that students are getting an unrealistic indication of their knowledge of mathematics. Indeed, the ***Standards*** may well have succeeded in generating in some the unwarranted feelings of confidence in their competence. Other groups, on the other hand, may need to be convinced by more than the glowing descriptions from those directly involved in the creation of the NCTM ***Standards.*** Objective, external, independent tests are needed to determine whether the current reforms do lead to better performance in mathematics by students exposed to them rather than by students studying in previous programs. If there is no substantive evidence that supports such a conclusion convincingly, the ***Standards*** ultimately will join prior attempts at reform by becoming merely a temporary experiment that failed. The problem of finding the way to improve the mathematical education will then still need to be resolved.

The NCTM claim is that a six-part vision has been created.

- Mathematical power for all in a technological society

  But there is no valid evidence to show that, in the desire to expose all children to the new technology as early as possible, we are not creating such dependence on calculators that no one will be able to do the most elementary calculations mentally. Further, the ablest are not being challenged, and it is not even clear that others are, so that, in the long run, all will fail to meet the needs of a technological society that will require deep understanding, ability, and flexibility.

- Mathematics as something one does-solve problems, communicate, reason

  But why ignore, or downplay, the abstract aspects of mathematics? These must be understood well enough so that when circumstances change and there is a need to solve a seemingly different problem, it is recognized as based on known principles. Further, why disregard the fact that much of the symbolism is. a valid means of communication, developed over the centuries to enable mathematical results to be represented and communicated more succinctly?

- A curriculum for all that includes a broad range of content, a variety of contexts, and deliberate connections

  But it fails to impart any real notion of the subject and allows for serious misinterpretation of requirements.

- The learning of mathematics as an active, constructive process

  Productive learning is generally an active process, and it occurred in the "traditional" curriculum in reviewing and trying to understand the theory, as well as in working out examples and exercises illustrating it. There is no agreement, even among psychologists studying the how-people-learn question, that discovery learning is more effective than information acquired passively (Anderson, Reder, & Simon, in press). Assuming even that it is, the discovery method ignores advances made by others and may be quite inefficient timewise. Indeed, it might be more constructive to provide some of the needed theoretical basis to allow more time for working on the solution of problems.

- Instruction based on real problems

  But what makes problems real? Doesn't this also provide specific instruction rather than take advantage of the abstract nature of mathematics? Indeed, Anderson, Reder, and Simon (in press), as noted earlier, present evidence supported by some psychologists that abstract instruction is at times even more successful than the concrete and specific. Further, the evidence shows that such instruction is more readily transferable to various other types of problems.

- Evaluation as a means of improving instruction, learning, and programs

  Such evaluation has generally been done by individual teachers as a part of their overall work.

On the whole, it seems unreasonable to propose such drastic changes on a systemic level when much of the material has still not been fully investigated.

References

Allen, F. (1997, February). *A program for raising the level of student achievement in secondary school mathematics* [On-line]. http://ourworld.compuserve.com:80/homepages/mathman/allen.htm

Anderson, J. R., Reder, L. M., & Simon, H. A. (1997, February). *Applications and misapplications of cognitive psychology to mathematics education* [On-line]. http://sands.psy.cmu.edu/personal/ja/misapplied,html

Council of Chief State School Officers. (1995). *Stateframeworks in mathematics and science: How are they changing across the states.* Washington, DC: Author.

National Research Council. (1996). *National science education standards.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics,* Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics,* Reston, VA: Author.

Ravitch, D. (1995). *National standards in American education: A citizen's guide.* Washington, DC: Brookings Institution.

Romberg, T. A. (1992). Problematic features of the school mathematics curriculum. In P. W. Jackson (Ed.), *Handbook of research on curriculum: A project of the American Educational Research Association.* New York: Macmillan.

Wu, H., *The mathematics education reform: What is it and why should you care?* (preprint).

A Mathematician Looks at National Standards

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Context

This section briefly addresses the historical, political, philosophical, psychological, and personal issues that form the context of my views of standard-based mathematics reform.

Why Do We Have National Standards?

In the early 1980s the popular perception arose that American education was in serious trouble. In international comparisons our students ranked low. In mathematics, the problem that captured the nation's imagination (quoted extensively in the national media) was the bus problem. Here is one version: 121 students are going on a trip, a school bus can hold 23 students, how many buses do the students need? A disturbingly large number of students would answer "$5\frac{6}{23}$", or even (rounding down) "5." Our kids could calculate, but they couldn't make sense.

The documents proclaiming the nation's concern had quasi-official sponsorship: *A Nation At Risk* was sponsored by the National Commission on Excellence in Education (1983), and *Educating Americans for the 21st Century* was sponsored by the National Science Board Commission on Precollege Education in Mathematics, Science and Technology (1983).

Note that, if everything had been going well, no one would have proposed national standards. National standards exist because there is a perceived need to change, and I will slide among the notions of "standards," "standards-based reform," and "current reform" acknowledging this relationship.

Politics

In 1986, the National Council of Teachers of Mathematics began the process of writing national standards for mathematics education. These standards appeared in three volumes: *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Assessment Standards for School Mathematics* (1995); the last somewhat supersedes the evaluation standards of the first volume. At the same time many states were rewriting their state guidelines. The California *Framework* (California Department of Education, 1992) has generated the most noise, but is not untypical: many if not most states in their mathematics guidelines were heavily influenced by the NCTM *Standards,* and many if not most states formulated their guidelines under the strong influence of outcome-based education.

Thus our national standards were born in an interesting political situation (Bass, 1994). They are not sponsored by a quasi-official body, but by the organization that represents teachers of K-12 mathematics and researchers in learning school mathematics. This sponsorship has obvious advantages (accountability to teachers, sustaining long-term interest) and obvious disadvantages (difficulty in involving other interested communities, credibility). While national standards are not intrinsically connected to outcome-based education, there is a strong association in the public mind between the two. In reaction to standards and their various implementations, there are turf battles among mathematicians, teachers, mathematics educators, parents, and business leaders—who should have the strongest voice in mathematics education? There is the classical American conflict over jurisdiction-what authority should national standards have? state standards? Decisions on education are ultimately made by local school boards, with states holding financial and accreditation sticks to keep school boards in line. There are efforts toward national consequences to local actions, such as national teacher accreditation, but it is not clear whether these will be meaningful on the local level.

The increasingly ascendant role of business is worth noting. We were a nation at risk in part because businesses found that our high school graduates needed serious remedial education. In many discussions of standards-based reform, the needs of business are cited as justification for changes in emphasis, e.g., businesses give a higher value to communicating technical knowledge clearly than to doing arithmetic efficiently and correctly. The National Research Council (NRC) launched state coalitions for mathematics and science education (which have recently become independent through their own national organization) to bring business, practitioners (scientists and mathematicians), teachers, and education researchers together. The agenda for my state coalition's July 1996 meeting has four reports. One is the treasurer's report, and the other three are titled "The roles of postsecondary education in workforce development," "The roles of business in workforce development," and "The roles of K-12 education in workforce development." These topics would not have appeared five years ago.

The political context of standards-based reform makes it important to distinguish among several notions that are often confused: what is actually written in the various standards documents, what people think is written in the various standards documents, and how what people think is written in the various standards documents is actually implemented. Underlying these notions is the definition of "standard," which I will not attempt.

Theory

Carefully reading through the various ***Standards*** volumes, I oscillated wildly among enthusiastic approval, confusion, and strong disagreement. For a long time I found this puzzling, until the chance e-mail receipt of a paper on cognitive psychology focused my attention on underlying assumptions. The choices for underlying assumptions in education are many; they are often contradictory; and they are often unstated, especially in documents not meant for specialists in education research.

Theoretical beliefs about education have both philosophical and psychological components, and it is not always possible to tease them apart. There are oatological and epistemological considerations. There are issues of political philosophy-Is the goal of education training or learning? training or learning for what purpose? There are conceptual issues: What are the units of learning-facts? tasks? cognitive processes? Are there any units at all? Most philosophical/psychological theories have consequences for education; even logical positivism had an influence in the over-logicization of the New Math. When William of Ockham defends Platonism against constructivism in the *Journal of Research in Mathematics Education* (*JRME*) (Orton, 1995), we have a welcome glimpse of the theory wars raging beneath the surface. Even a benignly titled article in the *JRME* (picked semirandomly) such as "Mental computation performance and strategy use of Japanese students in grades 2, 4, 6 and 8" (Reys, Reys, Nohda, & Emori, 1995) necessarily has implicit theoretical underpinnings.

I see four basic questions in mathematics education. Two of them-What is mathematics? What does it mean to learn mathematics?-will have different answers in different theoretical contexts. Consider a simple answer to the first question (having, of course, its own theoretical ground), that mathematics is what mathematicians do (this is, of course, the answer I prefer.). This is not much help, since mathematicians do so many different things, since much of the mathematics useful to those who use mathematics is essentially ignored by mathematicians, and since toward the boundaries it becomes problematic to decide who is a mathematician-what about theoretical physics, for example, operations research, or statistics?

Two other basic questions are What mathematics should children learn? and How should they learn it? These questions cannot be answered without reference to the first two questions. But these are the questions that any set of standards needs to answer. The need to accommodate different underlying philosophical and psychological theories is, I believe, what gives the various *Standards* documents their confusing nature; this is unavoidable.

I should state here my own theoretical predilections. I tend to like constructivism, but also distrust rigid adherence to ideology-my constructivism is radical enough to lead me to distrust intellectual constructs, including those of constructivism itself. With Bishop Berkeley I believe there are times when kicking a stone is a good philosophical argument. With Wittgenstein, I am a great fan of the notion of "use," giving that word the broadest sense possible, and cannot understand the notion of "meaning" without it.

Doing Mathematics

I came to mathematics somewhat late; the first real calculus course I took was after graduating from college. I came to mathematics late because school mathematics (in an honors track in an academically demanding high school) did not seem interesting. This gives me a predilection to side with education reform. Because I came to mathematics late, I am perhaps more aware of what I went through in internalizing mathematics (or, if you prefer, becoming acculturated) than most mathematicians. Gratifyingly, what I think of as the necessary processes of and attitudes toward mathematics, not just for mathematicians but for anyone who can be said to have a basic mathematical education, are richly reflected in the *Standards*. Let me state them here, with two caveats. Caveat 1: The language is generally mine and not necessarily the language of the *Standards*. Caveat 2: This is my own personal list and makes no claims to being exhaustive.

Making sense. The first thing that struck me about the current reform movement years ago was the emphasis on making sense. It is this emphasis that was lacking in my own school experience and led to my perception of mathematics as boring and barren. The movement from mathematics as received knowledge to mathematics as perceived knowledge is a basic and necessary move and can be made within most philosophical orientations. (It is not, however, compatible with certain fundamentalist notions of knowledge, with obvious political repercussions.)

Reification. The objects of mathematics are real objects, in a psychological, not necessarily ontological, sense-they feel real, we act as though they are real. For example, "number sense" is based on reification. For another example, many young children have not reified the notion of fraction-for them, 1/2 implicitly carries with it the question "1/2 of what?" When the concept of "1/2" takes its place in the number system as just one of many rational numbers, to be thought about and used as we think about and use all rational numbers, it has been reified.[1] To take a third example, algebra cannot really be understood unless variables are reified-x is not a placeholder standing in for some unknown number, but an object in its own right. Reification cannot be forced, but its encouragement is a major part of the art of teaching mathematics. In many places we find reification in various guises in the *Standards,* but there are places where my emphasis on reification will lead me to disagree with the *Standards.*

Making pictures. Quasi-concrete mental imagery is a major intellectual resource available to mathematicians.[2] The importance of making pictures out of the most abstract situations was kept secret from school mathematics, and one of the great strengths of the current reform effort is not only its emphasis on imagery and metaphor-usually through the forms of physical models and diagrams-but its stress that different ways of picturing a situation should be encouraged. Connected to this is the encouragement of informal arguments from an early age; the early stages of mathematical justification are similar to the oral (but not written) practices of many research mathematicians in their reliance on pictures.

Justification. Learning how to write acceptable mathematical justifications was the hardest part of my becoming a mathematician-it is a social process, and different cultures have different standards of logical robustness. But this is not to say that mathematical justification is arbitrary—the rules, while more subtle than many of us choose to acknowledge, evolved for good reasons.

Insistence on mathematical justification at all levels is another great strength of current reform, as is the recognition that the practical definition of "sufficient justification" will change with a child's growing mathematical development. But scattered in the *Standards* are some notions of justification that seem counter to established mathematical practice.

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[1] Of course the rational numbers are themselves created through reification of particular rational numbers such as 1/2, which is itself not fully reified until the whole system is. Clearly this is a very complex process.

[2] and is used in even the most abstract mathematics. My own work, for example, is in the very abstract world of set-theoretic applications to Boolean algebra and general topology, but I cannot think clearly without using things like dots, lines, and circles. The great mathematician Erdös, in giving talks on infinite combinatorics, draws almost the same diagrams every time-a few dots, a few lines, a few circles- which encapsulate very different meanings in different contexts.

Applications. This widely used word does not seem to fully capture what really happens when mathematics is successfully used in another area. It is not that we simply apply a technique over here (from mathematics) to a situation over there (in real life, or in another field of study, or in another area of mathematics. One of the great strengths of the *Standards* documents is their stress on all three senses of the word "applications." Rather, the process is related to reification-. abstract situations permeate the situations to which they are being applied. So, for example, the geometric situation "areas of rectangles" is an instance of the arithmetic notion of multiplication, and in turn illuminates notions of probability; trees become instances of fractals; motion and distance become comprehended through the notions of differentiation and integration; and certain forms of turbulence are conflated with certain differential equations. It is a kind of double vision I am after here, in which students do more than move from one mode of thought to another freely-the different modes are, rather, different languages for the same phenomena.

That mathematics adds powerful systems to our intellectual resources is one of the main reasons I believe all children should learn serious mathematics.

Disposition. This term encompasses a very powerful set of notions, well articulated in the *Standards,* which has unaccountably been trivialized by opponents of reform to the parody of short-attention-span mathematics-all play and no work. Disposition, rather, is a cluster of intellectual character traits-thinking for yourself, being skeptical of others' claims, not believing something until you really understand it, knowing when you don't understand something, lack of wishful thinking,[3] persevering, learning from mistakes.

As a kid I was told that mathematics taught logical thinking (Euclidean geometry) and accuracy (arithmetic). I didn't believe this and still don't-those tightly defined compartments did not generalize easily. But standards-based reform seems to have a better chance at teaching the sort of intellectual integrity and clarity that I believe is inherent only in mathematics (other disciplines have their own forms of integrity and clarity, of course, and that is why children need a broad education). This is the other major reason I believe all children should learn serious mathematics.

Specifics

The context being set, we are ready to discuss specific issues.

A Problem

Although I am generally pleased by the major directions of the *Standards,* it is undeniable that the *Standards* documents are peppered with statements that are mathematically questionable. Generally these are not anything as simple as a straightforward mathematical mistake. Their best description is as something no one who really knew the mathematics would say-extremely difficult (even unsolved) mathematical problems may be suggested for exploration in a way that indicates that students should be able to solve them, or complex mathematical situations may be presented as if they were simple.

[3] Wishful thinking, the belief that something is true because it is convenient, is the source of most mathematical error.

This sort of carelessness is perhaps to be expected in such an accumulation of pages, but it has two important consequences. Some mathematicians have devoted enormous amounts of time to finding these glitches and cannot take the ***Standards*** seriously because of them. Other people, knowing less mathematics, can be misled. I would encourage the authors of future standards documents to be more careful.

Theory applied

Let me pick one series of the oscillations referred to in Theory, above, to demonstrate the theoretical tensions I *see* in the ***Standards*** and how I respond to them. Here are my reactions to parts of standard 12, geometry, grades 5-8 (pp. 112-1 15).

For the bulleted standards "explore transformations of geometric figures;" and "represent and solve problems using geometric models," I wrote, "Good." These standards represent a significant increase in mathematical content over the traditional curriculum.

For the quote

> Geometry is grasping space . . . that space in which the child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breathe and move better in it. (Freudenthal, 1973, p. 403)

I wrote, "Bull." Does anyone really live, breathe, or move better because they have learned geometry? This is contextualism at its most strained.

For the sentence "Discussing ideas, conjecturing, and testing hypotheses precede the development of more formal summary statements," I wrote, "?" What is a hypothesis if not a formal summary statement?

Three lines later, for the phrase "develop informal arguments," I wrote, "Yes." It is the absence of experience with informal arguments that made formal arguments so meaningless for so many students (including myself).

For the sentence "Students should learn to use correct vocabulary, including such common terms as *and, or, all, some, always, never,* and if. . . ***then,"*** I wrote "Great." The ability to use these words (= understand these concepts) precisely is one of the best gifts we can give to children and essential to any logical clarity of thought.

But two lines later, when we are advised that words like ***dodecahedron*** are important, I wrote, "Nah." This seems a holdover from the math-is-complicated-vocabulary school, in which vocabulary is emphasized and ideas are not. What is important is that a student at the appropriate level can describe these solids geometrically.

Where it is suggested that the Pythagorean theorem can be discovered "through explorations, such as the one suggested in figure 12.2," I wrote, "Overly optimistic"; figure 12.2 encapsulates a proof; and a somewhat hard one.

The sentence "Students can make conjectures and explore other figures to verify their reasoning" got another "Nah." How can any finite collection of figures verify reasoning? This is getting the mathematical process backwards.

One line later, a paragraph recommending dynamic geometry software got a "Good," as do the suggested explorations of the relations between perimeter and area, surface area and volume.

For "Which polygons will cover the plane and which ones will not? Why?" I wrote, "Careful." We are walking a tightrope here between the highly nontrivial (e.g., for convex polygons) and the trivial (e.g., rectangles and triangles). Where are we supposed to go?

For the claim that "students can also consider why the square is used as a unit of area and the cube as a unit of volume," I wrote, "Huh?" What sort of response is of value here?

For the paragraph on symmetry, I wrote, "Do more."

The last sentence, "Experience with geometry at the 5-8 level should sensitize students to looking at the world around them in a more meaningful way," is again contextualism at an extreme. There are better reasons for studying geometry seriously in middle school.

Several partial conflicts become apparent. On the one hand, there is a serious strengthening of the level of the subject matter. On the other, the issues of justification and verification appear in a strange fashion. Constructivist, contextualist, and traditionalist attitudes all jostle for space. No matter what the reader's orientation, her reactions will oscillate as mine did, as things she approves of appear, to be followed by things she disapproves of.

Technology

What should be done with technology? This serious question has not been sufficiently addressed in the *Standards,* perhaps because when NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1995) went to press, so little had been done with or was known about technology. But technology has been seized upon by opponents of standards-based reform, who are trumpeting in the popular media what they believe to be the failures of its use as evidence of the failure of reform. So any serious discussion of technology in the classroom will need to be reflected in the popular media as well-no easy task.

Technology is neither benign nor malignant, but it is powerful. Furthermore, like the HIV retrovirus, it is protean; by the time you thoroughly understand how to use one version of it, you are several technological generations out of date.

While the *Standards* contains some good instances of using technology (along with some bad ones), it does not provide a unified discussion of sufficient depth of the issues raised by technology and, in places, seems to assume that technology should be used in the classroom simply because it exists.

The most egregious instance of this is in NCTM's *Professional Standards for Teaching Mathematics* (1991, pp. 82-83) where Pete Wilder "has read that calculators should be emphasized at the middle school level [and] has been reluctant to use them. His supervisor, Tim Jackson, has been urging him to use calculators whenever possible." The boldface note in the margins reads, "The teacher is aware of the need to incorporate calculators into his teaching but is reluctant to do so."

This is troubling. Is Pete Wilder really aware of a need, or is he aware'that he is supposed to do something without understanding why? I think it is the latter, and the rest of the vignette (which focuses on specific activities) does not address this fundamental issue.

There are four basic questions to answer about any use of technology in the classroom. (With slight changes, these are the four basic questions about using anything in the classroom-I have modified them from the first four questions about assessment [NCTM, 1995, p. 4]) What mathematics is reflected in the use of technology? What efforts are made to ensure that the mathematics is significant and correct? How does the use of technology engage students in realistic and worthwhile mathematical activities? How does the use of technology elicit the use or enable deeper understanding of mathematics that it is important to know and be able to do? These questions are benchmarks for using technology. They should be widely known, and examples of both good and bad uses of technology, explicitly referring to these questions, should be widely distributed.

Meanwhile, the bad press that technology has been given makes it urgent to communicate what is really known about its effects in the classroom and to continue such studies. There are subtle methodological issues in any serious studies in this area, and conclusions can never be as clear-cut as local school boards would wish, but a serious conversation with the public must be attempted. In particular, the public deserves to know what is known about how use of calculators affects children's facility with arithmetic, and how graplhing calculators affect students' abilities to translate algebraic functions into pictures.

Vanguard attempts to use technology to substantially transform mathematics education-I am thinking specifically of the work of James Fey and Kathy Heid's group in high school algebra (1995) and Ed Dubinsky's group in abstract algebra, both using computer technology-need to be discussed widely (and dispassionately) in the mathematics, mathematics education, and school communities, apart from discussions focused on curriculum adoption.

Finally, we need to understand what support systems are needed to use technology thoughtfully, and this knowledge needs to be-disseminated widely.

Three Good Examples of Bad Things

In the name of tradition a lot of bad things happen in our schools, so it should be no surprise that in the name of reform some bad things have happened. I will present in this section three situations in the name of reform mathematics education that I find both problematic and typical of the mistakes that can happen under reform. One situation was observed; the other two are from reform documents, hence possibly hypothetical.

I will also propose benchmark questions for preventing such mistakes.

A classroom visit. The first situation was observed during a classroom visit I made to a fifth-grade teacher in a small rural community. She had drawn a complex pattern, reflected it across a line, duplicated it, and asked the kids to "color the inside."

There was no inside. Toward one edge of the paper the pattern curved tightly, so it looked like there was an inside, but as the kids moved along they began to realize that the pattern was opening up and didn't have a closed boundary. They had no idea what to do, and neither did she-"Oh well, just finish it the way you want," she said.

What was her content goal for this activity? She had none. Her knowledge of reform was that you did less arithmetic and more . . . well, more stuff. Her pattern had symmetry, and symmetry was mathematics, and that was enough for her.

She had, of course, missed an important opportunity: when does a figure have an inside? She didn't know enough mathematics to have thought of this, nor did she understand this was important mathematics when I suggested it to her as a possible extension.

When I think of the need to communicate clearly what mathematics is under reform, I think of this well-meaning, dedicated, good-hearted woman.[4]

When is the use of number in applications an instance of mathematics? The second example is from an early draft of the *Assessment Standards.* I know it is bad form to quote from nonfinal versions of documents (and hasten to add that this example did not make it into the final version), but this example is telling because not only one person but a committee of people thought it was good mathematics. (Having been on committees I also know how strangely the group mind can work.)

In this activity, a group of African-American middle-school girls are reporting on the statistics about minority men, higher education, and prison. They engage in some very clever guerrilla theater with the class, they make a cogent sociological point, but mathematically this is as empty an activity as those fifth-graders trying to color an interior that didn't exist-data presented without mathematical analysis is not statistics, just as coloring a bar graph does not make it art. Where were issues of variance? of sampling? of categories (e.g., a federal prison is not a county

who, *mea culpa,* worked with me for three summers, so I can't claim any easy answers to the problem she represents.

jail, and Harvard is not ITT)? Why weren't the kids pointed towards these questions? If this really happened, it is another missed opportunity.

Another instance of this sort of problem occurs in the middle grades communication standard (p. 79) where kids are asked how many hours they think teenagers watch TV a day and to compare their answers with the results from a national magazine. While the discussion goes on to say that "this exercise encourages students to . . . discuss appropriate survey techniques," I don't believe it does. The kids don't have access to information about what the magazine did, nor do they have the resources to conduct a comparable survey. Without such access and resources, this too becomes a exercise in social science, not mathematics, and a superficial one at that.

Too much mathematics; deviation from mathematical practice. The third activity is from NCTM's *Assessment Standards for School Mathematics* (1995, pp. 36-39). In it, students are asked to explore, using dynamic geometry software, the following (where P is a variable interior point of an acute triangle): (a) the sum of the distances from P to the sides of the triangle; (b) the sum of the distances from P to the vertices of the triangle, (c) the area of the pedal triangle, (d) the perimeter of the pedal triangle. (A pedal triangle is the triangle formed with vertices the end points of the perpendiculars to the three sides.) They are supposed to make conjectures, make convincing arguments, support their arguments with data, and "explain a situation where someone would want to know this information." They do this after having been led through a similar exploration when the triangle is equilateral.

This is a very troublesome example. Let me briefly summarize its problems.

The first problem is that, paradoxically, the situation is too mathematically rich. With no suggestions of what's worth looking for, how is a student to find anything? A good student can spend hours looking in the wrong direction (Why not? Mathematicians spend decades, even centuries, doing this.) and end up with a collection of aimless observations. Furthermore, the previous exploration of an equilateral triangle is misleading-the situation there is not like the general case.

The sequence of steps described-conjecture, make a convincing argument, support by data— isn't how mathematics works. First look at the data, then conjecture, then convincingly argue. Only when the problem is intrinsically finite can data really support a conjecture; this problem is intrinsically continuous and very far from finite. This notion that data can be used to justify a conjecture is one place where the *Standards* greatly deviates from mathematical practice, and it reappears throughout the ***Standards***.[5]

The level is wrong. Even if the student comes up with true conjectures, what would constitute a convincing argument? I assume the student is expected to concentrate on minimizing (for (a) and (b)) and maximizing (for (c)); I don't know what (d) is about. But this is hard mathematics. These are unexpected results. Their proofs are nontrivial. Helping students learn this stuff,

[5] I am of course aware that psychologically data can be a more compelling justification than abstract reasoning, but claim that (1) this is either because the person is easily convinced, or because s/he is at a developmental stage in which the abstract needs to be encapsulated in the concrete; and (2) one purpose of teaching mathematics is to get beyond this stage.

whether constructively or in straight lecture, takes a lot of thought from the teacher.[6] Expecting students to do it on their own as part of assessment is inappropriate.

Expecting teachers to know what this is about is also inappropriate. Few teachers-few mathematicians, for that matter-will have had a chance to be familiar with this material. If something like this is suggested for either curriculum or assessment, the mathematics needs to be clearly explained.

How to avoid similar examples. These examples essentially fail because they don't answer at least one of four basic questions -what is the mathematical point? what is an acceptable mathematical justification? can we expect kids to do this? have we provided enough mathematical explanation for teachers? While these questions are implicit in many standards-based documents, we need to pay more careful attention to them.

Stacking the Deck

In NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989), before the individual standards are explicated for each different level (K-4,5-8,9-12) there is a summary chart on facing pages, one labeled "Increased attention," and one labeled "Decreased attention."

I have no quarrel with anything that is supposed to receive increased attention. The suggested curriculum is good authentic mathematics, and the instructional practices are clearly pointed toward making mathematical sense of things. Despite claims that standards-based reform means a lowering of standards, if everything that is supposed to receive increased attention really does, our current students will in many ways know much more mathematics by the time they graduate from high school than my generation did.

My quarrel is, instead, with the pages labeled "Decreased attention." The deck is rhetorically stacked, so that "decreased" can easily become "no." Bad words appear, such as "rote," "isolated," "routine," "by type"-everyone knows these: are bad words-and by association everything on these pages becomes suspect. But in fact this material is a mixed bag.

Let me deal with each level separately. To make this section easier to follow, I will put in boldface the notions that are slated to receive reduced attention.

K-4 (NCTM, 1989, p. 21). Personally, I never want to see anyone use **key words** ever again; this practice is indefensible. Estimation should have context; **rounding** is seldom useful. **Division facts** are really multiplication facts and should not be treated in an isolated fashion. But I do think there are times when **worksheets** and **written practice** are helpful, and when kids need to focus on **paper-and-pencil computations.** There are times when **you** do have to **tell** the class

6 For a constructivist explication of(b), see the Airport Problem section o:f "Optimization" in ***Connected Geometry*** (Educational Development Corporation, 1996), whose title indicates one possible reason why "someone would want to know this information."

something (e.g., π).[7] Often in mathematics -almost always in arithmetic-there really is **only one answer,** although there may be many ways of getting there, and sometimes there really is a **best method.** I like **long division** for two reasons: it is an early and well-motivated example of a complicated algorithm, and it lays important groundwork for algebra, both in the obvious sense of division of polynomials and in the more subtle sense that understanding it contributes to a general mathematical sophistication. For similar reasons, I want kids to do **paper-and-pencil computation with fractions. If early attention to reading, writing, and ordering numbers symbolically** is done in context, as in whole language, then what could be wrong with it?

5-8 (NCTM, 1989, pp. 71, 73). **Manipulation of symbols** is terribly important, as a skill in its own right, in order to do other interesting work, and as a step in the reification of symbols. Algorithms, formulas, vocabulary, facts and relationships need to be remembered, and for most of us that means consciously **memorizing** them. Some questions really do have **only yes, no, or a number** as responses. Here is a very important one at a more advanced level: what is $e^{\pi i}$? The answer (-1) is a profound piece of mathematics.

9-12 (NCTM, 1989, p. 127). About a quarter of what is listed here to be de-emphasized strikes me as very important. Under algebra, **simplification of radical (and other) expressions, factoring,** and **operations with rational expressions** are instances of algebraic manipulations that are themselves necessary steps in the reification necessary to understand algebra-being able to freely manipulate algebraic expressions is cognitively similar to number sense, and I am disturbed that it seems to be absent on the "Increased attention" side. **Geometry from a synthetic viewpoint** is important, and can be done by tlhe increased attention given to the development of short sequences of theorems and to deductive arguments. **Two-column proofs** should not only get decreased attention but be eliminated. I agree that analytic geometry and functions should not be **isolated,** but should be integrated with the rest of the curriculum. As for **Euclidean geometry as a complete axiomatic system,** yes, it should appear only as a piece of history, but my reason for this is somewhat maverick-if it is presented in a way that can be absorbed by ninth or tenth graders, then some things have to be fudged (which astute students will notice), and you end up with so many axioms that inquiring minds will wonder why you bothered in the first place. **Applications of trigonometric sum, difference, double-angle, and half-angle identities to specific examples** is important: the mere fact that these identities exist is remarkable, and students should have some immersion in them. There is nothing wrong with **using formulas to model real-world problems--that** is the essence of mathematical modeling. And **expressing function equations in standardized forms** is an important conceptual step in turning algebra into geometry. (It even shows up on page 101 of the NCTM's *Professional Standards for Teaching Mathematics,* 1991).

What's going on. The motivation of these lists is clear and even commendable. In general the thrust is to get away from rote exercises -1 am not the only adult who has no fond memories of page after page of trigonometric identities, and the cartoon "Hell's library" (in which every book is labeled "Word problems") has been widely distributed; someone must find it funny. But just

[7] There are good activities for motivating the hypothesis that, over all circles, perimeter/diameter is constant. But is this hypothesis true and exactly which number is that constant? That is what has to be told.

because something can be taught, and often was taught, by rote methods does not mean it is bad in itself. Much that is essentially good, even fundamental (such as algebraic manipulation), is being tarred with the brush of the bad. As long as there is no distinction between what should really be thrown out and what needs to be taught differently, important school mathematics will be in danger of disappearing from school curricula, either at the district or at the individual classroom level. Many mathematicians and parents, even teachers, are convinced that this has already happened (and will tell you all about it through various Web sites and e-mail lists). I am not so sure, but I am worried.

0.31 x0.588

Let me focus on a particular problem to which "instructional time should not be devoted" (NCTM, 1989, p. 96) as an example of the importance of mathematics that the ***Standards*** either de-emphasizes or throws out, and how such material can and should be incorporated in standards-based reform. This is the paper-and-pencil computation of 0.31×0.588.

Why is such a problem important? After all, anyone not a calculation prodigy, unlucky enough to face such a problem in real life, would use a calculator.

But this is irrelevant. To compute 0.31×0.588 by hand requires either a deep understanding of place value or sophisticated skill in symbol manipulation or both, and that is what this problem is really about. Would I have children work sheet after sheet of such problems? No, not even without time pressure. But would I have them work a few problems like this in small groups, reporting to the class how they solved them, and then work a few on their own to make sure they understand what's going on? Absolutely. As part of the standards relating to fractions, decimals, and arithmetic, I would expect all children in the class be able to do problems like this-not necessarily quickly, but correctly-throughout their lives.

Nearly everything that I would rescue from the "Decreased attention" charts has similar justifications, and can be handled in similar ways.

Content

If the summary charts in the NCTM's *Curriculum and Evaluation Standards for School* ***Mathematics*** (1989) are radical and might raise fears of a diluted curriculum, the actual boldface lists of topics defining each standard are both conservative and ambitious-there are even two tracks in 9-12, for college-intending and others. Topics slated for decreased attention in the summaries indeed appear (e.g., synthetic geometry), so we know in some cases that "decreased attention" does not mean "no attention." Reasoning ranges from informal to very formal indeed (including axiomatic systems and mathematical induction). Even infinite series is in there.

I have only three quarrels with this material, all at the 9--12 level. Two quarrels are that symbolic manipulation isn't appreciated sufficiently and that perhaps more mathematics is proposed than can realistically be achieved. Should kids planning on college really "prove elementary theorems within various mathematical structures, such as groups and fields," "represent finite graphs using matrices," "solve problems using linear programming and difference equations," and "interpret probability distributions including binomial, uniform, normal, and chi square"? Almost none of this is beyond the capabilities of motivated high school students (I'm not so sure about the groups and fields), but all of it? Along with everything else?

The third quarrel is with the words "verify" and "validation" that appear many times in the 9-12 standards. I'm not sure what they mean, and what is expected of students when they are used.

Applications

While NCTM's *Curriculum and Evaluation Standards-for School Mathematics* (1989, p. 77) reminds us that "not all problems require a real-world setting," there is a strong impetus in current reform (based in contextualist theory) to try to root classroom mathematics in real-world problems, especially in middle schools that attempt integrated curricula.

The *Standards* documents are themselves fairly balanced on applications-real-world applications are no more (and, I hasten to add, no less) standards-based than theoretical mathematics. That the momentum toward applications-based curricula is done in the name of standards-based reform is unfortunate.

There is one crucial place in NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) that can give rise to this misapprehension, the discussion of why "the educational system of the industrial age does not meet the economic needs of today" (pp. 3-4). Three of the four new social goals serve business needs: the need for mathematically literate workers; lifelong learning (which is connected with "changes in technology and employment patterns" and not learning for its own sake), and equity (which "has become an economic necessity"; maybe that is what it takes to finally gain what should be a right). The next section goes on to establish "learning to value mathematics" as the first new goal for students. Perhaps we have something very close to a political contradiction here-can we simultaneously serve the needs of Boeing and create a society of, say, Thomas and Thomasina Jeffersons?

For a beautiful example of an applied problem that involves very deep mathematics, see "Lightning Strikes Again!" from *Measuring Up* (Mathematical Sciences Education Board, 1993, pp. 115-124) in which fourth graders have an opportunity to move from simple arithmetic calculations to working out the intersection of two circles.

For a beautiful example of serious and difficult mathematics motivated by a simple-sounding application, see the airport problem in *Connected Geometry* (Educational Development Corporation, 1996).

Pedagogy

Here, as with applications, the *Standards* do not say what they are charged-by both supporters and detractors-with saying. They do not say that all mathematics learning should take place through activities in small heterogeneous groups in which students develop all of the ideas, with the teacher acting only as a moderator. Yes, there is a constructivist orientation in the ***Standards***, but nowhere is it exclusive, and NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) reminds us continually that all forms of instruction are useful (although this is contradicted somewhat by the bias in the "Decreased Attention" pages).

But occasionally a more dogmatic attitude creeps in which is disturbing. For example, in NCTM's *Professional Standards for Teaching Mathematics* (1991), Rich says that he was "really reluctant to use that activity because it didn't seem like exploration. It made me feel that I would be directing the students toward a single result" (p. 142). But there are many times when directing students toward a single result is exactly what is called for. Furthermore, just because students are going to inevitably find the same result doesn't mean it isn't exploration. And, finally, sometimes exploration isn't called for.

Where the constructivist bent is seen most clearly is in NCTM's *Professional Standards for Teaching Mathematics* (1991), where most of the vignettes are about teachers becoming more constructivist in their methodology. This is understandable. Even now, many teachers have few sources of information on constructivist methodology, and there was a clear need for such information in 199 1.

It should be noted that the key issue in many of these vignettes is how to guide exploration and discussion. Contrary to parodies of constructivism, children are not left to their own devices, nor do they work exclusively in small groups.

Assessment

As a mathematician, I am not used to thinking comprehensively about assessment, and I learned a lot from reading NCTM's *Assessment Standards for School Mathematics* (1995). The basic notions in this document seem unassailable, and I was especially pleased to see the emphasis on performance assessment and citations of assessments from other countries.

But assessment is another place where what the *Standards* say is not what they are perceived as saying. Somehow there is a perception that standards-based assessment is inherently trivial, does not allow for arithmetic calculation or algebraic manipulation by hand, invites subjective judgment, and is designed to make all children look good.

I believe that these misperceptions have several roots. One is a key-word approach, in which certain terms (e.g., "open-ended," "equity") are given different meanings than they have in context. Another is a not unreasonable concern that something that seems difficult (e.g., creating a robust rubric for a problem with complex or multiple solutions) may not be possible. The third is a philosophical position (which neither I nor the writers of the *Standards* share) that there is something called objective assessment that can be used to categorize students and place them in appropriate educational programs. One sign of this philosophical difference is that the *Standards* say very little about assigning grades, while several of the critics of reform do not speak about assessment but about grading systems.

This last desire-to put kids in the appropriate classes--has roots in real, even poignant, situations.* Correct placement is indeed very difficult, as is teaching outliers. Perhaps this is one issue that is not sufficiently addressed.

There is one very important issue in NCTM's *Assessment Standards for School Mathematics* (1995) that is handled somewhat cavalierly, and that is the issue of time. Having begun to use some alternative assessments myself, sparingly, and with only two classes of about 25 students each a semester, I can tell you how time-consuming this is. I can't imagine my son's junior high school teachers-7 classes a day , about 30 students in each-doing it on a regular basis. As with the plethora of interesting ideas for curriculum at the 9-12 level, this seems too much to expect.

Teacher Preparation

I could pick some nits, but basically NCTM's *Professional Standards for Teaching Mathematics* (1991) outlines a solid mathematical background for mathematics teachers, which is most welcome.

I am puzzled, however, by the comment, "Since the spirit and content of the coursework described above can be very different from traditional courses, every effort should be made to develop new courses that reflect these differences" (p. 3139).

Except for the call for manipulatives in probability and statistics (which would be good for all students), I don't really see much if any difference between what is recommended for teachers and what we teach in our regular courses. There is a danger that an entirely different track for future high school teachers would be perceived as lower level than the regular mathematics track,' and I know from experience that in courses created for teachers there is often pressure from the students to be relevant to exactly what they will teach This can get pretty strange-our preservice students regularly complain about having to learn transformational geometry, even when we assure them that they will be teaching it themselves. They didn't learn it themselves in high school, why should they believe us? I suggest that a mix of courses within the mathematics department, some with a preservice emphasis, others for all mathematics and mathematics education majors, may be the best solution.

Equity

Racial equity is a serious issue for this society, which faces the great contradiction of a national rhetoric steeped in equity and historical roots steeped beyond inequity in genocide and enslavement. As for women, in no society have we had an easy time of it.

So I naturally welcome the emphasis in the *Standards* on equity (even with the corporate sponsorship on page *4* of the *Curriculum and Evaluation Standards).*

[8] As a formerly precocious child and the mother of a son with learning disabilities, I have too much familiarity with both ends of this particular spectrum.

[9] We may wish that danger away, but wishing will not make it so.

I have, however, a major concern about equity.

This is concern about the essentialist view, which seems to have its attractions in education—women think like this, African American men think like that-and is closely connected to cultural stereotyping. We need to guard very carefully against essentialism, even as we recognize that, yes, our society is made up of different cultures, these cultures have different rules, and when rules collide there are problems. The desire for easy answers here is what makes essentialism attractive, but there are no easy answers.

Concern for equity has given rise to one of the most emotional critiques of standards-based reform, the claim that it hurts disadvantaged, especially minority, kids. The charge is that trivial curricula and overly easy assessments give the impression that these kids are learning, when in fact they are not. In this view, the various standards about equity are viewed as hypocritically creating demands for false entitlements ("I have a right to pass algebra" and not "I have a right to learn algebra"). As far as I know, these charges have not been directed at the national standards, but at the California *Framework.* Those making them are quite sincere and are armed with stories of parents and teachers begging the schools to deviate from the *Framework* and teach their children substantive mathematics.

support

The final topic I wish to discuss is the last section of NCTM's *Professional Standards for Teaching Mathematics* (1991), the section entitled "Responsibilities." This sets forth the responsibilities of policymakers in government, business, and industry, the responsibilities of schools, the responsibilities of colleges and universities, and the responsibilities of professional organizations.

My only comment here is that most of these groups were not seriously consulted, so no matter how laudable the recommendations, they are necessarily moot.

Summary and Conclusion

The ambiguity of the notion of "standards," coupled with the (never quite explicit) clash of theoretical positions, would make any standards document impossible to agree with completely. Within these constraints, the NCTM *Standards* generally stresses what is mathematically important and is to be applauded for seriously attempting to create a culture of doing mathematics in the classroom. I have some disagreement with content emphasis (symbolic manipulation, difficult arithmetic, algorithms) and method (some of the notions related to proof and justification); other mathematicians will have other complaints. There are places where the documents could be written better-more carefully, less ambiguously, or with less bias. But the overall framework is a good one, especially if it continues to be revised, and especially if those in charge of the revision process listen seriously to mathematicians, educators, and teachers with diverse viewpoints.

The debate has been muddied, however, by confusing ***Standards*** documents with other reform documents, with various interpretations of reform, and with classroom practices justified in the name of reform. The extremism of much of the rhetoric that attacks or justifies reform is a serious problem. What we have learned from studies in mathematics education needs to be communicated to the general public as clearly as possible, especially on such contentious issues as constructivist pedagogy, technology, and assessment.

As a mathematician, I have focused on the *Standards* documents, knowing that they are only a part of the picture. And as a mathematician I like to end papers with questions. I will end this one with two, whose answers need a very different expertise than I can bring to the table: How are standards actually implemented? and What overall systemic changes have been/should be made so that the standards movement can succeed?

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References

Bass, H. (1994). Education reform from a national perspective: The mathematics community's investment and future. Notices *AMS, 41, 92* 1-926.

California Department of Education. (1992). *Mathematics framework for California public schools, kindergarten through grade twelve.* Sacramento: Author.

Educational Development Corporation. (1996). *Connected geometry.* Dedham, MA: Janson Publications.

Fey, J., & Heid, *K.* (1995). *Concepts in algebra: A technological approach.* Dedham, MA: Janson.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, Holland: Reidel.

Mathematical Sciences Education Board. (1993). *Measuring up: Prototypes for mathematics assessment.* Washington, DC: National Academy Press.

National Commission on Excellence in Education. (1983). *A nation at risk: The imperativefor educational reform.* Washington, DC: U.S. Government Printing Office.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics.* Reston, VA: Author.

National Science Board Commission on Precollege Education in Mathematics, Science, and Technology. (1983). *Educating Americans for the twenty-first century: A plan of action for improving the mathematics, science and technology education for all American elementary and secondary students so that their achievement is the best in the world by 1995.* Washington, DC: National Science Foundation.

Orton, R. E. (1995, May). Ockham's razor and Plato's beard. *Journal for Research in Mathematics Education, 26,* 204-229.

Reys, R. E., Reys, B. J., Nohda, N., & Emori, H. (1995). Mental computation performance and strategy use of Japanese students in grades 2, 4, 6 and 8. *Journal for Research in Mathematics Education 26,* 304-326.

Comments About NCTM's *Curriculum Standards*

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As chair of the Commission on Standards for School Mathematics for the National Council of Teachers of Mathematics (NCTM) I have been asked to comment on NCTM's vision for the reform of school mathematics.

Background

To put this work in perspective, let me go back 12 years when we started to develop the *Curriculum Standards,* published in 1989, and summarize what we thought we were trying to do. At the very beginning of the document it says that "all students need to learn more, and often different, mathematics and that instruction in mathematics must be significantly revised" (p. 1). The document was designed to reflect the organization's vision of a school mathematics curriculum to meet this need.

The key notion in that statement is *all students.* If students are to be mathematically literate and productive citizens in the twenty-first century, they all need to have a good mathematics background. This concern was voiced in *A Nation at Risk* (National Commission on Excellence in Education, 1983) and *Educating Americans for the Twenty-first Century* (National Science Board Commission, 1983). The authors of those documents claimed that competing in today's global economic environment depends on a workforce knowledgeable about the mathematical, scientific, and technological aspects of the emerging information age. Furthermore, they argued that our schools were not adequately preparing very many of our students to participate meaningfully in the real world of work, personal life, higher education, and the country's social and political institutions. In particular, in our increasingly multicultural society the participation and achievement of both women and minorities in mathematics and science lagged behind that of white males.

This strongly voiced concern was in response to the reality of a decade ago. At that time about 40% of the American students studied no more mathematics than what was typically covered up through Grade 8-shopkeeper arithmetic. These students were expected to learn only paper-and-pencil calculations and routines for whole numbers, common fractions, decimals, and percents.

Another 30% of our students were expected to take an additional two years of high school mathematics-one-year courses in algebra and geometry. The assumption was that this mathematics was sufficient for general college entrance. Most elementary and middle school teachers would have had this background. Another 20% of students were likely to be going to college and studying areas that required more mathematics. These students would take another year or two of mathematics in high school and, it was assumed, would then take some mathematics courses in college. Finally, about -10% of the population, it was assumed, would take more mathematics courses-four years of mathematics in high school (perhaps including advanced placement courses). This group would include potential engineering, science, and mathematics majors in college. Most high school mathematics teachers have this mathematical background.

Our long-term objective was to change the percentages—40%, 30%, 20%, and 10%—by focusing our work on the needed changes for the 90% of the population of American students who took the least mathematics.

More, and Often Different, Mathematics

The argument was that eight years of arithmetic for 40% of the population, and two years of high school mathematics for another 30%, simply was no longer adequate. The basis of this argument was that jobs were changing. Today no one makes a living doing paper-and-pencil calculations. Calculators and computers have replaced shopkeeper calculations in business and industry. Additionally, these electronic tools are capable of doing massive calculation tasks quickly and displaying information in a variety of ways-as a result, the skills that need to be emphasized in mathematics courses are no longer the same as in previous generations. The fields that use new technologies are growing rapidly and often require a deep understanding of traditional mathematical topics as well as some topics not in current school courses (e.g., discrete mathematics, mathematical modeling, statistics).

Instruction Must Be Significantly Revised

In most mathematics classrooms, daily instruction follows a five-step sequence-review of homework, explanation and illustration of a problem type by the teacher, work by students independently on a set of similar problems, summarization of work and responses to questions by the teacher, and assignment of homework consisting of similar problems. We argued that this sequence needs to be changed, because to learn something involves more than to be shown a procedure and then be asked to repeat it. Learning involves investigating, formulating, representing, reasoning, reading, using strategies to solve problems, proving assertions, and reflecting on how the mathematics is used. Classrooms need to become discourse communities where conjectures are made, arguments presented, strategies discussed, and so forth.

Scheffler's (1975) denunciation of the traditional mechanistic approach to teaching basic skills and concepts illustrates the difficulties with the traditional perspective about school mathematics:

> The oversimplified educational concept of a "subject" merges with the false public image of mathematics to form quite a misleading conception for the purposes of education: Since it is a subject, runs the myth, it must be homogeneous, and in what way homogeneous: Exact, mechanical, numerical, and precise-yielding for every question a decisive and unique answer in accordance with an effective routine. It is no wonder that this conception isolates mathematics from other subjects, since what is here described is not so much a form of thinking as a substitute for thinking. The process of calculation or computation only involves the deployment of a set routine with no room for ingenuity or flair, no place for guesswork or surprise, no chance for discovery, no need for the human being, in fact. (p. 184)

In a similar vein, Polya (1957) argued that the

> teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge, and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. (p. v)

In summary, NCTM's intent was to set in motion a lengthy process to change the way in which mathematics has been organized and taught in American schools. At that time it was clear what we did not want-the routine, dull, unimaginative instruction happening in most classrooms, which filtered out too many students from further study of mathematics. I must admit, however, that we did not have a clear vision of what it was we wanted as an alternative or how to achieve that reform. Rhetoric about the importance of solving problems, or about the need for students making conjectures and building arguments, or about doing something other than hours of routine calculation with little understanding, does not make such changes actually happen. In fact, we did not have a clear vision of what implementation of such slogans actually would mean in America's classrooms, nor how long it would take.

Curriculum Standards

The 40 Curriculum ***Standards*** were grouped into three levels corresponding to Grades K-4,5-8, and 9-12. Four of the standards at each level are common standards about the mathematical processes of problem solving, communication, reasoning, and connections; and the rest (28) deal with mathematical content. Each standard is a relatively brief statement and has the following format:

STANDARD-:

In grades ______, the mathematics curriculum should include ________________ so that students can-

- ______________________,
- ______________________;
- ______________________

These statements are the core of the document. They indicate a vision of the mathematical content that all students should have an opportunity to learn. Furthermore, these are the criteria that states and schools are to use to judge the quality of their mathematics curriculum and their textbooks. Let's look, for example, at the following standard:

STANDARD 6

NUMBER SYSTEMS AND NUMBER THEORY

In grades 5-8, the mathematics curriculum should include the study of number systems and number theory so that students can-

- understand and appreciate the need for numbers beyond the whole numbers;
- develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers;
- extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers;
- understand how the basic arithmetic operations are related to one another;
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations. (NCTM, 1989, p. 91)

Each of the standards was written to indicate elements of a mathematical content domain that ought to be in the curriculum in those grades. The bulleted items indicate the key appropriate elements in that domain. We expected educators to use the standards when they examined a curriculum plan or textbooks.

Introductions, Examples, Next Steps

There are three other parts of the document. First, there is an introduction for the whole document and introductions to the K-4,5-8, and 9-12 sections. These introductions were written to set the stage for the reader of the ***Curriculum Standards.*** They contain brief statements about goals, the need for change, and assumptions on which the standards were based. One topic in the overviews to each level of the standards has been misunderstood; namely, the summary tables indicating "increased and decreased attention." In particular, "decreased attention" does not mean that the content is omitted. The message we were trying to convey in these tables was that the emphasis in traditional instruction was that students were to become proficient at using procedures, without necessarily understanding them. We were trying to shift that emphasis. For example, many students (and most adults) can multiply a two-digit number by another two-digit number and produce a correct answer, but are unable to give any reason for why the procedure works other than to say, "That's what I was taught to do."

Second, following each standard, we wrote two subsections: an explanation of the standard, entitled "Focus," and a "Discussion" containing examples that could be used in lessons. These subsections have proven to be problematic. Whenever anyone starts giving examples, someone will object because "you didn't include my favorite example." Also, some of the examples are good, and some of them not so good. In fact, when I now read some examples I ask myself, "Why didn't we include something else?" Next, I get comments from persons who say, "I've read this example, and that meant. . . . " Unfortunately, that is not what we intended. We realize that persons bring to any task their background, their experience, their use of language and terms, and so forth. Unfortunately, their understanding of terms does not always match ours. Most of the comments and criticisms of the Curriculum *Standards* are not about the standards, but about the examples.

Third, there is a final section to the document titled "Next Steps." We expected educators to use the document as a starting point for an open discussion about what would be included in the school mathematics curriculum. In fact, Mathematical Sciences Education Board (MSEB), following the publication of the *Curriculum Standards,* advocated a year of national dialogue. Unfortunately, this never happened. The document was meant to be a background document for changing the content of school mathematics, not the final word.

Other Comments

First, all publishers now claim that their materials meet the standards. Such claims are a marketing tool with little substance behind them. They are what Preston (1996) called "puffery in advertising." Such claims are legal, but often deceptive. There is not a textbook currently being marketed that truly reflects the standards.

Second, NCTM is in the process of developing a revised set of curriculum standards. Development of the current document began in 1986. The first draft of this document was printed for review in 1987, revised in 1988, and published in 1989. NCTM said it would prepare a revision every 10 years. The revisions being considered include the following:

- Sections will be reorganized to fit more closely with school 'practices. There will be sets of standards for Grades pre-K-2,3-5,6-8, and 9-12.
- The four basic process standards-problem solving, communication, reasoning, and connections-will be retained. But their role cutting across the content standards will be emphasized. They were not supposed to be considered independent of content. Some readers claimed they developed problem solving activities, for instance. Students do not just solve problems. They solve problems within particular mathematical domains.
- A fifth common standard with respect to procedures or routines will probably be included. In retrospect, although the use of procedures, and the understanding of those procedures, appeared in many standards, these concepts were buried in the bullets and examples.

- There will be an emphasis on content strands (number, algebra, geometry, statistics, and so forth) across levels. For example, at present there are several different standards on number in the Grades K-4 and 5–8 standards but nothing at 9-12. An examination of the coherence of ideas in a strand across the grades is needed.
- And then, of course, the examples that were included will be updated and their quality and appropriateness verified.

Third, when we finished writing the ***Curriculum Standards***, we thought the document would occasionally be read by graduate students, but have little overall impact. After all there have been documents like this produced fairly regularly in the history of education that have had little real impact. I believe this document succeeded because it filled a political void and because it was written in very general terms. It only indicated ideas that we needed to think about when considering the content in the school mathematics curriculum.

The biggest surprise for me was the wide acceptance of the curriculum standards at a very general level. I have heard politicians say, "We need to do something in mathematics. This looks like a reasonable set of ideas . . . solve problems, communicate, reason . . . deal with numbers," and so on, without understanding the message. Some of the acceptance was due to the way in which the document was written. Some had to do with the fact that it was written by NCTM, the group whose members are those responsible for teaching students in the public schools. In the past, teachers had little voice in what they taught. In fact, the organization argued, teachers have long needed something that they could put in front of administrators and other policymakers to say, "Here is what we ought to be doing as we consider our curriculum or adopt textbooks, rather than being told by publishers what is available." Teachers have found the document empowering. For me this fact has made the effort truly worthwhile.

References

National Commission on Excellence in Education. (1983). A *nation at risk: The imperative for educational reform.* Washington, DC: U.S. Government Printing Office.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Science Board Commission on Precollege Education in Mathematics, Science, and Technology. (1983). *Educating Americans for the twenty-first century: A plan of action for improving the mathematics, science, and technology education for all American elementary and secondary students so that their achievement is the best in the world by* 1995. Washington, DC: U.S. Government Printing Office.

Polya, G. (1957). How *to* solve *it.* New York: Doubleday.

Preston, I. L. (1996). *The great American blowup: Puffery in advertising and selling.* Madison: University of Wisconsin Press.

Scheffler, I. (1975). Basic mathematical skills: Some philosophical and practical remarks. In *The NIE Conference on Basic Mathematical Skills and Learning. Contributed position papers* (Vol. 1, pp. 182-189). Los Alamitos, CA: SWRL Educational Research and Development.

A Commentary on the Profound Changes Expected by the National Science Standards

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The goal of this paper is to provide a commentary on the national standards from the viewpoint of a faculty member who teaches in the physical sciences at a major research university. The commentary is directed towards Senta Raizen's paper, *Standards for Science Education* (1997), the National Research Council's (NRC) *National Science Education Standards* (1996), and the American Association for the Advancement of Science's (AAAS) *Benchmarks for Science Literacy* (1993). Our first observation is that the standards are brilliant. The *Standards* represent a complete and deep framework of goals that will profoundly change the capabilities of our society if they are reached, and they **can** be reached. However, severe problems may impede the attainment of these goals. The most severe problem is embedded in the mechanism for reaching these goals because the path to the goals requires the participation of the people who created the problem. The standards call for a systemic reform of the educational process, but the implementation of the reforms requires a change in attitudes and expectations. Our attitudes and expectations are the result of our past experiences and our understanding of how society expects things to be done. Changing attitudes and expectations is among the most difficult things an individual can be asked to do. The individual must understand in his or her soul:

Why should I change?
How do I change?
Can I change?
Will the change bring success?

These are the central questions for each individual involved in the educational system. However, the standards do little to address any of them. The standards are focused only on the goals, the definition of what success **is.**

The standards purposely do not help the individual understand how to accomplish the goals. They intend systemic reform to use a bootstrap approach. To achieve scientific literacy in students, teachers must change the nature of the learning experience so it incorporates scientific inquiry and self-discovery. The *NSES* recognize that teachers themselves must be scientifically literate if they are to succeed in modeling a scientific approach to problems. However, teachers were once students who were taught in ways that did not foster scientific literacy. Consequently, systemic reform requires that both teachers and students reach scientific literacy concurrently. The *NSES* recognize correctly that this problem is so fundamental that change will occur slowly over time and will require great patience. Since we are an impatient society, there needs to be a clear vision of what scientific literacy means and what it will look like when we reach it. If this vision is not clear, there will be no answers to the questions: Why should I change? and Will the change bring success? If we do not see or experience any models of successful reform that **reify** educational reform for us, we will not be able to answer the questions, How do I change? and Can I change? (*Reify* will be used extensively in this document. Its definition is "making the abstract concrete." Reification plays a central role in Judy Roitman's discussion of the mathematics standards.) If it is clear to people that change will benefit them as individuals, they will be willing to participate in the struggle. However, there is a limited window in which to discover a personal vision before patience is lost and the systemic reform effort becomes just another educational fad.

A number of points will be developed in this paper, and they all focus on the definition of what must be done to implement the standards successfully. First, the grain size of the systemic reform standards is purposely large (Raizen, 1997) in order to ensure that disagreement over details does not obscure the important ideas of what must be accomplished. However, this large grain size contains the seeds for the demise of the reform effort, because failure to appreciate the exact nature of what the standards intend will result in programs that do not attain the profound changes in skills and attitudes that are required of students. In particular, the standards do not define the problem they are trying to solve, nor do they (define scientific literacy with the precision required for implementation in a classroom. This paper attempts to provide a clearer understanding of how the framers of the standards would define scientific literacy, which is their central goal. It argues that, in addition to the normal teaching of content, concepts, skills, and appreciation for the beauty and power of science, student experiences must both reify the curriculum and make fundamental changes in student attitudes and skills that are at the heart of scientific literacy. This paper will further argue that a central problem of systemic reform is adjusting the relative amount of control and freedom that must be present in all student classrooms, including those in colleges and universities that not only teach our future teachers but also should be teaching our current teachers. It concludes with a question of whether we are ready to engineer a massive systemic reform or whether there are still crucial questions that must be answered before the foundations are strong enough to support massive systemic reform.

Grain Size

One of the key ideas a successful researcher learns is that the first and most important step in performing original scientific work is clearly defining the problem. It is therefore particularly striking that, although both the *Benchmarks* and the *NSES* have the problem of scientific literacy as their central focus, neither really defines the problem they are trying to solve, nor do they define the scientific literacy they are trying to attain. Definitions are not given in fundamental terms that allow an understanding of how to implement the solutions or how to recognize solutions.

A state mathematics supervisor said, "One of the brilliant characteristics of the *Curriculum Standards* is that the grain size is big enough that the bullets are not damaging" (McLeod et al., 1996, p. 116). The *Standards* clearly define the goals that must be achieved by all concerned, but they do not define many of the details that are required to put the standards into practice. Although this approach draws broad support for systemic reform, the framers of the *Benchmarks* and the *National Science Education Standards* postpone the difficult implementation issues where the different participants might not agree at all. Although the standards provide many examples of curriculum materials, they still do not define the details required to implement the standards in a classroom or the assessments needed to document student success. As with the Rorschach test, faculty will see different messages about the goals and attitudes underlying the framing of the standards. They will always have their own perception of what scientific literacy is. This large-grained approach provides the freedom to develop an implementation that matches the needs of local districts and represents a strength of the standards. However, for the vision of systemic reform to be clear, it is crucial to have models that give the educational enterprise examples of how to attain the standards and assessments that give credible measures of student success. If the vision is not clear, the implementation will fail.

There are two predominant definitions of a scientifically literate person:

1. A person who possesses a reasonable knowledge of the scientific concepts that control the world and issues that the typical citizen faces and who also possesses an appreciation for the beauty and power of science, mathematics, and technology.
2. A person who possesses not only a knowledge and appreciation of science but also develops the attitudes and problem-solving skills that typify scientists as well as the inclination and ability to apply them in their everyday lives.

The second goal is far more ambitious. Many educators feel it is unattainable. The definition we choose should be the one whose attainment solves the problem perceived by parents, educators, business leaders, and politicians. Better yet, it is the definition that maximizes human development. If we have not precisely defined the problem, we cannot define what we expect of a scientifically literate populace. We must also realize that our secondary school teachers will acquire attitudes and skills that reflect the definition we choose and that they will teach toward that definition.

The most important consequence for our definition of scientific competence is determining the depth and breadth of student accomplishment that meets the baseline standards and the teaching approach that is needed to realize them. Whom are we teaching? Future scientists or all students? Are we teaching all students at the same depth? Should all students have experience with exercises requiring more sophisticated problem-solving skills, or will experience with one-step problems where novice strategies work be sufficient? Should all students be exposed to problems requiring abstract thoughts, or should those problems be given only to excellent students as challenges? Do the baseline standards require only that students appreciate the richness and excitement of the natural world, or must students be involved in scientific inquiry deeply enough that they actually experience the logical thinking of a scientist? Do the standards aim for incremental improvements in our school systems, or do they aim for profound changes in the ways our children think? Are science courses teaching only science, or are the skills learned in science applicable to the everyday lives of everyone? Since educators hold different ideas on what students are capable of learning and what the objectives of education should be, their individual viewpoints will provide different answers to these questions. Perhaps the defining criterion should be, What must we do to maximize human development?

Objective reading of both the *Benchmarks* and the *NSES* leaves little doubt about the views of their framers. They both set the reform goal as the attainment of **scientific literacy** by **all** students. The NRC *National Science Education Standards* define having scientific literacy as being able to

- experience the satisfaction of understanding the natural world
- use scientific thinking in making personal decision
- participate intelligently in societal decisions on science and technology
- attain skills and knowledge that are required for being productive in our current and future economies. (1996, p. 13)

This last statement makes it clear that the writers of the *Standards* are optimistic about the ability of all students to master science. The *Standards* state that business requires even entry-level workers to have the ability that learning science can provide to learn, reason, think creatively, make decisions, and solve problems (p. 12). Workers should have the skills required to solve the problems given to them without constantly returning and asking, "What do I do next?" The *Standards* speak of creating a community of scholars and life-long learners. These goals require a profound change in student attitudes. In both documents, a central standard for attaining literacy is engaging students in meaningful inquiry, including experiences requiring inquiry over extended periods of time, so students experience the highs and lows of success and failure that characterize authentic problem solving. Novice problem-solving strategies fail whenever authentic problems are encountered, because there are always unanticipated consequences that become readily apparent when students put simplistic ideas into practice (Schoenfeld, 1987). The need to plan is never so much appreciated as when students have to repeat work because they purposely decide to "go for it" without planning, only to find problems that could have been anticipated. Genuine inquiry brings the perspective and insights to scientific ideas that are necessary if the ideas are to become part of a person's useable knowledge and skills (Newmann, 1995). Inquiry, which is central to the process of reification, is a central feature of the standards.

It is crucial to recognize the importance of mathematics, science, and technology to the development of student abilities and attitudes. Science is the one subject area in our schools that allows doing something concrete. Not only do children enjoy it, they also have the opportunity to transform thoughts into action with their own hands and initiative. Successful completion of a project can be very rewarding, because there is an opportunity for students to acquire ownership for their accomplishments. They also have the opportunity to fail and discover what to do next. Unfortunately, there are few places in the curriculum where students experience these attitudes and skills. Students will face countless problems during their lives, and we would all like our students to master them. Will they have the confidence and experience to take on important problems, or will they react emotionally and say the problems are bigger than they can handle? Will they have the discipline to carefully gather the facts, define the problems, and plan solutions, or will they act impulsively and try something, anything? Will they think critically about what the experts are telling them, or will they accept recommendations without question? Will they exercise good judgment about working cooperatively with others, or will they do the entire task themselves? Will they be paralyzed by small. problems, or can they be counted on to think and exercise initiative? Successful people have experiences that give them confidence to tackle challenges. They know how long to spend on a problem and when to get help. They have a "can-do" attitude. Is this attitude inherited, or can it be learned? If it can be learned, can it be taught in our schools?

The answers to these questions are clear to the framers of the Standards. They expect that successful implementation of the standards will provide the experiences and develop the attitudes that **all** students will need for success in life. This vision is very different from what most people think science courses are intended to do. Traditionally, science courses are intended to develop the chosen few who will be scientists and provide the remaining multitudes with enough insight that they will have a basic understanding and appreciation of the world around them. Where is the "can-do" attitude developed for the multitudes? If one looks at the possibilities in the core curriculum, there are not many subjects that potentially offer the same depth that mathematics, science, and technology can provide. It is a remarkable and sad, but true, fact that many people have found that participation on sports teams has instilled the attitudes required for success. Perhaps it is only sports where students have the meaningful experiences that give them the insights into the attitudes that foster success. One need only observe the permeation of our business and political vocabulary with sports terms to understand the close relationship between the attitudes required for professional success and sports. Perhaps that direct connection between extracurricular-s and success in life is responsible for the 60% of citizens who feel that extracurriculars deserve their emphasis and resources in contrast to the 35% who feel that money should be diverted to academics (Toch, 1996). Would it not be wonderful if the new standards caused intellectual fulfillment and sense of accomplishment to displace some of the intellectually less meaningful experiences? The lessons of success and failure, the lessons of teamwork, and the lessons of meaning can be taught outside of sports if we bring authentic scientific experiences to our students.

Achieving Scientific Literacy

If the skills and attitudes learned through scientific inquiry and problem solving can play central roles in a person's everyday life, how do we design an educational strategy that will create an environment where those skills and attitudes are learned? There are probably many answers to this question: problem-based learning, cooperative and collaborative learning, discovery-based learning, mastery learning, topic-oriented approaches, holistic education, etc. Despite the multitude of possible approaches, the standards make it clear that successful strategies must contain some common central elements. Two of these efforts merit special consideration because they are so central to the success of the systemic-reform initiative.

Reification

Science, technology, and mathematics are powerful vehicles to develop student abilities to connect the concrete world around them with the abstractions that allow them to see connections and make generalizations necessary for creativity, perspective, understanding significance, and seeing the big picture in their everyday experiences. These abilities have their roots in reification. It is a key idea in Judy Roitman's paper on mathematical standards. A central failing in our current educational system is that, although our students have learned definitions, algorithms, and facts that define science and mathematics, reification has not occurred. The definitions, algorithms, and facts remain isolated and have not been. incorporated into everyday skills. They do not spring from memory when a creative insight is required or a new situation is encountered. Students do not have ownership. Our lower-level courses are ineffective if reification has not occurred. Reification is a central goal of the standards; :it essentially defines what scientific literacy should be. It is the foundation for common sense about how the world works that we find abundantly in the people we need for leaders. Its attainment requires schools to provide inquiry experiences in mathematics, science, and technology at all grade levels, and it is crucial for university faculty to ensure that these experiences continue at the undergraduate level as well.

One of the central beauties of the national standards is that they include mathematics, science, and technology. A tragic failure, especially of the NRC *National Science Education Standards,* is the lack of integration between mathematics and science in the structure of the standards and their examples. Although the standards do explicitly include coordination of the mathematics and science curricula as standard C in the program standards, the examples and descriptions in the content standards and professional development standards contain only superficial connections. To successfully reify student experiences, it is crucial to have an integrated program of mathematics, science, and technology. The integration should not be done at the expense of mathematical rigor. It is difficult for most students to attain mastery of abstract mathematical and scientific concepts because they are not concrete and they are not a part of our intuition. The content standards build the experiences necessary to master abstract concepts as the students progress through the educational program. The elementary grades concentrate on providing students with concrete experiences that will develop an intuition for what happens in the world.

Technology is introduced in these early grades in the form of building projects. There is no better way to see how something works than to build and manipulate the levers, gears, chains, etc. in a real machine or device. Progressing from technology to science increases the level of abstraction that is required. Scientific examples are introduced after the technological ones, first the concrete and then the abstract. The factors that control many scientific effects have a larger mental component than machines. The number of atoms in a gas as well as their pressure, temperature, and volume are concrete and intuitive concepts, but their interrelationships require mental pictures. Technology such as computer animations can provide assistance in helping the students to form mental pictures that interrelate physical quantities, but they cannot substitute for the mental pictures that must form if reification is to occur. As one progresses, mathematics enters and the level of abstraction increases. Mathematics enters the curriculum to support scientific and technological experiences for two reasons: (1) because it is the only way for students to understand the relationships that define many physical phenomena, and (2) because it is important for students to develop the ability to master abstract ideas. Success at this stage opens the door to extending the abstraction to mathematical ideas that do not have explicit connections with the physical world but represent pure abstract mathematics. In addition to supporting the connection that science and technology have between concrete examples and the abstract ideas required to understand them, it is crucial for students to have experiences where the abstract ideas are first encountered and the concrete pictures are added as a way to understand the abstractions. As Judy Roitman observes, this extension gives the opportunity to teach how mental pictures can help in the reification of abstract mathematics. It also opens the door to understanding how to handle complex systems where many factors interact. Not only must one understand how the individual parts work, but the interactions lead to new behaviors that are often crucial to the system's operation. It is important for students to develop a perspective that includes the details of the individual components and the "big picture" that affects us.

Attitudes

Systemic reform is intended to be a sustainable revolution in the nature of the learning process. In order to be successful, it needs to result in fundamental changes not only in the attitudes and expectations of individual students, teachers, administrators, and parents, but also in the traditions of the society that feeds those attitudes and expectations. Changes in attitude and tradition are the most difficult to achieve because they require changes in the ways that individuals view the world. As previously noted, it is necessary for individuals to see the need for change, see how to change, believe that they can make the change, and be convinced that the change will profoundly improve student skills and attitudes. The changes need to occur in elementary, secondary, and postsecondary schools.

It is clear to people who subscribe to systemic reform that the architects of the standards expect the standards will result in this revolution if they are implemented successfully. Volumes have been written about current student attitudes and what must be done to change them (Tobias, 1990a, 1990b). All of the reform strategies assume that more responsibility for learning needs to transfer from the teachers to the students (Barr & Tagg, 1995; MacGregor, 1990; Heller, Keith, & Anderson, 1992; Heller & Hollabaugh, 1992). Our society has placed a high premium on providing our children with a safe and supportive environment that will teach them the lessons of life. With the best of intentions, we have tried to raise our children by anticipating their needs and telling them how to function successfully in the world so they will not have to experience the problems and failures as often as we did. Consequently, most of their experiences have been passive ones where they listen and the teachers teach, they watch and the media show and tell. When they arrive at a college or university, they expect faculty to "make them learn" (Katz, 1996). They and we have believed that, if they go through the system and do as they are told, they will pop out the end of the pipeline as successes and have a good job waiting. Problem solving starts with looking at their textbook and notes, going to reference materials in the library, or checking the Internet for some place where the answer is available. If the course is being taught by a "good" teacher, he/she will have covered all of the important problems. If answers cannot be found, one finds an authority or a generally smart person who already knows the answer or can figure it out quickly. Hard problems are solved by putting the conditions into a computer and letting it chug through the work. If it does not work, we just need a more powerful computer. Thinking independently is far down the list of options for many students because they have little confidence that it will be successful. In fact, they **expect** to get only partial credit for almost anything they do in life.

In contrast to the *NSES,* the *Benchmarks* emphasize the importance of student attitudes. In a separate chapter on the habits of the mind, the *Benchmarks* discuss the values, attitudes, and skills that define how students think and act and what students consider important in their lives. These habits of the mind transcend the individual parts of the core curriculum and encompass a broader change in students than those associated with better problem-solving and thinking skills. There are a number of specific changes that are expected in these *Benchmarks:*

- They state explicitly that, to be scientifically literate, a student must have the knowledge; the quantitative, communication, manual, and critical response skills; **and the attitudes and inclinations** required to solve problems (pp. 282-283, emphasis added).
- They stress the importance of linking quantitative and estimation skills with learning about the real world so students develop intuitive feelings for what is reasonable (p. 288). Mathematics must be brought in at all grade levels when science, technology, social studies, health, physical education, etc. are taught. The results should be checked against estimates based on student intuition so real world connections are made and students become accustomed to constantly checking their results throughout the problem-solving process.
- They emphasize the development of the manual and observational skills that connect the mind with the world (p. 292). These skills include measuring, repairing, troubleshooting mechanical and electrical devices, building structures, constructing devices, keeping notebooks, making electrical connections, taking things apart, and using common tools, audiovisual equipment, calculators, and computers.
- Finally, they state that scientifically literate adults respect and use the clear and accurate communication skills that are characteristic of scientific work. They should be and expect others to be quantitative in their assertions and arguments. They should recognize when vague and unsubstantiated arguments are used when quantitative ones are possible and relevant (p. 295).

Most importantly, these habits of the mind are only acquired when students are personally involved in inquiry.

Even more significantly, the Benchmarks require students to go beyond merely acquiring these ideas, skills, and attitudes. They require that **all** students are actually **likely** to use them and to make the necessary connections between them in new situations when it is appropriate. This expectation to use the lessons of science, mathematics, and technology in new contexts is one of best examples of how profound a change in student skills and attitudes is intended by the *Benchmarks*. Why is such sophisticated mastery of scientific ideas expected from all students? Is it an actual expectation or simply an aberration that entered the *Benchmarks* without broad support? The answer permeates this chapter in the *Benchmarks* where it is acknowledged that even though most students will not be scientists (p. 287), they must still internalize these habits of the mind because they are important and applicable to **everyday life:**

- The scientific attitudes associated with critical thinking are particularly important in relation to medical, political, commercial, and technological. claims. Individuals must make informed decisions about medical treatments. Parents must make decisions about when to seek medical attention for their children. Adults must decide whether a physician's diagnosis of their elderly parent's illness makes sense. Consumers must determine whether the salesperson, advertisement, or repairperson is being honest. Voters must determine which of the political arguments offers the best hope for the future. Managers must decide between two vendors of a product about which he/she does not have a clue. Do we ask questions and develop a picture that allows us to be part of the decision process, or do we give up and say we need a rocket scientist? Do we have the perspective and confidence to make choices intelligently?
- Everyday life involves quantities and numerical relationships. Although there are many situations where answers are known, it is more typical that the answer is not known. It is important to be comfortable with the estimation process and have the common sense to help judge whether something makes sense.
- Although students will rarely use scientific instruments, the products of modern society have become so sophisticated that the skills of a scientifically literate person will extend to using the tools of everyday life. In particular, it is expected that manipulative skills will unite with scientific and mathematical skills to help people to solve problems and increase their understanding of how the world works throughout their entire lives. When something does not work, scientifically literate persons will have the judgment required to either fix it themselves, if it can be done with ordinary troubleshooting techniques, or get the assistance of experts if necessary.

It is easy to overlook or trivialize the message of this chapter in the *Benchmarks* if one does not **believe** that **all** students are able to attain these standards. It is crucial to recognize that the foundations of systemic reform are based on this very assumption. *If* it is false, the *Benchmarks* lose their meaning and importance.

The NRC *National Science Education Standards* have similar views on the attitudes and depth of skills that must be part of the standards. They do not express them in a separate chapter, but the same ideas are integrated throughout the standards. The NRC *National Science Education*

Standards have the same goal of establishing high levels of scientific literacy for all people in the United States. They define scientific literacy as the knowledge and understanding of scientific concepts and processes required for personal decision making, participation in civic and cultural affairs, and economic productivity. Scientific literacy expects that a person can ask and answer questions that arise in their everyday lives. It expects people to describe, explain and even predict natural phenomena (*NSES*, p. 22). The *Standards* explain in the sections on teaching methods, professional development, and the content standards that scientific literacy is developed by having students engage in scientific inquiry so they internalize the depth of understanding, creativity, insights, judgment, logical skills, modeling, picture making, clear communication, skepticism, and discovery as well as the ability to work in groups, the ability to see alternative viewpoints, and the ability to monitor their progress and self-correct as necessary. They also expect that the emphasis on inquiry will give students the disposition to use the skills, abilities, and attitudes associated with science.

These goals are largely the same as those in the *Benchmarks*. Interestingly, the *National Science Education Standards* do not explicitly state why they are expecting all students to engage in the depth required for meeting the *Standards*. They do not make the same argument that scientific skills and attitudes are transferable to the everyday lives of everyone, but they emphasize the science context for skill development. This difference is puzzling. It suggests that the *NSES* do not share the same expectation that problem-solving skills learned in the classroom will transfer to the everyday problems that individuals face in their lives and professions.

Another major difference between the *National Science Education Standards* and the *Benchmarks* is the central role of the teacher in the *National Science Education Standards*. The *NSES* recognize that even though every part of the system must be involved in systemic reform, the changes in teaching are the foundation for all change (p. 28). The way that teachers view science and technology and the way they view and understand their students and their students' abilities deeply affect their approach and effectiveness. They must first of all believe that all their students can master the standards. They must have a deep appreciation for the tenets that underlie the standards and science. They must be able to guide students through their teaching, nurturing, and modeling. They must provide opportunities for students to attain the habits of the mind. They must decide on the delicate balance between breadth of topics and depth of understanding. They must have the creativity and insight to incorporate inquiry into their courses in ways that are challenging, interesting, and relevant. They must balance individual, small group, and large group work. They must encourage examination of alternative viewpoints. They must assist in developing judgment about when to struggle with a prolblem and when to proceed. They must judge when and how much to help and when to let the students work things out for themselves. They must invent assessment mechanisms that do justice to the new-found skills students are acquiring and recognize the creative nuances in a project about which students are thrilled and for which they have genuine ownership. These tasks require teachers with judgment, insight, and experience that they can hone only by engaging in authentic inquiry themselves. It will be impossible to develop a community of learners in schools if students see large discrepancies between what should be happening in schools and what is actually happening (*NSES*, p. 50).

Impediments to Change

The Control versus Ownership Continuum

The *Benchmarks* and the *NSES* are permeated with the theme that, for systemic reform to be successful, responsibility for learning needs to be increasingly placed on the shoulders of the students. Almost all educators and teachers will nod their heads in agreement, but very few actually understand the true meaning of that statement. These educators require the large grain size of Senta Raizen's bullets so they are not damaging. All educators are blinded to various degrees by an Atlas complex, the belief that they control all learning (Finkel & Monk, 1983). The weight of the world of learning is on their shoulders. We feel a great responsibility for ensuring that students learn the material at a depth that allows them to see the beauty and structure of our fields, and we try very hard to build controls into our courses that ensure students will achieve the depth of mastery we feel is essential. We really enjoy our time on stage in the role of Atlas. We test students to define whether they have mastered the material, and we pronounce judgment on the merit of those students. Our tests are artificial inducements to learn, but our judgments can shape how students think and feel about themselves for a lifetime. This need to control the learning process can prevent us from seeing and understanding the different viewpoints, goals, and attitudes about teaching expressed in the standards. If we have complete control over learning, students can own only what we give them; they must be able to accept what we give them and make it their own. If students have control over learning, the ownership is very different because they discover ideas themselves. Reification is most effective when students are responsible for their own learning.

Perhaps the most important challenge in the implementation of systemic reform is developing models that span a continuum of strategies. We certainly have a great deal of experience with control strategies. We have found they are very effective in covering content and ensuring that the students have been taught correctly. We have also learned that they fail terribly in retaining the knowledge and developing a "can-do" attitude. Our experience with strategies that abandon lecturing and embrace discovery suggest that reification is optimized but content coverage is sacrificed. Do students taught with a discovery-based strategy develop the attitudes expected by the standards? Do some of the student discoveries harbor misconceptions? Will the limitations in content coverage be solved later as the students use their new-found skills to fill in the missing pieces? Perhaps a blend of the control and discovery approaches is best. The answers to these questions require work by motivated teachers, and it is with them that the excitement of systemic reform rests. The only clear answer is that our old methods based on control are not successful in creating the outcomes our society demands. Why are the outcomes different from when we were students? Neither the *Benchmarks* nor the *NSES* gives us the answers.

Some guidance in exploring the question of how to optimize the learning environment is provided by analogy with watching students engaged in scientific research. When students work on original research, content is sacrificed to the extreme.. Students enter a deep mineshaft into a very specific part of a subspecialty. Teachers give up a great deal of control and are amazed at the phase change in student maturity that occurs as a result of research. The changes become evident in the first two years of research, and after four years students have developed sophisticated capabilities. It is most amazing and satisfying to see this development in students who have less-than-adequate backgrounds or thinking skills. Do misconceptions arise? Certainly, but they are easily managed. In fact, we know that misconceptions are common when we learn a new field ourselves. We are constantly on the lookout for the parts that do not make sense because they represent the opportunities for insight and discovery as we explore a new field. Students must learn the same process on their own. Does reification occur? Yes. Are attitudes changed? Profoundly. That is what the Ph.D. is all about. It represents certification that a person has developed the skills and attitudes to do independent work and be a life-long learner. Can something like that occur in the K-12 system? That's what the framers of the ***Benchmarks*** and ***Standards*** thought but were reluctant to state explicitly because people with an ingrained Atlas complex may not be ready to hear that message.

Surely, we are not serious about all students in K-12 acquiring the kinds of skills and attitudes that typify a Ph.D. degree! Our current system almost certainly does not have that aim because it both underestimates students' capabilities and undershoots the challenges given in the teaching. Teachers who believe that students lack motivation and capacity and teachers who are accustomed to controlling learning will be systematic, sequential, simple, clear, logical, and forceful in developing subject matter so students cannot: fail to understand. The chemistry section on Gas Laws at the university level will be taught by separately looking at the P,V relationships first, then the V,T relationships next, etc., with adequate and simple problems to insure understanding. Students will leave the lecture saying, "Everything is so clear. That was a brilliant and enjoyable lecture," only to find that their understanding fades away quickly. They have no clue about how to approach a new problem or laboratory project. This approach is very different from giving a more authentic and challenging problem that requires understanding the Gas Laws as part of the problem. We badly undershoot, and we wonder why student interest in science plummets between sixth grade when they are excited and high school when they are terrified. Fear of failure can paralyze a student engaged in the inquiry and discovery process, which requires freedom and encourages mistakes in order to see the consequences. Teachers with the Atlas complex, and that includes all of us, have a particular problem when we are asked to grade and judge the students engaged in this process. The very act of grading removes some of the freedom required to have a successful inquiry and discovery process.

Assessment, Examinations, and Judgments

If we are to reach the higher motivational plane envisioned in the standards, assessment strategies must also evolve. We must decide what the role of assessment is. Is assessment a method for allowing students to see where they need more work? Is it a method for allowing teachers to see where students are in their understanding? Is it a summative judgment on how well the students have mastered the material? Is it a measure of the relative merits of individuals within a course? Is it a tool that we use to enforce hard work? Is it a way for employers, universities, and medical schools to judge who deserves to employed or admitted'? All of our teaching instincts are based on a reward and punishment system where examinations and grades define the accomplishments and even the worth of individual students. Grading on the curve ensures that limited numbers of students can succeed. The standards expect that all students will be successful. Furthermore, the standards require courses to aim at developing skills and attitudes that are not measurable with our traditional examinations:

- the ability to inquire;
- the ability to reason scientifically;
- the ability to use science for personal decisions;
- the ability to use science for taking a stand on societal issues;
- the ability to communicate scientifically.

The standards expect students to participate with teachers in refining their expectations based on the outcomes measured by the assessments. Students resist and lose respect for assessment procedures that inadequately allow them to demonstrate the sophistication and maturity they have obtained. They are discouraged when teachers either do not recognize or criticize something of which the student is proud. It is necessary to reward the behaviors that are the focus of a course. Assessments need to be an integral part of the course structure and teaching strategies, and they need to reflect the standards.. The traditional roles of teachers and students need to evolve as well. Teachers need to relinquish as much control and judgment as possible while students need to accept responsibility for acquiring life skills and attitudes that will shape their futures. Once again, the standards do not tell us how, but these changes must occur if systemic reform is to reach its potential.

Professional Development

The standards require experiences that are not part of our traditional teacher training. One of the central problems in accomplishing systemic reform is reforming the skills and attitudes of teachers and administrators. One cannot teach, model, or support what one does not know, feel, or accept. Teachers and administrators are products of a traditional school experience that has not taught, modeled, or supported the very principles the standards define. Teachers model the way they were taught, especially their most respected teachers. We all aspire to be Atlas. Few secondary school teachers have had the depth of scienti-fit experience that can transcend their education. The NRC *National Science Education Standards* put a particular emphasis on the importance of teachers acting as guides and models for scientific inquiry. They require teachers to have mastered the attitudes, skills, and habits of the mind *(NSES,* p. 28) required for performing authentic scientific inquiry. The *National Science Education Standards* address this issue directly in the section on professional development standards. Many of the standards match those developed for the students. They require teachers to experience an active investigation of some phenomenon in order to lay the necessary experiential, conceptual, and attitudinal foundation. Even elementary school teachers are expected to have at least one in-depth experience with active investigation. The *Standards* require courses for future teachers that are based on investigation, group work, and open-ended problems that are interesting and relevant. These experiences must allow teachers to reach the threshold required to be life-long learners. It is expected that supporting and sustaining teachers' skills should involve collaborations between colleges and universities, local schools, local industries and businesses, and the community. The *Standards* make it clear that these changes will not occur rapidly and that patience is essential.

Role of Universities and Colleges

From the viewpoint of a university professor, the tasks required of us are striking. People in chemistry, for example, view their primary mission as educating the chemists of the future, despite the fact that we are a service department with the vast majority of our students coming from other disciplines. It safe to say that our physics and mathematics colleagues have similar views. Our curriculum is largely based on the assumption that our true clients will be taking upper-class courses, participating in undergraduate research, and going on to graduate school. We teach our courses in a logical and sequential fashion. The first courses in subject areas are meant to provide breadth. We teach facts and simple problem-solving skills in lower-level courses with the expectation that we will teach students how to put the facts and simple problem-solving skills together for scientific inquiry at a later time, usually through an independent research project. The vast majority of our students never experience the feelings of true scientific inquiry. We almost certainly miss providing most science teachers of the future with opportunities to engage in the discovery process or to experience the excitement of true science. It is not surprising therefore that the students we see are missing the thinking and problem-solving skills and intuitions required to succeed in our courses. We have not effectively taught their teachers what science is and how it is practiced. Future teachers desperately need to experience both the inquiry and discovery processes if they are to understand science and model the attitudes and habits of the mind required to reach our children.

We have an important stake in the reform effort because it is becoming clear that the attitudes and skills of the American students we teach in the sciences have changed, perhaps profoundly, and our old teaching strategies are not adequate for reaching the broad range of students at the depth required to reach our goals. Students fail to learn course material even though they have the ability. Even our best students are not the equal of the best from years past (Wright, J. C. Personal observation and discussions with colleagues, 1997). There are many reasons for this problem, many of which continue to be under debate. Regardless of the reasons, we must do something about this problem. The number of these students has increased to the point where they can force teachers to lower the standards of accomplishment and to teach them in ways that are enjoyable (Sesame Street?) and that do not require great effort from students (Katz, 1996). Graduate students no longer have the same interest or ability in the physical sciences that characterized the field in earlier days (Wright, J. C. Personal observation and discussions with colleagues, 1997). Society as a whole does not attach the same importance to promoting scientific knowledge and has cut back sharply on funds for research. These changes have our attention. Many research faculty feel these changes threaten the viability of the scientific enterprise and our technological future.

Regardless of the definition of our problems, they are clearly characteristic of our society and require a coordinated effort from everyone if the changes are to result in solutions. The *Benchmarks* and the *Standards for Science Education* are powerful documents that together lay out the directions that must be taken by educators, students, administrators, parents, and the community. They are encyclopedic and visionary. The attainment of the standards would result in profound changes in the character of our educational system.

Conclusions

We cannot teach true aesthetic appreciation. It must be experienced through the beauty of art, music, or the world around us. The Benchmarks and the *NSES* make the same statement about science and mathematics. Students who engage in authentic inquiry and discovery will experience the aesthetic beauty of science and mathematics and the satisfaction of solving a difficult problem or creating something that came from their own minds. These rewards are far more meaningful than comments or grades from teachers, although they add power to the feelings. The universe provides the incentive and inspiration for learning. Self-discovery and creation build a respect and love for the universe that cannot be attained by passive activities. We can never destroy what we have come to love and respect. If we insist on teaching content and do not include the experiences that build the values of beauty and soul, the content will not be permanent. There will be little of importance that remains. Teachers must transcend their lesson plans and incorporate their own sense of the universe before they can create a learning atmosphere that is supportive and enriching. The ***Benchmarks*** and the *NSES* provide the goals for which we must strive. We must make them happen.

What is the best way to implement the standards? We must return to the basic questions that individuals involved in education must answer: Why should I change? How do I change? Can I change? Will the change bring success? These questions can only be answered by seeing exemplary models that individuals find credible and relevant to their own situations. Is the best implementation to mount a large-scale effort that unites school systems, cities, and even states to bring all of the parts together? It is if we understand the: fundamentals that are required for successful attainment of the standards in a controlled environment. However, if there are important elements of systemic reform that are not yet adequately developed or accepted, it would seem that a wiser dissemination strategy would be to both create many smaller-scale projects that are more focused and develop a more effective communication strategy that connects successful projects with preservice and inservice teachers.

The standards need to be transformed into workable teaching plans. This need is especially keen if more authentic, inquiry-based projects are to anchor the science, mathematics, and technology experiences. It is foolish to create a program to reach the moon if scientists have not done the basic thermodynamics on the rocket fuels or if the engineers who will be working on the rockets do not have or have not accepted the validity of the scientific information that is known. There is a right time to implement the big project, and it is the role of policymakers to know when that time has arrived. Teachers and administrators at the grassroots level need data, teaching toolkits, and menus of approaches that have good assessment information so they can understand the profound nature of systemic reform. They have clear examples of how these changes are implemented and how they work. They see credible data about how students are able to master course content and how student attitudes change.

Lynn Stoddard (1990) argued that there are already model systems that are effective in helping all students develop into human beings who are valuable contributors to society. The design framework develops three aspects of a student:

- their identity through a strong sense of self-worth;
- their social interactions that are characterized by concern and respect for others;
- their thirst and skills for learning, creating knowledge, and solving problems.

This framework uses the same methods that form the basis for the *Benchmarks* and the *NSES*, that is, putting the responsibility for learning on the students. This example is one of many successful implementations of active learning strategies. Not only is personal development advanced, content mastery is also optimized, despite the fact that it was indirectly addressed. If such approaches are known to be successful, we already have models. Why have educators not accepted them more readily?

Stoddard continued by answering these questions. Implementation is blocked by educators who believe that programs that are directed toward personal development of students cannot succeed in delivering content in a curriculum as effectively as programs that are focused on the curriculum itself. If a problem exists, we merely need to change the curriculum in clever ways. This approach is obvious, especially to people who require control over those things that are important to them. If we have control, we can turn out a quality product, a standardized student. What such people do not realize is that, by acquiring control over the product, we lose the soul of the student. Stoddard argued that this attitude stifles independent thinking and teaches the lesson that schooling is irrelevant to life. She argued that the pervasive atmosphere of control causes alienation, withdrawal, drug use, sex, crime, and suicide.

The difference between methods that are based on control and those based on ownership is often subtle but deep. The difference appears most clearly in lhow the minds of the students are affected. Our central problem in creating the profound changes that systemic reform must accomplish is changing the attitudes of all the people involved in the educational process. It is exactly the same problem that we have in creating a more effective learning environment for our students. Reification and changes in attitude must occur. The standards clearly spell out how the reform of student learning takes place. The same process must occur for everyone else including the policymakers in charge of systemic reform. Do they understand systemic reform deeply enough that it appears in the policies and programs that are established? Should we have large-scale programs to implement change, or should we have many smaller and more focused programs that link directly with the teachers who have seen the new paradigm and have the passion for change? Do we sow our seeds in large clumps expecting there should be a beautiful bouquet at some spots, or do we scatter them more widely and allow the flowers to spread out to fill the empty spaces? People involved in the educational system will be able to appreciate and understand how systemic reform ideas work only if they experience them personally. That is the only way that reification and attitudinal change occurs. That is the only way that the questions and skepticism will shift from a focus on the old paradigm based on control to the new paradigm based on ownership. Dissemination of systemic reform needs a small grain size, and Senta Raizen's bullets need to do damage. What are the best ways to disseminate systemic reform? What are the best levers for moving along reform? If we believe the standards, the answer is active learning. If we choose strategies and levers that provide the educational community with personal experiences of active learning strategies, we will reach their very souls. And we must do no less.

References

American Association for the Advancement of Science. (1993). *Benchmarks for science literacy.* New York: Oxford University Press.

Barr, R. B., & Tagg, J., (1995). From teaching to learning-a new paradigm for undergraduate education. *Change, 27*(6), 12-25.

Finkel, D. L., & Monk, G. S. (1983). Teachers and learning groups: Dissolution of the atlas complex. In K. E. Eble & J. F. Noonan (Series Eds.) & C. Bouton and R. Y. Garth (Vol. Eds.), *Learning in groups- New directions for teaching and learning,* Jossey-Bass, *14,* 83-97.

Heller, P., Keith, R., & Anderson, S. (1992). Teaching problem solving through cooperative grouping. Part 1: Group vs. individual problem solving. *American Journal of Physics, 60,* 627-636.

Heller, P., & Hollabaugh, M. (1992). Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups. *American Journal of Physics, 60,* 637-644.

Katz, M. J. (1996). Teaching organic chemistry via student-directed learning. *Journal of Chemical Education, 73, 440-445.*

MacGregor, J. (1990). Collaborative learning: Shared inquiry as a process of reform. In K. E. Eble & J. F. Noonan (Series Eds.), *The changing face of college teaching- New directions for teaching and learning.* Jossey-Bass, 42, 19-30.

McLeod, D. B., Stake, R. E., Schappelle, B. P., Mellissinas, M., & Gierl, M. J. (1996). Setting the standards: NCTM's role in the reform of mathematics education. In S. A. Raizen & E. D. Britton (Eds.), *Bold ventures: Case studies of U.S. innovations in mathematics education* (Vol. 3, pp. 13-132). Boston: Kluwer.

National Research Council. (1996). *National science education standards.* Washington, DC: National Academy Press.

Newmann, F. M. (1991). Linking restructuring to authentic student achievement. *Phi Delta Kappan 72*, 458-463.

Raizen, S. A. (1997) *Standards for science education* (Occasional Paper No. 1). Madison: University of Wisconsin-Madison, National Institute for Science Education.

Roitman, J. (1997). *A mathematician looks at national standards.* Pp. 25-43 in this paper.

Schoenfeld, A. H. (1987). What's all the fuss about metacognition? In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 189-2 15). Hillsdale, NJ: Erlbaum.

Stoddard, L. (1990, Spring). The three dimensions of human greatness, a framework for redesigning education. *Holistic Education Review, 3*, 4- 10.

Tobias, S. (1990a). *Revitalizing undergraduate science.* Tucson, AZ: Research Corp.

Tobias, S. (1990b). *They're not dumb, they're different.* Tucson, AZ: Research Corp.

Toch, T., et al. (1996, April 1). The case for tough standards. *U. S. News and World Report,* p. 55.

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