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ABSTRACT

This paper examines the literature in the growing field of education and inequality that is concerned with how individuals sort, and the consequences of this knowledge for the accumulation of human capital, equity, efficiency, and welfare. It argues that how individuals sort across neighborhoods, schools, and households (spouses) can have important consequences for the acquisition of human capital and inequality. It discusses the implications of different education finance systems for sorting and analyses the efficiency and welfare properties of these in static and dynamic frameworks. There is no overarching theoretical framework in the field of sorting. Rather, different models illuminate some of the particular interactions among variables. This paper discusses a few models in depth and presents complex quantitative analyses of some of them. Findings include the following: (1) For a sufficiently patient social planner, the state education system will be preferred; (2) for a sufficiently powerful exam technology, the exam mechanism will always dominate the market mechanism for both aggregate production and consumption; and (3) countries with greater inequality exhibit greater sorting at the household level, and fertility differentials are increasing in inequality. The paper concludes that much work remains to be done in the areas discussed. (Contains 69 references.) (WFA)

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# SORTING, EDUCATION AND INEQUALITY

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## ABSTRACT

This paper examines the education literature through the lens of sorting. It argues that how individuals sort across neighborhoods, schools and households (spouses), can have important consequences for the acquisition of human capital and inequality. It discusses the implications of different education finance systems for sorting and analyzes the efficiency and welfare properties of these in static and dynamic frameworks.

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## 1. Introduction

Individuals sort in a variety of fashions. The workplace, the school of one's child, the choice of neighborhood in which to reside, and the selection of a spouse are all important arenas in which a choice of peers and access to particular goods and networks is explicitly or implicitly made. The aim of this chapter is to review the subset of the literature in the rapidly growing field of education and inequality that is primarily concerned with how individuals sort and the consequences of this for the accumulation of human capital, equity, efficiency and welfare.

At first blush, sorting may seem like a rather strange lens through which to examine education. After all, this field has been primarily concerned with examining issues such as the returns to education, the nature of the education production function or, at a more macro level, the relationship between education and per-capita output growth.[1] A bit more thought, though, quickly reveals that sorting is an integral component of these questions. Who one goes to school with, who one's neighbors are, who one works with, and who is a member of one's household, are all likely to be important ingredients in determining both the resources devoted to and the returns to human capital accumulation.

It is interesting to note that in all these spheres there is at least some evidence indicating that sorting is increasing in the US. Jargowsky (1996), for example, examines the changing pattern of residential segregation in the US over the last few decades. He finds that although racial and ethnic segregation has stayed fairly constant (with some small decline in recent years), segregation by income has increased (for Whites, Blacks and Hispanics) in all US metropolitan areas from 1970 to 1990. This increased economic segregation, and the fact that schools increasingly track students by ability, suggests that there is likely to be increased sorting at the school or classroom level by income and ability. Kremer and Maskin (1996) find evidence for the US, Britain, and

[1] For a survey of the education production function literature see Hanushek (1986), for returns to education see, for example, Heckman, Layne-Farrar, and Todd (1996), for education and growth, see, for example, Benhabib and Spiegel (1994).

France that there is increased sorting of workers into firms, with some high-tech firms (e.g., silicon valley firms) employing predominantly high-skilled workers and low-tech firms (e.g., fast-food industry) employing predominantly low-skilled workers. Lastly, there is also some indication of greater sorting at the level of household partner (or "marital" sorting). Although the correlation between spousal partners in terms of years of education has not changed much over the last few decades (see Kremer (1997)), the conditional probability of some sociological barriers being crossed–e.g., the probability that an individual with only a high-school education will match with another with a college education–has decreased, indicating greater household sorting (see Mare (1991)).

This chapter will examine some of the literature that deals with the intersection of sorting, education and inequality. This review is not meant to be exhaustive, but to give a flavor of some of the advances in the theory and quantitative evidence. Furthermore, it should be noted that there is no overarching theoretical framework in this field. Rather, different models are interesting because of how they illuminate some of the particular interactions among these variables and others–for example, the role of politics, the interaction between private and public schools, or the efficacy of different mechanisms (e.g. markets versus tournaments) in solving assignment problems. Thus, rather than sketch the contribution of each paper, I have chosen to discuss a few models in depth. Furthermore, as a primary concern in this area is the magnitude of different effects, wherever possible I focus on the contributions that have attempted to evaluate these.

The organization of the chapter is as follows. I begin with the topic of residential sorting. Local schooling is prevalent in most of the world. This policy easily leads to residential sorting and may have important implications for education and inequality, particularly in countries like the US in which the funding of education is also largely at the local level. I also use this section to review the theory of sorting. Next, I turn to examining sorting at the school level. The papers here are different as they are primarily concerned with the interaction of public and private schools and with the properties of different mechanisms. Lastly I turn to recent work on household sorting and its consequences for education and inequality.

## 2. Sorting into Neighborhoods

Neighborhoods do not tend to be representative samples of the population of the whole. Why is this? Sorting into neighborhoods may occur because of preferences for amenities associated with a particular neighborhood (say parks), because of some individuals' desire to live with some types of people or not to live with some others (say ethnic groups who wish to live together in order to preserve their culture, or who end up doing so as a result of discrimination), and in response to economic incentives. This chapter will be primarily concerned with the latter, and in particular with the endogenous sorting that occurs in response to economic incentives that arise as a result of education policies.

Primary and secondary education is a good that is provided locally. In industrialized countries, the overwhelming majority of children attend public schools (in the US a bit over 91 percent in 1996 and similar percentages in other countries).[2] Typically, children are required to live in the school's district to attend school there, making a neighborhood's school quality a primary concern of families in deciding where to reside. Furthermore, in most countries at least some school funding (usually that used to increase spending above some minimum) is provided locally; this is particularly true in the US in which only 6.6 percent of funding is at the federal level, 48 percent is at the state level, and 42 percent is at the local level.[3]

Does it matter that education is provided at the local level? How does local provision of education affect the accumulation of human capital, its distribution, and efficiency in general? What are the dynamic consequences of local provision? How do other systems of financing and providing education compare? These are some of the question this section will explore. I will start out with a brief overview of the economics of sorting, much of which will carry through to the other sections as well.

### 2.1. Multi-Community Models: The Economics of Sorting

Characterizing equilibrium in models in which heterogeneous individuals can choose among a given number of potential residences, and in which these choices in aggregate

[2] Digest of Education Statistics (1999).

[3] The remaining percentages comes from other miscellaneous sources. These figures are for 1996-'97 (Digest of Education Statistics (1999)).

affect the attributes of the community, is in general a difficult task. Since Westhoff (1977), economists working with these often called "multi-community models" have tended to impose a single-crossing condition on preferences in order to obtain and characterize equilibria in which individuals either partially or completely separate out by type.[4] As will be discussed in further detail below, the single-crossing condition also has two other very useful implications: (i) It guarantees the existence of a majority voting equilibrium over $p$; (ii) In many models it allows one to get rid of "trivial" equilibria (e.g. one in which all communities are identical) when a local stability condition is employed.

A typical multi-community model consists of a given number of communities, each associated with a bundle $(q,p)$. These bundles consist of a good or input is provided in some quality or quantity $q$ *at the community level* and of a community level price $p$ of some (usually other) good or service. The latter can simply be a price associated with residing in the neighborhood, e.g., a local property tax. Thus, we can assess the indirect utility of an individual from these residing in a given community as $V(q,p;y)$ where $y$ is an attribute of the individual such as income, ability, parental human capital, wealth or taste. We will assume throughout that $q$ is "good" in the sense that $V_q > 0$, whereas $V_p < 0$.

Individuals choose a community in which to reside. In these models, equilibria in which individuals sort into communities along their characteristic $y$ are obtained by requiring the slope of indifference curves in $(q,p)$ space,

$$\left.\frac{dp}{dq}\right|_{v=\bar{v}} = -\frac{V_q}{V_p} \tag{2.1}$$

to be everywhere increasing (or decreasing) in $y$. This implies that indifference curves cross only once and that where they do, if (2.1) is increasing in $y$, then the slope of the curve of an individual with a higher $y$ is greater than one with a lower $y$ (the opposite if (2.1) is decreasing in $y$).

The assumption of a slope that increases (decreases) in $y$ ensures that if an individual with $y_i$ prefers the bundle $(q_j,p_j)$ offered by community $j$ to some other bundle $(q_k,p_k)$ offered by community $k$, and $p_j > p_k$, then the same preference ordering over these

[4] In games of asymmetric information (e.g., signalling models, insurance provsion, etc.), the assumption of single-crossing indifference curves is used in order to obtain either partial or completely separating equilibria.

bundles is shared by all individuals with $y > y_i$ ($y < y_i$). Alternatively if the individual with $y_i$ prefers $(q_k, p_k)$, then community $k$ will also be preferred to community $j$ by all individuals with $y < y_i$ ($y > y_i$).

Either an increasing or decreasing slope can be used to obtain separation.[5] Henceforth, unless explicitly stated otherwise, I will assume that (2.1) is increasing in $y$, i.e.,

$$\frac{\partial \left( \left.\frac{dp}{dq}\right|_{v=\bar{v}} \right)}{\partial y} = -\frac{V_{qy}V_p - V_{py}V_y}{V_p^2} > 0 \qquad (2.2)$$

We shall refer to equilibria in which there is (at least some) separation by characteristic as sorting or stratification.

Condition (2.2) is very powerful. Independently of the magnitude of the expression, the fact that it is positive implies that individuals have an incentive to sort. As we shall discuss in the next section, this will be problematic for efficiency since it implies that even very small private incentives to sort will lead to a stratified equilibrium, independently of the overall social costs (which may be large) from doing so.

There are many economic situations in which condition (2.2) arises naturally. Suppose, for example, that $q$ is the quality of education and that this is determined by either a lump sum or proportional tax $p$ on income. If individuals are, for example, heterogeneous in income (so $y$ denotes the income of the individual), then this condition would imply that higher-income individuals are willing to pay more (either in levels or as a proportion of their income, depending on the definition of $p$) in order to obtain a greater quality of education. This can then result in a equilibrium stratified along the dimension of income. Alternatively, if the quality of education is determined by the mean ability of individuals in the community school, $p$ is the price of housing in the community, and individuals are heterogeneous in ability $y$, then (2.2) will be met if higher-ability individuals are willing to pay a higher price of housing in order to obtain higher quality (mean ability) schooling, allowing the possibility of a stratified equilibrium along the ability dimension.

It is important to note, given the centrality of borrowing constraints in the human capital literature, that differential willingness to pay a given price is not the only crite-

[5]Note that although either assumption can be used to obtain separation, the economic implications are very different, If increasing, then in a stratified equilibrium higher $y$ individuals would obtain a higher $(q, p)$ whereas, if decreasing, the high $(q, p)$ bundle would be obtained by lower $y$ individuals.

rion that determines whether sorting occurs.[6] Suppose, for example, that individuals are unable to borrow against future human capital or, less restrictively, that individuals with lower income, or lower wealth, or whose parents have a lower education level face a higher cost of borrowing. Then even in models in which there is no other incentive to sort (e.g., in which the return to human capital is not increasing in parental assets or, more generally, in which $V_q$ is not a function of $y$), there will nonetheless be an incentive to sort if the cost of residing in communities with higher $q$'s (i.e., the effective $p$ that individuals face) is decreasing in $y$. So, for example, if individuals with lower assets face a higher effective cost of borrowing (they are charged higher rates of interest by banks), then they will be outbid by higher-asset individuals for housing in communities with a higher $q$'s.

In many variants of multi-community models not only does (2.2) give rise to stratified equilibria, but it also implies that all locally stable equilibrium must be stratified.[7] In particular, the equilibrium in which all communities offer the same bundle, and thus each contain a representative slice of the population, is locally unstable.[8]

There are many local stability concepts that can be imposed in multi-community models. A particularly simple one is to define local stability as the property that the relocation of a small mass of individuals from one community to another implies that under the new configuration of $(q,p)$ in these communities, the relocated individuals would prefer to reside in their original community. More rigorously, an equilibrium is locally stable if there exists an $\varepsilon > 0$, such that, for all possible combinations of measure $\delta$ $(0 < \delta \leq \varepsilon)$ of individuals $y_i \in \Lambda_j^*$ (where $\Lambda_j^*$ is the set of individuals that in equilibrium reside in community $j$), a switch in residence from community $j$ to $k$ implies

$$V(q_k(\delta), p_k(\delta), y) \leq V(q_j(\delta), p_j(\delta), y) \quad \forall y \in \Lambda_{jk}, \forall j, k \tag{2.3}$$

where $(q_l(\delta), p_l(\delta))$ are the new bundles of $(q,p)$ that result in community $l = j, k$. Thus, condition (2.3) requires that, for all individuals who switch residence (the set $\Lambda_{jk}$), at the new bundles they should still prefer community $j$. This condition is required to hold

[6] For human capital models in which imperfections in credit markets play a central role, see Fernández and Rogerson (1998), Galor and Zeira (1993), Ljungqvist (1993), and Loury (1981), among others.

[7] In many settings this gives rise to a unique locally stable equilibrium.

[8] Note that this zero sorting configuration is always an equilibrium in multicommunity models as no single individual has an incentive to move.

for all community pairs considered.[9]

To see why the equilibrium with no sorting is rarely locally stable, consider, for example, the relocation of a small mass of high $y$ individuals from community $j$ to $k$. In models in which the provision of the local good is decided by majority vote, this will tend to make the new community more attractive to the movers (and the old one less attractive) since the median voter in community $k$ will now have preferences closer to those of the high $y$ individuals whereas the opposite will be true in community $j$. In models in which $q$ is an increasing function of the mean of $y$ (or an increasing function of an increasing function of the mean of $y$), such as when $q$ is spending per student or the average of the human capital or ability of parents or students, then again this move will make community $k$ more attractive than community $j$ for the high $y$ movers. Thus, in all these cases the no-sorting equilibrium will be unstable.

In several variants of multi-community models existence of an equilibrium (other than the unstable one with zero sorting) is not guaranteed.[10] For example, in a model in which the community bundle is decided upon by majority vote and voters take community composition as given, a locally stable equilibrium may fail to exist. The reason for this is that although there will exist (often infinite) sequences of community bundles that sort individuals into communities, majority vote need not generate any of these sequences. Introducing a good (e.g. housing) whose supply is fixed at the local level (so that the entire adjustment is in prices) though will typically give rise to existence.[11]

Condition (2.2) also has an extremely useful implication for the political economy aspect of multi-community models. Suppose that $p$ and $q$ are functions of some other variable $t$ to be decided upon by majority vote by the population in the community (say a local tax rate). They may also be functions, as well, of the characteristics of the (endogenous) population in the community. An implication of (2.2) is that independently of whether $p$ and $q$ are "nicely" behaved functions of $t$, the equilibrium outcome of majority vote over $t$ will be the value preferred by the individual whose $y$ is median in the community.

The proof of this is very simple. Consider the (feasible) bundle $(\widetilde{q}, \widetilde{p})$ preferred by

[9] See, e.g., Fernández and Rogerson (1996). If communities have only a fixed number of slots for individuals as in models in which the quantity of housing is held fixed, then this definition must be ammended to include the relocation of a corresponding mass of individuals from community $k$ to $j$.

[10] See Westhoff (1977) and Rose-Ackerman (1979).

[11] See, for example, Nechyba (1997).

the median $y$ individual in the community, henceforth denoted $\widetilde{y}$. An implication of (2.2) is that any feasible $(q, p)$ bundle that is greater than $(\widetilde{q}, \widetilde{p})$ will be rejected by at least 50 percent of the residents in favor of $(\widetilde{q}, \widetilde{p})$, in particular by all those whose $y$ is smaller than $\widetilde{y}$. On the other hand, any feasible bundle with a $(q, p)$ lower than $(\widetilde{q}, \widetilde{p})$ will also be rejected by 50 percent of the residents, namely all those with $y > \widetilde{y}$. Thus, the bundle preferred by $\widetilde{y}$ will be chosen by majority vote.[12]

It is also important to note that even in the absence of a single-crossing condition, to the extent that education is funded in a manner that implies redistribution at the local level, wealthier individuals will have an incentive to move away from less wealthy ones. This is by itself a powerful force that favors sorting but often requires a mechanism (e.g. zoning) to prevent poorer individuals chasing richer individuals in order to enjoy both a higher $q$ and a lower $p$.

For example, a system of local provision of education funded by a local property tax implicitly redistributes from those with more expensive housing to those with less expensive housing in the same neighborhood. The extent of redistribution, though, can be greatly minimized by zoning regulations that, for example, require minimum lot sizes.[13] This will raise the price of living with the wealthy and thus greatly diminish the amount of redistribution that occurs in equilibrium. In several models, to simplify matters, it will be assumed that mechanisms such as zoning ensure perfect sorting.

### 2.2. The Efficiency of Local Provision of Education

The simplest way to model the local provision of education is in a Tiebout model with (exogenously imposed) perfect sorting. In this model, individuals with different incomes $y_i$ but with identical preferences over consumption $c$ and quality of education $q$ sort themselves into homogeneous communities. Each community maximizes the utility of its own representative individual subject to the individual or community budget constraint. Let us assume that the quality of education depends only on spending per student (i.e., the provision of education exhibits constant returns to scale and there are no peer effects). Then, perfect sorting is Pareto efficient. Note that this system is

[12]See Westhoff (1977) and Epple and Romer (1991). Also see Gans and Smart (1996) for a more general ordinal version of single crossing and existence of majority vote.

[13]See Fernández and Rogerson (1997b) for an analysis which endogenizes zoning, sorting, and the provision of education.

identical to a purely private system of education provision.

The model sketched in the paragraph above often guides many people's intuition in the field of education. This is unfortunate as it ignores many issues central to the provision of education. In particular, it ignores the fact the education is an *investment* that *benefits the child* and potentially affects the *welfare of others* as well. These are important considerations as the fact that education is primarily an investment rather than a consumption good implies that borrowing constraints may have significant dynamic consequences; the fact that education primarily affects the child's (rather than parental) welfare raises the possibility that parents may not be making the best decisions for the child; and the potential externalities of an agent's education raises the usual problems for Pareto optimality.

Below I explore some departures from the assumptions in the basic Tiebout framework and discuss how they lead to inefficiency of the stratified equilibrium. This will make clear a simple pervasive problem associated with sorting, namely that utility-maximizing individuals do not take into account the effect of their residence decisions on community variables. I start out by discussing the simplest modification to the basic Tiebout model–reducing the number of communities relative to types.

Following Fernández and Rogerson (1996), consider an economy with a given number of communities $j = \{1, 2, ...N\}$, each (endogenously) characterized by a proportional income tax rate $t_j$ and a quality of education $q_j$ equal to per-pupil expenditure, i.e., $q_j = t_j\mu_j$. Individuals who differ in income $y_i$, $i \in I = \{1, 2, ...\overline{I}\}$ (with $y_1 > y_2 > ... > y_{\overline{I}}$), simultaneously decide in which community, $C_j$, they wish to reside. Once that decision is taken, communities choose tax rates via majority vote at the community level. Individuals then consume their after-tax income and obtain education.[14]

Assume for simplicity that individual preferences are characterized by the following separable specification:

$$u(c) + v(q) \tag{2.4}$$

[14] Very often the literature in this field has implicitly adopted a sequencing such as the one outlined above. Making the order of moves explicit as in Fernández and Rogerson (1996) allows the properties of equilibrium (e.g., local stability) to be studied in a more rigorous fashion. It would also be of interest to examine properties of models in which communities act more strategically and take into account the effect of their tax rate on the community composition. There is no reason to believe that this modification would generate an efficient equilibrium, however.

so that the sorting condition (2.2) is satisfied if $-\frac{u''(c)c}{u'(c)} > 1, \forall c$. We will henceforth assume that the inequality is satisfied, ensuring that individuals with higher income are willing to suffer a higher tax rate for higher quality.[15]

Suppose that the number of communities is smaller than the number of income types.[16] In such a case the equilibrium will generally not be Pareto efficient. The clearest illustration of this can be given for the case in which individuals have preferences such that an increase in the mean income of the community ceteris paribus decreases the tax rate that any *given* individual would like to impose. As the preferred tax rate of an individual is given by equating $u'(c)y_i$ to $v'(q)\mu_j$, this is ensured by assuming $-\frac{v''(q)q}{v'(q)} > 1$ (note that this is the parallel of the condition on $u$ that generates sorting).[17]

As discussed previously, the result of majority vote at the community level is the preferred tax rate of the median income individual in the community. A few things to note about the characteristics of equilibrium. First, in equilibrium no community will be empty. If one were, then in any community that contained more than one income type, those with higher income would be made better off by moving to the empty community, imposing their preferred tax rate, and engaging in no redistribution. Second, in a locally stable equilibrium communities cannot offer the same bundles and contain more than one type of individual (as a small measure of those with higher income could move to one of the communities, increase mean income there and end up with the same or a higher income median voter who has preferences closer to theirs'). Lastly, if communities have different qualities of education (as they must if the communities are heterogeneous), then a community with a strictly higher $q$ than another must also have a strictly higher $t$ (as otherwise no individual would choose to reside in the lower quality-higher tax community).

In the economic environment described above all locally stable equilibria must be stratified, i.e., individuals will sort into communities by income. In such equilibria,

[15] Most assumptions here are for simplicity only, e.g., preferences need not be separable and introducing housing and property taxation rather than income taxation would allow a sorting equilibrium to be characterized by higher-income communities having lower tax rates (but higher tax inclusive prices) and higher $q$. We forego the last option as it simply complicates matters without contributing additional insights.

[16] Note that type here is synonymous with income level. Hence the assumption that there are fewer neighborhoods than types is a reasonable one to make.

[17] This assumption implies that an increase in the mean income of the community that does not change the identity of the median voter will result in a higher $q$ and a lower $t$ ensuring that all residents are made better off.

communities can be ranked by the quality of education they offer, their income tax rate, and the income of the individuals that belong to them. Thus, all stable equilibria can be characterized by a ranking of communities such that $\forall j$, $q_j > q_{j+1}$, $t_j > t_{j+1}$, and $\min y_i \in C_j \geq \max y_i \in C_{j+1}$.

To facilitate the illustration of inefficiency, assume for simplicity that there are only two communities $j = 1, 2$ and $I > 2$ types of individuals.[18] A stratified equilibrium will have all individuals with income strictly greater than some level $y_b$ living in $C_1$ and those with income strictly lower than $y_b$ living in $C_2$ with $q_2 > q_1$ and $t_2 > t_1$.

Suppose that in equilibrium individuals with income $y_b$ live in both communities. It is easy to graph the utility

$$W_b^j \equiv u(y_b(1-t_j)) + v(t_j \mu_j) \tag{2.5}$$

of these "boundary" individuals as a function of the community in which they reside and as a function of the fraction $\rho_b$ of these individuals that reside in $C_1$. Let $\rho_b^*$ denote the equilibrium value of the boundary individuals residing in $C_1$. Note that a decrease in $\rho_b$ from its equilibrium value that does not alter the identity of the median voter in either community will make individuals with income $y_b$ better off in both communities as mean incomes will rise, qualities of education increase, and tax rates fall in both communities. Thus in order for this equilibrium to be locally stable, it must be that such a decrease makes $y_b$ individuals even better off in $C_1$ relative to $C_2$, reversing the outward flow and reestablishing $\rho_b^*$ as the equilibrium. Thus, as shown in Figure 1, the $W_b^1$ curve must cross the $W_b^2$ curve from above.[19]

This equilibrium is clearly inefficient. Consider a marginal subsidy of $s > 0$ to all individuals with income $y_b$ who choose to reside in $C_2$.[20] Given that without a subsidy these individuals are indifferent between residing in either of the two communities, it follows that a subsidy will increase the attractiveness of $C_2$ relative to $C_1$. Consequently, some $y_b$ individuals will move to $C_2$, thereby increasing mean income in both communities. For a small enough subsidy such that the identity of the median voter does not

[18] See Fernández and Rogerson (1996) for a generalization of this argument to many communities.

[19] Note that we are assuming for the range of $\rho_b$ shown that neither of the communities' median voters are changing.

[20] If income is unobservable, then a small subsidy to all individuals who reside in $C_2$ would have to be paid.

change in either community, the overall effect will be to decrease tax rates and increase the quality of education in both communities, thus making all individuals better off. Thus, it only remains to show that the subsidy can be financed in such a way to retain the Pareto improving nature of this policy. A simple way to do so is by (marginally) taxing those $y_b$ individuals who remain in $C_1$.[21] This tax will only further increase their outflow from $C_1$ to the point where they are once again indifferent between residing in both communities. As shown in Figure 1, the tax serves to further increase the utility of this income group (and consequently everyone else's). This last point suggests that a simpler way of producing the same Pareto-improving results is a policy that foregoes the subsidy and simply taxes any $y_b$ individual in $C_1$. This would again induce the desired migration and increase mean income in both communities.

Fernández and Rogerson (1996) examine these and other interventions in a model with many communities. The principle guiding the nature of Pareto-improving policies is not affected by the number of communities considered; policies that serve to increase mean income in some or in all communities by creating incentives to move relatively wealthier individuals into poorer communities will generate Pareto improvements.[22]

The possibility of Pareto improvements over the decentralized equilibrium in the model above arises as a result of individuals not taking into account the effect of their residence decisions on community mean income. In the next example, the inefficiency of equilibrium results from individual residence decisions not internalizing diminishing returns.

Consider a multi-community model with two communities, $C_1$ and $C_2$, and a total population (of parents) of $N = 2$. Parents differ in their human capital, $h_i$, and potentially in their own income $y_i$. To simplify matters we assume that the initial distribution is confined to two values $h_1$ and $h_2$ with $h_1 > h_2$ and total numbers of parents of each type given by $n_1$ and $n_2$, respectively, such that $n_1 + n_2 = 2$.

We assume that each community has a fixed number of residences, $N/2 = 1$ each available at a price $p_j$, $j = 1, 2$. Let $\lambda_1$ be the fraction of high human-capital parents who choose to live in $C_1$ (and thus $\lambda_2 = n_1 - \lambda_1$) and let $\mu_j$ be the mean human capital of parents that reside in $C_j$. Thus, $\mu_j(\lambda_j) = \lambda_j h_1 + (1 - \lambda_j) h_2$.

[21] Again, if income is not observable, it is possible to preserve the Pareto improving nature of this policy by (marginally) taxing all $C_1$ residents.

[22] The exact specification of these policies, however, depends on the number of communities involved in a rather odd fashion as explained in Fernández (1997).

Parents decide in which community to live, pay the price $p_j$ of residing there, and send their children to the community school. Parents care about aggregate family consumption, which is given by the sum of their own income and the child's future income, $I$, minus the cost of residing in the community and a lump-sum transfer $T$.

The child's future income is an increasing function of the human capital she acquires. This depends on her parent's human capital and on local human capital $q$ which is assumed to be an increasing function of the mean human capital in the neighborhood. As the latter is simply a linear function of $\lambda_j$, we denote this function as $q_j = Q(\lambda_j)$, $Q' > 0$. Thus,

$$I_{ij} = F(h_i, Q(\lambda_j)) \tag{2.6}$$

with $F_h, F_q > 0$ and where $I_{ij}$ indicates the income of a child with a parent of human capital $h_i$ that resides in neighborhood $j$.

Hence, parents choose a community in which to reside that maximizes

$$u(y_i + I_{ij} - p_j + T) \tag{2.7}$$

subject to (2.6) and taking $p_j$, $T$ and $q_j$ as given. Note that if parental and local human capital are complements in the production of a child's future income, then (2.7) obeys (2.2), and hence individuals will sort.[23] Henceforth, I will assume this is the case, that is,

$$\frac{\partial \left( \left. \frac{dp}{d\lambda} \right|_{u=\bar{u}} \right)}{\partial h} = F_{hq} Q' > 0 \tag{2.8}$$

Given (2.8), the only locally stable equilibrium is that with maximal sorting. Individuals with human capital $h_1$ live in $C_1$, characterized by a higher $p$ and a higher $q$ than that in $C_2$; individuals with $h_2$ live in $C_2$. If the number of one of these types exceeds the space available in a community (i.e., 1), then that type is indifferent and lives in both communities. Thus, in equilibrium $\lambda_1 = \min(1, n_1)$.

In order to close the model, we need to specify housing prices. Rather than determining the price by specifying the microfoundations of the housing market, as in many

[23]See de Bartolome (1990) for a two-community fixed housing stock model in which there is complementarity between spending on education and ability but in which peer effects matter more for low ability students.

models in the literature we simply solve for the price differential such that no individual would wish to move.[24] Depending on whether $n_1$ is greater, smaller or equal to 1, there are three different possible configurations as in the first case $h_1$ types must be made indifferent ($p_1 - p_2 = y'_{11} - y'_{12}$), in the second $h_2$ types must be made indifferent ($p_1 - p_2 = y'_{21} - y'_{22}$), whereas in the third each type must be at least as well off in its own community than in the other ($y'_{11} - y'_{12} \geq p_1 - p_2 \geq y'_{21} - y'_{22}$). Rather than include landlords or define the structure of house ownership by agents, we simply assume, as in de Bartolome (1990), that housing rents are rebated to individuals in a lump-sum fashion so that each individual receives $T = \frac{p_1+p_2}{2}$ regardless of the community of residence.[25]

Is the decentralized equilibrium efficient? Rather than characterizing Pareto improving policies, I will confine my discussion here to investigating whether the unique locally stable decentralized equilibrium (that with maximum sorting) maximizes productive efficiency.

The tensions that exist in this model are easy to define. On the one hand, parental and local human capital are complements, suggesting that future output is maximized by sorting, i.e., efficiency requires concentrating high-human capital parents in the same community, precisely what occurs in equilibrium. On the other hand, there is an externality to individual residence decisions that is not being taken into account, namely potentially decreasing returns to the concentration of high human-capital individuals in the same neighborhood. In particular, individuals do not take into account whether an additional unit of high human capital on the margin increases local human capital more in the community with a high or low concentration of $h_1$. Similarly, they do not take into account whether a marginal increase in local human capital will add more to total output by being allocated to a community with a high or low concentration of $h_1$.

To see this more formally, consider the total future income $Y$ generated by a community given that a fraction $\lambda$ of high human-capital parents live there:

[24] See Wheaton (1977) and de Bartolome (1990).

[25] See Benabou (1993) for a multi-community model in which individuals can acquire high or low skills or be unemployed. The costs of acquiring skills are decreasing in the proportion of the community that is highly skilled but this decrease is larger for those acquiring high skills. This leads to sorting although ex ante all individuals are identical. As in the model discussed here, there will be maximal sorting by (ex post) high-skill individuals. The interesting question is this paper is how the decentralized equilibrium compares to one with no sorting given that neither is efficient (since in both cases individuals ignore the externality of their skill acquisition decision on the costs faced by others).

$$Y(\lambda) = \lambda F(h_1, Q(\lambda)) + (1-\lambda) F(h_2, Q(\lambda)) \tag{2.9}$$

Note that if future income is concave in $\lambda$, then it is maximized by allocating high human-capital parents so that they constitute the same proportion in both communities, i.e., $\lambda_1 = \lambda_2$. If, on the other hand, future income is convex in $\lambda$, then maximum sorting will maximize future income, i.e., as in the decentralized equilibrium $\lambda_1 = \min(1, n_1)$.

Taking the appropriate derivatives yields:

$$\begin{aligned} Y'' \;=\; & 2\left[F_q(h_1, Q(\lambda)) - F_q(h_2, Q(\lambda))\right] + \left[\lambda F_q(h_1, Q(\lambda)) + \right. \\ & \left.(1-\lambda) F_q(h_2, Q(\lambda))\right] Q'' + \left[\lambda F_{qq}(h_1, Q(\lambda)) + (1-\lambda) F_{qq}(h_2, Q(\lambda))\right] Q'^2 \end{aligned} \tag{2.10}$$

Let us carefully examine the terms in (2.10). The complementarity of parental and local human capital in the production of children's human capital guarantees that the expression in the first square brackets is positive. Thus, this factor pushes in the direction of convexity of $Y$ and thus in favor of sorting. Recall from (2.8) that it is only on the basis of this factor that sorting occurs in equilibrium. If there is decreasing returns to community mean human capital in the formation of local human capital, however, i.e., if $Q$ is concave (and thus $Q'' < 0$), then $Q''$ times the expression in the second square brackets will be negative, imposing losses from concentrating parents with high human-capital in the community. Lastly, there will be an additional loss from sorting if there is decreasing returns to local human capital in the production of future income, i.e., if $F_{qq} < 0$, as this implies that the term in the third square brackets is negative. Thus, decreasing returns to community mean human capital in the formation of local human capital and decreasing returns to local human capital in the production of children's future income suggest that $Y$ is concave, and hence that efficiency would be maximized by having parents with high-human capital distributed in both communities in the same proportion.[26]

It is important to recall that maximum sorting will take place as long as $F_{hq}$ is positive but otherwise independently of its magnitude. Hence a very small amount of complementarity (again, the expression in the first square brackets) and private gain

[26]If $Y$ is not globally concave nor convex, then the equilibrium will still produce too much sorting and some redistributing of high human capital individuals towards a more equal distribution will be efficiency enhancing.

could easily be swamped by the concavity of $F$ and $Q$ and social loss.

The model presented above is one in which all sorting is taking place because of peer effects–that is, people want to live with individuals with high human capital as it increases the earnings of their children. As local human capital and parental human capital are complements, high human capital parents outbid others to live in a community where the level of local human capital is highest, leading to stratification by parental human capital levels. Note that income and the perfection or imperfection of capital markets actually played no role in producing the results above.[27]

The above analysis also suggests that if spending on education $E$ were an additional factor in the production of future income but not a factor that individuals sorted on, i.e., $F(h, Q(\lambda), E(\lambda))$ with $F_E > 0$, $E' > 0$ and $F_{hE} = 0$, then sorting would occur for the same reasons as before, but even a policy of enforced equalization of spending across communities would not stop individuals from sorting.

Unfortunately, there has been very little work done to assess the significance of the inefficiencies discussed above. Although much work points, for example, to the importance of peer effects in learning, whether the appropriate cross-partial is negative or positive remains in dispute (i.e., we do not even know whether it would be efficient, all considerations of diminishing returns aside, for children to sort by aptitude, for example, or for them to mix).[28] Similarly, we do not know whether quality of education (say spending) and parental human capitals are complements. This, to my view, makes models in which the main imperfection lies in the functioning of the capital market (and sorting on grounds of minimizing redistribution) relatively more attractive.[29]

[27]The fact that utility depends only on total net family income and that the latter is not influenced by spending allows us to abstract from issues of borrowing and lending as long as parents have sufficient income to bid succesfully for housing.

[28]For example, Henderson, Mieszkowski, and Sauvageau (1978) argue for a zero cross partial and dimishing returns whereas Summers and Wolfe (1977) for a negative cross partial.

[29]These borrowing constraints may not allow families to borrow to send their child to private school, for example. Alternatively, they may not allow poorer families to borrow to live in (wealthy) neighborhoods with higher quality public education. The general failure of these credit markets is that parents are unable to borrow against the future human capital of their children.

### 2.3. Comparing Systems of Financing Public Education: Dynamic Considerations

The choice of education finance system matters for various reasons. First, and foremost, different finance systems tend to imply different levels of redistribution. In economies in which there is imperfect access to financing the acquisition of human capital, redistribution can play an important role in increasing the human capital levels of children from lower-income families. Different finance systems may also may have important consequences for who lives where and thus for the identity of a child's peers and for the use of the land market.

There have been several papers written in this area that examine primarily the static consequences of different systems of financing education.[30] Fernandez and Rogerson (1999b), for example, examine five different education finance systems, and contrast the equity and resources devoted to education across these systems assuming that the parameters of the education finance system are chosen by majority vote. They calibrate their benchmark model to US statistics and find that total spending on education may differ by as much as 25% across systems. Furthermore, the trade-off between redistribution and resources to education is not monotone; total spending on education is high in two of the systems that also substantially work to reduce inequality. A political economy approach to the contrast of different education finance systems has also been pursued by Silva and Sonstelie (1995) and Fernandez and Rogerson (1999a) who attempt to explain the consequences of California's education finance reform, whereas Nechyba (1996) and de Bartolome (1997) both study foundation systems. There is also a growing empirical literature devoted to examining how changes in state-level education finance systems affect education spending, including Downes and Schoeman, (1998), Loeb (1998), Hoxby (1998), Evans, Murphy, and Schwab (1997, 1998), and Manwaring and Sheffrin (1997).

The papers mentioned above, however, are only indirectly concerned with the consequences of sorting and they are all static models. In this section, by way of contrast, we will focus on dynamic consequences of sorting in response to different education finance systems. To facilitate the theoretical analysis, we will focus on two extreme systems: a

[30]See Inman (1978) for an early quantitative comparison of education finance systems in the context of an explicit model.

pure local system with perfect sorting and a state system with uniform school spending per student across communities.[31]

This section presents two models.[32] The first, based on Fernández and Rogerson (1997a, 1998) uses a Tiebout model in which perfect sorting, from a static perspective, is efficient. It then examines the trade-off imposed by switching to a state financed system. The model is calibrated to US statistics, allowing one to determine whether these trade-offs are quantitatively significant. The main trade-off this analysis illustrates is that between a system that loosely speaking allows individuals to consume bundles that are "right" for them given their income versus a system that imposes a uniform education bundle across heterogeneous individuals, but allows for more efficient use of resources from the perspective of future generations. In particular, in an economy in which borrowing constraints prevent individuals from financing their education and missing insurance markets does not allow children (or parents) to insure against income or ability shocks, a state system may result in a more efficient production of next period's income (again in a sense that will be made rigorous below) than in a local system in which the possibilities for redistribution are only at the local level.[33] The trade-offs are found to be quantitatively significant.

The second model is based on Benabou (1996). This is a purely theoretical analysis that contrasts the short versus long-run consequences of a local compared to a state system in which the main trade-off is between human capital being complementary in production at the economy wide level but parental human capital and spending on education being complementary at the local level.

The simplest contrast between the dynamic consequences of these two extreme forms of education finance–local versus state–can be examined in the familiar Tiebout model of *perfect* sorting in which income is the only source of heterogeneity among individuals. This allows us to abstract away from complications that would be introduced by the political economy of tax choice at the local level when individuals are heterogenous, by changes in residence over time with the dynamic evolution of the income distribution,

[31] See Fernández and Rogerson (forthcoming) for a dynamic analysis of a foundation system.

[32] Other dynamic analysis of education finance systems include Cooper (1998), Durlauf (1995), Glomm and Ravikumar (1992), and Saint-Paul and Verdier (1993).

[33] It may be objected that this analysis confounds two things–the amount of redistribution (or insurance) and the system of education. In reality, education always entails some redistribution and a multidimensional political economy model would be required to allow one to differentiate between redistribution directly through income and through education.

by housing (and the inefficiencies that stem from taxing this good), peer effects, or simply from diversity in tastes.[34] Note that by considering a Tiebout system with perfect sorting, we can reinterpret what follows as contrasting a purely private system of education with a state financed one.

Following Fernández and Rogerson (1997a), consider a two-period overlapping generations model in which each person belongs to a household consisting of one old individual (the parent) and a young one (the child). Parents make all the decisions and have identical preferences described by

$$U(c, y') = u(c) + Ew(y') \tag{2.11}$$

where $y'$ is next period's income of the household's child and $E$ is the expectations operator.

In the first period of life, the child attends school and obtains the quality of education $q$ determined by her parent's (equivalently community's) spending. In the second period, the now old child receives a draw from the income distribution. A child's income when old is assumed to depend on the quality of schooling and on an *iid* shock $\xi$ whose distribution $\Psi(\xi)$ is assumed to be independent of $q$. Thus,

$$y' = f(q, \xi) \tag{2.12}$$

Once the adult's income is determined so is the community of residence as adult. The adult (now a parent), then decides how much of her income to consume and how much to spend on her own child's education. Letting $v(q) \equiv \int w(f(q,\xi))d\Psi$, we can now write preferences exactly as in equation (2.4). Assuming that $v$ is well behaved, under a local system individuals will set spending on education to equate the marginal utility of consumption with the marginal utility of education quality (i.e., $u'(c) = v'(q)$), implying a local tax rate $\tau(y)$ and $q = \tau(y)y$.

We next turn to the determination of spending on education in a state-financed system. We assume that all individuals face the same proportional income tax rate $\tau_s$ that is used to finance public education $q = \tau_s\mu$ and that individuals are unable to opt

[34] See, however, Fernández and Rogerson (1998) for a more complex dynamic model in which the sorting of individuals into communities endogenously evolves over time along with housing prices and the housing stock.

out of public education to a private system.[35].

The first-order condition for utility maximization now equates the ratio of the marginal utility of consumption and the marginal utility of education quality to the ratio of the mean relative to individual income, i.e., $\frac{u'(c_i)}{v'(q)} = \frac{\mu}{y_i}$. Note that this condition reflects the fact that under a state financing system, unlike in the local system, the relative price of a unit of education (in terms of foregone consumption) is not the same across individuals. Lower-income individuals face a lower price than higher-income individuals. In a local finance system, on the other hand, this relative price equals one for all individuals. Under majority vote, concavity of $u$ and $v$ imply that the preferences of the individual with median income in the population determines the choice of $\tau_s$.

Letting $g_t(y)$ be the income distribution of old individuals at the beginning of period $t$, under either education finance system an equilibrium at the end of period $t$ generates a beginning-of-period income distribution for period $t+1$, $g_{t+1}$. Let $F(g(y))$ be the income distribution that results in the following period given this period's distribution of $g(y)$. A steady state in this model then consists of an income distribution $g^*$ such that $g^*(y) = F(g^*(y))$.

Calibrating this simple model involves making choices over the education quality technology and preferences. There is a large and controversial literature that surround the education production function and there is no consensus on the form it should take.[36] Guided primarily by simplicity, a convenient specification is $y' = Aq^{\theta}\xi$, which yields an elasticity of future income with respect to education quality that is constant and equal to $\theta$. Evidence presented by Card and Krueger (1992), Wachtel (1976), and Johnson and Stafford (1973) suggest an elasticity of earnings with respect to education expenditures close to 0.2. We assume that $\xi$ is lognormally distributed such that $\log \xi$ has zero mean and standard deviation $\sigma_{\xi}$.

Our specification of preferences comes from noting that across US states the share of personal income devoted to public elementary and secondary education has remained roughly constant over the 1970-1990 period.[37] This property will be satisfied if the indirect utility function takes the form $\frac{c^{\alpha}}{\alpha} + E(\Phi(\xi))\frac{q^{\alpha}}{\alpha}$ where $\Phi(\xi)$ is some function of

[35]Introducing a private option into this system greatly complicates the analysis as existence of equilibrium is not ensured. See Stiglitz (1974).

[36]See Coleman et al (1966), Hanushek (1986), Card and Krueger (1992), and Heckman, Layne-Farrar, and Todd (1996).

[37]See Fernández and Rogerson (2001a) for evidence.

$\xi$. This requires a utility function of the form

$$\frac{c^\alpha}{\alpha} + E(\frac{b}{\alpha} y^\gamma) \tag{2.13}$$

with the restriction that $\theta\gamma = \alpha$.

Under local financing the preferences above imply a constant and identical tax rate across individuals, $\tau^* = \frac{1}{1+\kappa}$, where $\kappa = (bA^\gamma E(\xi^\gamma))^{\frac{1}{\alpha-1}}$. If a parent's income in period 0 is $y_0$, it follows that the child's income, $y_1$, is given by $\log y_1 = \log A + \theta \log \tau^* + \theta \log y_0 + \log \xi_1$. Given $\theta < 1$, it follows that $\log y_t$ has a limiting distribution that is normal with mean and standard deviation:

$$\mu_\infty = \frac{\log A + \theta \log \tau^*}{1-\theta} \quad \sigma_\infty = \frac{\sigma_\xi}{(1-\theta^2)^{1/2}} \tag{2.14}$$

We calibrate the steady state of the local model to match US statistics. We choose $A$ and $\sigma_\xi$ such that $\mu_\infty$ and $\sigma_\infty$ match the mean and median of the US family income distribution, respectively \$23,100 and \$19,900 in the 1980 census. The remaining parameters to be set are $b$ and $\alpha$, as the value of $\theta$ is already determined by the elasticity of earnings with respect to $q$.

For any given $\alpha$, we set $b$ to match the fraction of personal income devoted to public elementary and secondary education (in 1980 equal to 4.1 percent), that is, to yield a tax rate $\tau^* = 0.041$. This determines, for a given value of $\alpha$, a value of $b$ given by $b = \frac{[(1-\tau^*)/\tau^*]^{\alpha-1}}{A^\gamma E(\xi^\gamma)}$. To set $\alpha$, we draw upon two pieces of information. The first is the price elasticity of expenditures on education. In our model this can be computed at the equilibrium price (in terms of the consumption good), which here has been set to one. A survey of the literature by Bergstrom, Rubinfeld and Shapiro (1982) suggests an elasticity between $-0.5$ and $-0.25$, yielding $\alpha$ between $-1$ and $-2$. The second is from Fernández and Rogerson (1999) who model a foundation education finance system and use it to match the distribution of spending per student in California prior to the Serrano reform. They find an implied value for $\alpha$ equal to $-0.2$.

One of the main questions we are interested in asking is whether a local system will outperform a state system. Obviously, there is no reason to expect that individuals of all income levels will prefer one system over another nor that different generations will agree on the relative merits of the two systems. In order to have a measure of aggregate welfare, we use the sum of individual utilities, or equivalently, the expected utility than an agent would obtain if she were to receive a random draw from the equilibrium income distribution. Thus, we use

$$V_{rt} \equiv \int U_{rt} g_{rt}(y) dy \tag{2.15}$$

as our measure of aggregate welfare at time $t$ where $r = L, S$ (i.e., local $L$, state $S$) indicates the education finance regime.[38]

To provide a measure of welfare change at time $t$ that is unaffected by monotone transformations of the utility function, we examine the proportion by which the income distribution in the *steady state* of the *local* regime would have to be changed so that it provided the same aggregate welfare as the state system in period $t$. Given that the functional forms adopted are homogeous of degree $\alpha$ in income, this amounts to finding the value of $\Delta_t$ such that $(1 + \Delta_t)^{\alpha} V_L = V_{S_t}$ where the local system is evaluated at its steady state and the state system in period $t$.

If $\alpha$ is negative (as our calibration procedure suggests), then preferred tax rates under a state system are increasing in income (under a local system, as noted previously, they are independent of income) and only equal to the local tax rate for those individuals with income such that $y_i = \mu$. Since the median voter's income is lower than mean income, it follows that the tax rate will be lower under the state system than the (identical) tax rate chosen by each income group under the local system. This implies that in the first period, given that the income distribution is the same as in the local system, aggregate spending on education will decrease.

The table below shows the tax rate, mean and median income in the steady state of state finance regime. The last two columns report the first period gain in aggregate welfare (i.e., prior to the change in mean income) which we denote by $\Delta_1$ and the steady-state gain in aggregate welfare, denoted $\Delta_{\infty}$. Despite the fact that for $\alpha$ strictly negative spending on education will decrease in the first period of reform (relative to its value in the local system), we find that steady-state mean income is always higher than in the local steady state. Furthermore, aggregate welfare increases in period 1 as well as in every subsequent period relative to the initial local finance system steady state.[39]

[38]Note that this is equivalent to a utilitarian welfare measure or one chosen "behind the veil of ignorance" (i.e., an individual's welfare if her parents were a random draw from the income distribution in that system).

[39]The new steady state is typically reached in five periods.

Table 1

Steady State Comparisons of Local vs State

| $\alpha$ | $\tau_s$x$10^2$ | $\mu_\infty$ | median $y$ | $1+\Delta_1$ | $1+\Delta_\infty$ |
|---|---|---|---|---|---|
| 0 | 4.10 | 25,300 | 21,900 | 1.006 | 1.108 |
| −0.2 | 4.01 | 25,100 | 21,800 | 1.007 | 1.104 |
| −0.5 | 3.91 | 25,000 | 21,600 | 1.009 | 1.101 |
| −1 | 3.82 | 24,000 | 21,500 | 1.011 | 1.101 |
| −2 | 3.74 | 24,000 | 21,400 | 1.015 | 1.105 |

As shown in the Table above, the first period gains are relatively small (around 1 percent).[40] The steady-state gain is surprisingly constant across parameter values, even though the tax rate is changing relative to the local steady state by as much as 10 percent.[41] More generally, the "static" welfare gain might well be negative. In a model with housing, for example, the unbundling of the education and residence decision that a state system allows relative to a local system will in general imply an increase in housing prices in relatively poorer communities and a decrease in wealthier ones. Thus, lower-income individuals will end up paying higher property prices than previously, and the transition to the new steady state may well involve some losses in early periods. In the more complicated model studied by Fernández and Rogerson (1998), this change in housing prices and the fact that agents preferred tax rates differ, implies a small decrease (.3 percent) in aggregate welfare in the first period of the policy reform.

The more complicated analysis in Fernández and Rogerson (1998) gives rise to an even starker illustration of differences in short and long-run welfare. In that paper spending on education affects the mean (but not the variance) of the lognormal distribution from which individual income is assumed to be a random draw. Comparing across steady states of a local relative to a state system of financing education, we find that, *given* an individual's income, each individual prefers a local system to a state

[40]More generally, the "static" welfare gain might well be negative. In a model with housing, for example, the unbundling of the education and residence decision that a state system allows relative to a local system will in general imply an increase in housing prices in relatively poorer communities and a decrease in wealthier ones. Thus, lower-income individuals will end up paying higher property prices than previously, and the transition to the new steady state may well involve some losses in early periods. In the more complicated model studied by Fernández and Rogerson (1998), this change in housing prices and the fact that agents preferred tax rates differ, implies a small decrease (.3 percent) in the first period of the policy reform.

[41]See Fernández and Rogerson (1997a) for a sensitivity analysis for other parameter values.

system. However, an individual's income is of course not the same across systems since the probability with which any particular level is realized depends on spending on education, which in turn depends on the system of financing education. It is taking the new distribution of income that results into account that yields a higher steady-state welfare level under the state system.

Next I turn to an analysis based primarily on Benabou (1996). Consider an economy populated by OLG dynasties indexed by $i$ who spend some amount of time $\nu$ working and the remainder $1-\nu$ passing on education to their single child. The law of motion for the evolution of future descendants' human capital is given by

$$h_{t+1}^{i} = \kappa \xi_t^i((1-\nu)h_t^i)^{\delta}(E_t^i)^{1-\delta} \tag{2.16}$$

reflecting an inherited portion as given by $h_t^i$ (the parent's human capital) and an unpredictable portion given by an *iid* shock $\xi_t^i$. The shock is assumed to be distributed lognormally such that $\ln \xi_t^i \sim N(-s^2/2, s^2)$ and thus $E(\xi_t^i) = 1$. Formal schooling, $E_t^i$, is the other input into the production of next period's human capital. This is financed by taxing at rate $\tau$ the labor income of local residents. Hence,

$$E_t^i = \tau Y_t^i \equiv \tau \int_0^{\infty} y dm_t^i(y) \tag{2.17}$$

where $m_t^i$ is the distribution of income (and $Y_t^i$ is its average) in the community $\Lambda_t^i$ to which family $i$ belongs at time $t$. [42]

The production sector is made up of competitive firms with constant returns to scale CES technology given by $Y_t = (\int_0^{\infty}(x_t^r)^{(\sigma-1)/\sigma}dr)^{\sigma/(\sigma-1)}$, $\sigma > 1$ where $x_t^r$ denotes intermediate input $r$. Each worker must specialize in an intermediate input. As there are an infinite number of inputs, and each faces a downward sloping demand curve for its services, each worker will choose to specialize in a different intermediate input such that $r(i) = i$ and supply that input in the quantity $x_t^i = \nu h_t^i$. Thus aggregate output simplifies to

$$Y_t = \nu(\int_0^{\infty} h^{(\sigma-1)/\sigma} d\mu_t(h))^{\sigma/(\sigma-1)} \equiv \nu H_t \tag{2.18}$$

[42]In Benabou (1996), individuals choose how much time to spend work relative to educating their children so as to maximize the discounted value of future generations log of consumption (the dynastic utility function). Given the assumption of log preferences, all individuals choose the same $\nu$. They also choose a constant value of $\tau$. See the Appendix in Benabou (1996) for details.

where $\mu$ denotes the distribution of human capital in the entire labor force $\Lambda$. Note that the complementarity between inputs in the production function implies that a worker's earnings depend both on her own human capital and on an economy-wide index of human capital, $H_t$. That is, $y_t^i = \nu(H_t)^{1/\sigma}(h_t^i)^{(\sigma-1)/\sigma}$. This interdependence is also reflected in the per capita income of each community as $Y_t^i = \int_0^\infty y dm_t^i(y) = \nu(H_t)^{1/\sigma}(\int_0^\infty h^{(\sigma-1)/\sigma} d\mu_t^i(h)) \equiv \nu H_t^{1/\sigma}(L_t^i)^{(\sigma-1)/\sigma}$ where $\mu_t^i(h)$ is the distribution of human capital in the community $\Lambda_t^i$.

Incorporating the definitions above into the law of motion for the evolution of human capital (2.16) yields:

$$h_{t+1}^i = K\xi_t^i(h_t^i)^\alpha(L_t^i)^\beta(H_t)^\gamma \tag{2.19}$$

where $K = \kappa(1-\nu)^\delta(\nu\tau)^{1-\delta}$, $\alpha = \delta$, $\beta = (1-\delta)(\sigma-1)/\sigma$, and $\gamma = (1-\delta)/\sigma$. Note that this function exhibits constant returns to scale, i.e., $\alpha+\beta+\gamma = 1$ and that the law of motion incorporates a local linkage $L_t^i$ because education is funded by local funds, and a global linkage $H_t$ because workers (the inputs) are complementary in production.

The relative merits of a local versus a state system of education can be studied in this framework by comparing the benefits of a system in which individuals are completely segregated into homogeneous jurisdictions such that $L_t^i = h_t^i$ with one in which all communities are integrated and hence $L_t^i = H_t$.

Intuitively, the trade-off between the two systems is clear. On the one hand, complementarity and symmetry of inputs in production suggests that total output is maximized if individuals are homogeneous, pointing towards the benefits of a more homogenizing system such as a state-financed one. On the other hand, the fact that parental human capital and community resources are complements (i.e., the marginal return to an extra dollar spent on formal education is increasing in the level of parental human capital), suggests that at a local level assortative grouping of families is beneficial. The relative merits of the two systems, as we shall show, depend on the time horizon.

To analyze the pros and cons of the two systems, we need to derive the dynamic path of the economy under each education-finance policy. We do this under the assumption that the initial distribution of human capital at time $t$ is lognormal, i.e., $\ln h_t^i \sim N(m_t, \Delta_t^2)$. The cost of heterogeneity at both the local and global level then can be seen in that $H = (E[(h)^{(\sigma-1)/\sigma}])^{\sigma/(\sigma-1)} = e^{-\frac{\Delta^2}{2}}E[h] < E(h)$ and $L^i = e^{-\frac{\Delta^2}{2}}E[h^i] < E(h^i)$.[43]

[43] Recall that if $y \sim N(m, \Delta^2)$ and $y = \ln x$, then $x \sim \log normal$ with $E(x) = e^{m+\frac{\Delta^2}{2}}$ and $Var(x) =$

Noting that $\ln H_t = m_t + \frac{\Delta^2}{2\sigma}(\sigma - 1)$, the law of motion implied by (2.19) under a local finance regime, i.e., $h^i_{t+1} = K\xi^i_t(h^i_t)^{\alpha+\beta}(H_t)^\gamma$, implies that the distribution in the following period will also be lognormal with

$$\begin{aligned} m_{t+1} &= \ln K - s^2/2 + m_t + \gamma\frac{(\sigma-1)}{\sigma}\frac{\Delta^2_t}{2} \\ \Delta^2_{t+1} &= (\alpha+\beta)^2\Delta^2_t + s^2 \end{aligned} \tag{2.20}$$

Similarly, under a state-finance regime, (2.19) implies $\widehat{h}^i_{t+1} = K\xi^i_t(\widehat{h}^i_t)^\alpha(\widehat{L}_t)^{\beta+\gamma}$. Thus, if the initial distribution of human capital is described by $\ln \widehat{h}^i_t \sim N(\widehat{m}_t, \widehat{\Delta}^2_t)$, then $\ln \widehat{L}_t = \widehat{m}_t + \frac{\Delta^2}{2\sigma}(\sigma - 1)$ and next period's distribution of human capital is also lognormal with

$$\begin{aligned} \widehat{m}_{t+1} &= \ln K - s^2/2 + \widehat{m}_t + (\gamma+\beta)\frac{(\sigma-1)}{\sigma}\frac{\Delta^2_t}{2} \\ \widehat{\Delta}^2_{t+1} &= \alpha^2\widehat{\Delta}^2_t + s^2 \end{aligned} \tag{2.21}$$

(where ˆ is used to denote the state-finance regime).

We examine the implications of both regimes on per capita human wealth $A_t \equiv \int_0^\infty h d\mu_t(h)$. Under a local finance regime, $A_{t+1} = K\int_0^\infty h^{\alpha+\beta} d\mu_t(h) H^\gamma_t$, which, using (2.20) implies:

$$\ln\frac{A_{t+1}}{A_t} = \ln K - \left((\alpha+\beta)(1-\alpha+\beta) + \frac{\gamma}{\sigma}\right)\frac{\Delta^2_t}{2} \tag{2.22}$$

The first term represents the growth rate of a standard representative agent economy. When agents are heterogeneous in terms of their human capital, however, $\alpha + \beta < 1$, $\gamma < 1$, and Jensen's inequality imply $\int_0^\infty h^{\alpha+\beta} d\mu_t(h) < A_t^{\alpha+\beta}$ and $H^\gamma_t < A^\gamma_t$. These differences are reflected in the last term of (2.22) which captures the decrease in growth due to heterogeneity as a product of the current variance times a constant term that measures the economy's efficiency loss per unit of dispersion, $\Pi = \left((\alpha+\beta)(1-\alpha-\beta) + \frac{\gamma}{\sigma}\right)$. These losses reflect the concavity of the combined education production function $h^{\alpha+\beta}$ and the complementarity $1/\sigma$ of inputs in production which has weight $\gamma$ in the economy-wide aggregate $H$.[44]

$e^{2m+2\Delta^2} - e^{2m+\Delta^2}$. Furthermore, $[E(x^{(\sigma-1)/\sigma})]^{\sigma/(\sigma-1)} = e^{-\frac{\Delta^2}{2\sigma}}E(x)$.

[44]Note that the same reasoning implies that heterogeneity in human capital is a source of gain when

For the state-finance system, similar derivations yield

$$\ln \frac{\widehat{A}_{t+1}}{\widehat{A}_t} = \ln K - \left(\alpha\,(1-\alpha) + \frac{(\beta+\gamma)}{\sigma}\right) \frac{\widehat{\Delta}_t^2}{2} \tag{2.23}$$

and thus $\widehat{\Pi} = \left(\alpha\,(1-\alpha) + \frac{(\beta+\gamma)}{\sigma}\right)$. The interaction of heterogenous agents at the local level imposes a loss of $\beta/\sigma$ and the concavity of the parental human capital contribution to production function (i.e., $\alpha < 1$) implies losses from heterogeneity along with the usual losses stemming as before from the complementarity in production in the economy-wide aggregate $H$.

The analysis above implies that for **given** rates of resource and time investment in education, $\tau$ and $\nu$, in the short run a state-finance education system will lead to lower human capital accumulation than a local system. To see this, note that

$$\phi \equiv \Pi - \widehat{\Pi} = \beta\left(1 - 2\alpha - \beta - \frac{1}{\sigma}\right) = -\delta(1-\delta)(1-\frac{1}{\sigma^2}) < 0$$

implying that the drag on growth from heterogeneity is greater in a state-financed system. That is, two economies that start out with the same distribution of human capital in the first period will have a greater level of human capital in the second period under a local regime than under a state finance regime.

In the long-run, however, the conclusion is different. The handicap to growth from heterogeneity under a state regime tends to get reduced, as individuals have access to the same formal education system, whereas this source of heterogeneity is maintained under a local system in which education funding depends on family human capital. Solving for the long-run variances of the two systems given the same initial conditions yields: $\Delta_t^2 = \Delta_\infty^2 + (\alpha+\beta)^{2t}(\Delta^2 - \Delta_\infty^2)$ and $\widehat{\Delta}_t^2 = \widehat{\Delta}_\infty^2 + \alpha^{2t}(\Delta^2 - \widehat{\Delta}_\infty^2)$ where $\Delta_\infty^2 = \frac{s^2}{1-(\alpha+\beta)^2}$ and $\widehat{\Delta}_\infty^2 = \frac{s^2}{1-\alpha^2}$.

Note that we can write $\ln A_t$ as $\ln A_0 + t\ln K - \frac{\Pi}{2}\left((\Delta^2 - \Delta_\infty^2)\frac{1+(\alpha+\beta)^{2t}}{1-(\alpha+\beta)^2} + t\Delta_\infty^2\right)$ and similarly $\ln \widehat{A}_t = \ln A_0 + t\ln K - \frac{\widehat{\Pi}}{2}\left((\Delta^2 - \widehat{\Delta}_\infty^2)\frac{1+\alpha^{2t}}{1-\alpha^2} + t\widehat{\Delta}_\infty^2\right)$. Hence, taking the limit of these expressions as $t \to \infty$, we obtain that in the case of no uncertainty in which initial endowments are the only source of inequality (i.e., $s^2 = 0$), in the long run the

agents are substitutes in the production function or when the inputs of the community do not consist solely of education funds but also, say, peer effects that on aggregate imply increasing returns to scale in human capital at the local level.

two economies grow at the same rate (namely $\ln K$) and converge to a constant ratio of per capita human capital levels,

$$\ln \frac{\widehat{A}_\infty}{A_\infty} = \left( \frac{\Pi}{1-(\alpha+\beta)^2} - \frac{\widehat{\Pi}}{1-\alpha^2} \right) \frac{\Delta^2}{2} = \Phi \frac{\Delta^2}{2}$$

where $\Phi \equiv \left( \frac{\Pi}{1-(\alpha+\beta)^2} - \frac{\widehat{\Pi}}{1-\alpha^2} \right) = \left( \frac{\sigma}{2\sigma+\delta-1} \right) \left( \frac{1-\delta}{1+\delta} \right) \left( \frac{\sigma-1}{\sigma} \right)^2 > 0$.

If there is uncertainty in the generation of human capital, then for $t$ sufficiently large,

$$\ln \frac{\widehat{A}_t}{A_t} \approx \Phi \frac{s^2}{2} t$$

and the growth rate of the state-finance education system exceeds that of the local regime by $\Phi \frac{s^2}{2}$. Hence state-financing raises the long-run levels of human capital by $\Phi \frac{\Delta^2}{2}$ when there is no uncertainty and raises the long run growth rate of human capital by $\Phi \frac{s^2}{2}$ when there is uncertainty. Thus, in the long run a state system always does better. Whether a local or state education system is preferable will depend on how we discount different generation's welfare. For a sufficiently patient social planner, the state education system will be preferred.

## 3. Sorting into Schools

At some level it is possible simply to repeat much of the analysis of the preceding sections but refer to schools rather than neighborhoods. Obviously little additional insight would be gained by doing this. A topic which did not have a natural place in the previous section is how the possibility of attending a private rather than a public school matter.

Introducing private schooling in a model which includes public schooling is in general problematic since in these models the funding of public schools is usually decided by majority vote at the local level making it difficult to obtain existence of majority vote equilibrium.[45] The problem lies in the fact that those individuals who opt out of public schooling prefer (in the absence of externalities) to provide zero funding for private schools.

[45] See, though Epple and Romano (1996), Fernández and Rogerson (1995) and Glomm and Ravikumar (1998) for some related attempts.

Epple and Romano (1998) provide a model that allows one to study some of the interactions between the private and public provision of education in an economy where the demand for education depends both on ability and on income. They sidestep the problem of funding for education by assuming that the quality of a school depends only on the mean ability of its students. Although their theoretical results are somewhat incomplete given the difficulty of characterizing equilibria in an economy in which individuals differ in more than one dimension, their model nonetheless provides an extremely useful framework to begin thinking about sorting into schools.[46] The rest of this section is primarily dedicated to a discussion of their model.[47]

Consider an economy in which students are assumed to differ in ability $b$ and in income $y$. A school's quality is determined solely by the mean ability, $q$, of the student body. Student's care about the quality of the school as their utility depends on their achievement $a$, a function of their own ability $b$ and school quality. They also care about private consumption which will equal their income minus the price $p$ they pay for schooling. Public schools are free and financed (so that costs are covered) by proportional income tax rates, $t$. Letting $y_t$ denote after tax income, individuals maximize:

$$V = V\left(y_t - p, a(q,b)\right) \tag{3.1}$$

The authors characterize the equilibrium distribution of student types $(y, b)$ across public and private schools assuming that types are verifiable. Preferences are assumed to be single crossing in income in the $(q,p)$ plane, i.e., (2.2) holds. That implies that, for the same ability level, students with higher income will be willing to pay a higher price to attend a school with higher mean ability. Preference for quality is also assumed to be non-decreasing in ability; that is, $\frac{\partial\left(\left.\frac{dp}{dq}\right|_{V=\overline{V}}\right)}{\partial b} \geq 0$ .

All schools have the same cost function consisting of a fixed cost and an increasing, convex variable cost in the number $N$ of students $c(N)$. Public schools all offer the same quality schooling. The number of public schools simply minimizes the cost of operating the public sector which is financed by a proportional income tax on all households. Private-sector schools, on the other hand, maximize profits and there is free entry and

[46]Furthermore, for the interesting case of Cobb-Douglas preferences, their characterization holds as will be discussed later.

[47]See also Caucutt (forthcoming) for a discussion of how different policies matter when students sort (in a complex fashion) across schools by ability and income.

exit.

Private schools maximize profits taking as given the competitive utility $V^*(y,b)$ the student could obtain elsewhere. Schools can condition prices on ability and income. Thus, the profit maximization problem of a private school is to choose prices as a function of ability and income and the proportion of each type of student it wishes to admit (recognizing that there is a limit to the number of students of each type) taking into account the effects that these choices have on school quality and on cost via the types and number of students admitted.

The solution to private school's $j$'s maximization problem is characterized by a first order condition that, for an interior solution for that student type, equates the effective marginal cost of admitting the additional student $i$ of type $(b_i, y_i)$ to its reservation price. Note that when a school admits a student with ability $b_i$, its quality changes by $\frac{b_i - q_i}{N_j}$. The effective marginal cost of admitting this student is thus the increase in cost $c'(N)$ resulting from the fact that an additional student is being admitted minus the change in marginal revenue due to that student's effect on the school's quality.[48] The reservation price of a particular type of student is given by the maximum price $p_i^*$ the school can charge (given its quality) so as to leave the individual at her market utility. Note that this implies that some student types will not be admitted since their reservation price is too low to cover their effective marginal cost.

The equilibrium that emerges from this model has some nice properties.[49] As shown in Figure 2, there will be a strict hierarchy of school qualities $q_n > q_{n-1} > ... > q_0$, with the public sector (denoted by $j = 0$) having the lowest-ability peer group. Define the boundary loci between two schools as the set of types who are indifferent between the two schools (a curve with zero measure). Students who are on the boundary loci between two private schools will be charged their effective marginal costs; all other students will be charged strictly more than their effective marginal costs. This follows from the fact that students on the boundary are indifferent between attending either of the two schools competing for them, which drives down the price that each school can charge to that ability type's effective marginal cost. Furthermore, since a type's

[48]Thus the effective marginal cost can be negative for a relatively high-ability student leading to the possiblity of negative prices (e.g. fellowships) in equilibrium.

[49]"Epple, Newlon, and Romano (forthcoming) adapt this model to study ability tracking (or streaming) in public and private schools. Epple and Romano (1999) use a modified version of the model to study voucher design.

effective marginal cost is independent of income, their price will only depend on their ability. For students within the boundary loci, on the other hand, the fact that they are not indifferent over which school they attend leaves the school with some monopoly power which the school exploits by increasing the price. Hence, in general, the price charged to students within a school's boundary loci will depend both on ability and income. Note though that competition and free entry among schools implies that a school's profit is equal to zero.[50]

Lastly, it is also possible to characterize the type of students that will attend each school in equilibrium. The single-crossing condition in income ensures that if an individual with income $y_i$ attends a school with quality $q_j$, then all individuals with the same ability but greater income will attend schools of at least that level of quality and all individuals with lower income will attend schools with no greater quality.[51]

Thus, this model yields stratification by income. Stratification by ability need not follow, although the authors are able to find conditions (unfortunately on equilibrium variables) such that schools will also be stratified by ability.[52] Note that, as public schools have the lowest quality level, they will be composed of low-income individuals. If stratification by quality also holds, then public schools will consist of the lowest income and lowest ability students.

To understand the normative implications of the model, first suppose that no public option exists. Given the number of private schools, the allocation of types into schools is Pareto efficient. This is because private schools internalize the ability externality in their choices and there is perfect price discriminate over income income. The equilibrium number of school is not generally efficient, however, because the finite size of schools implies entry externalities. Furthermore, public sector schooling in this model in general implies Pareto inefficiency even given the equilibrium number of schools. Zero pricing by public schools independently of ability implies that the allocation of types

[50]As usual with model with fixed costs, free entry does not imply zero profits due to the integer problem. We will ignore that qualification here.

[51]This property of equilibrium does not follow immediately from single-crossing since schools can discriminate by types and thus a higher quality school may charge an individual with higher income a higher price. This behavior, however, will not disrupt income stratification because effective marginal cost depends only on ability and schools are sure to attract all types willing to paymore than effective marginal cost.

[52]In their working paper (1993), Epple and Romano show that for a Cobb-Douglas specification of utilty $U = (y_t - p)a(q, b)$ the equilibrium yields stratification by ability.

among public and private sector schools is inefficient.[53]

A very different issue in sorting into schools is studied by Fernández and Gali (1999). This paper is primarily interested in the properties of different assignment mechanisms under borrowing constraints. They examine a perfectly competitive model in which schools that vary in their (exogenous) quality each charge a market-clearing price to agents who vary in their ability and income. Schools have a fixed capacity and agents are assumed to be unable to borrow. In this model, the assumption that ability $a$ and school quality $q$ are complements in the production of output $x(a, q)$ implies that a social planner (or perfect capital markets) would assign the highest ability student to the highest quality school, the next highest ability student to the next highest quality school and so forth. A perfectly competitive pricing mechanism does not produce this outcome. Instead, lower ability but higher income individuals are able to outbid higher ability but lower income agents for a place in a high quality school.

This equilibrium outcome above is contrasted with an exam mechanism that assigns students to school based on their performance on the exam. The exam score is assumed to be an increasing function of expenditures on education (e.g., better preparation, tutors, etc.) and innate ability. The exam technology is such that the marginal increment in expenditure required to increase a given score is decreasing in ability.

The authors find that an exam mechanism will always produce greater output. However, as expenditures under an exam system are wasteful, aggregate consumption need not be higher. The authors show, nonetheless, that for a sufficiently powerful exam technology (one that is sufficiently sensitive to ability relative to expenditures), the exam mechanism will always dominate the market mechanism for both aggregate production and consumption.

## 4. Household Sorting

People sort not only into neighborhoods and schools, they also at the household level by deciding whom to "marry" or more generally who to match with. Although there is a small literature that analyzes the economics of matching (e.g. Becker (1973) and Burdett and Coles (1997)), there has been very little analysis, empirical or theoretical, of how

[53]See Epple and Romano (1998) for a fuller discussion.

this interacts with other general equilibrium variables such as growth and inequality.[54]

What are the consequences of household sorting for the transmission of education and inequality? Following Fernández and Rogerson (2001b), I will set down a rudimentary model that allows us to examine this issue. This model will leave exogenous several important features of the decision problem (such as who to match with and fertility), but it will simplify the analysis of key features of the transmission process.[55]

Consider an OLG model with two types of individuals–skilled ($s$) and unskilled ($u$)– in which the level of skill is also synonymous with the level of education (college and non-college respectively). These individuals meet, match, have children, and decide how much education to give each of their children.

Given a population at time $t$ whose number is given by $N_t$ and some division of that population into skilled workers, $N_{st}$, and unskilled workers, $N_{ut}$, where $N_t = N_{st} + N_{ut}$, let $\beta$ denote the fraction of the population that is skilled, i.e., $\beta_t = \frac{N_{st}}{N_t}$. Rather than endogenize matches, we assume an exogenous matching process in which a fraction $\theta$ of the population matches with probability one with someone of the same type, whereas the remainder match at random. As there are two types of individuals, this gives rise to three types of household matches indexed by $j$ which we shall denote by high ($h$) when it is between two skilled, middle ($m$) when the match is between a skilled and an unskilled, and low ($l$) between two unskilled.

The matching process specified above yields $\lambda_{ht} \equiv \theta\beta_t + (1-\theta)\beta_t^2$ as the fraction of matches that are high, $\lambda_{mt} \equiv 2(1-\theta)\beta_t(1-\beta_t)$ as the fraction of matches that are middle, and $\lambda_{lt} \equiv \theta(1-\beta_t) + (1-\theta)(1-\beta_t)^2$ as the fraction that are low. Of course, $\lambda_{ht} + \lambda_{mt} + \lambda_{lt} = 1$. Note that $\theta$ equals the correlation of partners' education levels.

Families have $n = \{0, 1, ...\overline{n}\}$ children. We allow the probability $\phi_{nj}$ with which they have a particular number $n$ to depend on the family type, so that average fertility $f$ for a family of type $j$ is given by $f_j = \sum_{n=0}^{\bar{n}} n\phi_{nj}$.

Children are either "college material" (whereupon if they went to college they would become a skilled worker) or they are not and sending them to college would still produce an unskilled worker. We denote these types as either high or low "aptitude" and allow

[54] Some exceptions are Cole, Mailath, and Postlewaite (1992) and Kremer (1997).

[55] See Fernández, Guner and Knowles (2000) and Fernández and Pissarides (2000) for models that endogenize several of these features.

the probability $\gamma_j$ that a child is of high aptitude to depend on her parental type.[56] If a high aptitude child is sent to college, she earns the skilled wage, $w_s$; otherwise she earns the unskilled wage $w_u$.

Lastly, we come to the education decision. We assume that the cost of college is given by $\nu > 0$. Capital and insurance markets are imperfect in that parents cannot borrow to finance the college education of their children but must finance it from their earnings. Insurance (as to which type of child a family might have) is also assumed not to be available. The assumption of not being able to borrow for a college education is not necessarily meant to be taken literally. Rather we have in mind the local primary and secondary education system described earlier whereby education is financed to a large extent at the local level and minimum lot sizes (or higher borrowing costs), for example, constrain the quality of education that less wealthy parents are able to give to their children.

Parents choose per family member consumption level $c$ and the number, $r$, of their high-aptitude children to educate so as to maximize the utility function below:

$$U = \begin{cases} (c - \overline{c}) & \text{for } c < \overline{c} \\ (c - \overline{c}) + \frac{r}{(2+n)} w_s + \frac{(n-r)}{(2+n)} w_u, & \text{otherwise} \end{cases} \tag{4.1}$$

implying that subject to a minimum per family member consumption level of $\overline{c}$, parents will send a high-ability child to college if they can afford to (and it is economically advantageous to do so). The family budget constraint is given by $(2+n)c + r\nu \leq I_j(\beta)$, $0 \leq r \leq a$, where $a$ is the total number of high aptitude children the family has, and

$$I_j(\beta) = \begin{cases} 2w_s(\beta) & \text{for } j = h \\ w_s(\beta) + w_u(\beta) & \text{for } j = m \\ 2w_u(\beta) & \text{for } j = l \end{cases} \tag{4.2}$$

Lastly, wages are determined in a competitive market as the appropriate marginal

[56] One should consider aptitude to reflect family background in the sense of making it more probable that a child will obtain a college education. It should be noted that this is not really standing in for a genetically determined process since in that case we would have to keep track of whether a particular match consisted of 0, 1, or 2 high-aptitude individuals.

revenue products of a constant returns to scale aggregate production function given by:

$$\begin{aligned} F(N_s, N_u) &= N_u F(N_s/N_u, 1) \equiv N_u F(\frac{\beta}{1-\beta}, 1) \equiv N_u f(\beta) \\ f' &> 0, \quad f'' < 0 \end{aligned} \tag{4.3}$$

Hence, wages are solely a function of $\beta$ and given by $w_s(\beta) = (1-\beta)^2 f'(\beta)$ and $w_u(\beta) = f(\beta) - \beta(1-\beta)f'(\beta)$. Note that (4.3) implies that skilled wages are decreasing in the ratio of skilled to unskilled workers whereas unskilled wages are increasing. Also note that no family would want to send their child to college if the fraction of skilled workers exceeds $\overline{\beta}$, where $\overline{\beta}$ is defined by $w_s(\overline{\beta}) = w_u(\overline{\beta}) + \nu$.

To solve for the steady states we need one additional piece of information, $\Gamma_j(z_j(\beta))$, the average proportion of children sent to college by families of type $j$. This will depend on how constrained each family is (this may differ according to family size and how many high aptitude children they have), $z_{nj}$, which in turn depends on wages and hence on $\beta$. Hence,

$$\Gamma_j(z_j(\beta)) \equiv \frac{1}{f_j} \sum_{n=1}^{\bar{n}} \phi_{nj} \left[ \sum_{a=0}^{z_{nj}} \binom{n}{a} \gamma_j^a (1-\gamma_j)^{n-a} a + \sum_{a=z_{nj}+1}^{n} \binom{n}{a} \gamma_j^a (1-\gamma_j)^{n-a} z_{nj} \right] \tag{4.4}$$

where the first summation term within the square brackets is the number of children that attend college from families of type j with $n$ children that are not constrained (as the number of high-aptitude kids they have is fewer than $z_{nj}$) and the second summation is over the number of children that attend college from constrained families of type $j$ with $n$ children.[57]

The steady states of the economy are the fixed points of the dynamic system below:

$$\beta_{t+1}(\theta) = \frac{N_{st+1}}{N_{t+1}} = \frac{\sum_j \Gamma_j(z_j(\beta_t)) f_j \lambda_{jt}(\beta_t;\theta)}{\sum_j f_j \lambda_{jt}(\beta_t;\theta)} \tag{4.5}$$

i.e., a level of $\beta$, such that $\widehat{\beta} = \beta_t = \beta_{t+1}$. We restrict our attention to those that are

[57] If a family of type $j$ with $n$ children is not constrained, we simply indicate this by $z_{nj} = n$.

locally stable, i.e., $\left.\frac{\partial \beta_{t+1}}{\partial \beta_t}\right|_{\beta_t=\widehat{\beta}} < 1$.

Note that in general there may be multiple steady states. To see why this is so, consider what may happen if we start out with a low level of $\beta$. In this case, low type families (and perhaps middle types as well) will be relatively constrained since unskilled wages are low. Thus, this will tend to perpetuate a situation in which $\beta$ is low next period as well and thus a steady state with a low proportion of unskilled wages (and high inequality). If, on the other hand, the economy started out with a high level of $\beta$, unskilled wages would be high and hence low type families would be relatively unconstrained, perpetuating a situation of high $\beta$ (and low inequality).

How does the degree of sorting affect this economy? If the change in sorting is sufficiently small that the degree to which constraints are binding is unaffected (i.e., the $\Gamma_j$'s are constant), then

$$\begin{aligned}\frac{d\widehat{\beta}}{d\theta} &= \frac{\widehat{\beta}(1-\widehat{\beta})[f_h(\Gamma_h-\widehat{\beta})-2f_m(\Gamma_m-\widehat{\beta})+f_l(\Gamma_l-\widehat{\beta})]}{D} \\ &= \frac{\widehat{\beta}(1-\widehat{\beta})[(f_h\Gamma_h-2f_m\Gamma_m+f_l\Gamma_l)-\widehat{\beta}(f_h-2f_m+f_l)]}{D}\end{aligned} \tag{4.6}$$

where $D = \sum_j f_j\lambda_j(\widehat{\beta};\theta) + \sum_j f_j \frac{\partial\lambda_j(\widehat{\beta};\theta)}{\partial\beta}(\widehat{\beta}-\Gamma_j)$. It is easy to show that local stability requires $D > 0$.

The expression in (4.6) is easy to sign for a few cases. Suppose all the $\Gamma_j$'s are the same, i.e., $\Gamma_j = \Gamma$. In that case, $\widehat{\beta} = \Gamma$ and the extent of sorting does not affect the personal income distribution (though it does the household income distribution) as wages are unchanged.[58]

Suppose next that average fertility is the same across all groups, i.e., $f_j = f$. In this case, the sign of (4.6) is given by the sign of $\Gamma_h + \Gamma_l - 2\Gamma_m$. The intuition behind this is simple. Note that the effect of an increase in sorting is to destroy middle-type matches and replace these by high and low ones. In particular, for every two middle matches destroyed, one high and one low match are created. Since average fertility is the same across family types, the effect of increased sorting depends on whether the fraction of children sent to college on average by two middle-type marriages ($2\Gamma_m$) is

[58]Recall that we are assuming that constraints are unaffected by the change in sorting.

smaller than the combined fraction of children that go to college on average in one high and one low type family ($\Gamma_h + \Gamma_l$). Thus, if the relationship between parents' education and children's education is linear, changes in sorting will have no effect on $\widehat{\beta}$; if concave, increased sorting will decrease $\widehat{\beta}$; and the reverse if the relationship is convex.

Lastly, making no assumptions about fertility or the $\Gamma_j$'s, a sufficient condition for an increase in sorting to decrease $\widehat{\beta}$ is $f_h\Gamma_h - 2f_m\Gamma_m + f_l\Gamma_l \leq 0$ and $f_h + f_l - 2f_m \geq 0$ (with at least one inequality strict). The first expression is the counterpart of the expression in the preceding paragraph. That is, subject to no change in population growth it ensures that there will be fewer skilled individuals in the following period. The second expression ensures that the population growth rate will not decline as a result of the increased sorting (thereby potentially giving rise to a larger *proportion* of skilled people despite the fall in their growth rate).[59]

The above discussion assumed that the $\Gamma_j$'s remained invariant to the change in sorting. Note, however, that these may well change as constraints become more or less binding as a result of the change in wages.[60] Hence, even if fertility is exogenous, the sign of $f_h\Gamma_h - 2f_m\Gamma_m + f_l\Gamma_l$ is in general endogenous since the $\Gamma_j$'s are endogenous variables.[61] Thus, whether the expression is concave or convex may itself depend on $\beta$.

Fernández and Rogerson (2001b) explore the effect of increased sorting on inequality by calibrating the model above to US data. They use the PSID to obtain a sample of parents and children and group all individuals with high school and below into the unskilled category and everyone who has had at least some college into skilled. The correlation of parental education ($\theta$) equals .6. Average fertility is given by $f_h = 1.84$, $f_m = 1.90$, and $f_l = 2.26$ (from PSID and Mare (1997)). For any average fertility number, the two integers that bracket the average are chosen as the only two possible number of children to have, with the appropriate weights used as the probabilities (e.g., $\phi_{1h} = .16$, $\phi_{2h} = .84$).

To calibrate the model, we need to know the $\gamma_j$'s. These are not available in the data but what is computable from the PSID are the $\Gamma_j$'s (i.e., the fraction of children

[59]The opposite signs on the two expressions is a sufficient condition for increased sorting to increase $\beta$.

[60]In a more general model where household incomes were continuous, then a change in $\theta$ that affected $\beta$ for a constant set of $\Gamma$'s, would also necessarily affect the $\Gamma_j$'s.

[61]In a more complex model in which fertility and/or matching are endogenized, then one can perform a similar exercise by changing technology such that the skill premium for any $\beta$ is higher or by changing the cost of search.

of each family type that on average attend college). These are given by $\Gamma_h = .81$, $\Gamma_m = .63$, and $\Gamma_l = .30$. Note from (4.4) that any value of $\Gamma_j$ can be decomposed into an assumption about how "inheritable" education is (the $\gamma_j$'s) and a corresponding assumption about how binding borrowing constraints are (the $z_{nj}$'s). The table below shows various such decompositions for $\Gamma_l$ (for the other $\Gamma_j$'s it is assumed that the constraints are not binding and hence $\Gamma_j = \gamma_j$).

Table 2

Aptitude Profiles Under Various Scenarios $z_{nm} = n, z_{nh} = n$

| | $z_{nl} = n$ | $z_{nl} = 2$ | $z_{2l} = 2, z_{3l} = 1$ | $z_{nl} = 1$ |
|---|---|---|---|---|
| $\gamma_h$ | .81 | .81 | .81 | .81 |
| $\gamma_m$ | .63 | .63 | .63 | .63 |
| $\gamma_l$ | .30 | .303 | .334 | .401 |

Fernández and Rogerson (2001b) use the second column as their benchmark. Note that this implies the existence of very mild constraints. Only low-type families with three high-ability children are affected and these are fewer than 1 percent of low-type families.

This information along with the $\Gamma_j$'s allows us to compute the steady state, yielding $\widehat{\beta} = .60$. To obtain wages, we use a CES production function $y = A[bN_s^{\rho} + (1 - b)N_u^{\rho}]^{\frac{1}{\rho}}$ and match the steady-state ratio of skill to unskilled wages to 1.9 (Katz and Murphy (1992)) and obtain $\rho = .33$ by matching an elasticity of substitution between skilled and unskilled workers of 1.5 (see survey by Katz and Autor (1999)). Lastly, for ease of interpretation of our results, we choose a value of $A$ to scale steady-state unskilled wages to some "reasonable" value, which we set to be $30,000$. This is purely a normalization.

It is important to note that the steady-state of the calibrated model fulfills the sufficient conditions such that an increased $\theta$ leads to a lower proportion of skilled individuals. Hence, from a theoretical perspective, we know that an increase in sorting will lead to higher skilled waged and lower unskilled ones. The quantitative impact is given in the table below. The first row reports mean years of education (in which the skilled group and unskilled group have been assigned the mean from their PSID sample). The second row gives the coefficient of variation of education. The last entry is the standard deviation in log income–our measure of inequality in the personal income distribution.

The first column of the table reports the result of the calibration.[62] The second column reports the effect of an increase in sorting to .7 assuming that the values of $\Gamma$ are unchanged. The third column does the same but assumes that the decrease in the unskilled wage means constraints are tightened for low-type families and that those with three children can only afford to send a maximum of one of them to college.

Table 3

Effects of Increased Sorting on Steady State

| | $\theta = .6$ | $\theta = .7$ | |
|---|---|---|---|
| | $\Gamma_l = .30$ | $\Gamma_l = .30$ | $\Gamma_l = .27$ |
| $mean(e)$ | 13.52 | 13.48 | 13.40 |
| $cv(e)$ | .134 | .135 | .137 |
| $\widehat{\beta}$ | .600 | .589 | .568 |
| $w_s/w_u$ | 1.900 | 1.95 | 2.07 |
| $std(log\ y)$ | .315 | .330 | .361 |

The main message of the table above is that changes in sorting can have large effects on inequality and that seemingly small changes in average years of education or in its coefficient of variation can underlie large changes in the income distribution. As shown in the table, a change in sorting from .6 to .7 will increase the standard deviation of log income by a bit under 5 percent in the absence of any assumption about borrowing constraints.[63] If as a result of the approximately $600 drop in $w_u$ that results (and consequently a $1200 drop in low-type family income) constraints tighten, this leads to an increase in inequality of almost 15 percent. In both cases, the effect on the standard deviation of log family income is large: 8.3 percent and 19 percent respectively.[64]

[62] Note that the standard deviation of log income is about half of what it is in reality for the US. It is not surprising that our model is not able to produce as much variation as in the data as there are only two wages.

[63] Note that these results, therefore, are independent of which column we choose from Table ?? as our benchmark.

[64] The results for the $\theta$ increase to .8 follow a pattern similar to the one above. The change in the mean and standard deviation of the education distribution are small, as before but the change in income distribution are large.

The analysis above also points out the dangers with assuming intergenerational processes are linear. Kremer (1997), for example, assumes that the years of education a child acquires is a linear function of average parental years of education as given by:

$$e_{i,t+1} = \kappa + \alpha \frac{(e_{i,t} + e_{i',t})}{2} + \xi_i \tag{4.7}$$

where $e_{i,t+1}$ is the education level for the child, $e_{i,t}$ and $e_{i',t}$ are the education levels of the two parents, and $\xi$ is a normally distributed random shock that is *iid* across families, with mean 0 and standard deviation equal to $\sigma_\xi$. Parents are all assumed to have two kids and an (exogenous) assortative matching of individuals takes place yielding $\theta$ as the correlation between the education levels of parents.

Note that within the framework of Fernández and Rogerson (2001b), the assumptions of a linear transmission process and the same fertility across all parent types would yield no effect of an increase in sorting on inequality. In Kremer's model, this is not the case as although the mean of the distribution is unaffected, the inclusion of a shock implies that greater sorting will increase inequality. To see this, note that with constant parameter values the distribution of education converges to a normal distribution with steady state mean and standard deviation given by $\mu_\infty = \frac{\kappa}{1-\alpha}$ and

$$\sigma_\infty = \frac{\sigma_\xi}{[1 - \alpha^2(1+\theta)/2]^{.5}} \tag{4.8}$$

respectively. Thus an increase in $\theta$ while not affecting the mean, increases the variance of the distribution of education.

To investigate the effects of sorting within this model, Kremer uses PSID data to run the regression suggested by (4.7), and finds $\alpha$ equals .4. Parents' correlation in years of education, as we saw previously is .6. This implies, using (4.8), that even a large increase in the correlation of parental education, say from .6 to .8 will only increase the standard deviation of the distribution of education by about 1 percent. Furthermore, if we assume as Kremer does that log earnings are linear in years of education (i.e., $y_{i,t+1} = a + be_{i,t+1}$), then exactly the same conclusion applies to the distribution of earnings.

The very different conclusions obtained by Kremer relative to Fernández and Rogerson emphasize that importance of certain features of the data (i.e. fertility differentials and non-convexities in the transmission process) as well as the endogeneity of wages. Furthermore, as shown in Fernández and Rogerson, borrowing constraints can greatly multiply the magnitude of any effect of increased sorting that takes the shape of the transmission process as given, rather than endogenous.

In light of the above, it is of interest to ask how inequality, fertility and sorting are related in a model in which these variables are endogenous. Fernández, Guner and Knowles (2000) develop a simple two-period search model in which individuals are given multiple opportunities to match with others. As before, there are two types of individuals (skilled and unskilled) distinguished only by their educational attainment. In the first period we assume that agents meet others from the population in general. In the second period agents meet only others who are similar to themselves in terms of skill level.[65] Agents characteristics (income) are fully observable, as is the quality of the match. The latter is assumed to be a random draw from a quality distribution, and is fully match specific. If agents decide to keep their first period match, they are unable to search in the second period.

Having matched, individuals decide how many children to have (at a cost per child $t$ that is proportional to income $I$) and devote the rest of their income to consumption. Thus individuals maximize:

$$\max_{c,n}\left[c+\gamma\log(n)+K+q\right], \tag{4.9}$$

subject to

$$c \leq I(1-tn),\ t>0,$$

where $n$ is the number of children, $I > \gamma$ is household income, $q$ is the quality of the match and $K$ is a constant. Plugging in the optimal decisions for an individual (and choosing $K$ such that the sum of the constants is zero) allows us to express the indirect utility function as $V(I,q) = I - \gamma\log I + q$.

Assuming a constant returns production function allows us as before to express wages solely as a function of the ratio of skilled to unskilled workers and to express household income as in (4.2). The cutoff match quality that a high wage worker will accept in order to match with a low wage individual in the first period is an increasing

[65] One could just as easily simply assume that the first period one meets a more representative sample of the population relative to the second period in which it is biased towards individuals who are similar.

function of $w_s$ and a decreasing function of $w_u$.[66]

Children face two costs to becoming a skilled worker. First, there is a constant monetary cost of $d$. Second, there is an individual-specific (additive) psychic cost (e.g., effort) of $\delta$ with a cumulative distribution $\Psi(\delta)$. The return to being a skilled worker is the probability of matching with a skilled worker and obtaining household income $I_{ss}$ (in which wages are assumed to be net borrowing and repaying $d$) plus the probability of matching with an unskilled worker and obtaining household income $I_{su}$. These probabilities depend on the probability that in the first period a particular type of worker is met and on the cutoff quality of the match a skilled worker will accept (and hence on the fraction of individuals that are skilled in the population, i.e., $\beta$). A similar calculation holds for the return to being an unskilled worker.[67]

If there were no borrowing constraints, then all families would have the same fraction of children become skilled so that the net return to being a skilled worker equalled that the return to being an unskilled worker plus $\delta^*(\beta)$ (the equilibrium psychic cost such that no worker with $\delta_i > \delta^*(\beta)$ is willing to become skilled). If, however, there are borrowing constraints such that the amount that an individual can borrow depends (positively) on family income, then families with higher household income will have a higher fraction of their children become skilled.

How does inequality matter? It is easy to show that as family income increases, fertility declines. Thus fertility differentials are increasing with inequality. Furthermore, as wage inequality increases, skilled workers become pickier about the quality of the match required to make them willing to match with an unskilled worker.

As before, this model will in general have multiple steady states. If the economy starts out with a low proportion of skilled workers, the skill premium will be high, skilled workers will be very picky about matching with unskilled workers, and hence there will be a high level of sorting. Given borrowing constraints, only a small fraction of children from low-income households will become skilled implying that in the next period a similar situation will tend to perpetuate itself–a high level of inequality, high sorting, and high fertility differentials. The opposite would be true if instead the economy starts out with a high level of skilled workers. In this case inequality is low,

[66]The skilled worker will always be the one whose cutoff quality level is binding as her income is greater.

[67]Note that unlike Fernández and Rogerson (2001b), the return to being skilled/unskilled depends also on how this decision affects the type of match one will obtain.

high-skilled agents choose a low cutoff quality for matching with unskilled agents so sorting is low, fertility differentials are low, and borrowing constraints are not very binding. This leads again to a high proportion of skilled workers the following period.

We take the implications of this model to the data. Using a sample of thirty three countries we examine the relationship between sorting and inequality and find that, as the theory predicts, these are positively correlated. Countries with greater inequality exhibit greater sorting at the household level. Furthermore, as also predicted by the theory, fertility differentials are increasing in inequality.[68]

## 5. Concluding Remarks

This chapter has reviewed some of the principal contributions to the literature that examines the links between sorting, education and inequality. Much work remains to be done in all of the areas discussed in this chapter: education finance systems and residential sorting, schools, and household sorting. In particular, it would be of interest to see more work that examined how different education systems matter, and provided an empirical basis on which to assess different policy proposals. At the school level, very little is known about how parents, teachers, students, administrators and the community interact in producing schooling of a particular quality. I think that the largest challenge here is the creation of a convincing multiple principal-agent model that endogenizes the quality of the school in response to information constraints, the availability of alternative options, and the system in which it is embedded. In addition, it would be of interest to study the incentive effects of external standards (e.g. national or state level exams) that allow schools to be "graded" against one another. Finally, work on household sorting is still at an embryonic level, both theoretically and empirically.[69] A notable omission form the models discussed above is the role of gender: they do not distinguish between the education and income distributions of men and women. It would be of interest to examine how these matter and to investigate, empirically and theoretically, the role of woman's large increase in labor force participation and educational attainment.

[68]Kremer and Chen (1999) examine the relationship between fertility and inequality for a large sample of countries and find that fertility differentials and inequality are positively correlated.

[69]See Greenwood, Guner and Knowles (1999) for recent work in this field.

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