DOCUMENT RESUME

ED 453 177		SP 039 977
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TITLE	Teacher Opportunities To Learn: Case Study of a Content Institute in Mathematics.	
PUB DATE	2001-04-00	
NOTE	32p.; Paper presented at the Annual Meeting of the American Educational Research Association (Seattle, WA, April 10-14, 2001).	
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)	
EDRS PRICE	MF01/PC02 Plus Postage.	
DESCRIPTORS	Elementary Secondary Education; *Faculty Development; Higher Education; Inservice Teacher Education; *Institutes (Training Programs); *Mathematics Education; Mathematics Teachers; *Pedagogical Content Knowledge; Problem Solving; *Summer Programs; Teacher Improvement	
IDENTIFIERS	Alaska; Subject Content Knowledge	

ABSTRACT

This study examined teacher opportunities to learn content subject matter at a summer mathematics institute. The Alaska Partnership for Teacher Enhancement was formed as a collaborative between a university, four school districts, the State Department of Education, the Anchorage Education Association, and General Communication, Inc. It offered summer institutes to improve teachers' content area knowledge. Researchers studied the mathematics institute, which included teachers from all grade levels. A university mathematics faculty member developed and delivered the institute curriculum. Three local teachers who had delivered mathematics professional development institutes in the past were also involved. Participants included teachers and teacher's aides with varied mathematics backgrounds. Researchers observed the institute, took notes on activities, collected activity sheets and journals, and interviewed presenters and participants. Overall, presenters viewed mathematical content through the lens of pedagogical content knowledge. They valued flexibility and responsiveness because they did not know who their participants would be. Presenters determined content and were extremely responsive to students, using various informal assessment strategies to determine student needs. There was an emphasis on inquiry-based activities. All activities included discussions about how to take learning back into the classroom. Participants considered the institute very successful. (SM)

Teacher Opportunities To Learn: Case Study of a Content Institute in Mathematics

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Paper presented at AERA Annual Meeting, Seattle, WA 2001

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Purpose of the study

As states have increasingly implemented standards-based assessment and accountability policies, more attention has been given to the need for teacher professional development. In particular, attention and funding have been focused on the need for increased subject-matter content knowledge, one of several recommendations articulated by reform organizations such as the National Commission on Teaching and America's Future (1996). Among the activities designed to address this need are summer "content institutes," intended to upgrade teachers' knowledge and typically lasting from several days to two weeks.

Yet, we know little about these opportunities. What do teachers have the opportunity to learn? If these are, indeed, opportunities to learn content subject matter, what ideas, information, and disciplinary methods are presented to teachers?

Moreover, little is known about how such decisions are made. Who is involved in deciding the nature of the content? How do those who initiate, organize, and facilitate these institutes conceive of content? What are they hoping teachers will learn? How do they mediate between the prerequisites of state and national standards and the expressed needs of their teacher-participants?

The purpose of this study is to shed light on questions such as these through an analysis of one case, a summer mathematics institute. It is understood that the findings are limited by the small scope. Nonetheless, the level of detail given through the qualitative approach lends an authenticity to the analysis that suggests illustrative areas for further inquiry.

Framework

"Content" is not a well-established construct and may be interpreted in a variety of ways. Schwab (1964) distinguished three different dimensions of disciplinary knowledge: the **substance** or ideas and information that constitutes the body of knowledge; the **structure** of discipline or how the ideas and information are organized; and the **syntax** or how knowledge is discovered, tested, validated, and overturned or revised.

Within Schwab's categories, variations of content definition are possible, depending on how one answers certain questions. For example, regarding *substance*, what ideas are deemed to be more foundational to the discipline than others? In mathematics, which numerical concepts form the linchpin for the understanding of others? (Ma, 1999) What skills should be considered basic? Within *structure*, ideas may be organized around integrated units, in a spiral curriculum, or according to a more hierarchical scope and sequence. The *syntax* of a discipline may be seen as more or less inquiry-based, depending upon the predisposition of the practitioner. While a pure mathematician may see the discipline as an open world of inquiry, an engineer is more likely to see a given set of truths and formulas.

Moreover, questions about content must be considered in context. The content defined for mathematics courses in colleges of arts and sciences will not be the same content appropriate to K-12 schooling, including all three of Schwab's dimensions, substance, structure, and syntax. Content knowledge required of professional mathematicians will not be congruent with the content necessary for public school teachers. The purpose of disciplinary study must be considered when deciding upon the appropriateness of content.

Thus, teachers have particular content needs, based upon the situation of schooling. Teacher knowledge involves a complex set of understandings that interact with each other. At the heart of the matter is that the primary purpose for teachers in studying a discipline is to improve their pedagogy. For teachers, content and pedagogy are inextricably linked in what Shulman (1986) terms **pedagogical content knowledge.**

Shulman (1987) identifies a model of pedagogical reasoning and action that describes the process that teachers utilize to transform content knowledge into usable information for the improvement of their practice. This process entails a cycle of learning that includes *comprehension* of subject-matter purposes, structures, and ideas; *transformation* of ideas into curricular representations and adaptations; *instruction* with students in the classroom; *evaluation* of effectiveness; and *new comprehension*. These stages are not meant to be in a linear sequence and may occur in various combinations and cycles. Shulman explains how this model is helpful in order to understand the role of content for teachers:

> Saying that a teacher must first comprehend both content and purposes, however, does not particularly distinguish a teacher from non-teaching peers. We expect a math major to understand mathematics or a history specialist to comprehend history. But the key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge that he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students. (p.15).

Research Question and Significance

The critical questions are: What content do teachers have the opportunity to learn in a summer content institute? How is the content decided?

These questions are significant because of the growing understanding of the need to improve teacher content knowledge and because of the increasing investment in teacher professional development. This investment is premised, in part, on the growing body of evidence that suggests that teachers' knowledge of their subject matter is correlated with student learning (Monk, 1994; Mullens, Murnane, & Willett, 1996).

Context and methodology for the study

The Alaska Partnership for Teacher Enhancement (APTE) was formed in 1999 with a Title II grant from U.S. DOE. A collaborative that includes the University of Alaska Anchorage (UAA), four school districts, the state department of education, the Anchorage Education Association, and General Communications, Inc., APTE organized and offered 4 content institutes in the summer of 2000 for the expressed purpose of improving teachers' content area knowledge. Each institute would last 10 days. The mathematics institute that is the focus of this study was one such institute. It was held in Anchorage at UAA.

The mathematics institute was meant to include teachers at all grade levels and from all of the cooperating school districts, which included three rural villages and the urban Anchorage district. A UAA faculty person in the mathematics department of the College of Arts and Sciences was selected by the APTE director to organize, develop, and deliver the institute curriculum. This person, in turn, hired a team of three teachers from Anchorage School District who had delivered district mathematics professional development institutes together in the past few years as part of a district initiative called the Mathematics Consortium. The team consisted of an elementary, middle, and secondary teacher, each of which was well-versed in national and state standards. Over the years, they had developed an extensive binder of activities that they had used in their institutes.

As it happened, the eleven participants who attended the institute presented a distinct bimodal profile. Seven of the participants were from an elementary school in a rural village, were Native Alaskans, and by their own admission, had limited to weak mathematical backgrounds. Only two of the seven were certified teachers; the other five were teacher aides. The other four participants were middle school teachers from Anchorage, Caucasian, and had, by their own admission, very strong mathematical backgrounds. None of the participants were known to the presenters until the first day of the institute.

Researchers observed five days of the 10-day institute, took field notes on activities, collected sample activity sheets and journal entries from participants, and interviewed presenters and participants. The data were described and analyzed utilizing the framework described above.

Summary and Analysis of Data

In what follows, we present our summary and analysis of data. We start by describing the course goals, and then move into an integrated description of how various data reflect *substance, structure, syntax, and pedagogical content knowledge*. Most of the activities observed and documented involved the interaction of these four aspects of content knowledge. Thus, instead of writing about these four areas as distinct and separate aspects, we describe the integration of these aspects in a collective narrative form.

To begin the work of developing the institute, the UAA faculty coordinator and the lead teacher of the presenter team got together to write a Course Content Guide (CCG). This is an official university document that details the learning goals and outcomes. The following goals were articulated in that document:

1) Standards for Mathematics: Participants will be able to demonstrate knowledge of the national, state, and local standards for mathematics and apply national, state, and local standards for mathematics to develop lessons (including assessments) for the classroom;
2) Exploring a Content Strand of the NCTM Standards: Participants will be able to increase their mathematics knowledge base in one of the NCTM Content Strands;
3) Technology: Participants will be able to use technology to create more effective lessons;
4) Mentoring and Peer Collaboration: Participants will be able to recognize the importance of teacher as learner and critique lessons created for their peers.

On the surface, it appears that only one of the goals, Goal 2, addressed content per se. In this goal, it is unclear what the content will be, other than the fact that only one strand will be the focus. How this focus would be determined was left unstated. There was, however, elaboration of the outcomes for this strand. They were stated as follows:

- Participants will explore using multiple representations;
- Participants will explore using hands-on activities;
- Participants will explore using manipulatives;
- Participants will explore using visuals;
- Participants will explore using writing/literacy.

Here can be seen a full emphasis on pedagogy, with little mention of content.

According to the presenter team, this document provided only the most general of structures for their decisions about what would happen during the institute. When asked at the end of the institute what their goals had been, the following comments were made:

> Everytime we met, I asked that question over and over again. Even after [the UAA faculty coordinator] told us, I still needed to ask, what is this that we're supposed to be doing?
>
> We wanted to teach them, show them, model to them, good mathematics and good mathematics instruction that's based on the NCTM Standards. That's kind of our overall feeling. Good mathematics AND how to teach it.
>
> The bottom line is that we weren't sure of the big picture exactly. So we knew that we would do something we could handle, that we knew how to do, that we could provide for them.

Thus, the evidence suggests that there was a loose linkage from the APTE intention of a content-centered institute to the UAA faculty coordinator to the team of presenters who were actually responsible for designing and implementing the activities. Perhaps feeling somewhat left on their own, the presenter team decided to rely upon what they knew, the experiences and materials they had shared together over the last few years of giving institutes. This institute would be an adaptation of work done for the Mathematics Consortium, with an emphasis on the NCTM Standards.

The presenters decided to organize the institute around what they called "big ideas": pattern blocks, base ten blocks, Pascal's triangle, data analysis, and the five NCTM process standards (problem solving, reasoning & proof, communication, connections, and representations). When asked about the content of the institute, these "big ideas" were what they cited. When it was suggested that the first three of the big ideas mentioned could be considered tools and activities rather than content, they gave the following replies:

> One of the reasons we chose the base ten blocks and the pattern blocks was because of the wide applications in all the different [content] areas.
>
> If you look at base ten blocks, there's all of this content that you can teach from them. Starting at place value, on up to algebraic thinking, you can teach a lot of content with base ten blocks. We know when we say base ten blocks to each other, that we're going to do adding, subtracting, multiplying, dividing, fractions.
>
> We know where we're going, so we don't need to have that discussion anymore. Maybe it's kind of a shorthand for us because when we say base ten blocks, we're saying place value, adding, subtracting - we're doing numeration and computation.
>
> It's a starting place. The valuable thing about saying pattern blocks and base ten blocks is that we know all the things you can do with it, with the content, but what we don't know is what the participants know, so that's why we can't say we're going to teach this, this, and this, because if they come and they've already had two years of working with pattern blocks, we're going to change our focus. Because we don't have to teach them, say, equality. We've learned different ways we can use the blocks.

Two points are noteworthy in this exchange. First, it is clear that the presenters viewed mathematical content through the lens of *pedagogical content knowledge*. This was especially the case when they were talking about using manipulatives to enhance their presentation of content. They saw the tools (e.g., pattern blocks, base ten blocks, etc.) as being representative of the *structure* or the starting place for organizing the *substance* of the content for instructional purposes. It is this perspective which allowed them to have the flexibility to respond to their participants and adjust instruction according to their needs.

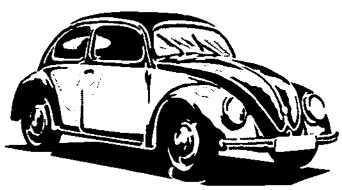
This flexibility is the second point worthy of note. The presenters placed a premium on flexibility and responsiveness because they had learned from experience that they wouldn't know ahead of time who their participants would be or what they would need to learn. In this way, they were not only dealing with the practicalities of an unpredictable summer workshop situation, but also might be said to be modeling a form of pedagogical content knowledge that their participants might use in their own classrooms. They elaborated:

> Now that we've worked together for so many times, we know not to have objectives. We know that we need to be as flexible as we can possibly be. So we didn't say we want to make sure that this, this and this happens. We said we'll use pattern blocks and see who we get in the class, and work from there.
>
> You have to be responsive to your students. All the different groups that we've taught over the last five, six, seven or eight years, they've all been different. They're all teachers, but sometimes you have more elementary, sometimes you have more secondary, people with a stronger math background or a weaker one. Sometimes you have groups that are more talkative, or groups that are quieter. So we discuss where we are with the group and what we want to do – to adjust to who they are. We did that in this institute the very first day.

The presenters' responsiveness to their students was, in fact, one of the most striking features of this institute. Faced with the bimodal group of participants described above, they quickly realized the need for varying seating arrangements, differentiating instruction, working in small groups, providing extra time for some or challenging extensions for others. There were occasions when the elementary teachers would continue working, for example, on using base ten pieces to represent numerical operations while the middle school teachers would be working on factoring with algebra tiles. Or the elementary teachers would be spending extra time with understanding how to teach fractions with pattern blocks while the middle school teachers were exploring graphing calculators.

Differentiation also included some self-selection of activities. Each morning would begin with problems to solve, labeled as elementary or middle school activities, placed at one of the three table clusters. Anybody at elementary was welcome to try middle school activities or vice versa. And indeed several did work across grade levels. The invitational nature of the setting allowed for student choice.

Further, many of the problems that were presented each morning reflected opportunities to learn variations of the substance of mathematical content. One example problem was:



A REAL LIFE MATH PROBLEM!!!!

> Fred Meyer just opened a gas station at their Muldoon store. To encourage and retain new customers, they are offering the following promotion. With every purchase of 8 gallons of gas, your "Frequent Fueler Card" will be punched. After you accumulate 10 punches, you will get $5 off any purchase of $25 or more in the store. Presently unleaded gas is sold for $1.549 per gallon. I would suspect that after initial sales promotions, the price will rise. My question

> is this. How much above the price of gas at other stations could I pay and still end up saving money at Fred Meyer? (I regularly shop at Fred Meyer and routinely make purchases $25 or more.) How can you represent mathematically or model this situation?

Within this problem, students were given the opportunity to explore various forms of mathematical substance. For example, several of the primary and elementary teachers explored the problem using number sense to make comparison while some of the upper elementary and middle school teachers used linear equations to represent the information.

As the institute progressed, there was more and more of a sense of open invitation to steer the learning. Said one presenter: "They were starting to feel so comfortable at saying, "I'm not comfortable with where I am with this topic, I need more, I'm not quite understanding this, can you help me…" They could identify where they needed to focus." And the presenters would modify their plans accordingly on a daily basis.

At the same time that the presenters made efforts to differentiate, however, they also recognized some common needs for the group as a whole and deliberately organized whole class activities and discussions in small cross-grade level groupings. This was especially true as participants engaged in discussions of the readings focused on the NCTM process standards and made charts which described and elaborated each standard. It was also true of most of the work done with patterning and with data analysis. The benefits of such heterogeneous groupings were described by the presenters:

> Almost everybody, including the middle school folks, needed to have a hands-on experience with these materials. They'd seen them but not used them, or didn't know how to use them.
>
> We also think it's important that they see the development of the ideas, so they can see how this develops for kids in elementary, on through to middle school. That continuum is very important. We like to have this kind of mix. We think that's important.

The presenters used various informal assessment strategies to determine the needs of their participants. A primary source of feedback was the daily journal kept by each participant. Each day, the presenters asked the participants to write an entry explaining for what aspects of the days work they felt in "equilibrium," and for what aspects they were in "disequilibrium." This opened a channel for communication about learning strengths and needs. Other feedback was obtained through the almost continuous interaction the three presenters had with their eleven participants, in small groups as well as whole groups, through observations, and ongoing discussions.

Underlying such communication and feedback was a noticeable and growing sense of comfort and sharing among the participants. The presenters consciously worked on creating a learning community through group activities and by explicitly discussing it with the participants. One of the many participant-created charts that came to adorn the classroom wall was entitled "Learning

Mathematics is a social experience." It detailed the brainstorming done by groups on the methods and purposes of developing a learning community.

Throughout the institute, there was an emphasis on inquiry-based activities. Seldom, if ever, were presenters observed to "give answers." Rather, they maintained a posture of questioning, facilitating, and guiding. Participants were, in effect, asked to "live" the process standards, as they were placed in the position of having to problem-solve, reason, communicate, connect, and represent their thinking. The presenters frequently encouraged participants to come to the overhead to show what they were thinking, to model their thinking with manipulatives, and to respond to others' ideas. Some specific observations of inquiry-based activities were: describing, extending, and generalizing patterns with pattern blocks; modeling computational story problems with base ten blocks; finding and predicting patterns in Pascal's Triangle; conducting a survey, graphing and interpreting the results.

One specific example of the inquiry-based activities that represented both *substance* and *syntax* was the construction of a Pascal's Pyramid using gumdrops and toothpicks (See Appendix A). Students were prompted to construct the pyramid and then answer questions concerning the features and mathematics found within the pyramid (See Appendix B).

This activity provided a focus on the "ideas and information" of mathematics (*substance*) by exposing students to geometric content. More specifically, using the Geometry Standard as presented in the *Principles and Standards for School Mathematics* (NCTM, 2000) the following content objectives were addressed:

- Analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- Specify locations and describe spatial relationships using coordinate geometry and other representations;
- Use visualization, spatial reasoning, and geometric modeling to solve problems (p. 164).

The organization of such a connected set of geometric knowledge around a rich task is an example of using the *structure* of mathematics to pedagogical advantage. In addition, this activity provided an opportunity for students to problem solve, estimate, and "discover, test, and validate" mathematical knowledge (*syntax*). Thus, in this and other learning activities during the institute, the instructors integrated multiple forms of content.

Moreover, virtually all activities included discussions about how to take this learning back into the classroom, how to make it work for students. The impetus for such conversations came from the participants as well as from the presenters. Said one presenter:

> We stay in touch with our participants, to know where their student population is, where they are going to take this and try it – when they are practicing an activity, they'll be saying, now I'm comfortable with this, but how will I take this to my kids – so it's important for us to always respond to that. I've been asked how will I explain this, what's the language that I should use with kids, or what steps should I take. We always try to help them think how they're going to apply.

As the participants took part in such engaging activities, the presenters were conscious of addressing the cultural differences in the room. As noted, seven of the participants were Native Alaskan, four Caucasian. The main concern expressed by the presenters was to try to give equal "air time" to all. Here's what one presenter said:

> The first thing that struck me was the quiet [of the Native Alaskans]. Whenever we presented questions, we knew in what order the Anchorage teachers would answer, and when they were done, that was the end of the conversation. So we gently encouraged comments from people who were not speaking, made it a point that at some point everyone made it up to the overhead projector to do a problem. We began to try to check with the Native people first, help them feel comfortable with where they were. Also, we'd have sharing just at your table, so it's a smaller group, you're not talking to the whole group...One day, everyone got two chips, and you could only speak if you put in a chip. That really cut down on the people who are very, very verbal. It worked great...The journals helped because people were able to express their thinking and comfort level in their journals. Whereas if we'd just had the discussion, it wouldn't have come out.

Participant perspective

Overall, the institute was judged to be very successful by all participants. On the last day, virtually all participants were expressing gratitude and a sense of accomplishment. The written evaluations were very high. One Native participant said:

> When we started, I always thought that in math you can only work with numbers. When I came here, I learned there's lots of ways you can work it, not just by numbers. You can use those different kinds of manipulatives in different ways...I always taught with numbers. I never thought of using base ten blocks or tiles or fraction pieces. When I came here, I realized how to use those things... I learned how you can add, subtract, multiply, divide with them.
>
> Most of the in-services I go to, they're all talking. This was mostly hands-on. I understood better.

Some journal comments:

> I learned that teachers can gain insights into students' thinking and their grasp of mathematical concepts by examining questions and interpreting their representations.
>
> I learned that my methods are not the only ones to solve problems.
>
> Doing hands-on experimenting will cement the learning method to the minds of the little people.

Effective problem solving involves experiencing disequilibrium. Otherwise there is little room or motivation to expand learning.

I really like how we began by looking at process standards. I now realize that they are just as important, if not more important, than the content itself.

In addition to teaching kids math, we need to teach them how to think mathematically.

No follow up was planned for the institute.

Reflection

Returning to our original question, what content do teachers have the opportunity to learn in a summer content institute? In the mathematics institute studied, the emphasis was on the process of "mathematizing" and on modeling appropriate pedagogical content knowledge with an integration of the substance, structure, and syntax of mathematical content. Using the starting points of structure and syntax rather than substance it was nonetheless evident that various forms of substance were embedded in the selected activities and throughout many discussions. Perhaps a better answer to this question lies in determining what types of pedagogical content knowledge were evident and in what balance.

This institute was vested with a charge to teach "mathematical content", but somewhere along the way that charge became ill-defined, lost in translation, or left up to the instructors to interpret as deemed fit. What was left was the best judgment and experience of three teacher-presenters who acted in a way that preserved flexibility for responding to participants. Not knowing ahead of time who these people would be or what their backgrounds and needs would be, the presenters, developed starting places and "big ideas" that would allow them freedom to use their own expertise to respond to their teacher-students. In this sense, this case illustrates how proficient teacher-presenters can make professional decisions for the sake of their particular circumstances when given a loose enough structure within which to move. Would the particular participants in this institute have been better served if there had been more rigid objectives and content specifications? Given the success of the learning community created, it is hard to believe that would be so.

What about the emphasis on process in this institute? Is that somehow antithetical to content? Or is it, in fact, an aspect of content itself? To use Schwab's framework, process may clearly be seen as part of the discipline's syntax. In mathematics as well as other disciplines, it matters how the "truth" is determined. The substance to be learned is learned through a process that is itself part of the content. That is, mathematical thinking and process is not apart from content; it is an essential part of it. It may be seen as the disciplinary means to a disciplinary end, greater understanding of substance.

And what about the emphasis on manipulatives and related problem-solving activities? Is this somehow apart from content? Not if we give credence to our presenters' ideas about using the manipulatives and problem-solving as the avenues that bring multiple content ideas together.

They are, in fact, using the tools of the trade as a form of structuring the learning. This is Schwab's third aspect of disciplinary knowledge. Using mathematical representations as an entry point and linking device for content may be seen as highlighting mathematical structure.

Is the balance correct between content and pedagogy? This is certainly arguable, depending on one's point of view. Some may see an overemphasis on pedagogy, but they would probably not be teachers. Matters of balance aside, what may be seen in this institute is that teachers were indeed working through Shulman's stages of comprehension and transformation. All along, participants and presenters were engaging in the conversations about how this might be done in the classroom. It was a recursive rather than a linear conversation, looping back and forth between comprehension and transformation. It is unfortunate that no follow-up was planned for these participants, making it a matter of hope rather than design that the remainder of Shulman's learning cycle will be enacted.

In short, the following conclusions arose from the data:

- The opportunity to learn for teachers in this institute was founded on a deliberate responsiveness to their interests, social as well as academic needs, and specific teaching contexts. National and state standards were employed in a process-oriented approach that stressed content syntax and structure as the entry points for content substance. The content emerged from the ongoing decision-making of teacher-presenters rather than from pre-determined objectives.
- The content was decided by the teacher-presenters. This may account for what may be viewed as an overbalance of pedagogy over content. And yet, who better to teach teachers than teachers? And as Shulman points out, how can authentic teacher learning ever be separated from pedagogy?

In closing, this case offers a view of a content institute where content is not divorced from process or pedagogy. This seems inevitable and desirable in the context of teacher professional development. The case also depicts problems with professional development. For example, why was a greater effort not made to determine who the participants would be and to do a needs assessment beforehand so that the institute agenda could be better planned? Why was no follow-up planned? How could policy intention and delivery be better linked, while preserving the discretion of the presenters to respond to their participants?

There are well-established and research-based standards for teacher professional development. Those planning summer institutes would be well-advised to understand and implement them. This would undoubtedly improve the effectiveness of such initiatives in improving teacher learning and classroom instruction, even though they may entail a greater investment of time and money. If we believe that improving teachers' content understandings will improve students' content understandings, and if we are serious about wanting to improve students' content understandings, then surely this is an investment we must make.

Finally, there is a question that begs to be asked. How do existing content institutes such as the mathematics institute discussed here fit into the larger questions about school improvement and the purposes of schools? In this age of information where knowledge is multiplying every few years, do we have a proper perspective about the role of schools vis-à-vis content? Do we know what the proper balance between disciplinary substance, structure, and syntax should be for the twenty-first century? If we do not look into such questions about the desired nature of content, how can we know if content institutes are on target? It could be that we are using outdated preconceptions of content to prepare for the future. But that is another study. Or perhaps another educational era.

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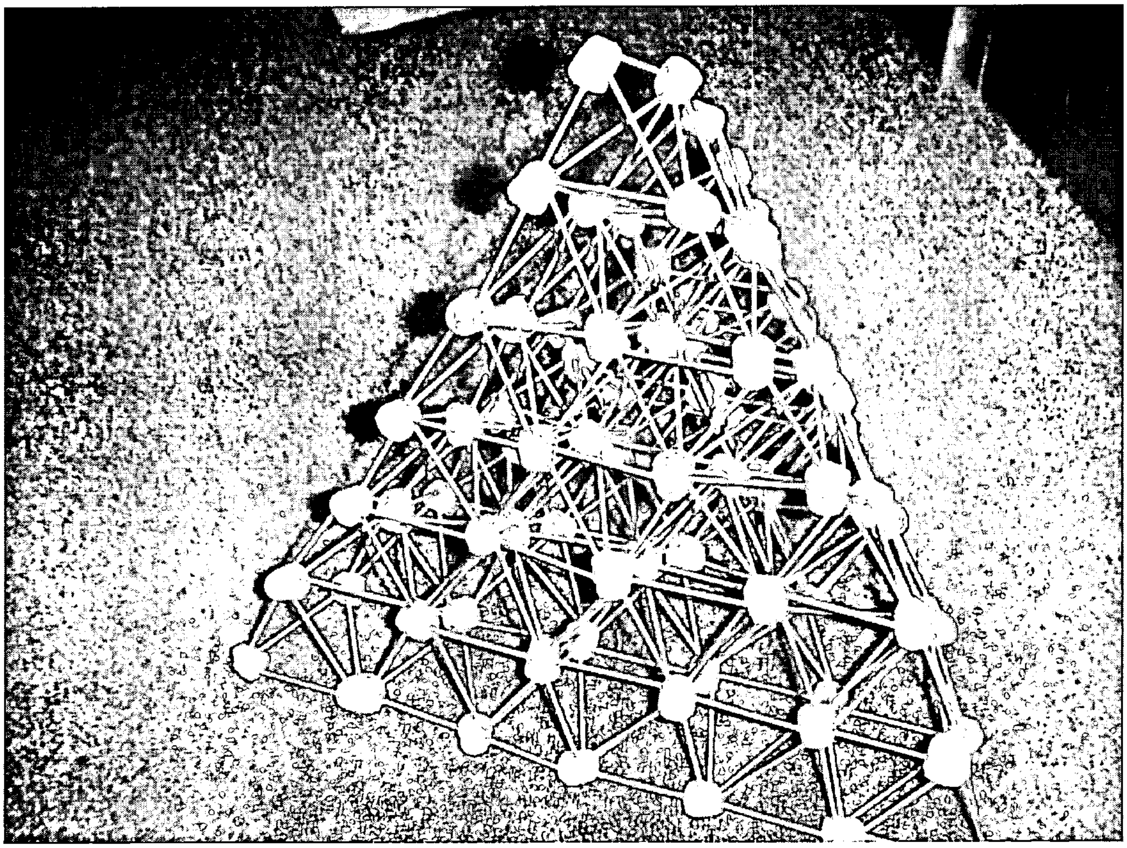
Appendix A
Example of a Gumdrop Pascal's Pyramid



Appendix B
Pascal's Pyramid Worksheet

PASCAL'S PYRAMID WORKSHEET

NAME:______________
PER.:___ DATE:________

Assume a perfectly built 7 layer pyramid for all questions unless otherwise stated.

A. GUMDROPS (Questions 1-3 Hint: Go layer by layer. Look for a pattern.)

1. How many gumdrops in your pyramid? ________
2. How many gumdrops would you need for a 7 layer pyramid?________
3. How many gumdrops would you need for a 10 layer pyramid?________
4. How many gumdrops have 12 toothpicks in them?________
5. How many gumdrops have 9 toothpicks in them?________
6. How many gumdrops have 6 toothpicks in them?________
7. How many gumdrops have 3 toothpicks in them?________
8. How many gumdrops entirely in the middle?________

B. PYRAMID

9. How many sides does your pyramid have?________
10. How many edges does your pyramid have?________
11. How many corners does your pyramid have?________

C. TOOTHPICKS (Hint: Try to do it a couple of ways. Discuss it with one another. Make sure you're not missing any!)

12. How many toothpicks in your pyramid?________
13. How many toothpicks would you need for a 10 layer pyramid?________

D. TRIANGLES (Hint: Be systematic and orderly)

14. Three gumdrops and three toothpicks form a small equilateral triangle. How many of these small equilateral triangles in your pyramid?________

E. SMALL PYRAMIDS

15. Four gumdrops and six toothpicks form a small pyramid (like the top two layers). How many small pyramids in your seven layer pyramid?________

F. OTHER SHAPES

16. Name other shapes (besides triangle and pyramids) that you see in your pyramid. Try to draw a sketch of each one.

F. EXTRA CREDIT

17. How many different triangles of all different sizes are in your pyramid?__________

Teacher opportunities to learn: Case study of a content institute in mathematics.

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Paper presented at AERA Annual Meeting, Seattle, 2001

Purpose of the study

As states have increasingly implemented standards-based assessment and accountability policies, more attention has been given to the need for teacher professional development. In particular, attention and funding have been focused on the need for increased subject-matter content knowledge, one of several recommendations articulated by reform organizations such as the National Commission on Teaching and America's Future (1996). Among the activities designed to address this need are summer "content institutes," intended to upgrade teachers' knowledge and typically lasting from several days to two weeks.

Yet, we know little about these opportunities. What do teachers have the opportunity to learn? If these are, indeed, opportunities to learn content subject matter, what ideas, information, and disciplinary methods are presented to teachers?

Moreover, little is known about how such decisions are made. Who is involved in deciding the nature of the content? How do those who initiate, organize, and facilitate these institutes conceive of content? What are they hoping teachers will learn? How do they mediate between the prerequisites of state and national standards and the expressed needs of their teacher-participants?

The purpose of this study is to shed light on questions such as these through an analysis of one case, a summer mathematics institute. It is understood that the findings are limited by the small scope. Nonetheless, the level of detail given through the qualitative approach lends an authenticity to the analysis that suggests illustrative areas for further inquiry.

Framework

"Content" is not a well-established construct and may be interpreted in a variety of ways. Schwab (1964) distinguished three different dimensions of disciplinary knowledge: the **substance** or ideas and information that constitutes the body of knowledge; the **structure** of discipline or how the ideas and information are organized; and the **syntax** or how knowledge is discovered, tested, validated, and overturned or revised.

Within Schwab's categories, variations of content definition are possible, depending on how one answers certain questions. For example, regarding *substance*, what ideas are deemed to be more foundational to the discipline than others? In mathematics, which numerical concepts form the linchpin for the understanding of others? (Ma, 1999) What skills should be considered basic? Within *structure*, ideas may be organized around integrated units, in a spiral curriculum, or according to a more hierarchical scope and sequence. The *syntax* of a discipline may be seen as more or less inquiry-based, depending upon the predisposition of the practitioner. While a pure mathematician may see the discipline as an open world of inquiry, an engineer is more likely to see a given set of truths and formulas.

Moreover, questions about content must be considered in context. The content defined for mathematics courses in colleges of arts and sciences will not be the same content appropriate to K-12 schooling, including all three of Schwab's dimensions, substance, structure, and syntax. Content knowledge required of professional mathematicians will not be congruent with the content necessary for public school teachers. The purpose of disciplinary study must be considered when deciding upon the appropriateness of content.

Thus, teachers have particular content needs, based upon the situation of schooling. Teacher knowledge involves a complex set of understandings that interact with each other. At the heart of the matter is that the primary purpose for teachers in studying a discipline is to improve their pedagogy. For teachers, content and pedagogy are inextricably linked in what Shulman (1986) terms **pedagogical content knowledge.**

Shulman (1987) identifies a model of pedagogical reasoning and action that describes the process that teachers utilize to transform content knowledge into usable information for the improvement of their practice. This process entails a cycle of learning that includes *comprehension* of subject-matter purposes, structures, and ideas; *transformation* of ideas into curricular representations and adaptations; *instruction* with students in the classroom; *evaluation* of effectiveness; and *new comprehension*. These stages are not meant to be in a linear sequence and may occur in various combinations and cycles. Shulman explains how this model is helpful in order to understand the role of content for teachers:

> Saying that a teacher must first comprehend both content and purposes, however, does not particularly distinguish a teacher from non-teaching peers. We expect a math major to understand mathematics or a history specialist to comprehend history. But the key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge that he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students. (p.15).

Research Question and Significance

The critical questions are: What content do teachers have the opportunity to learn in a summer content institute? How is the content decided?

These questions are significant because of the growing understanding of the need to improve teacher content knowledge and because of the increasing investment in teacher professional development. This investment is premised, in part, on the growing body of evidence that suggests that teachers' knowledge of their subject matter is correlated with student learning (Monk, 1994; Mullens, Murnane, & Willett, 1996).

Context and methodology for the study

The Alaska Partnership for Teacher Enhancement (APTE) was formed in 1999 with a Title II grant from U.S. DOE. A collaborative that includes the University of Alaska Anchorage (UAA), four school districts, the state department of education, the Anchorage Education Association, and General Communications, Inc., APTE organized and offered 4 content institutes in the summer of 2000 for the expressed purpose of improving teachers' content area knowledge. Each institute would last 10 days. The mathematics institute that is the focus of this study was one such institute. It was held in Anchorage at UAA.

The mathematics institute was meant to include teachers at all grade levels and from all of the cooperating school districts, which included three rural villages and the urban Anchorage district. A UAA faculty person in the mathematics department of the College of Arts and Sciences was selected by the APTE director to organize, develop, and deliver the institute curriculum. This person, in turn, hired a team of three teachers from Anchorage School District who had delivered district mathematics professional development institutes together in the past few years as part of a district initiative called the Mathematics Consortium. The team consisted of an elementary, middle, and secondary teacher, each of which was well-versed in national and state standards. Over the years, they had developed an extensive binder of activities that they had used in their institutes.

As it happened, the eleven participants who attended the institute presented a distinct bimodal profile. Seven of the participants were from an elementary school in a rural village, were Native Alaskans, and by their own admission, had limited to weak mathematical backgrounds. Only two of the seven were certified teachers; the other five were teacher aides. The other four participants were middle school teachers from Anchorage, Caucasian, and had, by their own admission, very strong mathematical backgrounds. None of the participants were known to the presenters until the first day of the institute.

Researchers observed five days of the 10-day institute, took field notes on activities, collected sample activity sheets and journal entries from participants, and interviewed presenters and participants. The data were described and analyzed utilizing the framework described above.

Summary and Analysis of Data

In what follows, we present our summary and analysis of data. We start by describing the course goals, and then move into an integrated description of how various data reflect *substance, structure, syntax, and pedagogical content knowledge*. Most of the activities observed and documented involved the interaction of these four aspects of content knowledge. Thus, instead of writing about these four areas as distinct and separate aspects, we describe the integration of these aspects in a collective narrative form.

To begin the work of developing the institute, the UAA faculty coordinator and the lead teacher of the presenter team got together to write a Course Content Guide (CCG). This is an official university document that details the learning goals and outcomes. The following goals were articulated in that document:

1) Standards for Mathematics: Participants will be able to demonstrate knowledge of the national, state, and local standards for mathematics and apply national, state, and local standards for mathematics to develop lessons (including assessments) for the classroom;
2) Exploring a Content Strand of the NCTM Standards: Participants will be able to increase their mathematics knowledge base in one of the NCTM Content Strands;
3) Technology: Participants will be able to use technology to create more effective lessons;
4) Mentoring and Peer Collaboration: Participants will be able to recognize the importance of teacher as learner and critique lessons created for their peers.

On the surface, it appears that only one of the goals, Goal 2, addressed content per se. In this goal, it is unclear what the content will be, other than the fact that only one strand will be the focus. How this focus would be determined was left unstated. There was, however, elaboration of the outcomes for this strand. They were stated as follows:

- Participants will explore using multiple representations;
- Participants will explore using hands-on activities;
- Participants will explore using manipulatives;
- Participants will explore using visuals;
- Participants will explore using writing/literacy.

Here can be seen a full emphasis on pedagogy, with little mention of content.

According to the presenter team, this document provided only the most general of structures for their decisions about what would happen during the institute. When asked at the end of the institute what their goals had been, the following comments were made:

> Everytime we met, I asked that question over and over again. Even after [the UAA faculty coordinator] told us, I still needed to ask, what is this that we're supposed to be doing?
>
> We wanted to teach them, show them, model to them, good mathematics and good mathematics instruction that's based on the NCTM Standards. That's kind of our overall feeling. Good mathematics AND how to teach it.
>
> The bottom line is that we weren't sure of the big picture exactly. So we knew that we would do something we could handle, that we knew how to do, that we could provide for them.

Thus, the evidence suggests that there was a loose linkage from the APTE intention of a content-centered institute to the UAA faculty coordinator to the team of presenters who were actually responsible for designing and implementing the activities. Perhaps feeling somewhat left on their own, the presenter team decided to rely upon what they knew, the experiences and materials they had shared together over the last few years of giving institutes. This institute would be an adaptation of work done for the Mathematics Consortium, with an emphasis on the NCTM Standards.

The presenters decided to organize the institute around what they called "big ideas": pattern blocks, base ten blocks, Pascal's triangle, data analysis, and the five NCTM process standards (problem solving, reasoning & proof, communication, connections, and representations). When asked about the content of the institute, these "big ideas" were what they cited. When it was suggested that the first three of the big ideas mentioned could be considered tools and activities rather than content, they gave the following replies:

> One of the reasons we chose the base ten blocks and the pattern blocks was because of the wide applications in all the different [content] areas.
>
> If you look at base ten blocks, there's all of this content that you can teach from them. Starting at place value, on up to algebraic thinking, you can teach a lot of content with base ten blocks. We know when we say base ten blocks to each other, that we're going to do adding, subtracting, multiplying, dividing, fractions.
>
> We know where we're going, so we don't need to have that discussion anymore. Maybe it's kind of a shorthand for us because when we say base ten blocks, we're saying place value, adding, subtracting - we're doing numeration and computation.
>
> It's a starting place. The valuable thing about saying pattern blocks and base ten blocks is that we know all the things you can do with it, with the content, but what we don't know is what the participants know, so that's why we can't say we're going to teach this, this, and this, because if they come and they've already had two years of working with pattern blocks, we're going to change our focus. Because we don't have to teach them, say, equality. We've learned different ways we can use the blocks.

Two points are noteworthy in this exchange. First, it is clear that the presenters viewed mathematical content through the lens of *pedagogical content knowledge*. This was especially the case when they were talking about using manipulatives to enhance their presentation of content. They saw the tools (e.g., pattern blocks, base ten blocks, etc.) as being representative of the *structure* or the starting place for organizing the *substance* of the content for instructional purposes. It is this perspective which allowed them to have the flexibility to respond to their participants and adjust instruction according to their needs.

This flexibility is the second point worthy of note. The presenters placed a premium on flexibility and responsiveness because they had learned from experience that they wouldn't know ahead of time who their participants would be or what they would need to learn. In this way, they were not only dealing with the practicalities of an unpredictable summer workshop situation, but also might be said to be modeling a form of pedagogical content knowledge that their participants might use in their own classrooms. They elaborated:

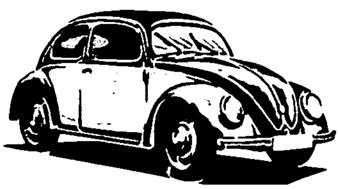
Now that we've worked together for so many times, we know not to have objectives. We know that we need to be as flexible as we can possibly be. So we didn't say we want to make sure that this, this and this happens. We said we'll use pattern blocks and see who we get in the class, and work from there.

You have to be responsive to your students. All the different groups that we've taught over the last five, six, seven or eight years, they've all been different. They're all teachers, but sometimes you have more elementary, sometimes you have more secondary, people with a stronger math background or a weaker one. Sometimes you have groups that are more talkative, or groups that are quieter. So we discuss where we are with the group and what we want to do – to adjust to who they are. We did that in this institute the very first day.

The presenters' responsiveness to their students was, in fact, one of the most striking features of this institute. Faced with the bimodal group of participants described above, they quickly realized the need for varying seating arrangements, differentiating instruction, working in small groups, providing extra time for some or challenging extensions for others. There were occasions when the elementary teachers would continue working, for example, on using base ten pieces to represent numerical operations while the middle school teachers would be working on factoring with algebra tiles. Or the elementary teachers would be spending extra time with understanding how to teach fractions with pattern blocks while the middle school teachers were exploring graphing calculators.

Differentiation also included some self-selection of activities. Each morning would begin with problems to solve, labeled as elementary or middle school activities, placed at one of the three table clusters. Anybody at elementary was welcome to try middle school activities or vice versa. And indeed several did work across grade levels. The invitational nature of the setting allowed for student choice.

Further, many of the problems that were presented each morning reflected opportunities to learn variations of the substance of mathematical content. One example problem was:



A REAL LIFE MATH PROBLEM!!!!

Fred Meyer just opened a gas station at their Muldoon store. To encourage and retain new customers, they are offering the following promotion. With every purchase of 8 gallons of gas, your "Frequent Fueler Card" will be punched. After you accumulate 10 punches, you will get $5 off any purchase of $25 or more in the store. Presently unleaded gas is sold for $1.549 per gallon. I would suspect that after initial sales promotions, the price will rise. My question is this. How much above the price of gas at other stations could I pay and still end up saving money at Fred Meyer? (I regularly shop at Fred Meyer and routinely make purchases $25 or more.) How can you represent mathematically or model this situation?

Within this problem, students were given the opportunity to explore various forms of mathematical substance. For example, several of the primary and elementary teachers explored the problem using number sense to make comparison while some of the upper elementary and middle school teachers used linear equations to represent the information.

As the institute progressed, there was more and more of a sense of open invitation to steer the learning. Said one presenter: "They were starting to feel so comfortable at saying, "I'm not comfortable with where I am with this topic, I need more, I'm not quite understanding this, can you help me..." They could identify where they needed to focus." And the presenters would modify their plans accordingly on a daily basis.

At the same time that the presenters made efforts to differentiate, however, they also recognized some common needs for the group as a whole and deliberately organized whole class activities and discussions in small cross-grade level groupings. This was especially true as participants engaged in discussions of the readings focused on the NCTM process standards and made charts which described and elaborated each standard. It was also true of most of the work done with patterning and with data analysis. The benefits of such heterogeneous groupings were described by the presenters:

> Almost everybody, including the middle school folks, needed to have a hands-on experience with these materials. They'd seen them but not used them, or didn't know how to use them.
>
> We also think it's important that they see the development of the ideas, so they can see how this develops for kids in elementary, on through to middle school. That continuum is very important. We like to have this kind of mix. We think that's important.

The presenters used various informal assessment strategies to determine the needs of their participants. A primary source of feedback was the daily journal kept by each participant. Each day, the presenters asked the participants to write an entry explaining for what aspects of the days work they felt in "equilibrium," and for what aspects they were in "disequilibrium." This opened a channel for communication about learning strengths and needs. Other feedback was obtained through the almost continuous interaction the three presenters had with their eleven participants, in small groups as well as whole groups, through observations, and ongoing discussions.

Underlying such communication and feedback was a noticeable and growing sense of comfort and sharing among the participants. The presenters consciously worked on creating a learning community through group activities and by explicitly discussing it with the participants. One of the many participant-created charts that came to adorn the classroom wall was entitled "Learning

Mathematics is a social experience." It detailed the brainstorming done by groups on the methods and purposes of developing a learning community.

Throughout the institute, there was an emphasis on inquiry-based activities. Seldom, if ever, were presenters observed to "give answers." Rather, they maintained a posture of questioning, facilitating, and guiding. Participants were, in effect, asked to "live" the process standards, as they were placed in the position of having to problem-solve, reason, communicate, connect, and represent their thinking. The presenters frequently encouraged participants to come to the overhead to show what they were thinking, to model their thinking with manipulatives, and to respond to others' ideas. Some specific observations of inquiry-based activities were: describing, extending, and generalizing patterns with pattern blocks; modeling computational story problems with base ten blocks; finding and predicting patterns in Pascal's Triangle; conducting a survey, graphing and interpreting the results.

One specific example of the inquiry-based activities that represented both *substance* and *syntax* was the construction of a Pascal's Pyramid using gumdrops and toothpicks (See Appendix A). Students were prompted to construct the pyramid and then answer questions concerning the features and mathematics found within the pyramid (See Appendix B).

This activity provided a focus on the "ideas and information" of mathematics (*substance*) by exposing students to geometric content. More specifically, using the Geometry Standard as presented in the *Principles and Standards for School Mathematics* (NCTM, 2000) the following content objectives were addressed:

- Analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- Specify locations and describe spatial relationships using coordinate geometry and other representations;
- Use visualization, spatial reasoning, and geometric modeling to solve problems (p. 164).

The organization of such a connected set of geometric knowledge around a rich task is an example of using the *structure* of mathematics to pedagogical advantage. In addition, this activity provided an opportunity for students to problem solve, estimate, and "discover, test, and validate" mathematical knowledge (*syntax*). Thus, in this and other learning activities during the institute, the instructors integrated multiple forms of content.

Moreover, virtually all activities included discussions about how to take this learning back into the classroom, how to make it work for students. The impetus for such conversations came from the participants as well as from the presenters. Said one presenter:

> We stay in touch with our participants, to know where their student population is, where they are going to take this and try it – when they are practicing an activity, they'll be saying, now I'm comfortable with this, but how will I take this to my kids – so it's important for us to always respond to that. I've been asked how will I explain this, what's the language that I should use with kids, or what steps should I take. We always try to help them think how they're going to apply.

As the participants took part in such engaging activities, the presenters were conscious of addressing the cultural differences in the room. As noted, seven of the participants were Native Alaskan, four Caucasian. The main concern expressed by the presenters was to try to give equal "air time" to all. Here's what one presenter said:

> The first thing that struck me was the quiet [of the Native Alaskans]. Whenever we presented questions, we knew in what order the Anchorage teachers would answer, and when they were done, that was the end of the conversation. So we gently encouraged comments from people who were not speaking, made it a point that at some point everyone made it up to the overhead projector to do a problem. We began to try to check with the Native people first, help them feel comfortable with where they were. Also, we'd have sharing just at your table, so it's a smaller group, you're not talking to the whole group...One day, everyone got two chips, and you could only speak if you put in a chip. That really cut down on the people who are very, very verbal. It worked great...The journals helped because people were able to express their thinking and comfort level in their journals. Whereas if we'd just had the discussion, it wouldn't have come out.

<u>Participant perspective</u>

Overall, the institute was judged to be very successful by all participants. On the last day, virtually all participants were expressing gratitude and a sense of accomplishment. The written evaluations were very high. One Native participant said:

> When we started, I always thought that in math you can only work with numbers. When I came here, I learned there's lots of ways you can work it, not just by numbers. You can use those different kinds of manipulatives in different ways...I always taught with numbers. I never thought of using base ten blocks or tiles or fraction pieces. When I came here, I realized how to use those things... I learned how you can add, subtract, multiply, divide with them.
>
> Most of the in-services I go to, they're all talking. This was mostly hands-on. I understood better.

Some journal comments:

> I learned that teachers can gain insights into students' thinking and their grasp of mathematical concepts by examining questions and interpreting their representations.
>
> I learned that my methods are not the only ones to solve problems.
>
> Doing hands-on experimenting will cement the learning method to the minds of the little people.

Effective problem solving involves experiencing disequilibrium. Otherwise there is little room or motivation to expand learning.

I really like how we began by looking at process standards. I now realize that they are just as important, if not more important, than the content itself.

In addition to teaching kids math, we need to teach them how to think mathematically.

No follow up was planned for the institute.

Reflection

Returning to our original question, what content do teachers have the opportunity to learn in a summer content institute? In the mathematics institute studied, the emphasis was on the process of "mathematizing" and on modeling appropriate pedagogical content knowledge with an integration of the substance, structure, and syntax of mathematical content. Using the starting points of structure and syntax rather than substance it was nonetheless evident that various forms of substance were embedded in the selected activities and throughout many discussions. Perhaps a better answer to this question lies in determining what types of pedagogical content knowledge were evident and in what balance.

This institute was vested with a charge to teach "mathematical content", but somewhere along the way that charge became ill-defined, lost in translation, or left up to the instructors to interpret as deemed fit. What was left was the best judgment and experience of three teacher-presenters who acted in a way that preserved flexibility for responding to participants. Not knowing ahead of time who these people would be or what their backgrounds and needs would be, the presenters, developed starting places and "big ideas" that would allow them freedom to use their own expertise to respond to their teacher-students. In this sense, this case illustrates how proficient teacher-presenters can make professional decisions for the sake of their particular circumstances when given a loose enough structure within which to move. Would the particular participants in this institute have been better served if there had been more rigid objectives and content specifications? Given the success of the learning community created, it is hard to believe that would be so.

What about the emphasis on process in this institute? Is that somehow antithetical to content? Or is it, in fact, an aspect of content itself? To use Schwab's framework, process may clearly be seen as part of the discipline's syntax. In mathematics as well as other disciplines, it matters how the "truth" is determined. The substance to be learned is learned through a process that is itself part of the content. That is, mathematical thinking and process is not apart from content; it is an essential part of it. It may be seen as the disciplinary means to a disciplinary end, greater understanding of substance.

And what about the emphasis on manipulatives and related problem-solving activities? Is this somehow apart from content? Not if we give credence to our presenters' ideas about using the manipulatives and problem-solving as the avenues that bring multiple content ideas together.

They are, in fact, using the tools of the trade as a form of structuring the learning. This is Schwab's third aspect of disciplinary knowledge. Using mathematical representations as an entry point and linking device for content may be seen as highlighting mathematical structure.

Is the balance correct between content and pedagogy? This is certainly arguable, depending on one's point of view. Some may see an overemphasis on pedagogy, but they would probably not be teachers. Matters of balance aside, what may be seen in this institute is that teachers were indeed working through Shulman's stages of comprehension and transformation. All along, participants and presenters were engaging in the conversations about how this might be done in the classroom. It was a recursive rather than a linear conversation, looping back and forth between comprehension and transformation. It is unfortunate that no follow-up was planned for these participants, making it a matter of hope rather than design that the remainder of Shulman's learning cycle will be enacted.

In short, the following conclusions arose from the data:

- The opportunity to learn for teachers in this institute was founded on a deliberate responsiveness to their interests, social as well as academic needs, and specific teaching contexts. National and state standards were employed in a process-oriented approach that stressed content syntax and structure as the entry points for content substance. The content emerged from the ongoing decision-making of teacher-presenters rather than from pre-determined objectives.
- The content was decided by the teacher-presenters. This may account for what may be viewed as an overbalance of pedagogy over content. And yet, who better to teach teachers than teachers? And as Shulman points out, how can authentic teacher learning ever be separated from pedagogy?

In closing, this case offers a view of a content institute where content is not divorced from process or pedagogy. This seems inevitable and desirable in the context of teacher professional development. The case also depicts problems with professional development. For example, why was a greater effort not made to determine who the participants would be and to do a needs assessment beforehand so that the institute agenda could be better planned? Why was no follow-up planned? How could policy intention and delivery be better linked, while preserving the discretion of the presenters to respond to their participants?

There are well-established and research-based standards for teacher professional development. Those planning summer institutes would be well-advised to understand and implement them. This would undoubtedly improve the effectiveness of such initiatives in improving teacher learning and classroom instruction, even though they may entail a greater investment of time and money. If we believe that improving teachers' content understandings will improve students' content understandings, and if we are serious about wanting to improve students' content understandings, then surely this is an investment we must make.

Finally, there is a question that begs to be asked. How do existing content institutes such as the mathematics institute discussed here fit into the larger questions about school improvement and the purposes of schools? In this age of information where knowledge is multiplying every few years, do we have a proper perspective about the role of schools vis-à-vis content? Do we know what the proper balance between disciplinary substance, structure, and syntax should be for the twenty-first century? If we do not look into such questions about the desired nature of content, how can we know if content institutes are on target? It could be that we are using outdated preconceptions of content to prepare for the future. But that is another study. Or perhaps another educational era.

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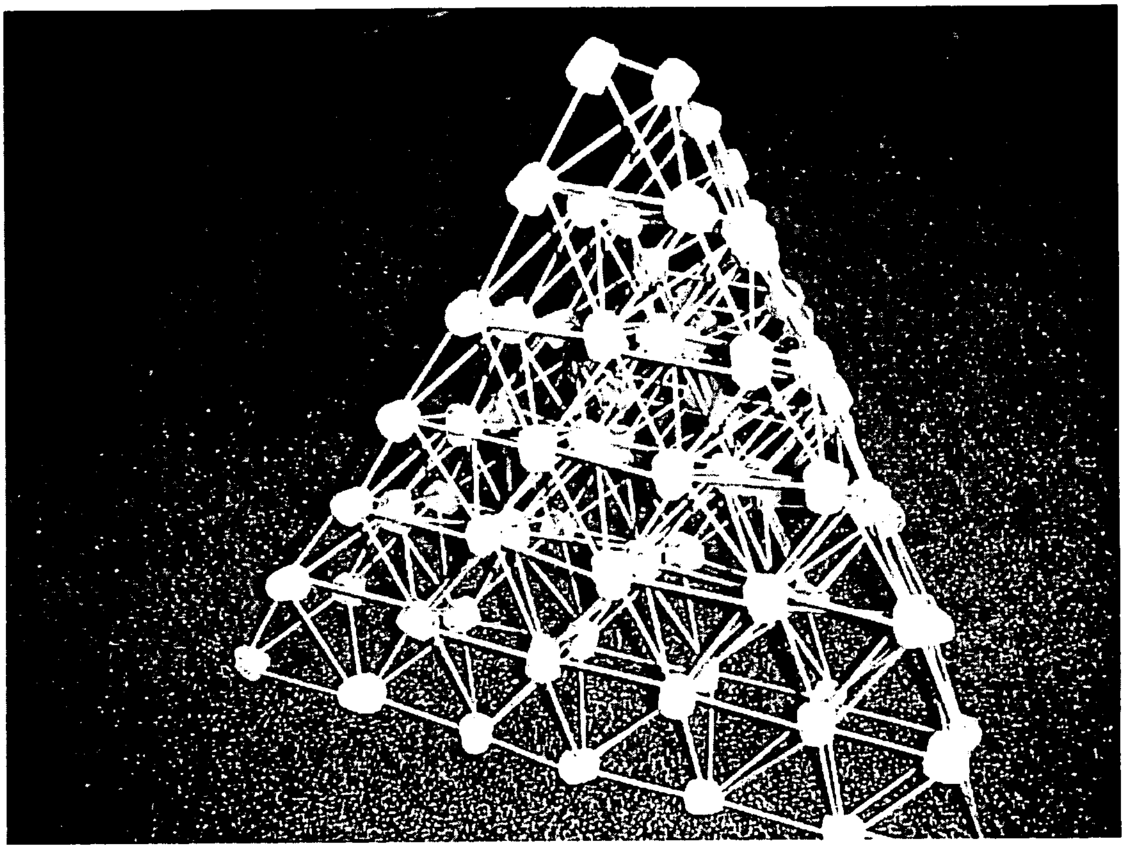
Appendix A
Example of a Gumdrop Pascal's Pyramid



Appendix B
Pascal's Pyramid Worksheet

PASCAL'S PYRAMID WORKSHEET

NAME:____________________
PER.:___ DATE:__________

Assume a perfectly built 7 layer pyramid for all questions unless otherwise stated.

A. **GUMDROPS** (Questions 1-3 Hint: Go layer by layer. Look for a pattern.)

1. How many gumdrops in your pyramid? __________
2. How many gumdrops would you need for a 7 layer pyramid?__________
3. How many gumdrops would you need for a 10 layer pyramid?__________
4. How many gumdrops have 12 toothpicks in them?__________
5. How many gumdrops have 9 toothpicks in them?__________
6. How many gumdrops have 6 toothpicks in them?__________
7. How many gumdrops have 3 toothpicks in them?__________
8. How many gumdrops entirely in the middle?__________

B. **PYRAMID**

9. How many sides does your pyramid have?________
10. How many edges does your pyramid have?________
11. How many corners does your pyramid have?________

C. **TOOTHPICKS** (Hint: Try to do it a couple of ways. Discuss it with one another. Make sure you're not missing any!)

12. How many toothpicks in your pyramid?________
13. How many toothpicks would you need for a 10 layer pyramid?________

D. **TRIANGLES** (Hint: Be systematic and orderly)

14. Three gumdrops and three toothpicks form a small equilateral triangle. How many of these <u>small</u> equilateral triangles in your pyramid?________

E. **SMALL PYRAMIDS**

15. Four gumdrops and six toothpicks form a small pyramid (like the top two layers). How many small pyramids in your seven layer pyramid?________

F. **OTHER SHAPES**

16. Name other shapes (besides triangle and pyramids) that you see in your pyramid. Try to draw a sketch of each one.

F. **EXTRA CREDIT**

17. How many different triangles of all different sizes are in your pyramid?__________

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EFF-088 (Rev. 9/97)