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AUTHOR	Moreno-Armella, Luis
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ABSTRACT

This paper suggests that the systems of representations that we use in mathematics have a cultural origin and concludes that the knowledge produced with the help of these systems of representation likewise has a cultural origin. This assertion forces a reformulation of the issue of objectivity in terms that differ from those inherited from epistemological realism. Information about cognition and historical development, cognition and context, and cognition and computing tools is included. (Contains 16 references.) (DDR)

ON REPRESENTATIONS AND SITUATED TOOLS

Luis Moreno-Armella
Departamento de Matemática Educativa, Cinvestav
lmorenoa@data.net.mx

Introduction

Since ancient times, philosophers have dealt, in their attempt to explain the issues that arise from the theories of knowledge, with the idea of *representation*. To illustrate the importance of this notion, we can mention that the *Encyclopaedia Britannica* has 1741 references of the various ways in which the idea of representation is used in different disciplines. Representation, however, is a slippery concept. And because of its long past, using Seeger's words (Seeger, 1998, p.311), "it does not seem possible just to define what is understood as representation and make a fresh start".

In modern times, Kant's epistemology introduced an important distinction at the core of what is understood by representation. In his *Critique of Pure Reason* (1797), Kant asserts that whenever a person is in contact with the world, the impressions he/she receives are put through an organizing process by the innate cognitive structures. In much the same way as a liquid is given shape by the container that holds it, so are the sensory impressions shaped by the innate cognitive structures that process that information. Our knowledge of the world is therefore just an *interpretation,* as given by our intellect, of that external reality. In Kant's view, knowledge of reality is not an isomorphic copy of that reality. The idea of knowledge as a reflection in a mirror is abandoned. Knowledge is now considered a *representation*, a *map* of a territory, not the territory itself, as is described by Korzybsky (Le Moigne, 1995, p.69). Kant believed that all objects of sensation must be experienced within the limits of space and time. All objects therefore have a spatial-temporal location. Because space and time are the backdrop for all sensations, he called them *pure forms of sensibility.* This approach to the theory knowledge had a profound impact from its very inception. The cognizing subject was given a central role in the production of knowledge; this became the foundation for contemporary constructivist epistemology —which answers many of the issues raised against the Kantian school.

In Western culture, it is in the Renaissance that the individual comes to the foreground, and with it the conception of knowledge as a phenomenon centered in the individual subject. It would not be until much later, in the first decades of the twentieth century, when, as a consequence of various social and cultural movements of the nineteenth century, that new conceptions of the human being arose. These new conceptions rejected the inmutability of ideas such as "time" and "space"; they introduced a contingent element in the nature of knowledge, and with it, a more diverse interpretation of the term representation.

In order to understand a human being, it would no longer be enough to study his present: it became necessary to take into account the various genetic domains that constituted his history. Different theories then emerged, such as that of epistemological constructivism and the social-cultural approaches, all of which –from different cognitive perspectives–, gave a primary importance to the genetic domains.

We should comment that the existence of these two different approaches: one which emphasizes the role of the *subject* in cognitive activity, and the other which puts the emphasis on the *social-cultural dimension,* has been reflected in the educational field as an almost irreconcilable tension between cognition and culture. In our view, this is an unfortunate state for education; as Noss and Hoyles (1996, p.107) explain: "if teaching is to figure alongside learning, we have as much to gain from Vygostky as we do from Piaget". We need to be able to articulate these different points of view in a way which would allows us "to treat mathematical learning as both a process of active individual construction and a process of enculturation" (Cobb et al., 1997). This is not an easy task.

On Signs and Representations

The production of signs and representations is crucial in order for human beings to be able to assimilate what is external to them and communicate the results from those assimilations, to other human beings.

The use of signs and representations inside a culture gives them a conventional character and an agreed meaning. It is thus that a ring may be an *indication* that the person wearing it, is married. Depending on the nature of the relationship between the sign and the object represented, signs are differentiated, according to Pierce, into *icons, indexes, and symbols* (Deacon, 1997, pp.70-71). Because this denomination *is dependent on the relationship* of the sign with an object, no sign is intrinsically an icon, an index, or a symbol. "The differences between iconic, indexical and symbolic relationships derive from regarding things either with respect to their form, their correlations with other things, or their involvement in systems of conventional relationships" (p.71).

We would like to suggest then, that the systems of representations that we use —specially those we use in mathematics— have a cultural origin and therefore, so does the knowledge which is produced with their help. It is important to point out that this assertion does not compromise the objectivity of knowledge. It does force us, however, to reformulate the issue of objectivity in terms which differ from those inherited from the epistemological realism.

There is wide evidence that suggests that the sign systems play a fundamental role in the development and use of concepts. They act as *mediators of thought.*

Sign systems offer users the possibility to approach a problem in diverse ways (according to the sign system being used). In a learning situation, signs are part of the structuring elements in the interaction between the subject and the emerging concept. By changing a system of representation we could highlight different characteristics of a concept. Metaphorically speaking, we could say that each system of representation allows us to see a different facet of the object-concept being studied.

On Cognition and Historical Development

Theories of learning have to respect a fundamental principle: cognition is mediated by tools either material or symbolic (Wertsch, 1993). Technology, in all its forms, modifies, substantially, the process of knowledge production.

Learning involves the construction of representations. It is through the construction of representations of an observed phenomena, (or of a mathematical concept) that we make sense of the (mathematical) world. *Representations become mediational tools for understanding.*

It is possible to support these ideas with some historical examples. Once I say this I have to make clear that I am not suggesting to transfer this support from a genetic domain to another one.

The decimal representation of numbers is, perhaps, one of the best examples of how an adequate symbolic representation becomes an instrument with which to explore: The reasoning is (almost) impossible without this representational system.

The geometric continuum appeared, in Euclidean mathematics, as an abstraction of the physical continuum. Because of the characterization of continuity as neverending divisibility, it was possible to conclude that the continuum was not made of indivisibles. On the other hand, number was the prototype of discreteness; number was a collection of units (and the unit was not a number).

This scenario changed radically with the work (1585) of Simon Stevin (Waldegg, 1993). The Greek concept of number had developed as a result of an abstraction process applied to the material world. Stevin challenged the Greek viewpoint to accommodate the utilitarian matter of measurement in the real material world. In his work (Moreno, L. & Waldegg, G.), he identified number and magnitude, attributing numerical properties to continuous quantities and continuity to numbers. From this point on, *it is not possible to separate the concept and its symbolic representation*. For example, the infinite divisibility of number corresponds to the operation of division made possible by the decimal notation. The new concept of number was partially possible through a reflective abstraction. Number was understood as *"that through which the quantitative aspects of each thing are revealed"*. Therefore, arithmetic operations are sustained, at a first moment, on the actions that are carried out on quantities (Waldegg, 1996). Afterwards, the symbols used in the decimal notation are identified with real numbers. In a sense, this means that symbols becomes icons. In fact, cultural icons. This narrative also seems to illustrate Damerow (1988) viewpoint on cognitive structures. According to Damerow, the initial conceptual change "might be exogenous to the cognitive system and historically as well as culturally determined. Part of individual development would then consist of the effort to appropriate culturally developed cognitive structures" (Nicolopoulou, 1997, p.208).

Cognition and Context

When the epistemological realism is applied to the educational field, it is accompanied by a conception of cognition as that of a mechanism which extracts information from a stable and objective world. As has been pointed out, this view of realism deeply constrains the possibilities for the study of learning mechanisms as they happen in a local, socially constructed environment (i.e. in the classroom). It is from there that the theories of situated cognition have emerged. In the field of mathematics, however, situated cognition has presented us with a formidable challenge (Noss & Hoyles, 1996, p. 36). Mathematics does not accept propositions anchored to fixed referentials, which are dependent on the accidents of the context.

Street mathematics, as it has been called, take advantage of the meaning of the context in which problems arise. In school, the manipulations of syntax make it possible to carry out operations of the type "the price of an apple multiplied by the number of apples". We cannot help at this point but to remind the reader of the state of the mathematics of magnitudes before the emergence of Algebra (see Galileo's Dialogues concerning Two New Sciences).

Nunes et al., (1993) point out, that there is evidence that the pragmatic schemes of street mathematics can be generalized: users resort to the symbolic system provided by money in order to carry out activities of situated transference. Thus, a person's situated knowledge can be used as support for expressing more general relationships as well as for inducing from there a reflection on the activity. We enter here into the complex world of abstraction. We have given this illustration because it gives some insight into the solution to the problem of how the representational systems being used can be used to cross the gap between a situated exploration and the need to systematize.

On cognition and computing tools

Our goal in this section is to *articulate a reflection* on the ways in which computational tools mediate the construction of mathematical concepts.

Computing environments provides a window for studying the evolving conceptions of students and teachers, as they use the tools provided by that environment.

We have noticed that –according to the students– *mathematics* refers mainly to a set of symbolic expressions. Knowledge of this mathematics means to be able to use algorithms to transform a symbolic expression into another. Now the presence of graphing tools tends to shift the attention from symbolic expressions to graphical representations. At this point, it is important to highlight the importance of articulating the different systems of representations. These are tools for understanding and mediating the way in which knowledge is constructed.

Our efforts to articulate a reflection on computational tools lead us to consider the phenomenology we can observe on the screens of calculators and computers. The screen is a space controlled from the keyboard but that control is very much one of action at distance. The desire to be able to interact with the screen objects provide a motivation for struggling with the complexities of a computing environment (Pimm, 1995, p.36). On this respect, Balacheff and Kaput (1996) have talked about a "new mathematical realism" due to the new experiences while working within a computational environment. We suggest that this new realism is due, mainly, to the nature of computational representations. Computational representations are *executable representations*. Consider, for instance, the environment provided by Cabri-Geometre. Therein one can transform (by dragging) a triangle into another while trying to invalidate a property (for instance, that the bisectors always intersect in an interior point of the triangle). The impossibility of invalidating the property leads to a proposition we call a "theorem". The Cabri environment is well suited to close the gap between the notion of drawing and the notion of geometrical object.

There is another attribute of executable representations on which we want to cast light. It is the fact that they serve to *externalize* certain cognitive functions which formerly were executed by people. That is the case, for instance, with the graphing of functions. Now the student has the opportunity to transform the graph into an object of knowledge. This is similar to what the Greeks did with writing. They used the writing system not only as an external memory but as a device to produce texts on which to reflect. As Donaldson (1993, p. 342) has said, the Greek's critical innovation consisted of "*externalizing the process* of oral commentary on events".

The interaction between diverse executable representations facilitates the construction of *situated abstractions* (Noss&Hoyles, 1996, pp. 105-107; Sacristan, 1997). Situated abstractions refers to the understanding and encapsulation of processes within the context in which they have been explored. Let us explain: At a first moment students can make some observations situated within the computing environment they are exploring, and they could be able to express their observations by means of the tools and activities devised in that environment. That is the case, for instance, when the students try to invalidate (by dragging) a property of a geometrical figure and they cannot. That property becomes a theorem expressed through the tools facilitated by the environment. As a result we have a *situated proof* (Sacristan, A. op.cit. ; Sacristan, A. Noss, R. & Moreno, L, 1999, in preparation)

We retain the use of the adjective "situated" just to call attention to the role of the environment and the tools employed. A situated proof is the result of a systematic exploration within an (computational) environment. It could be used to build a bridge between situated knowledge and some kind of formalization.

T. Nunes et al. (1993) have observed that Brasilian children used the money system to transfer their arithmetic strategies from one context to another. The money system, paper and coins, worked as a universal referent. Similarly, observing our students while working in a computing environment, we noticed they could articulate the results of their explorations in a way that could be taken beyond the environment in which they were found. The students purposely exploited the tools provided by the computing environment to explore mathematical relationships and to "prove" theorems (in the sense of situated proofs). Let us illustrate this point with the case of continuous non-differentiable functions. While working with the algebraic calculator TI-92, the students observe the dynamic drawing process of the polynomial approximations to Weiestrass' function, and begin to understand the randomness "hidden" in such a function. They realize this characteristic of the function under consideration, thanks to the dynamics provided by the executable representation of the Weierstrass' function.

Drawing by hand and using a computing device to draw, are different cognitive activities. Of course, the nature of the mediational tools applied in each case, support this assertion. We suggest that the executable nature of the computer's representations "orients" the reflections of the students to structural considerations: An apt tool for exploring the present example is the zoombox. Through its use, students may discover the degree of complexity of the function, perhaps only from a visual point of view. The tool is used, here, as a microscope, and not as a magnifying glass. We think this difference is important: What one can see by using a magnifying glass belong in the same structural domain as what one can see with naked eyes. The microscope, on the other hand, allows one to enter a new structural domain. It is not just a matter of scale, but of obtaining a new object of knowledge. This way, students can generate and articulate relationships that are general to the computational environment in which they are working.

Note. The field work, supporting our assertions, is part of the project "La incorporación de nuevas tecnologías a la cultura escolar" (referred to below) and of several joint works with Ana Sacristan (see references).

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References

Cobb, P. et al. (1997). Mathematizing and Symbolizing: The Emergence of Chains of Signification in one First-Grade Classroom. In Kirshner, D.& Whitson, J. (Eds.), *Situated Cognition, social, semiotic and psychological perspectives, pp. 151-233.* New York: Erlbaum.

Damerow, P. (1988). Individual development and cultural evolution of arithmetical thinking. In Strauss, S. (ed) *Ontogeny, Phylogeny and historical development*, Norwood, New Jersey.

Deacon, T. (1997). *The Symbolic Species: The co-evolution of language and the brain*. W.W. Norton& Company.

Donaldson, M. (1993). *Origins of the modern mind*. Harvard University Press.

Le Moigne, J.L. (1995). L*es Epistemologies Constructivistes*, PUF, Paris, France.

Moreno, L. & Waldegg, G. (1999). An epistemological history of number and variation, (accepted for publication, MAA, Katz, V. (ed))

Nicolopoulou, A. (1997). The invention of writing and the development of numerical concepts in Sumeria: some implications for developmental psychology. In Cole, M. et al (Eds.), *Mind, culture and activity*, Cambridge University Press.

Noss, R. y Hoyles, C. (1996). *Windows on mathematical meaning*. Netherlands: Kluwer.

Nunes, T.,& Schliemann, A.,& Carraher, D. (1993). *Street mathematics and school mathematics*, Cambridge University Press.

Pimm, D. (1995). *Symbols and meanings in school mathematics*. London: Routledge.

Sacristan, A., Noss, R. & Moreno, L. (1999, in preparation). *Situated abstractions–situated proofs: constructing generalizations and testing conjectures in an infinite processes microworld.*

Sacristan, A. (1997). *Windows on the infinite: Constructing meanings in a Logo-based microworld*. Ph.D. Thesis, Institute of Education, University of London.

Seeger, F. (1998). Representations in the mathematics classroom: Reflections and constructions. In Seeger,F., Voigt,J. and Waschescio,U. (eds), *The culture of the mathematics classroom*, Cambridge University Press.

Waldegg, G.(1993). La Notion du Nombre avant L'Etablissement de la Science Analytique. Proceedings *First European Summer University in History and Epistemology in Mathematics Education*, pp. 115-122. Montpellier, France.

Waldegg, G. (1996). Contribución de S. Stevin a la Construcción del Concepto de Número. *Educación Matemática, 8* (2), 5-17.

Wertsch, J. (1993). *Voces de la mente*. Madrid: Visor Distribuciones.

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