DOCUMENT RESUME

ED 465 813 TM 034 202

TITLE	Delaware Student Testing Program Item Sampler: Released Mathematics Items, Grades 3, 5, 8, and 10.
INSTITUTION	Delaware State Dept. of Education, Dover.
PUB DATE	2002-01-00
NOTE	89p.; Document Number 95-01/02/02/01.
AVAILABLE FROM	For full text: http://www.doe.state.de.us/aab/index.htm.
PUB TYPE	Guides - Non-Classroom (055)
EDRS PRICE	MF01/PC04 Plus Postage.
DESCRIPTORS	Academic Standards; Elementary Secondary Education; Information Dissemination; *Mathematics Tests; State Programs; State Standards; Study Guides; Test Construction; *Test Items; Test Wiseness; Testing Programs
IDENTIFIERS	Delaware; *Delaware Student Testing Program

ABSTRACT

The Delaware Student Testing Program (DSTP) is designed to assess progress toward the Delaware Content Standards. Every year a certain number of items are removed from the test and then selected for public release. This booklet contains items released from the 1998 administration of the DSTP mathematics component tests. Taken as a whole, these items represent a broad cross section of item types and topics measured. They also show the evolution in the construction of items and their rubrics. There may not be sample items for each grade for the four categories of mathematics concepts, but at each grade, no more than half a dozen items can be released. Each released item is accompanied by an answer key, whether an answer option or an item-specific rubric. For the mathematics items, concepts and procedures are measured in: (1) number concepts; (2) patterns and algebra; (3) geometry; and (4) statistics and probability. (SLD)

DELAWARE

STUDENT TESTING PROGRAM

ITEM SAMPLER

RELEASED
MATHEMATICS ITEMS

Grades 3, 5, 8, and 10

January 2002

Delaware Department of Education

Document Number 95-01/02/02/01

Harcourt Educational Measurement
A Harcourt Assessment Company

MATHEMATICS

The Delaware Student Testing Program (DSTP) is designed to assess progress toward the Delaware Content Standards. Items on the mathematics portion of the test are carefully developed to measure concepts and procedures in:

- Number Concepts (computation, estimation, and measurement);
- Patterns and Algebra;
- Geometry; and
- Statistics and Probability.

Every year, a certain number of items are replaced with carefully matched new items. Some of the items removed from the test are then selected for public release. This booklet contains items released from the 1998 administration of the DSTP mathematics component tests. These items were selected for release because, taken as a whole, they represent a broad cross-section of item types and topics measured. They also show the evolution in the construction of items and their rubrics. Each grade may not have sample items for the four categories of concepts.

At each grade level, no more than a half dozen items can be released. The richness and scope of the test can only be appreciated by considering this set of released items as a whole.

Each of these released items is accompanied by an answer key – simply the correct answer option in the case of multiple choice items or a complete two- or four-point item-specific rubric in the case of constructed response items. Each item is also identified as to the standard assessed and embellished with a brief discussion of the mathematics involved.

Parents of a school-aged child may find it useful to read through appropriate parts of the booklet with their children. Teachers in Delaware public schools should study these items with colleagues, both at their own and adjacent grade levels. Each item should be reviewed with the Delaware mathematics content standards in mind.

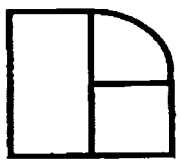
Math Standard #8 (Grade 3)

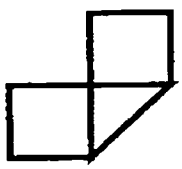
Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

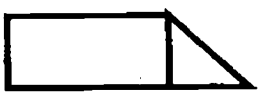
Item #1

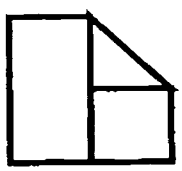
Tasha has these shapes.

Which of the following figures can she make if she uses each shape only once and no shape covers another?

a. 

b. 

c. 

d.  *

Commentary:

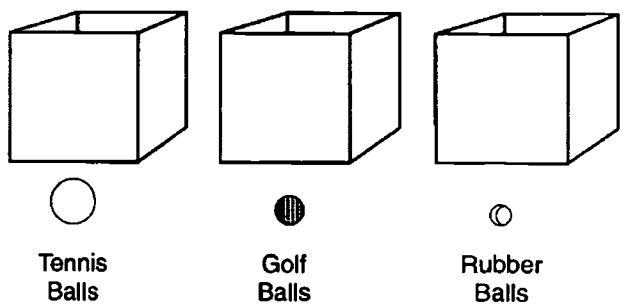
Students should eliminate options b. and c. if they carefully read the question, which states that each shape must be used only once, and no shape can cover another one. This item assesses the third grader's ability to mentally transform simple geometric shapes. In the correct answer, d, while both the triangle and the square remain in the same orientation in which they are initially presented in the item prompt, the rectangle has been rotated through a ninety degree angle and yet remains congruent with the original rectangle.

Math Standard #5 (Grade 3)

Students will develop an understanding of ESTIMATION, MEASUREMENT, AND COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

Item #2

All the boxes are the same size. Linda has three different sizes of balls as shown in the picture below.



If she fills each box with the kind of balls shown, which box will have the fewest balls in it?

a. The box with the tennis balls. *

b. The box with the golf balls.

c. The box with the rubber balls.

d. Each box will have the same number of balls.

Commentary:

Students must understand that the bigger the balls, the fewer the number of balls is required to fill up the box. The inverse relationship of size and number is one of the fundamental ideas in the development of the concept of volume or space filling. Students might approach this problem using a more or less concrete strategy. For example, some students might attempt to draw the balls in the boxes and perhaps even count the balls to determine the correct answer. Other students might appeal to personal experience and understand the answer at a more abstract level.

Math Standard #5 (Grade 3)

Students will develop an understanding of ESTIMATION, MEASUREMENT, and COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting appropriate tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

Item #3

Chris bought the following items at the store. How much did the items cost in all?

3 oranges
1 box of cookies
1 loaf of bread
1 carton of juice

Scoring Rubric:

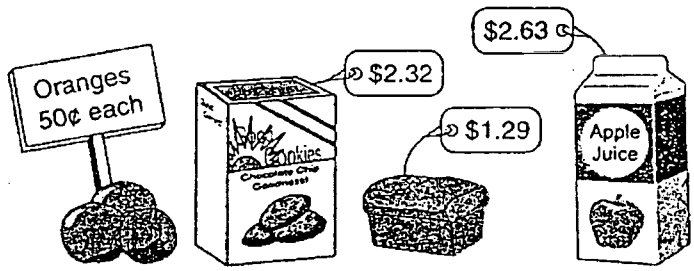
2	$7.74
1	$6.64 (calculated cost for a quantity of one rather than three oranges).
0	Any other answer.

Commentary:

This item represents a realistic situation in which a sum is required but one of the items must be added repeatedly. Attention to the details of the task is required for a score of 2 points. Because the repeated quantity, the oranges, cost 50¢ each, a subtotal for the oranges ($1.50) could easily be achieved mentally as a first step in the solution process.

Student Response

Score Point: 0



3 oranges $1.50 total $8.74

1 box of cookies $2.32

1 loaf of bread $1.29

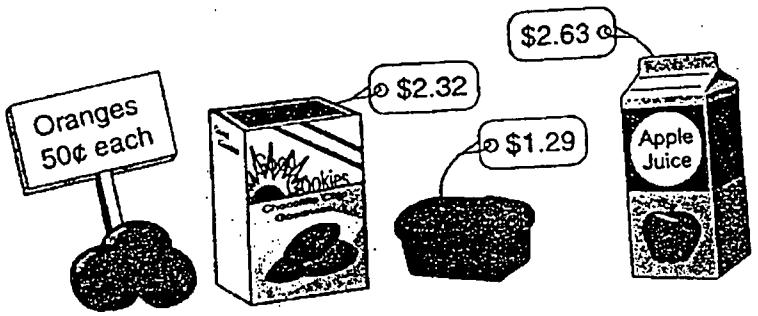
1 carton of juice $ 263

Comment:

Although this student did determine the correct price for the three oranges ($1.50) and attempted to find the total for all of the items, she or he was unable to correctly add the different prices. Since a premium was put on a correct numeric answer for this item, this response earns 0 point.

Student Response

Score Point: 1



3 oranges

1 box of cookies

1 loaf of bread

1 carton of juice

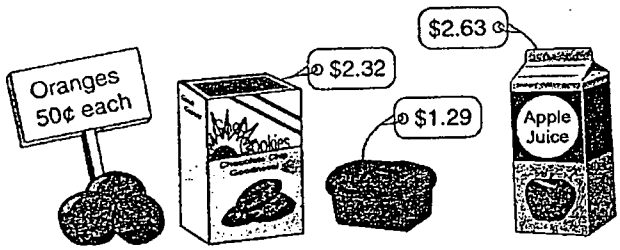
1
1.50 ¢
1.29
2.32
+2.63
$6.74

Comment:

The student calculated the correct cost based upon the incorrect assumption that only one orange was to be purchased rather than three. The student demonstrated proficiency in place value. This response earns 1 point.

Student Response

Score Point: 2



3 oranges

1 box of cookies

1 loaf of bread

1 carton of juice

I got the answer from adding all the number all together.

2 1
50¢
50
50
2.32
1.29
2.63
$7.74

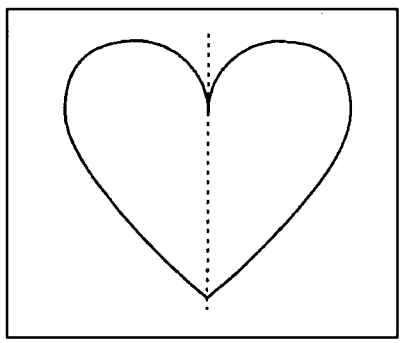
Comment:

The student explained his or her process and showed his or her work to obtain the correct cost for all the items. We note that this student did not calculate the price of all three oranges as a first step but, rather, regarded this as a purchase involving six separate items. Another successful strategy would be to calculate the cost of the three oranges as $1.50 and then enter this subtotal into a calculation of the final cost. This response earns 2 points.

Math Standard #8 (Grade 3)

Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

Item #4



Some shapes have line symmetry. This means that you can fold the shape so that it folds back onto itself perfectly. The heart shape to the right has one line of symmetry.

In the space below, draw <u>two</u> different shapes that have one line of symmetry. Mark the line of symmetry on each shape with a dotted line.

Scoring Rubric:

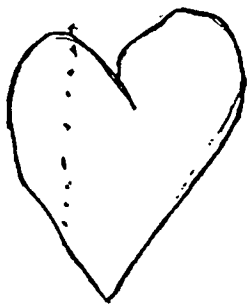
4	Two examples provided with one line of symmetry drawn on each. (Figures will be sketches as will the lines of symmetry and should be judged accordingly. Intent rather than precision of drawing is to be evaluated. Lines of symmetry need not be dotted.) Figures may have more than one line of symmetry without penalty.
3	Two symmetric figures provided but line of symmetry properly executed in only one of the two figures.
2	One symmetric figure with line of symmetry properly drawn. Second figure absent or non-symmetric.
1	Non-symmetric figure(s) or symmetric figure(s) with line(s) of symmetry improperly drawn. i.e., the response shows evidence that an attempt was made at drawing symmetric figures but without clear success.
0	Some work but without apparently understanding of the goals of the task.

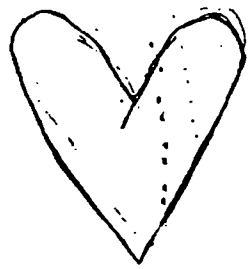
<u>Commentary:</u>

This item requires students to demonstrate their understanding of line or mirror symmetry. Line symmetry is defined and illustrated and then the student must create two additional examples. In many cases, responses will include familiar geometric figures such as rectangles or circles, but some students respond by creating more elaborate figures mirrored across a central line of symmetry.

Student Response

Score Point: 0



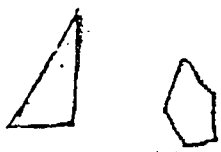


Comment:

This student apparently did not understand either the question or the concept of line symmetry. The original shape is reproduced twice with the original line of symmetry shifted first left and then right. There is no evidence of the mathematical concept of symmetry and so this response earns a score of 0.

Student Response

Score Point: 1



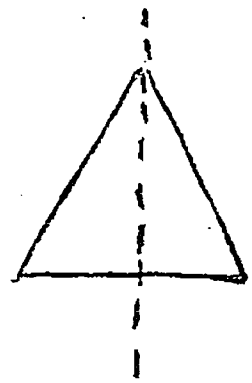
The Triangle you can flold it upand dawn. The sam with the pentagon.

Comment:

The student didn't demonstrate a complete understanding of the concept of line of symmetry, however, the phrase "*fold it up and down*" indicates some basic understanding. The pentagon does have a line of symmetry; the triangle, as drawn, does not. However, this student failed to attempt to draw the line of symmetry on the pentagon and so this response earns only 1 point.

Student Response

Score Point: 2

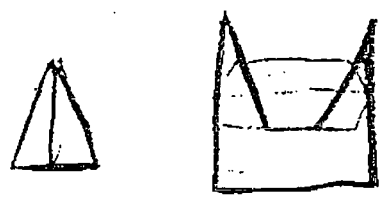


Comment:

The student drew a perfect example of a line of symmetry using an isosceles triangle. However, the second example shows a line of non-symmetry. Perhaps the student intended to rotate the triangle from a vertical to a horizontal position but failed to maintain the symmetry of the figure in the transformation. This failure does cast some doubt on the depth of the student's understanding of line of symmetry. The student might have understood that a shape can be folded along the dotted line and both parts must match, however, only one correct example is demonstrated. This response earns 2 points.

Student Response

Score Point: 3



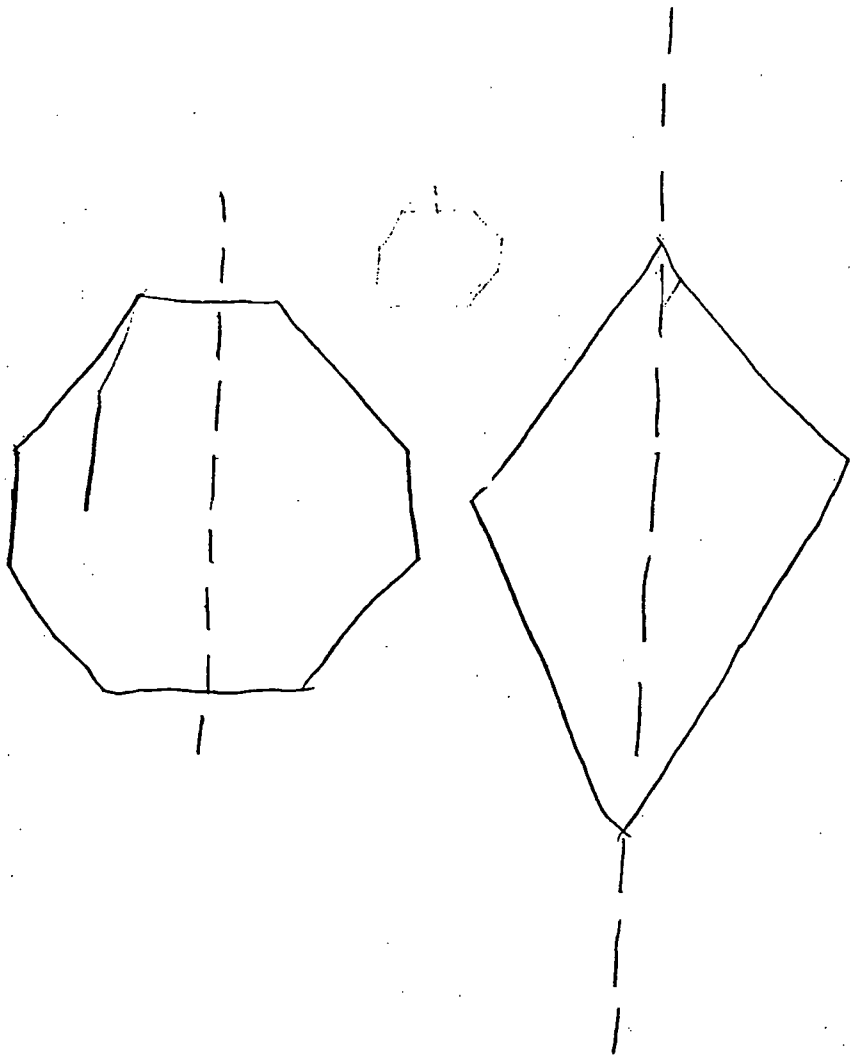
I did Thes Shapes because They are the only ones that have 1 line of Semetry.

Comment:

Two different symmetrical shapes are given, but only the first triangle has a line of symmetry properly drawn. The student seems to understand reflection symmetry but did not complete the second drawing in omitting to draw the line of symmetry. This response earns 3 points.

Student Response

Score Point: 4



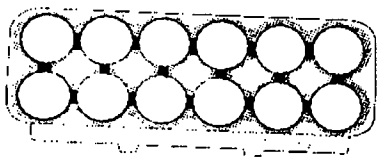
Comment:

Two correct examples of line of symmetry are illustrated. The student accurately drew dotted lines to fully demonstrate his or her understanding of the concept. This response earns 4 points.

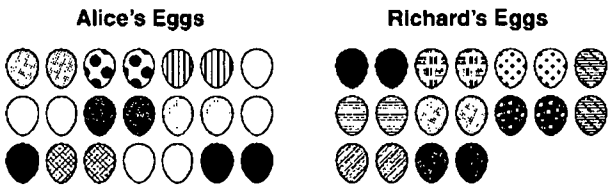
Math Standard #6 (Grade 3)

Students will develop NUMBER SENSE by solving problems in which there is a need to represent and model real numbers verbally, physically, and symbolically; to use operations with understanding; to explain the relationships between numbers; to apply the concept of a unit; and to determine the relative magnitude of real numbers.

Item #5



Alice and Richard are coloring eggs and putting them back into the egg cartons. Alice colored 21 eggs and Richard colored 18 eggs. (There are 12 eggs in a carton.)



How many cartons will they need to hold all the eggs? Explain how you got your answer.

Scoring Rubric:

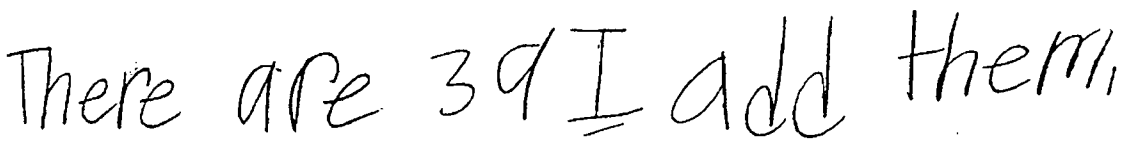
4	Correct answer (4 cartons) with clear verbal or pictorial explanation. (This may or may not involve finding the total number of eggs. For example, the problem might be successfully solved by grouping the egg images into groups of 12.)
3	Correct answer (4 cartons) but sketchy explanation or incorrect answer of 3 cartons with explanation that describes the number of cartons *filled.*
2	There is an (unsuccessful) attempt to divide eggs into cartons. Perhaps total number of eggs (39) is correct but attempt to divide into cartons is flawed.
1	An attempt is made to count eggs (perhaps even successfully), but there is no evidence of an attempt to divide eggs into cartons.
0	Trace evidence of work but without clear connection to problem situation.

Commentary:

This item addresses several components of number sense. The student must use several operations with understanding including addition and division but is able to do this in a context involving a physical representation of number. A great variety of solution strategies have been observed. For example, some students actually identified the first, second, and third dozen eggs in the diagram and found that three full cartons were needed and a fourth with only three eggs. Other students found the sum directly and compared this to the number of eggs in three dozen.

Student Response

Score Point: 1



Comment:

The student counted the total number of eggs correctly but didn't partition the eggs into cartons. The student did not attempt to divide the eggs in groups of 12, which would have given the required number of cartons. This response earns 1 point.

Student Response

Score Point: 2

4½ you will need 4 ½ because if you keep counting to twelve that's what you would get.

Comment:

This student showed some understanding of how to solve the problem but did not reach a correct numerical conclusion. The student employed the strategy of counting by 12s since each carton can hold 12 eggs. Clearly this student understood that there would be extra eggs left over that would not fill an additional carton but erred in a) claiming that four cartons would be filled (there would be only 3 actually filled) and that b) the extra eggs constitute one-half of an additional carton (3 eggs would be left over which would fill just ¼ of a carton). This response earns 2 points.

Student Response

Score Point: 3

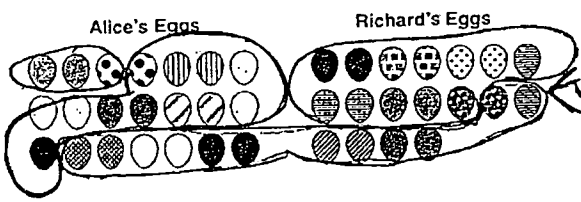
4. I got four for an answer. First, I added 21 + 18 = 39 eggs. Then I counted on my fingers until I got to 39.

Comment:

This response contains a correct answer (4 cartons) and a sketchy description of the process by which this answer was derived. Perhaps the student counted to 12 three times (on his or her fingers) and found that three cartons would be filled with some eggs left over, requiring, therefore, a fourth carton. We can, however, only speculate about the nature of the student's thinking given the limited evidence in the response. This response earns 3 points.

Student Response

Score Point: 4



How many cartons will they need to hold all the eggs? Explain how you got your answer.

They will need four I Showed my work up here

Comment:

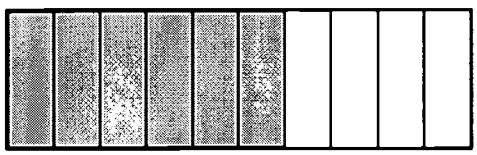
The student showed his or her reasoning and work on the drawing by grouping 12 eggs to fill a carton. The student understood that a fourth carton is necessary to hold all the eggs. A previous answer of 3 cartons was crossed out to indicate that the extra 3 eggs would go in a new carton. This response earns 4 points.

Math Standard #6 (Grade 5)

Students will develop NUMBER SENSE by solving problems in which there is a need to represent and model real numbers verbally, physically, and symbolically; to use operations with understanding; to explain the relationships between numbers; to apply the concept of a unit; and to determine the relative magnitude of real numbers.

Item #1

Which fraction represents the shaded part of the figure below?



a. $\frac{6}{10}$

b. $\frac{6}{12}$

c. $\frac{6}{6}$

d. $\frac{6}{4}$

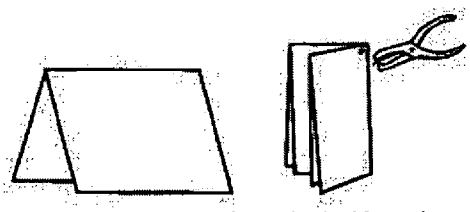
Commentary:

This item requires that students understand the concept of a whole and its parts. The visual aid will make this item accessible to all students if they see that all parts are equal and therefore only 6 parts are shaded out of a total of 10 parts.

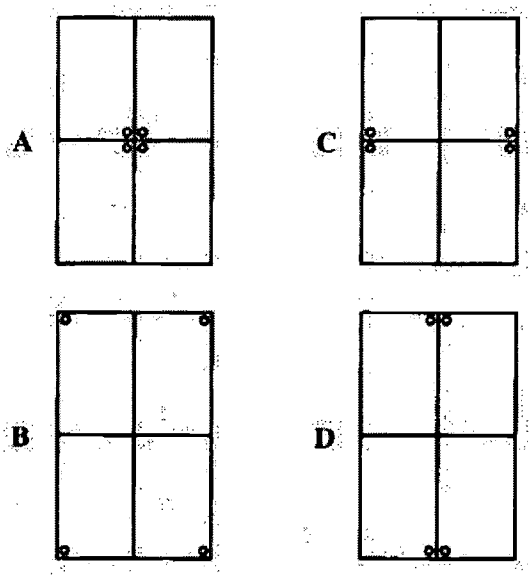
Math Standard #8 (Grade 5)

Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

Item #2



A piece of paper is folded in half and then is folded in half again. A hole is punched in the corner nearest all folds. Which picture shows how the paper will look when it is unfolded?



Commentary:

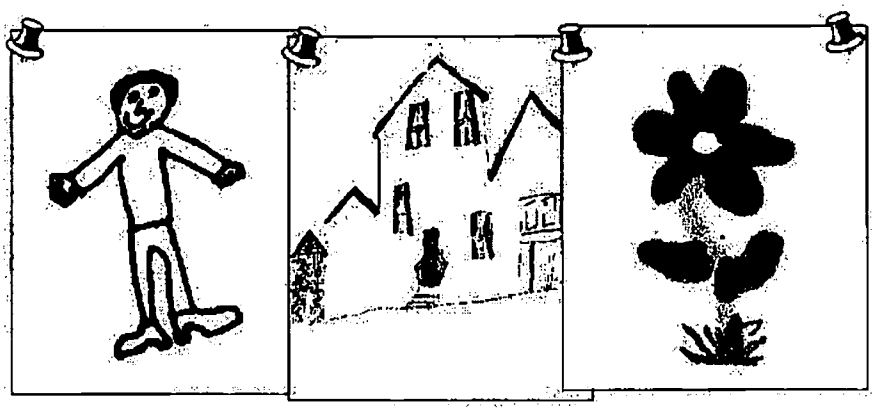
This item challenges students to understand geometric plane figures and their symmetries. The students must transfer the information in the given drawing to what the unfolded paper will look like. Special attention should be placed on the location of the punched hole in order to eliminate the distracters, b and c.

Math Standard #10 (Grade 5)

Students will develop an understanding of PATTERNS, RELATIONSHIPS, and FUNCTIONS by solving problems in which there is a need to recognize and extend a variety of patterns; and to analyze, represent, model, and describe real-world functional relationships.

Item #3

Pictures are hung in a line like the example below. Pictures next to each other share a tack. How many tacks are needed to hang 28 pictures this way?



a. 27

b. 28

c. 29 *

d. 56

Commentary:

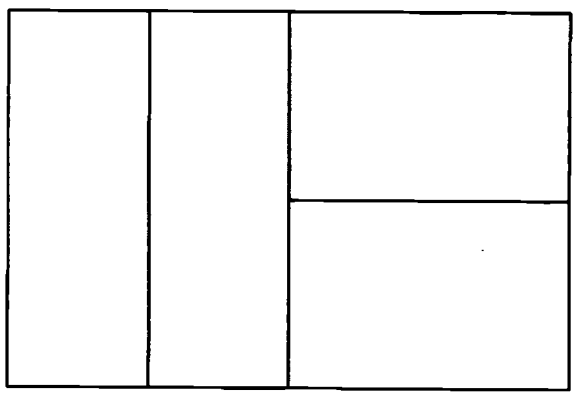
This item requires students to first translate the information in the pictorial setting to a number relationship. Using rather informal notation, students model a real-world relationship. Four tacks are required to hang 3 pictures, five tacks would be required to hang 4 pictures, then six tacks will be required to hang 5 pictures, and so on... Students should discover the pattern after several examples are considered: the number of tacks required is one more than the number of pictures to be hung. Therefore 29 tacks would be needed to hang 28 pictures.

Math Standard #6 (Grade 5)

Students will develop NUMBER SENSE by solving problems in which there is a need to represent and model real numbers verbally, physically and symbolically; to use operations with understanding; to explain the relationships between numbers; to apply the concept of a unit; and to determine the relative magnitude of real numbers.

Item #4

John split a cake into four pieces. The pieces are *not* all the same shape. Do you believe the four pieces are the same size? Please explain.



Scoring Rubric:

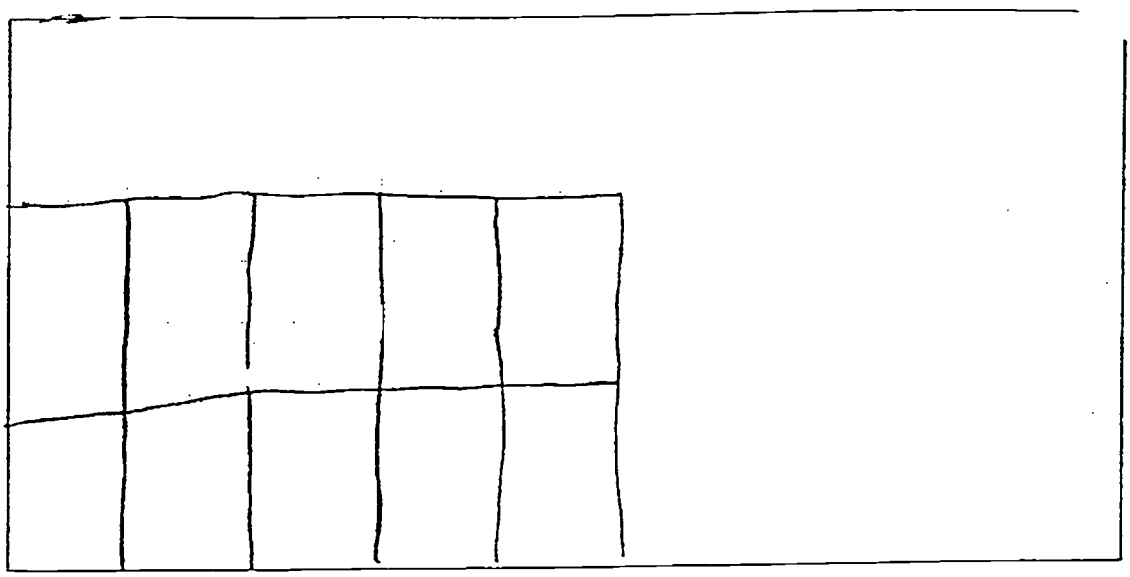
4	Explains that the four pieces are the same size because they are each half of a half (or one-quarter) of the full cake. Or the student might recreate a picture of the rectangular cake in the answer booklet and use an area or measurement model, i.e., counting the number of answer grid squares in each piece of the cake.
3	Agrees that the four pieces are all the same size but the explanation is weak or incomplete. For example, a response in this category might describe the process of cutting the cake but fail to describe the ultimate equality of half of a half: "I cut the cake into two pieces of the same size and then cut one of these in half the long way and the other in half the other way."
2	Agrees that the four pieces are the same size but without coherent (or perhaps any) explanation.
1	Asserts that the resulting pieces are not the same size, with or without rationale.
0	Attempts a response but does not clearly address the task.

Commentary:

Operations with fractions must be built upon a firm foundation of conceptual understanding. Using an area model of fractions as in this item, students are asked to compare fractional pieces of different shapes but with the same area. A strong, somewhat abstract, response may involve the recognition that half-of-a-half represents one-quarter. Other students may cut each shape into smaller units in order to compare the shapes' areas.

Student Response

Score Point: 0

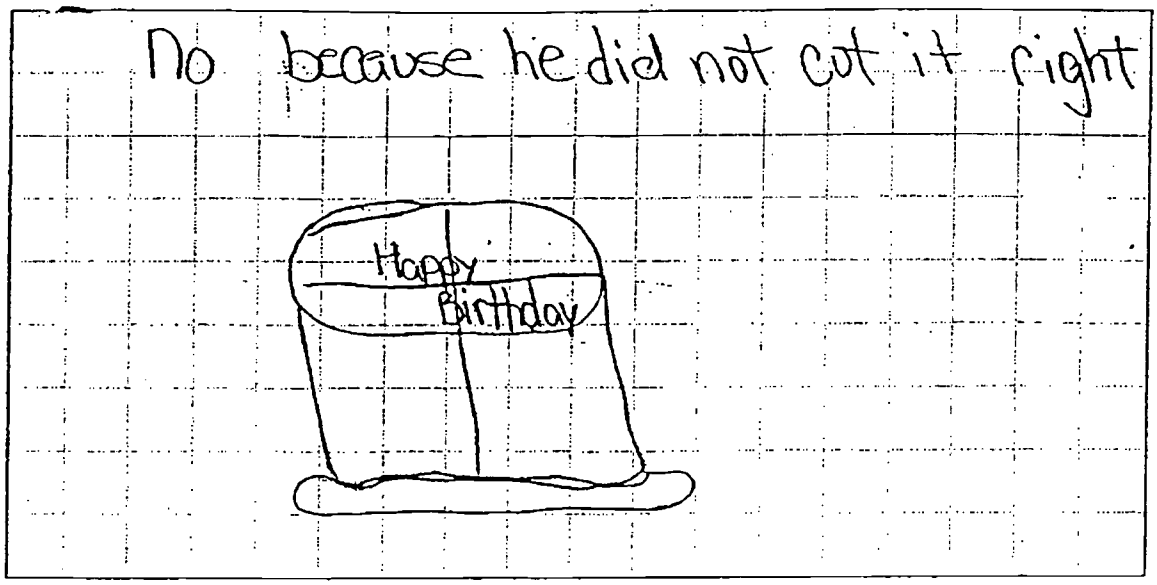


Comment:

There is an attempt at dividing the cake (rectangle) into equal parts but it doesn't address the task. It is not clear how this division of the rectangle into ten congruent pieces bears on the question of the equality of the four pieces given in the question. It may be that this student has reconceptualized the task given into a simpler one, i.e., divide the cake into (some number of) equal pieces. When comparing the size of different objects or figures, students are required to transition from concrete measurement to a more visual/abstract reasoning about measurement. This response earns 0 point.

Student Response

Score Point: 1

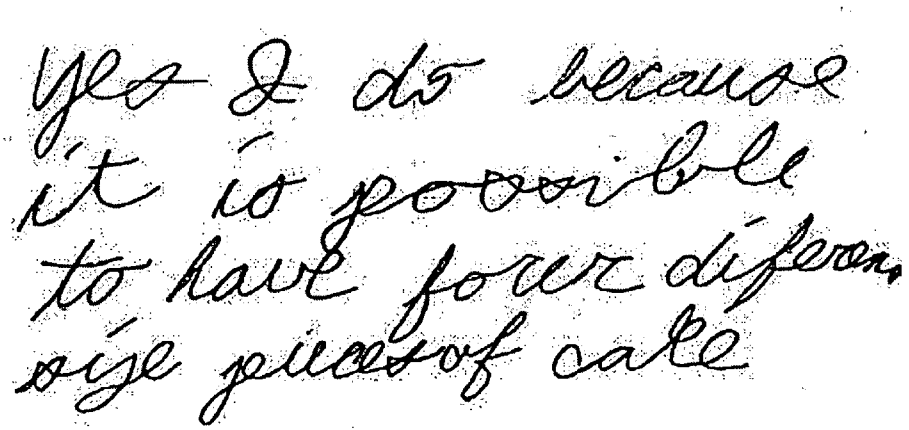


Comment:

The response indicates that this student is limited to thinking about fractions using models with congruent parts. Rather than address the question of whether non-congruent parts might indeed have the same area, the student used a familiar everyday activity to demonstrate the correct way of showing fourths. This response might suggest that this student has not progressed beyond a very concrete and basic stage of reasoning about fractions.
This response earns 1 point.

Student Response

Score Point: 2

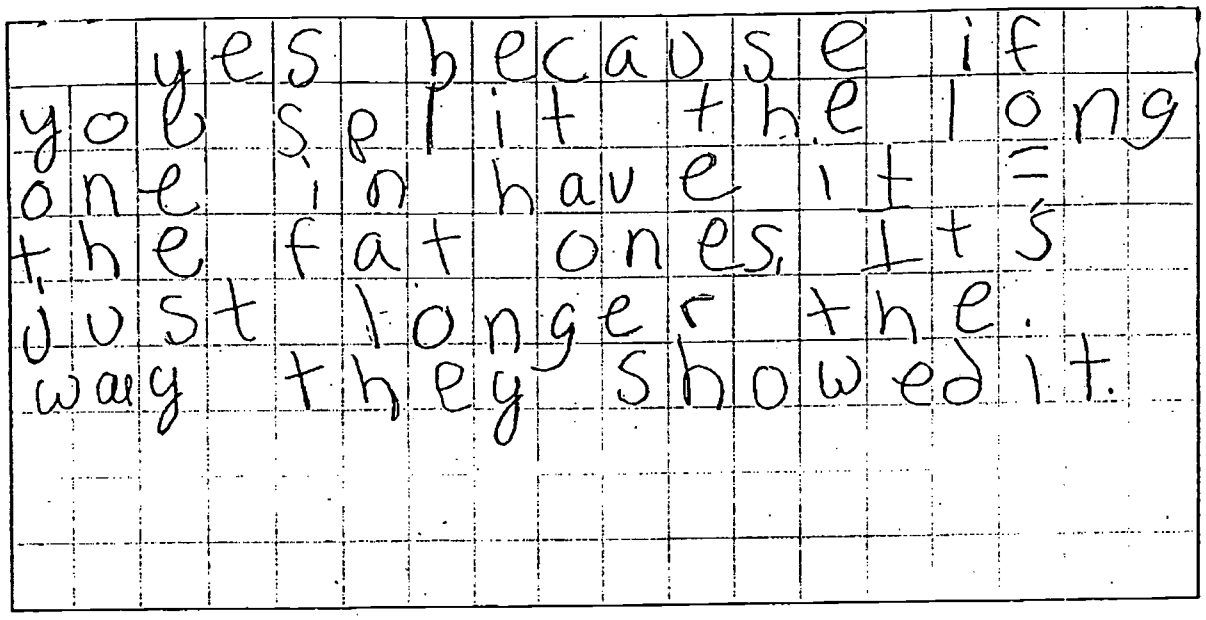


Comment:

This response demonstrates some understanding of the concept, that "*it is possible to have four different size pieces of cake*", however, they can be equal. The idea of a quantity being represented in different formats (shape) is important and this student shows that he or she is beyond the concrete stage and has some experience with abstract reasoning but does not explain adequately. This response earns 2 points.

Student Response

Score Point: 3

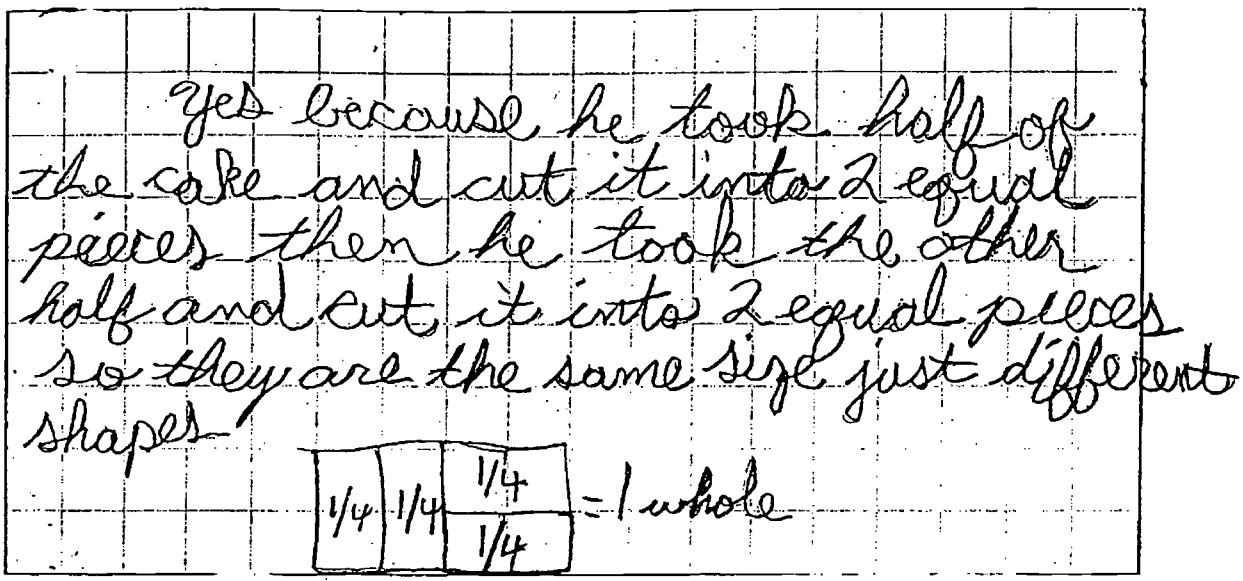


Comment:

The student response demonstrates an understanding of the concepts but with an explanation that is incomplete and somewhat difficult to apply in reconstructing the student's thinking. The idea of "*split the long one*" (2 left rectangles) "*equals the fat ones*" (2 right rectangles) describes the process the student used. Presumably the student found that the "long" and the "fat" rectangles could be decomposed into two smaller rectangles that were congruent with one another. It is interesting to note that this student can extend his or her reasoning beyond visual representation when stating "*It's just longer the way they showed it*".
This response earns 3 points.

Student Response

Score Point: 4



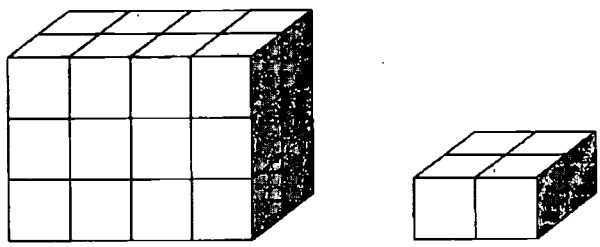
Comment:

This response shows a complete understanding of the concept of non-congruent fractional pieces, and contains a clear explanation supporting the answer. This student illustrates his or her answer with a picture of the cake but the narrative explanation contains evidence of a more sophisticated abstract reasoning about fractions. In essence, this student argues that a half of a half equals a half of a half no matter the shape of the halves.
This response earns 4 points.

Math Standard #5 (Grade 8)

Students will develop an understanding of ESTIMATION, MEASUREMENT, AND COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

Item #1



This first stack of blocks gets knocked down. Four of these blocks are then used to begin a second stack with the base shown above. How high will the second stack be if all the blocks are used?

a. 4 blocks

b. 6 blocks *

c. 8 blocks

d. 10 blocks

Commentary:

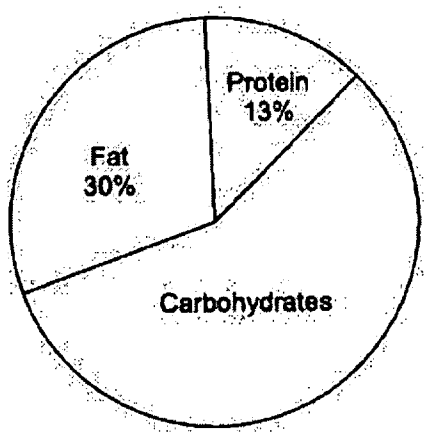
This item requires students to study the first figure in order to determine the total number of blocks in the figure. The second stack can be built by dividing by 4 the total number of blocks in the first figure. This would give the number corresponding to the height of the second stack. By grade 8, students should be proficient with factors and products, especially multiples of 4. An alternative strategy would be to visualize the first stack of blocks as two side-by-side stacks each with 2x2 cross-section just like the second stack. This first stack could then be decomposed and reassembled into a 2x2 stack that is 6 blocks tall. There are several very different ways that students can think about this item, allowing for the different levels of reasoning and justification.

Math Standard #5 (Grade 8)

Students will develop an understanding of ESTIMATION, MEASUREMENT, AND COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

Item #2

The pie chart shows the types of food that a teenage boy requires for a healthy diet.



If a teenage boy needs 2800 calories a day for a healthy diet, how many of the calories should be carbohydrates?

Scoring Rubric:

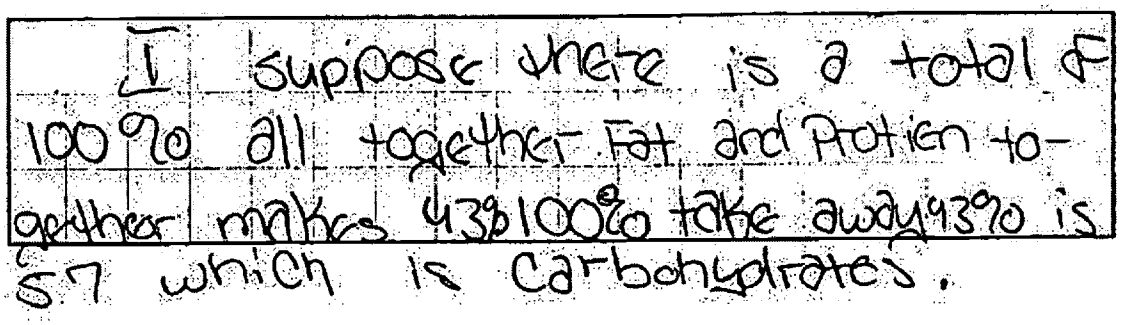
2	1,596 (calories)
0	Any other answer.

Commentary:

This item illustrates the former format of the 2-point rubric in which the item was scored either as correct (2 points) or incorrect (0 points). This format is no longer used in DSTP mathematics assessments. The intent of this real-world context item is for students to demonstrate their basic knowledge of reading a graph and then using the information to show proficiency when computing with percents.

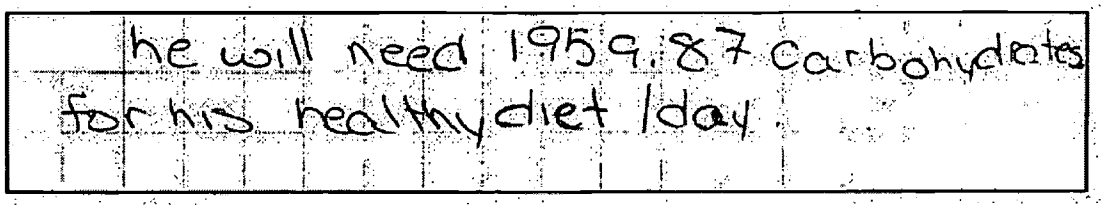
STUDENT RESPONSE

Score Point: 0



Comment:

This response demonstrates no evidence of the mathematical concept being assessed. The student attempts to find the correct percent for carbohydrates but in the process gets completely confused and reports that 5.7 of the total amount of calories are from carbohydrates. This response suggests a severely flawed knowledge of and inability to apply the concept of percent. This response earns 0 point.

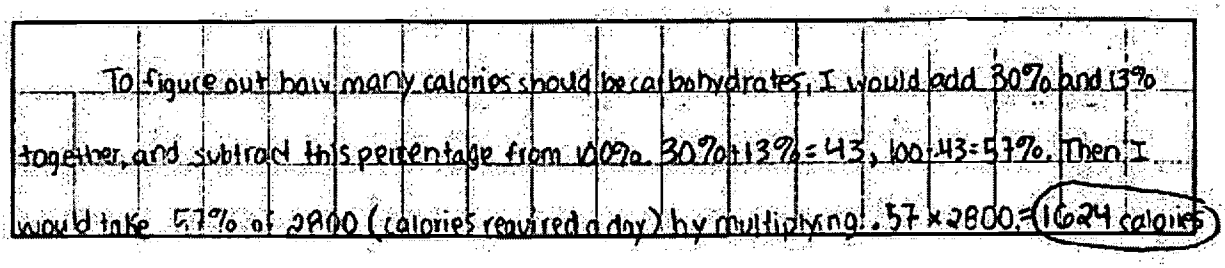


Comment:

This response suggests an understanding of the concept, simply because the given answer is reasonably close to the correct answer. We infer that the student made a computational error. However, there is no supporting work and because this item was scored as either correct or incorrect under the original dichotomous scoring, this response earns a score of 0.

STUDENT RESPONSE

Score Point: 0

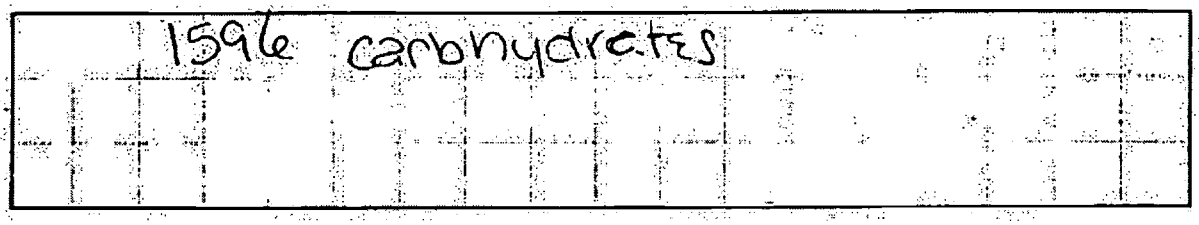

To figure out how many calories should be carbohydrates, I would add 30% and 13% together, and subtract this percentage from 100%. 30%+13%=43, 100-43=57%. Then I would take 57% of 2800 (calories required a day) by multiplying: .57×2800=1624 calories

Comment:

This response demonstrates a more-or-less full understanding of the concept. The student gives a clear explanation of his or her reasoning but makes a slight computational error to obtain the wrong number of calories from carbohydrates. This item would earn 1 point in a new 0-1-2 point rubric. Dichotomous scoring (0 or 2 points) has been replaced by 0-1-2 point scoring so that papers such as this with minor flaws in application can be given appropriate credit for the demonstration of essential knowledge.

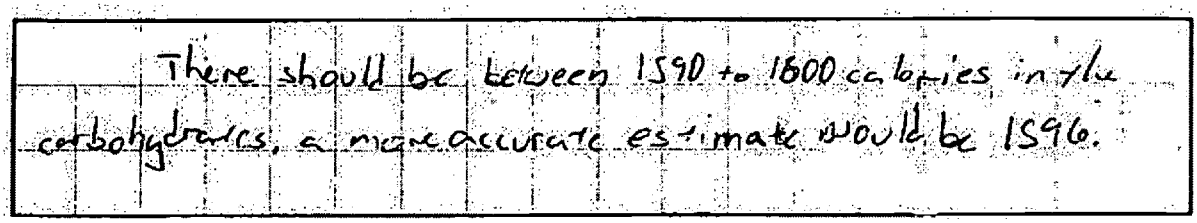
STUDENT RESPONSE

Score Point: 2



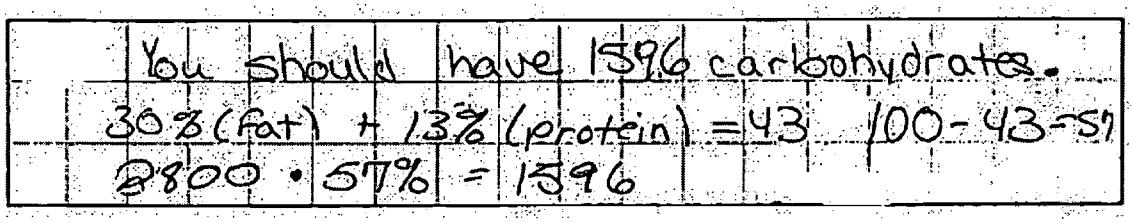
Comment:

This response does not show any work, however, the correct answer is stated and all the computations could have been done on a calculator. The item didn't ask for an explanation of how to obtain the answer, so based on the rubric, this response earns 2 points.



Comment:

This response demonstrates full understanding of the concept. The student not only gives a range for the answer but actually states the correct answer of the required 1596 calories from carbohydrates for a healthy diet. This response earns a score of 2 points.



Comment:

This response demonstrates full understanding of the concept being assessed. The student shows his or her simple and clear step-by-step work to support the correct answer. Although mathematical communication skills were not being assessed in this instance, this is a good example of a correct answer clearly communicated. This response earns 2 points.

Math Standard #7 (Grade 8)

Students will develop an understanding of ALGEBRA by solving problems in which there is a need to progress from the concrete to the abstract using physical models, equations and graphs; to generalize number patterns; and to describe, represent and analyze relationships among variable quantities.

Item #3

Leon goes to college in Pennsylvania. Leon's family lives in Massachusetts. Last month Leon's mother made 5 daytime calls to him. This is how the charges looked on the bill:

$ 0.24	1 minute
$ 1.20	5 minutes
$ 2.40	10 minutes
$ 3.60	15 minutes
$ 5.76	24 minutes

Write an equation that could be used to find the cost of an n minute daytime phone call to Leon from his mother. In your equation, let c be the cost of the call and n the number of minutes the call lasts.

Scoring Rubric:

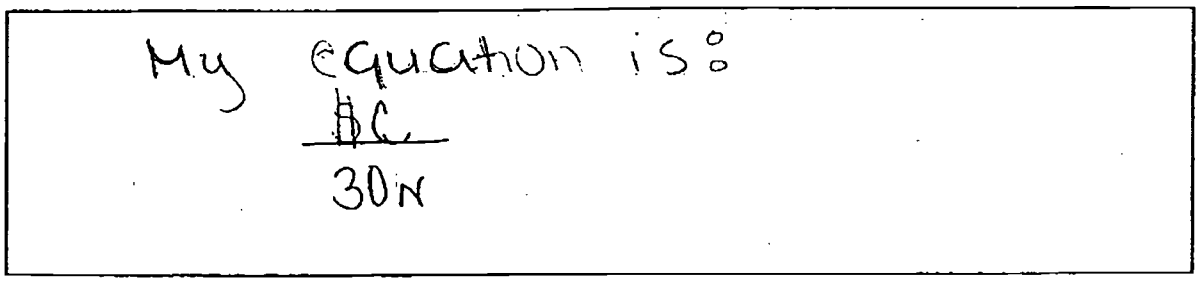
2	$c = 0.24n$
1	An equivalent answer (e.g. "twenty-four cents times the number of minutes") not using the specified variables or $c = 24n$.
0	Any response that fails to describe the proportional nature of the relationship between minutes and total cost.

Commentary:

This is an item in which the student is asked to write a simple linear model to describe a familiar situation. However, the student must discover that the relationship is directly proportional – for each of the phone calls, one minute costs $0.24 – in each of the cases given. A premium is placed on achieving a formal algebraic representation, correctly reasoned informal descriptions are awarded one rather than 2 points.

Student Response

Score Point: 0



Comment:

There is no evidence of an understanding of the mathematical concepts assessed by this task. The data is not used to even attempt to write a relationship between the dollar amounts and the minutes. This response earns 0 point.

STUDENT RESPONSE

Score Point: 1

The day time cost is 24 cents a minute. So you do 24 X the number of mins.

Comment:

This response demonstrates an understanding of the mathematical concept by writing the equivalent of the required equation. However, translating this statement into an equation would have shown a complete mastery of the concept and greater attention to the requirements of the task. This response earns 1 point.

$X = .24 \cdot m$ Example $.24 \times 5 = X$

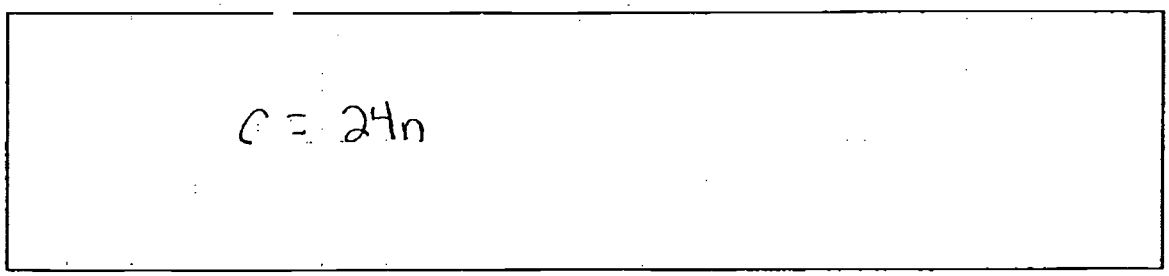
$\$.25 \cdot 5 = \$1.20$

Comment:

This response demonstrates some understanding of the concept using variables. An example using the given data supports the correct answer, however, the specified variables *c* and *n* were not used. This response earns 1 point.

STUDENT RESPONSE

Score Point: 1

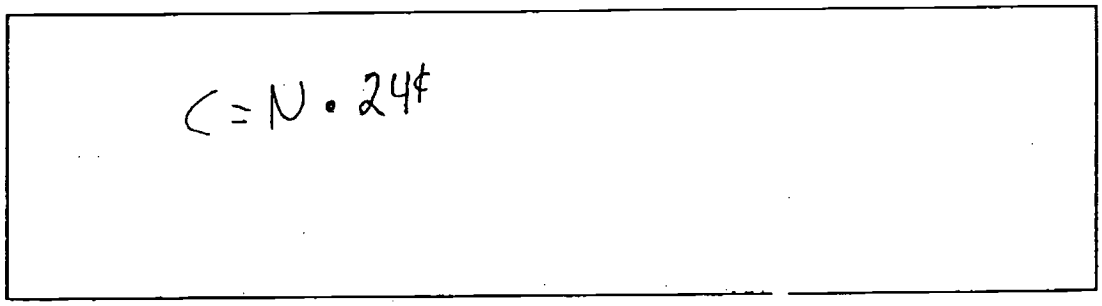


Comment:

This response demonstrates a basic understanding of the concept using the specified variables. The student converted dollars into cents to derive a correct equation but has not been explicit about this change of units. This response earns 1 point.

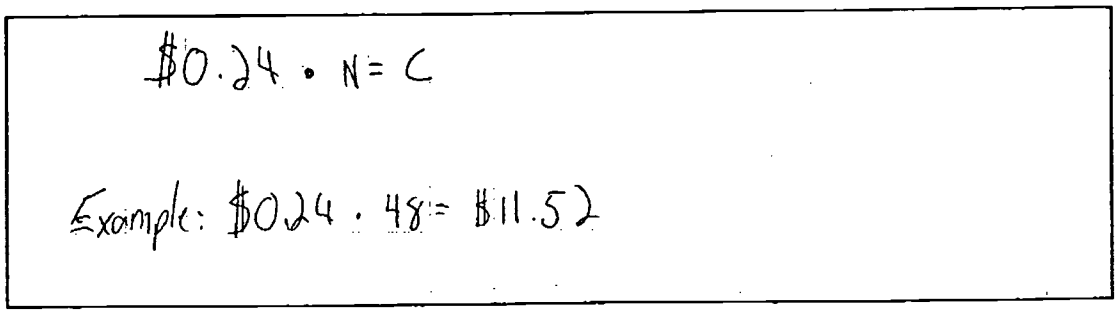
STUDENT RESPONSE

Score Point: 2



Comment:

This response demonstrates the basic understanding of the concept using the specified variables. The student converted dollars (the cost for one minute) to cents but labeled it so there is no ambiguity in the scoring. This response earns full credit since the question didn't ask for explanation but simply the equation relating the cost and the number of minutes.
This response earns 2 points.



Comment:

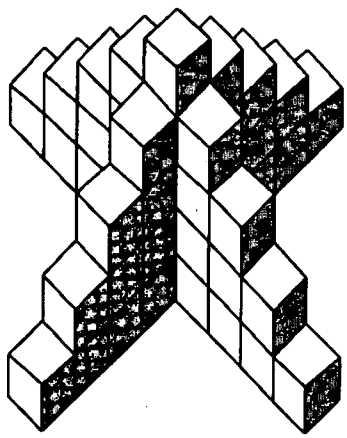
This response demonstrates full understanding of the concept using the specified variables. An example predicting the cost for a 48-minute call is given based on the equation. This suggests that this student understood the generality of the derived equation.
This response earns 2 points.

Math Standard #10 (Grade 8)

Students will develop an understanding of PATTERN, RELATIONSHIPS, AND FUNCTIONS by solving problems in which there is a need to recognize and extend a variety of patterns; and to analyze, represent, model, and describe real-world functional relationships.

Item #4

Use this diagram to answer the following question.



This tower is 5 cubes high.

a. How many cubes are needed to build this tower?

b. How many cubes are needed to build a tower like this 10 cubes high?

c. Explain how you determined your answer to part b.

Scoring Rubric:

4	Correctly identifies the 5-high tower as requiring 45 cubes (a.) and the 10-high tower as requiring 190 cubes (b.). The explanation (in c.) may describe the solution using models of addition, multiplication, and difference (e.g., four times the quantity $1 + 2 + 3 + \ldots + n–1$ plus n) but must make it clear how the computations were carried out. Note: the explanation may be presented without narrative as long as the solution process is evident.
3	Correct answer to parts a. and b. but explanation is incomplete or is hard to follow.
2	Part a. is correct. May be errors in part b. and explanation is incomplete or even absent.
1	Errors in part a. Part b. wrong or even missing.
0	Some work but generally without clear understanding of the goals of the calculation. For example, student might simply count visible blocks in the figure.

Commentary:

Finding patterns in both algebraic and geometric situations is an important mathematical activity. This item represents a situation involving a symmetric three-dimensional construction. Students are asked to count the cubes in the given structure and then to generalize to a larger structure. The fact that some of the cubes are hidden in the picture requires that students find a pattern rather than simply count all of the cubes. The symmetry of the structure encourages students to discover any one of several patterns.

STUDENT RESPONSE

Score Point: 0

a. 25 cubes

b. 50 Cubes

c. I times Answer A by 2.

Comment:

There is no evidence of an understanding of the concepts in this response. It is unclear how the student derived/concluded that the 5-cubes high tower is built with 25 cubes. Also, this student erroneously concluded that a tower which is twice as tall would use twice as many cubes. This response earns 0 point.

a. it takes 40 cubes to make this tower

b. about 100 cubes

c. I had added up 10 x 10 and that determaned my answer

Comment:

This student apparently determined that each "wing" of the tower required ten cubes so, given four wings, the tower would require 4x10 = 40 cubes altogether. The student failed to count the five cubes in the central tower. An estimate was then used in order to determine an answer for part b. This response earns a score of 0.

STUDENT RESPONSE

Score Point: 1

a. 46 cubes

b. 92 cubes

c. I add 46+46 to get 92 because you double your cubes.

Comment:

This response demonstrates some minimal understanding of the mathematical concept. Even though, the answer to part a. is incorrect (by one cube), parts b. and c. are consistent with part a. given that the student assumes a linear relationship between a 5-cubes and a 10-cubes high tower. This assumption is incorrect but demonstrates some very limited reasoning and/or visual interpretation. This response earns 1 point.

STUDENT RESPONSE

Score Point: 1

a. 41 cubes

16
10
10
10
+ 1
41

b. 82 cubes 41 · 2 = 82

c. To make a tower 5 cubes high, you need 41 cubes to make the tower. To make a tower 10 cubes high, you multiply 41 by 2 and get 82 cubes (since the tower increase upwards by 5 cubes)

Comment:

This response demonstrates a reasonable understanding of the concepts with supporting work in part a. The student used a combination of counting visible cubes and symmetry to derive the answer in part a. However, the student forgot 4 cubes (middle ones) for the 5-cubes high tower and assumes that a factor of 2 (twice as high) would give the correct answer for part b. This student would have achieved a better estimate of the required cubes to build a 10-cube high tower had s/he extended the procedure used in part a. (for example: counted the number of cubes on one side, multiplied by four but missed the 9 hidden middle cubes to obtain 181 cubes.) This response earns 1 point.

STUDENT RESPONSE

Score Point: 2

a. You need 45 of these cubes to build this.

b. You will need 90 cubes to build a tower of 10 cubes high.

c. You use the amount from part A than I timesed it by 2 and that is how I got the answer of 90 cubes because 45 x 2 = 90

Comment:

This response shows a correct answer to part a., but without support. Again, the student assumed that a 10 cube high tower would require twice as many cubes as a 5 cube high tower but failed to check his or her answer by applying the same kind of pattern or extending the given figure. This response earns 2 points.

a. 45

b. 190

c.

Comment:

This response shows correct answers for the first two parts only. There is no explanation of how the student reasoned to obtain these answers. The explanation is key and very important when scoring this item, which assesses the student's communication and reasoning skills. This response earns 2 points.

STUDENT RESPONSE

Score Point: 2

a. 57 cubes. because one section has 15 and all the other 3 have 14

b. 217.

c. I added 10, 9, 8 ..... 3, 2, 1 3 got 55 then added 3 54's

Comment:

This student has developed and applied a reasonable strategy but made a single conceptual error that results in incorrect (but reasonable) answers to parts a. and b. The student determined that one wing plus the central column of the given figure was comprised of 15 cubes. S/he erred in subtracting just the one visible central cube rather than the entire central column in order to determine the number of cubes in the other three wings of the tower. This flawed strategy was then applied to the case of the ten cube tall tower.
This response earns 2 points.

STUDENT RESPONSE

Score Point: 3

a. 45

b. 180

c. 10 center blocks
45 on each side of the ten, so you times 45 time 4 which equals 180 plus the ten center blocks.

Comment:

This response demonstrates a complete understanding of the concepts and clear explanation of how to derive the number of cubes for a 10-cube high tower using the 5-cube high tower. It is unfortunate that the student's work contains a computational error.
This response earns 3 points.

STUDENT RESPONSE

Score Point: 3

a. 45 cubes were used to build the tower.
b. 190 cubes are needed to build a tower 10 cubes high.
c. I made a tower like the one shown using 10 cubes in the center the working it down from 9,8,7,6 and so on. Then I counted all the cubes and got 190.

Comment:

This response demonstrates an understanding of the concepts by giving the correct answers to parts a. and b., but without supporting work. Part c. described a valid process of deriving the answer from the pattern "9,8,7,6,..." but is, however, incomplete. This response earns 3 points.

STUDENT RESPONSE

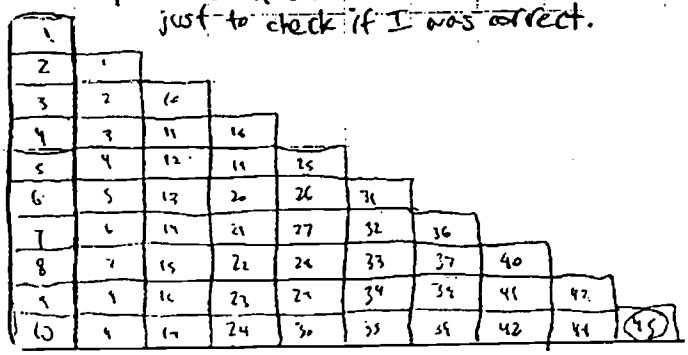
Score Point: 4

a. You need 45 cubes to build this tower.

45 · 4 = 180 + 10 = 190

b. You would need 190 cubes to build this tower 10 cubes high.

c. I figured since there were 45 blocks just to make a 5 cube high tower, then those 45 probably only make up one of the sides for the 10 cube high tower, So I multiplied 45 · 4 to find out how many cubes were used for all the sides, 180, then added the middle 10 cubes, and got 190 cubes. I made one of the stabs just to check if I was correct.



Comment:

This response demonstrates a complete understanding of the concepts. It contains the correct answers with adequate supporting work. The explanation in part c. is easy to follow. This student has shown great originality of thought in discovering a relationship between the number of cubes in a 5-cube high tower and the number of cubes in only one of the four sides of the 10-cube high tower. Nor did the student forget the 10 central cubes. This response earns 4 points.

STUDENT RESPONSE

Score Point: 4

a. 45 cubes

b. 190 cubes

c. I figured the middle would have 10 cubes. Then, I realized from the first example, that the wing parts went down from 5 to 4,3,2,1 So I did the same, but from 10. I added 9+8+7+6+5+4+3+2+1 and got 45. I then multiplied 45 by the 4 wings and got 180. Finally, I added the 10 original block to get 190.

Comment:

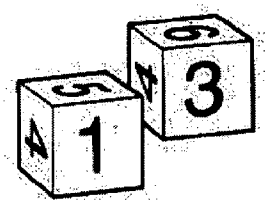
This response demonstrates a complete understanding of the concepts. The explanation is clear and well written and supports the correct solutions in parts a. and b. The student understood and saw the pattern in the number of cubes and the height of the tower as evidenced by the statement *"I realized from the first example ..."*
This response earns 4 points.

Math Standard #9 (Grade 8)

Students will develop an understanding of STATISTICS AND PROBABILITY by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions; to present convincing arguments; and to model mathematical situations to determine the probability.

Item #5

You and a friend are playing a game with number cubes. You roll 2 number cubes (with faces numbered 1 through 6) and find the product of the two numbers. If the product is even, you win, and if the product is odd, you friend wins.



If you roll the number cubes just once, who has the better chance of winning? Justify your answer mathematically (Hint: think about all possible outcomes).

Scoring Rubric:

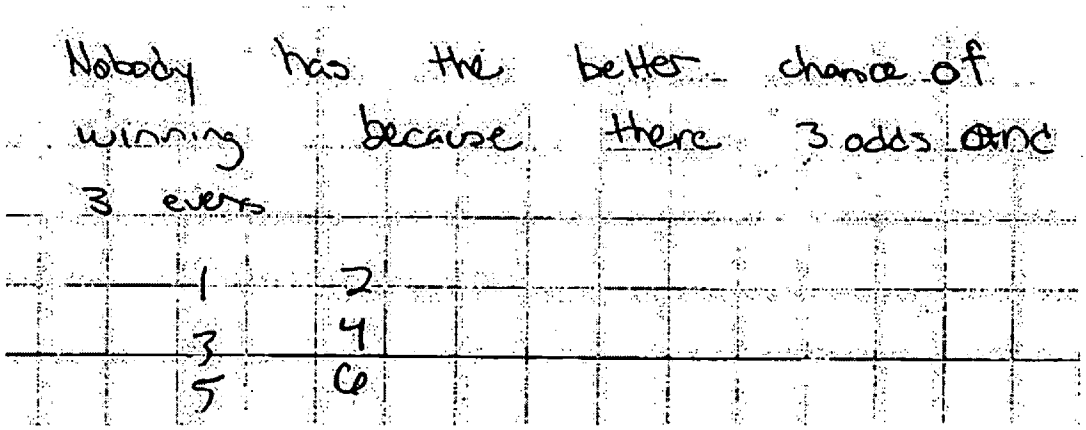
4	Correct answer (you win) with complete and compelling justification. This may take the form of a complete listing of the sample space, for example in a diagram, or an ordered list, and a count of favorable vs. non-favorable outcomes (there are 27 even outcomes and only 9 odd outcomes) or it may involve reasoning without listing, e.g., the product of even and even is always even, the product of even and odd is even, while only the product of odd and odd is odd.
3	Correct answer with explanation which is correct as far as it goes but is not quite complete. For example, a correct chart may be constructed but no discussion of the relative number of outcomes included.
2	Listing of sample space (or reasoning about products) incomplete. A correct or incorrect conclusion may result. For example, only half of sample space may be listed.
1	Only fragments of the sample space reproduced. Reasoning based on minimal data.
0	Some work but generally without clear understanding of the goals of the problem.

Commentary:

This item requires that students consider the entire sample space of possible outcomes and recognize that each of these outcomes is equally likely. Many students will have had instructional experience with problems involving the *sum* of two dice so this would involve a slight extension from a familiar situation. Some students may even find a more efficient solution strategy than enumerating the entire sample space by considering the characteristics of the products of even and odd numbers.

STUDENT RESPONSE

Score Point: 0

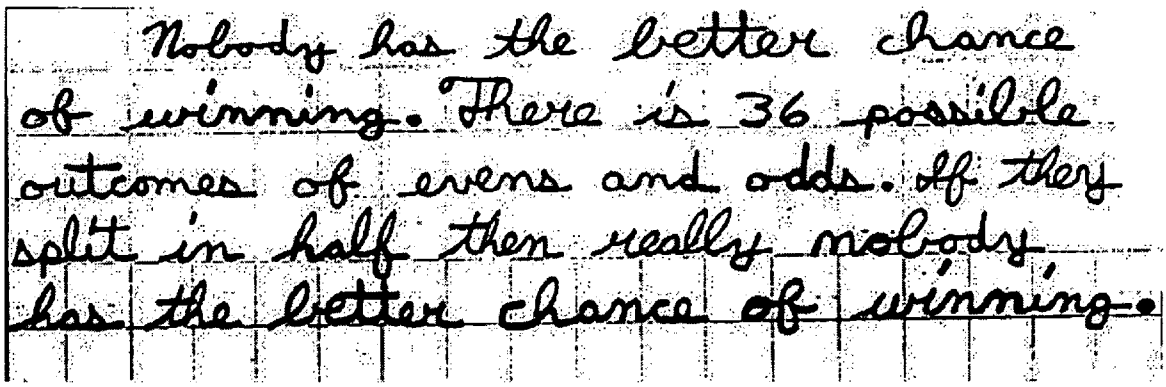


Comment:

This response demonstrates that this student has not fully understood the task or has chosen to ignore the complexity in the task and has reduced it to a much simpler situation involving the digits on a single die rather than the product of digits on a pair of dice. This failure to address the problem as posed merits a score of 0.

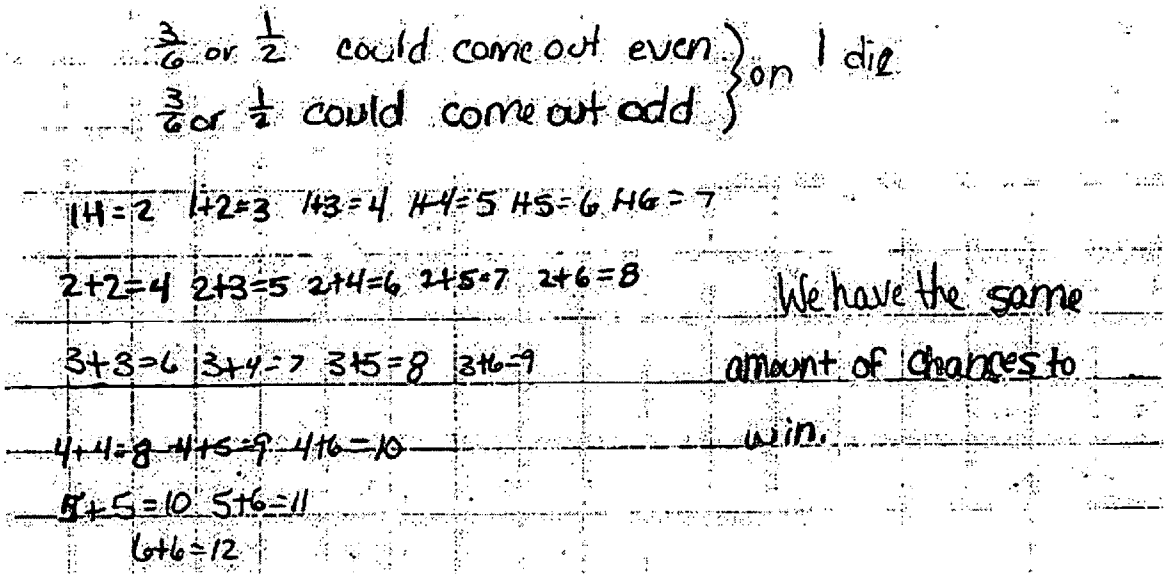
STUDENT RESPONSE

Score Point: 1



Comment:

This response demonstrates a basic understanding of sample space as well as even and odd numbers. The student understands that the sample space will contain "36 possible outcomes" but there is no evidence that she or he knows how this sample space can be created using the *product* of the digits on each die. It is, in fact, more likely that this student considered the *sum* of the digits rather than their product. Summing the digits from the dice would result in an equal distribution of even and odd outcomes. The response is based on some of the given information, not all of it. This response earns 1 point.



Comment:

This response demonstrates that the student did not read the question carefully when producing a partial list of the sample space using the *sum* of the two numbers instead of their product. This response earns 1 point.

STUDENT RESPONSE

Score Point: 2

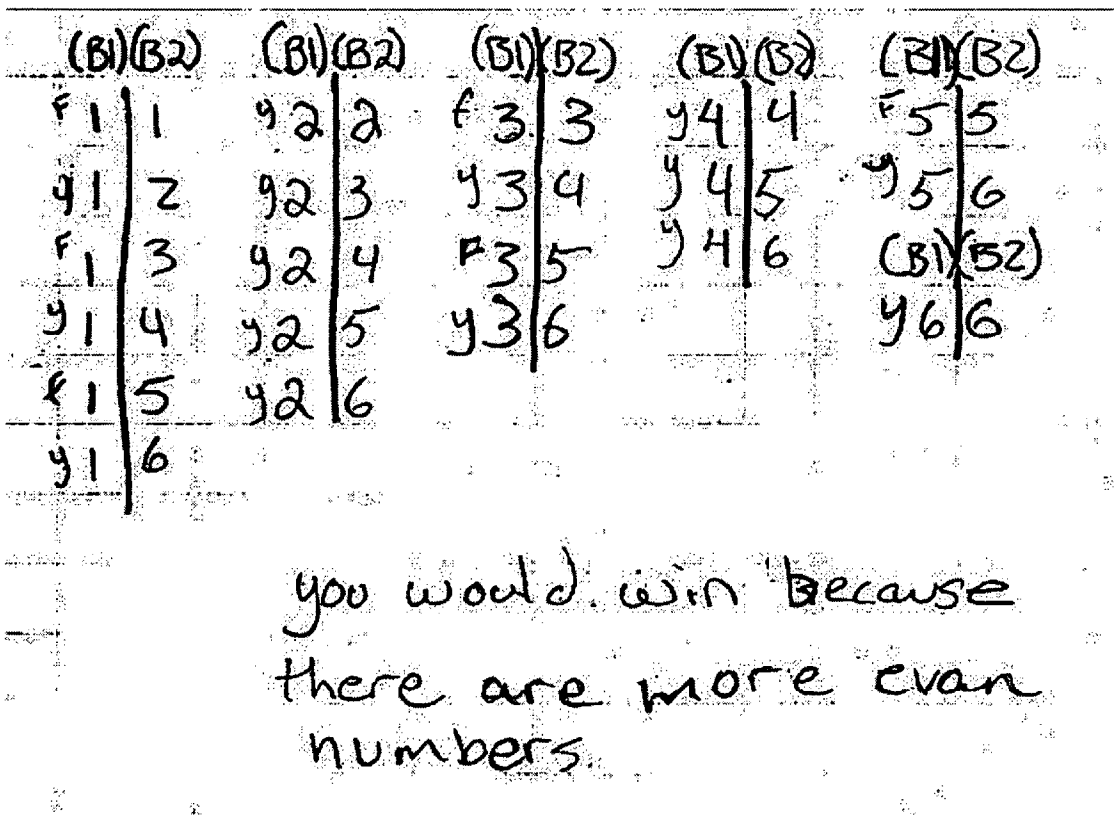
You would have the better chance of winning because more nombers that are multiplyed are Even. 8 of all the numbers multiply are odds.

Comment:

This response demonstrates some understanding of the concept. The correct answer that there are more even than odd products is stated with a partial attempt at comparing the number of outcomes for each type. This student's reasoning is not, however, fully communicated and there is little or no supporting work. As a consequence, this response earns only 2 points.

STUDENT RESPONSE

Score Point: 2



Comment:

This response demonstrates a substantial understanding of the concepts. The student lists an incomplete yet well organized and labeled sample space. The student fails to regard (2,1) and (1,2) – and all of the other symmetric outcomes – as distinct outcomes. This explains the partial sample space. There is some evidence that the student calculated the products for each pair of numbers to support the correct answer, i.e., pairs that would result in odd outcomes are labeled with an "f" presumably for "friend," and pairs that would result in even outcomes labeled "y" for "you" but a count of favorable outcomes is never recorded and so a correct conclusion is based on incomplete evidence. This response earns 2 points.

STUDENT RESPONSE

Score Point: 3

I would win, because the dice would most probably land on an even #.

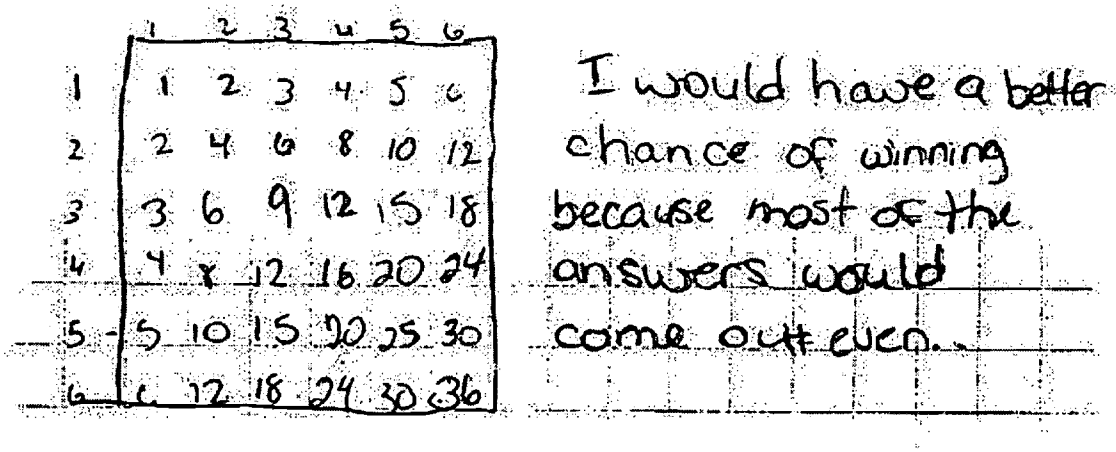
1	1	2 x anything = Even
2	2	4 x anything = Even
3	3	6 x anything = Even
4	4	3x(2)/3x(4)/3x(6) = Even
5	5	Anything times an
6	6	even equals and
		even.

Comment:

This response demonstrates abstract reasoning when solving this problem. A complete sample space is not provided but there is a clear understanding of even/odd products and how they can be applied to answer this task. The student knows that there will be more even products but this is not fully communicated in the response. Since the student did not fully answer the question this response only earns 3 points.

STUDENT RESPONSE

Score Point: 3

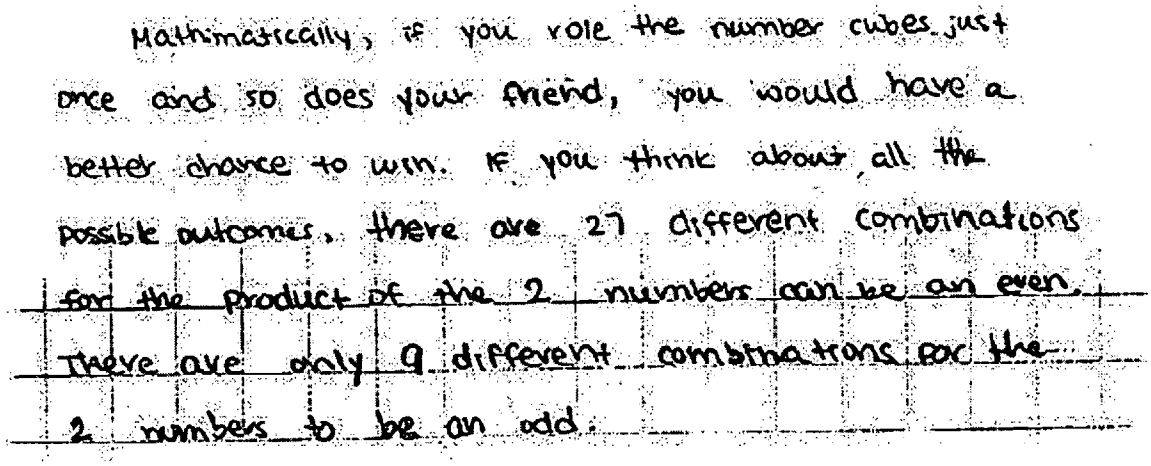


Comment:

This response demonstrates a full understanding of the concept listing all the possible outcomes for the products of 2 random numbers. The student states the correct answer referring to the even products outnumbering the odd products. The reason that this response does not earn the full 4 points is that the student does not communicate explicitly his or her thoughts, comparing the number of even and odd products (27 even outcomes and only 9 odd outcomes) or some indication (including visual link) between the chart and the final statement. This response earns 3 points.

STUDENT RESPONSE

Score Point: 4

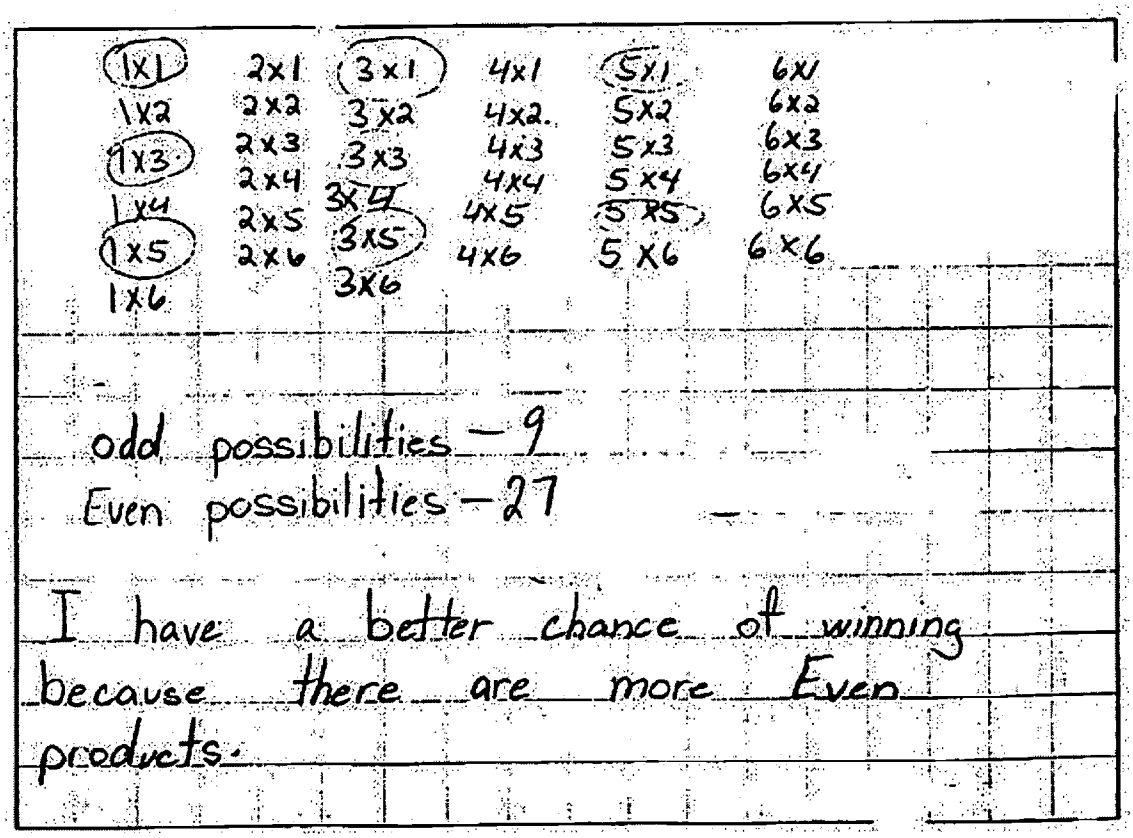

Mathmatically, if you role the number cubes just once and so does your friend, you would have a better chance to win. If you think about all the possible outcomes, there are 27 different combinations for the product of the 2 numbers can be an even. There are only 9 different combinations for the 2 numbers to be an odd.

Comment:

This response demonstrates full understanding of the concepts. The description of the answer shows that the student listed and investigated all the possible outcomes for the product of two numbers. This response also indicates that the student can write and communicate her or his thinking and reasoning without the need of extensive supporting work.
This response earns 4 points.

STUDENT RESPONSE

Score Point: 4



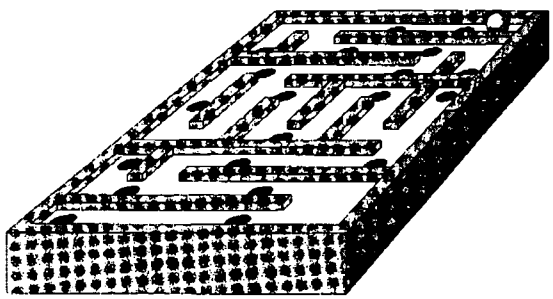
Comment:

This response demonstrates full understanding of the concept. A complete sample space and a comparison of the number of even and odd products support the correct answer. It is a clear easy-to-follow explanation of the answer. This response earns 4 points.

Math Standard #8 (Grade 10)

Students will develop SPATIAL SENSE and an understanding of GEOMETRY by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships between geometric figures.

Item #1



A 3-dimensional game consists of a series of mazes, a ball-bearing, and holes where the ball-bearing could fall through. If the metal ball-bearing has a volume of about 0.22 cubic inch, what is the smallest diameter for the holes on the game's surface?

A $\frac{1}{4}$ inch

B $\frac{1}{2}$ inch

C $\frac{5}{8}$ inch

D $\frac{3}{4}$ inch

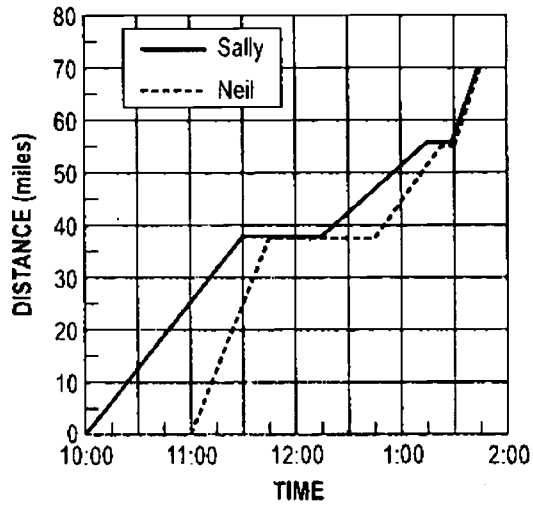
Commentary:

This item requires students to use the formula for the volume of a sphere and manipulate the equation to derive the radius and then the diameter. Students must demonstrate proficiency in basic operations including exponents. Success on this item ultimately requires that students distinguish between diameter and radius and that they recognize that the inverse of the cube is the cube root.

Math Standard #9 (Grade 10)

Students will develop an understanding of STATISTICS AND PROBABILITY by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions; to present convincing arguments; and to model mathematical situations to determine the probability.

Item #2



Between 11:45 A.M. and 12:15 P.M. —

a. Sally and Neil are resting in one place
b. Sally and Neil are traveling on a flat surface
c. Neil is traveling faster than Sally
d. Neil is ahead of Sally

Commentary:

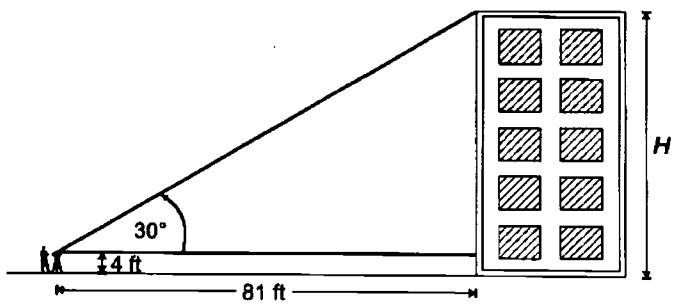
This item requires students to interpret a real-world graph. Students must interpret the word "resting" in the context of slope. That is, there is no distance covered between 11:45 A.M. and 12:15 P.M., this translates as a horizontal line for both Sally and Neil over the specific time period.

Math Standard #5 (Grade 10)

Students will develop an understanding of ESTIMATION, MEASUREMENT, and COMPUTATION by solving problems in which there is a need to measure to a required degree of accuracy by selecting appropriate tools and units; to develop computing strategies and select appropriate methods of calculation from among mental math, paper and pencil, calculators or computers; to use estimating skills to approximate an answer and to determine the reasonableness of results.

Item #3

In Dewey Beach, building codes restrict the height of building to 50 feet. Study the diagram; then determine by how much the building shown is above or below the code restriction.



Scoring Rubric:

2 The building approximately 50.77 feet tall or approximately 0.77 feet *over* code. (This answer might even be reported as 9 inches over code or between 8 and 10 inches over code.) Range .66 - .84 feet earns 2 points.

1 Solution procedure seems essentially correct but minor computational error – in either direction – is made. For example, the four feet to the transit might be neglected in which case the building would be calculated to be 46.77 feet tall and hence 3.23 feet under code.

0 Attempted solution is far from the mark. For example, respondent may simply try to "eyeball" solution from the drawing. The response earns 0 with no work when rounded out of range.

Commentary:

This item was based on a real-world context where trigonometry was required to solve the problem. Calculators were available to solve this item but students could solve it without one due to the 30 – 60 – 90 degrees angle relationship. A variety of student responses will show how students understood and used the given information to derive the correct solution. Some simply set-up an incorrect ratio, disregarding the 30° angle measure, while others knew they had to use 30° but did not know how to properly manipulate it to find the correct answer.

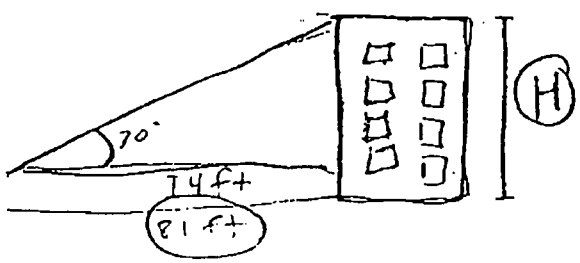
STUDENT RESPONSE

Score Point: 0

$\frac{81}{4} = h$ $h = 20.25$ the building is under code restriction

Comment:

The student response shows no understanding of the mathematical concept and is not able to use the figure as an aid to setup the problem. This response earns 0 point.



The building is very slightly under code restriction it is 48.6 feet tall.

Comment:

The student response doesn't show any computation, but simple reproduces the figure. There is an unsuccessful attempt to find the height of the building but little evidence of how this flawed answer has been derived. The student did not answer the problem as posed in that she/he did not calculate the difference between the building code restriction and the height of the building. While the use of a calculator is appropriate on this item, the lack of any record of how the calculations were set up and carried out makes it impossible to judge where the error in reasoning occurred in this response. This response earns a score of 0.

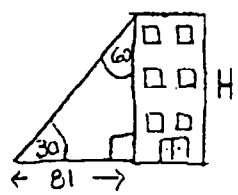
STUDENT RESPONSE

Score Point: 1

$\tan 30° \frac{H}{81} \cdot \frac{.5773}{1}$ 46 below by 4ft.

Comment:

This response shows a fundamental understanding of the problem by setting up a correct ratio and equation using trigonometry. The student has committed two errors in carrying out the computation, however. In the first place, the answer obtained from the trigonometric equation, has been truncated to 46 feet. (It is actually closer to 47 feet.) Also, the student has failed to add the four feet necessary to derive a correct answer which might suggest that the student didn't carefully read the question. A correct method is indicated but the student is ultimately careless in its execution. This response earns 1 point.



$\angle 30 = x \quad \angle 60 = x\sqrt{3} \quad \angle 90 = 2x$

$\angle 30 = 27\sqrt{3} \quad \angle 60 = 81 \quad \angle 90 = 54\sqrt{3}$

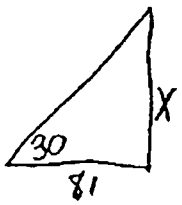
$27 \cdot \sqrt{3} = 46.8 = H \rightarrow$ building is 3.23 under the restricted height

Comment:

This response shows knowledge of the concept, using the relationship between the sides of a 30-60-90 degrees triangle. However, the labeling of the angles on the figure is confusing at best. The student forgot to add 4 feet to 46.8 which would have derived the correct answer. This response earns 1 point.

STUDENT RESPONSE

Score Point: 2



$\tan 30 = X/81$
$81 \cdot (.577) = (X/81) \cdot 81$
$X = 46.76$
$+ 4$
50.76

.76 ft. over limit

Comment:

This response shows a complete understanding of the concept, using both a correct figure and supporting work to derive the correct answer. The student uses the definition of a tangent, and manipulates the equation accurately. This response earns 2 points.

$H = ((\tan 30°) \cdot 81) = 46.76 \quad +4 = 50.76$

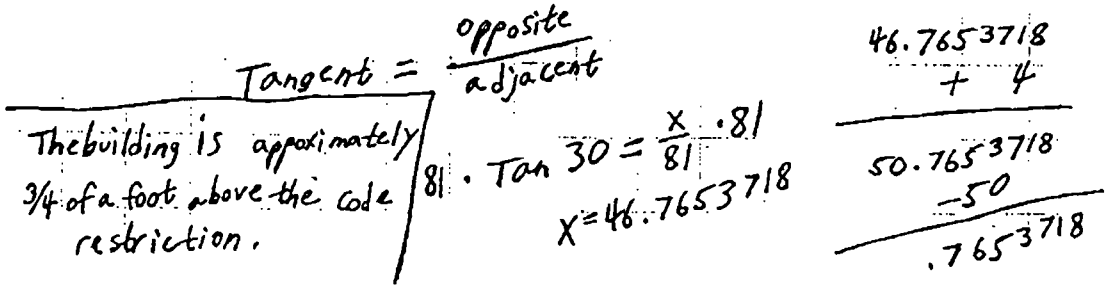
the building is .76 feet over code

Comment:

This response shows a complete understanding of the concept, using a correct equation. This response also demonstrates use of a calculator without truncating the answer too early. This response earns 2 points.

STUDENT RESPONSE

Score Point: 2



Comment:

This response shows a complete understanding of the concept with supporting work. It is an easy to-follow procedure. This response earns 2 points.

Math Standard #9 (Grade 10)

Students will develop an understanding of STATISTICS AND PROBABILITY by solving problems in which there is a need to collect, appropriately represent, and interpret data; to make inferences or predictions; to present convincing arguments; and to model mathematical situations to determine the probability.

Item #4

A game involves two cubes with sides numbered from 1 to 6. After a player throws the two cubes, the two numbers are subtracted. If they are not equal, the smaller number is subtracted from the larger number.

If a player throws the cubes many times, what difference will probably occur most often? Provide a diagram and/or a written explanation that you could use to explain this to a friend.

Scoring Rubric:

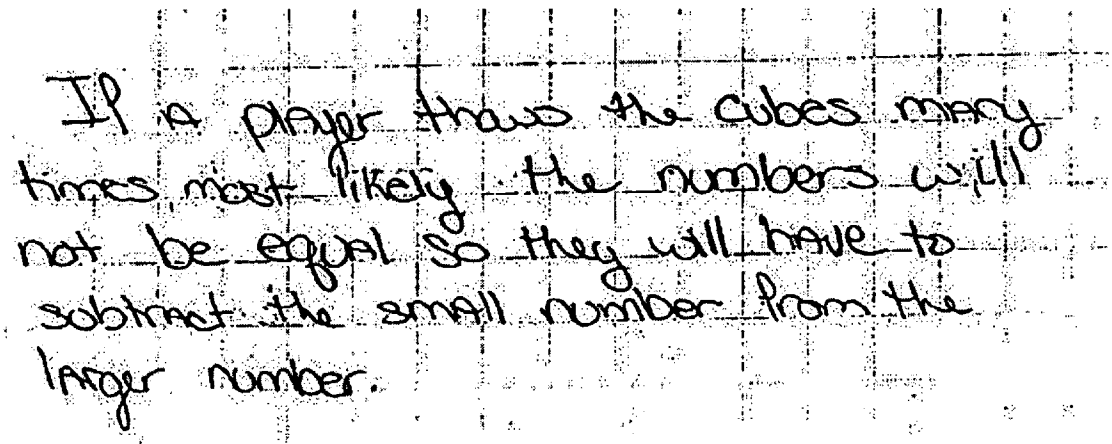
4	Diagram or listing of complete sample space and/or explanation involving a certain degree of quantification showing that the number 1 occurs in 10 out of the 36 possible outcomes and, therefore, more often than any of the other differences.
3	Incomplete or slightly incorrect description of the sample space (e.g., might ignore probability of rolling doubles or fail to enumerate symmetric outcomes) which still, however, results in correct conclusion that 1 is the most frequently occurring difference. The response is incomplete yet organized.
2	Incomplete or disorganized description of the sample space resulting in an incorrect answer.
1	Reasoning from a very limited number of outcomes or correct answer of 1 but unsupported by listing of sample space.
0	Some work but without much evidence that this work addresses the question.

Commentary:

This item requires the student to list the complete sample space and then using tallies or other methods to organize the supporting work to conclude that 1 is the difference that will occur most often. Many students will have had instructional experience with problems involving the *sum* of two dice so this would involve a slight extension from a familiar situation.

STUDENT RESPONSE

Score Point: 0

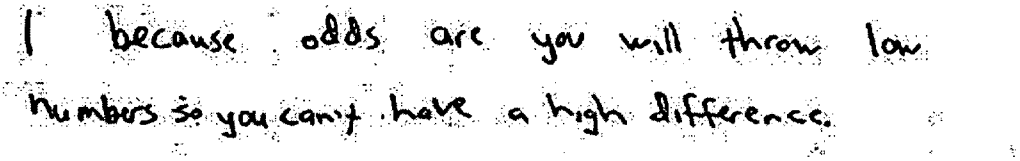


Comment:

This response demonstrates little or no understanding of the concepts. The student simply restated the question and there is no attempt to solve this problem. This response earns 0 point.

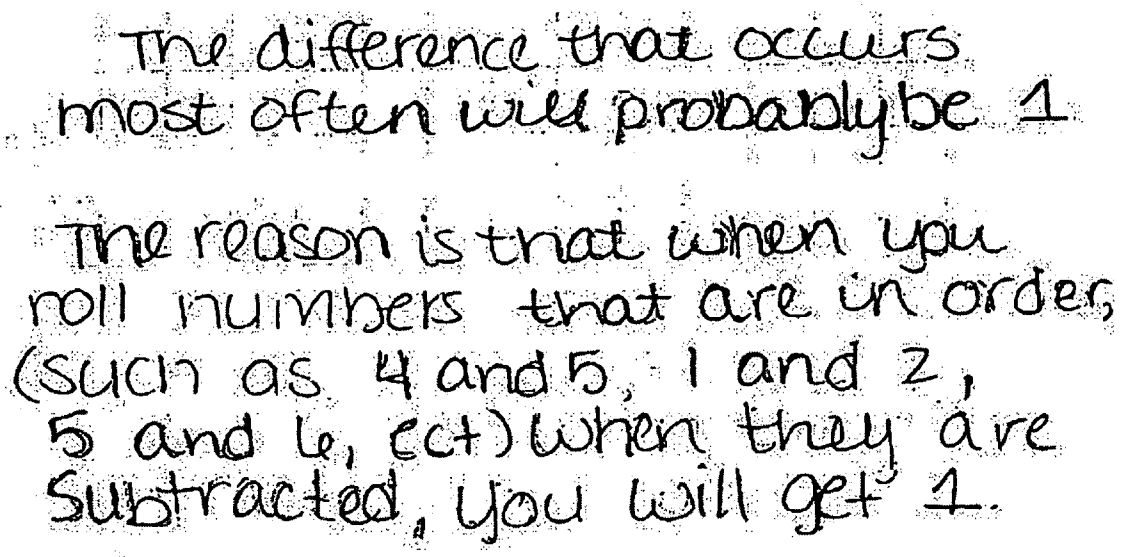
STUDENT RESPONSE

Score Point: 1



Comment:

The response states the correct answer that 1 is the *difference that occurs the most often* but the explanation is incoherent and even contradictory citing "high difference" rather than a difference of one. There is limited evidence that the student has some understanding of the mathematical concepts so the response only earns 1 point.

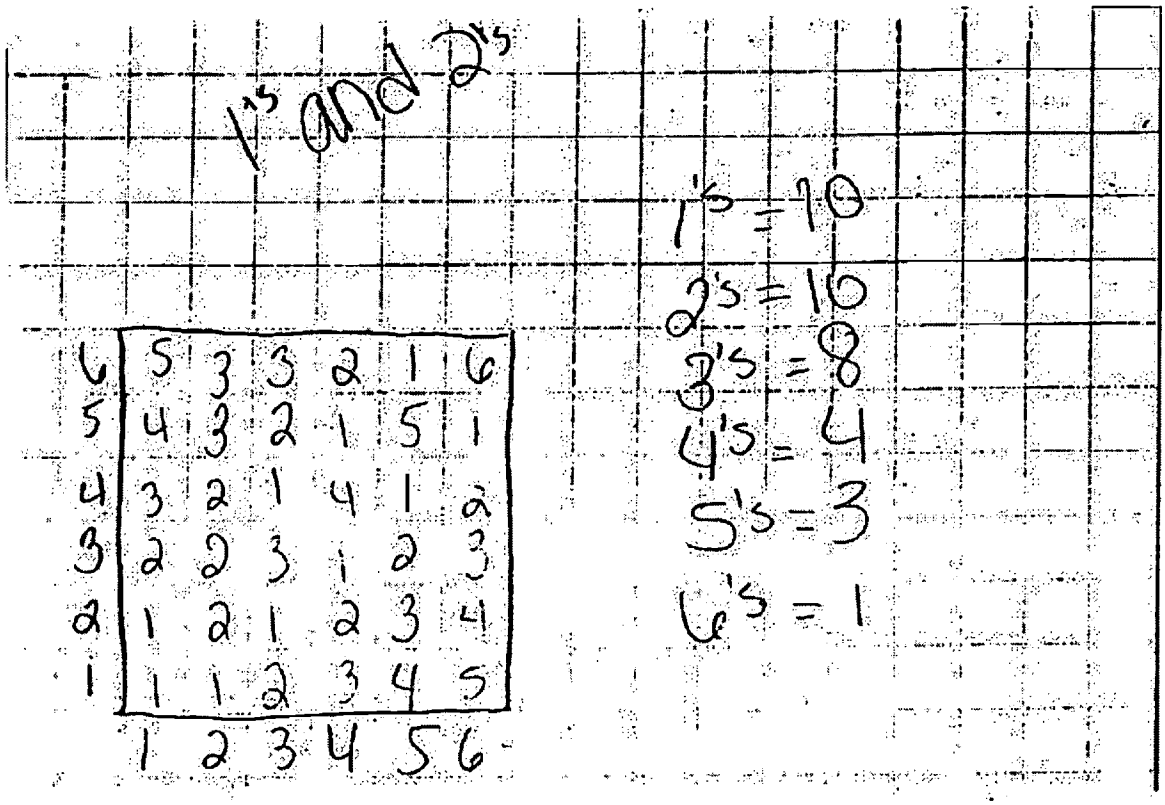


Comment:

The response demonstrates some limited understanding of the concepts. The student gives an incomplete rationale for the correct answer. Only a partial sample space made up of those outcomes supporting the conclusion is described. This response earns 1 point.

STUDENT RESPONSE

Score Point: 2

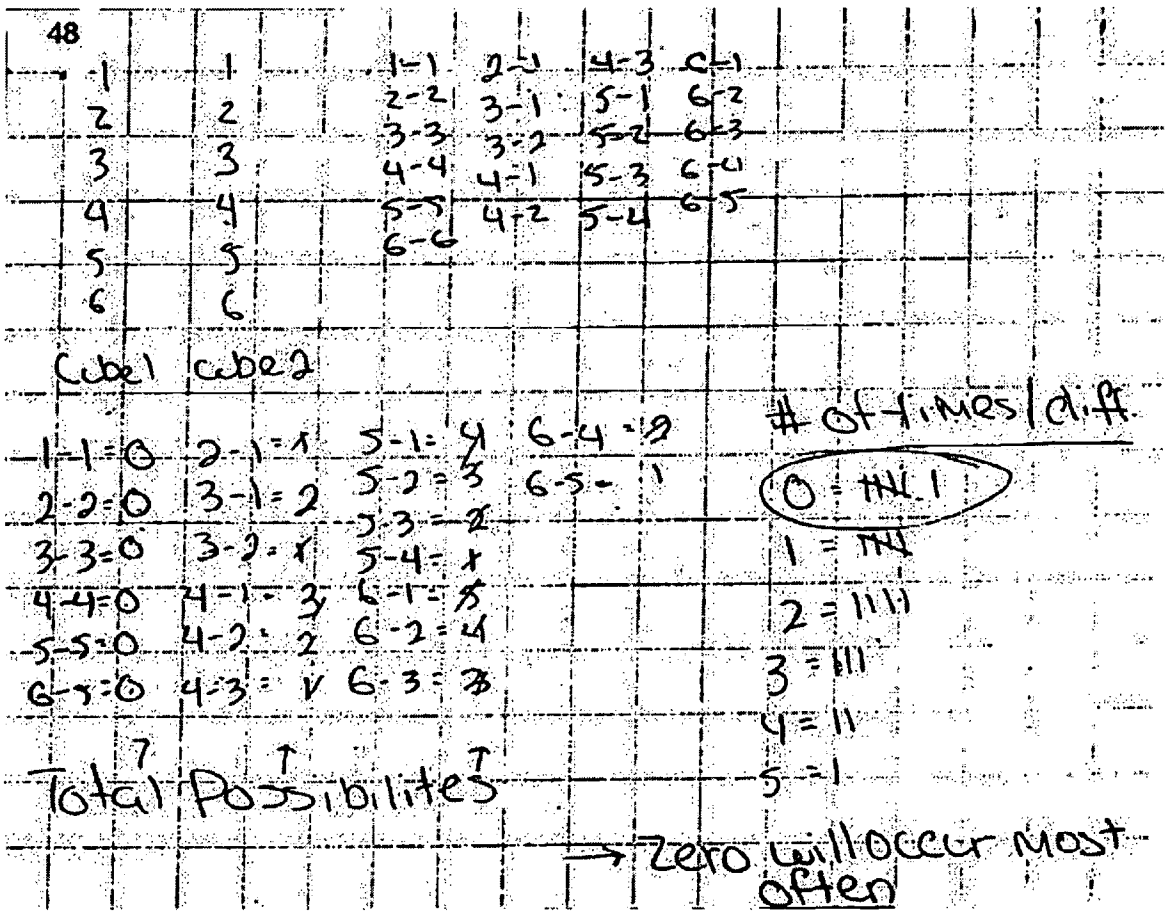


Comment:

This response demonstrates a flawed understanding of the concepts. The student's final answer, of "1's and 2's" are the most common answer, is consistent with the incorrect work. The chart is partially correct but ultimately contains too many mistakes. It is unclear how the student obtains some of these differences but apparently regards the difference between a number and itself as that number rather than as zero. This response earns 2 points.

STUDENT RESPONSE

Score Point: 2

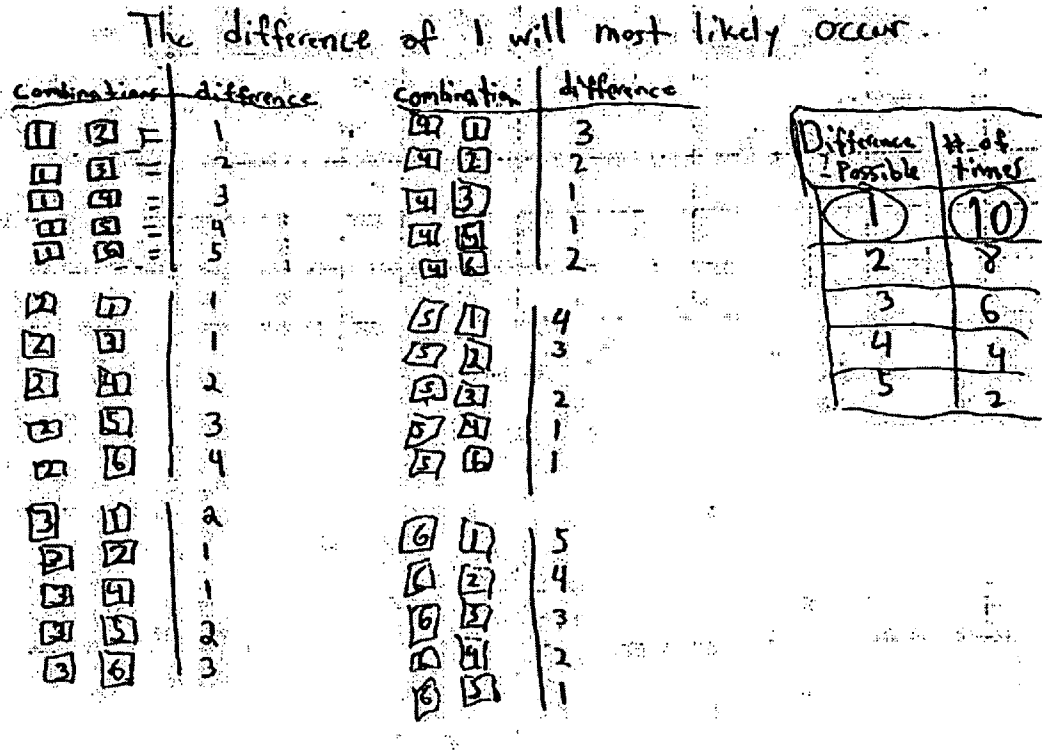


Comment:

This response demonstrates some understanding of the concepts. The student lists some of the combinations, omitting repeats of the same two numbers in a different order, therefore obtaining the wrong answer. The response does not earn additional points because the desired answer of a difference of 1 will occur most often is not stated instead "*zero will occur most often*" is the student's response. This response earns 2 points.

STUDENT RESPONSE

Score Point: 3

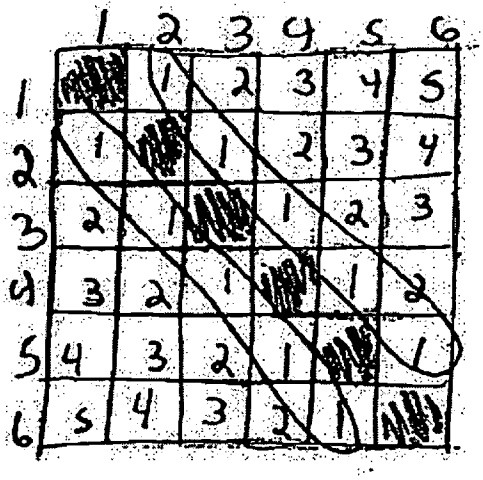


Comment:

This response demonstrates a full understanding of the concepts stating the correct answer with well organized supporting work communicated clearly. The student lists almost all the sample space omitting the doubles outcomes which do not change the final answer. For this minor omission of the doubles in the sample space, the response only earns 3 points.

STUDENT RESPONSE

Score Point: 4

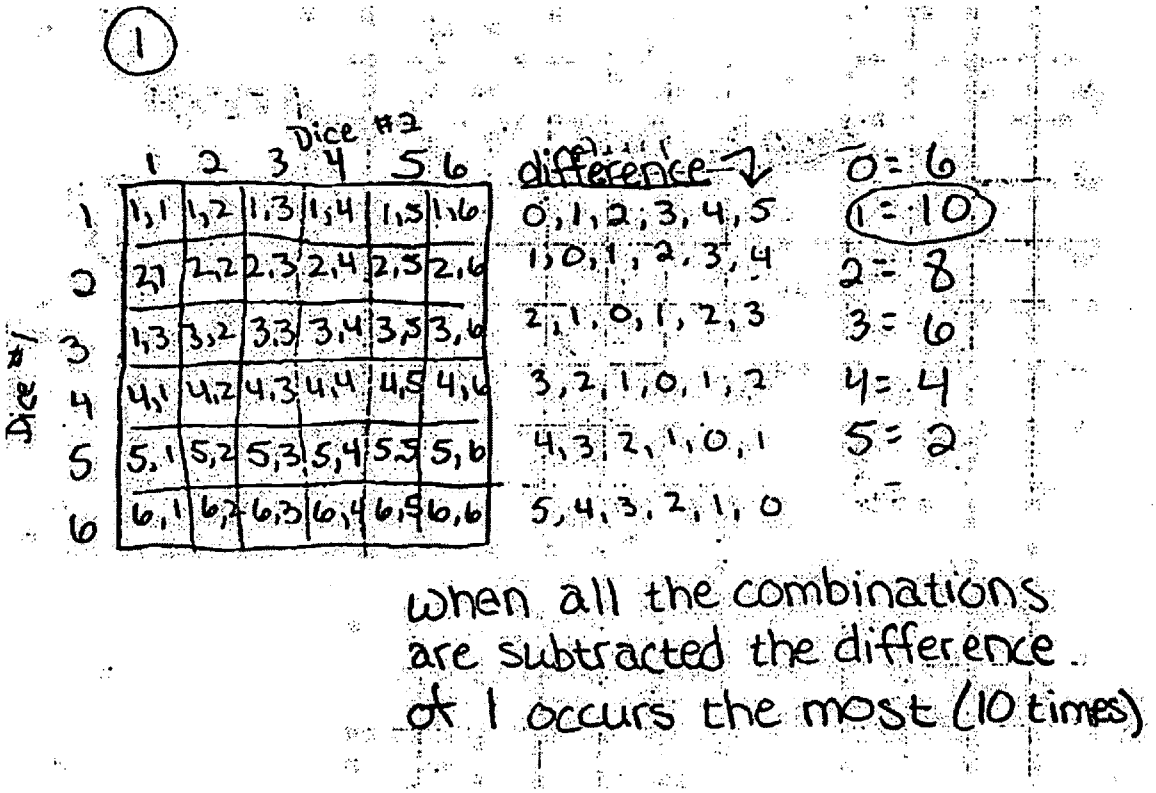


Comment:

This response demonstrates a complete understanding of the concepts. The student uses a table to illustrate all the differences, therefore all the combinations. The shaded diagonal indicates that there is no difference or a difference of 0 when rolling two identical numbers. The circled 1's indicate the correct answer, "*one is the most common answer*".
This response earns a score of 4.

STUDENT RESPONSE

Score Point: 4



Comment:

This response demonstrates a complete understanding of the concepts with full and clear support. The explanation is excellent. This response earns 4 points.

Math Standard # 10 (Grade 10)

Students will develop an understanding of PATTERNS, RELATIONSHIPS, AND FUNCTIONS by solving problems in which there is a need to recognize and extend a variety of patterns; and to analyze, model, and describe real-world functional relationships.

Item #5

The table below shows *thinking, braking* and *stopping* distances at different highway speeds.

Speed (mph)	Thinking Distance (ft)	Braking Distance (ft)	Stopping Distance (ft))
v		$\frac{v^2}{20}$	
10	10	5	15
20	20	20	40
30	30	45	75
40	40	80	120
50	50	125	175
60	60	180	240

For the values in the table, if speed is represented by v, then a formula for the braking distance would be $\frac{v^2}{20}$.

a. What formulas would represent *thinking distance* and *stopping distance*?

b. According to the formula, how many feet would it take to stop if a car is traveling at 55 mph?

c. The usual rule is to allow one car length (approximately 20 feet) of space between your car and a car in front for every 10 mph of speed. How good is this rule when compared to the data above? Explain your reasoning.

Scoring Rubric:

4 Correct answers to all parts.
   a. Thinking distance is v;
      Stopping distance is $v + \frac{v^2}{20}$ or $\frac{20v + v^2}{20}$
   b. 206.25 (feet)
   c. Explanation which indicates that the customary rule works for low speeds (through 20 mph) but fails to provide enough stopping distance at higher speeds. This is because the rule is linear but stopping distance is quadratic. (Student needn't use the terms linear and quadratic but should note that the rule fails for speeds above 20 mph).

3 Parts a. and b. answered correctly, but explanation in c. does not contain enough detail, i.e., doesn't identify 20 mph as the maximum speed for which the conventional rule works.

2 Parts a. and b. answered correctly, with, perhaps, minor computational errors in b. Response to part c. is inadequate or even missing.

1 Unable to write formula for stopping distance though perhaps able to approximate stopping distance at 55 mph through linear interpolation (207.5) from tabular values. Response within the range 205-210 earns 1 point.

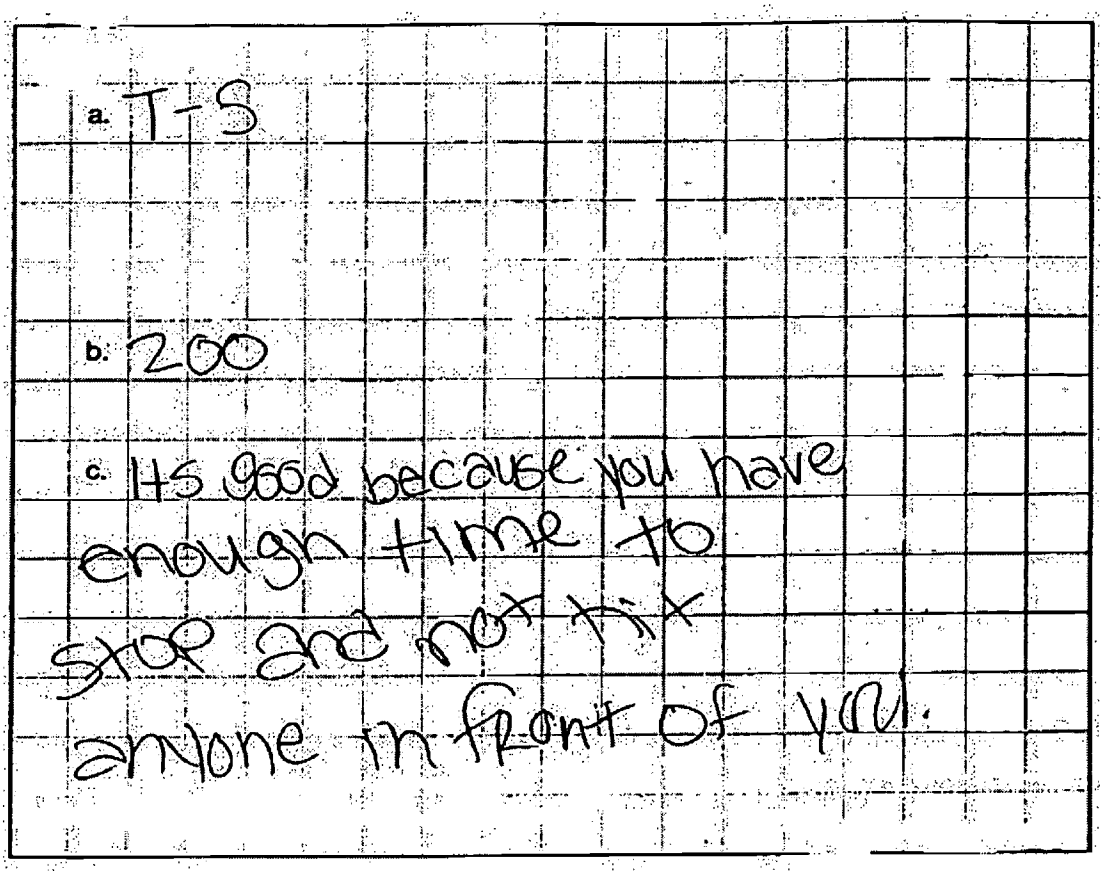
0 Some work but without much evidence that this work addresses the question.

Commentary:

A primary goal of high school mathematics is to promote the development of a variety of ways of modeling the world. This item features a quadratic model in a context that is important for young adults, that of the stopping distance of an automobile. The item involves scaffolding to promote access to the algebraic representation and then proceeds to require interpretation of mathematical results.

STUDENT RESPONSE

Score Point: 0

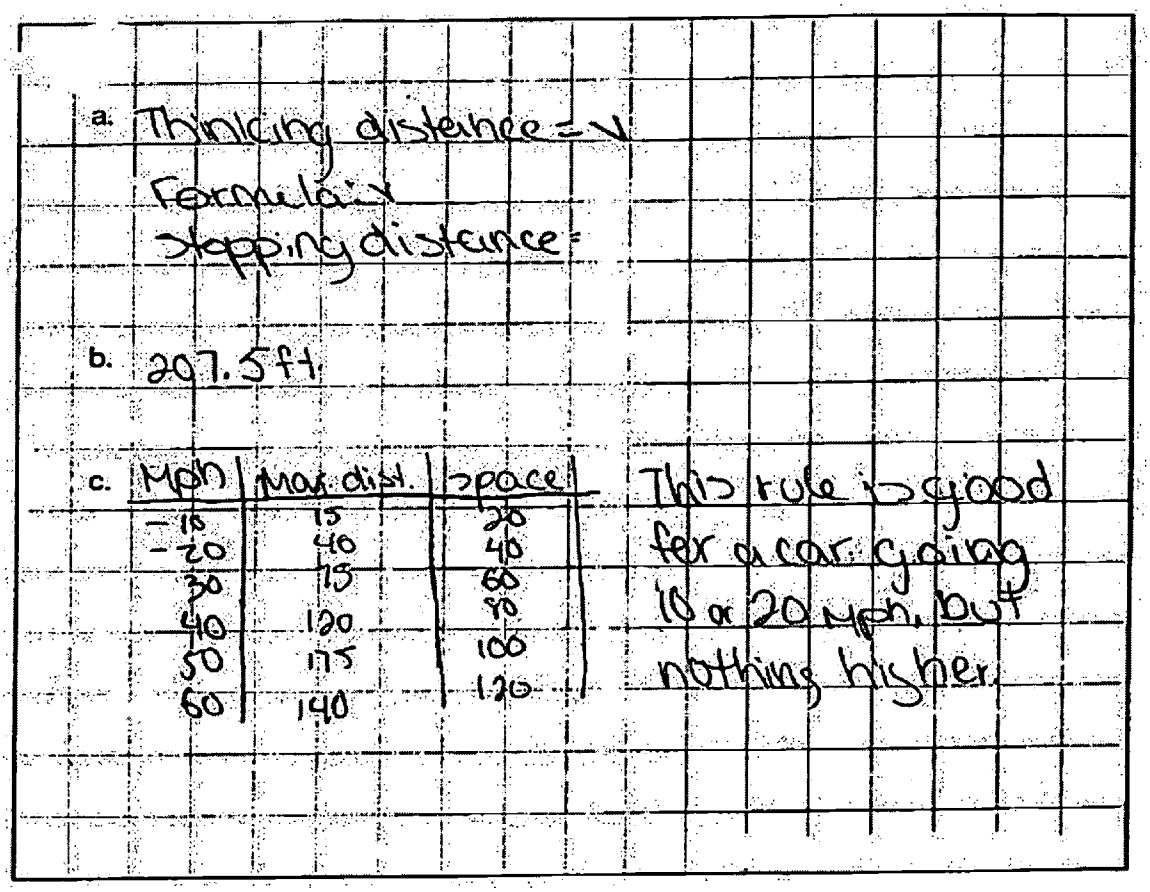


Comment:

This response does not demonstrate any evidence of the understanding of the mathematical concept. For example, the response to part a. simply involves the use of letters T and S presumably to represent Thinking and Stopping, but without any attempt to express these in terms of the speed of the car. The response to b. involves a weak interpolation from the table without explanation. This response earns 0 point.

STUDENT RESPONSE

Score Point: 1

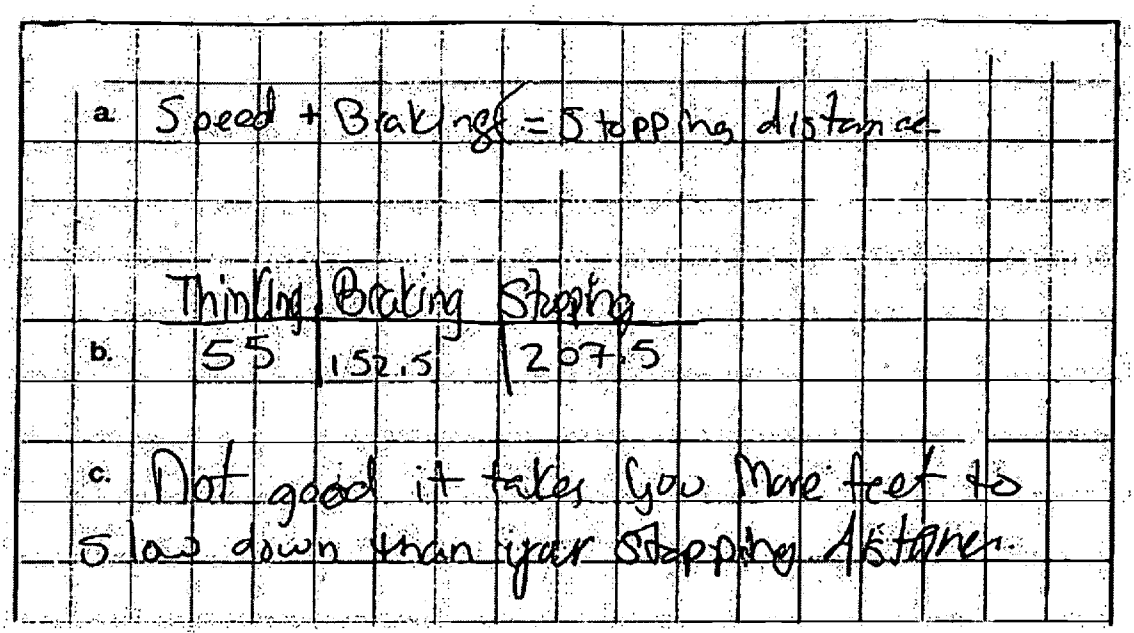


Comment:

This response demonstrates a minimal understanding of the concepts. The student writes a simple formula for Thinking distance but none for Stopping distance in part a. and there is no support for part b., which involves a simple linear interpolation – the student has taken the mid-point between the stopping distance for 50 mph and that for 60 mph. The student states the correct answer to part c. but the table is incomplete and not well organized.
This response earns only 1 point.

STUDENT RESPONSE

Score Point: 1

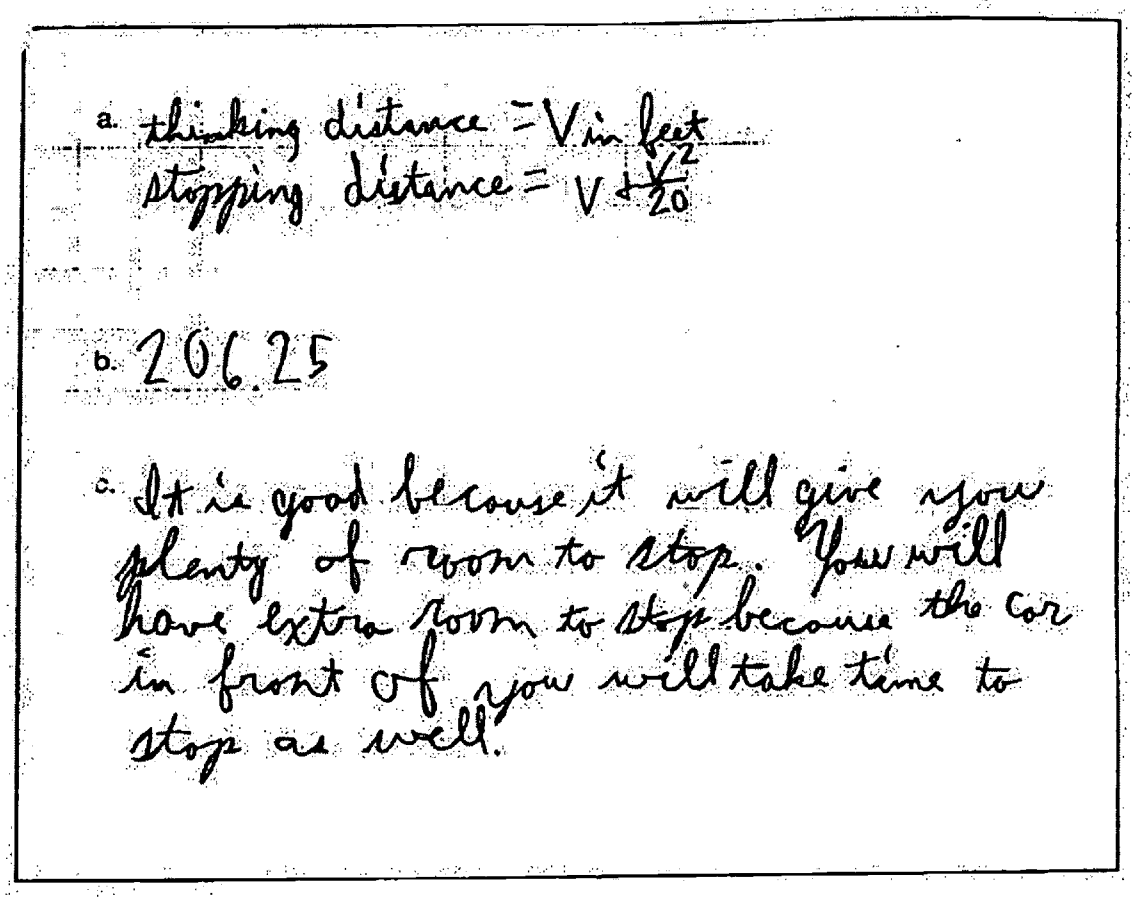


Comment:

This response demonstrates some understanding of the concepts, when attempting to answer all parts. Part a. describes the general relationship between the speed, braking distance, and the stopping distance but it does not answer the question, which asks for the two formulas with specified variables. A linear interpolation, and not the correct quadratic equation, was used to find the braking and stopping distance in part b. When these two incomplete and imperfect responses to parts a. and b. are combined with the weak explanation in part c., this response earns only 1 point.

STUDENT RESPONSE

Score Point: 2

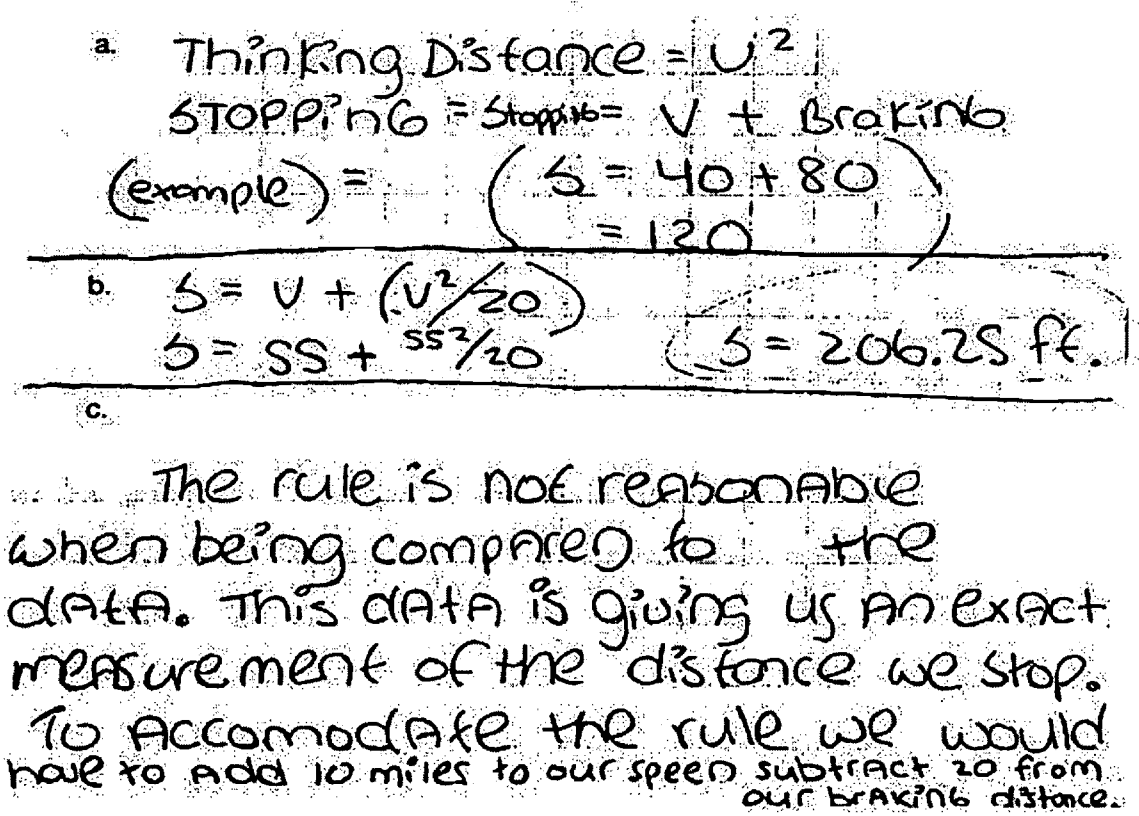


Comment:

This response demonstrates some significant understanding of the concepts. Parts a. and b. are answered correctly without work because the student can determine the formulas by an inspection of the given data. However, the student fails to consult either the table of data or the formulas derived for part a. and applied in part b. and gives a completely incorrect response to part c., so it can only earn 2 points.

STUDENT RESPONSE

Score Point: 2

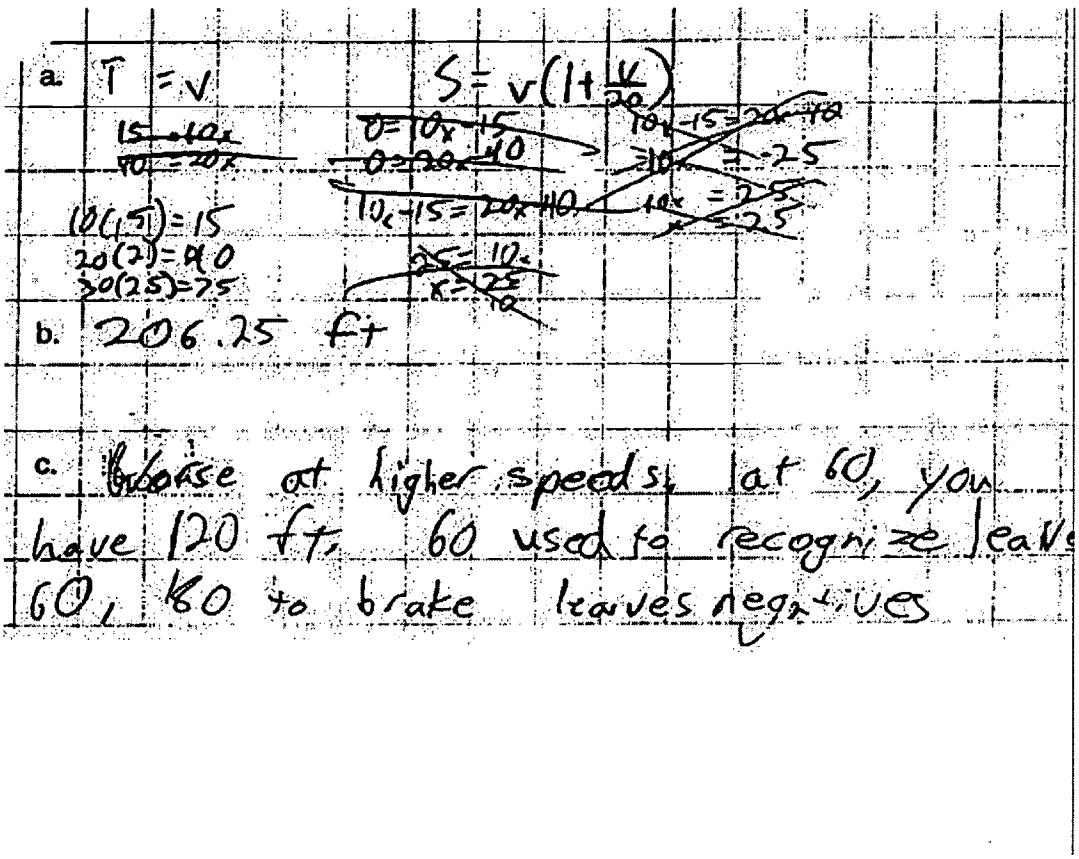


Comment:

This response demonstrates some understanding of the concepts. The student shows her or his reasoning when obtaining the correct formula for stopping distance using the given data and consequently determines the correct stopping distance for a car traveling at 55 mph. The student does not seem to understand part c. and therefore the explanation is incoherent and the response can only earn 2 points.

STUDENT RESPONSE

Score Point: 3

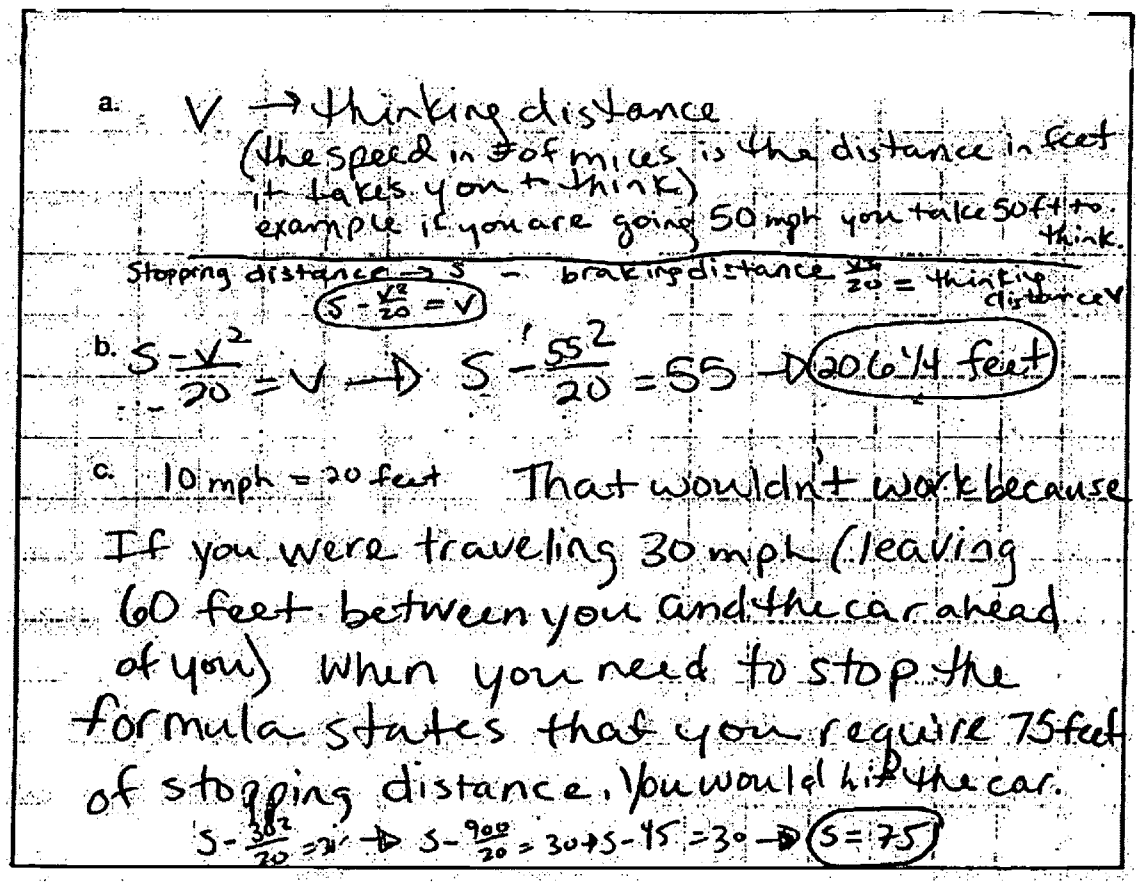


Comment:

This response demonstrates full understanding of the concepts used to determine the correct formulas for both the thinking and stopping distances. The student first attempted the formulas for Thinking and Stopping Distance although there seems to be some initial confusion as much of the preliminary work is crossed out. Finally, the student writes the formula for the stopping distance as a correct product of linear factors which may indicate that the student is very comfortable with quadratic functions. It is difficult to follow the student's explanation in part c. and furthermore the student does not identify 20 mph as the maximum speed for which the rule will work. As a result of this failure to communicate, this response only earns 3 points.

STUDENT RESPONSE

Score Point: 3



Comment:

This response demonstrates full understanding of the concept but the explanation in part c. is incomplete. It is interesting to note that this particular student determines the stopping formula using a different setup from many of the other responses. The student is proficient in manipulating and evaluating equations. The explanation for part c. is sufficient due to the computation supporting the statement. However, the student does not identify 20 mph as the maximum speed for which the rule works. For this reason, the response earns only 3 points.

STUDENT RESPONSE

Score Point: 4

a. $td = V$

$sd = V + \frac{V^2}{20}$

b. 206.25

c. The rule is good if you are traveling under 20mph. After that speed limit, the following distance is less than the braking distance.

Comment:

This response demonstrates full understanding of the concept. The student determines the correct formulas in parts a. and b. The formulas can be determined by an inspection of the data. The first is linear and the second is quadratic. The explanation for part c. is clear although concise and illustrates what data the student used – "*following distance is less than the braking distance*"- to determine her or his answer. This response earns 4 points.

STUDENT RESPONSE

Score Point: 4

a. V = thinking distance
V + (V²/20) = Stopping distance

b. 206.25 ft.

c. Not good. The 1st one at 10mph is fine b/c you still have 5ft left over. But when you get to 20mph you have just enough room. At 30 mph you'll have hit the other car.

Comment:

This response demonstrates full understanding of the concept. All of the answers in part a. and b. are correctly labeled, including the units. There is no need to provide support for part a. since the student can just determine the formulas by looking at the given data. Entering the correct numbers into a calculator would give the 206.25 feet required to stop a car traveling at 55 mph. The explanation for part c. is clear stating that at 10 mph there is extra space between the two cars; at 20 mph there is just enough space and at 30 mph there is NOT enough space between the two cars. One would extrapolate that there is not enough space between two cars when traveling at higher speeds. This response earns 4 points.

TM034202

U.S. Department of Education
Office of Educational Research and Improvement (OERI)
National Library of Education (NLE)
Educational Resources Information Center (ERIC)

NOTICE

Reproduction Basis

☒ This document is covered by a signed "Reproduction Release (Blanket)" form (on file within the ERIC system), encompassing all or classes of documents from its source organization and, therefore, does not require a "Specific Document" Release form.

☐ This document is Federally-funded, or carries its own permission to reproduce, or is otherwise in the public domain and, therefore, may be reproduced by ERIC without a signed Reproduction Release form (either "Specific Document" or "Blanket").