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ABSTRACT

Beginning in 1999, all of the Pennsylvania System of School Assessment (PSSA) had to be aligned with the Pennsylvania Academic Standards. This handbook describes the PSSA mathematics assessment. Following an overview of the PSSA, the second chapter focuses on the development and scoring of the mathematics assessment. Administering the assessment is described, with explanations of the permitted uses of calculators, rulers, and other tools in the assessment. A fourth chapter discusses the focus of the mathematics assessment and describes the reporting of school and student scores, setting performance standards for the assessment, and the use of scoring rubrics versus performance standards. The next three chapters discuss preparing for the assessment of grades 5, 8, and 11 respectively. Sample items, scoring guides, and sample student responses are presented for each grade. An appendix lists members of the Mathematics Assessment Advisory Committee. (SLD)

The Pennsylvania System of School Assessment

Mathematics Assessment Handbook

Pennsylvania Department of Education
Division of Evaluation and Reports

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INTRODUCTION

The *Pennsylvania System of School Assessment (PSSA)* assesses students to provide information about their achievement and that of schools and districts. In the 1998-1999 school year, the system began assessing the Pennsylvania Academic Standards for the first time.

This handbook focuses upon the 2000-2001 assessment. Assessment handbooks are also available for reading and writing. Copies of these documents can be obtained by contacting the Division of Evaluation and Reports, PDE, 333 Market Street, Harrisburg, PA 17126-0333. Past handbooks contained an appendix entitled "Testing Accommodations to Encourage Participation by Students with Disabilities in the Pennsylvania System of School Assessment." This information can now be found through accessing the Department of Education Web site: http://www.pde.psu.edu/.

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OVERVIEW OF THE PENNSYLVANIA SYSTEM OF SCHOOL ASSESSMENT

On October 21, 1998, the State Board of Education adopted final-form regulations for the new Chapter 4 of the Pennsylvania School Code. Upon conclusion of the regulatory review process, it was published in the January 16, 1999, Pennsylvania Bulletin as final rulemaking, binding on all public schools in the Commonwealth.

The new Chapter 4 replaced the previously adopted Chapters 3 and 5 and provided a new direction for the Pennsylvania System of School Assessment (PSSA). Beginning with the 1998-99 assessment, the PSSA became standards-based. This means that, beginning with the February/March 1999 testing, all of the PSSA had to be aligned with the Pennsylvania Academic Standards. The movement to standards was adopted as a major focus of the Chapter 4 regulations, and those for Mathematics and for Reading, Writing, Speaking and Listening were included as an Appendix to Chapter 4. These Academic Standards have been widely distributed and can be found on the Pennsylvania Department of Education Web site: http://www.pde.psu.edu/.

As outlined in Chapter 4, the purposes of the statewide assessment component of the PSSA are now the following:

1. Provide students, parents, educators and citizens with an understanding of student and school performance.

2. Determine the degree to which school programs enable students to attain proficiency of academic standards.

3. Provide results to school districts (including charter schools) and AVTSs for consideration in the development of strategic plans.

4. Provide information to state policymakers including the General Assembly and the Board on how effective schools are in promoting and demonstrating student proficiency of academic standards.

5. Provide information to the general public on school performance.

6. Provide results to school districts (including charter schools) and AVTSs based upon the aggregate performance of all students, for students with an Individualized Education Program (IEP) and for those without an IEP.

Although mathematics and reading standards were adopted at the grade 3, 5, 8, and 11 levels, the 1998-99 and 1999-2000 assessments for these content areas were conducted only at grades 5, 8, and 11, as specified in Chapter 4. The 1998-99 writing assessment was conducted exactly as in past years, with students at grades 6 and 9 writing essays, and scores produced only at the school level. During the 1998-99 school year a major writing assessment pilot study was carried out. The results of this study have guided the development of a new model for writing assessment to be implemented in the 2000-2001 school year. This implementation began in October of 2000 with the assessment of grade 6 and 9 students using two writing prompts. Individual student scores will be generated for the first time at these grade levels. The first grade 11 writing assessment will take place later in the 2000-2001 school year. As with grades 6 and 9, individual student scores will be produced. At this point in time, some of the details of the grade 11 model have not been worked out.

As was the case in the 1999-2000 school year, the 2000-2001 reading and mathematics assessments will occur in the spring. They each use a combination of census testing and matrix sampling procedures. Census testing requires all students to complete the same set of multiple-choice items and open-ended questions. Matrix sampling is accomplished by dividing a large set of items, both multiple-choice and open-ended, into several different test forms with an equal number of items on all forms. Matrix sampling helps to limit the time required for the assessment, provides for consistent administration procedures and reflects broad curriculum content.

As part of each test form in reading and mathematics, students are asked to respond to multiple-choice items. They also are asked to respond *in writing* to questions about reading passages and to explain *in writing* how they arrived at answers to mathematics problems. These open-ended questions, or *performance assessments*, are included so students can explain what they are thinking and doing.

DEVELOPMENT AND SCORING OF THE MATHEMATICS ASSESSMENT

ADVISORY COMMITTEE

Advisory Committees of Pennsylvania educators play a major role in the development of all PSSA assessment measures. In the area of mathematics, Division of Evaluation and Reports personnel have been greatly assisted in the development of assessment measures by a group of mathematics teachers, supervisors and other educators who constitute the Mathematics Assessment Advisory Committee. This committee has written virtually all of the multiple-choice and open-ended tasks included in the assessment. Committee work has also included development of the General Problem Solving Rubric and scoring guides for the open-ended tasks, providing assistance in categorizing items within the assessment framework and reviewing items for bias and technical adequacy. More than 200 persons from throughout the state have been involved in developing the assessment measures described in this document. A list of the present members of the Mathematics Assessment Advisory Committee is in Appendix A.

CONTENT FRAMEWORK

As a function of the adoption of Chapter 4, the content included in the 1998-99 PSSA and in all those occurring after this had to be based upon the framework specified by the Pennsylvania Academic Standards for Mathematics. These are stored in their entirety on the Web site previously listed. The entire listing of content for grades 3, 5, 8 and 11 is 18 pages long, with an additional 10-page glossary of terms. Shown on page 5 is a summary of the 11 content categories included in the standards and some descriptive information about each. It was taken from the table of contents preceding the standards.

ASSESSING THE STANDARDS CONTENT

The standards statements for Content Category 2.1, Numbers, Number Systems and Number Relationships, are shown in Figure 1. These were printed to be illustrative of all the standards statements in order to describe how the standards are being assessed.

It can be seen that the standards are printed as a cumulative list across grade levels. Students at the grade 11 level are responsible for knowing all of the content shown at all grade levels, those at the grade 8 level are responsible for knowing the content listed for grades 8, 5, and 3, etc. The assessment for a particular grade level, then, includes content listed at that grade level and may include content for the grade levels below it. No content for the grade level(s) above an assessed level will appear on that grade level's assessment.

In developing PSSA assessments aligned with the standards, Mathematics Assessment Advisory Committee members are directed to write items to assess student achievement of each standards statement (i.e., the statements listed as A, B, C, etc.), for the grade level being assessed and in some cases for statements that appear at a grade level(s) below this. It should be understood that, for example, grade 11 committee members who write items to assess a standards statement that appears only at the grade 8 level are assessing the grade 11 representation of this content, rather than that which is appropriate for grade 8 students.

A quick scanning of the standards statements leads to the accurate conclusion that they were written for the classroom rather than in a format readily conducive to state-level assessment. Verbs such as "explain," "demonstrate," "construct" and "describe" appear throughout the statements. Students'

abilities to carry out the great majority of these actions can be assessed in a paper and pencil format but only through the use of open-ended items. Since the number of such items that can be included is limited by the time they take to administer and score, only a few can be used at any one time. In order to assess as many statements as possible, the approach taken is to change the verb to one which is capable of being assessed with multiple-choice questions. In such multiple-choice questions, students are asked to "choose the correct explanation" rather than to "explain" and to "determine which is a correct demonstration" rather than to "demonstrate." Using this approach, the only statements that are not assessed are those few that involve the use of equipment. An example of a statement that is not assessed through the PSSA is 2.2.11.F, "Demonstrate skills for using computer spreadsheets and scientific and graphing calculators."

Content Categories of the Academic Standards for Mathematics

2.1 Numbers, Number Systems and Number Relationships
- Types of numbers (e.g., whole, prime, irrational, complex)
- Equivalent forms (e.g., fractions, decimals, percents)

2.2 Computation and Estimation
- Basic functions (+, -, x, ÷)
- Reasonableness of answers
- Calculators

2.3 Measurement and Estimation
- Types of measurement (e.g., length, time)
- Units and tools of measurement
- Computing and comparing measurements

2.4 Mathematical Reasoning and Connections
- Using inductive and deductive reasoning
- Validating arguments (e.g., if. . . then statements, proofs)

2.5 Mathematical Problem Solving and Communication
- Problem solving strategies
- Representing problems in various ways
- Interpreting results

2.6 Statistics and Data Analysis
- Collecting and reporting data (e.g., charts, graphs)
- Analyzing data

2.7 Probability and Predictions
- Validity of data
- Calculating probability to make predictions

2.8 Algebra and Functions
- Equations
- Patterns and functions

2.9 Geometry
- Shapes and their properties
- Using geometric principles to solve problems

2.10 Trigonometry
- Right angles
- Measuring and computing with triangles
- Using graphing calculators

2.11 Concepts of Calculus
- Comparing quantities and values
- Graphing rates of change
- Continuing patterns infinitely

Figure 1
2.1 Mathematics Standards

2.1 Numbers, Number Systems and Number Relationships			
2.1.3. GRADE 3	2.1.5. GRADE 5	2.1.8. GRADE 8	2.1.11. GRADE 11
Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills needed to:			
A. Count using whole numbers (to 10,000) and by 2's, 3's, 5's, 10's, 25's and 100's. B. Use whole numbers and fractions to represent quantities. C. Represent equivalent forms of the same number through the use of concrete objects, drawings, word names and symbols. D. Use drawings, diagrams or models to show the concept of fraction as part of a whole. E. Count, compare and make change using a collection of coins and one-dollar bills. F. Apply number patterns (even and odd) and compare values of numbers on the hundred board. G. Use concrete objects to count, order and group. H. Demonstrate an understanding of one-to-one correspondence. I. Apply place-value concepts and numeration to counting, ordering and grouping. J. Estimate, approximate, round or use exact numbers as appropriate. K. Describe the inverse relationship between addition and subtraction. L. Demonstrate knowledge of basic facts in four basic operations.	A. Use expanded notation to represent whole numbers or decimals. B. Apply number theory concepts to rename a number quantity (e.g., six, 6, 12/2, 3 x 2, 10-4). C. Demonstrate that mathematical operations can represent a variety of problem situations. D. Use models to represent fractions and decimals. E. Explain the concepts of prime and composite numbers. F. Use simple concepts of negative numbers (e.g., on a number line, in counting, in temperature). G. Develop and apply number theory concepts (e.g., primes, factors, multiples, composites) to represent numbers in various ways.	A. Represent and use numbers in equivalent forms (e.g., integers, fractions, decimals, percents, exponents, scientific notation, square roots). B. Simplify numerical expressions involving exponents and scientific notation and using order of operations. C. Distinguish between and order rational and irrational numbers. D. Apply ratio and proportion to mathematical problem situations involving distance, rate, time and similar triangles. E. Simplify and expand algebraic expressions using exponential forms. F. Use the number line model to demonstrate integers and their applications. G. Use the inverse relationships between addition, subtraction, multiplication, division, exponentiation and root extraction to determine unknown quantities in equations.	A. Use operations (e.g., opposite, reciprocal, absolute value, raising to a power, finding roots, finding logarithms).

TYPES OF ASSESSMENT MEASURES

The PSSA mathematics assessment employs two types of test items; multiple-choice and open-ended. These provide differing types of information about mathematics achievement.

Multiple-Choice Items

In years past, all items included in large-scale assessment programs such as Pennsylvania's Testing for Essential Learning and Literacy Skills (TELLS) were multiple-choice. Multiple-choice questions are also termed "selected-response" since students choose their answers from among those provided. Such items are an efficient means of assessing a broad range of curriculum. They are relatively easy to develop and can be administered in a short amount of time.

In the PSSA mathematics assessments, all multiple-choice items have only one correct response choice, and the student is awarded one point for choosing it. For all items, students are provided with four choices.

Multiple-choice questions can be used to assess a variety of skill levels, from short-term recall of specific facts or terminologies to problem solving. The great majority of the PSSA multiple-choice mathematics questions require students to carry out some process to find their answers, rather than simply recalling information from memory.

Open-Ended Tasks

Open-ended tasks were first introduced into Pennsylvania's mathematics assessments in 1993. These tasks require students to read a problem or task description and to write out their answers. Major components of such answers are both students' clear presentations of their computations and their explanations of the steps they followed in solving the problem. The types of tasks utilized do not always require solving a problem involving computations. Students may also be asked to perform such tasks as drawing a graph, shading some portion of a figure or listing object combinations that meet specified criteria.

For a number of years, the PSSA has used the term "open-ended" to apply to any mathematics question for which students provide a written solution. It is recognized that some of the questions being used would be called "constructed-response" rather than "open-ended" by some persons since they converge on only one or two answers rather than permit students to provide their own solutions from a multitude possible.

Open-ended tasks are especially useful for measuring students' problem-solving skills in mathematics. In most of these tasks students are asked to show how they arrived at an answer or what thinking was behind the answer they gave. The tasks present real-life situations that require students to solve a problem using math abilities learned in the classroom. There are no response choices on the page as with multiple-choice items. Students must read the task carefully, select the necessary information, devise a method of solution, perform the calculations, enter the response directly on the page on which the task is presented and explain the procedure to support the given response. This provides insight into the students' knowledge, mathematical abilities and reasoning processes. Samples of open-ended tasks, scoring guides and student responses are provided in later portions of this document.

THE SCORING OF OPEN-ENDED TASKS

PSSA open-ended mathematics tasks are scored in terms of a continuum of correctness. This continuum is defined by a "rubric," which outlines what the general requirements are at each scale point. Then, scoring guidelines based upon the general rubrics are developed to score individual tasks. The rubric approach to scoring has made it possible to include open-ended tasks in large-scale assessments, such as the PSSA. Large numbers of student papers can be scored in a highly reliable way by persons who have received training in applying the rubric.

PSSA open-ended mathematics tasks are of a variety of types and vary widely in complexity across grade levels. Some items require students to solve a problem with one correct answer; others permit a variety of correct answers. Some include only one part; others include a number of parts with correct answers necessary for each part. For most items students can only attain the highest score if they provide documentation of their work and an explanation that clearly indicates why they performed each step of their solution process. For some items, however, students are required to carry out non-numerical procedures, such as the drawing of a graph, with no explanation required.

Because of the diversity of item types it is difficult to provide a general scoring rubric for all PSSA items. In an attempt to communicate the overall approach to scoring being used, the General Description of Mathematics Scale Points (Figure 2) was developed. It is not a rubric but only a communication device to help think about the scoring of the diverse types of PSSA items.

Figure 3 provides a general rubric for scoring an open-ended task that requires students to solve a problem and produce a numerical answer. It gives a very good idea of what the six categories employed in scoring all such tasks entail, but in all cases a specific rubric, or scoring guide, must be developed which describes in detail what is expected for each category. All such scoring guides employ the same six categories. What differs is how these are defined. It should be emphasized that, although scoring guides do differ across tasks, the same overall philosophy prevails in the determining of student scores. To receive the highest score for questions which involve computations and the determining of one or more correct answers, students must show all of the steps they carried out to solve the problem and **must explain why they performed each of these steps**. Students who have not been asked to do this as part of their mathematics program will find it difficult to attempt this for the first time in responding to the PSSA. It is strongly recommended that, in preparing students for the PSSA, teachers use some open-ended questioning as a part of their classroom sessions and give students practice in both showing their work and explaining why each step was taken.

Figure 2
General Description of Mathematics Scale Points

5 - Advanced Understanding, Excellent

- Student provides a response that is exemplary. All numerical answers are correctly provided with appropriate labels and all necessary work is shown or described. All required procedures leading to non-numerical answers (e.g., drawing a graph, drawing a geometric figure that satisfies a list of designated criteria) are carried out in an excellent manner. If an explanation is appropriate, it demonstrates that the student has an exemplary knowledge of the concepts being assessed. It includes a clear explanation of why each step of the solution process was undertaken.

4 - Satisfactory Understanding

- Student provides a response that shows a knowledge of the concepts being assessed. In general, the student has met the requirements of the problem. Numerical answer(s) are correct, with work shown. Required procedures leading to non-numerical answers are adequate but not exemplary. If an explanation is appropriate, it demonstrates that the student has knowledge of the concepts of the problem.

3 - Almost Satisfactory Understanding

- Student provides a response that shows a general knowledge of what is being asked, but has some minor errors. The student may have made one small mistake, such as a calculation error leading to an incorrect numerical answer or a failure to correctly label answers. Most of the required procedures leading to non-numerical answers have been adequately carried out; a small number may have been left out or carried out incorrectly. If an explanation is appropriate, it demonstrates a general knowledge of the concepts being assessed, but is incomplete.

2 - Partial Understanding

- Student provides a response that shows some knowledge of the concepts of the problem, but this knowledge is limited. If a number of numerical answers are called for, the student may answer some parts correctly, but may not even attempt to answer other parts, or may do so in an inadequate way. If the problem calls for only one answer, this may be provided but with partial support. Only some of the required procedures leading to non-numerical answers are carried out correctly; others are not completed at all or are carried out incorrectly. If an explanation is appropriate, it may have been omitted completely or is adequate for only a portion of the problem.

1 - Minimal Understanding

- Student provides a response that shows little knowledge of the concepts of the problem. If a number of numerical answers are required, one may be correctly provided and this one with no or a minimal explanation. If the problem calls for only one answer, this may be provided with no support, or only work is shown that at best includes one correct procedure. Required procedures leading to non-numerical answers are generally incorrect, with only a small portion correct. If an explanation is appropriate, none is provided or that which has been written is completely inadequate.

0 - Incorrect

- Student cannot provide any correct answers or show evidence of having made any steps to solve the problem through correct procedures. Blank responses and Off-Task responses (e.g., profanity, unrelated drawings or comments) are scored as Incorrect responses. In addition, question marks and "I don't know" are scored as Incorrect since they indicate that the student has read the task and has responded to it.

Figure 3
Mathematics General Problem Solving Rubric

5 - Advanced Understanding, Excellent

- Correct answer with correct procedures/correct calculations shown or described and a written explanation that supports the work shown. The explanation tells what was done in the solution process and explains **why** the steps were done (or the reason(s) for the steps to be taken). No blemishes, that is, everything is correct. May have a minor omission in calculation or explanation where the omitted step or explanation may be of the level of 2 + 2 = 4 (something that is usually done mentally and considered trivial and understood).

4 - Satisfactory Understanding

- Correct answer with correct procedures/correct calculations shown or described and a written explanation which supports some of the work shown. May have minor omission in calculation or explanation (such as 2 + 2 = 4).

3 - Almost Satisfactory Understanding

- Correct answer with most correct procedures/calculations shown or described and no explanation. Some steps are missing, but you can follow what is being done.
- Correct answer with few correct procedures/calculations shown or described and some explanation. Some steps are missing, but you can follow what is being done.
- Incorrect answer with correct procedures shown or described and some explanation, but with one calculation or copying error carried through.

2 - Partial Understanding

- Correct answer with few procedures/calculations shown or described or some explanation. Too many steps are missing to follow what is being done.
- Incorrect answer with half or more correct procedures shown or described and some or no explanation. The student either did not proceed far enough or proceeded incorrectly.
- Incorrect answer with correct procedures shown or described and no explanation. May have no more than 2 calculation or copying errors.

1 - Minimal Understanding

- Correct answer with calculations, procedures or explanation that are either not legible or not understandable or missing or the procedure is incorrect. Less than a "2" score.
- No answer or an incorrect answer, but the student has provided some of the information critical to the solution. There is some indication that the student has read the item.

0 - Incorrect

- Incorrect answer in which the student attempts the task incorrectly or gives an incorrect or incomplete answer with an incorrect explanation or no explanation of the procedure or logic used in the solution. Nothing is correct.
- Blank responses and Off-Task responses (profanity, refusal to perform, unrelated drawings or comments such as "doodles") are scored as "Incorrect responses."
- Question marks and "I don't know" are scored as "Incorrect responses." The student has read the task and responded to it.
- No answer is treated as an incorrect answer.

A Category to Identify Exemplary Responses

In the 1994 and 1995 assessments, the rubric which was used for scoring tasks which were relatively complex in nature included five categories. The top category of this rubric, Satisfactory Understanding, did not differentiate student responses into degrees of quality. Rather, any student who presented an adequate solution accompanied by some explanation received the top score. While some of the student papers stood out as excellent, there was no way to reward this excellence. Students who met the basic requirements for a Satisfactory score and those who went well beyond these received the same score.

In 1996 a new category was added to the rubric, termed "Advanced Understanding." The intent of this category was to enable scorers to differentiate between papers which were merely satisfactory and those which were exemplary.

The use of this category in scoring papers from the 1996 assessment did partially accomplish the goal of locating student responses which were better than just Satisfactory. However, it became clear from this attempt that only for certain tasks was it possible for students to demonstrate a level of response which would be considered "Advanced" for students of their grade level.

Beginning in 1997, the highest category included "Excellent" in its title, with a definition which made it possible for student papers which stood out as the best of those produced to receive this top score. It then became possible to award scores of "5" for virtually all tasks of the assessment. Students who excel in responding to a particular task now receive a score of "5"; those who generally have accomplished the requirements of a task, but have done nothing exemplary either in their work or their explanation, receive a score of "4."

Student Explanations of Their Answers

The following example was developed by Mary Stover from Ephrata Area School District, a member of the Grade 5 Mathematics Assessment Advisory Committee. In this example, she has very effectively provided an annotated illustration of student responses that would receive scale points 1 through 5. In addition to these scale points, students who provided an entirely incorrect response would receive a "0" score and those whose papers were blank or who wrote an off-task response would be scored accordingly.

The task that is shown is less complex than the great majority of tasks for which the 6-point rubric would be used at any of the three grade levels. However, using a relatively simple task made it possible to focus upon the features of student responses rather than the complexities of mathematical processes.

Sample Problem:

Ed went to the store for school supplies. He bought a notebook, 6 pencils, a pack of notebook paper, 2 erasers and a pen. If he paid with a $10 bill, how much change should Ed have received? Show each step of your math work **and** write an explanation that indicates **why** you did each step.

PRICE LIST (includes tax)					
Eraser	$ .25	Notebook	$4.95	Dividers	$1.25
Pencils	2 for .30	Notebook Paper	1.50	Index Cards	.65
Markers	1.25	Assignment Pad	.75	Pen	.95

Correct answer: $1.20 change (spent $8.80)

Mary's Answer:

$4.95 notebook
.90 pencils
1.50 notebook paper
.25 eraser
.25 eraser
.95 pen
$8.80

$10.00
- 8.80
$ 1.20

First I added the prices of everything he bought to find out how much he spent. The pencils were 2 for 30¢ and he got 6, (6÷2=3) so he had to pay the 30¢ price 3 times (90¢). Total spent→$8.80. Then I subtracted the $8.80 from $10 to get how much money he had left (change).

Answer: $1.20

Credit: Advanced Understanding, Excellent (5) - A correct answer is given with computations to support it **and** she has written a complete explanation **telling what she did and why.** There is no doubt that she understands the procedures required to complete the problem correctly. She receives maximum credit because she has **correct computations and a thorough explanation.**

Jerry's Answer:

I added them all up and then subtracted to get the answer.

$4.95
.90
1.50
.95
$8.80

$10.00
- 8.80
$1.20

Answer: $1.20

Credit: Satisfactory Understanding (4) - A correct answer is given and computations are shown to support the answer. A written explanation is given; however, it is not very thorough. He seems to know the correct procedures to solve the problem, but he **has not** explained them very well nor did he explain why he did what he did.

Kim's Answer:

Subtract to get Ed's change.

$10.00
8.80
$1.20

Answer: ↓

Credit: Almost Satisfactory Understanding (3) - A correct answer is given, but her explanation is incomplete. She does not explain how she got the $8.80. She seems to know the correct procedures to solve the problem, but **has not** shown or explained all of the steps of her work or any of her reasoning.

Dan's Answer:

25¢ + 30¢ + 30¢ + $4.95 + $1.50 + 95¢ + 25¢ + 30¢ =
$8.80

Answer: $1.20

Credit: Partial Understanding (2) - A correct answer is given, but there is not enough computation work or a written explanation to help understand what he did or why. He receives **less credit because he has no written explanation**.

Tom's Answer:

$4.95
.30
1.50
.25
.95
$7.95

added

Answer: $7.95

Credit: Minimal Understanding (1) - An incorrect answer and an incomplete explanation are given. However, the work he has done shows that he knows what the first step of the solution should be. He is given credit only for attempting to carry out one correct procedure.

The reporting of results from the 2000 assessment for the first time included an indication of how well individual students performed on each of the 11 standards categories. Assessing students' performance in this many areas with at least a minimally acceptable level of reliability required some increase in testing time over that which occurred in the 1999 assessment.

The mathematics assessment for 2000 consisted of four testing sessions. In each of three of these sessions, between 22 and 24 multiple-choice items and one open-ended item were administered. These sessions made up the common portion of the assessment, with all students responding to the same items. In the fourth session, 15 multiple-choice items and one open-ended item were administered using a matrix sampling approach. Each student responded to one of 10 unique forms. This made it possible to produce reliable results for each of the 11 standards categories for schools. This is true since all of the items for a standards category across all matrix forms and the common form entered into the school-level results for the standards category.

Each testing session was estimated to take approximately one hour. However, a survey of about 100 school districts made it clear that the amount of testing time needed for a session varied from about 30 minutes up to 90 minutes or more, depending greatly upon students' reading levels.

The 2001 assessment will be of the same size as that which took place in 2000. In an effort to increase the validity of the test as a measure of mathematics, a concerted effort is being made to simplify both the wording and the format of each item. As an example of this, if the distracters for an item are numerical, wherever possible they will be listed in order from smallest to largest. The simplification process should have the added benefit of decreasing testing time, especially in schools where students are having some reading difficulties.

THE TWO WAYS OF ASSESSING STANDARDS CATEGORY 2.2

For a number of years, students have been permitted to use calculators in responding to the PSSA mathematics assessment. However, beginning with the 1999 testing, a section was introduced for which students were not permitted to use calculators. For the remainder of the common portion and for the matrix-sampled section, calculators were permitted. In the 2001 assessment the non-calculator section will consist of 7 items at grades 5 and 8 and 5 items at grade 11.

The inclusion of the non-calculator component has made it possible to examine and compare students' computation and estimation skills under two conditions. In the non-calculator component, students are asked to solve problems which are very straightforward. The focus is upon carrying out computations or estimations either within a context or without one, rather than upon attempting to solve a complex problem in which success depends upon choosing the appropriate procedures to follow. In the calculator component, the calculator is used as a tool in solving more complex problems. The student must first make decisions about which procedures are appropriate to the solving of the problem and then must carry out these procedures to arrive at an answer. Under both conditions the items to be used must reflect the description of content for Standards Category 2.2 at each level. Thus, for example, at the grade 5 level, some items have involved whole number and decimal computations and others have involved estimations.

CALCULATOR USAGE IN THE ASSESSMENT

Except for the items in the non-calculator component, students are permitted to use calculators throughout the assessment. A major reason for this is that the majority of the items and tasks have been developed with a strong emphasis upon problem solving in applied settings. For such tasks, the calculator truly is a tool. The calculations required are typically not ones which involve large numbers. Rather, what is important is the ability to make proper decisions about which operation to carry out. The calculator is of use in carrying out these decisions but is of secondary importance in solving such problems. All tasks can be solved without the use of a calculator. However, certain grade 11 tasks are much more difficult if a calculator is not available.

The type of calculator used should be appropriate to the grade level of the student involved in the testing. There is no need for a graphing calculator for a grade 5 student, but a graphing calculator is helpful to a grade 11 student. At the grade 8 level, scientific or fraction calculators have been shown to be helpful. Regardless of the grade level, a simple four-function calculator is sufficient for use in the assessment.

Students who will be participating in the Pennsylvania Mathematics Assessment should be prepared to use their calculators not as some special application but as part of their typical use of calculators in their mathematics program. A calculator which has been given to a student on the day of the assessment may actually hinder the student's performance. Practice in using a calculator is necessary. A student must be knowledgeable about how to use a calculator or it will be of little benefit to him or her in completing the mathematics tasks.

THE USE OF RULERS IN THE ASSESSMENT

In the 2000 assessment, for the first time, grade 5 students were required to use rulers to measure figures presented in the test. This was in order to directly address Standards Statement B for grade 5 under Standards Category 2.3, Measurement and Estimation. This statement reads as follows: "Select and use standard tools to measure the size of figures with specified accuracy, including length, width, perimeter and area." Packs of small rulers, scaled in both eighths of an inch and centimeters, were given to school districts for their students to use. This same approach will be used in the 2001 testing.

THE USE OF OTHER TOOLS IN THE ASSESSMENT

Students are not permitted to use mathematics books, dictionaries, or reference materials of any kind when they are responding to PSSA mathematics items. If such materials were allowed, students could use them, for example, to provide a correct answer to certain questions by looking up a definition (e.g., parallelogram) and then applying it. In addition, students may not use materials such as manipulatives that would give them an unfair advantage over students who do not have such materials available during the PSSA testing.

Students may, however, use colored pencils, hi-liters or similar items for highlighting questions and/or answer choices. These may make it easier for some students to focus their attention during PSSA testing and do not provide them with an unfair advantage. For the answer booklet, a No. 2 black pencil must be used.

Posters and charts displaying specific math-related information should not be displayed during testing. As an example, times tables should be covered if they are on the walls of a PSSA testing room. In general, anything that might assist students in answering **specific questions** should not appear in the testing situation. On the other hand, materials that are **general** in nature (e.g., general rubrics) are permitted.

THE REPORTING OF SCHOOL AND STUDENT SCORES

Beginning with the 1999 assessment, the major focus in reporting PSSA mathematics scores became how well schools and individual students perform on the 11 standards categories. In the 1999 assessment, individual student results were reported only as total scores. In the 2000 assessment, for the first time, individual student results were reported not only as total scores but also for each standards category. This same approach will be used for the 2001 assessment and for all of those to follow.

Shown on the next page is a sample Individual Student Report. The attempt in this report is, first of all, to provide an overall result indicating how well the student performed. Then, it is intended that a picture will emerge making it clear what the student's strengths and weaknesses relative to the standards categories are.

The mathematics results that are shown will be discussed. The same kinds of analyses can easily be made for the Reading results.

The report shows that the student achieved a score of 56 out of the 85 points on the grade 5 mathematics assessment. The 85 points are a function of 70 multiple-choice questions (70 points) and three open-ended questions (5 points apiece).

As is reported above the table containing the total scores, the student scored at the 59th percentile. This means that he or she scored at the same level or better than 59 percent of the students who took the grade 5 mathematics assessment. As is plotted to the right of the scores, the student scored about 4.6 points higher than the state average of 51.39.

The Mathematics table presented in the bottom half of the page shows how well the student scored on each standards category compared to the state average for each. A confusing aspect of the table is that the numbers in the left column of the "Points Possible" area actually total only 80 even though there are 85 points possible. This is due to some difficulty at only the grade 5 level presenting the results for the Mathematical Problem Solving Standards Category. As is shown in the gray area at the bottom of the table, there were 20 possible points for this standards category. Fifteen of these were from the three open-ended questions. The additional five were multiple-choice. These five points must be added to the 80 shown to produce the total of 85. Two other somewhat confusing parts of the table should be clarified. First of all, there were 14 points possible for Standards Category 2.2, Computation and Estimation. Eight of these points were for questions in which a calculator was not permitted; the remaining six were items for which it was permissible to use a calculator. The "8" and "6" printed in the right column are not added to the total points possible, since they are included in the 14 points for Standards Category 2.2. The second possible point of confusion is the gray area for Mathematical Problem Solving. There are 20 possible points for this area, but only five of them enter into the 85 total points. This is because the 15 points for the open-ended questions have already been counted within the content area represented by each question. The three open-ended questions are categorized two ways. First they are categorized according to their content (in this case, Number Systems and Relationships; Statistics and Data Analysis; and Geometry). In addition, only for purposes of providing diagnostic information, they are categorized as Mathematical Problem Solving. Their 15 points enter only once in the total score, but, for diagnostic purposes, they show up both in their content area and in Mathematical Problem Solving.

2000 Pennsylvania System of School Assessment
Grade 5 Mathematics and Reading: Individual Student Report

Name: STUDENT 5 District: PENNSYLVANIA SD School: PENNSYLVANIA ELEM

Highlights: In Mathematics, you scored as high or higher than 59 percent of 5th-grade students. You performed better in Geometry than in Algebra & Functions.

In Reading, you scored as high or higher than 59 percent of 5th-grade students. You performed better in Reading Critically than in Analyzing/Interpreting Literature.

Total Mathematics and Reading Results

Results for PSSA Mathematics and Reading are expressed as the number of points achieved. You can compare your points with the state average. Also shown is your *Percentile Rank*, which is the percent of students statewide who achieved the same or fewer points.

	Percentile Rank	Points Achieved	Points Possible	State Average	Difference Between Points Achieved and State Average -20 -16 -12 -8 -4 0 4 8 12 16 20
Mathematics Totals	69	56	85	51.39	*
Reading Totals	59	38	56	34.23	*

Results by Academic Standards

To assist in understanding how well you performed in each standard, the following results are presented:

- Points achieved and the maximum points possible
- Average points achieved for students statewide
- A graph that shows the difference between your points achieved and the state average

PSSA Results for Mathematics Academic Standards

Mathematics Academic Standards	Points Achieved	Points Possible	State Average	Difference Between Points Achieved and State Average -10 -8 -6 -4 -2 0 2 4 6 8 10
2.1 Number Systems & Relationships	7	11	5.64	*
2.2 Computation and Estimation	9	14	9.69	*
Without a Calculator	4	8	5.67	
With a Calculator	5	6	4.02	
2.3 Measurement and Estimation	7	9	5.36	*
2.4 Mathematical Reasoning	3	5	3.58	*
2.6 Statistics & Data Analysis	6	8	4.14	*
2.7 Probability & Predictions	3	5	3.00	*
2.8 Algebra & Functions	3	7	4.37	*
2.9 Geometry	8	11	6.00	*
2.10 Trigonometry	5	5	3.05	*
2.11 Concepts of Calculus	4	5	3.25	*
2.5 Mathematical Problem Solving *	9	20	9.44	*

*Academic Standard 2.5, *Mathematical Problem Solving*, includes some items that are also included in other Academic Standards.

PSSA Results for Reading Academic Standards

Reading Academic Standards	Points Achieved	Points Possible	State Average	Difference Between Points Achieved and State Average -10 -8 -6 -4 -2 0 2 4 6 8 10
1.1 Reading Independently	8	14	9.02	*
1.2 Reading Critically	11	14	8.22	*
1.3 Analyzing/Interpreting Literature	6	12	7.43	*
1.7 English Language Characteristics	6	8	4.58	*
1.8 Research	7	8	4.98	*

Please turn over for additional information

By examining the table's results, a pattern of strengths and weaknesses emerges. The student scored above the state average in some areas and below it in others. For example, as is printed in the "Highlights" area at the top of the page, the student performed better in Geometry than in Algebra and Functions. An interesting result is shown for Standards Category 2.2. The student scored below the state average for Computation and Estimation Without a Calculator and above the state average for this same area, but for items for which a calculator was permitted. This may point to some need for the student to review basic computational skills.

The results shown in the table should be viewed as only one indication of a student's performance in each of the standards category areas. Other performances of the student, both within the classroom and in other testings, should be consulted to either reinforce or refute the results shown. This is especially the case for areas for which only a few items are included in the testing (e.g., Mathematical Reasoning, Trigonometry). The results for these are less reliable than are those for such areas as Geometry and Numbers, Number Systems and Number Relationships. The more items that are included for an area, the more a student's results can be viewed as a measure of general performance. If only a few items are used, a student may score high because of specific items that he or she just happened to know. Conversely, a student may score low because of specific items that he or she just did not experience in instruction. The smaller the number of items, the more the results shown may be attributed to knowledge of specific items rather than to a general knowledge of the area tested.

SETTING PERFORMANCE STANDARDS FOR THE 2001 ASSESSMENT

In the PSSA assessment for the school year 2000-2001, performance standards will be set for the first time. Procedures will be followed to determine cut-off scores for total PSSA scores such that student scores can be classified as Advanced, Proficient, Basic or Below Basic.

The procedures that will be used to establish the four performance levels will involve judgment groups deciding, overall, what quality of student work is sufficient for a score to be classified as Advanced, Proficient or Basic. Both multiple-choice and open-ended task results will enter into the decisions.

SCORING RUBRICS VERSUS PERFORMANCE STANDARDS

The procedures that will be used to establish the four performance levels should not be confused with the categories being used for scoring individual open-ended tasks. It is best to think of the scoring rubrics as devices for sorting students' work into a number of categories. That work which is of high quality is termed Advanced, or Excellent, or "5." By awarding such a score there is no intent to imply that the student's work truly should be classified as Advanced overall, but rather that the work he or she accomplished on a particular task was at a high level. In fact, the task could be one that is not particularly demanding and because of this a relatively high proportion of students could have achieved an Excellent score on it. It is for the judgment groups to look at all parts of the mathematics assessment and to decide what overall level of performance is necessary for student scores to be classified as Advanced.

There is, therefore, no need for the scoring rubrics to match the four performance levels to be set in 2000-2001. Rather, all they must do is reliably sort student work on particular tasks so that those who responded at the highest level receive the highest scores. As such, the categories could just as reasonably be called "5," "4," "3," "2," "1" and "0" as long as they provide a reliable sorting process.

PREPARING FOR THE GRADE 5 MATHEMATICS ASSESSMENT[2]

Introduction

The PSSA test for grade 5 encompasses concepts up to and including grade 5. This test is a measurement of the complete elementary program and is not to be considered or evaluated as solely a fifth-grade test.

Preparation for the PSSA test cannot occur in a day or two, but must be an ongoing developmental process beginning at the primary levels. The recommendations that are reflected in this manual must become part of the guidelines for all elementary grades.

The Content of the Test

More test items will be placed on the grade 5 assessment for some standards categories than for others. The categories for which more items will be included are those which are most reflective of the curriculum at the grade 5 level. The approximate numbers of points on the test devoted to each standards category are as follows:

Standards Category	Approximate Distribution of Points on the Test*
2.1 Numbers, Number Systems and Number Relationships	15
2.2 Computation and Estimation	15
2.3 Measurement and Estimation	10
2.4 Mathematical Reasoning and Connections	5
2.6 Statistics and Data Analysis	8
2.7 Probability and Predictions	5
2.8 Algebra and Functions	7
2.9 Geometry	10
2.10 Trigonometry	5
2.11 Concepts of Calculus	5

* Fifteen of the 85 points are from open-ended questions. They are included within the content areas they assess. For example, if one of the open-ended questions assessed Standards Category 2.3, Measurement and Estimation, 5 of the 10 points for this standard would come from the open-ended question and the remaining 5 points would come from multiple-choice questions. The open-ended questions are counted only once in total scores but their points are combined together for curriculum analysis purposes into a separate Mathematical Problem Solving and Communication score.

[2]This section was developed in large part by Dorothy McCracken of Curwensville Area School District and John Wellington of Upper Darby School District. They are both members of the Mathematics Assessment Advisory Committee.

At the upper grade levels of the assessment, a formula sheet is provided since students are asked to apply a variety of formulas to the solving of problems. At the grade 5 level, there is no need for such a sheet.

However, students must know the formulas for **area** and **perimeter** of squares and rectangles, they must know the relationship between **radius** and **diameter** and they must know **how to compute means**.

In addition, students must have an understanding of the concepts and terms included in the standards through grade 5. This understanding may not be at the mastery level but should be at a high enough level that they are able to answer questions which require them to recognize the terms and apply them to the solving of a problem. A list of terms used in the Academic Standards through Grade 5 is shown below.

Terms used in the Academic Standards for Mathematics through Grade 5[3]

1. Acute Angle
2. Area
3. Composite Number
4. Congruent
5. Element
6. Equilateral
7. Expanded Notation
8. Factors
9. Hypotenuse
10. Isosceles Triangle
11. Least
12. Leg
13. Maximum
14. Mean
15. Median
16. Minimum
17. Mode
18. Multiples
19. Negative Number
20. Obtuse Angle
21. Parallel
22. Parallelogram
23. Pattern
24. Perimeter
25. Perpendicular
26. Polygon
27. Prime Number
28. Probability
29. Quadrilateral
30. Range
31. Right Angle
32. Right Triangle
33. Scalene Triangle
34. Similar
35. Symmetrical
36. Tessellation
37. Trapezoid
38. Tree Diagram
39. Triangle
40. Venn Diagram
41. Volume

[3]This list was developed by Elizabeth Aulbach of Central York School District, a member of the Grade 5 Mathematics Assessment Advisory Committee.

Suggestions for Teachers in Preparing Students for the Assessment

Review your results from the previous year. Be sure to look at the standard score for each mathematics strand—this might help you identify specific areas of weakness.

Students should know that they must be prepared to answer questions involving operations with whole numbers and decimals without the use of a calculator.

Proper use of the calculator cannot wait until the day of the test but must be part of the students' ongoing development of the use of technology in relation to mathematics.

Have students develop a mathematics vocabulary handbook. Also, refer to the "Terms to Know" from the Academic Standards, as listed on the previous page.

Model open-ended problems for your students, including the organization of the rubric. Specific examples have been provided in this manual.

Write some open-ended questions for your students and with them. Guide them in developing a rubric. Examining how a problem can be analyzed provides insight as to what is expected in the response.

Provide time for students to solve open-ended problems individually before sharing in small groups and discussing as a class. The sample problems in this manual are an excellent starting point.

Stress the requirements for rubric responses and review the sample responses in the manual with your students. Students will be asked to do the following when answering grade 5 open-ended questions:

1. Write each step of their math work.
2. Explain why they did each step.

Insist that students use mathematical terms in their explanations.

Communicate with teachers in lower grade levels. Encourage the teachers to develop and use open-ended problems with rubrics.

Most importantly: **ALL STUDENTS MUST TAKE THIS TEST SERIOUSLY!**

Sources for Obtaining Open-Ended Item Information

1. Your text and supplements. Most publishers have included open-ended problems in their latest publications or they can be purchased separately.

2. Web site for Pennsylvania Department of Education.

3. Intermediate Units have developed a program titled: "Performance Assessment Lesson Plan Topics."

4. National Council of Teachers of Mathematics (NCTM)—publications and Web site. Internet (nctm@nctm.org); World Wide Web (http://www.nctm.org/)

5. Pennsylvania Council of Supervisors of Mathematics.

6. Pennsylvania Council of Teachers of Mathematics.

7. Local subsidiaries of NCTM.

8. Each issue of the NCTM publications (Teaching Children Mathematics, Mathematics Teaching in the Middle School, and The Mathematics Teacher) have problems which can be used as extensions and open-ended items.

9. Local/District curriculum persons (i.e., math specialists).

Teacher Articles About Ways of Preparing Students for the Grade 5 PSSA Mathematics Testing

The following two articles were written by members of the Grade 5 Mathematics Assessment Advisory Committee. They were presented at one of the fall 1999 Pennsylvania Department of Education training sessions dealing with the writing and scoring of open-ended mathematics items.

Peggy Rhodes' article, entitled "A District Action Plan," focuses upon how to foster the use of open-ended mathematics items within a school district's mathematics curriculum. Joan Miller's article, entitled "Putting Problem Solving Techniques in Place in the Classroom," describes four instructional practices that can help a student develop proficiency in problem solving and application.

A District Action Plan

by Peggy Rhodes[4]

When Pennsylvania first started giving the state math assessment, it became obvious that we needed to make some changes in the way math was being taught in our district. Our scores on the open-ended questions indicated that our students needed help in this area. As a member of the Mathematics Assessment Advisory Committee to the Department, I had some experience in what might be done to address this concern.

When I approached the administration with several suggestions, they were very supportive. We decided that educating the teachers in terms of this type of questioning strategy was a good first step. I conducted a series of in-school workshops. They were organized by grade level and encompassed grades 1-6. At the first workshop, I introduced our staff to open-ended assessments. Our staff is typical—some knew about these assessment types and used them to some extent, some had limited knowledge and didn't use them, and some had no knowledge.

I had the staff do a sample question to familiarize themselves with what our students go through when they are faced with a similar problem. (I used the prompt 'Mrs. Smith's Coins,' a PSSA released item included in the 1998 Mathematics Assessment Handbook.) After that, I introduced the concept of rubric scoring. We scored sample training sets to show that papers using this non-traditional type of answer could, in fact, be scored consistently by different individuals using a rubric. The final portion of the workshop was dedicated to looking at the text books being used and finding questions that, with a bit of reworking, could be made into an open-ended assessment.

As I stated earlier, our staff is typical. Some teachers incorporated this type of assessment into their program, some felt it was useful but never got around to using it, and others didn't like the idea. We still needed to go further.

Phase II was instituted. The only way to ensure that every student was being exposed to these types of assessments and gaining experience with them was to make it an obligatory part of every math classroom. Realizing that the math curriculum was already full and that class time was at a premium, we didn't want another 'extra' that had to be fit in somehow. Also realizing that teachers' preparation time and familiarity with this kind of assessment tool was limited, we decided to develop the program so that all that remained was to set it in place in their daily plans.

I met with a math representative from each grade level, grades 1-6. We developed 6 open-ended assessments and rubrics for each grade, based on the text. This took about 1 day per grade. Because it was text based, the concept didn't have to be taught prior to the assessment. That had already been done in the regular class schedule. The idea went from being an 'extra,' to being something that complemented what was already being done in the classroom. One assessment was to be administered each grading period.

If we had stopped at this point, some improvement would have taken place, but not enough. It was decided to make the assessments mandatory. The building principals give every math teacher a class roster with all six assessments listed at the top. After the assessment is scored, the scores are recorded and returned to the administration. Before the next assessment is to be given, the lists are returned. By requiring scores to be turned in on a regular basis, we are insuring teacher accountability both in administering open-ended questions and in teaching problem solving.

[4] Peggy Rhodes teaches at Karns City Area School District and is a member of the Grade 5 Mathematics Assessment Advisory Committee.

By this procedure, several goals are achieved:

1. It is ensured that all students are getting practice in this area. It no longer depends on what teacher a student has while going through the system. All elementary math teachers will use this type of assessment at least 6 times during the year.
2. Because students are getting exposure from grade 1, this is a skill that is being constantly developed. We aren't waiting until the 5th grade test and hoping that the students understand the format and thinking skill involved.
3. Students are getting practice in application of math skills, not just drill in computation skills.

We aren't yet finished with the process. We meet as a grade level after each assessment to fine-tune and refine any problems that we notice. Although the prompts may not be as polished as those commercially prepared and the rubrics not statistically based, we feel that this has been a positive addition to our program. They are tailor-made to the needs of our staff and students.

Putting Problem Solving Techniques in Place in the Classroom

By Joan V. Miller[5]

Many students come to classrooms comfortable with computational skills, but the same students lack confidence when asked to apply these skills to solve problems or use them in real life situations. What can be done to change this? Math in the 21st century must empower students with logical reasoning skills and the ability to use math to problem solve. How can these skills be taught?

A skill is mastered only by practice. The more practice and exposure students have to concepts and application and problem solving the better. A math class rich in higher level thinking will afford children the opportunity to experiment and take risks, discuss strategies and procedures, view a variety of solutions to a problem and evaluate which one seems to be the best. Four different and simple instructional practices can help a student become proficient in problem solving and application. The four practices are: partner work, group work, child-authored problems, and real life situation projects. In all four scenarios calculator usage is permitted and encouraged.

Partner work makes problem solving safe. It is easier to take a risk if someone else thinks it is worth it. Two heads are always better than one. The less confident student does not sit at a desk stalled and not knowing how to begin when paired with a partner who can help get the problem started. This method keeps more kids on task in a productive fashion. Two stronger problem solvers paired together may even help each other discover that there is more than one method to arrive at the answer to the problem, or that there may be more than one correct answer to the problem.

Along the same lines group work also adds a safety net to problem solving. All the students learn from each other's ideas to tackle the problem. Often a variety of strategies and procedures are discussed and accepted or rejected. Students learn to defend their positions. Evaluation takes place. Students can solve problems independently or with partners and then come together as a group to verify answers and discuss the path they took to solve the problem. There may be one well worn path or a variety of new roads explored to reach the same destination.

[5] Joan Miller teaches at Central Bucks School District and is a member of the Grade 5 Mathematics Assessment Advisory Committee.

When students grow comfortable with problem solving, they can begin to create their own problems for classmates to solve. Problems may also be created for a younger class and then collected to be scored using a rubric created by the author of the problem. These problems can be developed with a partner or independently. Either way when students arrive at this point, they begin to develop a strong sense of ownership and pride in their work as problem solvers. Many are ready to develop rubrics for their own problems, or they can create problems and have someone else (a peer or teacher) write a rubric for it.

The following is an example of a problem created by a fifth grade student for classmates to solve:

> John went to a clothes store. He bought two pair of pants. They each cost $24. Then he went to the food store and bought 3 apples. They each cost 50 cents. Next he went to the shoe store. He bought two pairs of shoes. One cost $27.95 and the other cost $20.50. He went to work and made $10, but he spent $10 eating at a fast food restaurant. He had $20 total at the end of the day. How much money did John start with?

The last practice to encourage students' growth as problem solvers is to supply them with the opportunity to apply math to real life situations. This can be done by assigning projects. The projects should involve appropriate real life situations that will make math relevant for the child completing the project. Some topic ideas are: estimating with groceries; projects to redecorate and furnish his/her room given a budget; measuring and ordering correct amounts of supplies needed to remodel or redecorate; and planning for a large party, a special trip or a family vacation.

Some examples of real life mini-problems are:

> Doug is adding a sun-room to his home. Design a sun-room for him with a total area of 200 square feet. What could the dimensions be? What is the perimeter of the room if those dimensions are used?

> Mary's bedroom is 12 ft by 12 ft She wants to carpet it. How many square yards will she need?

Putting these four important practices in place in any classroom is important, but also takes time. Start with partner work on small problems and build up to the larger projects that are completed independently and scored. Spending time on these practices over the course of a school year will build the necessary confidence students need to apply computational skills that are already in place. More students will come to approach problem solving with an enthusiastic "can do" attitude when saturated with opportunities throughout the year to practice.

Sample Multiple-Choice Items from the 2000 PSSA

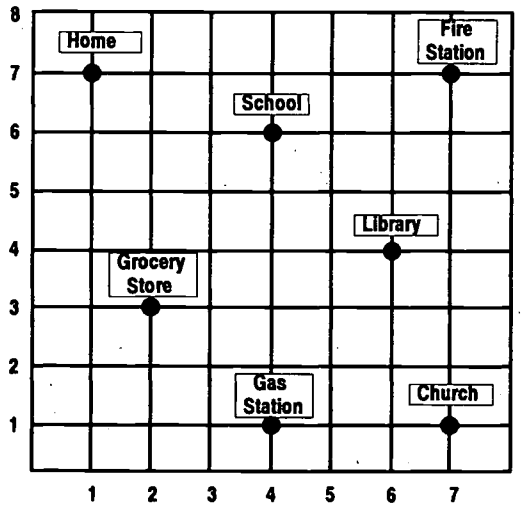
STANDARDS CATEGORY 2.2 (non-calculator)

1. 450 children each have 16 pieces of candy. Altogether how many pieces of candy do they have?

A 2812
B 7200 (correct answer)
C 7250
D 7400

STANDARDS CATEGORY 2.8

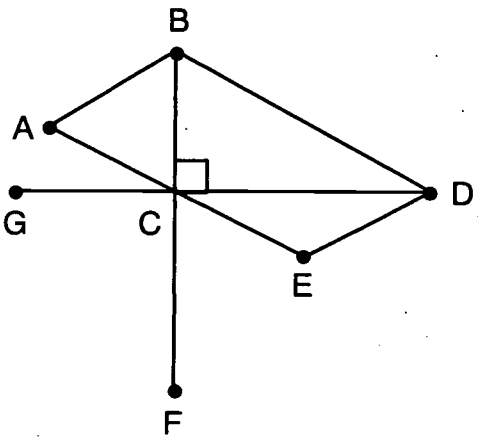
2. Which two places are found at coordinates (4,6) and (7,1)?



A school and home
B school and church (correct answer)
C library and fire station
D library and church

STANDARDS CATEGORY 2.10

3. Which line segment is the hypotenuse of a right triangle?



A $\overline{CD}$
B $\overline{BC}$
C $\overline{CE}$
D $\overline{BD}$ (correct answer)

Sample Open-Ended Tasks from the PSSA

Included in this section are samples of open-ended tasks for the grade 5 level. Also included for each is the specific rubric, or scoring guide, and copies of actual student responses that exemplify each score.

Two tasks are presented. In the first task, a boy and a girl are playing a game with two spinners. Students must first list all possible combinations of two numbers that would result if both spinners are spun at the same time. They must then determine which person would win the game under two different rules for combining the two scores. The task assesses Standards Category 2.7, Probability and Predictions. It also assesses Standards Category 2.5, Mathematical Problem Solving and Communication, as do all of the open-ended tasks.

In the second task, students are shown a checkerboard that measures eight squares by eight squares. A checker of radius 2 cm is shown to fit exactly inside each square, just touching the sides. Students must use this information to determine both the perimeter and the area of the checkerboard. This task assesses both Standards Category 2.3, Measurement and Estimation, and Standards Category 2.5, Mathematical Problem Solving and Communication. As is the case for many open-ended tasks, the checkerboard item assesses more than one content area. In this instance, students must have some knowledge of Standards Category 2.9, Geometry, in order to deal effectively with the need to find the diameter of the checker before computing the perimeter of the checkerboard.

Lorrie and Jim are playing a game with 2 spinners. Each of the spinners is numbered from 1 to 4. In each round of the game, both spinners are spun. If the product of the 2 numbers spun is odd, Lorrie gets a point. If the product is even, Jim gets a point. The winner of the game is the player with the most points.

A) List all the possible combinations of 2 numbers which could result when the 2 spinners are spun. For example, as you can see in the chart, in one round a "4" could be spun on Spinner 1 and a "1" on Spinner 2. There are 16 possible combinations. Complete the chart to find the remaining combinations and their products.

Spinner 1	4														
Spinner 2	1														
Product	4														

B) Look at all the combinations in the chart. Who has the better chance of winning the game? Write your answer in the box below. (Remember, Lorrie gets a point for each odd product and Jim gets a point for each even product.) **Explain** your answer.

C) Suppose Lorrie and Jim play the game again with new rules. In their new game, the 2 numbers which are spun are added, rather than multiplied. If Lorrie gets a point for each odd sum and Jim gets a point for each even sum, who has the better chance of winning the new game? Write your answer in the box below. **Explain** your answer. **Show any math work** you did to arrive at this answer.

This item requires students to model a situation by devising and carrying out experiments or simulations to determine probabilities. Students must make predictions based on experimental or theoretical probabilities and cite examples based upon these probabilities in a real-world situation.

Problem Solution:

In Part A student should list the remaining 15 combinations. These are: 1-1; 1-2; 1-3; 1-4; 2-1; 2-2; 2-3; 2-4; 3-1; 3-2; 3-3; 3-4; 4-2; 4-3; and 4-4. In Part B the student should indicate that Jim has the better chance of winning the game. In support of this the student should explain that the probability of obtaining an odd product is 4/16 while the probability of obtaining an even product is 12/16. In further support the student would point out that the only combinations which would produce an odd product are 1-1; 1-3; 3-1; and 3-3. In Part C the student should answer that in the new game both Lorrie and Jim have an equal chance of winning, since eight of the sums of the sixteen combinations are odd and eight are even. Again, the student could show the results of his or her computations, pointing out the eight combinations which have resulted in even sums and the eight which have resulted in odd ones.

Scoring is a function of how many points students achieve on each of the 3 parts. There are 3 points possible for each part as follows:

Part A

3 points	Must list all 15 combinations, including their products.
2 points	Must list 12 to 14 combinations, including their products (ignore repeats and incorrect combinations).
1 point	Must list 4 to 11 combinations, including their products (ignore repeats and incorrect combinations).

NOTE: Failure to compute 1 or more products correctly results in a drop of 1 point.

Part B

3 points	Must give correct name (Jim) based upon the products shown in Part A and a discussion of probability.
2 points	Must give correct name (Jim) based upon the products shown in Part A with some support (i.e., number of points each or the equivalent).
1 point	Must give correct name (Jim) based on the products shown in Part A.

Part C

3 points	Must indicate that the new game results in a tie, with some discussion of probability.
2 points	Must indicate that the new game results in a tie, with the total points shown only (8 even, 8 odd).
1 point	Must indicate that the new game results in a tie, with no support provided **OR** Carries out a correct attempt to fill in the chart, but arrives at an incorrect answer due to a counting/calculation error **OR** Provides an "appropriate answer" based on an incorrect chart. The chart **must** show addition.

5 - Advanced Understanding, Excellent

- Attains all 9 points.

4 - Satisfactory Understanding

- Attains 7 or 8 points.

3 - Almost Satisfactory Understanding

- Attains 5 or 6 points.

2 - Partial Understanding

- Attains 3 or 4 points **OR** 2 points in Part A.

1 - Minimal Understanding

- Attains 1 or 2 points **OR** shows minimal understanding.

0 - Incorrect

- Attains 0 points.

A) **LIST** all the possible combinations which could occur when the 2 spinners are spun. There are **16 possible** combinations. The first one has been completed for you on the chart below.

COMPLETE the chart to find the remaining combinations and their products.

Spinner 1	4	1	2	3	2	4	3	1	2	1	3	1	3	4	2	4
Spinner 2	1	4	2	2	3	3	4	1	1	2	3	3	1	2	4	4
Product	4	4	4	6	6	12	12	1	2	2	9	3	3	8	8	16
	5	5	4	5	5	7	7	2	3	3	6	4	4	6	6	8

B) **LOOK** at all the combinations in the chart above.

Who has the better chance of winning the game?

REMEMBER: **Lorrie** gets a point for each **odd product.**
Jim gets a point for each **even product.**
EXPLAIN your answer.
WRITE your answer in the box below.

Jim has a a better chance of wining the game because out of the 16 comoations 12 were even and 4 were odd.

Answer: Jim

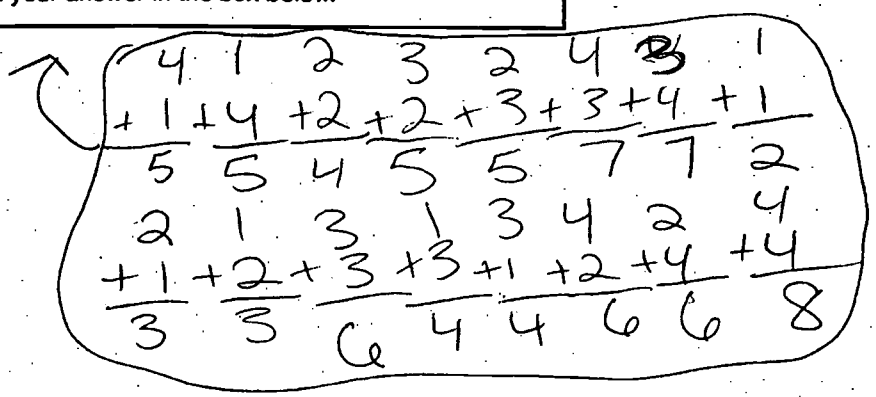
Response continued on the following page.

C) **SUPPOSE** Lorrie and Jim play the **game** again with **new rules.**

- The **2 numbers** which are spun are **added**, not multiplied.
- If the **sum** of the 2 numbers is **odd, Lorrie** gets a **point.**
- If the **sum** of the 2 numbers is **even, Jim** gets a **point.**

Who has the better chance of winning the new game? (Make sure you use your chart from Part A to help answer the question.)

SHOW any math work you did to arrive at your answer.
EXPLAIN your answer.
WRITE your answer in the box below.



They both have an even chance because the sums added up 8 for odd and 8 for even

Answer: Lorrie and Jim

5 - All nine possible points are earned. In part A all 15 combinations and products are correctly listed (3 points). In part B the player with the better chance of winning is chosen and the choice is justified with an accurate discussion of probability. A comparison is made between the number of possible odd and even products out of the total of 16 combinations (3 points). In part C a tie is indicated with a justification that is based on probability concepts (3 points).

A) **LIST** all the possible combinations which could occur when the 2 spinners are spun. There are **16 possible** combinations. The first one has been completed for you on the chart below.

COMPLETE the chart to find the remaining combinations and their products.

Spinner 1	4	4	4	4	3	3	3	3	2	2	2	2	1	1	1	1
Spinner 2	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Product	4	8	12	16	3	6	9	12	2	4	6	8	1	2	3	4

B) **LOOK** at all the combinations in the chart above.

Who has the better chance of winning the game?

REMEMBER: **Lorrie** gets a point for each **odd product.**
Jim gets a point for each **even product.**
EXPLAIN your answer.
WRITE your answer in the box below.

I take on the last row and gave Jim a point for each number, and in the middle row I gave Lorrie a point for each number, and for the last row if it was odd I gave Lorrie a point for each number and I gave Jim a point for each even number. Than I added them both and my reslats are that Jim won!

Jim = 12
Lorrie = 4

Answer: Jim

Response continued on the following page.

C) **SUPPOSE** Lorrie and Jim play the **game** again with **new rules**.

- The **2 numbers** which are spun are **added**, not multiplied.
- If the **sum** of the 2 numbers is **odd, Lorrie** gets a **point**.
- If the **sum** of the 2 numbers is **even, Jim** gets a **point**.

Who has the better chance of winning the new game? (Make sure you use your chart from Part A to help answer the question.)

SHOW any math work you did to arrive at your answer.
EXPLAIN your answer.
WRITE your answer in the box below.

4	4	4	4	3	3	3	3	2	2	2	2	1	1	1	1
1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
5	6	7	8	4	5	6	7	3	4	5	6	2	3	4	5

I added them together and gave Jim a point for each even number and gave Lorrie a point for each odd number. They both won!

Jim = 8
Lorrie = 8

Answer:
Jim and lorrie

4 - Seven points are earned. In part A all 15 combinations and products are correctly listed (3 points). In part B the player with the better chance of winning is chosen, but that choice is justified only by reference to the number of odd and even products as though they represented scores rather than probabilities (2 points). In part C a tie is indicated, but justification only refers to the total number of odd and even sums without demonstrating understanding of probability concepts (2 points).

A) **LIST** all the possible combinations which could occur when the 2 spinners are spun. There are **16 possible** combinations. The first one has been completed for you on the chart below.

COMPLETE the chart to find the remaining combinations and their products.

Spinner 1	4	4	4	4	2	2	2	2	1	1	1	1	3	3	3	3
Spinner 2	1	2	3	4	1	4	3	1	3	2	4	1	3	4	2	1
Product	4	8	12	16	4	8	6	2	3	2	4	1	9	12	6	3

B) **LOOK** at all the combinations in the chart above.

Who has the better chance of winning the game?

REMEMBER: **Lorrie** gets a point for each **odd product.**
Jim gets a point for each **even product.**
EXPLAIN your answer.
WRITE your answer in the box below.

Jim wins because he has more points than Lorrie. I figured it out by counting their points.

Answer: Jim

Response continued on the following page.

C) **SUPPOSE** Lorrie and Jim play the **game** again with **new rules.**

- The **2 numbers** which are spun are **added**, not multiplied.
- If the **sum** of the 2 numbers is **odd, Lorrie** gets a **point.**
- If the **sum** of the 2 numbers is **even, Jim** gets a **point.**

Who has the better chance of winning the new game? (Make sure you use your chart from Part A to help answer the question.)

SHOW any math work you did to arrive at your answer.
EXPLAIN your answer.
WRITE your answer in the box below.

Jim will win by one point. I added then this time and Jim won and so on

4 4 4 4 2 2 2 2
+1 +2+3+4+2+4+3+1
5 6 7 8 4 6 5 3

Answer: Jim

3 - Five total points are earned. In part A all 15 combinations and products are correctly listed (3 points). In part B the player with the better chance of winning is chosen, but the justification only refers to the number of odd and even products as though they represent scores rather than probabilities (2 points). In part C a tie is not indicated (0 points).

A) **LIST** all the possible combinations which could occur when the 2 spinners are spun. There are **16 possible** combinations. The first one has been completed for you on the chart below.

COMPLETE the chart to find the remaining combinations and their products.

Spinner 1	4	2	3	3	2	2	1	1	1	3	4	5	5	5	5	4
Spinner 2	1	2	2	1	4	1	4	5	1	3	3	2	3	4	5	4
Product	4	4	6	3	8	2	4	5	1	9	12	10	8	20	25	6

B) **LOOK** at all the combinations in the chart above.

Who has the better chance of winning the game?

REMEMBER: **Lorrie** gets a point for each **odd product.**
Jim gets a point for each **even product.**
EXPLAIN your answer.
WRITE your answer in the box below.

Lorrie

6

Jim

10

Answer: Jim

Response continued on the following page.

C) **SUPPOSE** Lorrie and Jim play the **game** again with **new rules.**

- The **2 numbers** which are spun are **added**, not multiplied.
- If the **sum** of the 2 numbers is **odd, Lorrie** gets a **point.**
- If the **sum** of the 2 numbers is **even, Jim** gets a **point.**

Who has the better chance of winning the new game? (Make sure you use your chart from Part A to help answer the question.)

SHOW any math work you did to arrive at your answer.
EXPLAIN your answer.
WRITE your answer in the box below.

Lorrie	2	3	6	8	6	7	7	6	5	9	10	6	13	2	3	1
Jim	3	8	9	5	3	9	7	7	6	3	1	9	3	2	3	1
Sum	5	11	17	13	9	16	14	13	11	12	11	15	16	4	6	2
	L	L	L	L	L	J	J	L	L	J	L	L	J	J	J	J

Lorrie has a better chance because there are more odd numbers than even.

Answer: Lorrie

2 - A total of 3 points are earned. In part A only 10 correct combinations and products are listed (1 point). In part B the player with the better chance of winning is chosen based on the products shown (1 point). The student's chart and work support the answer of Jim that is given. In part C an appropriate answer is provided based on the given chart of sums (1 point). The chart given is incorrect but the results are added and the number of possible odd scores is higher than the possible even scores. Given the student's work Lorrie would have the better chance to win.

A) **LIST** all the possible combinations which could occur when the 2 spinners are spun. There are **16 possible** combinations. The first one has been completed for you on the chart below.

COMPLETE the chart to find the remaining combinations and their products.

Spinner 1	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3
Spinner 2	1	1	2	3	4	3	4	2	3	4	2	4	3	2	3	4
Product	4	1	4	9	16	4	3	4	3	2	3	4	2	3	4	2

B) **LOOK** at all the combinations in the chart above.

Who has the better chance of winning the game?

REMEMBER: **Lorrie** gets a point for each **odd product.**
Jim gets a point for each **even product.**
EXPLAIN your answer.
WRITE your answer in the box below.

Beaces he DiD Move Spinner then number one DiD.

Answer:
Spinner 2

Response continued on the following page.

C) **SUPPOSE** Lorrie and Jim play the **game** again with **new rules.**

- The **2 numbers** which are spun are **added,** not multiplied.
- If the **sum** of the 2 numbers is **odd, Lorrie** gets a **point.**
- If the **sum** of the 2 numbers is **even, Jim** gets a **point.**

Who has the better chance of winning the new game? (Make sure you use your chart from Part A to help answer the question.)

SHOW any math work you did to arrive at your answer.
EXPLAIN your answer.
WRITE your answer in the box below.

Jim had the Bigger Number in the game

Answer: Jim

1 - A total of 1 point is earned. In part A 4 correct combinations and products are listed (1 point). In part B an incorrect response is given (0 points). In part C an incorrect response is given with no indication that sums were compared appropriately (0 points).

A) **LIST** all the possible combinations which could occur when the 2 spinners are spun. There are **16 possible** combinations. The first one has been completed for you on the chart below.

COMPLETE the chart to find the remaining combinations and their products.

Spinner 1	4	5	6	7	8	9	10	11	12	13	14	15	16	3	2	1
Spinner 2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Product	4	5	6	7	8	9	10	11	12	13	14	15	16	3	2	1

B) **LOOK** at all the combinations in the chart above.

Who has the better chance of winning the game?

REMEMBER: **Lorrie** gets a point for each **odd product.**
Jim gets a point for each **even product.**
EXPLAIN your answer.
WRITE your answer in the box below.

Jim does his numbers are bigger.

Answer: Jim

Response continued on the following page.

C) **SUPPOSE** Lorrie and Jim play the **game** again with **new rules.**

- The **2 numbers** which are spun are **added,** not multiplied.
- If the **sum** of the 2 numbers is **odd, Lorrie** gets a **point.**
- If the **sum** of the 2 numbers is **even, Jim** gets a **point.**

Who has the better chance of winning the new game? (Make sure you use your chart from Part A to help answer the question.)

SHOW any math work you did to arrive at your answer.
EXPLAIN your answer.
WRITE your answer in the box below.

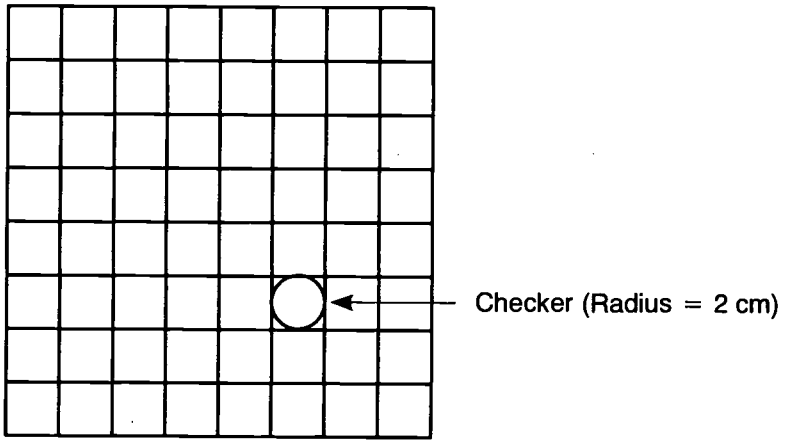
Jim does his number are still biger.

Answer: Jim

0 - No points are earned. In part A only 3 combinations and products are listed (0 points). Part B is incorrect based on the chart provided (0 points). In part C an incorrect response is given with no indication that sums were compared appropriately. There is no understanding of probability concepts demonstrated.

GRADE 5 SAMPLE TASK

Sally has a checkerboard that measures **8 squares by 8 squares**. A checker will fit exactly inside each square, just touching the sides. The **radius of each checker is 2 cm. The picture at the right shows this information.**



Checkerboard

A) What is the **PERIMETER** of the checkerboard?
WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the box below.
LABEL your answer.

B) What is the **AREA** of the checkerboard?
WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the box below.
LABEL your answer.

This item requires students to find perimeter and area of a square area using information about the dimensions of a circular object placed within the square.

Problem Solution:

Diameter of checker = 2 × 2 cm = 4 cm, so each small square is 4 cm by 4 cm.
Each side of the checkerboard = 8 × 4 cm = 32 cm. Perimeter = 4 × 32 cm = 128 cm.
Area = 32 cm × 32 cm = 1024 sq cm.

To obtain the highest scores, students must include work and an explanation of why each step was carried out. The explanation should include the connection of the radius to the diameter and the connection of the diameter of a checker to the size of the checkerboard.

5 - Advanced Understanding, Excellent

- Student answers Part A correctly (Perimeter = 128 cm) and Part B correctly (Area = 1024 sq cm). The work provided should be complete and the explanation should show an Excellent understanding of the concepts of the problem. As a part of this explanation the student should make it clear how knowledge of the radius length resulted in knowing the dimensions of the checkerboard squares and of the total checkerboard.

4 - Satisfactory Understanding

- Student correctly answers Part A and Part B with support for these answers. This support is not strong and clear enough to warrant a score of Excellent.

3 - Almost Satisfactory Understanding

A. Student correctly answers either Part A or Part B correctly with some support provided.

B. Student correctly answers both Part A and Part B but the support provided is inadequate but not incorrect.

C. The procedures the student has performed in <u>both</u> parts are correct and supported with work but the numerical answer is incorrect due to one computation error.

2 - Partial Understanding

A. Student shows the use of the radius in finding the checkerboard's dimensions, but uses this incorrectly (e.g., uses the radius, not the diameter) in computing both the area and the perimeter.

B. Student correctly answers either Part A or Part B but the support provided is inadequate but not incorrect.

C. The procedures the student has performed in <u>one</u> part are correct and supported with work but the numerical answer is incorrect due to one computation error. The other part is incorrect or not attempted.

1 - Minimal Understanding

- Student does not provide any correct answers but carries out at least one procedure which shows some knowledge of area or perimeter.

0 - Incorrect

A. Student gives a correct answer but all procedures shown are incorrect.

B. Student does not show any knowledge of area or perimeter.

Note: Student scores should be reduced one score point at the 4 or 5 level if labeling is missing or incorrect.

Grade 5 Sample Response for Category 5

A) What is the **PERIMETER** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

① 2 × 2 = 4cm ② 4 × 8 = 32 cm ③ 32cm × 4 = 128 cm

Step 1 was to find the length of a checker by multipling the radius by 2 to find the length of a square. Step 2 was finding the length of 1 side of the board by multipling the length of a square by 8. Step 3 was finding total perimeter by multipling the length of 1 side by 4.

Answer:
Perimeter = 128 cm

B) What is the **AREA** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

① 8 × 4 = 32 ② 32 × 32 = 164 + 960 = 1024 sq. cm.

Step 1 was finding the length of a side. Step 2 was multipling length x width to find the area

Answer:
Area = 1,024 Sq. cm

5 - Correct answers are provided in both parts (128 cm for part A and 1024 square centimeters for part B). All work is shown and the explanation demonstrates complete understanding of both perimeter and area as well as an explanation of how the radius of the checker was used to determine them.

Grade 5 Sample Response for Category 4

A) What is the **PERIMETER** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

Radius = 2 cm
checker = ⊖
2 cm
x 2
4 cm wide checker
8 blocks
x 4 cm
32 cm long & high
32 cm
x 4
128 (or) 32 + 32 + 32 + 32 = 128
because each side is the same and to find the perimeter you have to add all of the sides together

Answer:
Perimeter = 128 cm

B) What is the **AREA** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

Radius = 2cm
checker = ⊖
2cm
x 2
4cm wide checker
8 blocks
x 4 cm
32 cm long & high
32
x32
64
+960
1,024 area, because length Times width = area

Answer:
Area = 1,024 cm

4 - Correct answers are provided in both parts. Work is shown that indicates understanding for calculating both area and perimeter, but the explanation for using the radius is unclear. In addition the area is not labeled in square centimeters. Although both answers are correct the support provided does not demonstrate excellent understanding.

Grade 5 Sample Response for Category 3

A) What is the **PERIMETER** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

4 cm for each square, 8 squares
= 32 cm = l or w

l + l + w + w

l = length
w = width

32 cm × 4 = 128
perimiter

2 cm
+ 2 cm
4 cm diameter
× 8 squares ↕
32
4 squares
128 cm

because radius is only diameter is

Answer:
Perimeter = 128 cm

B) What is the **AREA** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

l = length
w = width

radius 2 cm is
radius + 2 cm
4 cm
× 8 squares ↕
32
× 8 squares ↔ l or w
256 cm

because radius is only half of circle, diameter because l × w = area and 4 × 8 =

Answer:
Area = 256 cm

3 - A correct answer is provided in part A with support. The student demonstrates correctly how to determine the perimeter from the given radius. Part B is incorrect.

Grade 5 Sample Response for Category 2

A) What is the **PERIMETER** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

I counted how many squares are on one side then I multiplied it by two. Then I multiplied that by four and got 64.

Answer:
Perimeter = 64 cm

B) What is the **AREA** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

I counted how many squares on one side multiplied it by 2 and knowing that length times width equals the area I multiplied 16x16 and got 256 sq. cm.

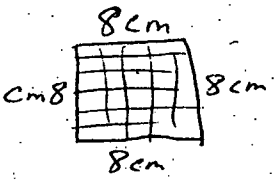
Answer:
Area = 256 sq. cm

2 - Both answers are incorrect because the student used the length of the radius as the side length of a square. The student does not demonstrate understanding that the radius of a circle is one half of the diameter. However, both the perimeter and area are shown correctly calculated based on that error.

Grade 5 Sample Response for Category 1

A) What is the **PERIMETER** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.



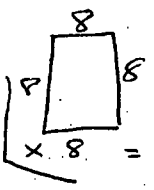
8
8
8
8
32

= 8 ×4 32

Answer:
Perimeter = 32cm

B) What is the **AREA** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.



= LXW = 8 ×8 64 cm²

Answer:
Area = 64 cm²

1 - Both answers are incorrect. The given information regarding the radius of the checker is ignored. The student shows no understanding of how the given information relates to the dimensions of the square. However, the work shown demonstrates understanding of perimeter and area.

Grade 5 Sample Response for Category 0

A) What is the **PERIMETER** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

Each squar is two cm

I added each squar vertically with the total of eight squars eight Pluse two equal 10 I did this to see what the answere would be.

$$\begin{array}{r} 8 \\ +2 \\ \hline 10 \end{array}$$

Answer:
Perimeter = 10 cm

B) What is the **AREA** of the checkerboard?

WRITE each step of your math work.
EXPLAIN why you did each step.
WRITE your answer in the **box below.**
LABEL your answer.

Each squar is two cm I added each squar horizontically with the total of eight I times it to two and my answere was 16 I did this to know the area of the checkerboard

$$\begin{array}{r} 8 \\ \times 2 \\ \hline 16 \end{array}$$

Answer:
Area = 16

0 - Both answers are incorrect. No understanding of either perimeter or area is demonstrated.

The Content of the Test

The PSSA eighth-grade test encompasses concepts up to and including grade 8. The approximate numbers of points of the test devoted to each standards category are as follows:

Standards Category		Approximate Distribution of Points on the Test*
2.1	Numbers, Number Systems and Number Relationships	10
2.2	Computation and Estimation	10
2.3	Measurement and Estimation	10
2.4	Mathematical Reasoning and Connections	5
2.6	Statistics and Data Analysis	10
2.7	Probability and Predictions	5
2.8	Algebra and Functions	15
2.9	Geometry	10
2.10	Trigonometry	5
2.11	Concepts of Calculus	5

* Fifteen of the 85 points are from open-ended questions. They are included within the content areas they assess. For example, if one of the open-ended questions assessed Standards Category 2.3, Measurement and Estimation, 5 of the 10 points for this standard would come from the open-ended question and the remaining 5 points would come from multiple-choice questions. The open-ended questions are counted only once in total scores but their points are combined together for curriculum analysis purposes into a separate Mathematical Problem Solving and Communication score.

The list on the following page includes terms used in the Academic Standards through grade 8. Students should be able to answer questions which require knowledge of these and also, of course, should be familiar with those previously listed for grade 5.

Terms used in the Academic Standards for Mathematics through Grade 8

1. Angle Measurement in Degrees
2. Bisector
3. Box-and-Whisker Plot
4. Combination
5. Complementary Angle
6. Coordinate Plane
7. Counter Example
8. Deductive Reasoning
9. Dimensions
10. Equation
11. Evaluate the Expression
12. Exponent
13. Exponential Relationship
14. Functional Relationship
15. Inductive Reasoning
16. Inequality
17. Irrational Number
18. Linear Function
19. Linear Relationship
20. Logical Reasoning
21. Number Line
22. Order of Operations
23. Percent
24. Permutation
25. Proportion
26. Pythagorean Theorem
27. Quadratic Relationship
28. Quartile
29. Random Sampling
30. Ratio
31. Rational Number
32. Regular Polygon
33. Reliability
34. Scale Model
35. Scientific Notation
36. Sequence
37. Slope
38. Square Root
39. Stem-and-Leaf Plot
40. Supplementary Angle
41. Transformation
42. Transversal
43. Unit Rate
44. Verbal, Symbolic Rules
45. Vertical Angle

Suggestions for Teachers in Preparing Students for the Assessment[6]

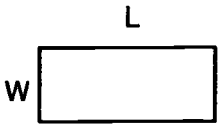
- Become familiar with the Pennsylvania Standards — the PSSA mathematics test reflects the mathematics standards.
- Encourage students to approach open-ended problems by starting with the type of problem that asks for "all possible combinations." (See the fifth-grade sample problem about Lorrie and Jim's Spinner Game.)
- Write some open-ended items for your students. Evaluate their responses by using a rubric (As a general guideline, you can use the General Rubric that is published in this handbook.)
- Brainstorm with your students what would be expected of them to earn an A, B, C, etc. Explain that these expectations form a "rubric." Form a "Good Work" chart.
- Review your school's results from the previous year. Be sure to look at the standard score for each mathematics strand — this will help you identify any specific areas of weakness. Also look at the results for the open-ended (performance) items. This will show you how well your students answer mathematics questions and explain their work.
- If possible, have your students use a type of scientific calculator that allows operations with fractions and mixed numbers. These types of models are beneficial in the instructional setting as well as in a testing situation. Students need to be taught how to use these types of calculators.
- Prepare your students to answer questions that are designated as non-calculator items. Typically, these items will not be long or tedious computations.
- Encourage all students to take the test seriously.

What Students Should Do in Preparing for the Assessment

- Become familiar with the meaning of each word in the List of Terms to Know from the Academic Standards through grade 8.
- Make sure they know when and how to use a calculator.
- Learn about what is expected of them on an open-ended item. This type of item is scored with a rubric and they can earn at least one point by showing a correct step toward solving the problem. To earn all the possible points, they will need to **have all of the correct work, show all their work and explain why they used each of the steps in their work.**
- The Formula Sheet provided to grade 8 students in the assessment is shown below. Students should become familiar with these formulas and know how to apply them to the solving of problems.

[6]This section was developed in large part by Ann Bacon of Abington School District, a member of the Grade 8 Mathematics Assessment Advisory Committee.

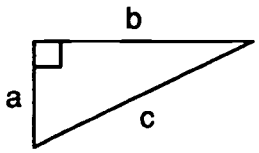
GRADE 8 FORMULA SHEET



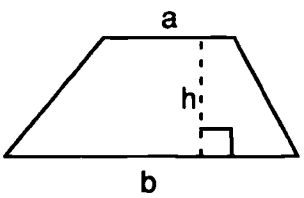
$A = LW$
$P = 2L + 2W$

Constant Motion $d = rt$

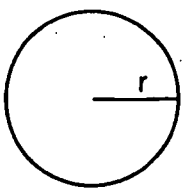
Simple Interest $I = prt$



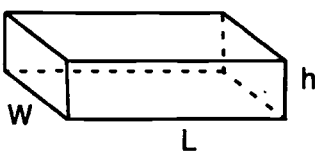
$a^2 + b^2 = c^2$



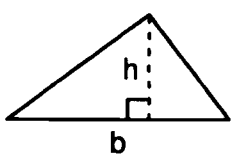
$A = \frac{h}{2}(a + b)$



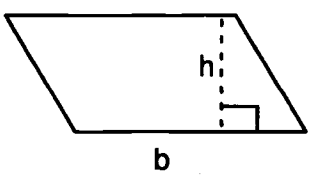
$C = 2\pi r$
$A = \pi r^2$



$V = LWh$
$A = 2LW + 2Lh + 2Wh$



$A = \frac{1}{2}bh$



$A = bh$

Teacher Articles About Ways of Preparing Students for the Grade 8 PSSA Mathematics Testing

The following articles were written by members of the Grade 8 Mathematics Assessment Advisory Committee. They were presented at one of the fall 1999 Pennsylvania Department of Education training sessions dealing with the writing and scoring of open-ended mathematics items.

The first two articles, by Diane Hurst and Carolyn Marchetti, were presented jointly as a Powerpoint presentation. Both authors described their use of "Magic Words" in having their students explain why they performed the steps they did in solving open-ended mathematics items. The presentations were separated for purposes of this handbook because each author had her own message to impart. Diane's article is entitled: "'Magic Words' for Explaining Mathematics Problem Solving Steps." Carolyn's title is: "Using 'Magic Words' and the 'Two-Column Approach' in Explaining Mathematics Problem Solving Steps."

Stephen Cicioni's article is entitled: "Using Journal Writing and the 'Yes/No Method' in Teaching Mathematics." Steve routinely uses these two methods in teaching his mathematics classes. Steve also gave this presentation in one of the fall 1998 training sessions. Participants in the session found it so valuable that he was asked to repeat it in a fall 1999 session.

"Magic Words" for Explaining Mathematics Problem Solving Steps

by Diane Hurst[7]

I am an 8th grade math teacher at Conestoga Valley Middle School. This is my 9th year of teaching, 8 of them with 8th graders. I am also a member of the PA Department of Education's Mathematics Assessment Advisory Committee for the past 7 years. I would like to share with you some "MAGIC WORDS."

What are MAGIC WORDS? There are MAGIC WORDS for teachers: "Where is your homework?;' "Is it Friday yet?;" "What day of the cycle is it?;" "When is the meeting over?."

There are also MAGIC WORDS for toddlers. I have two little boys, ages 4 and 2, and am learning a lot about MAGIC WORDS for them. MAGIC WORDS for toddlers: "excuse me;" "please;" "thank you;" and "why."

There are also MAGIC WORDS for students that are taking the PSSA math open-ended responses in 5th, 8th and 11th grade. It is important to have your students start using these MAGIC WORDS so they can be comfortable with them. As with any adventure in life, practice makes perfect or at least gives us an advantage. MAGIC WORDS for students: "to get;" "to find;" "to figure out;" "to show;" "because;" "since;" and "therefore."

If students use these words in their explanations they will produce solid responses. Trying to explain "why" you have done a particular math step is difficult but if students begin their thinking with the MAGIC WORDS it will be easier.

Students need to practice using the MAGIC WORDS before they take the PSSA test in April. They need to be familiar with open-ended items and how to respond to them, so I start early. Some suggestions for this practice are: journal entries, homework questions, quiz or test questions, group work. How you practice in class depends on you as a teacher. I share the rubric with students and even let them practice grading each other's papers. If students understand the process, they will be more likely to give the PSSA test all of their efforts.

Other ways to prepare your students for the PSSA:

- Encourage students to try PSSA open-ended items. Tell them to never leave them blank and to show ALL their work, even if it was done mentally. They need to understand what this means, as simple as it sounds.
- Tell students to write something in words. They should be using the MAGIC WORDS in their writing.
- Practice, practice, practice.
- As a teacher, you need to talk positively about the PSSA test. Students need to see you supporting the test in order for them to give their best effort.
- Review school PSSA scores from the previous year. The scores are given to school districts to help the teachers but it is incredible how many teachers have never seen the scores. There is a page in the scores that breaks down the questions by standards categories. This will help teachers know what subjects need extra attention in the curriculum. Ask your principal for a copy of your school's scores.
- Be familiar with the layout of the PSSA test.
- Though the PSSA test only has 3 open-ended items that enter into students' scores, these questions are worth about 18% of their scores. Students need to realize that even though they worked diligently on the 70 multiple-choice questions, skipping the open-ended will drastically lower their scores.

[7]Diane Hurst teaches at Conestoga Valley School District and is a member of the Grade 8 Mathematics Assessment Advisory Committee.

Each year my principal brings a copy of the PSSA scores to me as soon as they are released. It is an exciting moment in my day and the students notice my enthusiasm.

In conclusion, practice using the MAGIC WORDS and your students will impress you!

Using "Magic Words" and the "Two-Column Approach" in Explaining Mathematics Problem Solving Steps

by Carolyn Marchetti[8]

I have been a 7th grade math teacher for 10 years and a member of the 8th grade Mathematics Assessment Advisory Committee for 5. In those 5 years, I have used various ways to incorporate open-ended questioning in my classroom. I have also done numerous presentations on this subject. I would like to share with you what works best for me.

I'd like to begin with the following example of a math item that is typical of those assigned to my middle school students:

> **Directions:** Show or describe all your work, even if you used a calculator or did it mentally, AND explain WHY you did each step.
>
> Rick Jones is working part-time at Tommie's Restaurant as a busboy to save up for a haircut and perm. Rick's haircut will cost $18 and his perm will cost $45. Rick earns $5.25 an hour. How many hours must Rick work to pay for his haircut and perm?
>
> Note: Rick is too cheap to tip the hairstylist.

When students first encounter open-ended questions, they want to describe their steps ("I did this, then this," etc. . .). The purpose of these types of questions is to get students to **explain** rather than **describe**. Here is an example of a description for the problem above:

$$\begin{array}{r} 38.25 \\ +\ 17 \\ \hline \$55.25 \end{array} \qquad 4.25\overline{)55.25}\ \text{13 hours}$$

First I added $38.25 and $17 and got $55.25. Then I divided $55.25 by $4.25 to get my answer of 13 hours.

To help your students explain rather than describe, have them use the "magic words." These help guide them toward an explanation. These words are: "to get;" "to find;" "to figure out;" "to show;" "because;" "since;" and "therefore."

[8]Carolyn Marchetti teaches at Upper Daupher School District and is a member of the Grade 8 Mathematics Assessment Advisory Committee.

In addition to using these words, I have my students separate their paper into 2 columns—one for work and one for explanation. They must then number each step of their work and the corresponding explanation. This will ensure that they explain **each step**. I do not allow my students to use numbers in the explanation column—only words. This stops them from describing. An example of an explanation for Rick's haircut is shown below.

WORK	EXPLANATION
1. $\begin{array}{r} 38.25 \\ +\ 17 \\ \hline 55.25 \end{array}$	1. First I added the cost of the perm and the cost of the haircut to find the total Rick owes the stylist.
2. $4.25 \overline{)\ 55.25}$ (13 hours)	2. I then divided the total by the amount Rick makes per hour at the restaurant to figure out how many hours he will need to work.

You should incorporate open-ended items into your class regularly. When first starting, have students work in groups or with partners. Eventually, they should be doing open-ended items on their own.

Have the students use calculators when doing open-ended items. This way they can focus less time on the computations and more on the explanations. All open-ended items should have at least 2 steps. Here are some other suggestions:

- **Practice should occur the entire year, at ALL grade levels.**
- Open-ended questions should be addressed at least **once a week.**
- Some classroom ideas for incorporating open-ended mathematics items in your teaching:
  - homework;
  - put a question on tests or quizzes;
  - extra credit questions;
  - cooperative learning activity;
  - open-ended portfolio;
  - math journal.

Finally, here are some sources for open-ended items:

- released items from the PA Department of Education;
- Mathematics Assessment Handbook;
- multi-step word problems from your textbook;
- text resource books;
- Internet sites;
- teacher-made questions.

Good luck!

Using Journal Writing and the "Yes/No Day" Method in Teaching Mathematics

by Stephen Cicioni[9]

I have found journal writing and the "Yes/No Day" teaching strategies to be very beneficial to both my students and me. Both methods help students to prepare for the state assessment, especially the open-ended tasks. The journal writing is great preparation for the students when explaining why the steps they used in a solution were taken. Over a period of time, journal writing helps the students to communicate more clearly and effectively and encourages them to develop a better mathematical vocabulary. This aspect of mathematics is not only useful on the state assessment but is also in the Standards adopted by the NCTM, and should be a goal of mathematics teaching. The "Yes/No Day" method also helps students to better communicate mathematically and to use precise mathematical language in their questions. It encourages them to put more thought into the questions they ask. Instead of asking how to perform a task, they must express their thoughts to discover if their thinking is correct. It forces the students to think more about what they are doing, both procedurally and conceptually, thereby providing them with deeper mathematical understanding. I will describe both these methods in detail below.

The "Yes/No Day" Teaching Strategy

One of my favorite teaching strategies is to use a "Yes/No Day" with my classes. The premise is simple. It is a class in which my students may only ask me questions to which I answer yes or no. (Please note, as a middle school teacher, this only lasts for one period per class.) I do not lecture that period. My instruction might take the form of demonstrating a skill or solving a problem on the board. It is also very helpful in reviewing a topic I have already presented to the class or when reviewing for a test.

Sometimes, it is necessary for me to respond to a student's question with "Please rephrase your question (or statement)." My intention is to have the students draw conclusions about the topic. Some students have no difficulty asking questions that can be answered by me. Others get very frustrated in their inability to phrase a question appropriately. On occasion, I find it useful to suggest they start their questions with: "Is it correct to. . .?" or "Do you mean. . .?" or "Should I have. . .?"

One really important aspect of the lesson they eventually learn, and sometimes it takes a very long time, is to listen to the questions of their classmates and to my responses. Once they learn to do so, they can piggyback on each other's ideas. When one student stalls, another usually picks up the line of questioning.

Learning and using good vocabulary is another important aspect of the lesson. Students must phrase their questions properly, so there can be no misunderstanding about what they are asking. I find the students really have to work hard. They see it as a challenge. Even for students who have great difficulty asking questions, it is a very beneficial learning environment. Their classmates can model good questioning skills and proper vocabulary for them. Usually by the second or third time I have used this strategy, all students are able to participate.

[9]Stephen Cicioni teaches at Manheim Township School District and is a member of the Grade 8 Mathematics Assessment Advisory Committee. He wishes to thank his colleague, Diane DeRemer, for her contribution in preparing this article.

Using Journal Writing As A Window To The Mind

Over the past fifteen years or so, I have used journal writing with my students as a means to determine what they are learning in my class. As a middle school teacher, I am always curious to discover what my students are learning in my class compared to what I am teaching, (and thereby, thinking they are learning). Journal writing provides a great medium through which I can see what is going on in their minds. At first, I ask mostly procedural questions, and when they are more familiar with the process, I ask conceptual questions. Some examples of each will follow later. To get started, my first journal entry consists of these four questions: What are your strengths in math class? What are your weaknesses in math class? What do you need (expect) from me as your math teacher? What else, if anything, would you like me to know about you?

I have found journal writing to be an invaluable tool to both assess students' understanding and help students communicate their thoughts and ideas about mathematics. The following pages have been provided as a guide to help to incorporate this strategy into your teaching. It takes time for students to develop the skills necessary to be good journal writers. However, the time and effort will pay off for both you and your students.

Parameters For Student Journal Writing (This is given to the students.)

- Answers must be in sentence form.
- The written response should stand alone. I shouldn't have to reread the question to discover what your responses answer.
- Don't use words such as "it," "they," "these," "this problem," or "that answer."
- Have someone else read your entry. Does it make sense?
- If you include an example, it doesn't take the place of an explanation.
- Try to make your response as specific as possible. When in doubt—put it in.

Suggested Progression of Journal Writing

- Student observations of the class.
- Student "recap" in his/her own words the concept(s) taught.
- Student develops conceptual understanding or an algorithm after being given some introductory material.
- Student justifies procedures and answer(s) derived.
- Student develops concepts with no introductory material being given.
- Student explains his/her thought processes.

Some Possible Journal Entries

1. How do you know that an absolute value can't be negative?
2. Given: $|-3|$ *and* $-|-3|$
   a. Explain what each means.
   b. Explain one difference between them.
3. Write examples of a false equation, a true inequality and a false inequality.

4. State one similarity between an equation and an inequality.

5. State one difference between an equation and an inequality.

   (General guidelines: To question procedures—explain what to do. To question thought processes—explain why you made the decision to do what you did.)

6. Given: $2(4 + 3) - 5$

   Explain how you would simplify this phrase.

   (The next time, give a similar problem and ask them to explain how to simplify the phrase without using any numbers in the explanation.)

7. Solve $2 - |3x + 1| > 8$. State the solution set. Explain the thought process you used to solve this problem.

8. If x = the number of dimes and $x + 15$ = the number of quarters, translate to English what $x + \frac{1}{2}(x + 15) = 50$ represents.

9. When adding three odd numbers, will the sum be even or odd? Explain your answer.

10. When adding four odd numbers, will the sum be even or odd? Explain your answer.

11. When adding even and odd numbers, how do you tell if the sum will be even or odd?

12. When multiplying three odd numbers, will the product be even or odd? Explain your answer.

13. When multiplying four odd numbers, will the product be even or odd? Explain your answer.

14. When multiplying even and odd numbers, how do you tell if the product will be even or odd?

15. When subtracting two integers, how can you tell if the answer will be positive or negative?

16. Will the fraction $\frac{12}{7}$ be equivalent to a percent less than 100 or greater than 100? How can you tell?

17. Given an equation and an inequality:

    a. State one similarity in how you solve them.
    b. State one difference in how you solve them.

18. Given: $x^6 - y^6$

    a. How do you know it is the difference of two squares?
    b. How do you know it is the difference of two cubes?
    c. Would the way you approach factoring this problem change your answer? Should it?

19. Write a story problem so the proportion $\frac{a}{22} = \frac{52}{100}$ could be used to solve it.

20. If x = the cost of a shirt
$x + 15$ = the cost of a pair of pants
$2x$ = the cost of a sweater
$2(x + 15) + 3(2x) = 270$

Write a story problem for which this setup and equation would be used to solve it.

21. Given: a shirt costs \$17
a pair of pants costs \$25
$2(17) + x(25) = 209$

a. How much money did you spend altogether?
b. How much money did you spend on shirts?
c. How many shirts did you buy?
d. What does the "x" represent in the problem?

Seventh Grade Writing Assignment on Ratio and Proportion

The ratio of boys to girls in the science class is 2 to 3, and there are forty students in the class. Could you use a word ratio of boys to girls to find the number of boys in the class? Explain why or why not.

Sample Multiple-Choice Items from the 2000 PSSA

STANDARDS CATEGORY 2.2

1. A secretary can type 56 words per minute. How much time will she need to type a 4200 word report?

A 7 hours 30 minutes
B 1 hour 4 minutes
C 1 hour 28 minutes
D 1 hour 15 minutes (correct answer)

STANDARDS CATEGORY 2.7

2. There are 9 packages, 5 red and 4 green. There are calculators inside 4 of the red packages and inside 2 of the green packages. What is the probability of choosing a package containing a calculator from the entire group of packages?

A $\frac{4}{5}$
B $\frac{2}{3}$ (correct answer)
C $\frac{1}{2}$
D $\frac{4}{9}$

STANDARDS CATEGORY 2.8

3. Chris had the following equation for Math homework: $6x - 7 = 47$. Which of the following steps would be correct in solving the equation?

A $13x = 47$
B $6x - 7 + 7 = 47 + 7$ (correct answer)
C $6x - 7 + 7 = 47 - 7$
D $-x - 7 + 7 = 47 - 7$

STANDARDS CATEGORY 2.11

4. Which of the following **best** describes the pattern 4, 8, 12, . . .?

A $1 + n, 4 + n, 8 + n, \ldots$
B $n^2, n^3, n^4, \ldots$
C $n, 2n, 3n, \ldots$ (correct answer)
D $n, \frac{n}{2}, \frac{n}{3}, \ldots$

Sample Open-Ended Tasks from the PSSA

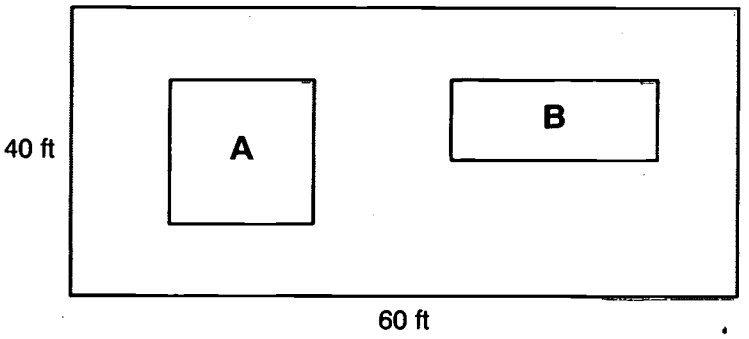
Two samples of open-ended tasks for the grade 8 level are shown on the following pages. Included with each task are: its specific rubric; a solution; and copies of actual student responses that exemplify each score point.

In the first task, "Courtyard," the student must determine the area of the entire courtyard and then subtract the grassy area from it to determine the area of the two cement pads for picnic tables. One pad is twice the size of the other. Since the student is working with three equivalent areas, the total of the cement pads can be divided by 3 and the size for each pad can be determined. This task assesses Standards Category 2.2.1, Numbers, Number Systems and Number Relationships, as well as Standards Category 2.2.5, Problem Solving and Communication.

In the second task, "Johnson Family Trip," the student must take into account the cost of taking a family trip with regard to all expenses of the trip and then determine the average cost per mile of the family outing. This task assesses Standards Category 2.2.5, Mathematical Problem Solving and Communication, and Standards Category 2.2.2, Computation and Estimation.

GRADE 8 SAMPLE TASK

A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



NOTE: Figure is NOT drawn to scale.

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

   AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

Problem Solution:

Area of courtyard is 60 feet by 40 feet. Apply area formula of $l \times w$, thus, $60 \times 40 = 2{,}400$ sq. ft.
Subtract area of grass: $2400 - 2040 = 360$ sq. ft. (area of both cement pads).
Area of pad A is twice the size of area of pad B, thus, $2B + 1B = 360$ sq. ft., $3B = 360$ sq. ft.
Solve for B: $B = 360/3 =$ **120 sq. ft.** (Area of pad B) and $2B = 2 \times 120 =$ **240 sq. ft.** (Area of Pad A)

The "why" might include the need to first find the total area of the pads which can be found by subtracting the area of the pads from the total area of the courtyard. Then, the student must see that if one area is twice another, he/she is dealing with three equivalent areas. Thus, there is a need to divide 360 into three parts. (The area of 360 is divided by 3. The area of the smaller pad is 120 and the area of the larger pad is twice that size, or 240.)

5 - Advanced Understanding, Excellent

- *Correct answer* with correct procedures/calculations shown OR described AND a correct explanation which tells "why" each step was taken.

4 - Satisfactory Understanding

A. *Correct answer* with correct procedures/calculations shown OR described AND an insufficient or no explanation.

B. *Correct answer* with <u>most</u> correct procedures/calculations shown OR described AND some explanation.

3 - Almost Satisfactory Understanding

A. *Correct answer* with most correct procedures/calculations shown or described AND no explanation.

B. *Correct answer* with few correct procedures/calculations shown OR described AND some explanation.

C. *Incorrect answer* with correct procedures shown OR described AND some explanation, BUT with one calculation or copying error carried throughout.

D. *Partially correct answer*, but student has switched the values for Pads A and B; work shown OR described AND some explanation.

2 - Partial Understanding

A. *Correct answer* with few correct procedures/calculations shown OR described OR some explanation.

B. *Incorrect answer*, with half or more correct procedures/calculations shown OR described AND some or no explanation. Other procedure(s) may be incorrect or missing. Student either proceeded incorrectly OR did not proceed far enough. For example, student gets to the 360 square feet, but, then doesn't know what to do or proceeds incorrectly, such as, dividing by 2 instead of 3 OR. . .?

C. *Incorrect answer* with correct procedures shown OR described AND no explanation, but no more than two calculation or copying errors carried through.

1 - Minimal Understanding

A. *Correct answer* with procedures, calculations and explanations that are not legible or not understandable or missing or incorrect.

B. *Incorrect answer*, with only one correct and critical procedure shown or described or explained (such as, 60 × 40). If there is a calculation (2,400), then, it must be correct to get credit OR student shows minimal understanding by clearly indicating that if area of pad A is twice the area of pad B, then, are dealing, ultimately, with 3 equivalent areas OR comes up with 360 with no support
OR . . .

C. *Incorrect answer* with correct procedures, BUT with three calculation or copying errors carried through.

D. *Correct OR Incorrect answer*, the student used the scale drawing concept. (Note: even though the drawing was NOT drawn to scale, some students attempted it in this manner and by their errors were able to get the correct answer.)

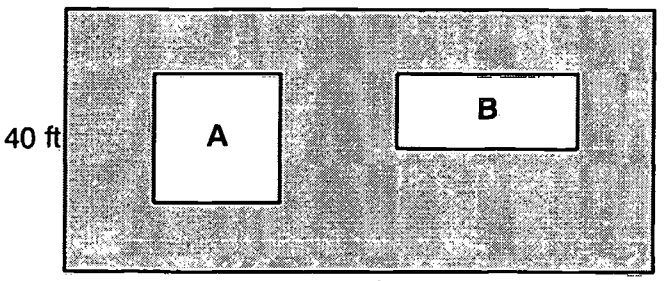
0 - Incorrect

A. *Incorrect answer* with no correct procedures, calculations or explanations shown or described. May, also, include a question mark or two (??) or "I don't know" written on page. Indication that the student HAS read the item.

*** Labeling error:** a "5" score becomes a "4" and a "4" score becomes a "3". Labeling errors do not affect scores of "3" or lower.

Note: Student only needs to label one of two answers, but cannot have one right and one wrong. For example: 120 square feet and 240, but NOT 120 square feet and 240 feet. . . .

79. A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



NOTE: Figure is **NOT** drawn to scale.

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

Pad A = 240ft²
Pad B = 120ft²

I arrived at this answer by first multiplying 40ft x 60ft to find the total area of the diagram.

60
x40
2400

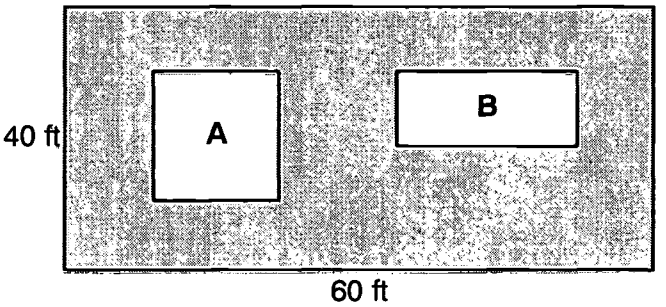
After completing this step, I subtracted 2040, which, according to the paragraph up top, is the total grass area, excluding the cement pads. My answer is 360ft² total Area of cement pads.

2400
- 2040
360

Since Pad a is twice the size of Pad b, I divided this number by 3. I got 120ft², the area of pad b. I then multiplied that by 2, to get the area for Pad A.

5 - Student has correct answer (240 ft² and 120 ft²) with all correct procedures and calculations shown and/or described with full explanation of reasons why each step was taken (The phrases: "to find;" "since;" and "to get" trigger those explanations).

79. A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



NOTE: Figure is **NOT** drawn to scale.

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

1. $60 \times 40 = 2400\ ft^2$
shaded $= 2,040\ ft^2$
$A = lw$

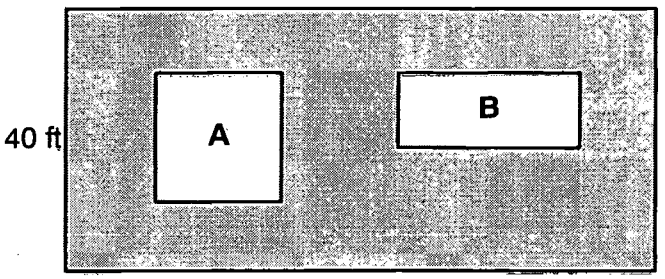
$2400 - 2040 = 360$
$A + B = 360$
$A = 2B$
$2B + B = 360$
$\frac{3B}{3} = \frac{360}{3}$
$B = 120\ ft^2$

$A + 120 = 360$
$A = 360 - 120$
$A = 240\ ft^2$

2. I made a systems of equations so that I could solve using substitution for one variable. Then I used the solution to find the value of the other variable. I chose this method because it was the easiest way to find the values of variables A and B.

4 - Student has correct answer (240 ft^2 and 120 ft^2) with all correct procedures and calculations shown with insufficient explanation of reasons why each step was taken. (Student is not specific in explanations regarding each step taken and only focuses on having taken an algebraic approach overall to the problem.)

79. A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



NOTE: Figure is **NOT** drawn to scale.

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

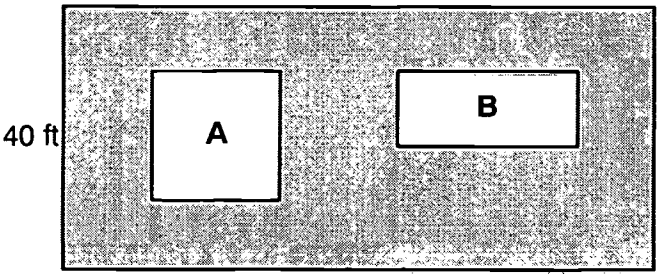
60
x40
2400
-2040
360
↓
120
3|360
3
60
60
0
↓
120
x 2
240
120
360

answer is pad A - 240
pad B - 120

this was they way I learnt to solve problems like this

3 - Student has correct *numerical* answer only (240 and 120) with all correct procedures and calculations shown with no explanation of reasons why each step was taken. Labels on answers are missing. (Square feet not indicated.)

79. A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



NOTE: Figure is **NOT** drawn to scale.

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

$$\begin{array}{r} 40 \\ \times\ 60 \\ \hline 2400 \end{array} \qquad \begin{array}{r} 2400 \\ -2040 \\ \hline 360 \end{array}$$

$360 \div 2 = 180$

$140 + 140 = 280$

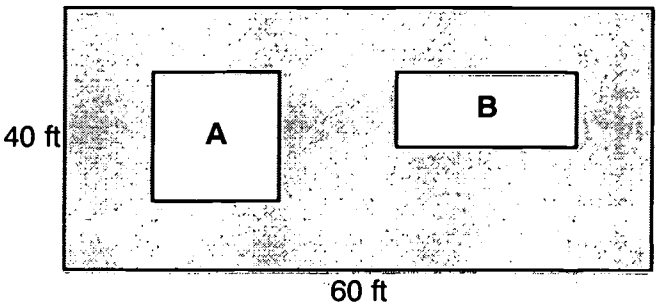
$$\begin{array}{r} 220 \\ 140 \\ \hline 360 \text{ sq ft.} \end{array}$$

Pad A = 220 sq ft.
Pad B = 140 sq ft.

At first cl read the problem then cl did 40x60 and got 2400 sq ft. and cl knew a portion of 2,040 was grass, Then cl subtract 2400 and 2040 and got 360. 360 represents the cement pads. Next cl had to divide 360 so that pad A would be twice as much than pad B. That is how cl got my answer

2 - Student has incorrect answer (220 sq ft and 140 sq ft) and has made a procedural error. Student has divided the 360 by 2 rather than 3, thus, committing an input error. Other procedures are correct and shown and/or described.

79. A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



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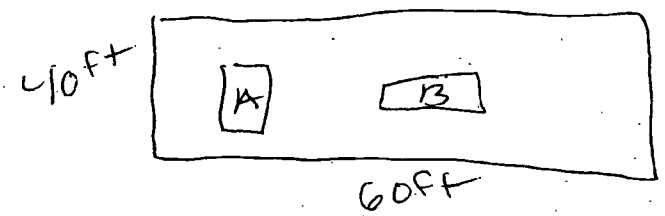
AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.



$A = LW = 40 \times 60$

$A = 2L + 2W = 80 \times 120$

$A = 9600$

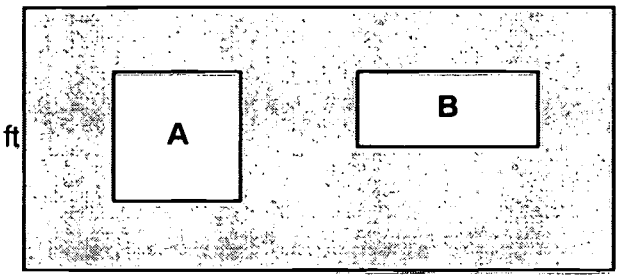
If the area of the grass is $\begin{array}{r} 9^{5}\cancel{6}00 \\ -\ 2,040 \\ \hline 7,560 \end{array}$

the answer is B = 2280 ft square

A = 4560 ft square

1 - Student has incorrect answer (2280 ft square and 4560 ft square) and has made several procedural errors, but does have one correct procedure shown. Student has correctly subtracted the area of the "grassy portion" from a perceived, though incorrect, courtyard area. Also, answers not in 2 to 1 ratio.

79. A rectangular courtyard is shown in the diagram below. The shaded portion represents grass, and rectangles A and B are cement pads for picnic tables and chairs. The area of pad A is twice the area of pad B. The remaining area is grass, which has an area of 2,040 square feet. What is the area of **each** cement pad?



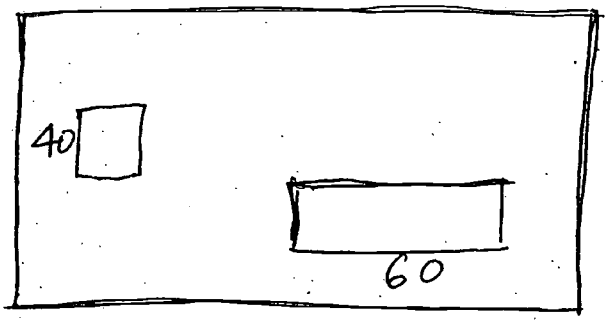
NOTE: Figure is **NOT** drawn to scale.

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.



3.14 x 40
3.14 x 40 x 40
3.14 x 1600
AREA = 5,024

3.14 x 60
3.14 x 60 x 60
3.14 x 3600
AREA = 11,304

0 - Student has incorrect answer (5,024 and 11,304) with incorrect procedures shown. (Student has used area formula for a circle rather than a rectangle.)

77

GRADE 8 SAMPLE TASK

Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

   AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

Problem Solution:

Find cost of gasoline: (13 × $1.25) = $16.25. Then, add $16.25 to $6.50 to $12.84 to get total cost of $35.59. Then, divide $35.59 by 221 to get **$.16** or **$.161** or **$.17** (per mile) OR "**about $.16**" or "**approximately $.16**" OR equivalent. (A less common approach: student finds cost per mile of gas, toll and food respectively, and, then, adds: $0.07353 + $0.02941 + $0.05810 = **$.16**/mile).

The "why" *must* deal with getting a total (by adding) and, then, getting an average by dividing. Mentioning the gas or rounding is not necessary, but may qualify for some explanation at a lower score point. . . .

Note: $ sign is a label. Thus, .16 is not $.16 and would drop a "5" to a "4" score and a "4" to a "3." Does not affect other score points. For example, .16$ will be acceptable if suspect dyslexia. $.16¢ or .16¢ is also a labeling error.

Also, for students who show or describe or explain dividing 35.59 by 221 as dividing 221 by 35.59 or something comparable and still getting $.16 as an answer, do not deduct from a "3" score down. If have it both ways, response could be a "4." If not, then, a "3" if all other requirements are met. It cannot be a "5."

5 - Advanced Understanding, Excellent

- *Correct answer* with all correct procedures/calculations shown OR described AND a correct explanation which tells "why" each step was taken. (Explanation need not address gasoline cost or rounding—just totaling and averaging.)

4 - Satisfactory Understanding

A. *Correct answer* with all correct procedures/calculations shown OR described AND insufficient or no explanation.

B. *Correct answer* with most correct procedures/calculations shown OR described AND some explanation.

3 - Almost Satisfactory Understanding

A. *Correct answer* with most correct procedures/calculations shown OR described AND no explanation.

B. *Correct answer* with few correct procedures/calculations shown OR described AND some explanation.

C. *Incorrect answer* with all correct procedures shown OR described AND some explanation, BUT with one calculation or copying error carried through.

2 - Partial Understanding

A. *Correct answer* with few correct procedures/calculations shown OR described AND some or no explanation.

B. *Incorrect answer* with half or more correct procedures/calculations shown OR described AND some or no explanation. Student EITHER did not proceed far enough or proceeded incorrectly. [e.g., found total expenses ($35.59 or 35.59), BUT did not attempt to find an average cost per mile OR divided 221 by $35.59 to get $6.2 or $6.21 OR divides by 3 to get $11.86.]

C. *Incorrect answer* with all correct procedures shown OR described AND no explanation, BUT with no more than two calculation or copying errors carried through.

1 - Minimal Understanding

A. *Correct answer*, BUT with procedures, calculations or explanation that are not legible or not understandable or missing or incorrect.

B. *Incorrect answer* with correct procedure to find total expenses, BUT incorrect procedure to find cost per mile with no more than two calculation or copying errors. Some or no explanation.

C. *Incorrect answer* with incorrect procedure to find total expenses, BUT correct procedure to find cost per mile with no more than two calculation or copying errors. Some or no explanation.

D. *Incorrect answer*, BUT with some information for solving task such as a correct partial procedure. [e.g.: 13 × $1.25 = $16.25 or 16.25 OR $1.25 + $6.50 + $12.84 = $20.59 or 20.59.]

E. *Incorrect answer* with correct procedures shown OR described, BUT with three calculation or copying errors carried through. Some or no explanation.

0 - Incorrect

A. *Incorrect answer* with no correct procedures, calculations or explanation shown or described. This includes $16.25 or 16.25 AND $20.59 or 20.59 with no work shown, described or explained.

B. *Question marks (????)*, "*I don't know*" or a written "*Absent*" on the response page. (Student has read or seen item.)

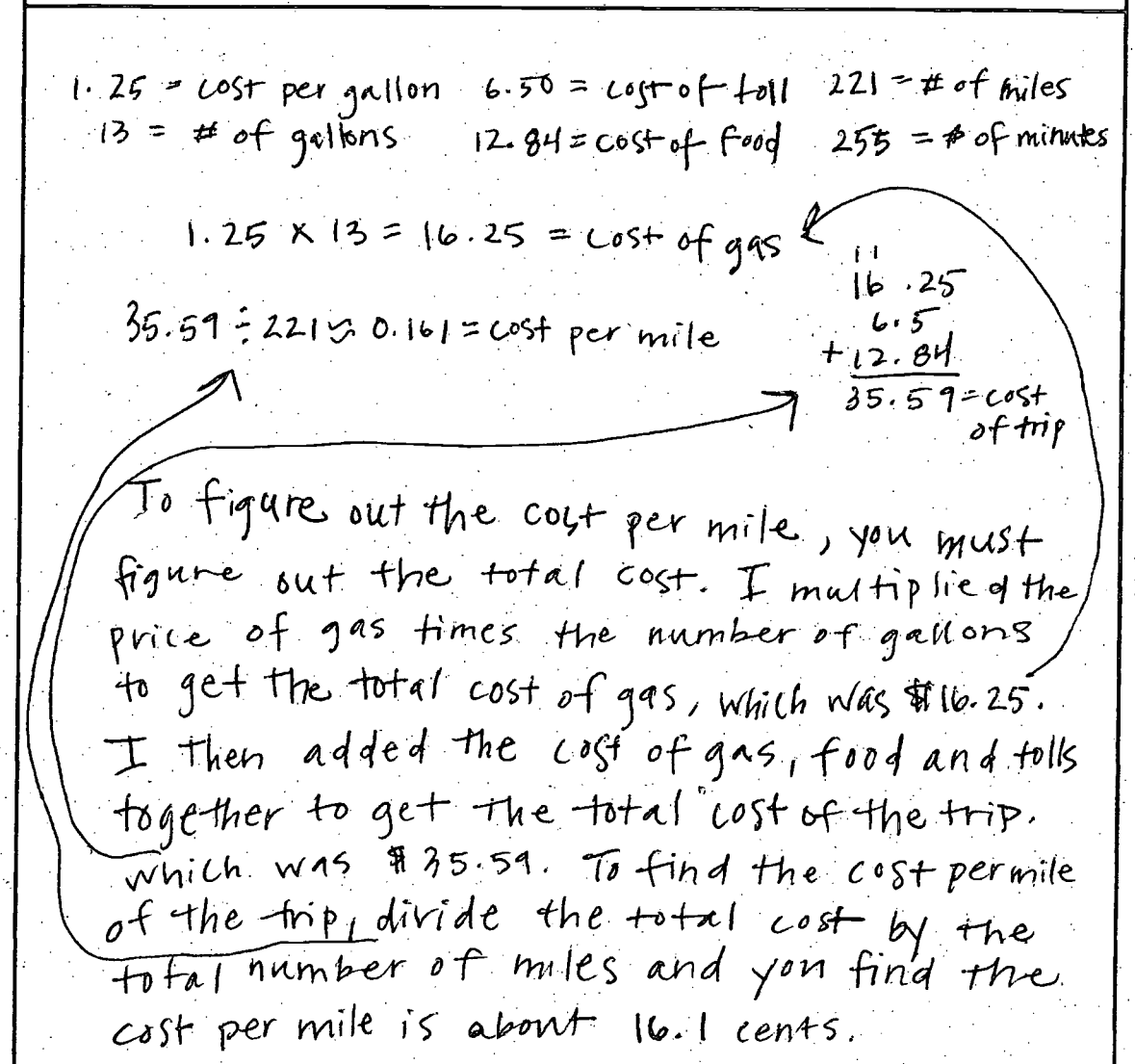
41. Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.



5 - Student has correct answer ("about 16.1 cents") with all procedures and calculations shown and/or described. Explanation is clear and complete. (The phrases: "to find;" "to figure out;" and "to get" trigger those explanations.)

Grade 8 Sample Response for Category 4

41. Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

221 miles

Total cost = $35.59

(13 x 1.25 + 6.50 + 12.84 = 35.59)

35.59 ÷ 221 = .161040724

= .16

the average cost per mile was 16 cents

I chose each of these steps because I thought it was the most less difficult way to find the total cost and divide it by the number of miles.

4 - Student has correct answer ("16 cents") with all procedures and calculations concisely shown. Explanation stating mathematical reasons for each step, however, is insufficient.

Grade 8 Sample Response for Category 3

41. Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

1.25
x 13
375
1250
$16.25

16.25
+ 6.50
$22.75

22.75
+ 12.84
$35.59

$35.59 ÷ 221 = $.15

First, I figured out how much gas was. After that, I added the cost of food and the turnpike. Next, to see the average cost per mile, I divided the full cost by the amount of miles and it come to be about 15¢

3 - Student has incorrect answer ($.15 rather than $.16) with correct procedures, but with one calculation error shown or described. Student has some explanation to remain at this score point (". . .to see" triggers that explanation).

41. Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

First I multiplied $1.25 and 13 to get $16.25 and add it to $6.50 and $12.84 and got $35.59

$16.25
$ 6.50
+$12.84
$35.59

Then I took 221 divided by $35.59 and got $6.21 as my final answer

$35.59\overline{)221} = \$6.21$

To check my answer of $6.21, I multiplied it by $35.59 and it answered as 221

2 - Student has incorrect answer ($6.21 rather than $.16) and has made a procedural error. Student has divided 221 by 35.59 rather than 35.59 by 221. Other procedures are correct and all calculations are shown. There is no explanation.

41. Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

It took #36.00 dollars for them to go from Pittsburg to Harrisburg. This is how I found out. I took 13 times #1.25 and got #16.25, and then I added the rest of it into #16.25.

I got #36.00 dollars,

1 - Student has incorrect answer ($36.00 rather than $.16) and has only a part of a procedure correct (finds cost of gas). Student may have known to add individual costs, but work is not shown and $35.59 is not determined. Additionally, student does not divide total cost by 221. The response is not totally incorrect.

41. Four members of the Johnson family took a trip from Pittsburgh to Harrisburg, a distance of 221 miles. It took them 4 hours and 15 minutes to make the trip. The car required 13 gallons of gasoline at $1.25 per gallon. The turnpike toll was $6.50, and they spent $12.84 for food. What was the average cost per mile based on the total expenses of gas, food and tolls for this trip?

For full credit, you **must** do the following:

1. show OR describe each step of your work, even if you did it in your head ("mental math") or used a calculator,

AND

2. write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

1.25 per g.
6.50 turnpike +
$12.84 food

221 m.
4 h.
15 m.
13 gal. of g.

273.69

1.25 per g.
6.50 tp
12.84
13 g. of g.

33.59
4
+ 15
13
65.59

16.3975
4)65.59

I plused all the numbers together then divided them to get the answer.

0 - Student has an incorrect answer with incorrect procedures shown. Student used hit and miss approach of just adding different combinations of numbers—none of them are correct.

The Content of the Test

The PSSA test for grade 11 encompasses concepts up to and including grade 11. The approximate numbers of points of the test devoted to each standards category are as follows:

Standards Category		Approximate Distribution of Points on the Test*
2.1	Numbers, Number Systems and Number Relationships	5
2.2	Computation and Estimation	10
2.3	Measurement and Estimation	10
2.4	Mathematical Reasoning and Connections	5
2.6	Statistics and Data Analysis	10
2.7	Probability and Predictions	5
2.8	Algebra and Functions	19
2.9	Geometry	11
2.10	Trigonometry	5
2.11	Concepts of Calculus	5

* Fifteen of the 85 points are from open-ended questions. They are included within the content areas they assess. For example, if one of the open-ended questions assessed Standards Category 2.3, Measurement and Estimation, 5 of the 10 points for this standard would come from the open-ended question and the remaining 5 points would come from multiple-choice questions. The open-ended questions are counted only once in total scores but their points are combined together for curriculum analysis purposes into a separate Mathematical Problem Solving and Communication score.

The list on the following page includes terms used in the Academic Standards for grade 11. Students should be familiar with these and should be able to apply this knowledge when solving problems. Since the grade 11 test assesses all content listed in the Standards, students also should be familiar with all the terms previously listed for grades 5 and 8.

Terms used in the Academic Standards for Mathematics through Grade 11

1. Absolute Value
2. Analytic Geometry
3. Arc
4. Arithmetic Series
5. Chord
6. Cluster Sampling
7. Compound Event
8. Consistent, Inconsistent (System of Equations)
9. Control Group
10. Deductive Proof
11. Direct Proof
12. Domain
13. Ellipse
14. Estimation
15. Experimental Design
16. Extrapolation
17. Finding Roots
18. Finite Sequence
19. Hyperbola
20. Indirect Proof
21. Infinite Geometric Series
22. Intercept
23. Interpolation
24. Lines, Curves of Best Fit
25. Logarithm
26. Matrices
27. Normal Curve
28. Odds
29. Opposite
30. Parabola
31. Proof by Contradiction
32. Radian
33. Raising to a Power
34. Rates of Growth/Decay
35. Reciprocal
36. Regression
37. Regression Equation of Best Fit
38. Secant
39. Sequences of Areas
40. Series
41. Simple Event
42. Standard Deviation
43. Statistical Measures of Center, Spread
44. Tangent
45. Treatment Group
46. Trigonometric Functions
47. Truth Table
48. t-Test
49. Two-Way Table
50. Validity of an Argument
51. Variance
52. z-score

Suggestions for Teachers in Preparing Students for the Assessment[10]

- Become familiar with the Academic Standards – the PSSA test reflects the mathematics standards.
- Review your school's results from the previous year. Be sure to look at the standard score for each mathematics strand — this will help you identify any specific areas of weakness. Pay particular attention to the results for the open-ended (performance) items. This will show you how well your students answer mathematics questions and explain their work.
- Have your students use a scientific or graphing calculator on a regular basis. Proper use of the calculator cannot wait until the day of the test.
- Make your students aware that there will be a section of the test where calculators **are not allowed**.
- Write some open-ended questions for your students. Evaluate their responses by using a rubric. Your rubric should be specific to the problem (see the examples in the handbook) but it should also reflect the categories explained in the General Problem Solving Rubric that is published in this handbook.
- Students should routinely use mathematics vocabulary in their explanations. Refer to the list of terms used in the Academic Standards for Mathematics through grade 11.
- Write some multiple-choice questions specifically for calculator usage.
- Write some sample questions for "mental math" solutions (i.e., no calculators allowed).
- Review the formulas given in the booklet and provide examples of their use for your students.
- Encourage all students to take this test seriously. Your attitude toward the test overtly and covertly communicates itself to your students.

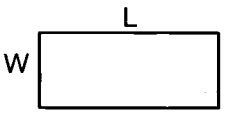
What Students Should Do in Preparing for the Assessment

- Become familiar with the meaning of each word in the list of terms from the grade 11 Academic Standards.
- The Formula Sheet provided to grade 11 students in the assessment is shown on the next page. Students should become familiar with these formulas and should know how to apply them to the solving of a problem.
- Know when and how to use a calculator.
- Learn what is expected of them in an open-ended item. This type of item is scored with a rubric and they can earn at least one point by showing a correct step toward solving the problem. To earn all the possible points, they will need to have all of the correct work, **show all their work and explain why they used each of the steps in their work**.

[10]This section was developed in large part by three Grade 11 Mathematics Assessment Advisory Committee members: Mary Moran (retired from Pleasant Valley School District), Dan Diefenderfer (Northampton School District) and M. Wayne Casto (Sullivan County School District).

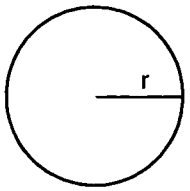
GRADE 11 FORMULA SHEET

Formulas that you may need to work questions on this test are found below. You may refer to this page at any time during the test. A calculator may be used on this test. You may use calculator π or the number 3.14.

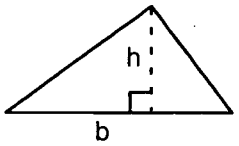


$A = LW$

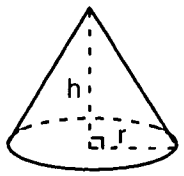
$P = 2L + 2W$



$C = 2\pi r \qquad A = \pi r^2$

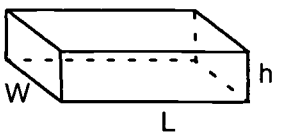


$A = \frac{1}{2}bh$



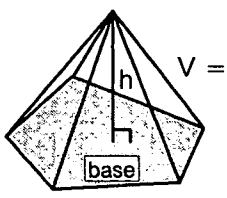
$A = \pi r^2 + \pi r\sqrt{r^2 + h^2}$

$V = \frac{1}{3}\pi r^2 h$

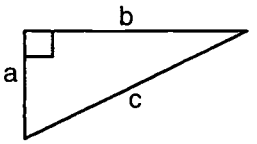


$A = 2LW + 2Lh + 2Wh$

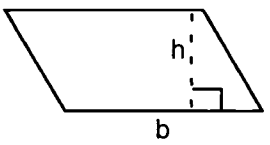
$V = LWh$



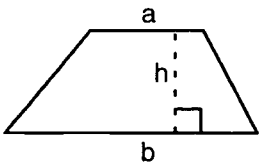
$V = \frac{1}{3}$ (Area of the base) × (Height)



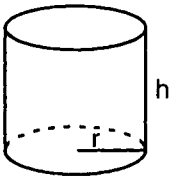
$a^2 + b^2 = c^2$



$A = bh$

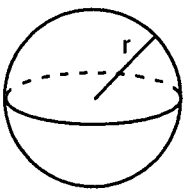


$A = \frac{h}{2}(a + b)$



$A = 2\pi r^2 + 2\pi rh$

$V = \pi r^2 h$



$A = 4\pi r^2 \qquad V = \frac{4}{3}\pi r^3$

Constant Motion $d = rt$

Simple Interest $I = prt$

Quadratic Formula

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Compounding Interest (n times per year) $\text{Amount} = p(1 + \frac{r}{n})^{nt}$

Permutations $P(n, r) = \frac{n!}{(n - r)!}$

Combinations $C(n, r) = \frac{n!}{r!(n - r)!}$

Standard Deviation $\sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N}}$

$\sin\theta = \frac{opp}{hyp} \qquad \cos\theta = \frac{adj}{hyp} \qquad \tan\theta = \frac{opp}{adj}$

Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos A$

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Nth Term of an Arithmetic Sequence $a_n = a + (n-1)d$

Nth Term of a Geometric Sequence $a_n = ar^{n-1}$

Sum of an Arithmetic Series $S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of a Geometric Series

$S_n = \frac{a - ar^n}{1 - r}$ *or* $S_n = \frac{a(1 - r^n)}{1 - r}$

Sum of an Infinite Geometric Series $S = \frac{a}{1 - r}$

$\text{Log}_b x = \frac{\text{Log}_c x}{\text{Log}_c b}$

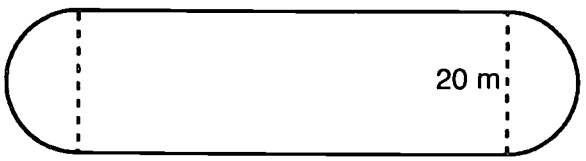
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$z = \frac{x - \mu}{\sigma}$

Sample Multiple-Choice Items from the 2000 PSSA

STANDARDS CATEGORY 2.3

1. The inside rail of a running track consists of a rectangle with a semicircle at each end as shown in the figure below. Find the approximate area surrounded by the track rail.



A 1200 m^2
B 2456 m^2
C 1514 m^2 (correct answer)
D 160 m^2

STANDARDS CATEGORY 2.4

2. In a group of 10 people, 7 people speak English, 4 speak French and 2 speak neither of the two languages. How many people speak both languages?

A 1
B 2
C 3 (correct answer)
D 4

STANDARDS CATEGORY 2.8

3. Which equation describes the relationship in the table?

x	y
1	3
2	6
3	11
4	18

A $y = x + 4$
B $y = 2x^2$
C $y = 3x$
D $y = x^2 + 2$ (correct answer)

Sample Open-Ended Tasks from the PSSA

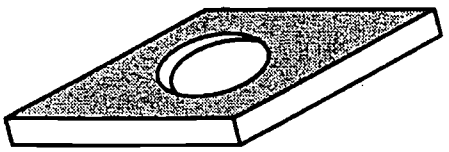
Two samples of grade 11 open-ended tasks are shown on the following pages. The specific rubric and student responses that exemplify each score point are included for each task.

The first task deals with finding the volume and weight of a metal washer. It assesses Standards Category 2.5, Mathematical Problem Solving and Communications, and Standards Category 2.3, Measurement and Estimation.

The second task deals with cost and profit from selling wreaths. It assesses Standards Category 2.5, Mathematical Problem Solving and Communications, and Standards Category 2.2, Computation and Estimation.

GRADE 11 SAMPLE TASK

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

Problem Solution:

Washer labeled with: W or L = 4 in.
Radius = 1 in or Diameter = 2 in.
Height = $\frac{1}{4}$ in.

Answers for the following may vary due to value used for π, rounding etc:

Volume of rectangular solid: = LWH
= (4)(4)(.25) cu in
= 4 cu in

Volume of cylindrical hole: = $\pi r^2 h$
= (3.14)(1)(.25)
= .785 cu in

Volume of washer: = 4 − .785
= 3.215 cu in

Weight of washer: = (3.215)(5 oz)
= 16.075 oz

5 - Advanced Understanding, Excellent

- Diagram of washer correctly labeled (L or W, R or D, and H)
- Correct numerical answers for Volume and Weight, correctly labeled (units)
- All work shown and fully explained with intermediate answers (e.g. Volume of rectangular solid, Volume of cylindrical solid, etc.) correctly labeled (units)
- Explanation includes why steps were performed
- Nothing incorrect

4 - Satisfactory Understanding

- Diagram of washer correctly labeled (L or W, R or D, and H)
- Correct numerical answers for Volume and Weight, correctly labeled (units)
- All work shown with some explanation
- Explanation is incomplete and/or addresses just what is being done, not why steps were performed
- Minor blemishes acceptable (e.g. incorrect units in work other than answers.)

3 - Almost Satisfactory Understanding

A. The response includes:

- Diagram of washer correctly labeled
- Correct Volume and Weight with or without correct labels (units)
- **Adequate work** shown and/or explained—some steps might be missing but you can tell what is being done or no verbal explanation

OR

B. The response includes:

- Diagram of washer correctly labeled
- Incorrect numerical answer for Volume or Weight due to **1 calculation**, copying or rounding error, with or without correct labels (units)
- Adequate work shown and/or explained

OR

C. The response includes:

- Diagram of washer at least **partially** labeled (at least 1 dimension correctly labeled)
- Correct Volume and Weight with or without correct label (units)
- Adequate work shown and/or explained

OR

D. The response includes:

- Diagram of washer **not** labeled
- Correct Volume and Weight with correct labels (units)
- **All work** shown with some explanation

2 - Partial Understanding

(NOTE: Labeling (units) of Volume and/or Weight no longer a factor in scoring):

A. The response includes:

- Correct numerical answers for Volume and Weight
- With or without labeling of diagram of washer
- Minimal work shown and/or explained

OR

B. The response includes (**Procedural error—Volume**):

- 1 out of 2 volumes correctly calculated with some attempt to use circular formulas (e.g. Area of circle, Volume of sphere, etc.)
- Corresponding values subtracted to find washer volume
- Corresponding weight correctly calculated
- Some correct labeling of diagram
- Supporting work shown and/or explained

OR

C. The response includes (**Procedural error—Weight**):

- Correct Volume of washer
- Weight incorrect due to procedural error or not attempted
- Some correct labeling of diagram
- Supporting work shown and/or explained

OR

D. The response includes:

- Incorrect numerical answers for Volume and Weight due to **more than 1** calculation, copying or rounding errors
- All correct procedures used
- Some correct labeling of diagram
- Adequate work shown and/or explained

1 - Minimal Understanding

A. The response includes:

- Correct numerical answers for Volume and Weight
- No work shown or explained
- With or without labeling of diagram

OR

B. "2B" or "2C" with calculation errors and/or unlabeled diagram

OR

C. The response includes:

- Volume of rectangular solid correctly calculated and Weight determined (Weight found before Volume of hole is subtracted)

  (4)(4)(0.25)(5) = 20 oz, **NOT** just (4)(5) = 20 oz
- With or without attempt at cylindrical volume
- Some correct labeling of diagram

- Supporting work shown and/or explained

OR

D. The response includes:
   - 1 volume correctly calculated
   - With or without attempt at 2nd volume
   - Some correct labeling of diagram
   - Supporting work shown and/or explained

0 - Incorrect

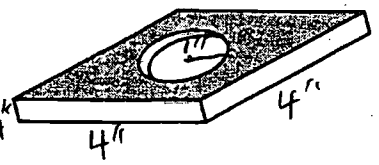
A. A response less than a "1"

ex.) Correct numerical answers with conflicting work

ex.) "I don't know" or "I don't understand"

ex.) Correctly labeled diagram with incorrect volumes, or volumes omitted

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

The volume of the washer would be the volume of the square washer minus the volume of the hole in the center because it is cut out.

volume of square washer = LWH

$(4)(4)(\frac{1}{4}) = 4 in^3$

The hole is similar to a short cylinder therefore use the cylinder volume equation

volume of hole = $\pi r^2 h$

$\pi (1^2)(1/4) = .79 in^3$

The total Volume = square − hole

$4 - .79 = 3.21 in^3$

weight is 5 oz / in^3 we have 3.21 in^3

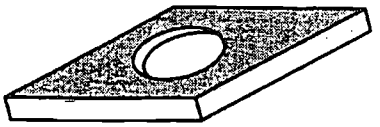
50 $5(3.21) = 16.05$ oz is weight of washer

Volume Answer:	Weight Answer:
3.21 cubic inches	16.05 oz.

5 - Student has labeled the diagram of the washer with the correct numbers. Student has the correct answers for the volume and the weight of the washer and each is correctly labeled (units). All work is shown and fully explained including the intermediate values such as the volumes of both the rectangular solid and the cylindrical solid. There is an explanation of why the steps were performed ("volume of the square washer minus the volume of the hole in the center because it is cut out").

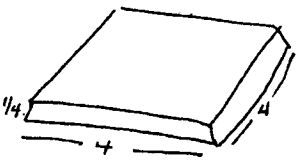
Grade 11 Sample Response for Category 4

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.



$h = \frac{1}{4}"$
$l = 4"$
$w = 4"$

To find the volume of the washer, you multiply the length times the width times the height & subtract the volume of the center, to find the weight, you must multiply the volume by 5oz.

$V = lwh$
$= (4")(4")(\frac{1}{4}")$
$= 4$ cubic inches

$r = 1$
$h = 1/4$

$A = \pi r^2$
$V = \pi r^2 h$
$= \pi(1)^2(1/4)$
≈ 0.79 cubic inches

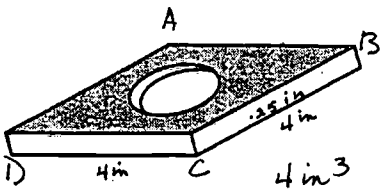
4.00
$-.79$
3.21 cubic inches
$\times$ 5 oz
16.05 oz.

Volume Answer:	Weight Answer:
3.21 cubic inches	16.05 oz.

4 - Student has labeled the diagram of the washer with the correct numbers. Student has the correct answers for the volume and the weight of the washer and each is correctly labeled (units). All work is shown and completely explained including the intermediate values such as the volumes of both the rectangular solid and the cylindrical solid. But, there is no explanation as to why the volume of the cylindrical solid is subtracted from the volume of the rectangular solid.

Grade 11 Sample Response for Category 3

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

$4in \times 4in \times .25in = 4in^3$

$\pi r^2 h$

$(3.14)(.1^2)(.25) = .7925\ in^3$

$4in^3 - .7925\ in^3 = 3.2075\ in^3$

Volume
$3.2075\ in^3$

$3.2075 \times 5 = 16.0375$

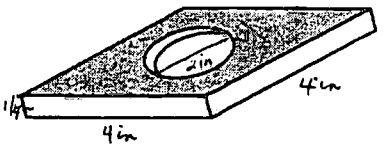
Weight
16.03750Z

Volume Answer:

Weight Answer:

3 - Student has labeled the diagram of the washer with the correct numbers. Student has incorrect answers due to one calculation error (.7925 in^3). Student has labeled the answers with the correct units. All work is shown, but there is no explanation.

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

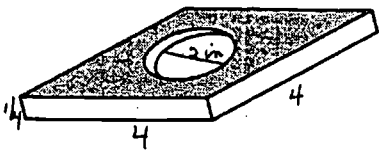
$V = 4 \times 4 \times \frac{1}{4} = 4 in^3 - \pi(4)(1/4) =$ 86 in³ = volume

$Wt = .86 in^3 \times 5 \text{ oz}/in^3 =$ 4.3 ounces = wt

Volume Answer:	Weight Answer:

2 - Student has labeled the diagram of the washer with the correct numbers. Student has the correct volume of the rectangular solid. But, the student uses the diameter of the circle, rather than the radius of the circle, in finding the volume of the cylindrical solid. This is an input error, and input errors are equivalent to procedural errors. Beyond that the student proceeds correctly—labeling the answers with correct units, subtracting the corresponding numbers to find the "volume" of the washer and finding the corresponding "weight." Supporting work is shown.

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

13.720z

Weight = L x w x H x (Weight metal) - hole weight

Weight = (4in x 4in x .25in)(5oz) - (3.14 [2])

W = 13.720z

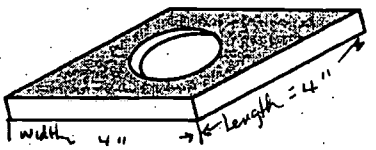
Volume Answer:

Weight Answer:

1 - Student has labeled the diagram of the washer with the correct numbers. Student has the correct volume and weight of the rectangular solid, and the weight is labeled correctly. But, then the student has two procedural errors in dealing with the cylindrical hole—using an incorrect formula for the volume, and not even attempting to find the corresponding weight. However, the student does have the supporting work for the volume and the weight of the rectangular solid.

Grade 11 Sample Response for Category 0

A machinist must make a square washer as shown in this figure. The width of the washer is to be 4 inches and the hole in the center has a diameter equal to half the width of the washer. It must be a quarter-inch thick.



Label the figure. Find the **volume** and the **weight** of the washer if the metal used to make it weighs 5 ounces per cubic inch.

Show your work and explain the steps you used to justify your answers. Do all work for this problem in the shaded region below. Remember you must show all the steps you used to solve the problem even if you have used a calculator. To receive the highest score, all calculation steps must be shown and verbally explained. Numerical answers must always be labeled.

$V = w \times h \times lt$

$(6 \times .25)$

$V = 5 \text{ cu in}$

$16 \times .25 = .80$

$42 \quad 506$

$\text{Weight} = 5 \text{ cu in} \times 5 \text{ ounce/cu in}$

25 ounces

Volume Answer:

Weight Answer:

0 - Student has some correct labeling of the diagram of the washer. But, even a completely correct diagram labeling is not enough for any points on this problem. The minimum is to get at least one of the volumes correct. Student correctly attempts to find the volume of the rectangular solid with $16 \times .25$, but then makes a calculation error ($16 \times .25 \neq 5$).

Dwayne and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

Problem Solution:

Algebraic Method:

Let x = minimum $ to charge per wreath
Gross income = 40x
Expenses/month = 50 (garage rental: electricity and heat)
140 (materials: 40 wreaths at $3.50/wreath)
Minimum Profit = 200 ($100 for each: Dwayne and Jonra)

Gross income ≥ **Expenses** + **Minimum profit**

$40x \geq (50 + 40(3.50) + 200)$
$40x \geq (50 + 140 + 200)$
$40x \geq 390$
$x \geq 9.75$

The wreaths should sell for a **minimum of 9.75** ($ not required)

Deductive Method:

A. Expenses (as listed above): 140 + 50 = 190
Minimum profit: + 200
Minimum amount needed/month: 390

Since 40 wreaths will be made/month, they must charge **at least** 390/40 = 9.75 for each wreath.

B. Garage expense/wreath = 50/40 = 1.25
Material costs = 3.50
Profits.wreath = 200/40 = 5.00
Gross Income = 9.75 charge/wreath

C. Each pays 70 + 25 = 95/month for materials + garage rental + 100 profit = 195
195 × 2 = 390, 390/40 = 9.75 min. charge/wreath

D. Each needs 100 + 25/month for profit + garage rental.
125 × 2 = 250
250/40 per wreath = 6.25/wreath
+ 3.50 materials/wreath
9.75 minimum charge/wreath

Trial and Error (maximum score of 4):

If sold wreaths for at least $9.75 (guess), the (check):
0.75 × 40 = 390.00
390 − (140 + 50) = $200.00 profit

5 - Advanced Understanding, Excellent

A. The response includes:
- Correct numerical answer (**$9.75 or 9.75**)
- All work shown AND fully explained
- Explanation includes why steps were performed
- Nothing incorrect (all quantities and variables, if applicable are defined. Simplistic labeling or trial and error method not acceptable at this level)

4 - Satisfactory Understanding

A. The response includes:
- Correct numerical answer (**$9.75 or 9.75**)
- Some explanation required (labeling acceptable)
- Explanation why steps were performed may be incomplete, unclear or might explain what is being done, but an attempt is made
- Work and/or explanation may contain minor blemishes

3 - Almost Satisfactory Understanding

A. The response includes:
- Correct numerical answer (**9.75**)
- All work shown with no explanation
- Work may contain minor blemishes

OR

B. The response includes:
- Correct numerical answer (**9.75**)
- Adequate work shown and/or explained (some steps are missing but you can follow what is being done — all appropriate mathematical procedures are used)

OR

C. The response includes:
- Incorrect numerical answer due to one calculation, copying or rounding error
- Equation must be set up correctly if used

- Adequate work shown and/or explained

2 - Partial Understanding

A. The response includes:
- Correct numerical answer (**9.75**)
- Some work shown and/or explanation
- Appropriate mathematical procedures used

OR

B. The response includes:
- Incorrect numerical answer due to procedural error
  1) Equation is set up with correct terms, but signs may be wrong and solved accordingly
  2) Prompt is solved to result in $100 profit: (190 + 100 = 290, 290/40 = 7.25)
- Adequate work shown and/or explained
- Work may contain one calculation or copying error

OR

C. The response includes:
- Incorrect numerical answer due to more than one calculation or copying error
- Algebraic method of solution used
- Adequate work shown and/or explained

1 - Minimal Understanding

A. The response includes:
- Incorrect numerical answer
- No supporting work or explanation

OR

B. The response includes:
- Total material costs: (40 × 3.50 = 140 — with or without calculation), **and**
  - Correct calculation of total expenses: (**140** + **50** = **190**)
  - With or without attempt to calculate incorrect minimum price/wreath
- Some supporting work shown and/or explained

OR

C. The response includes:
- Takes into account only 2 out of 3 factors with no mention of a 3rd factor:
  (supplies: 140 Garage: 50 Profit: 200)
  1.) Garage + Profit: 150/40 = 6.25/wreath
  2.) Garage + Supplies: 190/40 = 4.75/wreath
  3.) Supplies + Profit: 340/40 = 8.50/wreath
  (can also be calculated to result in $100.00 profit)
- Algebraic work shown and/or explained

OR

D. The response includes:
- Response is solved to result in $100 profit—$7.25/wreath
- Only some supporting work shown and/or explained

0 - Incorrect

A. The response includes:

- Work not meeting minimal requirements for score point of 1 or above
  ex.) Correct calculation of materials only: (40 × 3.50 = 140)
  ex.) Garage rental costs per wreath: (50/40 = 1.25)
  ex.) Profits per wreath: (200/40 = 5.00 or 100/40 = 2.50)

OR

B. The response includes:
   - Correct numerical answer(s) and/or selections with conflicting or irrelevant work/explanation

OR

C. The response includes:
   - Nothing correct

OR

D. The response includes:
   - "I don't know," or question mark (?)

64. Dwyane and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

First we need to see how much they need to spend each month.

40 wreaths x 3.50 and +50 for heat & elec.

140 + 50 = 190 dollars per month

then to determine price we need to make an equation. if x is the price, then:

$$40x - \overset{+190}{190} \geq \overset{+190}{200}$$

their expenses — their added desired profit

$40x \geq 390$

$x \geq 9.75$ they must charge

at least $9.75 for each wreath

5 - Response uses algebraic method by first calculating total production costs. An equation relating the income, less costs, to profits is constructed. The variable (x) is defined and the correct answer ($9.75) is found. All work is shown and clearly explained. The use of labeling is acceptable for the algebraic method when all parts of the equation are clearly defined.

Grade 11 Sample Response for Category 4

64. Dwyane and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) **why** you chose each of your steps.

100
+100
200 profit
need $250 at least (50 for rent)
+140 materials
390 at least

$3.50
× 40 wreaths
140 for materials

to make
$390
from 40 wreaths

$\frac{390}{40}$ = $9.75 each wreath, at least

4 - Response uses the deductive method to determine the correct answer ($9.75). All work is shown with some explanation. The use of labeling is not adequate for the method used.

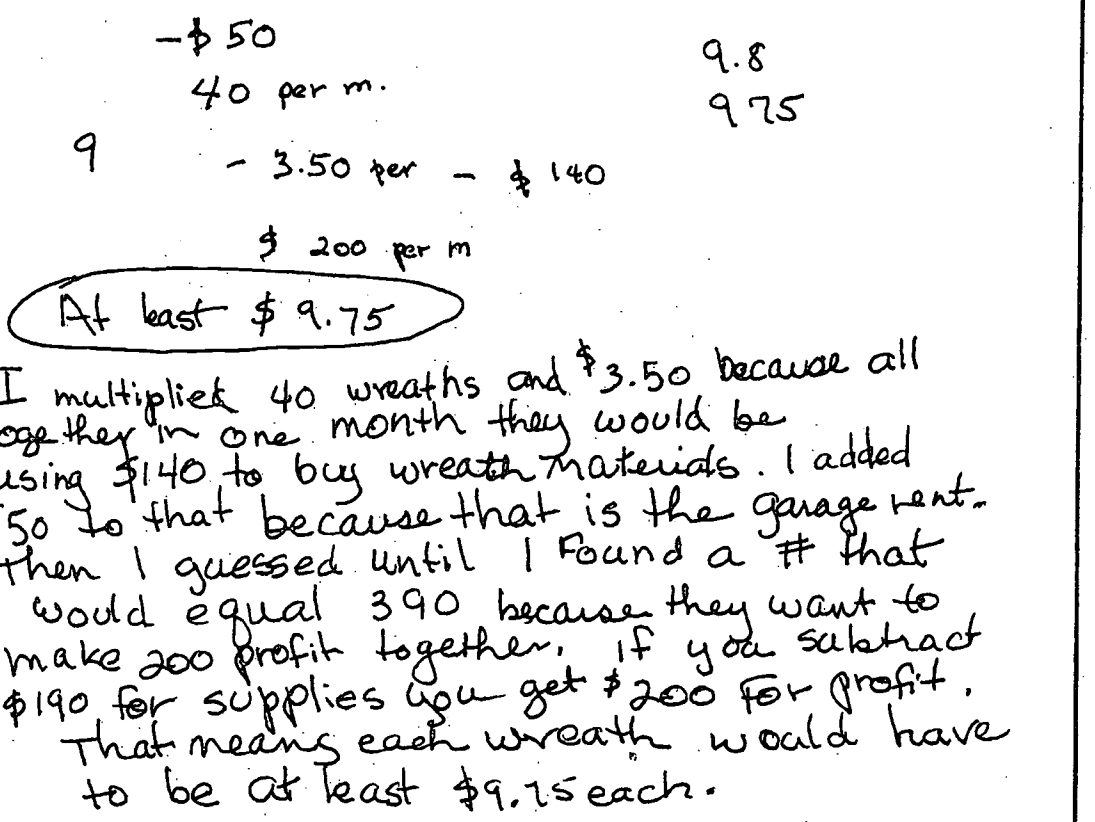
64. Dwyane and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) **why** you chose each of your steps.



3 - The response uses trial and error method to determine the correct answer ($9.75). Adequate work is shown and explained, but some work is missing (390 × 40 = 9.75). If trial and error is used, the response must show all of the steps in the trial and error process used to find the correct answer.

Grade 11 Sample Response for Category 2

64. Dwyane and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) why you chose each of your steps.

9.75

I added all the costs and the profits and divided by 40.

2 - The response uses the deductive method to determine the correct answer ($9.75). Some work is explained ("added all cost and profits and divided by 40").

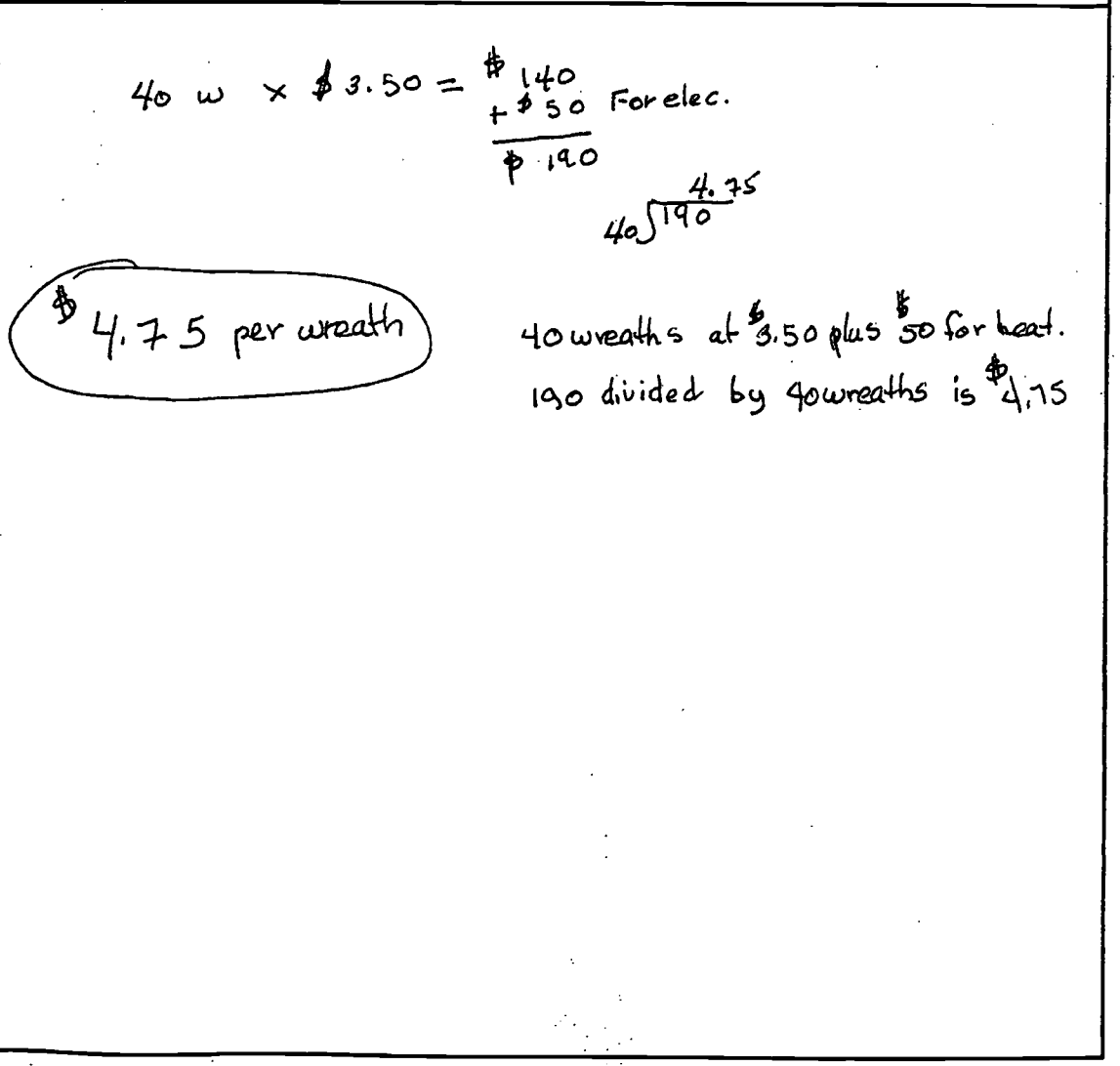
64. Dwyane and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) **why** you chose each of your steps.



1 - The response uses the deductive method to determine expenses/wreath ($4.75). Profits are omitted. Supporting work/explanation is provided.

64. Dwyane and Jonra decided that they are going to make and sell pine cone wreaths to make some extra money. Their parents agree to let them use the garage to make the wreaths if they pay $50 per month for electricity and heat. They estimate that they can make and sell 40 wreaths per month in the evenings and on weekends. They determine that the materials will cost about $3.50 per wreath. Each one of them wants to make at least $100 per month profit. What is the **least** amount Dwayne and Jonra must charge for each wreath?

For full credit, you **must** do the following:

1. Show all the steps you used to solve the problem. If you used a calculator or did some of the work in your head, you must write a description of the steps that you followed.

AND

2. Write an explanation stating the mathematical reason(s) why you chose each of your steps.

$50/month − 3.50/wreath + 40($) = $100 at least

40 wreaths / $3.5 materials ($13.25/wreath) − 50/month(electric) = $101.43

40 / $3.5 = $11.43

$11.43($13.25/wreath) = $151.45 − 50 for electric = 101.43

0 - The response "wreaths" havoc!!! The student attempts various strategies using the numbers given in the problem, but no strategy is correct.

APPENDIX A

MATHEMATICS ASSESSMENT ADVISORY COMMITTEE

Shown on the following pages are the present members of the committee. Also shown are the school district or university at which each member works, and, in parentheses, the grade level (5, 8 or 11) at which the member provides assistance in developing the state assessment.

Ann Allison (5) Keystone Central School District
Pamela Antonio (5) Bethlehem-Center School District
Glenn Aston-Reese (8) Trinity Area School District
Elizabeth Aulbach (5) Central York School District
Ann Bacon (8) Abington School District
Robert L. Baker (11) Conewago Valley School District
George Baranik (5) Harmony Area School District
Susan Barcelinno (5) Franklin Regional School District
Sandra Basanavage (5) Pennsbury School District
Renee Blanchard (5) Erie City School District
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David Bolton (8) West Chester University
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Stephanie Bricker (11) Fleetwood Area School District
Lorrie Bucklen (5) Ligonier Valley School District
Kelly Byrne (11) Octorara Area School District
Wayne Casto (11) Sullivan County School District

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Renetta F. Deremer (11) Hollidaysburg Area School District
Peter Desipio (11) Upper Darby School District

Frank Dessoye (11) Wilkes-Barre Area School District
Dan Diefenderfer (11) Northampton School District
Karen DiPrinzio (5) Great Valley School District (retired)
Madell Dobrushin (5) Pittsburgh School District
Robert Drupp (5) Southern York County School District
Diana Fitzgerald (11) Greensburg-Salem School District
Paulette Fleegle (8) Chestnut Ridge School District
Donna Gaffney (8) Springfield School District
William Garrett (11) Norristown Area School District
Charles Gerhart (11) Schuylkill Valley School District
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Victoria Harris (8) Penn Manor School District
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Judith Hudelson (5) Warwick School District
Diane Hurst (8) Conestoga Valley School District

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H. Jack Pencil (8) Bedford Area School District
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Darlene Plyler (8) United School District
Camala Potter (5) Bradford Area School District
David Rader (11) Wissahickon School District
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Joel Reed (11) Woodland Hills School District
Peggy Rhodes (5) Karns City Area School District
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Deborah Snavely (8) Manheim Central School District
Dennis Snyder (8) Hamburg Area School District
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