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ABSTRACT

The level of preparation for college entrance tests, university mathematics placement tests, and beginning college mathematics courses was studied for students who completed the Standards-oriented curriculum "Contemporary Mathematics in Context" (A. Coxford, J. Fey, C. Hirsch, H. Schoen, E. Hart, B. Keller, and A. Watkins, 2001), the high school curriculum developed by the Core-Plus Mathematics Project (CPMP). Eight CPMP field test schools provided SAT scores for their students, and 15 test schools provided ACT results. A total of 164 CPMP students and 177 precalculus students took a college placement test used at a major university. Freshman college mathematics course grade data for each year from 1995-1996 through 1998-1999 were gathered for the graduates of two high schools, one of which used CPMP. Neither the SAT nor the ACT assessments were well aligned with the content goals of the CPMP curriculum, but CPMP course 3 (third year of the curriculum) students appeared to be as well prepared for SAT mathematics as other high school students, and CPMP course 4 students appeared to be better prepared. The ACT was more problematic for CPMP students through course 3, but even those students were not disadvantaged on the ACT. The pattern of success of CPMP students on the college placement test was somewhat better than that of traditional students, in spite of placement criteria that ignored students' conceptual understanding and facility with applications if their algebraic symbol manipulation skills were below criterion level. These results provide evidence in support of the feasibility of curricula that embody Standards recommendations. (Contains 22 references.) (SLD)

# PREPARATION OF STUDENTS IN A STANDARDS-ORIENTED MATHEMATICS CURRICULUM FOR COLLEGE ENTRANCE TESTS, PLACEMENT TESTS, AND BEGINNING MATHEMATICS COURSES

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# Preparation of Students in a Standards-Oriented Mathematics Curriculum for College Entrance Tests, Placement Tests and Beginning Mathematics Courses

In response to calls for mathematics curriculum reform (National Council of Teachers of Mathematics, 1989, 1991, 2000; National Research Council, 1989, 1990), K-12 curricula have been developed that contain new and re-organized mathematical content. These curricula are also designed to support a teaching approach and classroom environment that emphasize students' engagement in making sense of mathematical ideas largely through problem solving. This pedagogical focus is consistent with recent research on teaching and learning for student understanding (e.g., Bransford, Brown & Cocking, 1999; Cobb, Wood & Yackel, 1993; Stigler & Hiebert, 1999; Fennema & Romberg, 1999). Some evaluation and research evidence is emerging that suggests a positive impact of at least some of these curricula on student learning (Senk & Thompson, In press). However, most of this research is at the elementary and middle school levels. There is very little evidence concerning how well Standards-oriented curricula prepare students for college and university mathematics.

In this report, we examine the level of preparation for college entrance tests, university mathematics placement tests and beginning college mathematics courses of students who complete the Standards-oriented curriculum, *Contemporary Mathematics in Context* (Coxford, Fey, Hirsch, Schoen, Hart, Keller, & Watkins, 2001). This is the high school curriculum developed by the Core-Plus Mathematics Project, and it will be referred to as CPMP or the CPMP curriculum. Our goal is to provide a description of the strengths and weaknesses of the mathematical preparation of CPMP students as compared to students who complete a more traditional high school mathematics curriculum.

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## BACKGROUND

### *Organization of the Curriculum*

In each year of the CPMP curriculum, mathematics is developed along four interwoven strands: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. These strands are connected within units by common topics such as symmetry, functions, matrices, and data analysis and curve-fitting. The strands also are connected across units by mathematical habits of mind such as visual thinking, recursive thinking, searching for and describing patterns, making and checking conjectures, reasoning with multiple representations, inventing mathematics, and providing convincing arguments. The strands are unified further by fundamental themes of data, representation, shape, and change. The choice of curriculum organization was influenced by the importance of connections among related concepts and procedures in developing deep understanding of mathematics (Skemp, 1987). This curriculum organization serves to break down the artificial compartmentalization of traditional "layer cake" curricula in this country and addresses weaknesses identified in the recent TIMSS findings (Schmidt, 1998). In addition, developing mathematics each year along multiple strands also seems to capitalize on the different interests and talents of students and helps to develop diverse mathematical insights (Hirsch & Coxford, 1997). Table 1 provides an overview of the scope and sequence of instructional units in the CPMP curriculum.

Table 1.
*Table of Contents for the Four-Course Core-Plus Mathematics Project Curriculum*

| **Course 1** | | **Course 2** | |
|---|---|---|---|
| 1 | Patterns in Data | 1 | Matrix Models |
| 2 | Patterns of Change | 2 | Patterns of Location, Shape, and Size |
| 3 | Linear Models | 3 | Patterns of Association |
| 4 | Graph Models | 4 | Power Models |
| 5 | Patterns In Space and Visualization | 5 | Network Optimization |
| 6 | Exponential Models | 6 | Geometric Form and Its Function |
| 7 | Simulation Models | 7 | Patterns in Chance |
| CAPSTONE | Planning a Benefits Carnival | CAPSTONE | Forests, the Environment, and Mathematics |

| **Course 3** | | **Course 4** | |
|---|---|---|---|
| 1 | Multiple-Variable Models | 1 | Rates of Change |
| 2 | Modeling Public Opinion | 2 | Modeling Motion |
| 3 | Symbol Sense and Algebraic Reasoning | 3 | Logarithmic Functions and Data Models |
| 4 | Shapes and Geometric Reasoning | 4 | Counting Models |
| 5 | Patterns in Variation | 5 | Binomial Distributions and Statistical Inference |
| 6 | Families of Functions | 6 | Polynomial and Rational Functions |
| 7 | Discrete Models of Change | 7 | Functions and Symbolic Reasoning |
| CAPSTONE | Making the Best of It: Optimal Forms and Strategies | 8 | Space Geometry |
| | | 9 | Informatics |
| | | 10 | Problem Solving, Algorithms, and Spreadsheets |

Evidence mainly from the national field test suggests that students in the CPMP curriculum score better on measures of conceptual understanding, applications, and problem solving than students at comparable points in the traditional college preparatory curriculum. Comparisons on measures of paper-pencil algebraic skill yield mixed results (Schoen & Hirsch, In press). Algebraic symbol manipulation skills are usually the main source of concern about preparation for college mathematics. In the next section, we discuss CPMP's algebra strand and how it compares to other traditional and reform approaches to algebra.

*Perspective on Algebra*

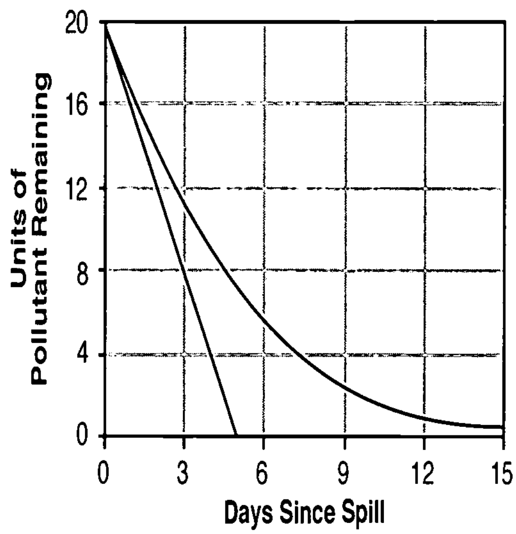
Traditional algebra curricula have focused on symbolic manipulation and procedures to solve rational and polynomial equations. In contrast, current reform efforts view algebra as a useful tool to describe mathematical and real-world situations. Reform efforts in algebra use a variety of approaches to teaching, including presenting algebra from a function perspective, as a way to generalize, as a tool for problem solving, and as a tool for modeling (Bednarz, Kieran, & Lee, 1996). While the CPMP curriculum reflects all four of these approaches to teaching algebra, an emphasis is placed on both a function and modeling approach.

The function approach focuses the study of algebra on quantitative relationships and their real-world applications (Heid, 1996). Students study relationships between quantities including causality, growth, and joint variation and learn to represent these relationships using symbols, tables, graphs, and verbal descriptions (Kaput, 1999). The modeling approach to algebra requires students to use mathematics, specifically functions, to describe relationships among real-world variables. In a modeling approach students use an equation, table, graph, or other mathematical object to describe a physical phenomenon. Modeling involves collecting data related to the phenomenon. Once the data is collected, students create a mathematical model to represent the situation and then use the model to reason or generalize about it. Modeling is a cyclic process since interpretation involves validating the model and may result in reformulation of the model (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Kaput, 1999).

In the CPMP curriculum, algebra is presented as a useful tool to model quantitative relationships and numerical patterns, and there is a focus on the importance of variables and functions. Students use algebraic concepts such as rate of change and functions to explore situations like the exponential launch activity in Figure 1 (Coxford et al., 2001). A lesson launch is intended to set the stage by generating discussion and thought about the main ideas of the upcoming lesson. In this case, the context is an experiment that simulates pollution of a lake by some poison and the following clean-up efforts. The launch also helps the teacher connect the situation to students' background knowledge.

**Think About This Situation**

One of the problems of our complex modern society is the risk of chemical or sewage spills that can pollute rivers and lakes. Correction requires action of natural or human cleanup processes, but both take time. The graphs below show two possible outcomes of a pollution cleanup effort following an oil spill.




(a). What pattern of change in pollution level is shown by each graph?

(b). Which graph shows the pattern of change that you would expect from a pollution cleanup effort? Test your idea by running the pollution clean-up experiment several times and plotting the (*time, pollutant remaining*)

(c). What sort of equations relating pollution $P$ and time $t$ would you expect to match your plot of data? Test your idea using a graphing calculator or computer.

*Figure 1.* Launching the Study of Exponential Decay (*Exponential Models* Unit, Course 1)

In CPMP, algebraic ideas are presented and linked through tabular, graphic, and symbolic representations, often using technology. By working with families of functions in a variety of contexts, students are expected to build a rich understanding of functions and functional relationships (Huntley et al., 2000). Through investigations and activities, students use mathematics, and algebra in particular, to represent real-life situations. They then use their representations to interpret, explain, and predict outcomes in the real-life context. Students' knowledge of families of functions and their properties aids them in modeling activities. The Checkpoint in Figure 2 illustrates the types of understandings that CPMP students are expected to develop about exponential functions in the second lesson of the Exponential Models unit of Course 1 (Coxford, et al., 1997). Pedagogically, a checkpoint serves as a framework for sharing the mathematical ideas developed by small groups of students during an investigation.

**√ Checkpoint**

In this lesson, you have seen that patterns of exponential change can be modeled by equations of the form $y = a(b^x)$.

(a). What equation relates *NOW* and *NEXT* $y$ values of this model?

(b). What does the value of $a$ tell you about the situation being modeled? About the tables and graphs of $(x, y)$ values?

(c). What does the value of $b$ tell you about the situation being modeled? About the tables and graphs of $(x, y)$ values?

(d). How is the information provided by values of $a$ and $b$ in exponential equations like $y = a(b^x)$ similar to, and different from, that provided by $a$ and $b$ in linear equations like $y = a + bx$ ?

Be prepared to compare your responses with those from other groups.

*Figure 2.* Summarizing and Formalizing Mathematical Learning

The CPMP curriculum incorporates graphing calculators throughout. In the study of algebra, the calculators serve as a tool for visualizing and manipulating tabular and graphical representations and for collecting data. The use of technology makes a focus on modeling and functions accessible and enjoyable for students with a broad range of interests and aptitudes. By using calculators and computers, students are able to study realistic situations and mathematically complex problems without being hindered by a lack of manipulative skills (Huntley et al., 2000).

In traditional algebra curricula, students are taught symbol manipulation via direct methods coupled with numerous practice exercises. CPMP approaches symbol manipulation by developing a conceptual foundation for manipulation skills. Early in the curriculum, symbolic forms are connected to graphic and numeric representations and formal symbolic manipulation is developed later in the curriculum. Steps in manipulation procedures are justified through symbolic reasoning and connections to contexts and other representations. Roughly, one-third of the CPMP units through Course 3 are mainly algebraic, and many other units make substantial algebraic connections. When students reach Course 4, they revisit symbol manipulation in a more formal and traditional manner, having the conceptual foundation already laid for them in earlier courses. All seven units in the Course 4 path designed for preparation for calculus contain substantial algebraic content.

A feature following each unit in Course 4 is relevant for symbol manipulation in the research reported here. At the end of each lesson in Course 4, a two-page feature called Preparing for Undergraduate Mathematics Placement (PUMP) provides multiple-choice test items similar to those typically found on university mathematics department placement tests. PUMPs serve to provide practice and item format familiarity that improves CPMP students' fluency in algebraic manipulation skills, especially as measured by traditional placement tests.

*Transition to College*

The present system for gaining entrance to college, being appropriately placed in college mathematics courses, and succeeding in those courses has existed with few substantial changes for several generations. Its familiarity and acceptance by most stakeholders in our society make the transition to college a major barrier to substantially changing the high school mathematics curriculum. While some changes are slowly working their way into the components of this system, it is still largely based on a traditional view of mathematics, of teaching, and of assessment.

The two widely used college entrance examinations in this country are the SAT and the ACT. In response to mathematics curriculum reform, both have begun to allow a calculator on their mathematics tests. The SAT I *Mathematics* test mainly measures mathematical reasoning and symbol sense, drawing on content from arithmetic, algebra and geometry. The SAT requires understanding of basic algebraic and geometric concepts typical of the first two years of traditional high school mathematics but measures little standard paper-and-pencil algebraic symbol manipulation. Both SAT *Mathematics* and ACT *Mathematics* include very little Statistics, Probability or Discrete Mathematics, content that has a substantial presence in most Standards-oriented high school curricula. Linn (1999) argues that SAT *Mathematics* emphasizes high-level reasoning in mathematical domains, but it is not tied to any particular instructional experience. Since high-level reasoning is an important reform goal as well, students are probably not disadvantaged on the SAT for having completed a Standards-oriented curriculum.

On the other hand, ACT *Mathematics* may be more problematic for students from reform curricula. Its content closely matches that of the traditional college preparatory mathematics curriculum through three years by measuring topics from pre-algebra, elementary algebra, intermediate algebra, coordinate geometry, plane geometry and trigonometry. ACT *Mathematics* items include a mix of algebraic, geometric and trigonometric concepts and procedures and fairly standard word problems usually intended to be solved using algebraic equations or inequalities. Recently, ACT *Mathematics* has begun to focus more on applications and problem solving, although these items are usually in a format and at a reasoning level similar to word problems in traditional textbooks. The ACT *Science Reasoning* test is also of interest since it is aligned with some reform goals. It requires students to retrieve information from graphs and tables, draw conclusions and predict results based on summaries of described experiments, and to compare two opposing views.

Beyond college admission, the mathematics departments of many large universities and some smaller colleges administer a placement test to their entering freshmen. The test is used as one source of information for advising students into the mathematics course that is best for them.

Controversial for some time, most of these tests are multiple-choice and focus largely, if not exclusively, on algebraic symbol manipulation skills. Such skills have been considered to be prerequisite to conceptual understanding and problem solving and the main determiner of success in traditional calculus. Many universities do not allow students to use calculators while taking these tests. It is typical for students from traditional high school curricula to score at levels which place them in a college course that is at the same or lower level than the one they just completed in high school. While it is too early to have much data, it seems reasonable to expect that students completing a Standards-oriented curriculum may be seriously disadvantaged on placement tests of this type. This study presents some of the first data on the topic.

After gaining admission and course placement, what of students' chances of succeeding in college or university mathematics courses? We know of no systematic studies of success rates in college mathematics courses of students from different high school curricula, but traditional college mathematics courses have expectations that may be quite different from Standards-oriented high school curricula. Fortunately for the potential success of the K-12 reform effort, a similar reform movement has been taking place in college calculus and other undergraduate mathematics as well. The fundamental goals of reform calculus are as follows (MAA, 1997).

- Calculus instruction should give the students an understanding of what mathematics is and how mathematics is done.
- Calculus instruction should develop students' conceptual understanding of important core content and not just manipulative skill. Students should also know how to apply the mathematics learned effectively.
- Students should be exposed to a broad range of mathematical problems. Being exposed to this broad range of problems provides exposure to a variety of mathematical techniques and methods.
- Students should make connections between different branches of mathematics.
- Students should be fluent in communicating their mathematical ideas.
- Students should be able to use various resources to learn mathematics.

Clearly, the above descriptors are well aligned with Standards-oriented high school curricula suggesting that reformed undergraduate mathematics courses are more familiar places for students from CPMP and other high school reform curricula. Furthermore, undergraduate mathematics reformers recognize the complexity of the mathematics learning process. New pedagogy and goals necessitate different types of assessment instruments. Examples of reform assessment tools are portfolios, open-ended problems on tests, student-constructed tests, projects, and interviews with instructors. Such a view of assessment suggests that changes may be coming, if slowly, in the kinds of assessments that university mathematics departments use for placing students into beginning courses.

## STUDIES OF THE TRANSITION TO COLLEGE

The CPMP evaluation team has been monitoring student outcomes in CPMP classrooms since the 1993-94 school year using a combination of a large-scale field test and more focused research studies. In this article, we report the results of three studies that have a bearing on students' level of preparation for college mathematics. The first study focuses on the SAT and ACT college entrance examinations, the second on a university mathematics department placement test, and the third on grades in beginning college mathematics courses.

### Performance on College Entrance Examinations

Most universities and colleges require applicants to complete either the SAT or the ACT college entrance examinations, and the results are used in the admission process as one indicator of potential for success in college. In this section, SAT and ACT scores of CPMP students are compared to those of students in parallel traditional mathematics courses.

*Instruments*

The SAT college entrance examination (SAT I) is comprised of two subtests, *Verbal* and *Mathematics*. The SAT *Verbal* and *Mathematics* score scales are standardized with a mean of 500 and standard deviation of 100. The ACT college entrance examination consists of four subtests, *English*, *Mathematics*, *Reading*, and *Science Reasoning*. ACT also reports a Composite score, the average of the four subtest scores, for each student. ACT subtest and composite scores are reported on a scale ranging from 1 to 36. Both the ACT and SAT allow, but do not require, students to use graphing calculators while completing the test.

*SAT Comparisons*

During the last year of the field test, eight CPMP field-test schools (6 suburban, 1 rural and 1 urban) provided us with SAT scores for all their students who completed the SAT. Two pairs of group means were compared, namely, (1) students who completed CPMP Courses 1 though 3 compared to those who completed traditional Algebra, Geometry and Advanced Algebra and (2) students who completed CPMP Courses1 through 4 compared to those who completed traditional Algebra through Precalculus. Since students could not be randomly placed into groups, analysis of covariance was used to compare Mathematics means after a least squares adjustment for differences in Verbal scores. The descriptive group statistics including adjusted SAT Mathematics means are given in Table 2.

Table 2.
*SAT Means, Standard Deviations and Least-Square Means in Mathematics for Four Curriculum Groups*

| | | SAT Verbal | | SAT Mathematics | | |
|---|---|---|---|---|---|---|
| | N | Mean | SD | Mean | SD | LS Mean |
| CPMP 3 | 148 | 527.0 | 91.4 | 516.4 | 93.3 | 520.2 |
| Adv. Algebra | 56 | 548.0 | 86.5 | 541.4 | 86.5 | 531.2 |
| CPMP 4 | 105 | 565.2 | 106.6 | 613.1 | 85.9 | 613.5[1] |
| Precalculus | 62 | 568.4 | 94.0 | 580.0 | 80.3 | 579.3 |

[1]Mean difference is significant ($F = 7.8$, $p = .006$).

After adjusting for SAT Verbal, the Mathematics mean of the CPMP Course 4 group was significantly greater than that of the Precalculus group. The adjusted Mathematics means of the CPMP Course 3 and Advanced Algebra students did not differ significantly.

*ACT Comparisons*

Fifteen CPMP field-test schools (10 suburban, 3 rural and 2 urban) provided us with ACT results for their students who completed the ACT during the last year of the CPMP field test. Since many of the field-test schools are in the midwest where the ACT is the predominant college entrance examination, the ACT data came from a much larger number of schools and students than the SAT data. The same group comparisons were made for ACT as for SAT. ACT English was used as covariate, since it is the ACT test that seems least likely to be impacted by different mathematics curricula. Furthermore, both the Mathematics and the Science Reasoning group means were compared after least squares adjustments for differences in English scores. The descriptive group statistics including adjusted ACT Mathematics and Science Reasoning means are given in Table 3.

Table 3.
*ACT Means, Standard Deviations and Least-Square Means in Mathematics and Science Reasoning for Four Curriculum Groups*

| | | ACT English | | ACT Mathematics | | | ACT Science Reasoning | | |
|---|---|---|---|---|---|---|---|---|---|
| | N | Mean | SD | Mean | SD | LS Mean | Mean | SD | LS Mean |
| CPMP 3 | 542 | 20.6 | 4.7 | 19.8 | 3.8 | 19.7 | 21.9 | 3.9 | 21.8[1] |
| Adv. Algebra | 246 | 20.4 | 4.6 | 20.3 | 3.9 | 20.4[2] | 21.1 | 3.7 | 21.2 |
| CPMP 4 | 191 | 24.5 | 4.9 | 25.0 | 4.7 | 24.6 | 25.5 | 4.6 | 25.2 |
| Precalculus | 99 | 23.0 | 4.3 | 23.8 | 3.5 | 24.4 | 23.9 | 3.6 | 24.5 |

[1]Mean difference is significant ($F = 8.4$, $p = .004$). [2]Mean difference is significant ($F = 6.9$. $p = .009$).

After adjusting for ACT English, the Mathematics mean of the Advanced Algebra group was significantly greater than that of the CPMP 3 group, while the Science Reasoning mean of the CPMP 3 group was significantly greater than that of the Advanced Algebra group. These differences in subtest means offset each other resulting in virtually equal adjusted group Composite means (CPMP 3: 20.93 , Advanced Algebra: 20.95). The CPMP 4 and Precalculus adjusted means did not differ significantly in either Mathematics or Science Reasoning.

## Performance on a University Mathematics Department Placement Test

A study was conducted to determine how the preparation for Calculus and other undergraduate mathematics courses attained by students in the CPMP curriculum differs from that attained by comparable students in more traditional curricula. A university mathematics department placement test was the criterion measure.

### *CPMP Course 4 Precalculus Sequence*

The Course 4 calculus-preparatory sequence for mathematics, engineering and the physical or biological sciences is outlined in Table 4. As is the case in traditional Precalculus courses, some of these ideas have already been introduced in previous CPMP courses but are dealt with more formally and deeply in Course 4. As in all CPMP courses, the topics are usually developed in the context of modeling realistic problem situations and then examined in terms of their underlying mathematical structure. Although use of graphing calculators is assumed, increased attention is given in Course 4 to analysis of symbolic representations of functions and associated symbolic manipulation and reasoning skills.

Table 4.
*Contents of the Seven Calculus-Preparatory Units in CPMP Course 4*

| | |
|---|---|
| 1 | Rates of Change<br>Instantaneous Rates of Change<br>Rates of Change for Familiar Functions<br>Accumulation at Variable Rates |
| 2 | Modeling Motion<br>Modeling Linear Motion – Vectors<br>Simulating Linear and Nonlinear Motion – Parametric Equations |
| 3 | Logarithmic Functions and Data Models<br>Inverses of Functions<br>Logarithmic Functions<br>Linearizing Data |
| 4 | Counting Models<br>Methods of Counting<br>Counting Throughout Mathematics<br>The Principle of Mathematical Induction |
| 6 | Polynomial and Rational Functions<br>Polynomial Functions<br>Polynomials and Factoring<br>Rational Functions |
| 7 | Functions and Symbolic Reasoning<br>Reasoning with Exponential and Logarithmic Functions<br>Reasoning with Trigonometric Functions<br>Solving Trigonometric Equations<br>The Geometry of Complex Numbers |
| 8 | Space Geometry<br>Representing Three-Dimensional Shapes<br>Equations for Surfaces |

*Sample*

The comparison of interest is the set of field-test students who completed the above sequence of seven units in CPMP Course 4 versus students at the end of a more traditional Precalculus course. In fact, during the field test, CPMP classes in the calculus-preparatory sequence did not typically complete all seven units. The sixth of these seven units, "Functions and Symbolic Reasoning," contains content that is especially crucial for calculus and is prominent on most placement tests. It includes both conceptual and symbolic manipulation work with logarithms, verifying trigonometric identities, solving trigonometric equations, and reasoning and calculating with complex numbers in trigonometric form. Thus, the decision was made to include in the comparison Course 4 students who completed six of these seven units, including "Functions and Symbolic Reasoning." Finally, since this placement test is designed to be taken by students when they first enter a university, only those Precalculus and Course 4 students who indicated on a written survey that they intended to enter a four-year college or university after graduating from high school were included in the comparison.

To summarize, the CPMP students (N = 164) are all those in the 1998-99 Course 4 field test who completed at least six of the calculus-preparatory units of Course 4 as part of their four-year study of the CPMP program. The Precalculus students (N = 177) are all college-intending students in the field test who were at the end of a traditional Precalculus course that was the fourth course in a college-preparatory sequence that included Algebra, Geometry, and Advanced Algebra. Both groups are composed of students who fell mainly in the $75^{th}$ to $95^{th}$ national percentile, on average about $85^{th}$, on standardized mathematical achievement tests at the beginning of high school. The very best mathematics students in these schools were likely enrolled in an AP Calculus course as seniors.

*Instrument*

A placement test that is presently used at a major university was administered to students in field-test schools at the end of CPMP Course 4 or at the end of traditional Precalculus. This multiple-choice placement test, compiled from a bank of items developed by the Mathematical Association of America, is used to make recommendations to entering freshmen concerning the college mathematics course that would be best for them. A graphing calculator that does not do symbolic manipulation, such as the TI-82 or TI-83, is allowed on this test.

This test contains three subtests—Algebra (15 items), Intermediate Algebra (15 items), and Calculus Readiness (20 items). The first two subtests consist almost entirely of algebraic manipulation tasks such as simplifying and factoring algebraic expressions, solving equations and inequalities, and finding equations for lines given sufficient conditions. The third subtest measures some of the important concepts and processes that underlie calculus such as reasoning with logarithmic and exponential equations, trigonometric functions and identities, composition of functions, rational functions and their domains, systems of nonlinear equations, and area under a curve.

*Group Differences by Mathematical Content*

Placement subtest means and standard deviations for each group are given in Table 5. On the Algebra subtest, the means of the Precalculus and CPMP Course 4 groups were virtually identical. On the Intermediate Algebra subtest, the mean of the Precalculus group was greater than that of the Course 4 group. The only statistically significant difference in means was on the Calculus Readiness subtest. That difference favored the CPMP students.

Table 5.
*Means and Standard Deviations for CPMP Course 4 and Precalculus Students on the Three Subtests of the University Mathematics Placement Test*

| | | Algebra | | Intermediate Algebra | | Calculus Readiness | |
|---|---|---|---|---|---|---|---|
| Group | N | Mean | SD | Mean | SD | Mean | SD |
| CPMP 4 | 164 | 11.5 | 2.6 | 9.2 | 3.4 | 12.9[1] | 4.7 |
| Precalculus | 177 | 11.4 | 2.3 | 9.6 | 3.2 | 10.5 | 4.3 |

[1]Mean difference is significant ($t = 4.93$, $p < .01$)

To further examine the group differences in performance by mathematical content, we ran a $t$-test of the difference between group mean percent correct for each item. This allowed us to identify all items for which the CPMP Course 4 and Precalculus mean percent correct differed substantially. We discuss these items next.

*Algebraic Test Items*

All Beginning Algebra and Intermediate Algebra test items for which CPMP Course 4 and Precalculus mean percent correct differed at the 0.01 level of significance are given in Figure 3. Group item means differed significantly on only two of the 15 Beginning Algebra test items, and the mean differences were significant on seven of the 15 Intermediate Algebra items (in all, four in favor of the CPMP students and five in the other direction).

| Beginning Algebra Test Items | |
|---|---|
| CPMP > Precalculus | Precalculus > CPMP |
| *BC 1. CPMP 91%; Precalculus 76%*<br>What is the equation of the line which goes through the points (0, 3) and (1, 5)? | *BP 1. CPMP 78%; Precalculus 91%*<br>The inequality $5x - 4 < 2x + 6$ is equivalent to: |
| **Intermediate Algebra Test Items** | |
| CPMP > Precalculus | Precalculus > CPMP |
| *IC 1. CPMP 55%; Precalculus 38%*<br>If $x > 0$, then $\sqrt{25x^2 - 9x^2} = ?$ | *IP 1. CPMP 59%; Precalculus 73%*<br>Subtract: $\frac{1}{b} - \frac{4}{a} = ?$ |
| *IC 2. CPMP 90%; Precalculus 78%*<br>If a rectangle has vertices (0, 0), (4, 0), (0, 3) and (4, 3), then the length of a diagonal is approximately: | *IP 2. CPMP 55%; Precalculus 69%*<br>Add: $\frac{b}{2a} + \frac{b}{3a} = ?$ |
| *IC 3. CPMP 65%; Precalculus 50%*<br>Which of the following best approximates the solution of the equation: $x^2 - 5x = 4$? | *IP 3. CPMP 64%; Precalculus 79%*<br>$\sqrt{27x^6y^9} = ?$ |
| | *IP 4. CPMP 44%; Precalculus 62%*<br>One of the factors of $15x^2 + 7x - 2$ is: |

*Figure 3.* Beginning Algebra and Intermediate Test Items on Whose Means for CPMP Course 4 and Precalculus Students Differed at the 0.01 Significance Level

Differing emphases of the CPMP and traditional curricula help to explain most of the differences in item means. The emphasis on multiple representations and functions in the CPMP curriculum may explain why CPMP students were better able to find an equation of a line through two given points (BC 1). A similar explanation may apply for IC 2 where students have to identify the opposite vertices of a rectangle in order to find the length of the diagonal. The other two items favoring CPMP were symbolic, but some understanding may help students avoid common errors. In IC 1, many students are tempted to take the square root of each term in the difference under the radical sign. Failure to move all terms to one side of the quadratic equation and set that side equal to zero is the common error in IC 3. It is likely that most CPMP students solved such an equation using either the graph- or table-building capabilities of their calculators, thereby avoiding the many pitfalls inherent in using the quadratic formula.

All the items on which the Precalculus students scored higher are straightforward uses of symbol manipulation algorithms that are commonly emphasized in traditional curricula. These include operations with rational expressions (IP 1 and IP 2) and factoring trinomials with leading coefficient greater than one (IP 4), topics which are deliberately de-emphasized in the CPMP curriculum in order to spend more time on conceptual understanding and problem solving. Another of these items required an answer in simplest radical form (IP 3), a topic that is less familiar to CPMP students who nearly always have a calculator available.

*Calculus Readiness Test Items*

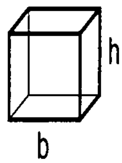
All Calculus Readiness test items for which the CPMP Course 4 and Precalculus mean percent correct differed at the 0.01 level of significance are given in Figure 4. Group item means differed significantly on 12 of the 20 Calculus Readiness items (11 in favor of the CPMP students and one in the other direction).

Consistent with evaluation findings described earlier, CPMP students performed at a higher level than Precalculus students on measures of conceptual and application outcomes. A perusal of the group item data and the items themselves illustrates both the magnitude and nature of the differences. Virtually all the items on which CPMP students did better measure understanding or application of mathematical concepts. Many items on which group means differed most involve graphical or diagram interpretation and/or verbally stated applications (RC 1, RC 2, RC 3, RC 7, RC 8, RC 9, and RC 11), both areas of emphasis in the CPMP curriculum. The large difference in understanding of exponents indicated by item RC 6 may be due to the early, conceptual introduction to exponents in CPMP Course 1 and frequent revisiting of situations involving exponential growth and exponential functions throughout the curriculum. The other Calculus Readiness items on which CPMP students did better (RC 4, RC 5, and RC 10) are symbolic, but they require understanding of key ideas and not just recall of procedures.

## CPMP > Precalculus ($p < .01$)

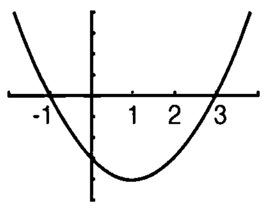
*RC 1. CPMP 65%; PreC 45%*

The box pictured below has a square base and a closed top. Express its surface area in terms of $b$ and $h$.




*RC 2. CPMP 73%; PreC 48%*

If $f(x)$ is a function whose graph is the parabola shown, then $f(x) > 0$ whenever:




*RC 3. CPMP 63%; PreC 40%*

A certain deer population increases by a factor of 1.2 each year. (For example, if there are 100 deer now, a year from now there will be 120.) Over a 12-year period, by what factor does the deer population increase?

*RC 4. CPMP 74%; PreC 56%*

If $\dfrac{(3x-1)(x+1)}{(x-1)} = 0$, then $x = ?$

*RC 5. CPMP 77%; PreC 62%*

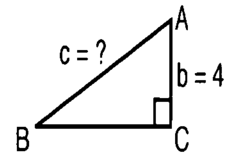
If $f(x) = 3x - 2$ and $g(x) = x^2$, then $g(f(x)) = ?$

*RC 6. CPMP 49%; PreC 29%*

If $2^{6,000} = 10^x$, then $x$ is

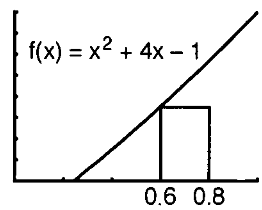
*RC 7. CPMP 66%; PreC 50%*

In the right triangle shown: $\sin B = 0.47$ and $b = 4$. Find $c$.




*RC 8. CPMP 77%; PreC 45%*

The area of the rectangle pictured below is:




*RC 9. CPMP 73%; PreC 48%*

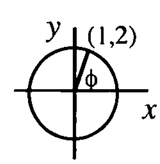
The point of intersection, in the first quadrant, of the line $y = 3x + 1$ and the curve $y = 2x^2$ has the $x$-coordinate equal to:

*RC 10. CPMP 67%; PreC 47%*

$\text{Cos}\,(90° - \phi) = ?$

*RC 11. CPMP 53%; PreC 37%*

The measure, in radians, of the angle $\phi$ shown in the circle below is:




## Precalculus > CPMP ($p < .01$)

*RP 1. CPMP 40%; PreC 56%*

$\cos\phi \cot\phi \sec^2\phi = ?$

*Figure 4.* Calculus Readiness Test Items Whose Means for CPMP Course 4 and Precalculus Students Differed at the 0.01 Significance Level

The item that favored precalculus students, RP 1 in Figure 4, involves recall of trigonometric definitions followed by simplifying a product of three trigonometric fractions. This difference in means is not unexpected. Because of the emphasis on circular functions (sine and cosine) as mathematical models, students worked less in CPMP than in precalculus classes with secant, cosecant, and cotangent functions.

The main educational significance of this test lies in its use as a tool to help place entering freshmen in beginning university mathematics courses. Course placements are considered next.

*Course Placements*

Typically, mathematics departments establish criteria in the form of "cut scores" to facilitate the placement or advisement of students into various freshmen mathematics courses. For the test used in the present study, Calculus I is recommended if (1) a student has a total score of 35 or higher. A precalculus course is recommended if (2) the combined score on the algebra and intermediate algebra subtests is at least 20. A more basic course is recommended if (3) neither of criteria 1 and 2 is met. These three criteria are applied in the given order, of course.

Notice that the Calculus Readiness subtest score does not enter into criterion 2. Based on these criteria, the algebraic skills of students placed in the same course are likely to be more or less homogeneous, but their levels of conceptual understanding as measured by the Calculus Readiness subtest may be very different. In fact, that is the case in the present study. The Calculus Readiness mean and standard deviation are given in Table 6 for subsets of the high school curriculum groups that met each of the above three placement criteria.

Table 6.
*Calculus Readiness Mean and Standard Deviation of CPMP 4 and Precalculus Students Who Met Each of Three Placement Criteria*

| | Criterion 1 | | | Criterion 2 | | | Criterion 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| Group | N | Mean | SD | N | Mean | SD | N | Mean | SD |
| CPMP 4 | 83 | 16.6 | 2.3 | 25 | 9.9 | 3.1 | 56 | 8.8 | 3.4 |
| Precalculus | 69 | 14.8 | 2.6 | 44 | 8.7 | 2.5 | 64 | 7.2 | 2.8 |

For those meeting each of the three criteria, the Calculus Readiness mean of CPMP Course 4 students is one-third to over one-half a standard deviation higher than the precalculus students' mean. In fact, the Calculus Readiness mean of CPMP Course 4 students in the criterion 3 group is essentially the same as that of traditional precalculus students in the criterion 2 group.

Performance in College Mathematics Courses

The following study provides preliminary evidence on how high school graduates who experienced the CPMP curriculum in its pilot version perform in collegiate mathematics courses. Freshmen mathematics course grade data for each year from 1995-96 though 1998-99 were gathered for all graduates of two similar high schools in the same midwestern, suburban school district who enrolled in a major university. For purposes of this report, the pseudonyms Midwestern University, East High School and West High School are used. Both East and West High School's 1995 and 1996 graduates experienced a traditional high school college-preparatory mathematics curriculum with offerings through AP Calculus. This program continued at West High School. At East High School, all 1997 graduates who were not in an accelerated mathematics program and all 1998 graduates completed the CPMP pilot curriculum. Accelerated students among 1998 East graduates took AP Calculus as seniors after completing CPMP Courses 1 through 4 in previous years.

Located in a suburb with many affluent, well-educated residents, East and West High School buildings (enrollments of about 842 and 1,070, respectively) are just two miles apart and are demographically similar. Many adults in the community are professionals in upper management positions. Over 80% of the students are white with Asian Americans comprising the largest of several minority groups. Less than 10 students in each school are eligible for the free lunch program. Freshman college mathematics course grades of graduates of these two schools who matriculated at Midwestern University were analyzed using computer data files with school names, but no student names, attached. Thus, the form of the data precludes any connecting of data to individual students, but allows for the following analysis of four-year school trends in college mathematics course-taking and grades.

Midwestern University uses the Harvard Calculus and other textbooks in its beginning courses that reflect the undergraduate curriculum reform. Pertinent mathematics courses at Midwestern University are Precalculus, Calculus I, Calculus II, Calculus III, Introduction to

Differential Equations, and honors (all honors math courses open to freshmen). Precalculus is the most basic mathematics course offered. Typically, freshmen enrolled in Precalculus have completed three to four years of college-preparatory high school mathematics but not AP Calculus. Freshmen enrolled in Calculus I in fall semester have usually completed at least four years of high school mathematics through Precalculus or CPMP Course 4, and some may have taken a high school AP Calculus course. Spring-semester Calculus I classes would also include some students who successfully completed Precalculus in fall semester. Freshmen in Calculus II or Calculus III would be placed there mainly because of high AP Calculus Examination scores or success in the preceding college calculus course in fall semester. Freshmen with exceptionally strong high school mathematics backgrounds and AP Calculus Examination scores may take Calculus III in fall semester and differential equations in spring semester.

Table 7 gives the number of matriculants (under the year) at Midwestern University among the 1995, 1996, 1997 and 1998 graduates of East and West High Schools, the numbers of these graduates completing each mathematics course in their freshman year, together with grade point averages and course averages. The grade point averages were calculated using Midwestern University's system: A+ (4.3), A (4), A- (3.7), B+ (3.3), B (3), ..., D (1), D- (0.7), E+ (0.3), and E (0).

Table 7.
*Mean Grade-Point Averages (Number of Students) by School, Course, and Year of High-School Graduation*

| | East High School (CPMP in '97 & '98) | | | | West High School (Traditional) | | | |
|---|---|---|---|---|---|---|---|---|
| College Class | 1995 (50) | 1996 (74) | 1997 (87) | 1998 (72) | 1995 (34) | 1996 (57) | 1997 (45) | 1998 (35) |
| Precalculus | 3.18(4) | 2.29(6) | 2.74(13) | 2.98(6) | 1.46(7) | 3.00(4) | 2.60(5) | 2.97(3) |
| Calculus I | 2.86(14) | 2.60(19) | 3.08(32) | 2.89(25) | 2.33(7) | 2.82(13) | 2.58(15) | 2.87(7) |
| Calculus II | 2.67(14) | 3.33(12) | 3.17(19) | 3.49(12) | 2.45(6) | 3.21(18) | 2.63(8) | 2.29(8) |
| Calculus III | 2.66(5) | 3.10(4) | 2.95(6) | 2.99(8) | 2.50(2) | 3.17(11) | 3.34(6) | 2.34(5) |
| Intro. to Diff. Equ. | 2.15(2) | 4.00(1) | 4.00(2) | 3.30(2) | --- | 3.67(3) | 3.65(2) | --- |
| Honors | --- | 3.28(5) | --- | --- | 3.3(1) | 3.77(3) | 4.23(4) | --- |
| All Courses | 2.76(39) | 2.89(47) | 3.06(72) | 3.07(53) | 2.15(23) | 3.15(52) | 2.92(40) | 2.57(23) |

University mathematics course grades of East High School graduates for 1997 and 1998 when the CPMP pilot curriculum was in place are higher, on average, than both pre-CPMP (that is, 1995 and 1996) East graduates and 1997 and 1998 West High School graduates. The number of 1997 and 1998 East High School graduates matriculating at Midwestern University is greater than it was in the previous two years. The percent of course enrollments in Calculus I is greater for West High School in 1997 (88% compared to 82% for East High); but in 1998 when all East High School graduates had completed the CPMP curriculum, these percents were 89% for East High and 87% for West High.

## SUMMARY AND CONCLUSIONS

A continuing central issue in the development of any mathematics curriculum is how to properly balance conceptual understanding, procedural skill, and problem solving in curriculum materials and classroom activities. Curricula like CPMP are based on the premise that conceptual understanding, procedural skill, and problem solving can and should develop together, largely through activities in which students engage in making sense of mathematical situations. On the other hand, traditional high school curricula have often tended to be organized and taught as a sequence of algebraic manipulative skills ordered from basic to more complex. The assumption underlying this organization seems to be that a high level of proficiency at manipulating algebraic symbols should be developed before focusing curriculum and instruction on understanding the meaning and use of the symbols and processes. As a consequence of these competing viewpoints, students from Standards-based high school curricula like CPMP may be penalized by the content, format and administration procedures of tests designed to align with traditional curricula. In this article, we reported studies of the preparation levels of students at or near the end of the CPMP curriculum for three traditional hurdles to entrance and success in college mathematics, namely, college entrance examinations, university mathematics department placement tests and grades in beginning college mathematics courses. We discuss the results relative to each hurdle in the given order.

Neither the SAT nor the ACT college admissions mathematics test is well aligned with the content goals of the CPMP curriculum. Two of CPMP's content strands—statistics & probability and discrete mathematics—are seriously underrepresented on both tests. Nevertheless, CPMP Course 3 students appear to be as well prepared for SAT Mathematics as students at comparable points in traditional curricula, and CPMP Course 4 students are better prepared. The likely explanation is that while SAT Mathematics is not tied to any particular instructional experience, it emphasizes high-level reasoning in mathematical domains (Linn, 1999). High-level reasoning is a very important instructional focus throughout the CPMP curriculum.

ACT Mathematics is more problematic for CPMP students through Course 3. Its content is closely aligned to the traditional Algebra, Geometry, Advanced Algebra and Trigonometry curriculum. By the time students are into the more formal and symbolic CPMP Course 4, any earlier curricular preparation differences there may have been on ACT *Mathematics* have disappeared. Interestingly, too, even CPMP Course 3 students are not disadvantaged on the ACT test overall, since they score better than traditional Advanced Algebra students on ACT *Science Reasoning*. As a result, ACT *Composite* means are virtually equal for the two curricular groups.

University mathematics department placement tests typically focus on algebraic symbol manipulation skills. Course 4 students who completed six of the seven calculus preparatory units were as well prepared for the algebraic skills and better prepared for the calculus readiness concepts and applications than traditional Precalculus students. CPMP students' pattern of course placement was also somewhat better than the traditional students, in spite of placement criteria that ignored students' conceptual understanding and facility with applications if their algebraic symbol manipulation skills were below criterion level. As a result, in each placement criterion group CPMP students had higher mean scores on calculus readiness concepts and applications than traditional students in the same criterion group. These results suggest that university mathematics course placement is an area where students from Standards-oriented curricula may be penalized by a test and its interpretation that are based on a traditional skill-first view of mathematics.

The preliminary evidence presented in this report suggests that the CPMP curriculum does not harm, and may help, students' grades in beginning university mathematics courses, but several caveats concerning this evidence are important to keep in mind. First, the data is from one school district in which the pilot version of Course 4 was used. Second, the university mathematics curriculum in the study was largely based on undergraduate reform principles that align well with CPMP (MAA, 1997). The grade pattern may be different in a more traditional university mathematics program. Third, related to the placement test issue discussed in the above paragraph, CPMP students may have been placed in university courses below the level for which they were prepared.

We believe the results presented here provide evidence in support of the feasibility of curricula that embody Standards recommendations. More research is needed to study the effect of the "final" versions of emerging Standards-based curricula on student achievement outcomes in high school and post-high school settings. Ideally, such research would involve schools that have faithfully implemented the curriculum for at least a few years so that (1) teachers understand and take advantage of the curriculum's full scope and sequence and (2) both teachers and students are accustomed to the expectations of the classroom environment.

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