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ABSTRACT

This paper presents some ideas on how to utilize TI-83 Plus calculators to perform division of one polynomial (the divided) by another polynomial (the divisor) and how that procedure might be incorporated into a college algebra lesson. Four ways to obtain the quotient and remainder when dividing a polynomial by a first-degree polynomial are presented. Each program can be performed interactively using a TI-83 Plus calculator and some insights are provided into the synthetic division procedure as well as some other aspects of the mathematical method used. The paper includes several examples of problems together with a lesson plan for a college algebra course. (KHR)

Polynomial Division Using TI-83 Plus Calculators

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1. Introduction

In this short paper, I will present some ideas on how one could utilize TI-83 Plus calculators to perform division of one polynomial (**the dividend**) by another polynomial (**the divisor**), and how that procedure may be incorporated into a College Algebra Lesson. Let $f(x)$ be the dividend polynomial and $g(x)$ be the divisor polynomial. Assume that the degree of $f(x)$ is less than or equal to degree of $g(x)$. The division algorithm states that there are polynomials $q(x)$, **the quotient** and $r(x)$, **the remainder** such that,

$$f(x) = g(x)\,q(x) + r(x)$$

where the degree of $r(x)$ is less than the degree of $g(x)$. One can use the standard long division algorithm to perform polynomial division or use the *synthetic division* procedure. In the case of $g(x)$ being a linear polynomial (degree=1) with leading coefficient one (1), this procedure is well known and is available in any College Algebra textbook. I have created **a generalized synthetic division procedure [2]** that works for divisor polynomials of any degree. The preprint containing this procedure was enclosed with the Proceedings CDROM for the 13th Annual **T³** International Conference held at Columbus, Ohio in March 2000 [**1**].

2. Methods

Included below are four ways to obtain the quotient and remainder when one divides a polynomial by a first-degree polynomial of the form $(x-c)$. All these methods can be performed interactively using a TI-83+ calculator and will be a great hands-on practice for the students. These methods also provide some insights into the synthetic division procedure as well as to some other aspects of mathematical methods used. Alternatively, the methods can be used to produce TI83+ programs and run to obtain desired results. In **section 3**, I will provide two such program routines that one can utilize to do the synthetic division using some of the methods listed below.

Method 1: A Vertical Synthetic Division Table

One can manipulate the synthetic division on a TI-83 Plus calculator via an interactive synthetic division table. Suppose we want to divide $f(x)=a_nx^n+ a_{n-1}x^{n-1} +\ldots+ a_1x+a_0$ by $g(x)=x-c$. Create a table using **STAT -> 1:Edit ->** and enter coefficients of $f(x)$ on the **L1** List as follows. First row of **L2** will be 0. All rows of L3 are obtained by adding row elements of L1 and L2 (i.e. **L3=L1+L2**). To obtain the next row of L2 multiply previous row of L3 by c.

L1	L2	L3
a_n	0	$b_{n-1} = a_n$
a_{n-1}	$c\,b_{n-1}= c\,a_n$	$b_{n-2} = c\,a_n+a_{n-1}$
a_{n-2}	$cb_{n-2}=c(c\,a_n+a_{n-1})$	$b_{n-3} = c^2a_n+ca_{n-1}+a_{n-2}$
…	…	…
a_1	$cb_1=c(c^{n-2}a_n+c^{n-3}a_{n-1}+c^{n-4}a_{n-2} +\ldots+a_2)$	$b_0= c^{n-1}a_n+c^{n-2}a_{n-1}+c^{n-3}a_{n-2} +\ldots+ca_2+a_1$
a_0	$cb_0=c\,(c^{n-1}a_n+c^{n-2}a_{n-1}+c^{n-3}a_{n-2} +\ldots+ca_2+a_1)$	$R = c^na_n+c^{n-1}b_{n-1}+c^{n-2}a_{n-2} +\ldots+c^2a_2+ca_1+a_0$

Fill the columns L2 and L3 interactively using the above procedure imitating the standard synthetic division algorithm. This yields the quotient polynomial $g(x)= b_{n-1}x^{n-1}+ b_{n-2}x^{n-2} +\ldots+ b_1x+b_0$ and Remainder $R= c^n a_n+c^{n-1}b_{n-1}+c^{n-2}a_{n-2} +\ldots+c^2a_2+ca_1+a_0$. For example, when $f(x)= x^3+3x-7$ is divided by $x-3$, we obtain:

L1	L2	L3
1	0	1
0	3	3
3	9	12
-7	36	29

Hence the quotient is $g(x)= x^2+3x+12$ and the remainder is 29.

Alternatively, One can start with finding the remainder $R=f(c)$ and moving backwards to obtain the quotient polynomial as follows. Here the quotient polynomial is $g(x)= b_{n-1}x^{n-1}+ b_{n-2}x^{n-2} +\ldots+ b_1x+b_0$.

L1	L2
a_0	$f(c)=R$
a_1	$b_0= (R-a_0)/c$
a_2	$b_1=(b_0-a_1)/c$
...	...
a_{n-1}	$b_{n-2}=(b_{n-3}-a_{n-2})/c$
a_n	$b_{n-1}=(b_{n-2}-a_{n-1})/c= a_n$

Above example, in this format yields the following table. You need to calculate the remainder $f(c)$ using, for example, the function key on the calculator and tables or substituting c into the function $f(x)$ directly on the calculator.

L1	L2
-7	$f(3)=29$
3	$(29-(-7))/3=12$
0	$(12-3)/3=3$
1	$(3-0)/3=1$

Method 2: An Iterative Formula

One can also use the **sequence features** of TI83+ to do this division via defining a Recursive Sequence. Let $U(m) = a_{n-m}$, $m=0,\ldots,n$ be the sequence of coefficients of $f(x)= a_nx^n+ a_{n-1}x^{n-1} +\ldots+ a_1x+a_0$. Then one can define $V(m)$ to be the following sequence that will produce the quotient and remainder polynomials.

$$V(0)= U(0) = a_n$$
$$V(m)=cV(m-1)+U(m)$$
$$m=0,1,\ldots,n$$

Here the Remainder is $R=V(n)$ and quotient polynomial is $g(x)= V(0)x^{n-1}+ V(1)x^{n-2} +\ldots+ V(n-2)x + V(n-1)$. We can also define **U(m)**, by the **mth degree Taylor polynomials of f centered at x=0**, as

$$U(n-m) = a_m = (1/m!)\, f^{(m)}(0)$$

Where $f^{(m)}(0)$ is the value of m^{th} derivative of $f(x)$ evaluated at $x=0$. Recall that the numerical derivative is available on TI83+ under the math menu **MATH→8:nDeriv(**. For example, if Y1 stores the function f(x), we have:

$f^{(1)}(0)$ = nDeriv(Y1,X,0)
$f^{(2)}(0)$ = nDeriv(nDeriv(Y1,X,X),X,0)
$f^{(3)}(0)$ = nDeriv(nDeriv(nDeriv(Y1,X,X),X,X),X,0)
...
$f^{(m)}(0)$= nDeriv(...(nDeriv(nDeriv(Y1,X,X),X,X),...),X,0)

Method 3: A Derivative Approach

As mentioned in the above iterative formula procedure, one can obtain the coefficients of $f(x)$ using the nth degree Taylor polynomials centered as x=0. Hence the coefficients of the quotient polynomial as well as the remainder can be obtained via following formulas. This method is more time consuming to enter into the calculator interactively. It also takes a longer time to run in the calculator via a program. In view of this, this procedure is recommended only to demonstrate a use of derivatives in a calculus class.

$b_{n-1} = a_n = (1/n!)\, f^{(n)}(0)$
$b_{n-2} = c\, a_n + a_{n-1} = c(1/n!)\, f^{(n)}(0) + (1/(n-1)!)\, f^{(n-1)}(0)$
$b_{n-3} = c^2 a_n + c a_{n-1} + a_{n-2} = c^2 (1/n!)\, f^{(n)}(0) + c(1/(n-1)!)\, f^{(n-1)}(0) + (1/(n-2)!)\, f^{(n-2)}(0)$
...
$b_0 = c^{n-1} a_n + c^{n-2} a_{n-1} + c^{n-3} a_{n-2} + \ldots + c a_2 + a_1 = c^{n-1}(1/n!)\, f^{(n)}(0) + c^{n-2}(1/(n-1)!)\, f^{(n-1)}(0) + \ldots + c f^{(2)}(0) + f^{(1)}(0)$
$R = c^n a_n + c^{n-1} b_{n-1} + c^{n-2} a_{n-2} + \ldots + c^2 a_2 + c a_1 + a_0 = c^n (1/n!)\, f^{(n)}(0) + c^{n-1}(1/(n-1)!)\, f^{(n-1)}(0) + \ldots + c f^{(2)}(0) + f^{(1)}(0) + f^{(0)}(0).$

Method 4: A Matrix Procedure

One can create a column matrix C_1 containing the coefficients of the dividend polynomial, $f(x)$, of size $(n+1) \times 1$. Then attach column matrices to C_1 to create a square matrix of size $(n+1)\times(n+1)$. If we are dividing $f(x)$ by $x-c$, the i[th] columns of the square matrix will be $\mathbf{C_i = c^{i-1}\, C_1}$. Then the off diagonal sums of the square matrix up to the center off diagonal will produce the coefficients of the quotient polynomial. The sum of the elements of the **center off diagonal** is the remainder.

$$\begin{bmatrix} a_n & ca_n & c^2a_n & \ldots & c^{n-1}a_n & c^na_n \\ a_{n-1} & ca_{n-1} & c^2a_{n-1} & \ldots & c^{n-1}a_{n-1} & c^na_{n-1} \\ a_{n-2} & ca_{n-2} & c^2a_{n-2} & \ldots & c^{n-1}a_{n-2} & c^na_{n-2} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_1 & ca_1 & c^2a_1 & \ldots & c^{n-1}a_1 & c^na_1 \\ a_0 & ca_0 & c^2a_0 & \ldots & c^{n-1}a_0 & c^na_0 \end{bmatrix}$$

That is, if the quotient polynomial $g(x)= b_{n-1}x^{n-1} + b_{n-2}x^{n-2} + \ldots + b_1x + b_0$, then $b_{n-1} = a_n$, $b_{n-2} = c\, a_n + a_{n-1}$, $b_{n-3} = c^2 a_n + c a_{n-1} + a_{n-2}$, ..., and $b_0 = c^{n-1} a_n + c^{n-2} a_{n-1} + c^{n-3} a_{n-2} + \ldots + c a_2 + a_1$. Furthermore the Remainder is $R = c^n a_n + c^{n-1} b_{n-1} + c^{n-2} a_{n-2} + \ldots + c^2 a_2 + c a_1 + a_0$. If the column matrix is inverted as $C=[\, a_0\ a_1\ \ldots\ a_{n-1}\ a_n]^T$, then The remainder is simply the trace of square matrix and traces (sums) of lower diagonals up to the center diagonal provide the coefficients of the quotient polynomial.

Instead of creating a matrix, one can certainly do the same using lists defined via the columns of the above matrix. i.e. **L1** is defined to be the coefficients list. The other lists, **L2, L3,... , Ln** can be defined to be $\mathbf{Li = c^{i-1}\, L1}$.

3. Programs

In this section, I have produced two programs that can be used on the TI83+ calculator. Obviously, some programs will work faster than the others and are more efficient. If you have any suggestions to improve on these programs or alternative approaches, please do not hesitate to share them with me.

Program 1

The Following program can be used to divide any **polynomial of degree up to 6** by a polynomial of the form *x-c*. It can be extended very easily to any degree by adding additional columns as desired.

```
PROGRAM: SYNDIV1
:ClrAllLists
:Output(1,1, "ENTER DIVI POLY"
:Input "LIST OF 7 COEFFS", L1
:Lbl 11
:Input "ENTER A for X-A=", A
:L1(1) → L2(1)
:A*L2(1)+L1(2) → L2(2)
:A*L2(2)+L1(3) → L2(3)
:A*L2(3)+L1(4) → L2(4)
:A*L2(4)+L1(5) → L2(5)
:A*L2(5)+L1(6) → L2(6)
:A*L2(6)+L1(7) → R
:Disp "REMAINDER IS", R
:Disp "QUOTIENT IS", L2
:Goto 11
```

When above program is run on the TI83 Plus, we get following output screens. You need to clear the home screen before running the program {may try **ClrHome** too}.

ENTER DIVI POLY	
LIST OF 7 COEFFS	←This indicates that the dividend polynomial must be entered as a list of 7 elements containing its coefficients. We must enter these coefficients in descending order of powers for the program to work. i.e. { a_n a_{n-1} … a_2 a_1 a_0}
{0,0,0,1,0,3,-7}	←How the coefficients are entered
ENTER A FOR X-A=	←Enter A for the divisor polynomial written as (X-A) when prompted as this
3	←When dividing by $x-3$

The program will run and produce the following results

REM IS	
29	← This is the remainder when x^3+3x-7 is divided by $x-3$
QUOTIENT IS	
{0 0 0 1 3 12}	← This mean the quotient polynomial is $x^2+3x+12$.
ENTER A FOR X-A=	← The program prompts for more A values which is very useful when you are trying to find zeros of the polynomial by repeated division.

Program 2

The Following program can be used to divide any **polynomial of degree up to 5** by a polynomial of the form *x-c*. It can be extended very easily to any degree by adding additional columns as desired.

```
PROGRAM: SYNDIV2
:ClrAllLists
:Output(1,1, "ENTER DIVI POLY"
:Input "LIST OF 6 COEFFS", L1
:Lbl 22
:Input "ENTER A for X-A=", A
:A*L1 → L2
:A*L2 → L3
:A*L3 → L4
:A*L4 → L5
:A*L5 → L6
:List matr(L 1, L2, L3, L4, L5, L6, [B])
:[B](1,1) → LQUO(1)
:[B](1,2) +[B](2,1) → LQUO(2)
:[B](1,3) +[B](2,2) +[B](3,1) → LQUO(3)
:[B](1,4) +[B](2,3) +[B](3,2) +[B](4,1) → LQUO(4)
:[B](1,5) +[B](2,4) +[B](3,3) +[B](4,2) +[B](5,1) → LQUO(5)
:[B](1,6) +[B](2,5) +[B](3,4) +[B](4,3) +[B](5,2) +[B](6,1) → R
:Disp "REMAINDER IS", R
:Disp "QUOTIENT IS", LQUO
:Goto 22
```

Notice that LQUO is a list defined on the calculator to store the coefficients of the quotient polynomial.

4. Generalized Synthetic Division Procedure

Following is the step-by-step procedure of the generalized synthetic division algorithm [2]. Suppose we want to divide $f(x)=b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0$ by $g(x)=x^n - a_n x^{n-1} - \ldots - a_2 x - a_1$. Notice that we have arranged both polynomials in the descending order of powers of x. For simplicity, we also assumed that the leading coefficient of $g(x)$ is 1. As we did in the first order synthetic division procedure recall that one writes 0 as the coefficients of missing powers. Here $deg(f)=m$ and $deg(g)=n$.

Process

STEP 1. Solve the divisor $g(x)=0$ for its leading term to obtain the expression $x^n = a_n x^{n-1} + \ldots + a_2 x + a_1$

STEP 2. First row of synthetic division is the coefficients of $f(x)$ and $g(x)$ arranged as follows, a direct generalization of first order synthetic division.

$a_n \quad a_{n-1} \quad \ldots a_2 \quad a_1 / b_n \qquad b_{n-1} \quad \ldots b_2 \quad b_1 \quad b_0$

STEP 3. First column after the / will be all zeros (0) below b_m. Number of zeros will be equal to $deg(g)=n$. Now we are ready to get the first number of the Final row (will be called **Sum Row** here after for obvious reason), which is the sum s_1 of all numbers on that column (yes, it is just b_m). Now multiply $a_n\, a_{n-1} \ldots a_2\, a_1$ by s_1 and write the answers on second column onwards starting from last row going up one row at a time. Now add all numbers in second column to obtain s_2. Multiply $a_n\, a_{n-1} \ldots a_2\, a_1$ by s_2 and write the answers on third column starting from last row and going up one row at a time. Add this Column to get the sum number s_3. Repeat this process until a product adds a number to the last column [the Constant term Column], which happens at the top line below b_0. Add all the remaining columns to complete the sum row.

STEP 4. Last n numbers in the sum row are the coefficients of the remainder $r(x)$. As in first order synthetic division, create the quotient polynomial of degree m-n, $q(x)$ starting with the first sum number s_1 as the coefficient of x^{m-n} term and proceeding right along the sum row using the first m-n+1 numbers. Last n numbers proceeding left from the constant term along the sum row will produce the remainder polynomial $r(x)$.

Examples:

01. Suppose we are dividing $f(x)=x^5-8x^4+7x^3-3x^2+4x+7$ by $g(x)=x^2+5x+6$. In our **generalized synthetic division table**, we obtain the first row and the completed first column after vertical line as follows:

-5	**-6**	1	-8	7	-3	4	7
		0					
		0					
		1					

Multiplication by $s_1=1$ yields:

-5	**-6**	1	-8	7	-3	4	7
		0		-6			
		0	-5				
		1					

This adds up to give s_2=-13 and multiplication by s_2=-13 yields the next column and sum s_2=66.

-5	**-6**	1	-8	7	-3	4	7
		0		-6	78		
		0	-5	65			
		1	**-13**	**66**			

Proceeding this way we completes the following synthetic division table.

-5	**-6**	1	-8	7	-3	4	7
		0		-6	78	-396	1530
		0	-5	65	-330	1275	
		1	**-13**	**66**	***-255***	***883***	***1537***

Therefore our quotient is $q(x)=x^3-13x^2+66x-255$ and Remainder $r(x)=883x+1537$.

Implementation of this procedure on a TI-83 Plus calculator can be done using any of the procedures suggested above with obvious generalizations. These methods, of course, can be implemented much easier on a CAS calculator [such as TI-89 or TI-92], or using software such as DERIVE, MAPLE, or Mathematica. This will be dealt with separately in a paper by this author. Our focus here would mainly be on first-degree divisor polynomials.

5. A Lesson Plan for a College Algebra Course

In this section, I will produce a typical college algebra lesson that could be carried out with the aid of TI83 Plus calculator equipped with one of the programs listed above. For simplicity, I will use the program **SYNDIV1** (as labeled in **Section** 4) in the following examples.

Plan

- Introduce polynomial functions, and their graphs produced via TI 83 Plus
- Define zeros of polynomials and obtain some using TI83 Plus
- Division algorithm and synthetic division – use **Section 1** as appropriate
- Introduce Remainder Theorem and Factor Theorem, and produces examples for them using one of the above program
- Introduce Rational Zeros Theorem and produces examples using one of the above programs
- Introduce Descartes' Rule of Signs, Upper and lower bounds for zeros, and Fundamental Theorem of Algebra and produces examples for these results using a synthetic division program on TI-83 Plus.

Examples

Here are some examples to get you started on using these procedures and programs.

1. Divide x^3-5 x^2+7 by x-8.

Run SYNDIV1. When prompted for ENTER DIVI POLY LIST OF 7 COEFFS, enter {0,0,0,1,-5,0,7}. When prompted for ENTER A FOR X-A=, input 8. Your outputs are REM IS 199, QUOTIENT IS {0 0 0 1 3 24}. Therefore the Quotient = $x^2+3x+24$ and Remainder = 199.

2. Find all rational zeros of f(x)=$3x^3-2x^2+7x+3$.

By Rational zeroes theorem, possible rational zeros are ±1, ±3, and ±1/3. Run SYNDIV1. When prompted for ENTER DIVI POLY LIST OF 7 COEFFS, enter {0,0,0,3,-2,7,3}. When prompted for ENTER A FOR X-A=, input 1. Your outputs are REM IS 11, QUOTIENT IS {0 0 0 3 1 8}. Therefore the Quotient = $3x^2+x+8$ and Remainder = 11. The program stops at the prompt ENTER A FOR X-A=. We continue with other possible A values –1,1/3,-1/3,3,-3 to find that none leads to a remainder 0. Hence this polynomial has no rational zeros.

3. Find all rational zeros of $g(x)= x^4+6x^3+7x^2-6x-8$.

As in above example, enter {0,0,1,6,7,-6,-8} as the coefficient list. Then we enter potential rational zeros ±8/±1 = ±1, ±2, ±4, and ±8 one at a time as A values. Notice that 1, -1, -2, and -4 produce remainders of zero. Hence those are the 4 rational zeros of $g(x)$ and furthermore $g(x)$ factors completely as:

$$g(x)=(x+1)(x-1)(x+2)(x+4).$$

4. Find integers that are upper and lower bounds for the real zeros of the polynomial $h(x)= x^4-2x^3+x^2-9x+12$.

Run SYNDIV1. Enter {0,0,1,-2,1,-9,12}. Enter A=1, 2, ... until end up with all **no negative entries** in the remainder and quotients. In fact, A=3 produces Remainder=21 and Quotient as {0 0 1 1 4 3}. Hence 3 is an upper bound for the real zeros. Now enter A=-1,-2,... until you end up with **alternatively nonpositive and nonnegative entries** for quotient and remainder. As we see A=-1 produces {0 0 1 –3 4 –13} {25} which is alternatively nonpositive and nonnegative. Hence –1 is a lower bound for the real zeros of $h(x)$.

6. References

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