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ABSTRACT

This publication features articles that illustrate how several Northwest teachers are using problem solving to achieve rigorous and imaginative learning in their classrooms. Articles include: (1) "Open-Ended Problem Solving: Weaving a Web of Ideas" (Denise Jarrett); (2) "Teenager or Tyke, Students Learn Best by Tackling Challenging Math" (Suzie Boss); (3) "Want to Gain Parents' Trust? Let Them in on the Action" (Rosemary Shinohara); (4) "Reflections: Confessions of a Former Mathphobe" (Judith Carter); (5) "Connected Corner: Technology Tips" (Judi Mathis Johnson); (6) "Classroom Resources: A Library of Materials" (Amy Sutton); (7) "Discourse: Integrating Problem Solving and Projects" (Louis Nadelson); and (8) "Inquiry: A Way of Seeing" (Chris Sanchez). (KHR)

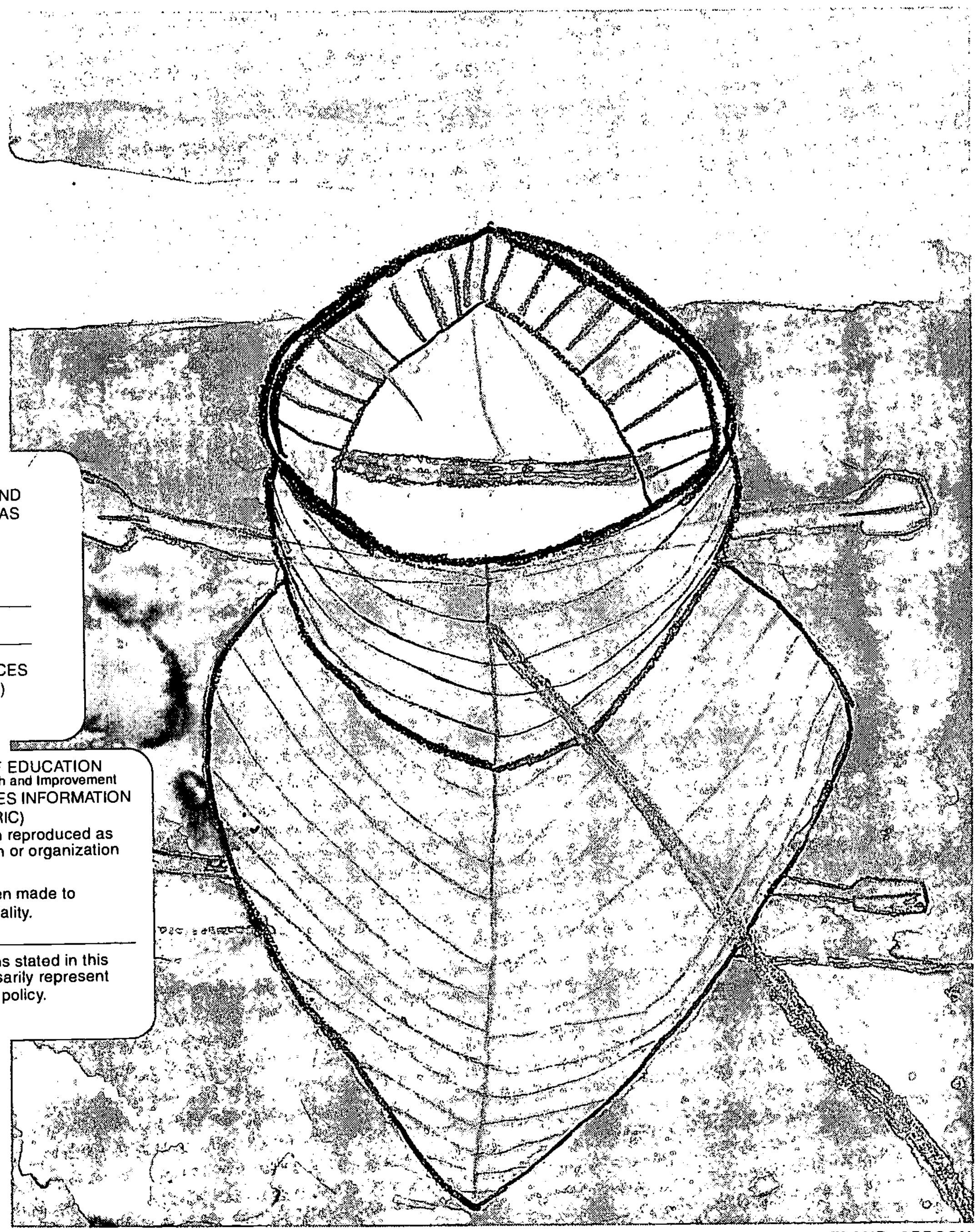
northwest

TEACHER

VOLUME 1 NUMBER 1

SPRING 2000

A MATH AND SCIENCE JOURNAL
DEVOTED TO RIGOROUS AND IMAGINATIVE LEARNING



REFLECTING ROWBOAT PAINTED BY LIZETH RODRIGUEZ, 9, OF PORTLAND, OREGON.

2

PROBLEM SOLVING: GETTING TO THE HEART OF MATHEMATICS

IDEAS CREATE A WEB OF UNDERSTANDING ■ EMBRACING CHANGE: INVOLVING STUDENTS IN CHALLENGING MATH ■ HOW TO WIN PARENTS' TRUST
TECHNOLOGY AND OTHER RESOURCE TIPS

Northwest Regional Educational Laboratory

NORTHWEST

TEACHER

A Math and Science Journal Devoted to Rigorous and Imaginative Learning

VOLUME 1 NUMBER 1

SPRING 2000

contents

PROBLEM SOLVING: GETTING TO THE HEART OF MATHEMATICS

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ON THE COVER:

This painting of a moored rowboat, with its reflection spreading across blue waters, is by Lizeth Rodriguez, 9, a student at Woodlawn Elementary School in Portland, Oregon. It reveals the artist's thinking about the concept of reflection and her attempt to solve the problem of representing this concept visually. Lizeth's rowboat was exhibited in the Young Artist Project of Pacific Northwest College of Art and Portland Public Schools.

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EDITOR'S NOTE

DENISE JARRETT jarrettd@nwrel.org

THE VASTNESS OF THE Pacific Northwest can increase teachers' feelings of isolation. By sharing stories from Northwest classrooms, this journal aims to bridge the distances, fostering a community of teaching professionals.

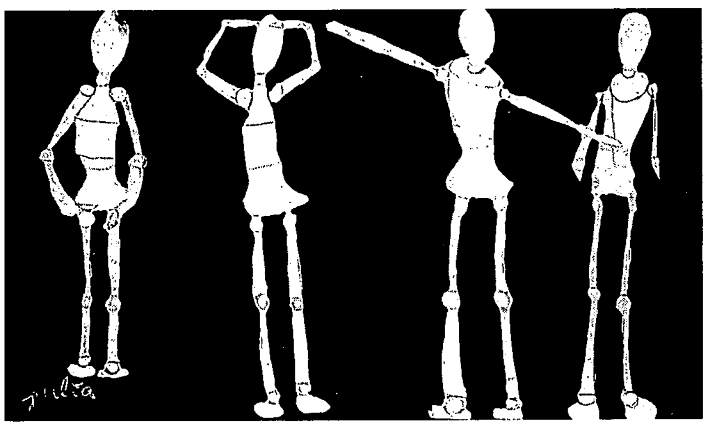
Welcome to the premiere issue of *Northwest Teacher*. We're very pleased to be launching this free journal for teachers in the Pacific Northwest. It's intended as a place where teachers from all grade levels who teach math or science can read about stories and issues in education that matter most to them.

Published three times a year, each edition of the journal will focus on a key topic in mathematics and science instruction, beginning in this issue with mathematics problem solving and, in upcoming issues, the impact of the standards movement, and teaching science through inquiry.

We know the teaching life leaves little extra time, so we're keeping the articles concise with practical and, we hope, insightful information. You'll find some good ideas for classroom activities and content here, but, rather than presenting "ready-made" lessons, *Northwest Teacher* strives to address the long view, taking a look at best practices and their theoretical underpinnings through the lens of real-life classrooms.

We recognize that, in this era of education reform and scientific discoveries about how people learn, teachers want succinct, readable, and accurate information about complex topics to help them make the most of new developments and grow professionally.

The Pacific Northwest, like other regions of the country, has a distinctive character that influences what takes place in classrooms. As a regional publication, this journal aims to speak to the interests and needs of teachers and students in Northwest classrooms.



Drawing by Julia Rudinsky, 7, courtesy of Innerscape Art Center, Portland.

Each journal provides an overview of the main theme to help readers reflect on the link between theory and the real world. Through interviews and anecdotes in feature articles, teachers, students, and parents share their concrete experiences about what it takes to teach and learn mathematics and science well. In columns, educators offer suggestions on using technology in the classroom and finding instructional resources. Guest editorials and letters from readers are two more ways that teachers' voices are heard in this publication.

We urge you to write to us with your ideas, tips on where good things are happening in Northwest schools, feedback on the magazine, and letters for the Discourse column.

If you are a Northwest educator and would like a free subscription to the journal, please contact us. Single copies are available free of charge to educators in Alaska, Idaho, Montana, Oregon, and Washington. To request a single copy, contact NWREL's Mathematics and Science Education Center by phone, (503) 275-0457; e-mail, math_ and_science@ nwrel.org; or visit our Web site, www.nwrel.org/msec/pub.html.

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Let us hear from you!

PROBLEM SOLVING IS AS challenging for teachers as it is for students. But you don't have to go it alone. NWREL's Mathematics and Science Education Center offers resources and technical assistance to help you.

In a recent position paper on the development of students' numerical power, the National Council of Supervisors of Mathematics notes that "all instruction should be couched in an environment of sense making, even if that means taking longer to solve problems .... [W]hen the majority of a child's time in mathematics is spent memorizing what the child considers to be nonsense, he or she soon abandons altogether his or her efforts to make sense of mathematics."

The Northwest Regional Educational Laboratory's Mathematics and Science Education Center embraces this recommendation and that of the National Council of Teachers of Mathematics to place problem solving at the center of mathematics education. It is for this reason that mathematics problem solving was selected as the theme of the inaugural issue of *Northwest Teacher*.

We believe that mathematical problem solving involves much more than the routine use of algorithms. Instead, problem solving should engage students in rigorous and complex tasks that require them to think, reason, communicate, and apply their understanding of number concepts and operations.

This view of mathematical problem solving has guided our research and development work over the past two years. The Center has developed the NWREL Mathematics Problem-Solving Model to provide resources and assistance in support of effective problem-solving instruction and assessment. Like the Six-Trait Writing™ Assessment Model (and other models in the NWREL family of trait-based assessments), the mathematics problem-solving

A WORD FROM THE DIRECTOR

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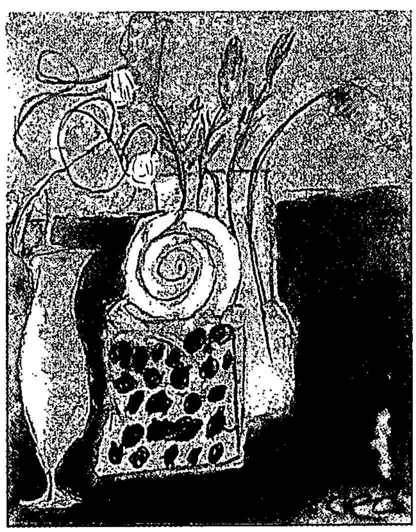


Cleo Hehn, 6, Innerscape Art Center.

model focuses on student learning by helping teachers to use information from formative evaluation activities to improve and adjust their instruction based on students' performance.

It is this approach to problem solving that the stories and columns in this issue of *Northwest Teacher* explore.

An overview of the Mathematics Problem-Solving Model is available at the Web site www.nwrel.org/msec/mpm. In addition to the model, NWREL's Mathematics and Science Education Center will offer a summer institute on problem solving, open by application to individuals who are interested in becoming "certified" trainers. For more information about the model and institute, contact Mathematics Associate Robert McIntosh at (503) 275-0689.

Two additional resources on problem solving will be available in fall 2000. A video will show what effective problem solving "looks like" and the roles students and teachers play. An upcoming monograph will synthesize the research on effective problem-solving strategies.

Watch our Web site for information on how to obtain these upcoming resources.

Our goal for this publication is captured in its subtitle: a math and science journal devoted to rigorous and imaginative learning. The articles illustrate how several Northwest teachers are using problem solving to achieve rigorous and imaginative learning in their classrooms. The columns offer resource ideas and information to support others in creating similar opportunities for learning.

We are excited about this new venture and look forward to hearing readers' reactions.

OPEN-ENDED PROBLEM SOLVING
Weaving a Web of Ideas

STORY BY Denise Jarrett PHOTOS BY Suzie Boss

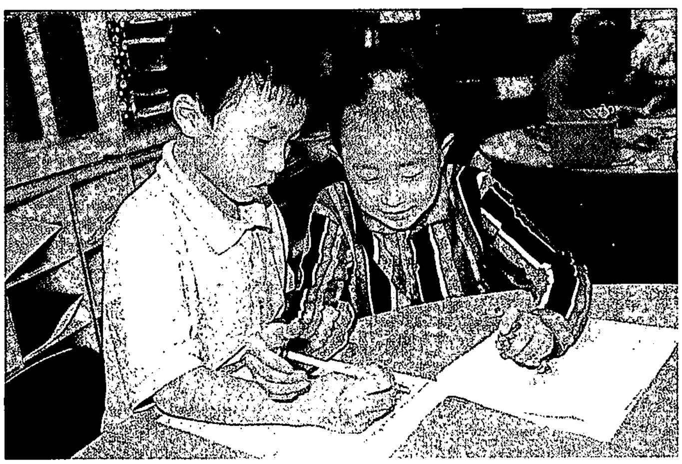
THE ALLURE OF PROBLEM solving isn't just for great mathematicians. Being absorbed in a difficult task is common to all of us, and, often, these moments prove to be some of our most rewarding experiences.

Take the case of Andrew Wiles. At 33, the Princeton University professor set out to prove Fermat's Last Theorem, joining a 350-year-old contest. Pierre de Fermat posed this challenge to the world in the 17th century by writing a note in the margin of his favorite book of ancient mathematics. He wrote that he'd developed a proof for a particularly knotty problem, but there wasn't enough room in the margin to write it all out.

Fermat's theorem stated that there are no whole number solutions for n greater than 2 for the equation $x^n + y^n = z^n$ (an extrapolation of the Pythagorean Theorem, $x^2 + y^2 = z^2$).

In the centuries since Fermat provoked the contest, legions of mathematicians have tried to build on the scanty hints he left behind. One of the first great breakthroughs came in the 19th century by French mathematician Sophie Germain. Like Wiles, she was transfixed by the problem, working on it single-mindedly for several years. Germain's work paved the way for more breakthroughs. But it remained for Wiles to solve at the end of the 20th century, drawing on all the power of modern mathematics (Singh, 1997).

The story of Wiles' pursuit of the elusive proof is dramatic and emotional. He worked on it for eight years, experiencing all manner of ups and downs. His struggle to find a solution epitomizes the power and allure of open-ended problem solving. His triumph in 1993-94 made for exultant headlines and a riveting BBC Horizon documentary film—the theorem even found its way into *Arcadia*, Tom Stoppard's play of that year about love, mathematics, and the nature of scientific discovery.



Young problem solvers buddy up at Clarendon Elementary School in Tacoma, Washington.

Wiles' story appeals, even to those who don't see themselves as mathematically inclined, because solving problems is a basic human drive. We may not understand the mathematics involved in Wiles' proof, indeed few mathematicians do, but we understand his enormous capacity for curiosity, perseverance, and resiliency.

We understand, perhaps, because we've all experienced that state of "flow," as psychologist Mihaly Csikszentmihalyi (say Chick-SENT-me-high) terms it, in which we're so focused on doing something that we lose track of time. These moments of immersion in a meaningful challenge are not only some of our most satisfying experiences in life, they are also the richest for learning.

The concept of flow is a useful analogy to describe the level of engagement Wiles must have experienced while grappling with Fermat's problem. This level of absorption is the kind of learning experience students are meant to attain through open-ended problem solving.

Czikszentmihalyi (1990) writes that flow occurs when a person's abilities are fully engaged in overcoming a challenge that is interesting and "just about manageable." During flow, the person is controlling the direction of his approach to the task, constructing his own learning as he stretches his abilities to master the activity, whether it be building a shed, quilting, writing a poem, or solving a math problem.

Freedom to make one's own decisions about how to approach a problem and what strategies to employ is also a key to open-ended problem solving. This requires a fundamental shift away from traditional methods that emphasize teacher-directed rote learning.

While the phrase *open-ended problem solving* may sound forbidding, it basically describes this heightened learning experience of being fully absorbed in a difficult and interesting task. Research shows open-ended problem solving to be particularly effective in promoting deep mathematical understanding (Hiebert, Carpenter, Fennema et al., 1996; Schoenfeld, 1992). A teacher's role is to make classroom conditions favorable for this kind of learning to take place.

weaving ideas

Czikszentmihalyi notes that "playing with ideas is extremely exhilarating" and when made a lifelong habit, it weaves a web of connected ideas, giving resiliency and snap to intelligence. Transfer this concept to the mathematics classroom, and the writings of James Hiebert, Professor of Education at University of Delaware, come to mind. Hiebert's research on problem solving has done much in the past 20 years to help identify and articulate the features and benefits of problem solving in the teaching and learning of mathematics. The real value of problem solving, he concludes, is in the ideas it produces.

When a student learns math by grappling with difficult and absorbing problems—rather than by simply memorizing and practicing predetermined procedures—she is free to "wonder why things are, to inquire, to search for solutions, and to resolve incongruities," he says. "This approach yields deep understandings of the kinds that we value" (Hiebert, Carpenter, Fennema, et al., 1996).

Thinking abstractly about ideas increases the flexibility of one's thinking capacity. (Though ideas within a "real-world" context are also valuable.) The National Research Council explains the importance of developing flexible thinking in its influential 1989 publication *Everybody Counts: A Report to the Nation on the Future of Mathematics Education:*

"Experience with mathematical modes of thought builds mathematical power—a capacity of mind of increasing value in this technological age that enables one to read critically, to identify fallacies, to detect bias, to assess risk, and to suggest alternatives."

To help young people to be better problem solvers is to prepare them not only to think mathematically, but to approach life's ever-changing challenges with confidence in their problem-solving ability.

open-ended problem solving

Open-ended problem solving involves problems that have multiple solution methods and answers. Teachers should choose problems that are just beyond the solver's skill level. The difficulty should be an intellectual impasse, notes Schoenfeld (1992), rather than a computational one. In fact, students who haven't yet mastered computations should be allowed to do open-ended problem solving. Nonroutine problems can provide ample opportunity to build computation skills while engaging the student in more challenging mathematics and higher-order thinking. All students deserve the opportunity to develop their problem-solving ability.

Working collaboratively is a key feature of open-ended problem solving, though students will also work individually. To solve a problem, students draw on their pre-

> "THINGS LEARNED WITH UNDERSTANDING ARE THE MOST USEFUL THINGS TO KNOW IN A CHANGING AND UNPREDICTABLE WORLD."
> —**JAMES HIEBERT AND COLLEAGUES** (1996)

vious knowledge and experience with related problems. They decide which solution method to follow, perhaps even constructing their own method, trying this and that, before arriving at a solution. They then reflect on the experience, tracing their own thinking processes and reviewing the strategies they attempted, determining why some worked and others didn't. Students will discuss the problem, identifying its features, considering possible solution methods, conjecturing, and explaining their thinking.

"Communication works together with reflection to produce new relationships and connections," writes Hiebert (1996). "Students who reflect on what they do and communicate with others about it are in the best position to build useful connections in mathematics."

(Even Andrew Wiles, who worked in seclusion most of those eight years, needed the direct collaboration of others to help him finally solve Fermat's Last Theorem.)

Through it all, the solver constructs a mathematical understanding of the problem that is both deep and flexible. By connecting her prior knowledge with new concepts and skills, she gains depth of understanding. These connections also create flexibility in her thinking, enabling her to extend her knowledge to new mathematical situations and beyond. Good problem solvers, says Robert McIntosh, Mathematics Associate for the Northwest Regional Educational Laboratory, can see past the surface features of problems to common underlying structures. They can monitor their own thinking strategies, recognizing when an approach or tactic is not being productive and modifying it as necessary. Self-awareness and the ability to reflect are essential for improving one's problem-solving ability. Furthermore, good problem solvers are resourceful, confident, and willing to explore. They're persistent and tolerate a measure of frustration.

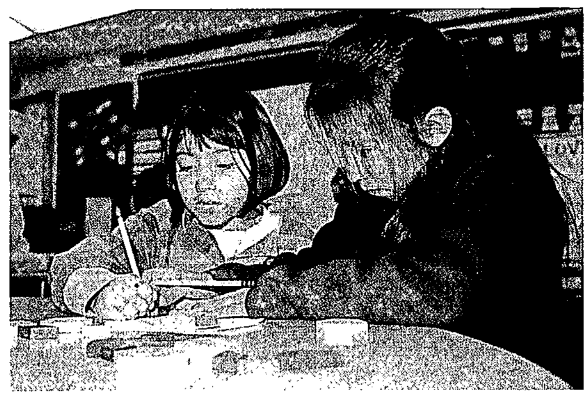
Japanese students are some of the most tenacious problem solvers. Indeed, when the U.S. members of the Third International Mathematics and Science Study viewed videotapes of Japanese math classes, they were amazed by students' perseverance (Stigler & Hiebert, 1999). More pointedly, another comparative study of first-grade students working a difficult task (in fact, the task was unsolvable, but the students didn't know that), reported that U.S. students gave up in about 15 seconds, while Japanese students didn't stop until the class came to an end an hour later (Stigler, 1999).

"To develop these abilities, students need ample opportunities to experience the frustration and exhilaration that comes from struggling with, and overcoming, a daunting intellectual obstacle," says McIntosh.

The beauty and utility of open-ended problem solving is just this: It leads to understanding that is transferable. And in this increasingly complex world, the ability to transfer knowledge and skills to meet changing conditions and challenges is essential.

standards: a statement of values

Standards, says Hiebert (1999), are simply a statement about what we most value. From our best judgment, we create education standards based on past experiences, research, advice from practitioners, and societal expectations. At least since the 1940s when George Pólya identified problem solving as an essential math skill—the *heart* of mathematics, in fact—problem solving has been a stated education priority. When the National Council of Teachers of Mathematics began issuing its standards for math teaching and learning in the 1980s, problem solving rose to the fore of standards reform. In its *Principles and Standards for School Mathematics* (NCTM, 2000), the council continues to identify problem solving as a core strand of math learning for all grade levels.



Manipulatives help these students solve a geometric problem.

Across the country, states are writing mathematics standards and assessments to include problem solving, acknowledging the importance of assessing students' ability to reason, communicate, make connections, and apply their knowledge to problem situations. Thus, problem solving is an area teachers are increasingly expected to teach and assess.

making the change

Teachers are often caught between daily pressure from colleagues, parents, and community members to uphold convention in the classroom, and pressure from administrators and policymakers to employ standards-based practices that show immediate and positive results on achievement tests. One must consider these opinions and mandates, but teachers who make meaningful changes are those who develop their own inner voice of authority (Wilson & Lloyd, 2000).

Teaching problem solving is an art mastered over a long period of time (Thompson, 1989). By reflecting on their personal understandings of teaching and learning—as well as their students' understandings—teachers develop inner authority for improving their instruction. Through reflection, teachers determine for themselves the value of particular reform innovations (Wilson & Lloyd, 2000).

Many teachers do recognize that nontraditional strategies are necessary to meet the learning needs of their increasingly diverse students. Embracing change can be unsettling, but these teachers venture into new territory, opening a world of discovery for themselves and their students. For they know that a young mind carefully nurtured may be the next big thinker to solve another of the world's mysteries.

Denise Jarrett is an education writer and editor of Northwest Teacher.

references

Csikszentmihalyi, M. (1990). *Flow: The psychology of optimal experience*. New York, NY: Harper Collins.

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1), 3-19.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A., & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, 25(4), 12-21.

National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

Schoenfeld, A.H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D.A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-367). New York, NY: Macmillan.

Singh, S. (1997). *Fermat's enigma: The epic quest to solve the world's greatest mathematical problem*. New York, NY: Walker.

Stigler, J.W. (1999, September). *Improving the quality of teaching in mathematics and science*. Paper presented at the meeting of the National Commission on Mathematics and Science Teaching for the 21st Century, Washington, DC.

Stigler, J.W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York, NY: Free Press.

Thompson, A.G. (1989). Learning to teach mathematical problem solving: Changes in teachers' conceptions and beliefs. In R.I. Charles & E.A. Silver (Eds.), *The teaching and assessing of mathematical problem solving*. (Vol. 3, pp. 232-243). Reston, VA: National Council of Teachers of Mathematics.

Wilson, M.S., & Lloyd, G.M. (2000). Sharing mathematical authority with students: The challenge for high school teachers [Editorial]. *Journal of Curriculum and Supervision*, 15(2), 146-169.

9

NONROUTINE PROBLEMS

If problem solving is at the heart of mathematics, then nonroutine problems are at the heart of problem solving.

True problem solving involves nonroutine, or open-ended, problems. Moving into the territory of nonroutine problems is full of unknowns. Solution methods and answers are not made explicit. Decisionmaking is shared between teachers and students. Teachers must predict what tactics and questions might come up in class, preparing for them as best they can.

Teachers need mathematical expertise to anticipate students' approaches to a problem and how promising those approaches might be. They must choose tasks which are appropriately difficult for their particular students. They must decide when and how to give help so that students can be successful but still retain ownership of the solution. Teachers will sometimes find themselves in the uncomfortable position of not knowing the solution to a problem. Letting go of the "expert" role requires experience, confidence, and self-awareness.

Teachers should choose tasks with nonroutine problems that (Hiebert et al., 1996):

- Make the subject problematic so students see the task as interesting and challenging
- Connect with students' present level of understanding, allowing them to use their knowledge and skills to develop methods for completing the task
- Allow students to reflect on important math ideas

confessions OF A former mathphobe

TO CONFRONT HER FEAR OF MATH, elementary school teacher Judith Carter took a math course for educators. For the first time, she was learning math in an environment that encouraged independent thinking and problem solving. Below, she reflects on the changes this experience prompted in her personal beliefs about teaching and learning.

When I entered a graduate program to become a teacher seven years ago, I had been aware for some time that as a student of mathematics I had been injured by the pedagogy of my teachers. In the classes I had taken from kindergarten to high school it was never acceptable to use pictures or objects to help me understand what the numbers and letters represented. No one let me talk about what I thought, or encouraged me to ask questions. In fact, to ask a question was taking a big risk that I might be seen as someone who "didn't get it." Most of all, no one said it was okay to struggle and even more, no one viewed struggling and confusion as part of problem solving. Had the pedagogical use of manipulatives, social interaction, reflective processing from various perspectives, and honoring the perseverance it takes to solve complex tasks been taught by any of my teachers, my mathematical experience and self-concept would have been tremendously different.

One of the main reasons I took this math class was to overcome my fear of math and change the image I had of myself as a math student. As a student of any subject I have always wanted to know "why" and math has been no exception. This class was all about finding "why." But what was so different was that we were encouraged to find out "why" in our own way and in our own time. Everyone's paths were explored and honored and, though I found that my path to "why" was often circuitous, I learned that the long way is not the wrong way. I struggled and I was confused, but the commitment that had led me to take the class in the first place kept reappearing in my persistence with each problem. Because perseverance was supported, by collaboration, by honoring diverse approaches, by being given time to solve problems, I found that my perseverance increased exponentially. There were some awesome moments of revelation in this class and I could feel the child in me want to jump out of my seat (I think I did this a few times) and shout, "I get it!" There were also occasions when the injured math student appeared, and I learned how to be patient with her. Perseverance and patience are now part of my new self-image.

I had always said I believed that all children can learn but, because of my own fears, I think there was a part of me that was still thinking, "but not me." So if I continued to believe this about myself, wouldn't I also believe it about some of my students? I know I can learn difficult math concepts and explain my thinking to others. This will have a profound effect on how I view my students because I know now that all children can learn when they are allowed to work in an environment that is safe for taking risks.

To feel safe and gain confidence in math, students need to have opportunities to process their thinking in a variety of ways: using manipulatives, writing, working with others, and presenting their work to the class. They need to be able to assess their own work and to reflect on it and to redo the work as they learn more. I hope that this year I can create an environment where students will see themselves as problem solvers and value their own path to "why."

(For a related story featuring Judith Carter, see Page 10.)

JUDITH CARTER
is a second-grade teacher at Clarendon Elementary in Portland.

AFTER EMBARKING ON A PROBLEM-SOLVING APPROACH, TWO TEACHERS CONCLUDE:

Teenager or Tyke, Students Learn Best BY TACKLING CHALLENGING MATH

STORY AND PHOTOS BY Suzie Boss

HEIDI EWER

If she sees a student struggling, Heidi Ewer offers support but doesn't give away any answers.

TACOMA, WASHINGTON—When she joined the staff of Wilson High School four years ago, Heidi Ewer was the new kid on the faculty.

Then only 27 years old and shorter than most of her students, she had only two years of teaching experience behind her, neither at the high school level. Ewer was assigned her first couple of years to teach pre- and remedial algebra.

"I dreaded it," she admits. "These kids had failed this subject matter over and over. Most of them had been through it three or four times already." She discovered that students were failing math largely due to behaviors, not from an inability to understand concepts. "They had poor attendance. They were bored. And they didn't care about grades. It was a struggle to find rewards or even consequences that would motivate them to learn. What would drive them internally?"

Ewer decided to depart from the traditional, textbook-based teaching methods. "Even I was getting bored with the routine of lecture and homework," she admits. At the University of Puget Sound in Tacoma, where she earned an undergraduate degree in math and a master's in teaching, she had studied the theory behind research-based teaching practices that actively engage students in learning. During the two years she spent as an elementary teacher, she had seen the benefits of hands-on learning. But she'd never seen such concepts modeled with secondary students.

making ideas central

A little apprehensive, she decided to add some new approaches to her repertoire. Rather than drill-and-practice desk work, she tried using cooperative learning—allowing teenagers to socialize a bit in class, so long as they focused on math. She created opportunities for students to learn by having them investigate ideas rather than memorize formulas. She assigned open-ended problems that could be solved in more than one way, using hands-on materials. She attended conferences and a workshop on math reform to build her confidence and gather new resources. And her students responded. "I could see the lights click on," she says.

Ever since, Ewer has stuck with her decision to use student-centered teaching methods that still make her a bit of a maverick at Wilson High, a racially diverse school of 1,800 students. More traditional teachers raise their eyebrows when she turns her students loose with tipsy-looking glass vases and beakers of water to investigate volumes. Colleagues wonder what's up when animated student conversations about how to harness a horse to maximize grazing space spill out of her doorway and echo down the hall. A few ask pointedly whether Ewer will have time to cover all the material in the textbook.

Even new students sometimes misinterpret what goes on in her classes, Ewer admits. "I have a reputation for teaching fun classes," in subjects such as geometry and advanced algebra. "Some students think that 'fun' means 'easy,'" she says with a laugh that means they'll soon learn otherwise.

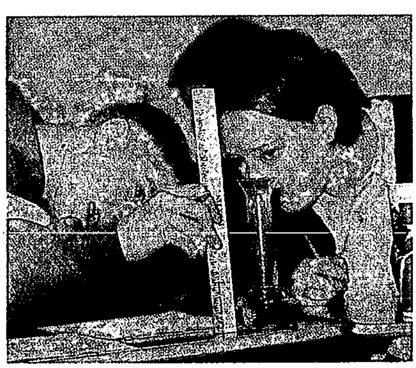
Teenagers living in Tacoma don't have to look far to find examples of mathematical problem solving out in the real world. There's the Tacoma Dome rising on the horizon, its convex roof built to allow unobstructed views for concert goers. Cargo ships docked at the harbor must have freight properly distributed in order to stay afloat in Puget Sound. Ewer routinely brings such real-world problems right into her classroom, asking students to determine how much cable would be needed to support, say, the Tacoma Narrows Bridge.

And students seem to welcome the challenge. Says a girl named Amanda, "When you're the one pouring the water and measuring the rate of change in volume, it's more real than reading something out of a book. You learn how to do it yourself. Eventually, when we have jobs, we'll need to know the best way to solve problems." Adds Matthew, "This way of learning makes you more open-minded. There's no one right way to solve these kinds of problems. That's changed my perspective—not just about math, but about life." And Evan, who has sought out Ewer's classes for three years in a row, says her approach "makes you think, and be open to other people's ideas. It's not enough to figure out a problem. She expects you to be able to explain your thinking in a written report. That means you really have to get the concept for yourself."

fostering perseverance

Although buoyed by her students' enthusiasm, Ewer is quick to admit that this way of teaching poses its own challenges. Students who are more sequential learners, for instance, often balk at open-ended problems. They're used to learning one method that will lead them to the right answer, and get frustrated when the rules change. During a recent problem-solving activity, Ewer saw tears welling up in a girl who can blitz through more traditional worksheets without batting an eye. Ewer came to her aid, coaching her to consider possible solutions. The teacher's dilemma: "How can I lead her without just giving her the answer?"

Letting students struggle is valuable, up to a point. "Struggling helps them see this as an investment of their own time and energy. It makes them more willing to learn," Ewer says. Struggling to solve problems requires students to use their intuitive skills to investigate concepts, she explains, and, in this way, they gain a deeper and more lasting understanding of the mathematics. "A student who has a good grasp of problem-solving strategies and concepts will be so much stronger than one with a knowledge of the material learned through memorization. They will know how to use their skills and confidence to attack new concepts and learn though their own method." But Ewer also peppers her students with encouragement: "Have confidence." "Listen to the wind (learn from what others are saying)." "Try—it's better than guessing."



LEFT: Problems that are based on real life hold students' interests.

ABOVE: Advanced algebra students put their heads together to solve a problem involving the volume of a glass vase.

The evolution of her own classroom practice reminds Ewer that students need a variety of approaches to help them learn. "It's like teaching reading to elementary students," she says, harking back to her student teaching days. "Should you use phonics or a more holistic approach? Some students will learn better each way. You need to pull in a variety of methods," she asserts, in order to serve all students well. Class discussions and collaboration also help students learn to appreciate one another's strengths and different learning styles. Ewer's brown eyes widen with delight, for instance, when Amanda observes, "Some days, I learn more from other students than I do from the teacher."

meeting the challenges

Occasionally, a student's behavior will interrupt the dialogue and collaboration that's central to the problem-solving process. "A lot of teachers will say they can't do this because of behavior problems," admits Ewer. "But I teach math and behavior." Rather than starting off with problems that involve glass, water, or model race cars, she builds up from paper and tape to more challenging materials. "I ask them to be accountable, and I have a solid behavior plan." When one boy acted edgy during a vase-and-water problem, Ewer kept him on track with good-humored reminders, direct eye contact, and a promise to remove him if he couldn't manage himself. In four years, she's never had to bounce a student out of class during a problem-solving lesson.

Time poses another classroom challenge. Wilson High used to offer a block schedule. With 110 minutes, Ewer could take her class from warm-up dialogue to hands-on work to wrap-up discussion. This year, the school is back to 55-minute periods. That means a constant race against the clock. She had to cut short a discussion one morning, just as students were making some keen insights, to shift to a hands-on problem. A closing discussion would have to wait for another day.

Although Ewer feels supported in her teaching approach by district administrators, her principal, and cohorts from other schools, she remains a bit of a loner within her own building. Eager to share the problem-solving materials she's gathered at workshops, she gets few takers. "I get kind of preachy," she admits, "like I'm on a pulpit."

But the students keep her passion burning bright. "I go into my classroom and do the best I can for my kids. I hope it pours out of my room so that other teachers will see the sparks. I hope they'll wonder what in the world we're doing in here and come find out for themselves."

Collaborative work allows Tacoma High schoolers to discuss ideas together.

JUDITH CARTER

Judith Carter coaxes children to keep trying until they "muddle through to understanding."

PORTLAND, OREGON—Gathered around their teacher's chair, the second-graders join in a spirited discussion about their favorite flavors of birthday cake. Their teacher, Judith Carter, uses the dialogue to introduce a delicious-sounding problem that the students will tackle in pairs. "Today you're going to make a pretend cake, a gourmet cake, the most special cake there is," says Carter. "It comes from the Tri-Cities Bakery. Can anyone guess what shape their cakes are? What words do we know that start with tri?" Hands shoot into the air as the children clamor to share their ideas.

facing her fears

When she was the tender age of her students, Judith Carter used to avoid raising her hand during math lessons. Asking a question when she felt confused would not result in a different explanation or open an alternate route to understanding. It would only reveal that she didn't get it.

"My teachers had one way of seeing math, one way of teaching," says Carter. Now 45, with sandy brown hair and eyes alight with intelligence, Carter doesn't sound bitter when she describes her own classroom experiences. In a matter-of-fact tone, she relates a tale of missed opportunities. Roles were clear-cut in the math classes of the '50s and '60s, she explains, not only in Vermont where she grew up, but all across the country. The teacher's job was to explain the concepts in the textbook. The students' job, to memorize and practice them. Answers were either right or wrong. It was a formula that left no room for uncertainty.

By middle school, Carter had adopted a survival strategy: "I kept quiet and did the minimum." Math was something she couldn't get away from fast enough. And for decades, she managed to keep the subject at a safe distance, earning a college degree in English and working in a variety of office jobs. She did wind up marrying a math teacher, but math was his thing. Not hers. Or so she thought.

In her late thirties, living in Washington with her husband and three children, Carter was ready for a career change. As part of her master's in teaching program at Evergreen State College in Olympia, she walked back into a mathematics class—only to discover that the universe had shifted. "In my math methods class, they gave us hands-on materials, manipulatives. That's when I realized there was another way to learn math than how I was taught." For the first time in years, she felt intrigued by math. But getting comfortable with the subject would take her a little longer.

Once the class has determined that Tri-Cities Bakery produces triangle-shaped cakes, Carter adds another dimension to the problem: Each big cake is assembled by fitting together smaller pieces that are cut into geometric shapes to identify different flavors. Lemon cake is cut into hexagons; blueberry, parallelograms; chocolate mint, small triangles; and strawberry, trapezoids. The students' task is to create a big cake containing at least two pieces each of the four flavors. "What do we have in the room that would help you solve this problem?" Carter asks. A boy points to the pattern blocks, which just happen to be in plain view. "Remember," Carter adds, "there's going to be more than one way to solve this problem. How do you feel when you have a problem to solve?" Hands wave. "Excited," says one girl, "because it's something I've never done before."

When she started teaching with the Tacoma School District six years ago, Carter spent a year as a Title I reading specialist. But ever since, she's been teaching all subjects, including mathematics. In Tacoma she taught fourth- and fifth-graders in a high-poverty urban school. When Carter's family relocated to Portland, she was hired as a second-grade teacher. She's now in her second year at Clarendon Elementary, a modern-looking school built with open pods instead of traditional classrooms that serves about 320 children in a racially diverse neighborhood. For about a third of the students, English is a second language.

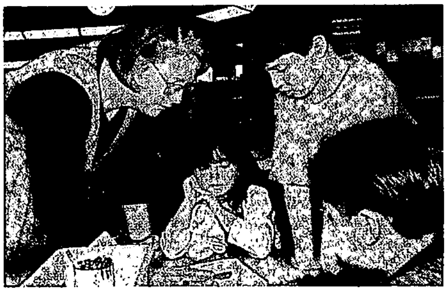
Both the Tacoma and Portland districts have adopted math programs that favor student-centered learning. Philosophically, Carter shares this commitment to improving how math is taught. She wants to be the kind of teacher who allows children to make discoveries and connect math to the world beyond the classroom. She embraces the concept of open-ended problem solving, which allows children to ask questions, develop strategies, and cement their own understandings.

echoes from the past

But as Carter has discovered, moving from theory to practice takes more than good intentions. "You can have a philosophy you believe, you can have great training in good practices, you can have a sound curriculum. But then you'll hear your own fears coming from your own mouth," she says.

"REMEMBER," **Carter adds,** "THERE'S GOING TO BE MORE THAN ONE WAY TO SOLVE THIS PROBLEM. HOW DO YOU FEEL WHEN YOU HAVE A PROBLEM TO SOLVE?" **Hands wave.** "EXCITED," **says one girl,** "BECAUSE IT'S SOMETHING I'VE NEVER DONE BEFORE."

"This is hard," complains a girl who's struggling to fit her pattern blocks into the shape of a large triangle. Her partner's stuck, too. Carter bends down to their eye level, asking questions. She doesn't want to spoon-feed them a solution, just start them thinking. She wants the youngsters to begin recognizing patterns and relationships, and developing their reasoning and communication skills. They'll have to use their computation skills, too, to figure out how


When children feel "stuck," Judith Carter asks probing questions to get them thinking.


Working in pairs, second-graders learn from each other's strategies.


Carter provides just enough help to further her students' thinking.

much their big cake will cost. By trial and error, the students discover that placing the largest pieces—the hexagons—is the hardest step. Once they're in place, the rest fit more easily. Carter slips away to the next table, leaving the two to finish on their own.

When she began teaching elementary math, Carter was startled to hear echoes of her old math teachers in her classroom. "I had to keep fighting the way I was taught," she says. She'd go home at night and debrief with her math-teacher husband. He encouraged her to keep going down the path of problem solving, even if it felt a little bumpy. "I had to stop myself from blurting out that my students' solutions were right or wrong. Sometimes, I'd have to take a student's solution home and think about it. I had to be willing to show my kids that I don't always know the answer. If I was going to do my kids justice," she says, "I realized I'd have to get over my own math phobia. And it was deep."

Last summer, Carter faced her fears head-on. She enrolled in a two-week, intensive seminar in algebraic thinking offered by the Mathematics Education Collaborative of Ferndale, Washington. Designed for educators, the course offered challenging content and promised to model an optimal learning environment. She would learn the "big ideas" of algebra in the same kind of classroom she wanted to create for her own students.

Carter had studied algebra in high school, of course, "but I remembered none of it. As elementary teachers, we need to know algebra so we can see the connections between what we teach and what our students will learn in middle school and high school. When we teach patterns during calendar time in second grade, we need to know that those patterns will come back later, when our kids learn algebra." Seeing those connections, Carter says, "helps us see the value of what we're doing."

During the two-week class, Carter's moods rose and fell like a stock market graph. "Some days I'd go home wondering, what have I gotten myself into? Then I'd go struggle over my homework." Because this was her personal quest, Carter refused her husband's help. "This was my struggle. I had to figure it out on my own."

The first time she was invited to walk up to the overhead projector and explain her solution to the rest of the class, Carter felt her old insecurities well up. "To put my thinking out there was scary," she admits. But to her amazement, once she had actually figured out a solution, she was excited to share her thinking.

a new view of mathematics

The biggest insight she gained that summer? "Being good at math is not something you're born with. Most people who are good at math work hard at it. They have a high tolerance for confusion and uncertainty. You have to muddle through. That's what learning is—muddling through to understanding. The teacher's challenge is to help students be okay in that space of feeling muddled. Don't let them feel defeated. Help them understand that it takes perseverance. The key work of teachers is to develop that quality in our students so they'll keep working at it, trying things, until they reach their own solution."

Lunchtime is fast approaching when Carter calls a halt. Her second-graders have been working hard for more than an hour. There's just time to regroup and invite students to explain their strategies. Carter moves her chair to the back in hopes of tempering her tendency to jump into the student-led conversation too soon. She sees so much rich learning going on during problem solving, and wants her students to see it, too. But they're only second-graders, *after all. Sometimes they need a little prompting. "Tell us," Carter says, leaning forward, "what did you discover?"*

When she finishes a problem-solving activity, Carter often feels exhausted. This is high-energy teaching, requiring her to plan everything from which materials she'll need to how she should pair students to maximize collaboration. It's not an activity she can fit into every day, or even every week. But now that she's developed her confidence to teach this way, she wouldn't dream of not making time for open-ended problem solving. She loves hearing the buzz of conversation that wells up during these activities. "Children can develop their mathematical power as early as second grade," she says. "If we don't give them a chance to be problem solvers, we're selling them short."

Suzie Boss is associate editor of NWREL's quarterly magazine, Northwest Education.

EMBRACING CHANGE

Teachers are constantly making changes to meet the evolving needs of their students and to try new instructional practices. And they exercise considerable control over the kinds of changes they decide to make (Richardson, 1990). Standards-reform measures, therefore, can't determine teachers' decisions directly but can serve as a resource for developing inspired but personally tailored innovations (Ball, 1996). To be most effective, teachers also need to keep abreast of developments in the field and educational findings from research (Richardson, 1990).

Research. Research is one basis for improving practice. With an understanding of the theory behind standards-reform measures, and an opportunity to discuss with others how theories agree or disagree with teachers' own experiences, teachers are more likely to make informed and effective changes. Otherwise, teachers may accept or reject reform-based practices on the basis of whether they fit one's personality or accommodate the daily pressures of classroom management and content coverage (Richardson, 1990).

Reflection. The role of reflection in improving practice is essential. Teachers need time and a context for reflecting on their own teaching experiences in light of what research reveals about teaching and learning. They need to be able to discuss their ideas with others; articulating ideas to colleagues helps teachers clarify their own beliefs and practices.

Risk. Moving in the direction of reform practices, such as open-ended problem solving, means confronting the uncertainties and complexities of learning and understanding.

"When we ask students to voice their ideas, we run the risk of discovering what they really do and do not know," writes Deborah Loewenberg Ball (1996). "In pursuing reform goals, deep anxieties about one's effectiveness and one's knowledge are likely to surface."

Teaching with an open-ended problem-solving approach requires more than mathematical knowledge and skill. Personal qualities, such as patience, curiosity, generosity, trust, and imagination, are essential. So are an interest in seeing the world from another's perspective, a sense of humor, tolerance for confusion, and concern for the frustration and shame of others, says Ball.

"As teachers build their own understandings and relationships with mathematics, they chart new mathematical courses with their students," she says, "and conversely, as they move on new paths with students, their own mathematical understandings change."

Ball, D.L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappa, 77*(7), 500-508.

Richardson, V. (1990, October). Significant and worthwhile change in teaching practice. *Educational Researcher, 19*(7), 10-18.

WANT TO GAIN PARENTS' TRUST? Let Them in on the Action

STORY BY Rosemary Shinohara PHOTOS BY Stephen Nowers

MAKING MOMS AND DADS classroom collaborators is a good way to persuade them of the value of open-ended problem solving and learning math without a textbook.

PALMER, ALASKA—When Jana DePriest began focusing her math classes on problem solving about five years ago, she had to work hard to overcome the fears of parents.

"Parents weren't seeing the math they knew when they grew up," recalls DePriest, a fourth- and fifth-grade teacher at Finger Lake Elementary School. "Change is hard."

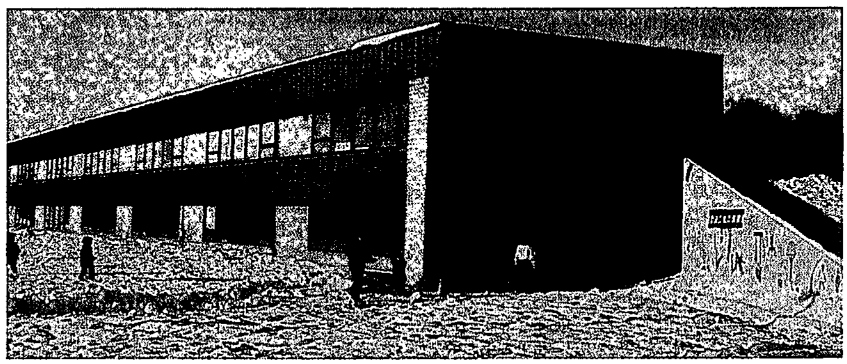
Some moms and dads still question what DePriest is doing, but usually not for long. She has won them over with a reputation for getting children engaged in math, and by a determined effort to keep parents informed and to invite them into her classroom.

"Parents in her class love her," says Dave Nufer, the Finger Lake principal. "Other teachers look to her as a mentor for ideas."

a need for change

DePriest, 37, is in her 13th year of teaching, and her sixth year at Finger Lake Elementary. The school serves rural subdivisions in the Matanuska-Susitna Borough, about 45 miles north of Anchorage in a fertile valley cut by two glacier-fed rivers. Finger Lake Elementary sits halfway between Palmer and Wasilla—once colony towns, now bedroom communities for Anchorage professionals. Most of the school's 500 pupils live in the new subdivisions and apartments that have bloomed like fireweed on former farmland. But cows still dot the slopes of remaining pastures, a reminder of the pioneer farmers who settled here in the 1930s to grow robust vegetables and livestock under summer's so-called midnight sun. In fact, DePriest and her husband, Ray, an educational technologist, live on Ray's childhood farm, growing hay in summer—their farm is now surrounded by housing developments and a golf course.

The shifting of the Mat-Su Valley landscape was gradual, over decades. But sometimes when change occurs, it's more like the ground moves and suddenly you're seeing it from a new perspective. That's what it was like for DePriest when she decided on a radical change in her teaching approach.



Finger Lake Elementary School in Palmer, Alaska.

As a beginning teacher, she recalls, "It was pretty much you followed a textbook page by page. I used manipulatives when the book told me to."

But when students got to harder problems, such as multiplying two digits by one digit, they relied on memorization, following a process by rote rather than fully understanding the concept of multiplication.

"I would ask, 'Why are you regrouping that one to the top?' They didn't have the answers. They didn't really understand what they were doing."

So DePriest read the research on using real-life problem solving to help students learn math, and began building it into her program.

"Right when I read it, I knew it would work. I still know it's the right change," she says.

DePriest is now one of a handful of teachers at Finger Lake who teach mathematics by involving students in open-ended problem solving, avoiding the district's textbooks and focusing instead on math problems that have a real-world context. She covers the material set out in the Alaska mathematics standards, which are similar to the national standards, but she does so primarily through hands-on activities. She uses a balance of paper-and-pencil tests, oral questioning of each student, and projects to evaluate where students stand. While she's questioning students, she's prodding them to think. Students reflect on their learning in journals which are turned in at the end of a unit of study. The projects and individual assessments allow DePriest to adapt her instruction to meet the needs of students at different levels of learning in her multigrade classroom.

DePriest job-shares, teaching science and math afternoons while teaching partner Lynda Chud handles language arts and social studies mornings. Whenever possible, DePriest combines her two subjects. This spring, the class explored the principles of flight. The students set up scientific experiments to collect and examine data on how different variables affect flight. They learned and applied distance, time, and rate formulas. They measured in centimeters and learned to use a stopwatch and a protractor.

winning parents' trust

In the beginning, many parents were skeptical of the change, worried that their children wouldn't be prepared for middle school. Most of the 21 teachers use Saxon math texts, a much more structured approach, says Nufer, the principal. Some parents whose children studied math from Saxon texts in earlier grades worried about the lack of apparent structure in DePriest's class. Saxon lessons follow a script. They involve repetition and memorization. Students bring home worksheets every night. This is many parents'

image of what mathematics learning is all about.

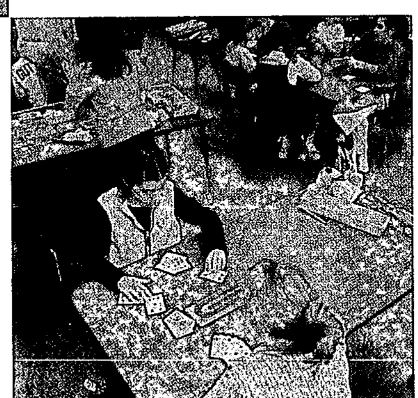
"When we were growing up, we had books we brought home," says Pam Wooding, mother of 10-year-old Mackenzie Deiman. When she learned Mackenzie didn't have a math text, she thought, "Whoa, how can you teach math without a book?"

With no text, Mackenzie's parents didn't know how to help her with her homework. That was last year, when Mackenzie was in fourth grade. Then, at a parent-teacher conference, DePriest explained to her that students must not simply memorize how to do problems, but be able to explain their thinking and describe their strategies for solving problems. Now Wooding is a convert. Wooding says she hated math as a child and figures it was too abstract. She has watched DePriest and Chud make math more real-

ABOVE: Jesse Eshleman examines her cards during a math game in Jana DePriest's class.

LEFT: Veronica Iversen counts money after a lunchtime opening of the class store.

BOTTOM: Students play cribbage and other math-oriented games on a recent afternoon.

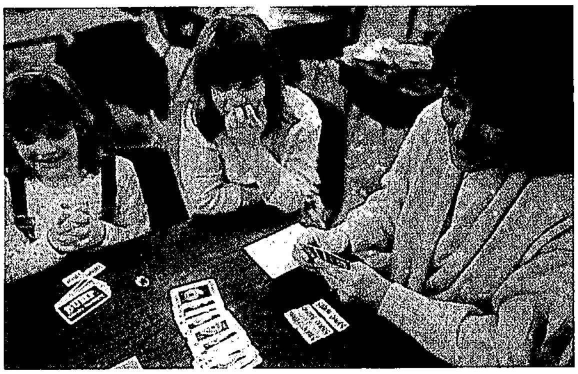


world by incorporating math into learning throughout the day.

Lisa Greenwood, mother of fourth-grader Emma, says she has no concerns that her daughter's missing anything in her unconventional math class. "I can see her applying math in everyday situations," Greenwood says. "Like how much dog food she would need for her puppies—how many pounds are in the bag."

math-filled days

To provide real-world contexts for problem solving, the teachers had students create a mini-society in which they made products and sold them in a market. Class members also run a real store at lunchtime from a table in the front lobby of the school. They sell erasers that look like dollar bills, pens that smell fruity, rulers, and clear plastic balls with reptiles inside. Students eagerly volunteer to be sales people, giving up their lunch breaks. Fourth-grader Raymond Hajduk explains: "You have to get all your homework done, and make sure your name isn't up on the board for missing assignments."



Teacher Jana DePriest plays a math card game with her students.

The students decide what merchandise to order, and, in the end, how to spend the profits. Raymond suggests they use the money to save mountain gorillas in danger of extinction. Someone else proposes that they could buy school equipment and throw a party. But all that will be after they pay back a $100 loan, plus 8 percent interest, to the teacher. In the meantime, there are sales to be made. "Thank you for doing business at the school store," Raymond tells a customer purchasing a pink, smelly pencil.

Parent volunteer Kathy Krell advises students in the operation of the class store.

involving moms and dads

Kathy Krell says she doesn't mind that her child doesn't have a math book because DePriest sends home detailed notes to parents explaining what's going on in her classroom. In fact, DePriest has made a conscious effort to inform parents through regular newsletters and by sending home thorough descriptions of math units. Her description of the recent unit on flight was four pages long. In it, DePriest laid out the math and science concepts and the learning goals, the state content standards that would be covered, the questions students would consider, and the criteria for student success. She included writing standards and a planning guide. She also gave parents a checklist for assisting their children in the final project that entailed building a flying machine and writing a paper about it.

In another note home to parents on multiplying and dividing large numbers, DePriest told parents five ways a child might divide 245 by eight. She included questions a parent could ask to check their child's understanding of a problem: Why does your answer make sense? How are these strategies similar?

"I felt like parents and kids thought I was making this stuff up off the top of my head," DePriest says. "I wanted them to know that just because I was not using a traditional textbook, I was still teaching math and had done my research."

Parents feel welcome to come to class, sit at their child's table, and dive into the work. Dad Steve Miller arrives as his daughter, Lindsey, and classmate Emma are puzzling over a graph that has five vertical bars, but no title, and no numbers.

"What's this a graph of?" he asks.

"We don't know, we're trying to figure it out," says Lindsey.

As clues, they have a story about a variety of events that affected earnings at the school store last year between January and May. Total earnings were about $300. The students' task is to figure out what the bar graph might be representing. DePriest wasn't looking for just one answer. Any reasonable solution would be okay. Miller picks up one of their calculators and starts helping. After adding up all the bars and dividing them into $300, his group decides the monthly revenues could be graphed in increments of $9. Others decide it makes more sense to count by 10s. Either answer could be right.

"I'm looking for their strategies and how they're thinking," DePriest says.

On another afternoon, students use their math problem-solving skills to determine the mean, mode, and median of data collected during a science experiment. Blowing up tubular balloons to different lengths, the students and a couple of parents tape the inflated balloons to sliding devices that are attached to strings stretching from wall to wall. Releasing the trapped air, they let the balloons fly, gathering data that will allow them to evaluate Newton's third law: For every action there is an equal but opposite reaction.

"This is fun," says Greenwood. "I wish they taught math like this when I was a kid."

DePriest also invites parents for math games, including cribbage and other card games, every couple of weeks on Friday afternoons. In one of her messages home, DePriest tells them:

"Math games develop students' sense of number and math operations. Math games provide students with many opportunities to develop their own strategies to solve arithmetic problems. Creating strategies to solve problems is an intellectually demanding endeavor because the students are combining mental math, arithmetic, and thinking ahead for game strategy all at once."

DePriest believes that some students can succeed at math through more traditional teaching methods, but that many more students will get the concepts if they see practical application. And if they're consistently asked to explain their thinking, they'll construct a better understanding of why a solution works. With parents largely onboard and students thriving in her problem-based classroom, DePriest is now developing ways to measure and document her students' progress.

"I know it's the right change, I can see the progress with the kids. But now I must document and be able to prove to parents just how much their kids are learning," she says.

Creating a student-centered classroom, in which students tackle challenging math problems for which there are no clear answers, is a time-consuming affair. So is communicating and interacting regularly with parents. But the payoff is big when struggling students develop into skilled problem solvers and proud parents express their gratitude. Changing one's practice can be a long and difficult process, but, says DePriest, "It's worth it."

Rosemary Shinohara is an education reporter and Stephen Nowers is a photographer with the Anchorage Daily News.

Tips for Parents

Young children are natural problem solvers, and early childhood settings—where children interact with one another and participate in decisionmaking—offer countless opportunities for children to grow in their problem-solving abilities. These important experiences help children learn to value different kinds of thinking, think logically and creatively, and take an active role in their world.

Here are a few suggestions for parents on encouraging children to be creative problem solvers:

Brainstorm. Invite children to be fluent thinkers by asking them to respond to questions that have many right answers.

Reflect. Help children to be flexible thinkers by asking them to comment on specific objects or situations.

Challenge. Encourage children to practice critical and logical thinking by asking them open-ended questions.

Listen. Encourage children to express their ideas. Asking questions about things that don't make sense is another way children express critical thinking.

TECHNOLOGY TIPS to facilitate learning

BELOW ARE TECHNOLOGY RESOURCES and Internet sites that can facilitate learning through mathematical problem solving.

A spreadsheet program that stores data so the numbers can be crunched, graphed, and analyzed is a practical classroom resource. However, teaching young students how to use a spreadsheet program can take valuable class time. Now students can learn the value of a spreadsheet while the focus of the lesson is on problem solving and decisionmaking.

Ice Cream Truck is a simulation program from Sunburst Technology for grades two through six (sunburstdirect. sunburst.com/cgi-bin/SunburstDirect.storefront/). Student groups decorate an ice cream truck, buy and sell ice cream, and travel to different locations. Daily and weekly sales results are stored in summary sheets. Students can create custom spreadsheets to compare, for example, each group's sales results or to organize and sort data. Spreadsheet data can be exported to a more powerful spreadsheet for graphing or sharing.

The program includes a journal. Students can record why they chose a particular location, why they purchased one item over another, or how they decided what to charge for the ice cream. Student journal reflections may be used again and again as students explore the simulation, developing an understanding of relationships between variables by practicing math in a real-world scenario. Students can, for example, determine how weather, marketing, location, and product variety affect profits.

Group interaction and journal writing foster students' verbal skills. In their journals, students can write about their sales plan, and, later, reflect on the results of their plan.

Lessons and copyable handouts are provided.

other resources

The Tenth Planet series of software products from Sunburst (www.tenthplanet.com) provide an entrance into problem solving for elementary students. Assessment is an integral part of each program. Electronic journals are also a key feature of this resource. As students solve problems related to the real world, they can save their results and write comments beside their work.

The spreadsheet program, Cruncher 2.0 from Knowledge Adventure (www.knowledge adventure.com) is the latest version of a very successful classroom product. It is designed for middle-schoolers. Colorful organization and larger fonts immediately make this an inviting number playground. The teacher's guide has copyable pages linked to prepared spreadsheets.

Geometer's Sketchpad from Key Curriculum Press (www.keypress. com/sketchpad) provides much more than just the technology tools for exploring geometric concepts. The built-in text windows enable students, from grades five through 12, to describe their problem-solving processes as well as their conclusions. The ability to export data to a spreadsheet allows additional data analysis.

internet resources

LOGAL Software offers mathematics, life science, and physical science simulation software programs, available by subscription. To sign up for a free trial, visit www.riverdeep.net.

To practice buying and selling stocks, students can use the free simulation at the Web site, www. mainXchange.com.

Teachers and students can access information and data on changing global conditions and print maps from National Geographic Society's Web site, www. nationalgeographic.com/ education.

JUDI MATHIS JOHNSON
is a nationally published software reviewer.

books and materials available from THE CENTER'S LENDING RESOURCE COLLECTION

THE NWREL MATHEMATICS AND SCIENCE EDUCATION CENTER'S RESOURCE COLLECTION is a lending library of teacher-support material. Search the collection and request items from the Web site www.nwrel.org/msec/resources/ or call (503) 275-0457. The only cost is to mail items back at library rate.

Following are titles on problem solving for all grades:

A Collection of Performance Tasks and Rubrics
Charlotte Danielson & others (1997)
The four books in this series (primary, upper elementary, middle, and high school) provide performance tasks and scoring rubrics for key math topics. Includes samples of student work to clarify tasks and anchor the points of the scoring rubrics.

Posing Open-Ended Questions in the Primary Classroom
Christina Myren (1995)
Open-ended math questions for K-2 children, with children's literature serving as a springboard for many of the lessons. Lessons contain an overview, materials list, description, student responses and assessment discussion, and references.

Projects to Enrich School Mathematics Level I
National Council of Teachers of Mathematics (1990)
Independent study projects for grades four through six. Topics encourage student research, investigation and communication of mathematics through reporting and writing. Projects supply teachers with materials adaptable to a variety of settings.

Elementary and Middle School Mathematics: Teaching Developmentally
John Van de Walle (1994)
A resource for major topics in the K-8 mathematics curriculum, this guide and instructional resource shows teachers how to help all children become confident "doers of mathematics." Key elements are the nature of mathematics as a science of pattern and order, an understanding of how children learn mathematics, a view of teaching mathematics as a problem-solving endeavor, and specific methods for integrating assessment with instruction.

United We Solve: 116 Problems for Groups Grades 5-10
Tim Erickson (1996)
Tasks provide a structure for learning to work together and assume high standards for student performance, communication, and understanding of mathematical ideas. Problems focus on proportional reasoning, generalizing from patterns, spatial reasoning, and visualization.

101 Short Problems
Jean Kerr Stenmark (1995)
Open-ended questions for grades four through nine. Leading questions to motivate students to go beyond the first answer. All problems are in English and Spanish.

The New Sourcebook for Teaching Reasoning and Problem Solving in Junior and Senior High School
Stephen Krulik & Jesse Rudnick (1996)
A resource that educates teachers about teaching and evaluating reasoning and problem solving, and offers reasoning and problem-solving challenges for students. Includes performance tasks, problem-specific rubrics, and recommendations for basing instructional decisions on accurate assessment of student problem solving, reasoning, and communication skills.

High School Mathematics at Work
Mathematical Sciences Education Board & National Research Council (1998)
Essays accompanied by illustrative tasks from workplace and everyday contexts for high school students. How to develop curricula so that students learn to solve problems they are likely to encounter in life.

Get It Together: Math Problems for Groups Grades 4-12
Tim Erickson (1989)
More than 100 mathematics problems that three through six people can solve together, with each person holding information needed to find the solution. Includes advice on how to introduce, facilitate, and assess cooperative learning in the classroom.

AMY SUTTON
is a math and science resource specialist for NWREL.

problem solving AND project-based learning IN high school mathematics

In my teaching experience, I've found that project-based learning is an ideal context for problem solving and constructivist learning, the foundation of my pedagogical philosophy. To meet my students' learning needs, I've adopted a project-based curriculum that integrates subjects, creates opportunities to apply skills, and allows for higher-order thinking and problem solving.

With project-based learning, students are able to apply their skills and understanding of concepts over an extended period that involves them in a variety of real-world problem-solving situations.

A wider range of student learning styles and multiple intelligences can be accommodated with a project-based approach. For many students, the hands-on and applied manner of extended projects helps them to find meaning and justification for learning, often engaging those students who struggle most in academic settings.

Building a project-based curriculum that emphasizes open-ended problem solving is not easy. It requires one to shift primary focus from the textbook and skill learning to a curriculum that emphasizes active learning, application, synthesizing knowledge, and transferring strategies and skills to new situations. One must be willing to experiment with teaching and learning, to be creative in curricular approaches, using strategies that may not be traditional in school mathematics.

One must be willing to accept failure, learn from mistakes, and realize that change takes time, committing to a view of teaching and learning that is different from the traditional paradigm.

LET US HEAR FROM YOU

SUBMIT LETTERS TO:

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The rewards can be significant. By facilitating students' learning of content knowledge, as well as their reasoning and problem-solving abilities, this approach can help prepare them to meet state standards and assessments.

In Washington, project-based learning can help students master the Washington State Essential Learnings, the state's summary of important skills and content that include communication, reasoning, making connections, problem solving, and conceptual and procedural knowledge.

After examining the Washington Assessment of Student Learning (WASL), I was not at all concerned by how well my students might perform (see www.osd.wednet.edu/chsscimath/default.html). By combining skill building with opportunities to actively apply knowledge in a project-based curriculum, my students met the Essential Learnings and have performed well on the WASLs.

Project-based learning energizes and challenges me as a teacher. It motivates me to examine curriculum critically to ensure that I develop meaningful and effective projects. I continually explore my role as teacher and curriculum developer, and this involves me in the excitement of research as well as engaging me and my students in new activities.

LOUIS NADELSON
is a math and science teacher at Capital High School and Evergreen State College in Olympia.

CHRIS SANCHEZ, 14, A STUDENT AT KELLOGG MIDDLE SCHOOL IN PORTLAND, OREGON, employs the tools of the artist in this non-representational painting, using geometric shapes, colors, asymmetry, and perspective to explore relationships between diverse elements. Chris' painting was included in an exhibit by the Young Artist Project of Pacific Northwest College of Art and Portland Public Schools. The scholarship-based program provides time, materials, and encouragement to talented young artists, enabling them to exercise their creativity and build their skills in after-school art classes.

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