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## ABSTRACT

This issue of Changing Schools explores what algebraic thinking is and why students need algebraic thinking. The efforts of Kansas and North Dakota to incorporate algebraic thinking into their state mathematics standards are described. Contents include: (1) "Algebraic Thinking: Preparing Students for Mastering Algebraic Concepts" (Vicki Urquhart); (2) "States Embrace Algebraic Thinking" (Vicki Urquhart); (3) "Colorado Conference Examines Needs of Learners with Limited English" (Shae Isaacs); and (4) "Diversity Training Can Decrease Violence" (Diane McIntyre Wilber). (KHR)

# Changing Schools

MAY, 2000 • MID-CONTINENT RESEARCH FOR EDUCATION AND LEARNING **McREL**

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## **Algebraic thinking:**

### **Preparing students for mastering algebraic concepts**

by Vicki Urquhart

All teachers struggle with how best to engage their students in learning course content and at the same time instill a genuine love for the subject matter and enthusiasm for putting it to use. Most educators today know that understanding mathematics is an indispensable life skill. By teaching algebraic thinking throughout the K–12 years, educators provide students with critical thinking skills they can use in other content areas.

### **What is algebraic thinking?**

According to McREL Principal Consultant Clare Heidema, mathematics teachers are beginning to look at algebra in a new way — not as a separate subject to be taught in the traditional ninth-grade Algebra 1 class, but as a way of thinking that can be integrated into teaching as early as kindergarten.

Shelley Kriegler of the Department of Mathematics at UCLA has identified two major components of algebraic thinking. The first is the development of mathematical thinking tools, such as forming analytical habits of mind during early childhood, particularly in the primary grades. The other component involves grasping the diverse nature of algebraic ideas. Kriegler examined algebraic ideas through three different lenses: “algebra as abstract arithmetic, algebra as a language, and algebra as a tool for the study of functions and mathematical modeling.”



*Katie Hoiby, a fifth-grader at Lincoln Elementary School in Minot, N.D., uses the “Hands-on Equations” program to solve an algebra problem.*

Algebraic thinking skills build during the elementary and middle school years in at least five areas and provide verbal, symbolic, and graphic formats to describe and solve problems later encountered in high school. Skills that can be developed early are:

- using pictures, graphs, tables, and equations to represent relationships;
- understanding numbering systems, especially positive and negative numbers;
- working with properties of operations (cumulative, associative, distributive) and order relations;
- using variables and open sentences to represent quantities and express relationships; and
- recognizing patterns and functions, including how a change in one quantity affects another related quantity.

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## Why do we need algebraic thinking?

Algebra has long been considered a “gatekeeper” for advanced math and science study in high school. We now recognize that students who pass algebra are more likely to be successful in school and continue their education beyond high school.

In nearly every state, K–12 standards reflect this awareness. A Colorado state standard exemplifies this trend, stating: “Students use algebraic methods to explore, model, and describe patterns and functions involving numbers, shapes, data, and graphs in problem-solving situations and communicate the reasoning used in solving these problems.” The National Council of Teachers of Mathematics (NCTM) Standards 2000 document emphasizes algebraic thinking as an essential strand in the elementary school curriculum.

Certain professions rely heavily on the understanding of algebraic concepts. Tom Davis, principal scientist for Silicon Graphics, Inc., detailed the need for such skills in the business world when he wrote to the president of the University of Rochester after that institution decided to cut back its math program. “It is becoming more and more difficult for us to hire students with sufficient background in mathematics,” Davis wrote. “To do computer graphics, calculus is not enough. We require a strong background in linear algebra . . .”

### “Algebra for all”

Nationwide, mathematics educators recommend a continuous K–12 experience in algebraic ideas, an approach known as “algebra for all.” For this reason, and to meet standards dealing with algebraic thinking, schools now prepare students as early as kindergarten by asking questions like these:

- How can pre-K–2 students use informal methods to solve real-world problems involving finding an unknown quantity, guessing a rule, or describing a pattern?
- What methods can students use in grades 3–5 to demonstrate understanding of simple problem situations using diagrams, models, and symbolic expressions translated from verbal phrases?
- Which approaches best allow sixth- to eighth-grade students to use algebraic problem-solving strategies to solve real-world problems involving linear equations and inequalities?

Since many state standards stress knowledge and understanding of formal algebra by the eighth or ninth grades, grasping basic concepts in elementary and middle school is vital.

This emerging view of algebra acknowledges the dynamic nature of mathematics in general and of algebra in particular. Because it treats mathematics as a human activity, students’ thinking is moved to the forefront. Emphasis in the classroom changes from whether an activity itself is algebraic to whether the underlying thinking and reasoning of students is algebraic. Through this approach, students begin to participate in discussions that involve algebraic thinking.

Teachers and parents can take advantage of problems that arise in everyday situations to spark algebraic discussions. If, during a trip to the bakery, three chocolate chip cookies and four peanut butter cookies cost 68 cents while four chocolate chip cookies and three peanut butter cookies cost 72 cents, a parent or teacher might ask, “Which is the more expensive cookie?” and “How much does each kind of cookie cost individually?” These types of exchanges develop a richer understanding of algebraic relations even before formal study begins.

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Changing Schools, May 2000

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## States embrace algebraic thinking

by Vicki Urquhart

All states in the McREL region incorporate algebraic thinking into their state mathematics standards, beginning as early as kindergarten. This issue of *Changing Schools* will focus on the efforts of Kansas and North Dakota.

### Kansas

In its state mathematics standards, Kansas identifies four broad areas, then separates them into specific grade-level benchmarks to be reached prior to high school algebra, at grades two, five, and eight. The state's summary of its algebra standard reads, "The student uses algebraic concepts and procedures in a variety of situations." According to Kim Gattis, education program consultant with the Kansas State Department of Education, every school in the district, as part of the accreditation process, must have a school improvement plan targeting math (including algebra) and reading.

Tammy McKean, elementary mathematics teaching specialist for the Wichita School District, said the district's goal for its students is, "to master content, to develop understanding, and to build confidence as mathematicians." McKean has been instrumental in designing and implementing a summer intervention program for students who fail to meet local benchmarks, which are aligned with state benchmarks. Students participate in an intense, three-week summer program focusing on numeration, patterns, and computation. Teachers also participate, in an extensive in-service program featuring best practices. The district provides teachers with sets of manipulatives and follow-up training.

*For more information on Wichita's approach, contact Tammy McKean at (316) 973-4404 or e-mail her at [tkmckean@feist.com](mailto:tkmckean@feist.com).*

### North Dakota

North Dakota standards enumerate six goals for K–12 mathematics programs in the state. These goals require students to

- be mathematical problem solvers,
- communicate mathematically,
- make mathematical connections,
- reason mathematically,
- be confident in their mathematic abilities, and
- use appropriate technology.

To achieve these goals, benchmarks related to algebraic reasoning are established for students from kindergarten through twelfth grade, calling for them to use algebraic concepts, functions, patterns, and relationships to solve problems.

Teachers Virginia Saathoff of Minot Public Schools and Mavis Kelly of Grand Forks Public Schools each use the "Hands-on Equations" program (Web site at [www.Borenson.com](http://www.Borenson.com)) in their respective fifth- and eighth-grade classes. Presidential Award winner Saathoff particularly values the appeal of this program — which includes 25 lessons, worksheets, answer keys, and game pieces — to visual and tactile learners. She believes programs for lower elementary grades establish confidence in using algebra that can be built upon later. Likewise, Kelly uses the program for its hands-on appeal and its four-step, problem-solving approach. Her students often solve algebraic puzzles using logic, a strategy they practice regularly. ☺

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## Colorado conference examines needs of learners with limited English

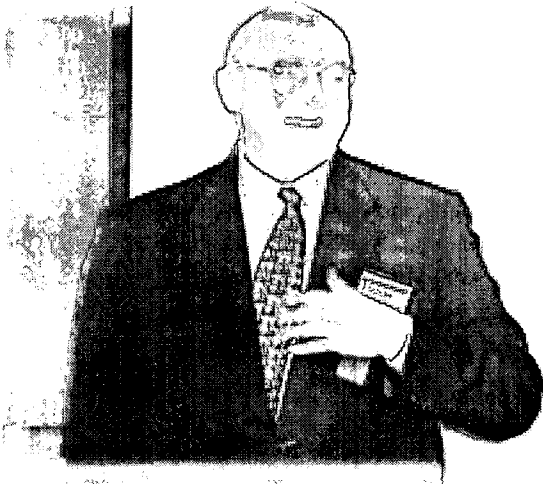
by Shae Isaacs

On April 10–11, more than 200 Colorado educators gathered for a conference dedicated to improving academic achievement among K–12 students with limited English proficiency.

*Education Summit: Improving Academic Achievement for English Language Learners* focused on assessment, accountability, civil rights, effective teaching strategies, and technology. Sponsors included McREL and the Region IX Comprehensive Center, in conjunction with the Colorado Department of Education and the Pacific Southwest Regional Technology in Education Consortium (PSR\*TEC).

Students for whom English is a new language are the most rapidly growing segment of the student population in the country, up 104 percent over the last decade, compared to a 14-percent increase in the overall student population during the same time period.

“We have to get away from the idea that students with limited English proficiency belong with the ESL or bilingual teacher,”



*David Ramirez of Pacific Southwest Regional Technology in Education Consortium delivers the keynote address at a conference on improving academic achievement for English language learners, telling participants, “School reform will fail unless it addresses issues of linguistic, cultural, and racial diversity.”*

explained McREL Senior Consultant Nilda Garcia Simms, one of the conference organizers. “In reality, these students spend most of their time in mainstream classrooms and it’s mainstream teachers, as well as principals and tech people, who need this information and these strategies.”

“I’m here to find out [about] the accountability issues with these state tests,” said participant Michele Wilson, an ESL teacher from Montrose County, where English language learners comprise about one-third of the student population. Wilson was referring to CSAP, the Colorado Student Assessment Program, on which second language learners have not performed well. Several conference sessions focused on CSAP’s implications for teacher practice and students with limited English. (See sidebar.)

“CSAP was designed to give districts feedback on their instructional programs,” explained Simms. “It wasn’t meant to be punitive; rather, it was intended to help answer the question, ‘How well are our students meeting Colorado standards?’” But Simms acknowledged that the test has resulted in confusion and concern regarding how and when to include second language learners in the assessment process. “CSAP is here — at least for now — and I strongly encourage teachers and principals to learn more about CSAP’s structure and to reflect on their teaching practices,” she said. “It’s important to think about not only the content students will need to know, but also about how students will be asked to demonstrate their knowledge.”

Other conference presenters shared creative uses of technology that support English language development and academic learning for English language learners. Kevin Rocap of PSR\*TEC shared information on the Project SMART distance learning network for the children of migrant workers, which allows

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## AROUND THE REGION

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them to continue taking the same classes with the same teachers as they move from area to area. Other applications of technology included global learning networks, mobile computer labs, and using the Internet to strengthen instruction in the bilingual classroom. ☺

*For more information on bilingual education, call Nilda Garcia Simms at (303) 337-0990 or e-mail her at [nsimms@mcrel.org](mailto:nsimms@mcrel.org). You may visit the PSR\*TEC Web site at [psrtec.clmer.csulb.edu](http://psrtec.clmer.csulb.edu). Visit [www.jump.net/~newman/tea/toc97.html](http://www.jump.net/~newman/tea/toc97.html) for more information on the Project SMART learning network.*



*University of Colorado Associate Professor Kathy Escamilla tells conference participants that Colorado did not report Spanish-language CSAP results to either the media or the legislature, even though in some districts Spanish-language results were better than English results, as well as being better than the state average.*

## Tips for helping English-language learners succeed on statewide tests

With more and more states introducing statewide assessments, the pressure is building for educators to assure the success of all students. But teachers and administrators know all too well that successful performance for students with limited English proficiency presents significant challenges.

*At Education Summit: Improving Academic Achievement for English Language Learners*, McREL's Nilda Garcia Simms offered strategies to help English-language learners succeed on CSAP, Colorado's statewide assessment. Most of these strategies can be used or adapted by teachers throughout the region McREL serves under its Regional Educational Laboratory contract:

- Familiarize yourself with the test's format and the kind of items on the test; then reflect on the type of experiences you are providing for your students. Students will respond better to something they have seen before.
- Tell students what will be expected of them.
- Teach students the academic vocabulary they will find on the test and familiarize them with the structure of the test (e.g., teach students how to follow test directions).

- As early as kindergarten, begin introducing students to the idea of rubrics by identifying specific criteria that will be used to measure their work. You can develop a rubric for anything — a language arts lesson, music, science.
- Practice essay writing on a regular basis so it becomes routine and isn't a surprise on the test. (Even if you are doing native language instruction, write! It's an important way for students to learn to organize their thoughts.)
- Introduce students to writing prompts. Have them figure out what a prompt is asking them to do, then do it. Introduce students to various writing genres (e.g., informative, narrative, persuasive) and have them practice writing in that genre.
- Find out what test accommodations (e.g., additional time) are allowable for English-language learners and students with special needs.
- Elicit support from parents. Send home sample test questions. Help parents understand that, even if they don't speak English, they can help their children learn. Critical thinking skills, deductive reasoning, and organizing one's thoughts can be practiced in any language.

## McREL conference to explore the power of accountability

McREL's fall conference, *The Power of Accountability to Transform Teaching and Learning*, planned for October 18–20, will connect practitioners with the latest thinking and research about the complex issue of school accountability. "We've developed a program of presenters with nationally recognized expertise in assessment and accountability to address questions that are on the minds of every educator in the country right now," said Tim Waters, McREL's executive director.

The conference will be organized around four critical questions:

- In what ways is the larger community both responsible and accountable for the education of children?
- What is the responsibility of educators for teaching and learning, and to whom should they be accountable?
- What kinds of information should be gathered to determine if teaching and learning are taking place?
- How can assessment results be used to improve teaching and learning?

McREL staff members and other presenters will share the most up-to-date, research-based thinking about these questions. Sessions will offer practical strategies educators can use in the classroom to instruct and assess learning in ways that maximize student achievement. ☺

# The Power of Accountability to Transform Teaching and Learning

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and Barbara Sizemore, founder of  
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To receive registration information, contact the McREL Conference Office at 303-694-4728, ext. 33, fax 303-694-4869 or e-mail [sblackstock@assnoffice.com](mailto:sblackstock@assnoffice.com). For updates, visit McREL's Web site at: [www.mcrel.org/conference/](http://www.mcrel.org/conference/)

## Diversity training can decrease violence

by Diane McIntyre Wilber

The more we see violence erupting in our schools, the greater our need for diversity training, said Teresa Delorme, a REACH certified diversity trainer, elementary principal, and McREL board member.



*Teresa Delorme*

“There’s a narrow vision right now in America stemming from many biases,” Delorme contended. “One reason students commit violent acts against others is because of their inability to deal with differences.

Our children must have teachers who understand their needs and who can help them understand and appreciate each other.”

A member of the Chippewa tribe, Delorme served as principal at three Native American schools before she joined Bismarck Public Schools in North Dakota in 1992. She has been principal at Riverside Elementary in Bismarck for three years, serving a student population that is highly transient and one of the most diverse in the district.

### Training creates new awareness

Until recently, Delorme said, she saw herself as open-minded and accepting of others. Then in early 1997, she enrolled in diversity training with the REACH (Respecting Ethnic and Cultural Heritage) Center in Seattle, Wash. “I had so many ‘ah-has’ during the training that I realized I and many other people had really missed the boat,” she explained. Delorme has since worked on these perceived shortcomings and is now a trainer for REACH.

In addition to her other duties at Riverside, she coordinates staff diversity training — based on the REACH model — for a consortium that includes the Bismarck Public Schools, nearby Mandan Public Schools, the United Tribes Technical College, and its affiliated elementary school. The consortium is funding the training with a four-year grant (in its final year) from Goals 2000, through the state Department of Public Instruction and matching district funds. Roughly 20 percent of Bismarck’s staff members have completed the training so far, and the consortium is hoping for another year of state funding.

### Infuse diversity training into curricula

“To make significant gains, teachers must move beyond teaching diversity as a separate subject once a year,” Delorme advised. The REACH training model addresses differences in culture and ethnicity and prejudices based on such aspects as appearance, size, speech, gender, and sexual orientation.

Teachers learn how to infuse diversity training into curricula on a daily basis. According to Delorme, the principles addressed in the training can easily be incorporated into core subjects and any lesson plan.

“Addressing diversity [issues] is no longer a threat to the teachers and they realize they can even have fun doing it,” she said. “If our students can develop an appreciation of and sensitivity to others, it’s so much more rewarding and helps prepare them to be successful in our society today.” ☺

*For more information about REACH, contact Director Colleen Almojuela at the Seattle REACH office, (206) 545-4977.*

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## Support for teachers

Heidema, a mathematics specialist, recommends that elementary teachers hone their own algebraic thinking skills through professional development sessions and other available resources. "Teachers need to learn about algebraic thinking through their own experiences with it, in order to make it a realistic part of their interactions with elementary school students," she explained. ☺

See sidebar for a brief list of resources. For a more complete list and a sample professional development course outline on algebraic thinking, contact McREL's Resource Center at (303) 337-0990.

## Resources:

*Teaching Children Mathematics*. (February, 1997). Focus Issue on Algebraic Thinking. National Council of Teachers of Mathematics.

*Algebra for Everyone*. (1990). Edwards, E. L., Jr. (Ed.). National Council of Teachers of Mathematics.

*The Nature and Role of Algebra in the K-14 Curriculum*. (1998). Proceedings of national symposium. National Research Council. National Academy Press.

Resource materials and professional development workshops related to algebraic thinking are available from McREL (online at [www.mcrel.org](http://www.mcrel.org)) and other organizations, such as the Consortium for Mathematics and its Applications (online at [www.comap.com](http://www.comap.com)) and National Council of Supervisors of Mathematics (online at [forum.swarthmore.edu/ncsm/index.html](http://forum.swarthmore.edu/ncsm/index.html)).

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