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ABSTRACT

This study provides an organized list of core mathematics standards and benchmarks that are recognized by most or all of the states in the central region which includes Colorado, Kansas, Missouri, Nebraska, North Dakota, South Dakota, and Wyoming. State standards were reviewed against the reference set of standards and benchmarks to determine what content from the reference document was commonly valued among the seven states in the central region. This report contains the table of benchmark grade range used by the seven central region states and lists exemplary mathematics standards commonly/rarely found in the central region. (KHR)

EXEMPLARY MATHEMATICS STANDARDS AMONG THE SEVEN STATES IN THE CENTRAL REGION

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INTRODUCTION

PURPOSE

This study provides an organized list of core mathematics standards and benchmarks that are recognized by most or all of the states in the Central Region (comprised of Colorado, Kansas, Missouri, Nebraska, North Dakota, South Dakota, and Wyoming) and that are found in standards documents that have been rated as exemplary by national organizations. In short, this list of standards and benchmarks embodies content that is highly valued nationally as well as within the Central Region. Current school reform efforts create a significant demand for curriculum materials that help students achieve specific and worthwhile standards. A shared set of standards and benchmarks should provide educators with a focus for their efforts to find or contribute high-quality materials that support exemplary standards and benchmarks.

METHOD

To ensure that the list of standards and benchmarks ultimately produced by this study would indeed be exemplary, analysts selected as the reference document a McREL study that identified only highly rated mathematics standards and benchmarks (Kendall, Snyder, Schintgen, Wahlquist, & Marzano, 1999). That project identified the mathematics content present in documents from a handful of states that were highly rated by national organizations for the quality of their standards. It was reasoned that such a list of content represented what reputable national organizations (specifically, the American Federation of Teachers, the Council for Basic Education, and the Fordham Foundation) deemed to be model content in mathematics. State standards were reviewed against the reference set of standards and benchmarks to determine what content from the reference document also was commonly valued among the seven states in the Central Region. The following state standards documents were examined:

- *Colorado Model Content Standards for Mathematics* (1995, June)
- *Kansas Curricular Standards for Mathematics* (1999, March)
- *Missouri's Framework for Curriculum Development in Mathematics: K–12* (1996)
- *Nebraska K–12 Mathematics Standards* (1998, February)
- *North Dakota Standards and Benchmarks: Content Standards: Mathematics* (1999)
- South Dakota Content Standards: Mathematics (1998, December)
- Wyoming Mathematics Content and Performance Standards (1998, June)

The process of comparison first was undertaken independently by two analysts. Each analyst indicated which states addressed the same content as that found in the reference document. Thus, for any benchmark in the reference document, anywhere from zero to all seven states were identified as sharing the same content. In the course of content analysis, benchmarks were mapped within two grades of the grade range of the reference document. (For further discussion, see the "Organization of Content" section.)

When both analyses were completed, a third analyst then compared the reviews in order to resolve discrepancies that were significant, specifically, those discrepancies that would have a bearing on whether the identified content would be included in the final list of commonly found standards and benchmarks. (The threshold for inclusion of content in the list was determined after a preliminary review of the results, which is discussed in the following section.) If the disparity between raters on any item was greater than three states (out of the seven states reviewed), however, the original raters consulted regarding that item in order to resolve the disparity to within at least two states, before it was submitted for resolution by the third analyst, if necessary.

THRESHOLD FOR INCLUSION

A preliminary comparison of the two independent reviews of state standards against the reference document indicated that if the list of standards and benchmarks was restricted to only content found to be common across every state, then the number of benchmarks identified would be fairly small and not very informative. However, if the content to be identified was expanded to content common to *any five of seven* states, then the number of benchmarks was considerably expanded and likely to be of greater use. Specifically, just 13 benchmarks were found to be common to all seven states in the preliminary analysis, while 68 benchmarks, after the final analysis and resolution of discrepant ratings, were common among any five of the states. Of these benchmarks, just 10, or 15 percent of the total, are common to five states; most, or 85 percent, are common to either six or all seven states. The threshold was not further lowered to content that might be present in *any four of seven states*. The fact that content is present in a simple majority of the states was considered insufficient to argue that the content is truly representative of the states in the region.

ORGANIZATION OF CONTENT

The resulting list of exemplary benchmarks commonly found in the Central Region begins on page 3. The benchmarks are organized by the standards used in the reference document. These standards do not differ significantly from the kinds of standards found in most mathematics standards documents, including the state documents reviewed here. These standards serve as useful organizers of information; the content of primary interest is found in the benchmarks. Benchmarks are presented in the grade ranges of K–2, 3–5, 6–8, and 9–12, which is consistent with the structure of the reference document. Table 1 shows the variation of grade ranges used in the seven state documents.

In the course of content analysis, benchmarks were mapped to no more than two grades from the range of the reference document; the content always was mapped to the most appropriate developmental level.

ADDITIONAL FINDINGS

The preliminary comparison of the independent reviews revealed other information that might be of interest to educators in the region. Specifically, a number of benchmarks present in the reference document (i.e., benchmarks identified as important mathematics content in highly rated

standards documents) were found to be either missing from all state documents or present in just one or two of them. These benchmarks are listed beginning on page 10.

Table 1. Benchmark Grade Ranges Used by the Seven Central Region States

STATE DOCUMENT	BENCHMARK GRADE RANGE
Colorado Model Content Standards for Mathematics	K–4, 5–8, 9–12
Kansas Curricular Standards for Mathematics	K–2, 3–4, 5–7, 8–10
Missouri's Framework for Curriculum Development in Mathematics: K–12	K–4, 5–8, 9–12
Nebraska K–12 Mathematics Standards	K–1, 4, 8, 12
North Dakota Standards and Benchmarks: Content Standards: Mathematics	K–4, 5–8, 9–12
South Dakota Content Standards: Mathematics	K, 1, 2, 3, 4, 5, 6, 7, 8, 9–12
Wyoming Mathematics Content and Performance Standards	K–4, 5–8, 9–11

EXEMPLARY MATHEMATICS STANDARDS COMMONLY FOUND IN THE CENTRAL REGION

Seven state standards documents were compared against a set of exemplary standards in mathematics. The benchmarks presented on the following pages were found to be present in highly rated standards documents and in standards documents of at least five of seven states in the Central Region. These benchmarks are organized by the following standards:

1. Uses a variety of strategies in the problem-solving process
2. Understands and applies basic and advanced properties of the concepts of numbers
3. Uses basic and advanced procedures while performing the processes of computation
4. Understands and applies basic and advanced properties of the concepts of measurement
5. Understands and applies basic and advanced properties of the concepts of geometry
6. Understands and applies basic and advanced concepts of statistics and data analysis
7. Understands and applies basic and advanced concepts of probability
8. Understands and applies basic and advanced properties of functions and algebra

1. Uses a variety of strategies in the problem-solving process

Grades K–2

- Uses whole number models (e.g., pattern blocks, tiles, or other manipulative materials) to represent problems

Grades 3–5

- *No commonalities found among five or more states*

Grades 6–8

- *No commonalities found among five or more states*

Grades 9–12

- Constructs logical verifications or counter examples to test conjectures and to justify algorithms and solutions to problems (i.e., uses deductive reasoning)

2. Understands and applies basic and advanced properties of the concepts of numbers

Grades K–2

- Counts whole numbers (i.e., both cardinal and ordinal numbers)
- Understands symbolic, concrete, and pictorial representations of numbers (e.g., written numerals, objects in sets)
- Understands basic whole number relationships (e.g., 4 is less than 10, 30 is 3 tens)

Grades 3–5

- Understands the relationships among fractions, decimals, mixed numbers, and whole numbers
- Understands the basic meaning of place value
- Understands the relative magnitude of whole numbers, fractions, decimals, and mixed numbers

Grades 6–8

- Understands the relationships among equivalent number representations (e.g., whole numbers, positive and negative integers, fractions, ratios, decimals, percents, scientific notation, exponentials) and the advantages and disadvantages of each type of representation
- Understands the characteristics and properties (e.g., order relations, relative magnitude, base-ten place values) of the set of rational numbers and its subsets (e.g., whole numbers, fractions, decimals, integers)
- Understands basic number theory concepts (e.g., prime and composite numbers, factors, multiples, odd and even numbers, square numbers, roots, divisibility)
- Understands the concepts of ratio, proportion, and percent and the relationships among them

Grades 9–12

- Understands the properties (e.g., relative magnitude, density, absolute value) of the real number system and its subsystems (e.g., irrational numbers, natural numbers, integers, rational numbers)

3. Uses basic and advanced procedures while performing the processes of computation

Grades K–2

- Adds and subtracts whole numbers
- Understands the inverse relationship between addition and subtraction

Grades 3–5

- Adds, subtracts, multiplies, and divides whole numbers and decimals
- Adds and subtracts simple fractions
- Uses specific strategies (e.g., front-end estimation, rounding) to estimate computations and to check the reasonableness of computational results
- Understands the properties of and the relationships among addition, subtraction, multiplication, and division (e.g., reversing the order of two addends does not change the sum; division is the inverse of multiplication)
- Understands factors and prime numbers

Grades 6–8

- Adds, subtracts, multiplies, and divides whole numbers, fractions, decimals, integers, and rational numbers
- Uses proportional reasoning to solve mathematical and real-world problems (e.g., involving equivalent fractions, equal ratios, constant rate of change, proportions, percents)
- Understands the properties of operations with rational numbers (e.g., distributive property, commutative and associative properties of addition and multiplication, inverse properties, identity properties)

Grades 9–12

- *No commonalities found among five or more states*

4. Understands and applies basic and advanced properties of the concepts of measurement

Grades K–2

- Understands the concept of time and how it is measured
- Knows processes for telling time, counting coins, and measuring length, weight, and temperature, using basic standard and nonstandard units

- Makes quantitative estimates of familiar linear dimensions, weights, and time intervals and checks them against measurements

Grades 3–5

- Understands the basic measures perimeter, area, volume, capacity, mass, and circumference
- Selects and uses appropriate units of measurement, according to type and size of unit
- Selects and uses appropriate tools for given measurement situations (e.g., rulers for length, measuring cups for capacity)

Grades 6–8

- Solves problems involving perimeter (circumference) and area of various shapes (e.g., parallelograms, triangles, circles)
- Selects and uses standard and nonstandard units and tools, depending on degree of accuracy required, to find measurements for real-world problems
- Uses measurement formulas (e.g., to calculate area, volume, surface area)
- Solves problems involving units of measurement and converts answers to a larger or smaller unit within the same system (i.e., standard or metric)

Grades 9–12

- Solves real-world problems involving three-dimensional measures (e.g., volume, surface area)

5. Understands and applies basic and advanced properties of the concepts of geometry

Grades K–2

- *No commonalities found among five or more stat*es

Grades 3–5

- Knows basic geometric language for describing and naming shapes (e.g., trapezoid, parallelogram, cube, sphere)
- Understands basic properties of figures (e.g., two- or three-dimensionality, symmetry, number of faces, type of angle)
- Understands that shapes can be congruent or similar
- Understands characteristics of lines (e.g., parallel, perpendicular, intersecting) and angles (e.g., right, acute)

Grades 6–8

- Understands the defining properties of three-dimensional figures (e.g., a cube has edges with equal lengths, faces with equal areas and congruent shapes, right-angle corners)

- Understands geometric transformations of figures (e.g., rotations, translations, dilations)

- Understands the mathematical concepts of similarity (e.g., scale, proportion, growth rates) and congruency

Grades 9–12

- Uses synthetic (i.e., pictorial) representations and analytic (i.e., coordinate) methods to solve problems involving symmetry and transformations of figures (e.g., problems involving distance, midpoint, and slope; determination of symmetry with respect to a point or line)

- Understands the basic concepts of right triangle trigonometry (e.g., basic trigonometric ratios such as sine, cosine, and tangent)

- Uses inductive and deductive reasoning to make observations about and to verify properties of and relationships among figures (e.g., the relationships among interior angles of parallel lines cut by a transversal)

- Uses properties of and relationships among figures to solve mathematical and real-world problems (e.g., uses the property that the sum of the angles in a quadrilateral is equal to 360 degrees to square up the frame for a building; uses understanding of arc, chord, tangents, and properties of circles to determine the radius given a circular edge of a circle without the center)

- Understands that objects and relations in geometry correspond directly to objects and relations in algebra (e.g., a line in geometry corresponds to a set of ordered pairs satisfying an equation of the form $ax + by = c$)

- Uses the Pythagorean Theorem and its converse and properties of special right triangles (e.g., 30°-60°-90° triangle) to solve mathematical and real-world problems

6. Understands and applies basic and advanced concepts of statistics and data analysis

Grades K–2

- Understands that observations about objects or events can be organized and displayed in simple graphs

Grades 3–5

- Organizes and displays data in simple bar graphs, pie charts, and line graphs

- Reads and interprets simple bar graphs, pie charts, and line graphs

Grades 6–8

- Understands basic characteristics of measures of central tendency (i.e., mean, mode, median)

- Uses data and statistical measures for a variety of purposes (e.g., formulating hypotheses, making predictions, testing conjectures)

- Organizes and displays data using tables, graphs (e.g., line, circle, bar), frequency distributions, and plots (e.g., stem-and-leaf, box-and-whiskers, scatter)

Grades 9–12

- Understands measures of central tendency and variability (e.g., standard deviation, range, quartile deviation) and their applications to specific situations)

- Understands different methods of curve-fitting (e.g., median-fit line, regression line) and various applications (e.g., making predictions)

7. Understands and applies basic and advanced concepts of probability

Grades K–2

- *No commonalities found among five or more states*

Grades 3–5

- *No commonalities found among five or more states*

Grades 6–8

- Determines probability using mathematical/theoretical models (e.g., table or tree diagram, area model, list, counting procedures, sample space)

- Determines probability using simulations or experiments

- Understands how predictions are based on data and probabilities (e.g., the difference between predictions based on theoretical probability and experimental probability)

Grades 9–12

- *No commonalities found among five or more states*

8. Understands and applies basic and advanced properties of functions and algebra

Grades K–2

- Recognizes regularities in a variety of contexts (e.g., events, designs, shapes, sets of numbers)

- Extends simple patterns (e.g., of numbers, physical objects, geometric shapes)

Grades 3–5

- Recognizes a wide variety of patterns (e.g., basic linear patterns such as [2, 4, 6, 8 . . .]; simple repeating and growing patterns) and the rules that explain them
- Knows that a variable is a letter or symbol that stands for one or more numbers
- Solves simple open sentences involving operations on whole numbers (e.g., _+ 17 = 23)

Grades 6–8

- Knows that an expression is a mathematical statement using numbers and symbols to represent relationships and real-world situations (e.g., equations and inequalities with or without variables)
- Understands various representations (e.g., tables, graphs, verbal descriptions, algebraic expressions, Venn diagrams) of patterns and functions and the relationships among them
- Solves linear equations using concrete, informal, and formal methods (e.g., using properties, graphing ordered pairs, using slope-intercept form)

Grades 9–12

- Uses a variety of models (e.g., written statement, algebraic formula, table of input-output values, graph) to represent functions, patterns, and relationships
- Understands the general properties and characteristics of many types of functions (e.g., direct and inverse variation, general polynomial, radical, step, exponential, logarithmic, sinusoidal)
- Uses a variety of methods (e.g., with graphs, algebraic methods, and matrices) to solve systems of equations and inequalities

EXEMPLARY MATHEMTICS STANDARDS RARELY FOUND IN THE CENTRAL REGION

Seven state standards documents were compared against a set of exemplary standards in mathematics. This section provides a list of benchmarks that were found in the reference document of exemplary standards, but appeared in just two or fewer state standards documents in the Central Region. These benchmarks are organized by the following standards:

1. Uses a variety of strategies in the problem-solving process
2. Uses basic and advanced procedures while performing the processes of computation
3. Understands and applies basic and advanced properties of the concepts of measurement
4. Understands and applies basic and advanced properties of functions and algebra

1. Uses a variety of strategies in the problem-solving process

Grades K–2

- Justifies the process he or she used to solve a numerical problem
- Makes organized lists, tables, or charts to solve a problem
- Uses "guess and check" to solve problems

Grades 6–8

- Formulates a problem, determines information required to solve the problem, chooses methods for obtaining this information, and sets limits for acceptable solutions

Grades 9–12

- Uses a variety of strategies (e.g., identifies a pattern, uses equivalent representations) to understand new mathematical content and to develop more efficient solution methods or problem extensions

2. Uses basic and advanced procedures while performing the processes of computation

Grades 6–8

- Knows when an estimate is more appropriate than an exact answer for a variety of problem situations

3. Understands and applies basic and advanced properties of the concepts of measurement

Grades 3–5

- Measures elapsed time to the nearest minute

4. Understands and applies basic and advanced properties of functions and algebra

Grades 9–12

- Understands basic concepts (e.g., roots) and applications (e.g., determining cost, revenue, and profit situations) of polynomial equations
- Understands formal notation (e.g., sigma notation, factorial representation) and various applications (e.g., compound interest) of sequences and series

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