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ABSTRACT

Noting that current research on children's mathematical development does not adequately detail how toddlers represent small numbers and the role that number words play in the development of number understanding, this study used a combination of methods to examine mathematical development in one toddler. Underlying the study was an Integrated Model (IM) of the development of preschoolers' mathematical abilities that considers both how children represent numbers and what they represent in their mathematical thinking. Three data collection methods were used to examine the mathematical competence and development of one child, Blake, from age 18 through 36 months: (1) naturalistic observations conducted by the child's mother; (2) the microgenetic method, in which elected nonverbal matching and nonverbal production tasks were administered at 26 and 30 months; and (3) a teaching experiment, the Kumon mathematics program, involving teacher-directed drill and practice with worksheets to present math concepts incrementally. Findings revealed that Blake initially used isolated number words in a nonfunctional manner. In time, he used them with discrimination to identify the numerosity of small visible collections. Later, Blake used number words with increasing effectiveness to identify a partially visible collection and then nonvisible collections. During this study, Blake was unable to recognize the numerosity of collections composed of sequentially presented sounds. Observational data and sample Kumon worksheets are appended. (Contains 43 references.) (KB)

Running Head: THE CASE OF BLAKE

The Case of Blake: Number-Word and Number Development

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The Case of Blake: Number-Word and Number Development

Current research on children's mathematical development does not adequately detail how toddlers represent small numbers (Mix, Huttenlocher, & Levine, 2001). The role that number words play in the development of number understanding is also unclear. Observing children in their homes provides a rich context for noting the intricacies of such development (Anderson, 1991; Blevins-Knabe & Musun-Miller, 1996; Fuson, 1988). Furthermore, mothers may be a significant source of support for learning about children's development (Anderson 1991; Blevins-Knabe & Musun-Miller, 1996). Unfortunately, only a few studies (e.g., Anderson, 1991; Baroody, 1987a; Baroody & Price, 1983; Fuson, 1988) have involved examining number and number-word development in the home environment during the preschool years. The purpose of the present study was to understand better this development by using a combination of methods, including a mother's observations of her toddler's mathematical progress.

THEORETICAL FRAMEWORK—AN INTEGRATED MODEL

Researchers have offered conflicting conclusions about the data on infants' (e.g., Sophian, 1998; Wynn, 1992, 1998) and preschoolers' mathematical abilities (Huttenlocher, Jordan, & Levine, 1994; Jordan, Huttenlocher, & Levine, 1992; Jordan, Huttenlocher, & Levine, 1994; Levine, Jordan, & Huttenlocher, 1992; Mix, 1999a; Mix, Huttenlocher, & Levine, 1996; Sophian, 1987, 1988a, 1988b; Sophian, & Adams, 1987; Sophian, Wood, & Vong, 1995). Our theoretical perspective builds on two efforts to fashion theoretical models that account for preschoolers' mathematical strengths but are more cautious than current nativists' models (e.g., Wynn, 1998). According to the first of these (Huttenlocher et al.'s 1994 Mental Models view), *how* children represent number changes dramatically during early childhood. According to the second (Resnick's 1992 model of mathematical development), *what* children represent also evolves--mathematical thinking evolves from concrete (context-specific) to abstract (general). In the Integrated Model (IM), these two theories are merged to take into account both how and what children represent (see Figure 1 for a comparison of the two models).

Pre-Transition One: An Approximate Mechanism for Representing Quantity

Proponents of the Mental Models view argue that the development of a numerical representation is more complicated than that suggested by the nativist view (see Mix et al., 2001). According to nativists, infants—from the start—differentiate between discrete and continuous quantities. Furthermore, they contend that infants can represent small discrete quantities *exactly*. In this view, conventional counting knowledge presumably builds directly on this existing pre-counting knowledge.

According to the Mental Models view, children may not initially differentiate between discrete and continuous quantities. Instead, Huttenlocher and colleagues (Huttenlocher et al., 1994) suggest that infants represent quantity with an approximation mechanism based on continuous amount rather than discrete number. Infants' approximate representation may reflect their tendency to focus on the overall amount in terms of density and area, which usually co-varies with number (Mix et al., 2001).

A Pre-Transition One phase is consistent with Resnick's (1992) view that mathematical thinking begins with *protoquantitative reasoning*: inexact or qualitative reasoning about uncounted amounts. At this level, "comparisons of amounts are made and inferences can be drawn about the effects of various changes . . . on amounts; but no numerical quantification is involved" (p. 403).

Figure 1. An integration of two models of mathematical development.

Unlike Resnick's model, the IM accounts for Transition 1 of the Mental Models View. As a result, unlike the former, the latter includes five logically plausible sublevels for protoquantitative thinking: Levels 0, 1A, 1B, and 1C (the highlighted portion of column 2 below). (These four levels may extend beyond Transition 1, 2, or even 3.) Unlike Resnick's model, the IM also accounts for Transition 3 and, thus, the possibility that level 3 or 4 reasoning could occur before or after children learn a written representation. Furthermore, the relations among Resnick's four levels may not be entirely linear. For example, the development of Level 3 thinking in one domain may prompt lower-level thinking in this and other domains. Unlike the Mental Models View, the IM accounts for levels of thinking within a phase. For example, between Transitions 2 and 3, there may be as many as four levels (Levels 1C, 2, 3, and 4). Moreover, unlike the Mental Models View, the IM includes content knowledge such as that highlighted in column 3 below. Finally, the IM, unlike Resnick's model or the Mental Models View, includes an explicit distinction between *nonverbal number recognition* (the rapid apprehension of number without counting) and *verbal number recognition* (rapid number apprehension associated with a number word).

Component 1—Mental Models View (Huttenlocher et al., 1994): Changes in the forms of representations (ways concepts are represented)	**Component 2—Based Partly on Resnick's (1992) Development of Mathematical Thinking Model: Changes in the representational content (kinds of conceptual entities)**	**Examples from Children's Understanding of Number Suggested by the IM**
Pre-Transition 1	Level 0: protoquantitative thinking involving qualitative reasoning about non-quantified entities	Objects can be identified and grouped (classified) by physical properties such as color.
	Level 1: protoquantitative thinking involving qualitative reasoning about inexact, nonverbally represented (uncounted but perceptually estimated) quantities	Collections of objects can be classified/identified by relative size such as their approximate area or total perimeter (contour length).
Transition 1: inexact (estimated) nonverbal representation → exact nonverbal-based representation	Level 1A: first transitional subphase to quantitative thinking involving qualitative reasoning about exact, nonverbally recognized and represented quantities	The nonverbally recognized collection ••• is more than the nonverbally recognized collection ••. May start to learn number words but not their meaning.
	Level 1B: second transitional subphase to quantitative thinking involving quantitative reasoning about exact, nonverbally recognized and represented quantities (tied only loosely to number words)	The nonverbally recognized collection ••• is *one* more than the nonverbally recognized collection ••. May recognize that numbers can be used to characterize/identify collections but still fuzzy about what, for example, "two" or "three" means.
Transition 2: exact nonverbal representation → exact, verbal-based representation	Level 1C: third transitional subphase to Level 2 thinking involving quantitative reasoning about exact, verbally subitized and represented quantities tied to number labels	The verbally recognized collection ••• or "three" is one more than the verbally recognized collection •• or "two." Clarifies the meaning of "two" (and perhaps somewhat later, "three") and so can consistently apply a number term discriminately.
	Level 2: (counting-based) quantitative reasoning about verbally recognized *or* counted collections (reasoning about numbers tied to a specific context)	The counted collection ••• ("one, two, three, —*three*") is one more than the counted collection •• ("one, two—*two*").
	Level 3: numerical reasoning (reasoning about numbers in the abstract)	"*Three* is more than *two* because it comes after *two* when counting."
	Level 4: abstract reasoning (generalizing about numerical relations)	"The later a number comes in the counting sequence, the larger the quantity it represents."
Transition 3: exact verbal representation → exact written representation	Level 3: numerical reasoning Level 4: abstract reasoning	

Transition One: The Development of Exact Nonverbal Numerical and Arithmetical Processes

According to the Mental Models view, as they mature, infants replace or augment their approximate representation with one built on discrete number. There is a fundamental change in children's representations when they begin to depict quantity based on exact number independent of the size or mass of each item. An exact representation of number might underlie advances across a range of numerical abilities (Mix et al., 2001). Some research indicates that preschoolers' ability to respond exactly in equivalence, ordination, and cardination tasks first appears around two-and-a-half years old (Huttenlocher et al., 1994; Jordan et al., 1994; Mix, 1999a; Mix et al., 1996; Mix et al., 2001). When this shift occurs, children no longer estimate the collection size but produce exact matches for collections of up to two items. Some young children even solve calculation problems correctly up to a total numerosity of two. The ability to respond precisely gradually extends to larger numerosity problems (Mix et al., 2001).

To explain this shift in representation, Huttenlocher and colleagues (Huttenlocher et al., 1994; Mix et al., 2001) proposed that children use mental models around the same time that they are able to represent images symbolically in areas such as language, model use, and pretend play. By acquiring a symbolic one-to-one representation of number in early childhood, children construct a mental version of the hidden collection by representing features of the collection that are necessary for calculation. Young children's mental representations do not preserve features that are not relevant, such as the color of the individual items or their spatial arrangement. The representations allow children to manipulate the mental symbols sequentially and spatially, ultimately leading to children's ability to add or subtract.

Existing evidence indicates that young children are more successful on nonverbal number and arithmetic tasks than on verbal ones (Huttenlocher et al., 1994; Jordan, Blantero, & Uberti, in press; Jordan et al., 1992, 1994; Levine et al., 1992). In this model, nonverbal numerical and arithmetical competencies, then, develop before assimilating verbal-based numerical and arithmetical knowledge (Transition Two) and probably provide a foundation for it. This account is consistent with Resnick's (1992) view that children reconstruct arithmetical and numerical knowledge at the protoquantities level with the aid of counting experiences, which provides a scaffold for the quantities level (narrowly applied verbal-based counting and computational knowledge).

Transition Two: The Development of Verbal-Based Numerical and Arithmetical Competencies

According to the Mental Models view, soon after attaining a symbolic ability, children learn conventional symbols and connect them to their discrete, preverbal representations. These connections might lead to children forming a verbal number recognition ability similar to that observed in older children and adults (Mix et al., 2001).

Drawing from Resnick's (1992) model, Transition Two permits the emergence first of (narrow) quantitative reasoning, then (broader) numerical reasoning, and ultimately, general or abstract reasoning. The first involves reasoning about specific quantities—numbers tied to a particular and meaningful context (e.g., three candies and two more candies is the same as two candies and three more candies). Numerical reasoning entails reasoning about numbers in the abstract—without reference to a particular and meaningful context (e.g., three and two more equals two and three more). Abstract reasoning involves recognizing general principles that apply to any context or numbers (e.g., the principle of additive commutativity: the order in which any two numbers are added does not affect their sum).

RATIONALE

Despite these theories of mathematical development, there is little research on the origins of children's mental models of discrete quantities. What still needs to be explained is why Huttenlocher and colleagues (e.g., 1994) observed a shift from inexact to exact responses between the ages of two and three. Put differently, if in fact a mental model of discrete quantity emerges at that developmental stage, what are its bases or precursors? We raise two related issues below.

First, recent research (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, in press) indicates that infants use perceptual cues, such as contour length, area, or volume to quantify collections. Thus, they might not even distinguish between collections of one and many items if these collections had the same perceptual cues. If so, this distinction might mark the first important step toward differentiating between discrete and continuous quantities. Furthermore, it seems likely that it would occur well before two-and-a-half and three-and-a-half years old, when Huttenlocher et al. (1994) hypothesized that children develop a symbolic ability, including a mental model of discrete quantities. Might the construction of a mental model for discrete quantities occur much earlier and evolve more slowly than suggested by the current Mental Models view?

The second issue is the need for uncovering how significant number words are in recognizing number as a means of categorizing. Another essential step toward understanding number is recognizing that—like color, size, and weight—it is an important basis for categorizing and, thus, identifying and comparing items (Mix et al., 2001). Although this process of categorizing probably starts before language develops, language provides labels that facilitate and accelerate this process. Consider Huttenlocher et al.'s (1994) conclusion that two-and-a-half-to three-and-a-half-year-olds who were not proficient counters (e.g., could not accurately *enumerate*: one-to-one object counting or were unable to *verbally produce one or two items*: create a collection of a specified number of objects) were successful on simple matching tasks—those involving similar items. However, it is not clear that such pre-Transition Two (pre-counting) children had not attained the meaning of *one* or *two*, as Huttenlocher and colleagues claimed.

METHODS AND DATA SOURCES

Our pilot efforts with young children convinced us that administering psychological tasks in the traditional manner might not provide an accurate picture of their competence. In general, the younger the child, the less willing they were to answer thoughtfully or at all as the task type moved from left to right in the following continuum: (a) child initiated—unguided response, (b) child initiated—guided response, (c) adult initiated—invited response, and (d) adult imposed—directed response. Therefore, using traditional research techniques, such as a cross-sectional design, would limit the value of the data collected.

Methods

To address the need for more valid data, we used a methodologically sophisticated and systematic effort to see if the IM adequately explained our participant's, Blake's, mathematical development. Blake was 18 months old when the study began in June 2000. We examined his mathematical competence and development using a long-term (14 month) case study based on naturalistic observations and the microgenetic method. Using the latter, we repeatedly administered selected tasks in a specific manner at different intervals, particularly during an apparent transition in development. Detailed knowledge of Blake's developmental readiness and performance in his natural environment (as documented by naturalistic data) can significantly improve the understanding of Blake's performance in a microgenetic study. The data from the microgenetic study can provide naturalistic observations with a clear direction (e.g., direct attention to essential behaviors or patterns of behavior). Using a combination of methods can provide richer data on number development than using any single method.

Naturalistic Observations

Blake's mother conducted the naturalistic observations. She recorded her observations of his everyday mathematical behavior on a personal digital assistant as they occurred. She transferred these notes to a computer file (see Appendix A for a table of these observations)[1]. She noted any mathematical advance, however minor, because a seemingly insignificant event might--in the context of a developmental history--be an important turning point or transition. Because Blake's actions determined the events, they did not occur at regularly scheduled intervals. The observations focused on mathematical developments that dealt with anything seemingly connected to numbers and operations on them. As this was a difficult enough task, Blake's mother did not document his geometry development.

The benefit of the naturalistic method is that Blake's mother was able to observe him during most of his waking hours. Many of the observations arose while they were preparing food in the kitchen, driving in the family van, and eating at restaurants. These settings provided authentic opportunities for Blake to exhibit his mathematical knowledge. Because the observations were not limited to one particular location or time, they show the pervasiveness of mathematics in Blake's life.

[1] The author, Blake's mother, began the study videotaping his play to capture any significant mathematical events. However, it was determined that many of the mathematical events occurred throughout the day in environments other than his room. Therefore, the videotape missed crucial data.

Microgenetic Methods

The microgenetic component involved administering the nonverbal matching and nonverbal production tasks to Blake at 26 and 30 months. Blake's mother took notes, particularly in regard to any developmental changes, and videotaped the sessions. After the sessions, the first author transcribed the conversations during the tasks to gain a more accurate description of Blake's knowledge.

Inexact or Exact Nonverbal Matching Task. The purpose of the matching task was to test simple one-to-one correspondence—to observe whether Blake could re-create collections of one to four items when a model collection remained visible. If Blake was unable to match, he should be unable to produce a collection once it was covered. This task ensures that failure on the production task was not due to Blake being unable to match.

We planned to have him participate for three days in a row to gauge learning effects or consistency. The materials for both tasks consisted of 20 black checkers divided evenly between the mother and Blake. Additionally, there were black and white drawings of a truck, fish, and snowman. The fourth mat was a blank page to ensure that the designs were not confusing or disrupting the children. We collated copies of the pictures so they were in two ordered stacks. The mother rotated the picture and numerosity in each session to make sure that the pictures did not affect Blake's response. We designed the tasks so each drawing had each numerosity, one through four by the end of the three days.

Two matching demonstrations were conducted. On the first demonstration trial, Blake's mother took one disk from the pile and placed it on her mat in full view of Blake. Next, she put one disk from Blake's pile and placed it on his mat in full view of him. She stated, "See, yours is just like mine," pointing to the disks on both mats.

The demonstration item was presented again, following the same procedure, except this time Blake was asked to place the appropriate number of disks on his mat after the disk was presented. If Blake did not respond or placed the wrong number of disks on the mat, he was corrected and the item was repeated one more time. After Blake's mother demonstrated the matching procedure with one item, she demonstrated the procedure with two. Next, his mother had Blake attempt matching one to four items in that order. (In pilot work with 2-year-olds, a random order did not work. Presenting a more difficult matching trial first resulted in off-task behavior.) She took one page, placed the checker on her mat, and gave Blake a copy of the same page. Blake's mother instructed him to make his mat (blank page, snowman, fish, or truck) just like hers, where quarter-sized checkers represented "checkers," "buttons," "bubbles," and "cans." During this task, Blake could put out any number of checkers (up to 10) on any trial. This procedure continued for the rest of the numerosities. On a score sheet, Blake's mother recorded exactly what Blake put on his mat. This provided data for the error analysis.

Inexact or Exact, Nonverbal or Quasi-Verbal Production Task. The production task is similar to the matching one. The purpose of this task was to gauge whether Blake could nonverbally use number recognition and produce collections of one to four items. Blake's mother introduced the task by saying, "Now we're going to play the 'Hiding Game.' Let me show you how it's played." The same demonstrations were conducted as in the matching task, except Blake's mother covered the disks. She covered her mat six seconds after creating a collection (see Huttenlocher et al., 1994). She used white cardboard the same size as the mat to cover the checkers. She also provided Blake a cover because we found in our pilot work that children enjoy modeling the researcher.

On the first demonstration item, Blake's mother took one disk from the box and placed it on her mat in full view Blake. She showed the disk long enough for him to get a good look and represent it in his mind.

Then, Blake's mother covered her mat. She said, "Make yours just like mine." Next, she put one disk from the box and placed it on Blake's mat in full view of him. The mother then stated, "See, yours is just like mine," pointing to the disks on both mats.

The mother presented the demonstration item again, following the same procedure, except this time she asked Blake to place the appropriate number of disks on his mat after she presented the disk. If he did not respond or placed the wrong number of disks on the mat, he was corrected and the item was repeated one more time. The same demonstration procedure was used with two disks.

After the demonstration items, Blake was asked to match the quantities of four test items, ranging in numerosity from 1 to 4. Blake's mother presented the matching test items sequentially based on difficulty. She rotated the pictures so the same picture was not used with the same numerosity. For each test item, Blake's mother took a set of disks from the box and placed it on her mat in full view of him. She then hid the disk collection under the cover. Blake indicated how many disks were under the cover by placing the appropriate number of disks on his picture. On a score sheet, Blake's mother recorded exactly what he produced.

Kumon Data

The third data source is a blend of the following methods: naturalistic, microgenetic, and a teaching experiment. Blake's mother enrolled him in the Kumon mathematics program. The Kumon curriculum involves teacher-directed drill and practice using worksheets to present math concepts incrementally. According to the Kumon technique, children master skills through repeated practice. Blake completed between one to five worksheets daily, focusing solely on enumerating collections of dots and pictures (see Appendix B for sample Kumon worksheets). Blake's enrollment was not initially part of the design of this study, particularly because the approach contradicts the authors' teaching philosophies. However, because Blake was having difficulty with speech, his mother used Kumon as a means for Blake to have more opportunities to communicate. As he participated in Kumon, Blake's mother took observational notes on advances. Interestingly, Blake's performance in Kumon oftentimes was inconsistent with his performance in everyday situations.

Scoring Data

For the microgenetic component, the first author analyzed the conversations in light of Blake's responses and behavior on the videotape. She looked for commonalities among the responses to determine how his behavior connected to his everyday mathematical conversations. In addition to analyzing the conversations, we scored his responses as right, wrong, or no response. Blake scored one point for each exact correct match or production. The total possible score on the matching and production task each ranged from 0 to 4. Responses were also analyzed qualitatively to evaluate whether Blake's response could be an estimate of the corresponding numerosity.

Figure 2. Skills, definitions, and examples arising in Blake's mathematical development.

Skills	**Definition**	**Example**
Pre-number recognition	Stating the verbal number name without quantitative significance	Alexis was giving Blake raisins. She said, "You can only have three." Blake pointed his finger to his highchair and said, "One" before Alexis put any raisins down.
Verbal number recognition	Stating the numerosity of the set without conspicuous enumeration	We were playing with straws in a restaurant. Alexis asked him, "How many straws?" He replied correctly, "Two."
Verbal counting	Stating the counting words in a sequence	He recited the correct numerical sequence up to 5 in Kumon. Alexis pointed to dots and as she pointed Blake said, "One, Two, Three, Four, Five."
Enumerating a collection	One-to-one object counting	This evening, we gave him 3 grapes. Kenneth said, "Can you count them?" He put them from my hand to Kenneth's. He said "One" as he put the first one in Kenneth's hand. Then he placed a second and said, "Two." After the third, he said "Three." Kenneth said, "How

		many?" and he said "Three."
Matching	Simple one-to-one correspondence by recreating collections of one to four items when a model collection remained visible	Researcher: "Ready. I'm going to do this with my mat. Can Blake make his mat just like mine?" (Blake grabs one checker and places it on the mat. Then he covers it up.) Researcher: "Good job." (He wants to keep the checker.)
Production	Verbal production: creating a collection based on a specified number of objects Nonverbal production: creating a collection based on the number of objects that were presented and then hidden Theoretically, this task is more difficult than matching because the child has to mentally represent the set.	He responded sporadically when I had two disks down. He did have four disks in his hand. I told him to put down two. He ended up handing me two and said, "Mama two."

As observations in the naturalistic, microgenetic, and Kumon components were recorded, the authors discussed the mathematical significance of the events and related them to research in the field (see Appendix A for the table of observations). The first author coded the observations by mathematical data to create an organizing structure. Next, this structure was used to group the observations by skills to compare and contrast them to previous results reported in the research literature. The skills documented in the observations were: *pre-number recognition*, *verbal number recognition*, *verbal counting*, *enumerating a collection*, and *matching and production* (see Figure 2 for definitions and examples of these skills and Figure 3 for a compilation of Blake's development in these areas from all three data methods). In this paper, we describe the evidence of Blake's sequence of development in each of these areas and how his development reflects current research. We conclude by refining the IM based on our findings.

RESULTS AND DISCUSSION

Verbal Number Recognition

Results

Below we discuss Blake's initial learning of isolated number words and his gradual use of them to identify small numerosities (collections of one to four items) in increasingly abstract contexts. More specifically, Blake initially used isolated number words in a nonfunctional manner. In time, he used them with discrimination to identify the numerosity of small collections that were visible. Later, Blake used number words with increasing effectiveness to identify a collection that was partially visible and next those that were not visible. During this study, Blake was unable to recognize the numerosity of collections composed of sequentially presented sounds.

Pre-Number Recognition

Everyday experiences. At 19 months, Blake was able to repeat the word "one," but he had not used the word "one" to characterize the numerosity of a collection on his own. Starting at 22 months, Blake was able to say the word "two" when asked "How many?" questions. For example, when asked, "How old are you?" he responded, "Two." (His parents wanted to be sure that he could answer how old he was by the time of his birthday.) Although this evidence indicates that he associated saying a number with the "How many?" question, he typically responded to such questions indiscriminately with the number "two."

Kumon. The Kumon program required using number words beyond "two." For example, when Blake was 27 months old, his mother showed him a worksheet with four items on it and told him that it was four. The next collection also had four. However, his mother did not label the numerosity for him. When asked "How many?" he mechanically repeated, "Four." Except for imitating his parents, Blake did not use "four" to label collections or otherwise show signs of understanding the concept of four (e.g., the difference between four and any number greater than two) in Kumon or daily life experiences.

Number Recognition--Visible Collections

Everyday experiences. By 27 months, Blake was finding many examples of two objects in his life, such as dogs, newspapers, tractors, etc. When Blake identified the collections on his own, his responses were accurate. The first time that Blake labeled a collection of one on his own came after finding collections of two on his own. Excluding imitation efforts, Blake first used "one" to describe a collection with a single item at 27 months. Blake and his mother were playing with straws in a restaurant. She asked him, "How many straws?" He replied correctly, "Two." She took one away and asked again, "How many straws?" Blake responded, "One."

By 28 months, he was holding up one finger and saying, "One." Almost on a daily basis, he also began to discriminately label pairs of objects as "two," *when he defined the collection.* That is, when he initiated the activity, he neither labeled one object or more than two objects as "two" nor two objects with some other number such as "three." As there was no obvious connection among the different things he labeled as "two," he apparently had a relatively abstract understanding of the number. The collections included things (a) he could not hold in his hands (e.g., smokestacks or airplanes in the sky), (b) not labeled "two" previously by his parents (e.g., smokestacks or remote controls.), (c) with different physical properties, such as different length (smokestacks vs. fingers).

Blake did not use what numbers he knew beyond "two" discriminately. Oftentimes, he used "a lot" or "five" to describe a collection greater than two. In fact, his concept of "many" did not change significantly over the course of the study. At 29 months, for example, Blake asked for more mushrooms (a vegetable that he really liked). His dad asked, "How many do you want?" Blake said, "Two." He quickly changed his mind and said, "Five." Furthermore, he apparently was unable to distinguish the difference between the quantities of three and five. At 30 months, Blake asked for five things by saying, "Five." His mother countered with, "How about three?" Blake responded, "Five." His mother replied, "How about three?" and gave him three. Blake happily exclaimed, "Five!" His mother corrected, "Three." Blake offered, "Four." This may be an example of the "one, two, many" view of numbers (Wagner & Walters, 1982).

Kumon. At 27 months, Blake shifted from using "two," to using "three" to label (indiscriminately) collections of two to five items depicted on his Kumon worksheets. For example, shown a picture of three items, he said, "Two" while shaking his head no, which apparently indicated "not two." It seems that he knew a collection of three was different from a collection of two. However, it does not provide evidence that he knew the relation between two and three (i.e., three is greater than two). If his mother told him, "That's not three," then he did revert to using "two" (indiscriminately).

At 27 months, when reminded by his parents, Blake would use the word "four" mechanically to characterize collections ranging from two to five in numerosity. One month later, at 28 months, Blake often used the word "five" to characterize collections of more than two items.

By 28 months, while Blake was becoming more accurate in his daily number recognition of collections of two, he became less discriminate during formal math time. During this study, Blake did not use "two" discriminately in Kumon. With his Kumon pictures, he sometimes labeled three or four objects as "two" and two objects as "three" or "four." These errors appeared to be due to inattention or lack of interest, because the tasks were imposed on him rather than being self-initiated.

Verbal Number Recognition—A Collection That Was Partially Visible

In this section, we detail Blake's behavior with collections that were not completely in view. As the Kumon instruction only involved recognizing the numerosity and enumerating objects pictured on a sheet, this (and the following sections) do not include any Kumon observations.

Everyday experiences with collections of two. Blake's number recognition ability for a collection that was not completely in view began to emerge around 29 months. His mother forgot that she had a soda on the table where Blake was sitting. When she took another soda out of the refrigerator, he saw it in her hand. From where she was standing, Blake could not see the one on the table and the one in her hand simultaneously, but he excitedly proclaimed, "Two!" Similarly, as Blake and his mother were driving home, he saw one crane on one block. After several turns, Blake saw a second crane (the first was now out of sight). He said, "Two." His mother asked him to what was he referring, and he said, "Cranes."

Blake continued to label collections of two that were not completely in view. His ability was not limited to a particular context; he labeled collections of blocks, pieces of chalk, cups, and flashlights. These episodes provide converging evidence of his ability to represent mentally a collection of two when both items were not in sight simultaneously. This supports the notion that he had some internal mechanism or mental model. He apparently could recall a similar item and mentally represent it and a second visible item as "two."

Everyday experiences with collections of three. Blake's ability to utilize number recognition for collections of three items was not reliable during this study. However, two episodes provide evidence that he might have been able to form a mental representation of three or at least "many" (more than two), despite the inability to use number words beyond "two" discriminately.

The first episode occurred when Blake was 30 months old. He and his mother were visiting a snack room with vending machines. Blake inserted two nickels into a vending machine and got a dime back. He looked discouraged. He said, "Two money? Two money?" He apparently wanted his two coins back, not just one.

Next, his mother gave Blake three dimes. Asked how many he had, Blake said, "Four. " He proceeded to put the four coins in the vending machine and got two coins back (a quarter and a nickel). He said, "Four! Four! Reach!" (He was reaching in the coin return trying to get his third coin.)

His mother then gave him two dimes and a nickel. He said, "Four." She corrected, "Three." When the quarter came out he said, "Four?" He looked at his mother, corrected himself, and said, "Three?" as he looked for the other two coins.

Lastly, Blake put three coins (two quarters and a nickel) in the machine and got three coins returned. He walked away without any comment. He apparently was not surprised that if three go in, three should come out. This episode is consistent with the interpretation that he mentally represented the coins. It is less clear how precise his mental representation was. The most conservative interpretation would be he knew that initially he did not have one or two coins and when he got two coins back (a number he knew reliably), he knew this was different than what he had originally. This exercise supports that he had some notion of three items--they are not two and they are not one. However, there was not any other overt evidence in his daily life that he mentally represented three. In order to establish that he accurately represented three, the task would have needed to contrast three coins with four.

It could be argued that Blake was only noticing that he got different coins—instead of noticing the numerosity of the collection. There is evidence that this was probably not the case. First, he was describing the collections by their numerosity, not their denomination (e.g., he did not ask for his pennies). Second, in his daily life, when he asked for pennies and he received a nickel, dime, or quarter he was equally happy. There was never a time in his daily routine that he got upset when he had a penny and his parents replaced it with a dime. Therefore, it seems unlikely that he was responding to a change in coins. More likely, he was responding to the change in numerosity.

A second episode occurred when Blake was 31 months. His mother opened one soda can, took a sip, and placed it on the table. She took two more soda cans out of the refrigerator.

Pointing to her hands, Blake said, "Two." His mother then took a sip of the soda on the table, moved across the room and took a sip from each of the other two sodas. Blake was unable to view the entire collection with one glance. His mother asked, "How many Cokes do I have?" He replied, "Five." He did not even look at the one behind him on the table. He apparently knew that there were more than two without viewing the entire collection together.

Number Recognition--A Collection That Was Not Visible

When he was 27 months, his mother asked Blake if he wanted milk or water. Blake responded, "Milk" then added, "Water." His mother asked, "Which do you want, milk or water?" Blake replied, "Two." It appears that he was using "two" to mean both. He had not yet used the word "too" to connote "also," so he probably was not intending, "milk and water too [also]."

It took four months until Blake demonstrated this ability again at age 31 months. Blake's family lived next door to a mother named Wanda and in front of another mother named Wanda. Blake's mother was talking about one of them. Blake asked, "Billy's Wanda?" His mother answered, "Another Wanda." Blake clarified, "Two Wanda." Neither of the two Wandas was in sight.

While in the family van, Blake and his mother were looking for his lost sandal. He never saw the first one. His mother told him that she found one, but she could not find the other. Hours later when she told him that she found the other sandal, he said "two" without seeing either of them.

In an incident similar to the first example of this section, Blake was wavering between wanting water and milk to drink. He was saying, "Milk" then, "Water" repeatedly. His mother kept asking him, "Do you want milk or water?" Finally, he said, "Two cups." Apparently, he was able to reason that if he wanted both milk and water, he would need two cups. This was the first sign of mental one-to-one correspondence between two collections. Asking for milk and/or water has been a familiar routine for him. It might be that the familiar routine facilitated his inference that he needed two cups. Perhaps his one-to-one correspondence schema, although weak, was emerging.

Number Recognition--Sound

At 28 months, Blake was unable to apply his number recognition skills to sounds over time. He looked perplexed when his mother asked him how many sounds she made (like ba, ba, ba). He was able to mimic the sounds. If she said, "Ba, ba," then he would repeat, "Ba, ba." If she said, "Ba, ba, ba," then he would sometimes repeat, "Ba, ba, ba." However, he would not repeat his mother's sounds for collections of sounds greater than three.

Discussion

According to Fuson (1988), the conventional count words are part of children's vocabularies almost from the time they begin to speak at 18 to 24 months. As the case of Blake illustrates, children's initial use of number words may be nonfunctional in the sense that they are not associated with quantifying a collection (number recognition or enumeration).

Consistent with Beilin's (1968) view that "two" is typically the first number word used, particularly to indicate a plurality, "one" was Blake's first number word, but not his first meaningful number word. The number name "one" is not required to indicate a singularity because the noun does that. For example, instead of saying, "Look at that one cat," most people would just say, "Look at that cat." However, once there is more than one object, a number name can enhance of the specificity of a statement. For example, a person could be referring to two or one hundred cats with the sentence, "Look at those cats."

Before young children begin to use number words, recent research (Clearfield & Mix, 1999; Feigenson, Carey, & Spelke, in press) indicates that infants use perceptual cues, such as contour length, area, or volume to quantify collections. Thus, they may not even distinguish between collections of one and many items if these collections had the same perceptual cues. However, by 28 months, Blake was able to distinguish collections of two items that did not provide perceptual cues (i.e., smokestacks and remote controls). Blake was clearly using a different mechanism for representing those collections than what research on infants suggests.

Several researchers have documented young children recognizing the numerosity of small collections before they are able to verbally enumerate them (e.g., Schaeffer, Eggleston, & Scott, 1974; Sophian, 1992; Starkey & Cooper, 1995). Furthermore, some researchers suggest that the development of counting depends on number recognition as evidenced by enumeration developing after number recognition. Klahr and Wallace (1976 cited in Starkey & Cooper, 1995) proposed that children use number recognition before they count because counting requires knowing the correct sequence of verbal labels, one-for-one tagging, and keeping track of the items counted and to be counted; these counting requirements are conventions learned from the social environment. On the other hand, verbal number recognition requires learning verbal labels but not order and place keeping.

Even though verbal number recognition emerges before verbal enumeration, the development of verbal number recognition is not complete before children are able to verbally enumerate collections (Starkey & Cooper, 1995). Blake's verbal number recognition development also mirrors Schaeffer et al.'s (1974) and Wynn's (1990, 1992) findings that children attach a discriminate meaning to number words one at a time starting with those for the very smallest collections. These findings also challenge the nativist position that number knowledge is a privileged domain because children do not gain a sudden insight into the meaning of number words or counting (Mix et al., 2001).

Blake's development in identifying colors might help explain his verbal number recognition development. Consistent with Sandhofer and Smith's (1999) observations, he initially used color words indiscriminately. At 21 months, Blake first learned the names of colors and labeled every color blue or yellow. Soon after, he became more discriminate with blue, yellow, and purple. After several months of inconsistent application, 25-month-old Blake's ability to discriminate among blue, yellow, red, green, purple, pink, orange, brown, and even gold blossomed in a matter of weeks. His breakthrough may have been the result of realizing that color was useful for classifying and identifying objects (Mix et al., 2001). Soon after he first expressed number words, Blake attempted to discriminate among the numerosities of small collections. However, his early performance was unreliable. Perhaps his use of number words mirrors his acquisition of color.

Blake may have realized that numbers, like colors, are useful in categorizing and identifying items in the environment. At 27 months—before children supposedly construct a mental model (Huttenlocher et al., 1994)—Blake learned to use number labels such as "one" and "two" to identify discriminately and reliably very small collections. This evidence suggests that number words may provide a scaffold for constructing a mental model for numbers.

The observations of Blake are consistent with the view that he formed an *inexact verbal representation* and then refined that to an *exact verbal representation*. The results are less clear about whether he first had an inexact nonverbal representation, next had an exact nonverbal representation, and then had a verbal-based representation. Blake's daily experiences did not offer evidence that he did or did not have a nonverbal representation. Instead, he began using number words indiscriminately, which eventually led to him using the words "one" and "two" to discriminately describe collections. If Blake began with an inexact nonverbal representation and then transitioned to an exact nonverbal representation, it would seem that, he would have rapidly applied the number names with an exact verbal representation once he learned the number names.

An alternative explanation could be that his learning of number words in a sing-song manner, without quantitative significance was developing around the same time as his mental models. Blake could have been developing mental representations of collections he experienced in his daily life. As his mental representations became more discriminate (the mental representation for one was different than the mental representation for two), he was able to apply the socially learned numerical conventions to his mental representations. Thus, the sing-song numbers attained quantitative significance. The amount of time that it took for him to develop an exact verbal representation might have been due to his connecting the word "two" to his exact mental model. Perhaps he had an *exact nonverbal representation* even while he appeared to have an *inexact verbal representation.*

Despite being able to discriminately identify collections of "one" and "two", Blake failed to accurately recognize the numerosity for collections greater than two. The observations provide evidence that Blake used the number labels for three, four, and five interchangeably when describing collections. However, when his mother and he were playing with balls, he said that he wanted, "One not two." However, the example does not provide evidence that he had knowledge of the relative magnitude.

Interestingly, when using the words "three," "four," and "five" to talk about age, Blake was able to distinguish among the labels. At 30 months, he began talking about things that he would do when he was older. His mother read a book to him about traveling throughout the United States. He would ask if they could go there when he was two. If his mother said, "No," then he would ask, "Three?" If she still said, "No," he would counter with, "Five?" When people would ask him how old he was, he would often say, "Two, no five, no three" (meaning two, not five, not three). When the boy next door told Blake that he was four, Blake smiled in amazement. Blake replied (referring to himself), "No four." He apparently knew that two was different from three, four, and five when it pertained to age. Although he could not use three, four, or five discriminately with collections, he appeared to do so with ages. This presents evidence that he was able to distinguish that the number *words* three, four, and five were different. However, it is clear that he had not linked the words to their referent collections.

Another difficulty for Blake during the study was quantifying sounds. He was sometimes able to repeat up to three sounds (like "ba, ba, ba") during the study. However, according to Mix (1999a), this ability does not mean that he was attending to the numerosity of the collection of sounds. She notes that people are able to sing "fa la la la la" in "Deck the Halls" without knowing the numerosity of the collection. The observation that, despite his ability to repeat sounds, Blake's failure to recognize the numerosity of collections of sounds during this study further reinforces that his repetition was unrelated to numerosity. This finding is consistent with Mix et al. (1996). They found that three-year-olds performed at chance on their auditory-visual matching task, even for the lowest numerosity tested. Their research supports the conclusion that only proficient counters were successful on matching the numerosity of collections of objects with sounds. This finding is in direct conflict with Starkey, Spelke, and Gelman's (1990) results that seven-month-old infants looked longer at the corresponding visual display while hearing a drum beat. It seems that if Blake did have the ability to match visual displays with sounds, he would have been able to transfer his verbal number recognition skills to sounds sooner.

Figure 3. Compilation of observations in a developmental timeline of skills and collection size.

Skill	Collection Size	Age 19	20	21	22	23	24	25	26	27	28	29	30	31
States verbal number name for the first time	"One"	e(1)												
	"Two"				E(1)									
	"Three"									K(3)				
	"Four"									K(1)				
	"Five"										K(1)			
Verbally recognizing the numerosity of a visible collection	1									E+(1)	e+(1)		M+(1)	
	2									e+(2) E+(1)	e+(5)			

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Verbally recognizing the numerosity of a partially hidden collection	2											e+(4)		e+(2)
Verbally recognizing the numerosity of a collection that was not visible	2									e(1)				e+(5)
Enumerating a collection	3										E(1)	E(1) e(1)		
	>3													e(1)
Nonverbal matching	1								M(1)					
	2								M(1)					
	4								M(1)					

Verbal production	1											E(1)		
	2												M(1)	

Key:

e = child initiated in everyday experiences

E = adult initiated in everyday experiences

k = child initiated in Kumon

K = adult initiated in Kumon

m = child initiated in microgenetic study

M = adult initiated in microgenetic study

+ = accurate, appropriate, and discriminate use

(n) = number of observations

Enumerating a Collection

Results

Blake began to learn the counting word sequence, a key foundation for enumerating a collection, relatively late in the case study. Indeed, in his daily routine, Blake's mother never observed him enumerating a collection of two. Instead, he would always use number recognition for collections of two. His progress in counting to three was different from that of counting larger collections, therefore we describe it in a separate section.

Pre-Enumeration: Reciting the Counting Sequence

Kumon. Although Blake knew the number words "one" to "four" at 27 months, he was unable to combine two or more different words because his speech had not developed enough. For example, when prompted to say, "More music please," he would say, "Please, please, please." He was able to say the words individually, but he could not say the words in sequence. When he counted objects that his mother pointed to, he gave a sound for each object. For example, when they counted three objects during Kumon, he would often say, "Two, two, two." When asked how many, he would typically guess, "Three."

By the end of 27 months, he was able to say "two three." This was the first time he strung together conventional number words. It may be that previously, some of his grunts were distinguishable to him--his own counting string. By the end of 27 months, his pronunciation had evolved, and his counting string sounded more like actual number words. By 28 months, he was able to recite the correct numerical sequence up to five. While his mother pointed to dots on his math sheet, he would say, "One, Two, Three, Four, Five."

Everyday experiences. At 27 months, Blake began to learn the counting sequence by rote. He was also able to supply the word "three" when his mother was counting and paused after saying, "Two." By 29 months, he knew to say the word "seven" after hearing the counting sequence up to "six." For example, he and his mother were counting parts of a truck. They said, in unison, "One, two, three, four, five, six." His mother stopped at the end of the six items, Blake said, "Seven." This was not numerically significant because it was an example of learning by rote (it is the same as him knowing that B comes after A in the alphabet), but it was an important step for learning the correct counting sequence. By 32 months, he began to recite the counting sequence from one to ten, becoming more accurate daily.

Enumerating Collections of Three

At 28 months, Blake enumerated three items without help. He pointed and recited the correct sequence with one-to-one correspondence. This was the first instance of him performing this without parental guidance, so it is not clear whether this performance was an accident (false positive) or evidence that this ability was emerging.

Blake did not enumerate three items on his own again until 29 months. During brunch, he grabbed three packs of Sweet-N-Low. His dad said, "Give Mama two." He started handing them to her one at a time. As he handed her one, he said, "One." Then he handed her another and said, "Two." As he handed her a third one he said, "Three (with a higher pitch)." His dad asked, "How many?" He replied, "Three."

Later that evening, Blake's parents gave him three grapes. His dad asked, "Can you count them?" He took one from his mother's hand and placed it in his dad's, saying, "One." Then he placed a second in his dad's hand and said, "Two." After moving the third, he said, "Three." His dad asked, "How many?" Blake replied, "Three." This provides converging evidence of his ability to enumerate a collection of three items by himself.

Enumerating Collections Larger Than Three

At 31 months, while Blake was spontaneously counting a collection by himself, he used several counting strings for the same collection. For his first two counts he said, "One, three, four, five" and " One, three, four, five." The third time that he counted the collection, he said, "One, two, three." At this time, Blake had not developed a stable counting string.

By 32 months, Blake liked to count things spontaneously, such as stairs, blinds, and grapes. However, he did not count with one-to-one correspondence. He also did not stop counting when the items were exhausted. Instead, he tended to continue counting until he reached "ten." If there were more items than numbers, he tended to continue counting by repeating, "six, seven, eight, nine, ten." Blake's behavior is indicative of a quasi-functional use of the number sequence because he did not use one-to-one correspondence and he did not use the standard counting sequence.

Kumon. At 32 months, his performance was different with his Kumon math sheets. He stopped his counting on those sheets after the last touch. A possible explanation for the difference in performance is that his mother may have given him an inadvertent cue that the counting was complete. Alternatively, the items on the Kumon sheet were more organized and homogeneous, perhaps providing more structure for Blake.

Discussion

When Blake initially began reciting the counting sequence with grunts at 27 months, it did not appear to have mathematical significance. By 28 months, Blake gradually learned to recite the number words "one" through "five" and usually emphasized the last number word counted by raising the pitch of his voice. However, he did not appear to attach any numerical meaning to those counts. This result is consistent with that of Fluck (1995). He found that about a quarter of two- to three-year-old's counts emphasized the final word, though it had no apparent cardinal significance. Instead, it appeared that children accented the final word to denote that they completed the counting task. Sophian (1992) offered that children might stress the last number counted because they learned this as a component of the counting procedure. Children later discover its cardinal significance.

Blake's early attempts at counting reflected Fuson's (1992) recitation context of counting, which is similar to children singing the alphabet. Ginsburg (1977) called such counting a "sing song." Sophian (1998) agreed that reciting counting strings demonstrates children's memory abilities more than their quantitative skills. Not until children begin to enumerate a collection with a counting string do they use their quantitative skills.

Mandler and Shebo (1982), citing Chi and Klahr (1975), concluded that children operate differently on the numerosities of one and two than on three and higher. Young children only gradually develop the ability to deal precisely with numbers greater than two (Mix et al., 2001). Blake's ability to determine the numerosity of a collection of two consistently on his own suggests the existence of a number recognition ability that is separate from verbal counting. Even though Blake could enumerate three items, there was not sufficient evidence to conclude he could verbally or nonverbally utilize number recognition for collections of three. Fischer (1992), agreeing with Beckman (1923 cited in Fischer) and Gelman and Tucker (1975 cited in Fischer), proposed that children might count collections of three objects before they use number recognition. Perhaps with very small numbers, children (nonverbally and verbally) apply number recognition first; for larger numbers, they must be able to accurately enumerate collections before they can use number recognition.

At 32 months, Blake was demonstrating a rudimentary ability to enumerate a collection. At that time, he was verbally counting without conceptual understanding (Fuson, 1992; Sophian, 1988a). Two-year-olds typically point to objects and say number words. The types of errors that they make involve pointing without saying a word, saying a word without making a point, giving extra points to an object, and leaving some objects without any points (Fuson, 1992).

Sophian (1988b) proposed that children only gradually acquire understanding of enumerating a collection, presumably through their counting experiences. For example, when Blake's dad asked him to give Blake's mother two sugar packets. Instead, Blake counted out three packs, exhausting the collection that he had in his hand. His verbal production error was a classic "no stop" error. That is, instead of stopping at two packs as requested, he counted all the items available (see Baroody, 1987b; Resnick & Ford, 1981; Wilkins & Baroody, 2000 for a discussion of this type of error and others). Blake may have failed to compare his count to the requested amount either because he did not have a cardinal-count concept (enumerating a collection of a previously specified amount) or because of a working memory difficulty. The incident did seem to indicate an understanding of the count-cardinal concept (stating the cardinality of the collection after verbal enumeration or number recognition) as it applies to three because Blake responded correctly to the "How many" question. The higher pitch may also be indicative of a cardinality principle (meaningful application of the cardinality rule; cf. Bermejo, 1996). However, Blake did not show evidence that he had a more developed understanding of verbal-based cardinality. In Fluck's (1995) study, none of the two- to three-year-olds showed any evidence of understanding the connection between counting and cardinality. Blake's progress also conforms to Fuson's (1992) findings. Most two- and three-year-old children who do not answer the "How many" question correctly can remember the word they said for the last object they counted.

Matching and Production

Results

Nonverbal Matching

Microgenetic. At 26 months, the authors administered a matching and production task for two consecutive days. Blake was not responsive, so we were unable to present tasks for three days in a row. After the second day, we determined that the third day would probably not yield any more data because of his lack of response.

First, one researcher hid a checker under his hand and then moved it from hand to hand. Blake was able to find the checker each time. This demonstrated his ability to locate hidden items. Therefore, we cannot attribute his performance on the matching and nonverbal production tasks to his lack of object permanence or working memory capacity.

Blake matched one checker with the researcher's one checker. However, by the time that the researcher moved to the next collection, Blake began asking for pretzels and not attending to the researcher's mat. Blake did not match two checkers with the researcher's two checkers. Blake's performance on three items was similar; he placed two checkers instead of three. Although he may have a lack of competence matching two or three items, it was evident that he was not attending to the researcher's mat, and was unwilling to comply with the task.

Surprisingly, when asked to match four checkers, Blake complied. During the testing, the authors regarded it as a fluke. However, upon closer inspection of the videotape, Blake appeared to be attending to the task more than the previous two collections. Blake placed four checkers on his mat. One checker went off the mat and he placed it back on the paper. Of course, this one example does not rule out that it was a lucky guess.

The next day, Blake initially complied with the researcher's request to match two checkers. However, after placing two he said, "More" then got more checkers. It may be that he was unable to match, but it might be that his desire for more checkers was greater than his desire to fulfill the task.

At 30 months, when his mother re-administered the nonverbal matching and nonverbal production tasks, Blake was uncooperative. On the matching task, he was able to respond to one item consistently by telling his mother that she had one (verbal number recognition instead of matching). Blake responded sporadically when his mother had two disks on the mat for him to match.

Nonverbal Production

Microgenetic. When Blake was 30 months, he was unsuccessful on the nonverbal production task. Even though this task was nonverbal, Blake's mother tried to elicit a response by asking, "How many?" He would not respond until she probed him and showed him the collection. Perhaps, he was unable to complete the nonverbal production with two items because he was not able to apply number recognition for the hidden collection, a competence he did not reliably exhibit in his daily experiences until 31 months.

Verbal Production

Microgenetic. At 30 months during one point of the microgenetic study, he had four disks in his hand. His mother told him to put down two (thus transforming the task from nonverbal matching to verbal production). Instead of following his mother's request, he handed his mother two and said, "Mama two."

Everyday experiences. At 29 months, Blake produced one item from his mother's verbal request. She told Blake to give her one packet of Equal, and he grabbed two. She repeated, "Can you give me one?" He said, "Uh huh [Yes]." He put one back and gave her one. When he first took two Equal packets, he may have grabbed them without concern for number, just as one might grab a handful of nuts. Perhaps his mother's reiteration of the request helped focus him on the concept of number.

Discussion

Blake utilized number recognition for collections of one and two before he could reliably create a matching collection or nonverbally produce items (as gauged in the microgenetic study). During the second administration, when Blake was 30 months, he was extremely accurate in ascertaining the numerosity of collections of two items. It was surprising that his ability did not translate to the nonverbal matching and production tasks. The mother administered these adult-imposed tasks during a time that Blake practiced Kumon math every night. However, these tasks were not similar to his Kumon activities, which only required verbal counting. He was excited about having "new math," as his mother characterized the tasks. Despite his mother's efforts in trying to make the conditions intriguing, his performance during these tasks did not seem to reflect the ability he demonstrated in daily life.

The data indicate that current methodology in this line of research is not sufficient to test the proposed model. Huttenlocher et al.'s (Huttenlocher et al., 1994; Jordan et al., 1994; Mix, 1999a; Mix et al., 1996; Mix et al., 2001) research recorded that preschoolers' ability to respond exactly first appears around two-and-a-half years old in equivalence, ordination, and cardination tasks. Huttenlocher et al. explained that when the mental model emerges, children begin to generate exact responses and produce exact matches for collections up to two items. However, it became evident during Blake's performance in the nonverbal matching and production tasks that these tasks did not capture the earliest signs of numerical equivalence.

The researchers designed these nonverbal production tasks to tap into nonverbal understandings. Research has found that most children learn number words between 18 and 24 months. Huttenlocher et al. (1994; Jordan et al., 1994; Mix, 1999a; Mix et al., 1996; Mix et al., 2001) do not adequately take into account the importance of these verbal labels in applying them to the numerosity of collections. Blake was able to use verbal number recognition for collections of one and two items before 30 months. He began using the word "two" to describe the numerosity of the collection before he could reliably match two items. When performing the microgenetic matching and production tasks, Blake transformed the tasks from nonverbal to verbal because he responded solely with number words. Nonverbal tasks are typically administered to assist children who may not have the verbal skills to respond. According to Jordan et al. (1994), responding with a number word might even be easier than responding by putting out objects where children not only have to represent an answer but also construct an array based on that representation. If they do not accurately map numerosities onto number words, however, then a verbal-response task should be more difficult than a nonverbal-response task. It seems that Blake's mapping of the numerosity of two onto the number word facilitated his performance in this task.

Blake did not reliably answer questions about the equivalence between the two mats. The pictures were the same, each had at least one checker, and the checkers looked the same. The only thing that was not the same was the numerosity. Blake may not have realized that the researcher wanted him to produce the same numerosity. Children apparently fail to see number as a means for categorizing. Even though they can represent the number of objects in a collection, children do not necessarily recognize the relation between numerically equivalent collections (Mix, 1999b).

Additionally, Huttenlocher et al. (1994; Jordan et al., 1994; Mix, 1999a; Mix et al., 1996; Mix et al., 2001) did not explain how the acquisition of the mental models for the larger collections might progress. Their nonverbal production tasks presented children with collections consisting of up to five items. The nonverbal matching and production tasks assumed that young children are capable of number recognition for small collections before they are able to enumerate them. This issue is important because Blake exhibited a different mechanism for different collection sizes. Specifically, his development of the concept of three was different from his development of two. He did not form an exact, verbal-based representation of three unless he counted a collection of three items. That is, the only times that he accurately labeled collections as three were times when he first enumerated the collections. For the numerosities of three and greater, he loosely tied the numerosities to number words. In other words, he appeared to randomly label collections greater than two as three, four, or five.

CONCLUSIONS

The Spiral Integrated Model

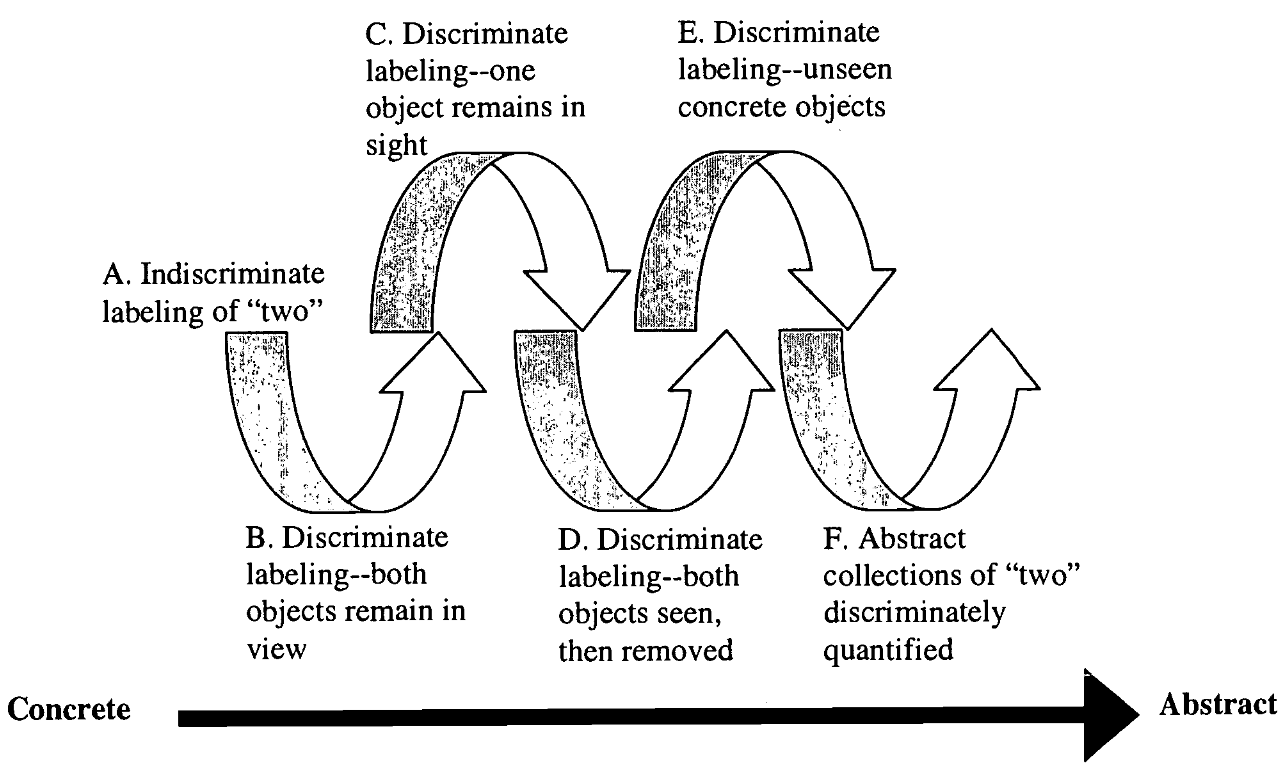
Despite the daily observations, there were no data on how Blake mentally represented objects before he was able to say number words. Furthermore, the formal nonverbal matching tasks did not provide enough insight because Blake did not reliably match collections. Therefore, the authors draw no generalizable conclusions about Blake's preverbal mathematical development.

Only when he started verbalizing was there evidence of his mathematical development. The data from the study support the realization (e.g., Anderson, 1991; Fuson, 1992; Mix et al., 2001; Resnick, 1992) that early mathematical behavior is not restricted to verbal and object counting (see Figure 3 for a developmental timeline of skills and collection size). In fact, Blake's initial mathematical development did not involve object counting at all.

This case study helps clarify the IM depicted in Figure 1. Specifically, the details in the observations map a developmental path that Blake followed. Current models (e.g., Huttenlocher et al., 1994; Resnick, 1992) represent mathematical development in a linear fashion. However, as the following section explains, a spiral model may fit the data of Blake's development better.

According to the Spiral Integrated Model (SIM), mathematical development progresses from concrete to abstract. It incorporates Resnick's (1992) assertion that quantitative reasoning first emerges in a narrow more concrete form. As children mature, they develop a broader more abstract numerical reasoning ability. In our proposed model, we extend the progression of number recognition from concrete to abstract, based on Blake's data (see Figure 4).

Figure 4. The development of number recognition for two items.



When children first begin to use number words (around 18-24 months, depending on their language development), they playfully recite the words in a singsong manner, without any numerical meaning (Ginsburg, 1977). In other words, during the stage when children state numbers without quantitative significance, a number word does not have a well-defined meaning.

Around their second birthday, children begin to use the word "two" to quantify a collection (point A in Figure 4). Perhaps verbal labels facilitate an exact mental representation. The representation might become more discriminate as children tell others their age and show finger patterns. At this time, children may demonstrate a bias toward two and label collections of different numerosities as "two." As they use the word "two" more discriminately, children may move to a more exact verbally supported representation.

The mental representation of two appears to move from concrete to abstract. First, children label two indiscriminately (point A in Figure 4); second, both objects must remain in view (point B); third, one object must remain in sight (point C); fourth, both objects may have been seen and then removed (point D); fifth, concrete objects do not have to be seen in order to be mentally represented (point E); finally, abstract collections (such as sounds) can be quantified (F).

The SIM also suggests a possible explanation for evidence that young children are more successful on nonverbal arithmetic tasks than on verbally presented story problems or symbolic "number-fact" tasks (Huttenlocher et al., 1994; Jordan et al., in press; Jordan et al., 1992, 1994; Levine et al., 1992). If children have not yet developed the abstract number recognition ability for each numerosity proposed (point E or F in Figure 4), they would not be able to solve verbally presented story problems or "number-fact" tasks. The nonverbal arithmetic tasks using concrete referents might draw on a more concrete number recognition ability (point D in Figure 4).

Our research results support that of others (e.g., Schaeffer et al., 1974; Wynn, 1990, 1992), indicating children learn abstract number recognition one number at a time. Blake followed the path of the SIM with the numerosity of two. He had not begun to use number recognition for collections of three. Perhaps, with the numerosity of three, Blake may spiral back to a more concrete form of number recognition before he would be able to make use of number recognition on a more abstract collection of that numerosity. Alternatively, given that number recognition for three appears to develop after enumeration of three, he may build on his number recognition abilities from a point that is more abstract.

Blake achieved point C in Figure 4 before he started enumerating collections of three. Perhaps at the onset of their reliable object counting ability, children's number recognition knowledge is still incomplete. The two skills of number recognition and enumeration seem to follow two separate developmental paths. Possibly, the two domains of number recognition and enumeration will begin to integrate with experience.

Lawler (1981) offered evidence that distinct domains of mathematical knowledge develop separately and eventually merge. When observing his own children, he explored how distinct structures of knowledge interacted and became integrated. Specifically, he noted that his daughter's development seemed to occur in microworlds: the money, count, and decadal world knowledge each developed separately. The decadal world initially developed because she was solving problems using angles while maneuvering a turtle in the computer software program, Logo. His daughter's decadal world knowledge facilitated her solving problems that did not require her to carry, like 33 plus 54. The knowledge from one microworld did not intersect with the other initially. Eventually, the separate microworlds fused into a more powerful organization that connected the knowledge.

Object counting and number recognition could be similar to Lawler's (1981) microworlds in that the domains may develop independently then later merge into a whole. By the end of this study, however, Blake had not integrated the two microworlds of number recognition and enumeration.

We also propose that the development continues in a spiral manner for enumerating collections. However, Blake's counting ability was not fully developed by the end of the study. He was only able to enumerate reliably collections of three objects that he could manipulate with his hands. We expect that as his counting development advances, he will be able to enumerate collections of three abstract items. Observations of Blake after this study do indicate that Blake's enumeration abilities became more abstract. However, the exact developmental path is still unclear.

Future Research

The SIM needs further testing. We are conducting a follow-up study to see if the developmental path of counting follows a similar trajectory as the number recognition development: children initially may be able to enumerate only small collections accurately, beginning with three. Their ability to enumerate may develop by counting three homogeneous objects that they can manipulate to maintain one-to-one correspondence. The ability may then develop to handle more abstract requirements, eventually leading to the ability to enumerate intangible concepts. After children are able to accurately apply number recognition to two items and enumerate three items, perhaps they begin each developmental spiral with the next higher numerosity.

It is likely that children follow separate developmental paths. It might be that Blake's development was anomalous; we need to observe more children carefully to see what paths their development follow. Do other children spiral through the number recognition and enumeration developmental milestones? Specifically, it would be beneficial to know what percentage of children enumerate three before they are able to use number recognition for three. Directly testing the implications of the SIM should lead to a better understanding about the origins of number recognition development and how it evolves.

This case study does not offer any evidence of nonverbal number recognition. It seems that the tests that are designed to detect nonverbal number recognition are too challenging for children who have not yet begun to speak. A test for nonverbal number recognition needs to be developed that does not privilege language.

Another area of research suggested by these results is the methodology used in studies of young children. Based on our initial findings, it appears that children may exhibit mathematical understandings in their every day life that they do not express in formal testing settings. Most of Blake's developmental milestones were not realized during planned math activities. Because Blake made many of his discoveries in the kitchen, Blake's mathematical knowledge would not have been documented as accurately without his mother's observations. Additionally, children may also appear to have skills that upon closer inspection are only a localized competence (Mix et al., 2001). A 15-minute examination of preschoolers' competence is too cursory to draw valid conclusions about their mathematical development. Even though it is not efficient to study children in their home environments, the data documenting their mathematical development are much more robust. These rich data can be invaluable in gauging how quickly and in what manner development occurs. Therefore, future research needs to find ways to capture the rich home-based data in an efficient manner.

Educational and Scientific Implications

Resolving the methodological issues and refining a developmental model will prove useful to educators. With the current interest in early childhood education, valid and reliable assessment methods are needed to identify children's mathematical development. If future studies support a spiral developmental model of mathematics then educators can design more sophisticated assessment measures.

Better assessment could also lead to curriculum that accelerates children's mathematics development. The developmental changes in mathematics that take place over the preschool period seem to be central to providing a foundation for future mathematics success. Advances in young children's preschool mathematical development may lead to success in formal mathematics (Sophian, 1987).

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Appendix A: Data Collected on Blake's Development

Date	Observations
6/9/00 age: 19 months	Alexis was giving Blake raisins. She said, "You can only have three." Blake pointed his finger and said, "One" before Alexis put any raisins down.
7/10/00 age: 20 months	Blake can sing "Frere Jacques." He can sing the entire song (without using the words clearly); every beat is clearly represented with a tone. I don't think he's mimicking because he'll start and finish singing it by himself. There are other songs that he can sing too. I was also surprised by his ability to "play" the piano. When we get to the end, he will play the three notes for "ding, dang, dong."
7/18/00 age: 20 months	Blake got 2 nuts out of a jar. He ate one; he had one in his hand. He wanted to grab something so he could not hold on to the nut. He gave it to me--I put it in my pocket. Once he was finished playing (about 3 minutes) he wanted the nut back. I gave him the one that he had given to me. He looked perplexed. He put his hand out for more as if I had more in my pocket.
7/19/00 age: 20 months	Kenneth had 3 tomatoes in his hand. He quickly moved them to his other hand when Blake was not looking. He opened the hand that originally had the tomatoes, but they were gone. He has a sign for "Where is?" He used that sign, which indicated to me that he was expecting the tomatoes to be there and looking for the missing tomatoes.
9/12/00 22 months	Blake's parents taught him to say, "Two" as the answer to, "How old are you?"
10/9/00 23 months	Blake was saying, "More" every time that he had more than one.
10/26/00 24 months	Blake grabbed two spoons and said, "Spoon." I replied, "Spoons." Then Blake said, "More."
10/26/00 24 months	Blake's grandmother asked him, "How many crackers do you have?" Blake responded, "More."
1/5/01 age: 26 months	Art: Would you like to play the hiding game? (Art places one checker on the table. Blake nods.) And now I'm going to cover it. (Art places two hands on the table, covering the checker with his right hand) Can you point to where the checker is? Can you point to where the checker is? Can you find the checker? Mom: Where's the checker? Art: Is the checker here (taps his right hand) Is the checker here (taps his left hand) (Blake points to his right hand) Ah, you won that. (Blake repeatedly taps on the table) Are you ready? Ready? I'm going to hide it. (Art covers the checker with his left hand) Where's the checker? Where's the checker? Is the checker here (Art taps with his left hand) Or, is the checker here? (Art taps with his right hand) Blake points to his left hand. Oh, you got it right again. Art: Ready? Ready? Here's the checker. (Blake grabs it) Oh, you're going to hide it this time. I'm hiding the checker. (Both Mom and Art prompt Blake to watch.) (Art

	hides checker under his right hand and then slides his right hand under his left—he changes the checker's position, but not the hand it's under) Where's the checker? Is the checker here? Or is it here? (Blake points to the correct position.)
1/5/01 age: 26 months	Art: Ready. I'm going to do this with my mat. Can Blake make his mat just like mine? (Blake grabs one checker and places it on the mat. Then he covers it up.) Good job. (He wants to keep the checker)
1/5/01 age: 26 months	Art: Here's your snowman and here's my snowman. (Art places two checkers on his mat.) Now, can you make your snowman just like mine? Blake: (Blake doesn't appear to be attending to the snowmen pictures. Blake places the checker from the last task on the snowman.) Pretzel. Mom: Make yours just the same. Are they the same? Blake: hmm hmm (meaning yes) Mom: Is that the same. Blake: hmm hmm.
1/5/01 age: 26 months	My fish is going to have some bubbles. (Art places three bubbles on the fish.) Can you make your fish just like mine? Blake: bubbles. (Blake places two checkers on the fish.) Pretzel. Mom: Pretzel? Are they the same? Art: Is your fish just like mine? Mom: Do you see his fish and your fish? Can you make them look the same? Let's do the truck. Blake: Truck. Blake.
1/5/01 age: 26 months	Art: Here's Blake's truck and here's my truck (Art places 4 checkers on the truck.) Blake, can you make your truck just like my truck? Blake: (Blake places 4 checkers on the truck. One checker goes off the paper and he places it back on the paper.) Art: Is your truck like mine? Blake: hmm hmm. (He takes 3 checkers off his paper)
1/6/01 age: 26 months	Art: Okay Mommy make yours just like mine. (Art places two checkers on his mat) Mom: Ready (Blake put two checkers on his mat.) Blake: More (Blake puts more checkers on his mat)
2/7/01 age: 27 months	He's been finding all sorts of examples of 2 objects in his life--dogs, newspapers, etc.
2/7/01 age: 27 months	He asked for two cups today--one for milk and one for water. I asked him if he wanted milk or water. He said, "milk" then he said, "water." I said, "Which do you want, milk or water?" Then he said, "Two."
2/7/01 age: 27 months	When counting, he gives a sound for each object. He has yet to develop his own counting sequence (his language is not refined enough to put three words together. For example, if I prompt him to say, "More music please." He'll say "Please, please, please." His counting is similar, "Two, two, two" If I ask, "How many?" He'll say, "Three."
2/16/01 age: 27 months	I was showing him the math sheets. I showed him one sheet with 4 items on it. I told him that it was 4. The next collection also had 4. When I asked him, "How many?" He said, "4."

2/16/01 age: 27 months	There was a sheet with 3 items. He said "3" to describe the collection. He then said, "2" while shaking his head. In his language, that means not 2.
2/16/01 age: 27 months	When doing math sheets, now that he uses 2 and 3 to describe sets, he is making more mistakes in labeling sets of 2.
2/20/01 age: 27 months	Yesterday he "counted" by saying "two three."
2/21/01 age: 27 months	He seems to be trying to figure out 3 right now. His progress seems to have regressed on 2. He sometimes labels sets of 2 objects as having 3. He seems to have "3" on his brain. He is more likely to use the word "3" to describe a collection. If we tell him "No", he responds by saying "2." He did not do this last week. He is not labeling any objects 4, except when we tell him 4 (he'll repeat the word 4).
2/22/01 age: 27 months	We were playing with straws in a restaurant. I asked him, "How many straws?" He replied correctly, "Two." I took one so one was left. I asked, "How many straws?" Blake said, "One."
3/18/01 age: 28 months	Blake appears to have a strong understanding of 2 objects, *when he defines them.* He has identified 2 objects in his world almost on a daily basis. This started during our last trip to Champaign when we saw the two smokestacks. He also consistently asks for his two blankets. He grabbed 2 remotes and said, "Two." He saw two jets and said, "Two!" He is now able to hold up 2 fingers and say, "2". He also can hold up one finger and say, "One."
3/18/01 age: 28 months	Blake is unable to subitize a sound over time. He looks at me as if I'm crazy when I ask him how many sounds (like ba, ba, ba). I've also tested him over two days by telling him that we can go downstairs after I sing 2 songs. He is desperate to go downstairs and I stall him by singing. You would think that he would jump after the second song (maybe even the first because he was so anxious). I even added a longer pause after the second song and he just sat there. After I sang for a long time, then he asked if we could go down. It seems that he was counting length of time instead of number of songs.
3/18/01 age: 28 months	He recited the correct numerical sequence up to 5 in Kumon. I pointed to dots and as I pointed he said, "One, Two, Three, Four, Five." He also changes the pitch on his last number word counted. When he sees a picture with several items (in his Kumon book), he oftentimes will say the number 5.
3/18/01 age: 28 months	He "counted" to three. He pointed and recited to 3.
3/18/01 age: 28 months	I gave him 3 raisins. He told me, "No. A Lot."
3/29/01 age: 29 months	I had a soda on the table where Blake was sitting. I was fixing lunch and grabbed another soda out of the refrigerator. Blake saw it in my hand. From where I was standing, he could not see the one on the table and me in the same glance. He excitedly proclaimed, "Two!"

4/8/01 age: 29 months	One tool was composed of 2 pieces that he put together. He separated them so he could take off two wheels. While we pointed, we said 1, 2, 3, 4, 5, 6. I stopped at the end of the six items, he said "7."
4/14/01 age: 29 months	As we were driving home, Blake saw one crane on one block, we made several turns and he saw a second crane (the first was now out of sight). He said, "Two." I probed him to see what he was referring to and he said "Cranes."
4/17/01 age: 29 months	He was playing with blocks he had one on a table. His basket of blocks was behind him. He turned around and grabbed a second one. Before he turned back around he said, "Two."
4/17/01 age: 29 months	When I put him to sleep last night he had a book on one side of his pillow and a ball on the other side. I placed them in his bed after he laid down. There was no way that he could see them simultaneously because he was lying down on his belly. He put his hand on each (so his hands could have provided additional stimulus) and said, "Two."
4/18/01 age: 29 months	We were at the restaurant today and I asked Blake if he could give me one packet of Equal. He grabbed two. I repeated, "Can you give me one?" He said, "Uh huh (Yes)." He put one back and gave me one. He didn't even hesitate to figure out that he had to put one back to get one.
4/18/01 age: 29 months	Later during brunch he grabbed 3 packs of Sweet-N-Low. Kenneth said, "Give Mama two." He started handing them to me one at a time. As he handed me one he said, "One." Then he handed me another one and said, "Two." He handed me a third one and said, "Three." (with his higher pitch) Kenneth said, "How many?" and he said, "Three."
4/18/01 age: 29 months	Also at brunch, Blake asked for more mushrooms. Kenneth said, "How many do you want?" He said, "Two." He quickly changed his mind and said, "Five."
4/18/01 age: 29 months	This evening, we gave him 3 grapes. Kenneth said, "Can you count them?" He put them from my hand to Kenneth's. He said "One" as he put the first one in Kenneth's hand. Then he placed a second and said, "Two." After the third, he said "Three." Kenneth said, "How many?" and he said "Three."
4/24 age: 29 months	We were playing with balls. He said that he wanted, "one not two."
5/8/01 age: 30 months	We attempted the matching and nonverbal production tasks. Blake was pretty much uncooperative. He was able to respond to 1 item consistently by telling me that I had one. Interestingly, when I covered it he did not answer how many until I probed him and showed him. He responded sporadically when I had 2 disks down. He did have 4 disks in his hand. I told him to put down 2. He ended up handing me two and said, "Mama two."
5/8/01 age: 30 months	He has started talking about things that he'll do when he's older. We read a book about traveling throughout the United States. He'll ask if we can go there when he's 3 or 5 or 2. When people ask him how old he is he'll say, "2, no 5, no 3." Meaning,"2 --not 5, not 3." When the boy next door told him that he was 4, he smiled in amazement. He said, "No 4"--referring to himself.

5/8/01 age: 30 months	Also, he consistently says "5" for items larger than 2.
5/21/01 age: 30 months	Blake loves money and usually carries it wherever he goes. There was a vending machine and he inserted his money as he always does. He put in two nickels and got a dime back. He looked discouraged. He said, "Two money!? Two money!?" He apparently wanted his two coins back--he did not want just one. I started brainstorming other combinations of coins that I had in my wallet that would also change the amount of coins that he got. I gave him three dimes. I asked how many he had; he said, "Four." He put them in the vending machine and got two coins back (quarter and nickel). He said "Four! Four! Reach!" (He was reaching in the coin return to get his third coin.) I gave him two dimes and a nickel. He said, "Four." I said, "Three." When the quarter came out, he said "Four?" He looked at me and corrected himself and said "Three?" as he looked for the other 2 coins. Lastly, he had three coins (2 quarters and a nickel) and he got three coins out. He walked away without any comment. He apparently was not surprised that if 3 go in, 3 should come out.
5/24/01 age: 30 months	Blake asked for 5 things. He said, "5" I said, "How about 3?" Blake: "5" I: "How about 3?" I gave him 3. Blake: "5!" He was happy as if he'd received 5. I said: "3" Blake: "4"
Approx 5/30/01 age: 31 months	I said, "Another Wanda." He said, "Two Wanda." Neither Wanda was in sight.
Approx 5/01 age: 31 months	Had two chalks. He put them away in separate cubbies so both were not in sight simultaneously. He said, "Two homes."
Approx 5/01 age: 31 months	I opened one soda, took a sip, and placed it on the table. I took out two more. Pointing to my hands, he said, "Two." I went back and took a sip of one on table and went to island and took a sip of the sodas. How many Cokes do I have? He said, "5." He didn't even look at the one behind him.
Approx 5/01 Age: 31 months	He had a cup of milk. I had a bottle of water. He said, "Mama's water." (indicating that he wanted my water.) I said, "Hold on, let me put it in a cup for you." I went to the cabinet to get a cup. He said, "Two cups." I got the second cup out.
5/24 Age: 31 months	He used the following counting strings for the same collection: "1345" "1345" "123"
Approx 6/01	He had a flashlight from our bedroom in his hand. I told him, "Why don't you go get yours." He left the one in our room, went to his room, and got it. As we were

age: 31 months	walking down the hallway, he said, "Two." He came into our room with his flashlight.
6/02/01 Age: 31 months	We were looking for his lost sandal. He never saw the first one. I told him that I found one, but I couldn't find the other. Much later when I found the other, he said "two" without seeing them.
6/21/01 Age: 31 months	He was wavering between wanting water and milk to drink. He was saying "milk", then "water" over and over. I kept asking him, "Do you want milk or water?" Finally he said, "Two cups."
7/7/01 age: 32 months	I told Blake, "Mommy has to go potty. Daddy has to go potty." Blake said, "Two potties."
7/15/01 32 months	He is beginning to recite the counting sequence from one to 10. He is becoming more accurate daily. He counts things spontaneously, such as: stairs, blinds, grapes, etc. However, they are not counted with one-to-one correspondence. He also does not stop when the items are exhausted. Instead, he tends to continue until he reaches 10. If there are more items than numbers, he continues only with, "6, 7, 8, 9, 10." That sequence is repeated.
7/15/01 32 months	We were counting objects on his Kumon math sheets. He stops his counting on these sheets after our last touch. Every time he finished counting, I asked him, "How many?" He responded with, "4" every single time. There were not four items on any of the pages.
7/24/01 32 months	Blake, "Chocolate?" Blake, "When three?" Alexis, "No." Blake, "Four?" Alexis, "No." Blake, "Five?" Alexis, "Maybe." Blake, with a big smile, "Five!"

Appendix B: Sample Kumon Worksheets

7A 107

7A 107a☆

Date ____________

Name ____________

Number of Dots 1

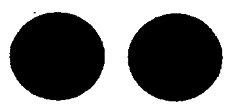
How many dots (●) are there?

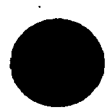
● □

● ● □

7A 107b

How many dots (●) are there?





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