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ABSTRACT

When students confront arithmetic or algebraic word problems, they develop ideas and notations during the processes of solving them by using various arithmetic strategies. Those ideas and notations are the basis for solving that type of problems. Is it possible to aid the development of students' algebraic thinking during their transition from arithmetic to algebra? We think that a way of doing so is by presenting them with different types of problems-as proposed by Bednarz and Janvier (1996)--and encouraging their reasoning and development of strategies linked to their arithmetic thinking. Hence, the identification of strategies used by fifth graders when solving algebraic word problems is of outstanding importance for finding effective ways to aid students during their transition from the arithmetic to the algebraic thinking. Results obtained in this research study show that with these types of mathematics word problems, students generate strategies that at certain moments become useful for their transition to the algebraic thinking. (Author)

IDENTIFICATION OF STRATEGIES USED BY FIFTH GRADERS TO SOLVE MATHEMATICS WORD PROBLEMS

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When students confront arithmetic or algebraic word problems, they develop ideas and notations during the processes of solving them by using various arithmetic strategies. Those ideas and notations are the basis for solving that type of problems. Is it possible to aid the development of students' algebraic thinking during their transition from arithmetic to algebra? We think that a way of doing so is by presenting them with different types of problems—as proposed by Bednarz and Janvier (1996)—and encouraging their reasoning and development of strategies linked to their arithmetic thinking. Hence, the identification of strategies used by fifth graders when solving algebraic word problems is of outstanding importance for finding effective ways to aid students during their transition from the arithmetic to the algebraic thinking. Results obtained in this research study show that with these types of mathematics word problems, students generate strategies that at certain moment become useful for their transition to the algebraic thinking.

Background

In the plan and programs for elementary school education (grades 1-6) in México, a methodological approach based on problem solving is proposed for the teaching of mathematics. Recommendations in these official documents emphasize dialogue, interaction, and confrontation of viewpoints as important activities for promoting learning and the construction of knowledge. It is argued that by implementing these activities students will view mathematics as a functional and flexible tool for solving problems posed to them (SEP, 1993, pp. 49-70).

However, elementary school teachers seldom put these recommendations into practice and, in the school environment, students perceive mathematics as complex and governed by rules under which their learning depends mainly on memorization. Students in the lower secondary level of education (grades 7-9) have a similar perspective about mathematics. When students in this level (ages 12 to14) solve algebraic problems, for instance, they usually have recourse to rules and procedures with no reflection on these; they believe that this type of activities represents the essence of algebra. Kieran (1988) reported that students in the lower secondary level of education are often unable to apply their knowledge of basic algebra when solving problems.

Solving algebraic word problems enhances students' attitudes for searching and producing conjectures that in a middle term will allow them to acquire algebraic notions such as equation, unknown, generalized number, variable, and function.

Therefore, by means of solving this type of problems, it is possible to help students make the transition from arithmetic to algebra.

The passage of students from the elementary to the lower secondary level of education underlies the development of new mathematical abilities. Several authors have reported that students encounter great difficulties during their transition from arithmetic to algebra—Filloy and Rojano (1989), Herscovics and Linchevski (1991), Kieran (1981, 1988), Kieran and Chaloug (1993), Booth (1984), and Bednarz and Janvier (1996), among others.

In the trend of research that approaches the area of transition from arithmetic to algebra, Filloy and Rojano (1989) introduced the concept of *didactic cut* to characterize the transition in the specific realm of solving equations. Herscovics and Linchevski (1991), in a similar trend of research, described that a *cognitive shift* occurs when students operate on the unknown. Bednarz and Janvier (1996) made a different contribution to this area of educational research: they investigated the conditions that allow students to construct algebraic arguments in a context of solving mathematics word problems.

In a previous research paper, Bednarz and Janvier (1994) presented an analysis of arguments produced by students when solving either arithmetic or algebraic problems. These researchers discovered that before introducing students to algebra, they put different stable profiles of arithmetical reasoning into play when confronted with different types of problems traditionally presented in algebra.

One objective in the investigation by Bednarz and Janvier (1996) was to clarify the conditions under which, in a context of solving word problems, students' algebraic thinking emerges and develops. They also studied two basic aspects for the characterization of such problems: nature of the problems—their structure—and the relative difficulty for students to solve them. Moreover, they noticed the importance of identifying the general structure of a problem according to the involved (known and unknown) quantities, the relationship between them (connection between quantities), and the type of relations implied (additive or multiplicative comparison).

Thus, according to the structure of problems, Bednarz and Janvier (1996, p. 123) classified them as arithmetic or *connected*, and algebraic or *disconnected*. With connected problems, it is possible to build bridges between known information and students can work from the known to the unknown. Disconnected problems allow no direct relation between known and unknown information to be established.

Hence, these authors could show the complexity of algebraic word problems usually posed to students during their regular courses of lower secondary education. They also emphasized that the knowledge of strategies applicable in the solution of arithmetic problems is the basis for the transition from arithmetic to algebra. In the selection of the word problems for our research, the characterization proposed by Bednarz and Janvier (1996) was of fundamental importance.

The Research Study

The purpose of our investigation was to identify arithmetic strategies that fifth graders have recourse to and the way they use them for solving mathematics word problems—above all, those problems related to algebra. The research activities for this study initiated with 35 fifth graders (ages 11 and12) of a public school in an urban area. Students in this group had an acceptable performance in mathematics, although they had little experience in problem solving.

Since working with 35 students would make it difficult to observe their interactions and their processes for solving problems, only 15 students were selected for the experimental phase: five high-achieving students, five average achievers, and five low-achieving students. We based our selection on the analyses of results obtained from a diagnostic questionnaire (DQ) applied to all students in this group. Then the selected students formed working teams, each with one student of each level of achievement.

The difficulties students had when trying to solve the selected problems we presented them were evident. Thus, we reconsidered the question posed by Bednarz and Janvier (1996): *Is it possible to help students make the transition from arithmetic to algebra?* We think that it is possible to aid the development of students' algebraic thinking during their transition from arithmetic to algebra. A way of doing so is by presenting students with problems of different nature—as proposed by Bednarz and Janvier (1996)—and encouraging their reasoning and development of strategies linked to their arithmetic thinking.

Methodology

During the first phase of the investigation, we looked for and designed problems to elaborate the DQ. As a part of the search, we revised mathematics textbooks of the last cycle of elementary education (grades 5-6) and of the first grade of lower secondary education (grade 7). The questionnaire students had to answer contained arithmetic word problems mainly. In the analyses of the responses by students, we noticed some arithmetic difficulties they had and we could identify some of their problem solving abilities as well—which were deficient.

Based on the identification of students' previous arithmetic knowledge, we designed 22 working sheets with an algebraic problem each (for examples of these problems, see Appendix) and planned the manner in which we should pose algebraic problems. The purposes for the elaboration of those working sheets were to:

- present students with problems traditionally studied in school algebra;
- observe the advantages of working in teams;
- identify arithmetic strategies students use; and
- enhance students' development of abilities to solve algebraic problems.

In the second phase of the investigation, during three weeks we had 15 working sessions of about 50 minutes each. All sessions were video-recorded and, additionally, the researcher took notes about features of each.

Students approached every proposed problem by working in a collaborative environment; that is, they worked in teams during this experimentation phase. Due to this way of working, it was easier to identify the strategies students used when solving word problems.

At the beginning of each working session, the researcher would ask students to read the statement of the problem carefully so that they could understand what was required. We discussed with them what it means (a) to understand the problem, (b) the selection and development of a strategy, and (c) the verification of a solution. Moreover, we suggested them to use a pen to do all their writing during the process of solving a problem.

Students would re-read the problem and decide the way to approach it by working in teams. The three students in each team discussed and tried strategies, checked their results, and registered their trials in the working sheets. An example of those discussions is the solution process in one of the teams for the following problem.

> The perimeter of a rectangular plot of land is 102 meters. The depth of the plot is double its width. What is its depth? What is its width? (Adapted from: Alarcón, Bonilla, Nava, Rojano, & Quintero, 1994, p. 162)

Researcher [R]: How did you solve the problem?

Student 1 [S1]: First, we divided 102 by 2 [student writes the division] and we got 51.

R: What did you do with that number?

Student 3 [S3]: Then, we divided 51 by 2 [student writes the division] and we got 25.5. This was not right because when multiplying 25.5 by 51 [student writes the multiplication] we got 1300.5 ... So it was wrong.

S3: Next, we decided to divide 102 by 3 [student writes the division] and we got 34.

R: Was this result useful?

Student 2 [S2]: We wrote 34 plus 34 [student writes the addition] and we got 68, but we did not have the width yet.

S2: We thought of 34 divided by 2 and we got 17.

R: How do you know that the solution you found is correct?

S1: Because we multiplied 17 by 2 [student writes the multiplication] and we got 34.

S1: Then, we added 68 plus 34, and we finally got 102.

When necessary, the researcher participated by asking questions and giving suggestions to students with the purpose of encouraging them in their work processes of solving the problems. Moreover, the researcher aided students to overcome difficulties that appeared at any point during the solution processes. This help provided to students consisted of asking questions to promote their reasoning and lead them to analyze the problem from a different perspective to the one they had adopted. The following are some of the questions asked.

Regarding the way of posing the problem:

Have you understood the problem?

Can you explain it to your teammates?

Can you explain what the problem is about with your own words?

What do you want to find?

Do you have any difficulty in understanding any part of the problem?

What part of the problem you do not understand?

What do you know?

What you do not know?

What are the data of the problem?

Did you identify the relevant information?

Does the problem have any information that is not relevant?

Can you make a sketch or draw a picture to interpret or illustrate the problem?

Regarding the solution process:

Do you have any idea on how to solve the problem?

Have you ever solve any similar problem?

How are you going to solve it?

What strategies could be useful to solve the problem?

What are the conditions of the problem?

What are you doing now?

Are you getting somewhere with that?

How does that relate with the solution?

Could a table or a graphic be useful?

Can you think of another way or method to solve the problem?

After solving the problem:

How do you know that the solution you found is correct?

What do you do with that result?

Does your answer make sense with respect to the conditions of the problem?

What strategies did you use?

Can you check or verify your solution?

Could you have solved the problem in a different way?

How would you do that?

Can you express with words what you have checked?

What do you think about the solution methods used by your classmates?

Thus, the role played by the researcher during this phase was that of facilitator and guide. After students had finished solving a problem, we did a group revision of the solutions found. This comparison was helpful for analyzing the solution processes and strategies used. Our intention was to leave no doubts as to the solution and verification of results. We must emphasize that this group revision allowed students to identify the strategies that emerged. Then, for solving similar problems they could have recourse to those strategies they thought were more useful.

During the working sessions, a collaborative and trusting environment was created. Students could freely exchange their ideas and they rotated the responsibilities of coordinating and presenting the teamwork. At the beginning of the experimental phase, high-achieving students took the initiative for coordinating the teamwork. As the working sessions progressed, average- and low-achieving students had a relevant participation both in the teamwork and in the group revisions of the work done.

During the third phase of the investigation, students had to answer a final questionnaire (FQ) individually. (We had previously designed the FQ with problems similar to those in the DQ.) Problems in the FQ aimed at: (i) exploring the individual progress of students during the experimental phase, and (ii) identifying strategies used by students for solving algebraic problems.

Discussion of Results

By working in teams, students could discuss different ways of interpreting the same problem and come to an agreement as to the most convenient way of approaching it. Besides, due to the group revisions and the use of the different solution strategies that emerged, they constructed new knowledge. At the end of the experimental phase, we observed that students could efficiently use the strategies analyzed during the group revisions. At the beginning, non-systematized trial and error was the strategy they used the most.

Moreover, the lack of reflection by students on the initial conditions of the problems when preferring the use of the basic arithmetic operations (addition, subtraction, multiplication, and division) was remarkable. Additionally, students systematically valued the results of their own activities as well as the solution criteria of their teammates. Confronting their solution processes was also an achievement.

The strategies systematically used by students during the experimental phase were the following:

S1. Propose a number and check it to find a solution.

S2. Divide one of the quantities into parts "to be distributed," followed by the search of numbers ending in 0 or 5, and, then, approximate to the desired quantity adding one by one.

S3. Base their work on the design of a drawing to find the solution.

S4. Construct a table for comparing the data and approximate the solution.

S5. Draw a number line to compare paths covered by a series of *jumps*.

S6. Mechanical use of basic arithmetic operations (addition, subtraction, multiplication, and division), that is, without any reflection on the initial conditions of the problem.

S7. Use of the rule of 3.

S8. Preference for the use of mental arithmetic without having to write the operations used (numeric answer).

At the beginning of the experimentation, the strategy most frequently used by students was *S1*. However, they soon realized the disadvantages of having to do several operations to find the solution. Hence, thereafter they preferred to change to the use of strategy *S2*. Few students used *S1* for answering the FQ (8.6%).

When using strategy *S2*, students first approximated the desired quantity by dividing; then, they approximated it with quantities ending in 0 or 5, and, finally, they tried with adding one by one to a number until they found the solution. Students preferred this strategy both in the experimental phase and in the FQ (49.3% used it in the FQ).

The solution of the following problem illustrates what has been just described above (see Figure 1).

> Seventy-eight (78) candies are distributed among Edgar, Juan, and Saúl. Juan receives 3 times as many candies as Edgar, and Saúl receives two candies less than Edgar. How many candies does each boy receive? (Adapted from: Alarcón et al., 1994, p. 166)

Some students found strategy *S3* very useful during the experimental phase, especially for grasping the problem posed. Nevertheless, students seldom had recourse to strategy *S3* in the FQ (0.6%). Only 4.6% of students had recourse to a table (*S4*) to solve some problems. To compare traveled distances in problems of "reaching," 8.6% of students based their work on a number line, strategy *S5*, a percentage similar to the one for *S1* and *S8*.

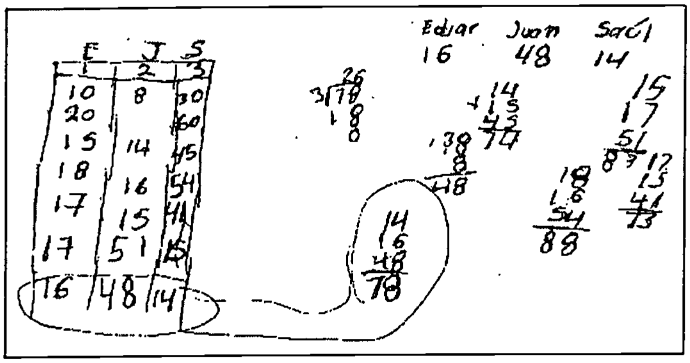

Figure 1

Strategy *S6* was used 15.3%. Generally, students did arithmetic operations with the data of the problem without any reflection on the conditions given at the beginning of it. It is evident that, in a school environment, students usually proceed so. School instruction promotes strategy *S7* to solve certain kind of problems. When the experimentation started, *S7* was preferred; however, students participating in this research study soon realized that *S7* was not useful to solve the proposed problems and its utilization was reduced (2.0% in the FQ).

The strategy of mental arithmetic, *S8*, was also important for students: once they had understood the conditions of the problem, they did operations without having to write them down. Although it is common for students to use this strategy in their daily work, generally teachers do not accept it and demand from students to write down the operations they carry out mentally. From the analyses of results of the FQ, besides observing that students had recourse to the arithmetic strategies that emerged during the experimental phase, we can claim that they did develop abilities to solve problems of different nature.

Conclusions

It is complicated for students to transit from arithmetic to algebra. The difficulties they have when solving arithmetical and algebraic word problems is a clear evidence of this. This evidence is more apparent when we realize that the strategies and reasoning of students for solving algebraic problems are fundamental for the construction of their algebraic thinking.

We think that it is possible to aid students' development of algebraic thinking during the phase of their transition from arithmetic to algebra. We think that a way of doing so is by presenting them with different types of problems—as proposed by Bednarz and Janvier (1996)—and encouraging their reasoning and development of strategies linked to their arithmetic thinking.

At the beginning of the experimental phase, we observed that students lacked concentration when they tried to solve algebraic problems. They even claimed that there was not enough information and that therefore they could not approach the proposed problems. Then, when students attempted to solve one of the problems posed to them, they tried different ways. Their main strategy was *S1*, but they soon realized that it was not necessary to do so many computations. Thus, as a first approximation, they started systematizing their strategies using quantities ending in 0 (by the easiness to operate with them, as can be seen in Figure 1).

Later, they continued with *controlled quantities*, that is, they had recourse to strategy *S2*. Eventually, this strategy became the most used. For instance, some students perceived the structure of the problem in an arithmetic context, and then they solved it by means of strategy *S2*.

Despite that in some occasions students did not take into account some portion of the information contained in the proposed problem, there was a positive evolution of most students to solve the problems as the experimental phase progressed. Moreover, students would check their answer to the problem according to the initial conditions of it. This was not easy, for students were used only to find a result without having to verify it. Thus, verification of the answer by students was an important accomplishment.

Now, with respect to the purpose of the investigation, the identification of strategies that fifth graders have recourse to for solving mathematics word problems, we can claim that students used informal arithmetic strategies—non-school strategies—to solve them. These strategies can evolve whenever students are stimulated to work with those relations and transformations immersed in the text of the problem. By means of an adequate intervention of the instructor, emphasizing correctly the arithmetic experience of the students, it is possible to make them come close to the algebraic thinking by designing ad hoc activities and problems.

By taking into account the complexity of word problems according to the characteristics proposed by Bednarz and Janvier (1996), we can select problems and rank them suitably for the design of a didactical intervention. Such a didactical intervention would support—in a controlled manner—the evolution of student' arithmetic reasoning toward their algebraic reasoning.

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Appendix

Examples of Problems Included in the Working Sheets

1. We want to distribute 100 chocolates among Saúl, Ricardo, and Nelson in such a way that Ricardo receives 4 times as many chocolates as Saúl, and Nelson receives 10 chocolates more than Ricardo. How many chocolates does each boy receive? (Adapted from: Alarcón et al., 1994, p. 165)
2. We want to distribute 91 bracelets among Rita, Luisa, and Candy in such a way that Rita receives 3 times as many bracelets as Luisa, and Luisa receives 3 times as many as Candy. How many bracelets does each girl receive? (Adapted from: Alarcón et al., 1994, p. 165)
3. An airplane flew 12 kilometers in 72 seconds. What was its average speed? How long will it take the airplane to fly a distance of 1800 kilometers? (Adapted from: Cárdenas et al., 1976, p. 199)
4. An automobile leaves from México City to Nayarit at an average speed of 40 kilometers per hour. Two hours later another automobile also leaves México City with the same destination at an average speed of 60 kilometers per hour. How long will it take the second automobile to reach the first one? (Adapted from: Preciado & Toral, 1971, p. 173)
5. There are three piles of flat bottle caps. The first pile has 5 bottle caps less than the third, and the second pile has 15 more than the third. The total number of flat bottle caps is 31. How many bottle caps are there in each pile? (Adapted from: Alarcón et al., 1994, p. 166)
6. The perimeter of a rectangular plot of land is 102 meters. The depth of the plot is double its width. What is its depth? What is its width? (Adapted from: Alarcón et al., 1994, p. 162)

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