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ABSTRACT

The purpose of this paper is to investigate how collaborative problem solving affects individual mathematical problem-solving ability. Quantitative and qualitative evaluations were conducted, focusing on four students in the same fourth grade mathematics class in Taipei, Taiwan, as they participated in two steps of individual problem-solving activities and one step of collaborative problem-solving activity. The paper begins with a review of the applicable theoretical notions on problem solving, and on collaboration as it relates to problem solving. Next, it explains the experiment's method and procedures, and then goes on to analyze and discuss its results. Finally, it presents some conclusions, along with a brief discussion of the project's implications for further research. The results show no quantitative differences between collaborative and individual problem solving. However, some qualitative differences were evident. (Contains 21 references and 5 figures.) (Author/AEF)

The Effects Of Collaborative Problem Solving On Individual Problem-Solving Ability

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Abstract

The purpose of this paper is to investigate how collaborative mathematical problem solving affects individual mathematical problem-solving ability. This paper begins with a review of the applicable theoretical notions on problem solving, and on collaboration as it relates to problem solving. Next it explains the experiment's method and procedures, and then goes on to analyze and discuss its results. Finally it presents some conclusions, along with a brief discussion of the project's implications for further research. The results show no quantitative differences between collaborative and individual problem solving. However, some qualitative differences were evident.

Introduction

"Problem solving" has been regarded and designated by many mathematics educators and researchers (see, for example, Goldin, 1982; Lester, 1982; Mayer, 1982; Schoenfeld, 1982, 1985) as an important activity in mathematics learning. Indicative of the perceived importance of this topic is its inclusion by the National Council of Teachers of Mathematics (NCTM) as a heading in its new standards for the mathematics curriculum, which list mathematics as problem solving along with mathematics as reasoning, mathematics as connection, an mathematics as communication (NCTM, 1989).

"Collaboration" too has been much advocated recently as a powerful factor both in learning in general and in problem solving in particular. This advocacy is based on social interaction frameworks (see, for example, L. S. Vygotsky, 1978), and is supported by the findings of research experiments (for example, Forman, 1981). These results confirm that for many types of problem solving the results of collaboration are superior to those of individual efforts. But how and why does collaboration function so effectively? More specifically, what are the mechanisms through which collaboration promotes problem-solving abilities? This has been the motivating question behind my research effort; and the purpose of this paper is to provide at least some partial, tentative answers.

In order to gain some answers, an experiment was designed and conducted that would enable the experimenter to observe both individual and collaborative problem-solving activities together, and to compare their results in terms of both quantitative and qualitative ethnographic perspectives. This paper will begin with a review of the applicable theoretical notions on problem solving, and on collaboration as it relates to problem solving. Next it will explain the experiment's method and procedures, and then go on to analyze and discuss its results. Finally it will present some conclusions, along with a brief discussion of the project's implications for further research.

Theoretical Background

It is not too overwhelming to say that the human civilization is a history of problem solving activities (Yang, 1994). Often in our daily lives we find ourselves working to solve problems in our lives, whether these problems are big or small, important or unimportant. We try to gain knowledge and skills both from school and in our daily lives in order to solve these problems. For example, we learn mathematics in order to solve those problems related to number, quantity, shapes, area, volume, and so on. And indeed, "mathematical problem solving" is both the main goal of mathematics learning (Brodinsky, 1977; LeBlance, 1977) and the most important activity of mathematics learning (Lester, 1980). In recent years advocacy from mathematics educators and researchers has resulted in increasing attention given to issues concerning mathematics problem solving and in the adoption of this activity as a major focus of a new set of standards for K-12 mathematics curriculum.

In traditional mathematical problem-solving activities in school settings, the most often used learning method is individual problem solving, rather than collaboration. Moreover, collaboration, often regarded as cheating, is commonly prohibited as a mathematical problem solving activity. Mason (1972) has the following impressive observation, "We isolate students by pitting them against each other competitively, and imposing on them a fierce decorum of silence and regimentation" (p. 6).

In view of the fact that collaboration has become a popular, effective, and efficient form of learning in daily life and in the work place, and has been advocated as a mode of working style to meet the rapid changes and big challenges of modern society (Senge, 1994), it seems reasonable to suppose that the mathematics curriculum should also increase its acceptance of collaborative-oriented problem solving in order to prepare the learners to collaborate with others in order to solve problems in daily life.

The significance of collaboration can be understood from two perspectives: first regarding it as a practical phenomenon, and then from the viewpoint of developmental psychology. In the following paragraph I shall consider each of these perspectives in more detail.

As a practical phenomenon, collaboration, or "working together", is in the very nature of society (Mason, 1972), and "inherent in everyday interaction" (Choi & Hannaffin, 1995, p. 62). As Forman and McPhail (1993) put it, "The everyday lives of adults are full of complex and ill-defined problems that require high-level reasoning and organizational skills. These problems are often solved in collaboration with other people. For example, a husband, wife, and babysitter may need to coordinate their weekly occupational and domestic work schedules in order to supervise one or more young children" (p. 213).

To acknowledge the fact that all human progress is a result of collaboration amounts not only to a simple affirmation that collaboration is the most common mode of social functioning (Resnick, 1987), but also to an indication of its status as the most magical of all human activities. In common-sense terms, progress is possible because "collectively, we can be more insightful, more intelligent than we can possibly be individually. The IQ of the team can, potentially, be much greater than the IQ of the individuals" (Senge, 1994, p. 239).

But how and why does collaboration function so universally, so effectually? The anthropologist Edwin Hutchin gives us an example that lets us begin to see how collaboration structures the efforts of a society, the formation of knowledge, and even the nature of cognition (in Resnick, 1987). In Hutchin's view, guiding a ship by triangulation on coastal landmarks entails both a complex interaction between helmsman and lookouts and a collaborative articulation of the fabric of knowledge itself, involving distance experts such as cartographers and gyrocompass-builders. Here we cannot speak of collaboration without saying something about the knowledge-structures involved, nor can we speak of the required knowledge without saying something about the collaborative processes through which it is put into effect.

When we look at the social deployment of knowledge, we find it inseparable from collaborative processes. That is to say, social learning-at any rate, social learning outside of school-takes place through collaboration. As Resnick (1987) remarks, "much activity outside school is socially shared. Work, personal life, and recreation take place within social systems, and each person's ability to function successfully depends on what others do and how several individuals' mental and physical performance mesh" (p. 41). Even if we choose to view learning as an individual process, we will find that it is collaboration that provides the "context in which supports for, constraints on, and challenges to an individual's thinking occur" (Forman & McPhail, 1993, p. 213). Yet there is reason to believe that knowledge is best measured as the collaborative achievement of a group. Barnes and Todd (1977), studying groups engaged solely in assigned talk, found that these groups attained cognitive levels higher than those attained by individual members. Under such a notion, knowledge is distributed in nature. To return to Resnick's analogy (1987) from Hutchin: "No individual in the system can pilot the ship alone. The knowledge necessary for successful piloting is distributed throughout the whole system" (p. 41). Resnick's conclusion is that most work and learning in a society is a matter of "shared cognition," not "individual cognition."

L. S. Vygotsky (1896-1934), from the perspective of developmental psychology, provided a conceptual framework to deal with this question through his analysis of the relationship between social interaction and higher mental processes. He felt that higher functions originate first in the social interactions, before they are integrated into the cognitive structure of the mind: "Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals" (Vygotsky, 1978, p. 57).

Moreover, according to Vygotsky's view of development as a dialectical process, the individual cognitive structure of mind, having been internalized through interaction with the social environment, is then promoted to a state in which it will itself influence the social environment. So there is an ongoing reciprocal interaction, a bi-directional relationship, between the individual and the social environment. On the one side, the social environment or cultural frame within which the individual participates and interacts plays a crucial role in the development of human cognition; on the other side, the individual self is also an essential source in the social environment for fostering the ongoing reciprocal processes of interaction.

Vygotsky further proposed that any learning exploits a "zone of proximal development" or "ZPD." Vygotsky defined ZPD as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, p. 86). Vygotsky observed that children learning mathematics seemed to be using the ZPD they created for themselves in the course of collaborative problem-solving, in such a way as to attain higher levels of achievement than they would have been able to manage while working along (Yackel, Cobb, Wood, Wheatley, & Merkel, 1991). ZPD can be fostered by collaboration with adult or more competent peers, who supply a context "in which support for, constraints on, and challenges to an individual's thinking occur" (Forman & McPhail, 1993, p. 213). Within the resultant ZPD a group can attain cognitive levels not attained by individual members, leading the child on ahead in his or her development.

Purpose of the Research

The above section has explored the practical phenomenon of collaboration and some theoretical notions from developmental psychology about why collaboration works. These perspectives constitute the general background underlying the author's interest in investigating how collaboration might affect individual ability in mathematical problem solving. More specifically, what are the mechanisms through which collaboration promotes individual mathematical problem-solving abilities? To carry out this investigation a research experiment was designed in terms of the following questions:

- Is individual mathematical problem-solving ability after collaboration significantly different from individual mathematical problem-solving before collaboration? If so, what, quantitatively speaking, are the differences? And what, qualitatively speaking, are the important characteristics of these differences?
- Is collaborative mathematical problem-solving ability significantly different from individual mathematical problem-solving before collaboration? If so, what, quantitatively speaking, are the differences? And what, qualitatively speaking, are the important characteristics of these differences?
- Is collaborative mathematical problem-solving ability significantly different from individual mathematical problem-solving after collaboration? If so, what, quantitatively speaking, are the differences? And what, qualitatively speaking, are the important characteristics of these differences?

Method

A quantitative evaluation and a qualitative ethnographical investigation were conducted, focusing on four students in the same 4th grade mathematics class as they participated in two steps of individual problem-solving activities and one step of collaborative problem-solving activity.

Participants

The data-collection phase of the research was carried out using students from a 4th-grade class in an elementary school in Taipei, Taiwan, Republic of China. All these students were Chinese-Mandarin speakers. A group of four students out of a class of thirty-five was selected to participate in this study. This group of four students represented four different levels of achievement: high, median-high, median, and low. The selection method for this class was as follows:

A copy of the research proposal was sent to the school's research office, which in turn solicited teachers interested in letting their students to participate in the project. The class group selected for this study was one taught by one of the interested teachers who taught, initiated, and guided mathematics collaborative problem solving and discourse. Because the participants were members of a class group that was already familiar with collaborative problem solving and discourse, and also with non-synchronous thinking-aloud skills (Yang, 1994), an adaptation of thinking-aloud (Ericsson & Simon, 1985), the research project did not necessitate introducing the participants to any unfamiliar models of collaborative problem solving. No problems involving the establishment of a collaborative relationship were anticipated, because the participating students had practiced collaborative discourse and problem-solving since they were in the third grade.

The design of the selection of participants proposed by the researcher was: All of the students in the class were grouped into four ranks, based on their fourth-grade mathematics achievements level. One student from each rank was randomly selected by means of a random number table. Four students, representing a mixture of mathematics achievement levels, high, median-high, median, and low, were assigned to each group. The design of the selection of group size is consistent with the suggestions of the way of grouping in cooperative learning (which suggests small group size of 3-5 persons) (Johnson & Johnson, 1994), The design of forming group members is consistent with Vygotsky's ZPD notion which suggests heterogeneous grouping (Vygotksy, 1978).

In the research report, the participants' responses necessarily be described. However, no real names were be used. Rather, the participant with high level achievement was given by a letter "H," the participant with median high level achievement was given by a letter "MH," the participant with median level achievement was given by a letter "M," and the participant with low level achievement was given by a letter "L".

Materials and Instrumentation

The purpose of the study is to investigate how collaboration might affect individual ability in mathematical problem solving. A vital preliminary to this project is to understand the context, Jen's Choice, where the participants of the research study were working with problem solving. Therefore, there is a need to introduce at the beginning of this section about Jen's Choice.

Jen's Choice

Jen's Choice is a Chinese-language computer-anchored 4th-grade mathematics instruction program, designed and developed by Dr. Hsin-Yih Shyu (1997) and associates at Tamkang University in Taiwan. Based on the notions of anchored instruction and situated cognition (CGTV, 1990), it provides an inquiry, and real life learning environment and authentic tasks for learner to solve mathematics and other cross-subject problems. The story describes that Jen, a fifth grader was accused by her classmates of stealing a watch from the class model student, because she showed her watch after the model student lost her watch. In order to find the giver of her watch, Jen's grandmother, to approve that the watch was really given by her grandmother, Jen, accompanied by her three friends, planned and took a trip to visit her grandmother. In order to do so, they had to encounter several plans and decisions, such as time planning, expense planning, among others, and these need them to apply 4th-grade mathematics and across-subject knowledge to solve problems and plans.

At the end of the story, two types of problems, one type is about life problem, the other is about mathematics problems, are provided to challenge the learner who view this program. In this research study, only mathematics problems are given to the participants to solve. The mathematics problems contain the following three mathematics problems:

Question #1: Up to now ("now" meaning before they buy tickets for the rides), how much money does each person have left?

Question #2: At the Children's Recreation Center, if everyone wants to ride the same rides, then how should they spend the remaining money on tickets? (Note that the cheapest ride at the recreation center costs three tickets.)

Question #3: Supposing that Jen's apartment is 15 minutes away from the Tamsui Station bus stop, and taking into consideration that she has to be home before 5:30 p.m., answer the following questions.

a) What is the longest possible time they can stay at the Children's Recreation Center?
b) What is the latest possible time they can leave the Children's Recreation Center? (supposing that it will take them 10 minutes to wait for a bus in front of the recreation center)
c) Which show can they see at Tomorrow's World Theater? 1:00 p.m., 2:00 p.m., or 4:00 p.m.

Because the nature of these three mathematics problems: authentic tasks, complex situations, real-life, therefore, it makes meaningful that Jen's Choice was used as instruments for test problem solving.

Procedures

The research experiment consisted of three steps. In step 1, participants, working individually on PC computer in a separated decent room, solved mathematics problems. First, each of the participants was shown ***Jen's Choice***. After that, three mathematics problems from this program were given to each individual participant to solve using paper and pencil. The participants were asked to show their mathematical problem-solving procedures, and also to engage in non-synchronous thinking aloud (Yang, 1994), an adaptation of thinking-aloud method (Ericsson & Simon, 1985) Thinking aloud is a research method that involves asking each participant in an experiment to speak out loudly, explaining as fully as possible, for every process he/she performs, how, why, and what he/she is thinking and doing. This is a vital method for investigating a participant's thinking processes. However, since the participant has to think aloud at the same time that he/she is doing a task, this procedure could interfere with the participant's thinking and acting. Therefore, the method of non-synchronous thinking aloud, was employed: participants were requested to 'think aloud', not while performing, but a little after completion of each segment of the task, in order not to interrupt the participant's thinking and doing the task. There was a facilitator with each of the participants to encourage non-synchronous thinking aloud and to remind the participants not to forget the non-synchronous thinking aloud.

In step 2 participants undertook group collaborative problem solving. One hour after the completion of step 1, the four participants were gathered together in a decent room having a PC computer, to solve, collaboratively, the same three problems they had solved individually in step 1. And as above, they were asked to engage in non-synchronous thinking aloud while, working as a group, they worked out their collaborative approaches to the problems with paper and pencil.

In step 3 participants again undertook individual problem solving, on a PC computer in a separated decent room. One hour after the completion of step 2, each of the four students was asked to solve, individually, the identical three problems again. All the procedures in step 3 were identical to those in step 1.

All three steps of the experiment were video recorded and audio taped. Cordless microphones were distributed to each participant to ensure that each participant's voice was clearly recorded and clearly heard by the others. These three groups of written solutions and video-recorded non-synchronous thinking-aloud protocols were compared and analyzed for quantitative and qualitative differences. Quantitative comparisons were made by evaluating the degree of correctness of the different final answers given to each of the three questions during each of the three steps. For qualitative comparison, these protocols were analyzed in terms of the following five components (Yang, 1994):

1. Problem perception, understanding and representation: evaluates the problem solver's understanding of what the problem is, and what the situation of the problem is; evaluates the problem solver's ability to draw a figure to introduce situational notation.
2. Mathematics knowledge and concepts: evaluates the participant's ability to access and apply the mathematical knowledge and concepts that are relevant to the problem.
3. Devising and carry out the plan: evaluates the problem solver's ability to devise a plan based on the problem, and to implement the plan step by step.
4. Consciousness and control of the plan: evaluates the problem solver's ability to consciously control what he/she is doing—to judge if it is meaningful or reasonable, and to adjust it when appropriate.
5. Looking back: evaluates the problem solver's ability to check the process of what he/she has done, identifying and correcting any mistakes or inappropriate solutions.

Results and Discussion

Figure 1 shows a summary of the total number of right answers for each of the three steps of problem solving.

Participant / Step	H	MH	M	L
	Total no. of right answers		(Total no. of questions = 3)	
Step 1 (individual p-s)	0	0	0	0
Step 2 (collaborative p-s)	¼			
Step 3 (individual p-s)	0	0	0	0

Fig. 1. Number of right answers achieved by the participants in each of the three steps of the experiment

It seems that the results for collaborative problem solving were not significantly better than those for individual problem solving. Furthermore, collaborative problem-solving experience did not seem to facilitate later individual problem solving. But these conclusions, representing strictly quantitative results, are inherently limited in scope; they reveal nothing about the actual processes through which the participants constructed their understanding. At this point the learning process remains a 'black box.' An additional, qualitative, analysis is required in order to identify and evaluate the changes in thinking that took place over the three steps of problem-solving activity. Because the space limitations in this paper, the findings of this additional analysis are only briefly summarized here (Figures 2 to 5):

Participant: H									
Question #	#1			#2			#3		
Step/Compo.	step 1	Step 2	Step 3	step 1	Step 2	step 3	step 1	step 2	Step 3
Problem Perception...		x							
Math knowledge...	x	x	x	x	x	x	X	x	x
Devising... the plan		x							
Conscious... the plan		x			x			x	
Looking back...									

Fig. 2. Components of problem solving that participant H performs

Participant: MH									
Question #	#1			#2			#3		
Step/Compo.	step 1	Step 2	Step 3	step 1	step 2	step 3	step 1	step 2	Step 3
Problem Perception…		x							
Math knowledge…	x	x	x	x	x	x	x	x	x
Devising… the plan		x							
Conscious… the plan		x			x			x	
Looking back…									

Fig. 3. Components of problem solving that participant MH performs

Participant: M									
Question #	#1			#2			#3		
Step/Compo.	step 1	Step 2	step 3	step 1	step 2	step 3	step 1	step 2	Step 3
Problem Perception…		x							
Math knowledge…	x	x	x	x	x	x	x	x	x
Devising… The plan									
Conscious… The plan									
Looking back…									

Fig. 4. Components of problem solving that participant M performs

Participant: L									
Question #	#1			#2			#3		
Step/Compo.	step 1	Step 2	Step 3	step 1	step 2	step 3	step 1	step 2	Step 3
Problem Perception…									
Math knowledge…	x	x	x	x	x	x	x	x	x
Devising… The plan									
Conscious… The plan									
Looking back…									

Fig. 5. Components of problem solving that participant L performs

One of the reasons that all of the participants had very low achievement in solving the three problems was that they did not fully understand the problems when individually solving these problems. For example, when Question #1, "Up to now (meaning before they bought tickets for the rides), how much money does each person have left?" can serve as an example here. In the first step, the participants misunderstood it as meaning either 'how much money did each of them have before they started the journey?' or 'how much total money did they, collectively, have left before paying for the rides?' Accordingly, they devised wrong plans, carried out wrong plans, and arrived at wrong things.

Doubtless the relatively high degree of complexity of the problems contributed to these frequent misunderstandings. Traditional educational approaches had accustomed the participants to less complex problems, for which all they needed was simply to put all of the numbers together without thinking too deeply. Thus, when they met with this experiment's much more complex problems, they were totally lost and confused, and tended to revert to just putting the numbers together. In such complex and challenging situations, collaboration does not seem to help, as can be seen in the results from the second and third problems. Even though the participants came to the experiment with considerable prior experience in collaborative problem solving and related mathematics knowledge and concepts needed for solving problems, collaboration at this time did not facilitate successful solutions.

Also, deficiencies in the categories of consciousness and looking back resulted most participants reaching wrong answers for almost all of the problems, even though they performed reasonable problem-solving procedures.

However, collaboration did facilitate control and monitoring of the collaborative problem-solving process. From the protocols, we were able to see that solutions and plans were revised due to the reminders from peer participants: when some peers were unable to find the errors, others, having found them, reminded those peers to be cautious and to rethink; the resultant back-and-forth arguments often resulted in reasonable solutions. This striking collaborative phenomenon may represent the most essential aspect of the collaborative function. As Forman (1981) said, "Both Vygotsky and Piaget would agree that social interaction provides the individual with feedback about his own thoughts and action which enables him to reflect upon and modify his behavior" (p. 2). Thus, The potential effectiveness of collaborative control in the problem-solving process is evident, for example, in the results from problem #1. Here, none of the participants were able, individually, before or after collaboration, to solve the problem correctly, or even to find the correct data to on which to do the calculation; but in collaboration, they were able to identify almost all the data needed to do the calculation.

Although the above findings do not support the conclusion that collaboration significantly promotes later individual problem-solving ability, the qualitative comparison data show that collaboration does have an effect on collaborative problem-solving strategies.

Conclusions

The aim of this research effort was to develop a deeper understanding of how collaboration affect individual mathematical problem-solving ability. In this research, we investigated how individual and collaborative mathematical problem solving took place in instructional contexts when anchored in authentic problem situations. The results of the research help us to see the different ways in which individual and collaborative problem solving approaches contributed to the participants' problem-solving strategies. These findings remain tentative, due to the participants' unfamiliarity with the style of the problem items (in particular, their high complexity), the small sample size, and the restricted components of observation. Further research designed to circumvent these limitations should achieve more complete and definitive results.

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