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ABSTRACT

This paper draws on a body of research to present representations of numbers which can assist children in learning mathematics. Sections in the paper deal with counting, quantity, rank, and place value. Also included is a model which shows an approach for teaching two-digit and three-digit numbers in kindergarten, first grade, and the first half of the second grade. (MM)

# Number Representations That Assist Children to Succeed in Mathematics

by

Calvin J. Irons

# Number Representations That Assist Children to Succeed in Mathematics

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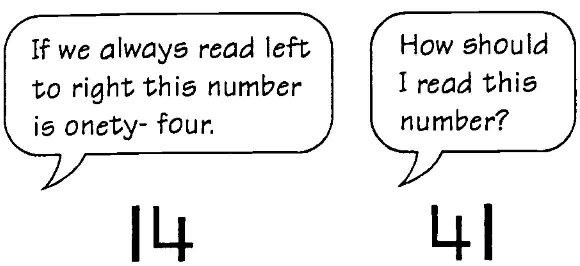
## Introduction

A thorough understanding of numbers is essential for success where mathematics is employed to solve problems. As Baroody (1998) states, "Numbers and numerals are basic and indispensable tools for mathematics." However, complete facility with even two-digit numbers is difficult for many young children to achieve (Fuson, 1992). The unique place-value features of our number system do not always make sense to children and often present challenges that require more time to assimilate than is traditionally allowed.

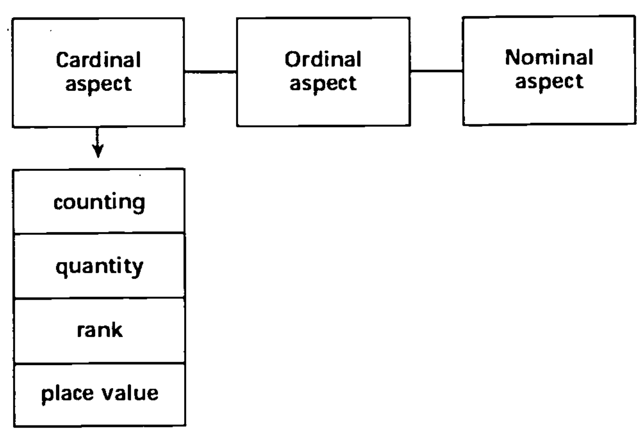



Number sense activities have helped to identify a range of representations that children use to work with numbers that are not always based on place value (Cooper, T. J., Heirdsfield, A. M., & Irons, C. J. 1996b). These representations provide a basis for children to naturally work with numbers, sometimes well before they are taught formal place-value. When children are confident with the place value representation of numbers, the other more natural ways of thinking are enhanced and expanded. Taken together, the representations provide children with flexible and powerful ways to think when they work with numbers.

## Cardinal Number Representations

In real-world situations, numbers are used in at least three different ways—cardinal, ordinal, and nominal (Reys, R., Suydam, M., Lindquist, M., Smith, N. 1998). The cardinal aspect of numbers refers to the size of a collection. This usage of numbers helps to answer questions such as, "How many in the class? What was the total amount collected?" The ordinal aspect of number is used to describe the position of an object in an ordered collection. This relates to questions like, "Who finished the race first? Who lives in the fifth house on the left side of the street?" The nominal aspect acts as a label rather than telling how many (cardinal) or which one (ordinal). Room numbers, telephone numbers, and bus numbers are some examples of numbers that act as labels.

This paper will describe the four representations, shown below, that children use to work with the cardinal aspect of numbers.

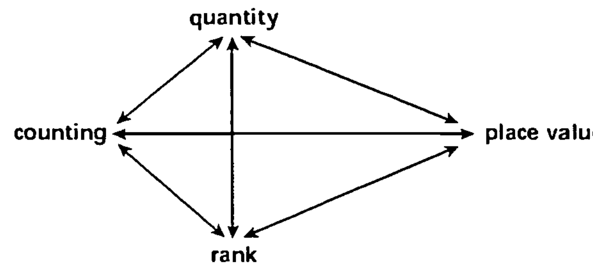


The most significant observation about the representations is the ease with which children initially work with numbers using the first three models without knowing anything about the symbolic or place-value form. They count, form ideas of a number quantity for certain key numbers such as five or ten, and work with the rank order of numbers without using any symbols. These representations encourage children to mentally manipulate numbers as they begin to calculate, even with numbers greater than ten, before place-value ideas are studied formally.

When children work with larger numbers or new ideas such as fractions or decimals, the first three representations continue to provide powerful ways to calculate and make estimates. Therefore, it is important to foster the use of all representations, rather than stressing just the one: place value. Work with fractions must focus on representations other than place value.

Place value is important and, because it is not always a natural model for children, requires a considerable amount of time to teach. However, it is not necessary to establish all of the intricacies of place value before children work with two-digit or larger numbers. Children can use many ideas from the other representations to assist them as they work with numbers. Once the first ideas of place value are established, these properties can be used with the other representations to support and enhance learning.

The kite-shaped diagram shows the relationship among the four representations. The diagram relates to each number range (two-digit numbers, then three-digit numbers, and so on). A modified kite, without place value, would be used in Pre-K and kindergarten to develop numbers to ten. So taken together, a series of kites, possibly layered one on top of the other, could be used to show the development throughout elementary school.

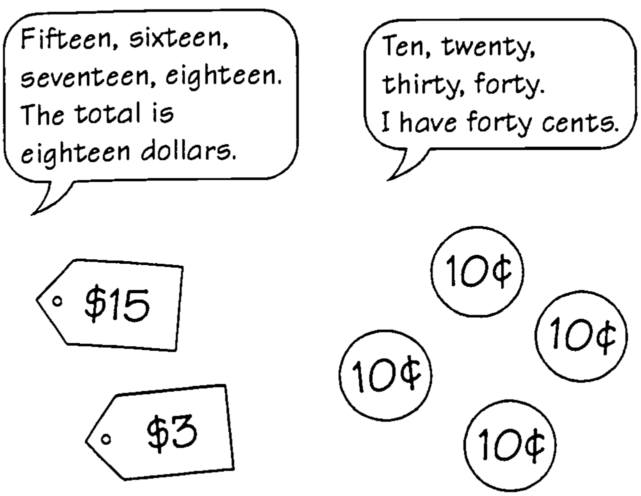



Counting is the first number representation children use. Shortly after they begin to count, children begin to form ideas related to quantity and rank representations. These seem to develop side by side. Place value is located to the right to indicate that it is more complex and is developed later. The diagram also shows children can be involved with some work using all of the first three representations before place value is introduced. Once children have begun place-value experiences, they can move back and forth using one representation or another as they manipulate numbers. Place-value ideas are then used to enhance and broaden the other representations.

**Counting**

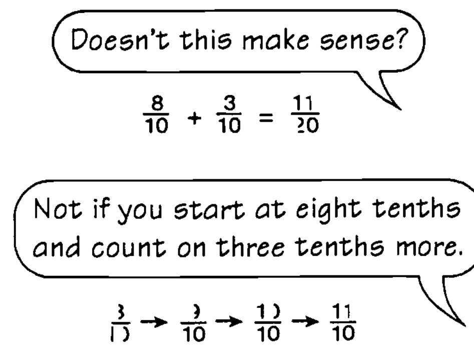
Counting is the first number skill that children learn. All young children love to count and they master rote counting with relative ease. Kindergarten children learn the order of the number names and usually can say the names in the correct order beyond twenty. Once children identify the pattern for saying two-digit number names, they can continue to one hundred. Rote counting is not the same as rational counting, or counting with one-to-one correspondence, and the ability to say the number names in a sing-song manner should not be the only indication that children possess all of the skills associated with counting. And, of course, counting is only one representation for number.

Number rhymes and games help children learn to count in other ways—by fives, tens, and twos. Through appropriate activities, children also become successful at counting-on from a number. These fundamental skills provide children with the initial confidence required to use numbers as they calculate and solve daily life problems, especially with money.




Counting does not need to be restricted to small whole numbers. Before children are introduced to three- or four-digit numerals, they can be involved in counting in hundreds, thousands, fifties, or twenty-fives. When children are confident with number symbols, counting can continue using jumps on a number line, on a hundred board, or with a calculator. Sophisticated counting strategies should be fostered by asking questions such as, "If you count in steps of three, starting at three, will you say the number forty-two? How do you know?"

Children are also confident about counting fractions such as halves, quarters, or tenths before they know how to write these numbers. Later, after symbols have been introduced, counting provides a strategy for working with these numbers. A counting approach is a good way to avoid some difficulties that children have with adding or subtracting fractions.




Counting is a number representation that all children learn early, and should be used frequently with a greater range of numbers. Because children successfully use counting when they work with all types of numbers, it should be fostered as a valued strategy well after children have become confident with place value.

Counting is the basis for the other three representations of number, but particularly for quantity and rank.

**Quantity**

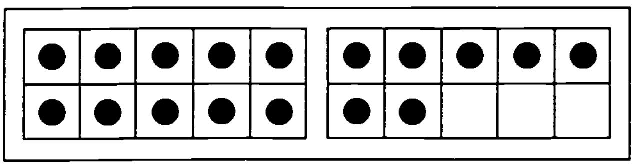
When children count, they begin to form certain benchmarks that help them work with numbers. In kindergarten, five and ten are important numbers for children. In many ways, these numbers become their personal friends when they start to calculate. They know, for example, that six is five and one while ten and one make eleven. Children begin to form pictures of five and ten and see them as whole quantities rather than collections of separate discrete entities. This idea of number is important for all numbers less than ten.

The ability to see a collection and quickly recognize the number in it (called subitization) helps children when they add, subtract, multiply, or divide. However, the picture that most children form of numbers such as five and ten seems to be more than simply subitizing.

Usually, children are not able to subitize collections greater than ten. However, they continue to think about and use the idea of quantity for many larger numbers. Young children see one hundred, fifty, twenty-five, and most multiples of ten as quantities. They are able to work with these numbers to calculate before they have been introduced to any formal algorithm. For example, young children know that twenty is two groups of ten and one hundred can be split into fifty and fifty.

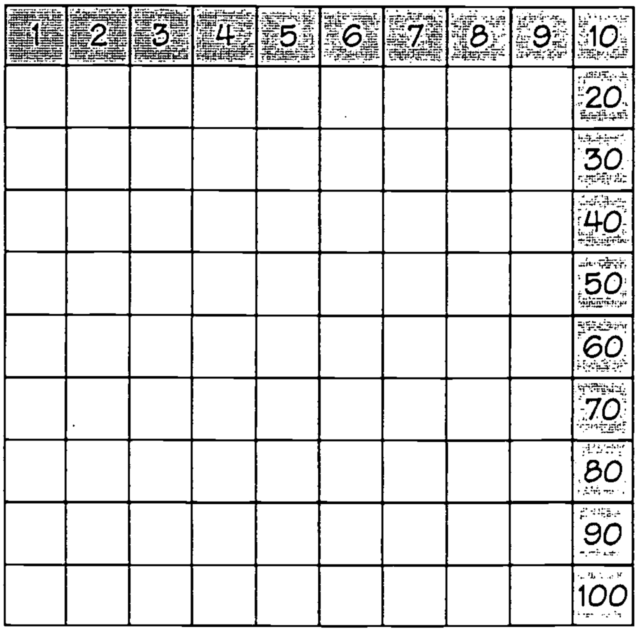
After children are introduced to place value, they continue to use the quantity representation to calculate. For example, when multiplying a number such as 25 by 6, most adults do not use the place-value properties of 25. They think 4 twenty-fives make 100 and 50 more is 150, or possibly they know that 6 twenty-fives equal 150. Twenty-five has become a personal friend for most people and they can use it in different ways from other numbers. The same is true for many other numbers, as long as the development of the quantity idea is encouraged throughout the early years of school.

Number frames are a good way to help children form and use ideas of quantities. Five frames and ten frames are organizers for children that help them form these important benchmarks. A double tens frame will help children build a benchmark for twenty and show how teen numbers relate to both ten and twenty.



A hundred frame—like a hundred board without numbers written on it—is a useful model in first grade. Children know, upfront, that there are 100 spaces in the frame and can use the frame to talk about various combinations that can be formed using the empty spaces. Gradually, numbers can be written on the frame beginning with 1 to 10 across the top row and moving on to multiples of ten down the right-hand column. These numbers can be used for a time to talk about combining quantities that are not labeled on the frame. For example, 20 and 30 make 50. At first, it is not necessary to think about the place value ideas that could be used to form this sum. Children can simply use the two quantities shown by the five rows of spaces.

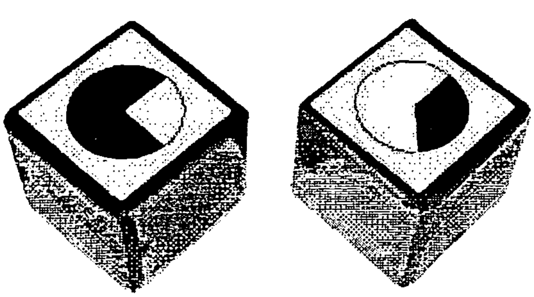
Two-hundred frames can be used to build a benchmark for 200.



Doubling is a powerful thinking strategy that helps children calculate, even with large numbers. When children quickly double two numbers they use the idea of quantity to determine the answer. By the end of first grade, they are able to double most numbers to ten, multiples of ten, numbers such as 15, 25, 35 and 45, and many teen numbers. Doubling two-digit numbers is possible without knowing all of the place-value properties because children use a quantity representation for numbers.

The quantity representation helps children work out the answer to addition problems such as 25 + 26 = ___ using a doubling procedure. Although this example might be considered to be difficult because it involves regrouping, many children, when prompted, say that they double 25 and then add one. In this situation, they have not used the place-value idea for 25, but the quantity representation. Money is a good way to support children when they work with the quantity representation. Coins are natural benchmarks that assist children when they calculate.

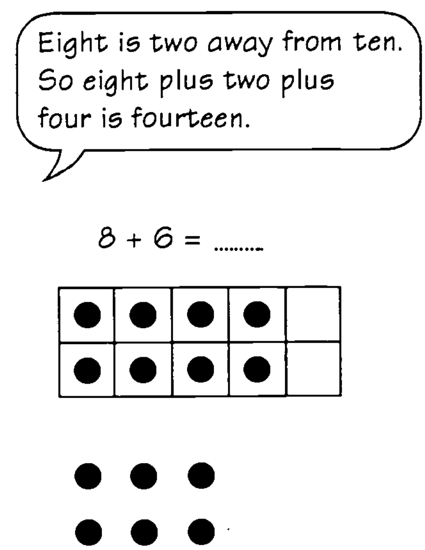
Another example of the quantity idea relates to the development of fractions. Before children are introduced to symbols, they do have an idea of fractions and can combine and separate them. An important benchmark for fractions is one whole and, later, one-half. Using pictures like those shown on the jumbo cubes below, children can discuss questions such as, "Is the total more than (less than or equal to) one whole? How do you know?"



**Rank**

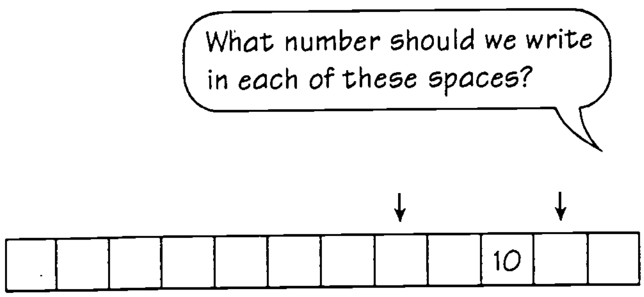
When children count, they also learn about the relative position of one number to another. For example, they know that 5 comes after 4 and 9 is just before 10. The counting sequence helps them work out the rank position of numbers such as 15 to the other teen numbers. They also learn and use facts such as 15 is midway between 10 and 20. Later, they use ideas such as 99, 95, and 90 are respectively one away, five away and ten away from 100.

One of the first places where children use the rank idea is to add when one of the numbers is close to 10. The ten frame helps children determine how far away the number is from 10 and then using compensation, work out the total.

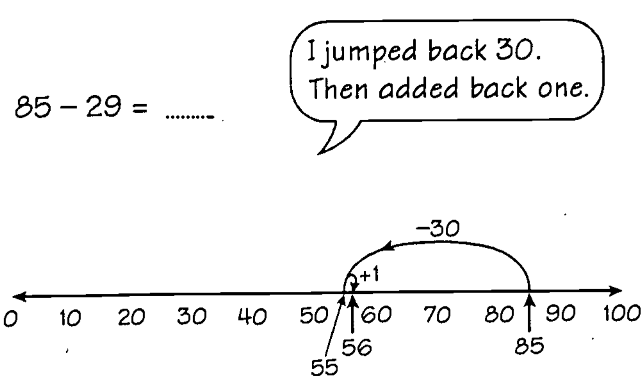



Children can work with the idea of rank without using place-value properties. When they add amounts such as 39 cents and 29 cents, they use the idea that 39 is one away from 40 and 29 is one away from 30 to mentally add 40 and 30 and then take off two cents. In this case, of course, symbols are used to represent the amounts, but place value is not the dominant strategy for adding.

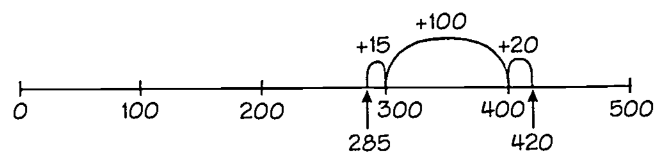
Number tracks and number lines are ideal models for reinforcing the rank idea. At first, the key benchmark numbers can be written on the tracks/lines. This will reinforce the role of counting to work out the numbers, either counting on or counting back from a benchmark. The spaces on a number track make this easy for kindergarten and first grade children.




When children are confident with place-value ideas, they use these properties and the rank representations to add and subtract two- and three-digit numbers. Number lines help children explain the strategies they have used. In the example below, a nearby number is used to subtract:




When children calculate larger numbers, it becomes more apparent that they use their knowledge of rank along with place value to help them. In the following example, 420 – 285 = ___, the child started at 285 and knew the distance to 300 was 15, jumped 100 to 400, and then knew that 420 was 20 more than 400. The difference is the sum of 15 + 100 + 20, or 135.




Rank representation does not need to be linear. Hundred boards also act as a model that supports the development of rank in two different directions. This is a key element that assists in the development of two-digit place value.

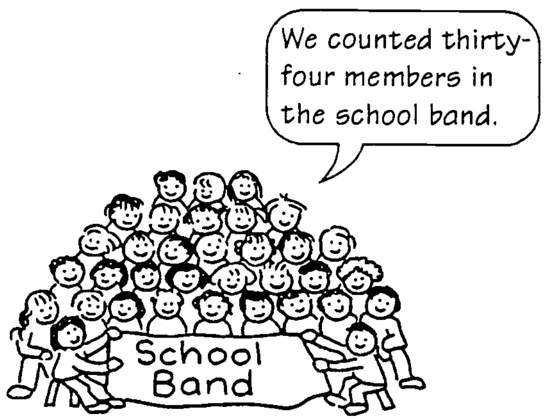
**Place value**

The importance of place value as a representation for number is well documented (National Council of Teachers of Mathematics, 2000). At the same time, research has shown that numbers are difficult to learn (Fuson, 1992). A recent illuminating article shows why the language complexities of two-digit numbers, particularly the teen numbers, cause problems when children move on to use these numbers (Sun, W. & Zhang, J. 2001). As described in the introduction, the confusion arises when children attempt to relate the number symbols to the words that match.

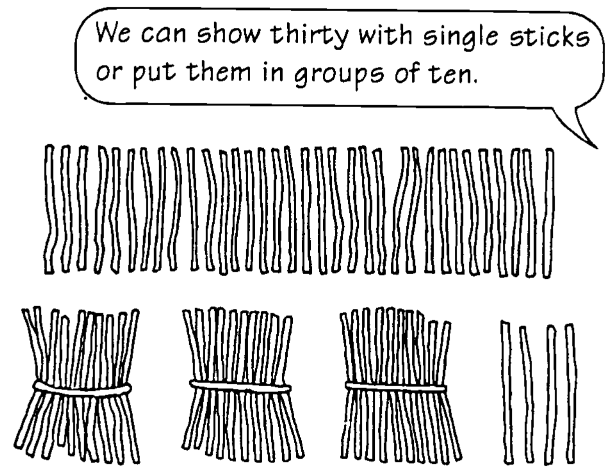
A good teaching sequence for place value must carefully develop all key components associated with this representation for number. Significantly, the sequence involves a range of language experiences that begin with the real-world experiences familiar to children and extends to appropriate activities with classroom materials to develop the language further. Gradually, mathematical language that stresses place value and then the symbols is taught. The illustrations below summarize this development (Irons, C & Irons R., 1989).

When children are confident with the place-value representation, they should use the number symbols in activities that then extend understandings related to counting, quantity ideas, and rank. In this way, children will be able to flexibly work in any of the contexts they might encounter where numbers are involved.

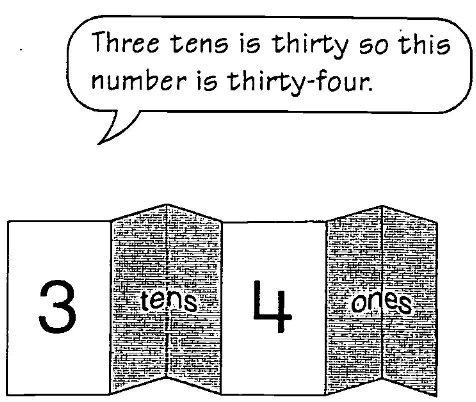
**Stage 1: Real-world language**



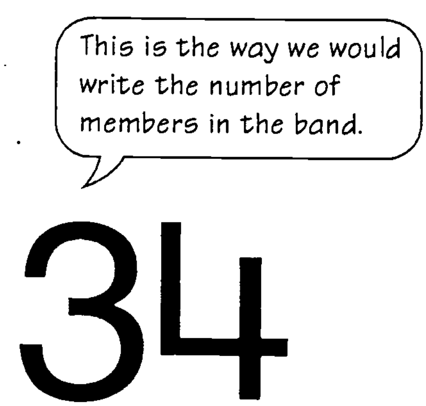

**Stage 2: Materials language**




**Stage 3: Mathematical language**



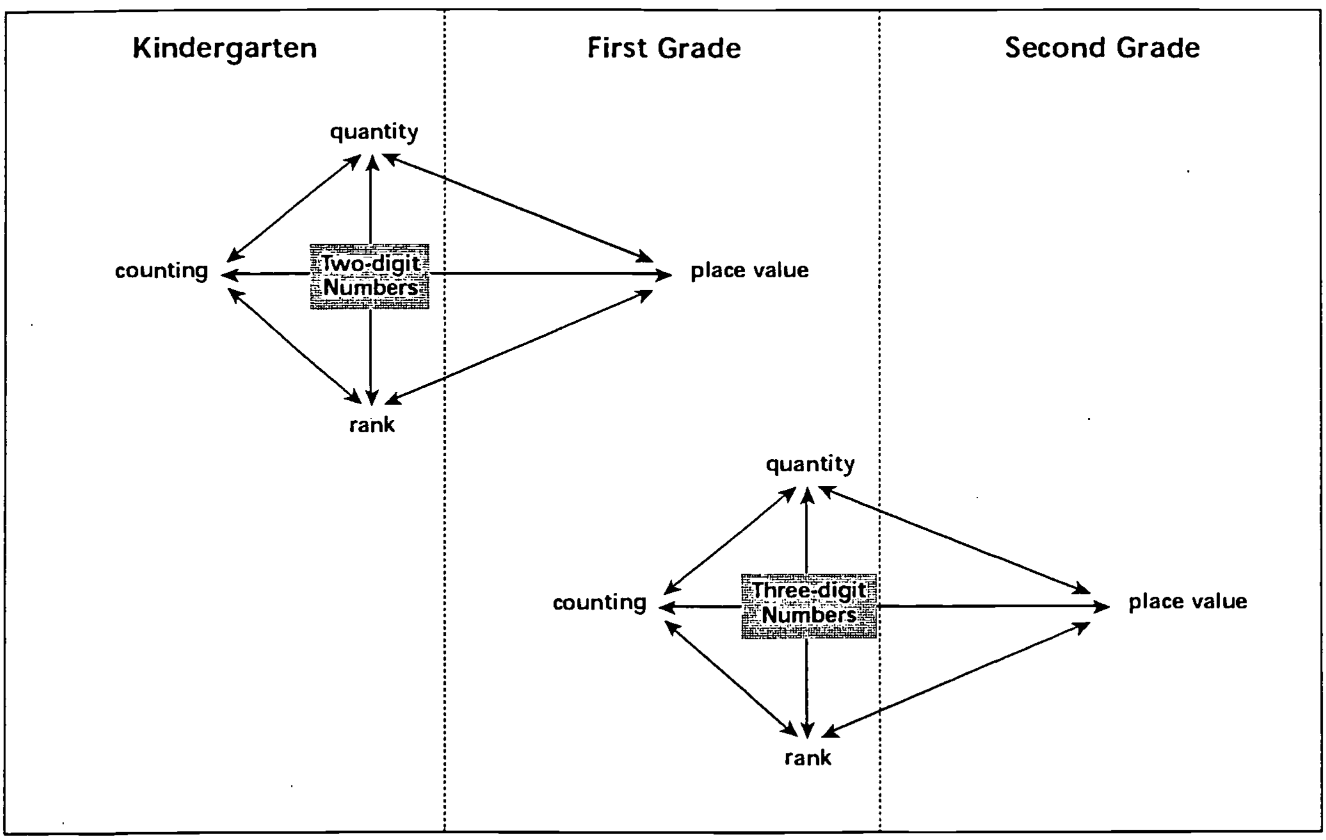

**Stage 4: Symbolic language**



## A Number Scope and Sequence (Kindergarten to Second Grade)

Each kite below shows the approach for teaching two-digit and three-digit numbers in kindergarten, first grade, and the first half of second grade. A similar approach would be used for larger numbers, fractions, and decimals in other grades. And, of course, there is a modified kite for numbers 0 to 10 that are studied in Pre-K and kindergarten. The first kite spanning kindergarten and first grade indicates that considerable work with two-digit numbers can begin in kindergarten, but caution needs to be exercised with teaching place value. Because of the complexities of place value, this is the focus of two-digit number work in first grade. The second kite shows how three-digit number work can span first and second grade with the emphasis on place value in second grade.

The flow shown should be taken as a guide. In some classrooms the sequence needs to move more slowly, while with many individual children the pace can be accelerated. A mathematics curriculum that includes all four number representations opens opportunities for children and allows them the flexibility to work in a greater range of ways and achieve success using strategies that make sense to them.



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