DOCUMENT RESUME

ED 453 166 SP 039 963

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TITLE	From Informal Strategies to Structured Procedures: Mind the Gap!
PUB DATE	2001-04-00
NOTE	20p.; Paper presented at the Annual Meeting of the American Educational Research Association (Seattle, WA, April 10-14, 2001).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Division; Elementary Education; Elementary School Mathematics; Elementary School Students; Foreign Countries; *Mathematics Education; *Mathematics Skills; *Teaching Methods
IDENTIFIERS	England; Netherlands

ABSTRACT

This paper explores written calculation methods for solving division problems used by students in England and the Netherlands at two points in the same school year. It analyzes informal strategies and identifies progression toward more structured procedures that result from different teaching approaches. Comparison of the methods used by fifth grade students in the two countries shows greater success with the Dutch approach, which is based on careful progression from informal strategies to more structured and efficient procedures. This success is particularly significant for the girls in the sample. For the English students, whose written solutions largely involve the traditional algorithm, the discontinuity between the formal computation procedure and informal solutions strategies presents difficulties. In addition to the influences of different teaching approaches, strategies and facilities are associated with the presence or absence of a context and the nature of the numbers involved. (Contains 32 references.) (Author/SM)

From Informal Strategies to Structured Procedures: mind the gap!

Paper presented at AERA Annual Conference, Seattle, 10-14 April 2001

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From Informal Strategies to Structured Procedures: mind the gap!

Abstract

This paper explores written calculation methods for division used by pupils in England (n=276) and the Netherlands (n=259) at two points in the same school year. Informal strategies are analysed and progression identified towards more structured procedures that result from different teaching approaches. Comparison of the methods used by year 5 (Group 6) pupils in the two countries shows greater success in the Dutch approach which is based on careful progression from informal strategies to more structured and efficient procedures. This success is particularly notable for the girls in the sample. For the English pupils, whose written solutions largely involved the traditional algorithm, the discontinuity between the formal computation procedure and informal solution strategies presents difficulties.

In addition to the influences of different teaching approaches, strategies and facilities are associated with the presence or absence of a context and the nature of the numbers involved.

Background

In recent years there has been widespread publicity for results of international testing of arithmetic in schools with countries like England performing less well than other countries in Europe and some Pacific Rim countries. Contributing to these variations in performance will be a diversity of factors including different attitudes towards education, different social pressures and different teaching approaches, as well as the content, timing and emphasis given to arithmetic teaching in the school curriculum (Macnab, 2000). Although comparisons are complex, children's written solutions for selected problems can shed light on some reasons for differences in attainment and this study identifies critical differences in calculating approaches in England and the Netherlands.

Different teaching approaches

As close neighbours in Europe, England and the Netherlands share many cultural characteristics but approaches to mathematics teaching have been subject to different pressures over the last two decades (Brown, 2001: van den Heuvel-Panhuizen, 2001) and this has resulted in contrasting teaching approaches to written calculations (Beishuizen and Anghileri, 1998). Different national requirements for the school curriculum put pressure on teachers to introduce specific written methods (Carpenter et al. 1997). In England, "understanding of place value is central to pupils' learning of number.... . Progression in understanding about place value is required as a sound basis for efficient and correct mental and written calculation" (SCAA, 1997). In Dutch RME approaches, on the other hand, calculating "is not based on the teaching of the place value concept in the first place but develops more gradually through the extension of counting strategies" (Beishuizen and Anghileri 1998; Thompson 1997).
In England, the National Numeracy Strategy places emphasis on mental calculations in the early years and proposes that working with larger numbers will necessitate the introduction of "informal pencil and paper jottings" that become "part of a mental strategy" (DfEE 1998: 51). By the age of 11 years, children are, however, required to know a standard written method for each operation as "standard written methods offer reliable and efficient procedures" (DfEE 1998). The standard written methods illustrated in the Framework for Teaching Mathematics (DfEE, 1999a) are little

different from the traditional algorithms that have been taught to successive generations. It has been acknowledged that "many children do not reach the stage of recording calculations the traditional way (by the age of 11 years)" (Cockcroft 1982: 77) and calls have been made for pupils to be encouraged to develop alternative methods (Thompson, 1997, Anghileri, 2000). However, the only documentation to omit explicit reference to the traditional algorithms is the new National Curriculum for England which refers to children having "efficient written methods" for calculating (DfEE 1999b).

In the Netherlands the Realistic Mathematics Education (RME) movement (Treffers and Beishuizen 1999; van den Heuvel 2001) has introduced some radical changes in the teaching of calculating methods with early focus on mental methods and later a development of different levels in written calculating. Research has led to the proposal of 'trajectories' whereby learning evolves as a process of gradual changes as "students pass various levels of understanding: from the ability to invent informal context-related solutions to the creation of various levels of short cuts and schematisation" (van den Heuvel 2001). A fundamental aspect of learning written calculations is "guided development from informal to higher-level formal strategies" which involves "reflection on strategy choice" in whole class discussion (Beishuizen and Anghileri 1998).

The RME approach asks children "to solve many real-world problems guided by interactive teaching instead of direct instruction in standard algorithms" (Beishuizen 2001: 119). Central are contextual problems that "allow for a wide variety of solution procedures, preferably those which considered together already indicate a possible learning route through a process of progressive mathematization" (Gravemeijer 2001). The Dutch approach places emphasis on the development from naïve skills such as counting and doubling, and involves holistic approaches to numbers within a calculation in contrast with the place value approach developed in the English curriculum (Beishuizen and Anghileri 1998; van Putten, Snijders and Beishuizen, in preparation).

The arithmetic operation of division

By focussing on pupils' strategies for division in late primary school (10 year olds), it is possible to highlight progression from mental methods and informal strategies to the more structured approaches that are adopted when written calculating procedures are introduced.

Two models for division, normally referred to as *partitive* division (how many sevens in 28?) and *quotitive* division (28 shared between 7) have formed the basis for analysing the division operation for whole numbers (Greer, 1992). Related to these models are two distinct procedures for written calculations: repeated subtraction of the divisor (becoming more efficient by judicious choice of 'chunks' that are multiples of the divisor) and sharing based on a place value partitioning of the number to be divided (used efficiently in the traditional algorithm). There are many informal strategies that will be built upon and Neuman (1999) includes counting, repeated addition, chunks (performed in different ways), reversed multiplication, dealing, estimate-adjust, repeated halving, repeated estimation, many of which will be incorporated into structured procedures for division calculations.

Whether the approach is informal or reflects a taught procedure, structuring the recording becomes beneficial as more complex problems are introduced. In considering pupils use of written recording, Ruthven (1998) identifies two distinct purposes: "to augment working memory by *recording* key items of information" and "to cue sequences of actions through *schematising* such information within a standard spatial configuration". The former may be identified with informal solution strategies that are often idiosyncratic and give little consideration to efficiency or ease of communicating to others. The latter suggests a taught procedure that will "direct and organise" (Anghileri, 1998) children's approaches and has as priorities efficiency and clarity of communication.

Formal written procedures for calculating can be difficult to reconcile with intuitive understanding (Fischbein et al. 1985, Anghileri and Beishuizen, 1998) and can lead to mechanical approaches which are prone to errors (Brown and VanLehn, 1980). Ruthven and Chaplin (1998) refer to "the improvisation of malgorithms" to describe pupils' inappropriate adaptations of procedures for the algorithm.

There is also evidence of 'conflict' between computation procedures and context structure (Anghileri, 2001a) and it is suggested that there is one primitive model for division in children's thinking, the partitive, and that the quotitive model is acquired with instruction (Neuman, 1999). Where problems are set in a context this may influence the solution strategy but research suggests that the quotitive model appears to influence more strongly written approaches with calculations such as 42 ÷ 6 interpreted as 'How many sixes in 42? (Anghileri, 1995; Neuman, 1999).

Comparing solution methods

This study considers pupils' written methods for solving ten division problems, using five word problems that vary in their semantic structure together with five 'bare' problems expressed only in symbols. Comparisons are made between the strategies used by English and Dutch pupils and their success for different problem types. By identifying the pupils solution strategies at two points in the school year (January and June) changes in approach are identified and related to instructional approaches in the two countries. Dutch pupils are introduced to written methods for division of large numbers in the second term of Group 6 and this would be common in all schools where mixed ability classes are taught mathematics by the class teacher. There was not such consistent practice in the English schools where, although many pupils work on division problems in Year 5, at the time of this study there was no common curriculum and experiences varied with teachers and textbooks and across different groups which were often streamed according to age and ability.

Sample

Nine and ten year old pupils (n=553) in twenty different schools were involved in the study. Ten English and ten Dutch schools with average class sizes were selected in and around small university cities in England and in the Netherlands

Although comparison is complex, the nature of the populations in the two localities appeared to share many common characteristics such as stability of population and general nature of employment in the area. Further criteria for selection of schools were high scores on standard national assessments (in the case of English schools) or use of specific textbooks (in the Dutch schools) related to their involvement in implementing a Realistic Mathematics Education (RME) curriculum. The English schools all had their most recently published Standard Assessment Test (SAT) scores in mathematics (average 72.5% at level 4 or above) well above the local (LEA) average of 54.3% and the national average of 53.2%. In the Dutch schools, teachers were using approaches to mathematics teaching centred on the use of RME textbooks. All schools were

selected so that the pupils were likely to have confidence to tackle novel problems and the ability to show some working to reveal their strategies. It is suggested that the range of strategies in more 'average' schools would show types similar to those found in this study with potentially different success rates.
Average ages of the children (n= 553) in January were similar for the two cohorts (Table 1)

Table 1: Means and standard deviations of ages of the two cohorts

Ages in years	Mean	Standard deviation
English (n=291)	9.79	0.28
Dutch (n=262)	9.90	0.44

The distribution was, however, significantly different due to national policies of the two countries. In England pupils' ages determine the class/grade they will join and it is rare to find any variation (Bierhoff 1996; Prais 1997). In the Netherlands the age range in most classes will be wider, reflecting a national policy for accelerating able pupils and holding back, for one or sometimes two years, those who do not reach the required standard.
Another difference that is not evident from the statistical data is the policy for pupils with Special Educational Needs. In England there is a policy to integrate pupils with SEN into mainstream classes whenever possible while in the Netherlands many such pupils will attend special schools. In the results of this study such influences need to be taken into account when interpreting the differences in performance of the two cohorts.

All pupils completed a test of mental arithmetic but a reduced cohort (n=534) completed division tests in January and also in June. Only pupils who were present for *both* division tests were included in this analysis [English (n=275) and Dutch (n=259)]. This reduced cohort showed no significant difference from the larger sample in age distribution and was evenly balanced for gender (Table 2).

Table 2: Gender distribution in the two samples

	Girls	Boys	Total
English	138	137	275
Dutch	128	131	259

Methodology
Pupils were tested twice, in January and in June of the same school year, so that changes would be evident in the calculating methods used. In the first round of testing each of the twenty classes completed a short timed 'speed test' of mental calculations in addition, subtraction and multiplication, based on Dutch national tests. This was followed by a written test with no time limit so that it could be completed by all the pupils. The tests were designed collaboratively by the English and Dutch researchers and administered by the researchers. Pilot tests were administered in schools which were not involved in the final testing and modifications were made where necessary. Problems were presented in workbooks and pupils were invited to complete the

problems in any order and try another problem if they were stuck. The teacher and the researcher assisted with reading the problems where necessary but gave no further guidance. When the pupils were tested for the second time, in June of the same year (5 months later), only the written test was used.

The speed tests

Pupils division strategies may be related to their ability in mental arithmetic. In order to assess the pupils performance they were given a short speed test which involved 5 columns each with 40 mental calculations of progressive difficulty. Column 1 involved addition from 1+1 to 54+27, Column 2, 3, 4 and 5 involved subtraction, multiplication, harder multiplication and harder subtraction, respectively, each involving a progression from easy to more difficult questions. After attempting some practice questions, pupils were timed for one minute each for columns 1-3 and 2 minutes each for columns 4 and 5. The number of correct and the number of incorrect responses were scored. These ten scores were used to select and compare the results of sub-samples of better and weaker pupils which are reported elsewhere (Anghileri 2001b)

The division tests

Two practice items were presented one at a time to the class and, after a minute of thinking, solution strategies were invited from the pupils. The researcher wrote pupils' suggestions clearly on the board so that at least three different strategies, including informal/intuitive approaches, were illustrated and these illustrations were left for the duration of the test. Pupils then worked individually on ten problems, each with space to show working and an answer, and were encouraged to record 'the way they think about the problems'.

Between the tests in January and June, all pupils will have had further experiences in arithmetic learning including some work on multiplication and division.

The division test items

The written test consisted of ten division problems; five illustrated word (context) problems and five symbolic (bare) problem with similar numbers. Problem types included 'sharing' and 'grouping' models, and involved single-digit and two-digit divisors, with and without remainders (table 3). The numbers were selected to encourage mental strategies and to invite the use of known number facts so that it would be possible to approach all the problems using intuitive methods. Some numbers were selected to include the potential for the common error of missing a zero in the solution.

Table 3: Ten problems used in the first test

context problem	problem type	bare problem	number type
1. 98 flowers are bundled in bunches of 7. How many bunches can be made?	grouping	6. $96 \div 6$	2-digit divided by 1-digit no remainder
2. 64 pencils have to be packed in boxes of 16. How many boxes will be needed?	grouping	7. $84 \div 14$	2-digit divided by 2-digit no remainder

3. 432 children have to be transported by 15 seater buses. How many buses will be needed?	grouping	8. 538÷15	3-digit divided by 2-digit remainder
4. 604 blocks are laid down in rows of 10. How many rows will there be?	grouping by 10	9. 804÷10	3-digit divided by 10 remainder
5. 1256 apples are divided among 6 shopkeepers. How many apples will each shopkeeper get? How many apples will be left?	sharing	10. 1542÷5	4-digit divided by 1-digit remainder

In June, the problems involving 96÷6, 84÷14, 538÷15, 802÷10 and 1542÷5 which were 'bare' in test 1 were given the contexts used in the first test. Again, the context problems were the first 5. The problems 98÷7, 64÷16, 432÷15, 604÷10 and 1256÷6 were now presented in 'bare' format as problems 6-10.

Results

Performance in the Mental Arithmetic Speed Tests

Prerequisite knowledge for learning division in school includes mental computation in addition, subtraction and multiplication. The number of questions completed and the number of errors in the timed test of mental calculations were recorded. Pupils' performances for the different calculations are recorded in table 4.

Table 4 Results for the speed tests

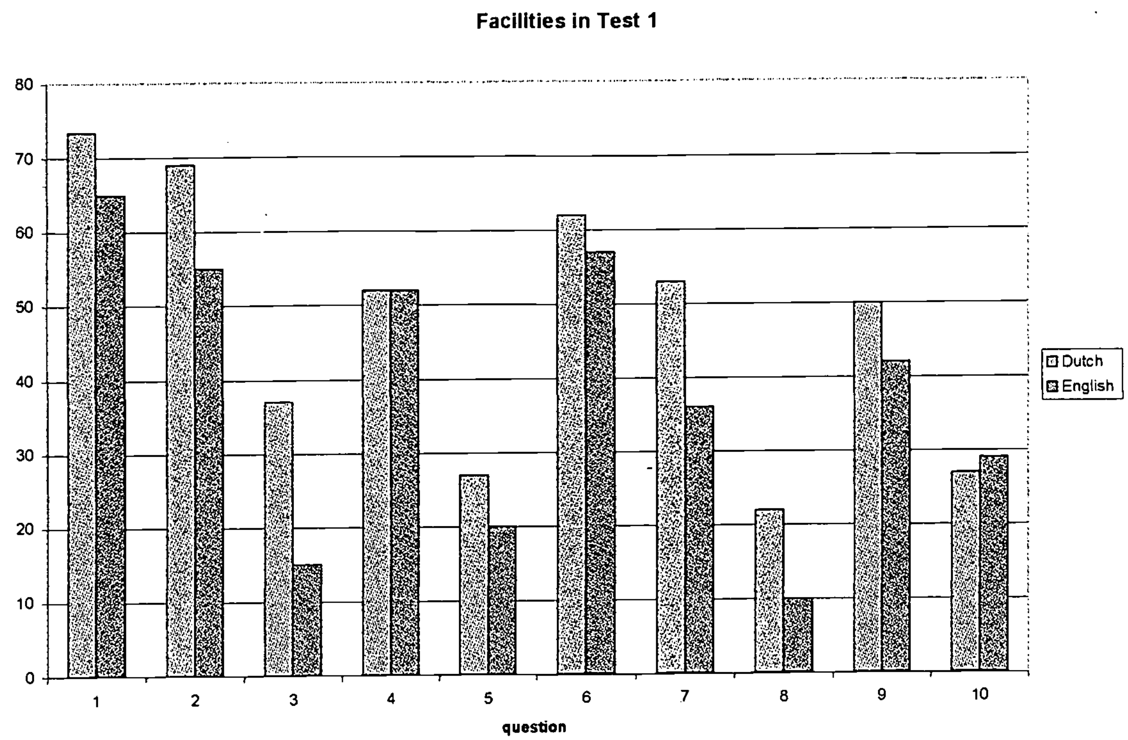
speed test	column 1 addition		column 2 subtraction		column 3 multiplication		column 4 multiplication		column 5 subtraction	
	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.	mean	s.d.
Dutch (n=262)	23.3	4.7	20.8	4.5	19.9	4.7	12.8	6.6	11.0	5.1
English (n=293)	18.7	5.4	13.6	5.5	13.9	5.7	7.5	6.0	4.2	4.2

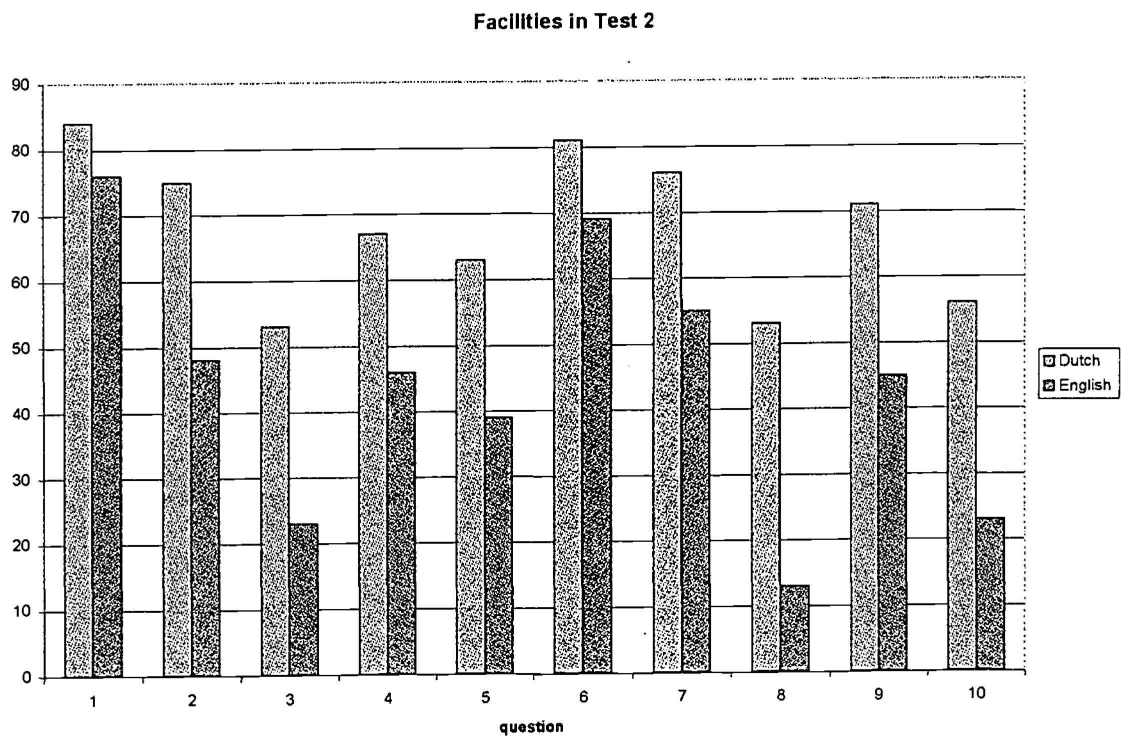
The scores were higher in every type of problem for the Dutch pupils who not only completed more questions but made fewer errors in their attempts. The highest scores were similar for both cohorts but the standard deviation shows greater variation among the English pupils. The better success of the Dutch pupils is not surprising as the emphasis given to mental arithmetic has for some time been greater in the Netherlands and speed tests of this type are familiar in schools. More recently there has been a growing emphasis on mental arithmetic in England but the pupils in this year 5 cohort will have experienced more focus on written calculation.

Performance In The Written Tests

In the first test in January the number of items with correct solutions was similar for the Dutch (mean 4.7: s.d. 2.9) and the English (mean 3.8: s.d. 2.7) but average Dutch scores were higher on all but one of the ten items (Figure 1a).

Figure 1: Scores for the ten questions in test 1 and test 2



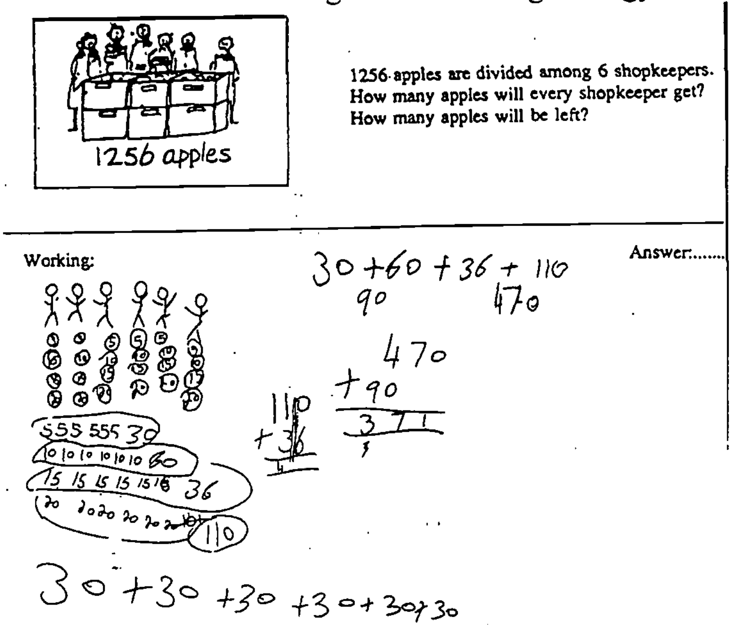


In the second test differences between the performances of the Dutch and English pupils were more marked with the English (mean 4.4: s.d. 2.6) and the Dutch (mean 6.8: s.d. 2.6). On all ten items the average score for the Dutch pupils was greater than that of the English pupils (Figure 1b).
The biggest difference appears for items 3 (538 children have to be transported by 15 seater buses. How many buses will be needed?) and particularly for item 8 (432÷15). Both items involve a two-digit divisor which is not normally encountered in Year 5 in English schools where the emphasis on formal procedures means that the long division algorithm would need to be introduced. Dutch pupils in Group 6 are taught a written methods that is equally appropriate for single-digit and multi-digit divisors. These differences will be discussed further in a later section.

Strategies for solving division problems

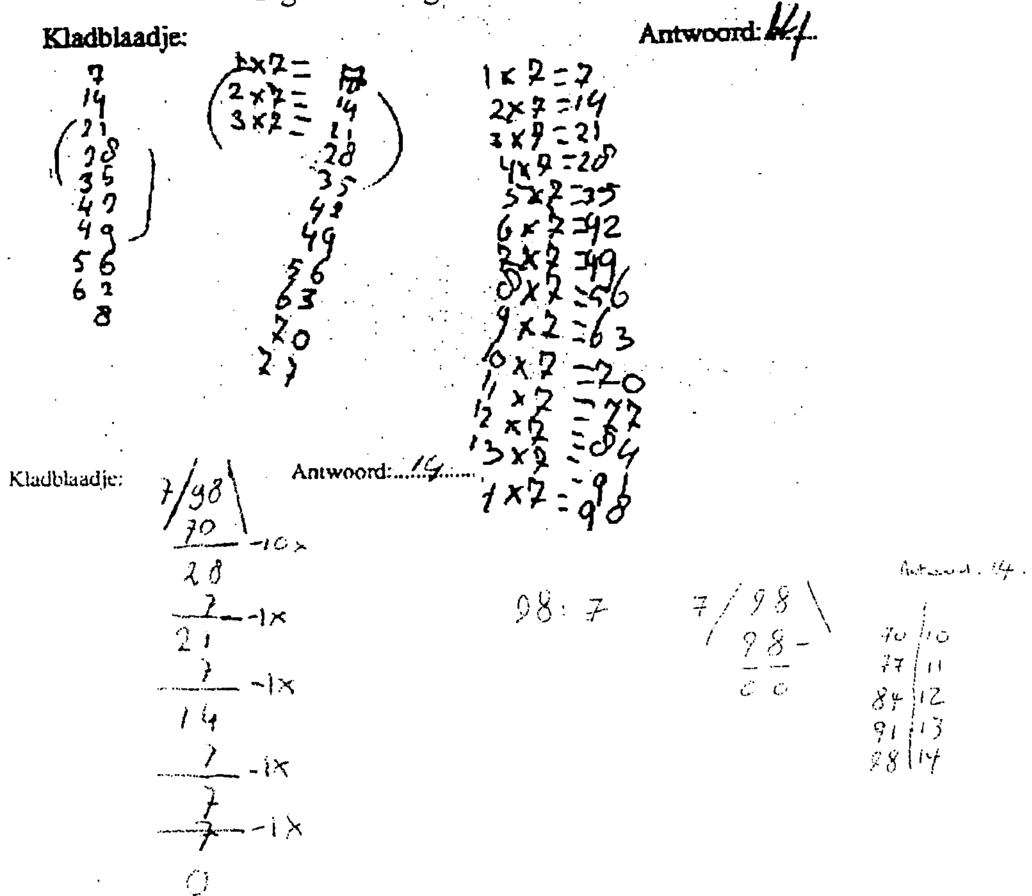
The pupils written methods ranged from inefficient strategies such as tallying or repeated addition to use of a standardised written procedure. There were marked differences in the ranges of strategies in the different countries and in the ways the pupils organised their calculations on paper and this led to complex initial classifications in order to represent important variations. Most of the strategies identified by Neuman (1999) were evident to some extent but the larger numbers involved in the study meant that such naïve strategies were sometimes adapted to include efficiency gains. Neuman's category of 'dealing' one at a time, for example, was similar to dealing/sharing using multiples of the divisor (Figure 2).

Figure 2: A sharing strategy for division



In the English sample there were examples of complex procedures where the recording was difficult to follow while many Dutch pupils showed clearer organisation in their recording methods that could be associated with a taught procedure based on repeated subtraction. Progression was evident in the Dutch strategies from inefficient strategies, through structured recording, to more formalised and efficient procedures and the Dutch approaches illustrated how similar procedures were used at different levels of efficiency by individual pupils (Figure 3).

Figure 3: Progression in Dutch solution strategies



In the English methods, informal strategies showed sound approaches but were often disorganised in their recording. There was stark contrast between the informal strategies that were recorded in idiosyncratic ways and formal recording which was most usually the traditional algorithm. No clear progression was evident from these informal strategies to the standardised procedure and attempts to use it for 2-digit divisors sometimes showed inappropriate adaptations.

Classification of these methods was somewhat different for the two cohorts of pupils as progression within Dutch strategies meant that methods that were essentially the same could involved different levels of efficiency. Sub-categories were initially identified in the Dutch cases to reflect such different levels within the same approach.

<u>Classification of Strategies</u>

Pupils were invited to record their thinking and a number of informal strategies were recorded as well as the more structured standard algorithm. Some pupils used 'low level' strategies like tallying and direct addition of the divisor, despite their inefficiency for large numbers, while others used more efficient 'high level' strategies involving the use of related number facts. Initially fifteen different categories were identified which were then grouped into 8 types:

1. Using tally marks or some symbol for each unit;
2. Repeated addition of the divisor;
3. Repeated subtraction of the divisor from the dividend;

4. Sharing with images of a distribution;

These four strategies involved long calculations with no evident attempt to gain efficiency despite the large numbers involved. These 4 strategies were grouped together as **1(S).**

5. Operating with the digits independently;
6. Partitioning the dividend into (thousands) hundreds, tens and units;

Both 5 and 6 involved ways to 'break down' the numbers using ideas of place value and were classed together as **2(P).**

7. Low level 'chunking' e.g. adding small subtotals (30 instead of 15) within long procedures;
8. Using doubling, or repeated doubling of the divisor;
9. Halving the divisor or the dividend;

Working with small multiples of the divisor (low level chunking) gained some efficiency but generally led to long calculations. This was also true for strategies involving halving and doubling and 7, 8 and 9 were classed together as **3(L).** For some calculations (e.g. 64÷16) halving and doubling were very efficient approaches and it was not appropriate to classify these as low level. In these cases doubling and halving were included in 4(H).

10. High level 'chunking' using efficient subtotals (e.g. 150 instead of 15) and shortened procedures **4(H).**
11. The traditional algorithm **5(AL).**
12. Mental calculation showing an answer but no working **6(ME).**
13. A wrong operation (e.g. 98 – 7 = 91) **7(WR).**
14. Unclear strategy **8(UN).**
15. No attempt (missing) **o**.

The traditional algorithm sometimes involved other strategies and informal jottings to support the calculations. It was, none the less, classed as a separate category because the solutions were structured by this approach and it was not possible to make a distinction between the algorithm as a strategy or as a procedure.

<u>Further classification of the Dutch chunking strategies</u>

In the initial strategies of the Dutch pupils there was evidence of progression within the categories identified above and further subcategories were defined so that improvements from the first test to the second could be more clearly identified. The pupils' working sometimes involved the initial writing of a list (l) of multiples (usually 2x, 4x, 8x and 10x) to facilitate the calculation and was sometimes formally structured as repeated subtraction using a structured written record (scheme) (a) (Table 5):

Table 5. Subcategories for the Dutch strategies

la	low level chunking with a scheme
lp	low level progressive chunking [involving some progression from low level to higher level chunking]
lpa	low level progressive chunking with a scheme
hl	high level chunking (informal) with a list of multiples
hal	high level chunking using a scheme and a list of multiples
ha	high level chunking using a scheme

These strategies reflect progression in the taught procedure where the same problem may be tackled at different levels according the individual pupils' understanding and choice of multiples. In the following analysis these are incorporates into the larger categories.

Comparison of success rates associated with different strategies

In order to make comparisons of the pupils methods meaningful the number of categories for the Dutch pupils was reduced so that 'high level chunking' and 'low level chunking' formed two classes with no subdivisions. The relative success rates for each of the strategies were compared (Table 6).

Table 6: Percentage of pupils using each strategy and success rates (in brackets)

	English				Dutch			
	test1		test2		test1		test2	
	attempt	correct	attempt	correct	attempt	correct	attempt	correct
1(S)	17%	7%	11%	6%	10%	4%	1%	1%
2(P)	5%	0%	3%	0%	7%	1%	6%	2%
3(L)	6%	2%	8%	2%	16%	7%	6%	5%
5(H)	8%	5%	7%	5%	41%	28%	69%	51%
7(AL)	38%	18%	49%	25%	4%	1%	3%	1%
8(ME)	9%	5%	11%	6%	9%	6%	11%	7%
9(WR)	3%	0%	2%	0%	5%	0%	1%	0%
10(UN)	4%	1%	3%	0%	2%	0%	1%	0%
o	9%	0%	8%	0%	8%	0%	2%	0%
total	**100%**	**38%**	**100%**	**44%**	**100%**	**47%**	**100%**	**68%**

There was improvement in facility for both cohorts and a general trend towards use of more efficient strategies although these are not altogether more effective. The most popular strategy for the Dutch pupils involved identification and use of large chunks, 5(H), usually in a structured procedure of repeated subtraction which was used for 69% of items in test 2 with 51% successful. This contrasts with the traditional algorithm 7(AL) used in 49% of the items in test 2 by English pupils with success in only 25% of all attempts. In the written recordings of the Dutch pupils, progression was evident with reduction in the use of low level strategies 1(S) from 10% in test 1 to only 1% in test 2. A similar change is evident for the English pupils but 22% persist with the low level strategies 1(S), 2(P) and 3(L) in the second test with low (8%) success rate. Working mentally 8(ME) was generally associated with problems involving division by ten and the table shows similar frequency of use with better success rates among the Dutch.

Repeated subtraction may be viewed as an intuitive approach to division but it was evident only in the Dutch children's methods suggesting that it is learned rather than used spontaneously as a strategy. In the second test repeated subtraction did not persist as an informal strategy but appeared as a structured procedure with the introduction of 'chunks' to improve efficiency. The accessibility of this procedure as a direct progression from more naïve methods could account for no Dutch pupils using tallying, sharing or repeated subtraction in the second test. English pupils, in contrast, used repeated addition in both the first and second tests and many (3%) of their attempts were impossible to decipher, 10(UN). General confidence appears to be better in the Dutch cohort in test 2 as only 2% of items were not attempted compared with 8% of items for the English cohort.

Comparing the English and Dutch facilities for division by a single digit

Better results for the Dutch pupils may be explained by the fact that they meet division by a 2-digit divisor in group 6 (Y 5) while most English pupils will meet only 1-digit divisors. There were, however, differences in those items involving only a single digit divisor. Improvements are similar for the items, 96÷6 and 98÷7, but for the 4-digit numbers, 1256÷6 and 1542÷5, the Dutch improvements were higher (Table 7).

Table 7 Success rates for the problems involving division by a single digit

	96÷6	1256÷6	98÷7	1542÷5	Average
English test1	69	22	60	31	45.5
Dutch test1	73	27	62	27	47.25
English test2	74 (+5)	24 (+2)	81 (+21)	41 (+10)	55 (+9.5)
Dutch test2	81 (+8)	56 (+29)	84 (+22)	63 (+36)	71 (+23.5)

The figure in brackets shows the % gains from test1 to test2.

Scores in the first test (January) were close for the English and Dutch sample with averages of 45.5 and 47.25 correct solutions over the four problems. Both cohorts of pupils were more successful in dividing a two-digit number that in dividing a four-digit number. In three of the four items the score was higher for the Dutch children while the English children were more successful with the problem 1542÷5. This could be due to English pupils greater familiarity with 5 as a divisor because of it relevance in place value teaching but the change in test 2, where the Dutch pupils did better, shows any advantage does not appear to persist.

In the second test (June) improvements are similar for problems involving the division of a two digit number, 96÷6 and 98÷7, with Dutch/English improvements +8/+5 and +22/+21 respectively for the two questions. For the problems involving division of four-digit numbers, however, the Dutch improvements are much higher than those of the English children with increases +29/+2 and +36/+10 respectively.

Looking at the most popular strategies used for these problems, English pupils used the algorithm with low success rate for the 4-digit numbers. The Dutch pupils used repeated subtraction with large chunks and although the success rate is not as high for 4-digit numbers, differences are less marked.

Table 8: Percentage use of most popular strategies for test 2

	Strategy	96÷6	1256÷6	98÷7	1542÷5
English test2	traditional algorithm	66 (51)	67 (21)	66 (52)	70 (34)
Dutch test2	repeated subtraction of large chunks	78 (69)	72 (50)	76 (69)	71 (52)

The figure in brackets is the percentage of correct attempts.

Errors by the English pupils included missing digits in the answer, but also many confused attempts often leading to impossible (and sometimes bizarre) answers.

Context and Bare Problems

Because half the problems were set in context and the other half were parallel numerical problems with no context it was possible to compare performances across the two formats. Comparison was not ideal as the context problems were questions 1-5, attempted first, and the numbers used in the two types of problem were similar but not the same. Results show a better overall performance on the context questions by both English and Dutch cohorts in test 1. Those numerical problems which were 'context' in test 1 became 'bare' in test 2 and vice versa. In test 2 the English pupils continue to be more successful with the context questions but there appears to be less difference for the Dutch (Table 9).

Table 9: Comparison of success in context and bare problems

English (n=275) correct solutions		
	Context	Without context
test1	41%	35%
test2	47%	41%
Dutch (n=259) correct solutions		
	Context	Without context
test1	52%	43%
test2	67%	68%

In the Dutch RME approach calculations are introduced through a contextual problem and generalisations are made to different numerical examples.
When changes in facilities for the 'context to bare' and for the 'bare to context' problems with the same numerical calculation were considered it was found that in both the English sample and the Dutch sample there were more improved results (incorrect in test 1 to correct in test 2) and fewer deteriorated results (correct in test 1 to incorrect in test 2) where the problem format changed from 'bare' to 'context' (Table 10).

Table 10: Improvements and deteriorations in questions that were attempted

	'Context' to 'bare'		**'Bare' to 'context'**	
English	Improved	23%	Improved	33%
	Deteriorated	32%	Deteriorated	29%
Dutch	Improved	55%	Improved	58%
	Deteriorated	21%	Deteriorated	19%

Some of the improvement could be attributed to the fact that changing from 'bare' to 'context' also involved a change in the order of the problems as the context problems were the first five. The number of attempts was greater for the context problems with 12% of 'bare' problems not attempted in test 2 by the English pupils compared with 5% of 'context' problems not attempted. The difference was smallest for the Dutch cohort in test 2 where 3% of 'bare' problems were not attempted compared with 1% only of the 'context' problems not attempted.

Boys and girls improvements

When considering improvements from test 1 to test 2 there is a significant difference in the performance of Dutch boys and girls but great similarity between English boys and girls. The Dutch girls made bigger gains (mean = 2.6) than the Dutch boys (mean = 1.5). An unpaired t-test for the Dutch cohort shows this is significant with $t = -3.14$ and $p = 0.0018$. For the English cohort there is some difference with mean gains of 0.64 (girls) and 0.50 (boys) but this difference is not significant, $t = 0.59$ with $p = 0.56$. When comparing the Dutch pupils' strategies and facilities, in test 1, the Dutch boys not only used high level chunking 5(H) more often (Table 11) but had more success with all the strategies they used and were successful in 52% of the items compared with the girls success in 42% of the items. In the second test the girls were still using more lower level strategies overall but shows greater use (70% of all items) of high level chunking 5(H) and greater success with this strategy (53% correct). The girls have 'pulled up' to the success level of the boys with both successful in 68% of the items.

Table 11: Strategies used by boys and girls in the first and second tests

	test1		test2		test1		test2	
	attempts	correct	attempts	correct	attempts	correct	attempts	correct
strategy	**Dutch girls**				**Dutch boys**			
1(S)	11%	5%	2%	1%	8%	4%	1%	1%
2(P)	8%	1%	7%	2%	6%	1%	4%	2%
3(L)	16%	7%	6%	5%	16%	7%	6%	5%
5(H)	37%	24%	70%	53%	45%	31%	68%	50%
7(AL)	5%	2%	3%	1%	2%	1%	2%	1%
8(ME)	6%	3%	8%	5%	12%	8%	14%	9%
9(WR)	4%	0%	1%	0%	5%	0%	2%	0%
10(UN)	1%	0%	1%	0%	2%	0%	1%	0%
o	12%	0%	2%	0%	5%	0%	2%	0%
total		**42%**		**68%**		**52%**		**68%**
	English girls				**English boys**			
1(S)	17%	7%	12%	7%	15%	6%	9%	5%
2(P)	5%	0%	4%	0%	3%	0%	2%	0%
3(L)	6%	2%	9%	2%	6%	2%	7%	2%
5(H)	8%	5%	6%	5%	8%	6%	7%	4%
7(AL)	38%	18%	48%	25%	39%	16%	49%	24%
8(ME)	9%	5%	8%	5%	12%	6%	14%	6%
9(WR)	3%	0%	2%	0%	3%	0%	2%	0%
10(UN)	4%	1%	3%	0%	4%	1%	2%	0%
o	9%	0%	7%	0%	11%	0%	9%	0%
total		**38%**		**45%**		**37%**		**42%**

The Dutch boys showed no working in 14% of items in test 2 compared with Dutch girls (8%). The English cohort show very similar results with 14% of items attempted by English boys showing no working compared with English girls attempts (8%). About two thirds of all attempts were correct except for the English boys who were correct in less than half of these items.

Discussion

Difficulties associated with different strategies

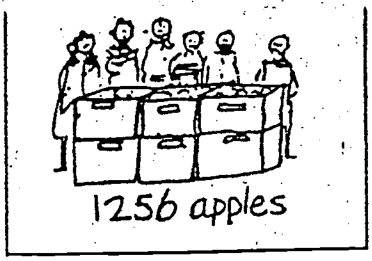
The low level strategies of tallying, adding and sharing generally showed pupils had understanding of the nature of division as these approaches were often correct and could lead to a solution. These 'low level' strategies were sometimes successful for the smaller numbers (98÷7 and 96÷6) but where larger numbers were involved (e.g. 432÷15) few pupils worked through to an answer although the approach would ultimately have led to a solution. In all cases these strategies were inefficient and errors occurred.

In low level chunking pupils showed some attempt to gain efficiency with repeated addition particularly in the problems involving division by 15 where subtotals of 30 or 60 were used. The processes used were long and many of the pupils lost track of their working where their written recording was poorly structured. Some came close to completion, for example, where pupils found how many 30s were in 432 but forgot to double the result to get the correct answer.

High level chunking was a more successful strategy often showing good understanding, not only of the division problem, but also of the relationships between numbers. Where the number 432 was 'chunked' as 300, 60, 60 and 12, division by 15 could be efficiently accomplished. The number 1256 was also considered in chunks as 1200 and 56 to give a quick method of finding 1256÷6. Where this high level chunking failed, poor organisation of the written record appeared to lead to confusion and some correct calculations were not used appropriately to find an answer. Some examples showed clearly the way a place value approach based strictly on (thousands, hundreds) tens and units can lead to difficulty. Some pupils attempted to solve 1000÷6, 200÷6, 50÷6 and 6÷6 adding the results (Figure 4). 'Chunking' involves thoughtful partition of the dividend 1256 into numbers associated with the divisor 6.

Figure 4: Place value partitioning can lead to difficulties

Figure 4: Place value partitioning can lead to difficulties



1256 apples are divided among 6 shopkeepers.
How many apples will every shopkeeper get?
How many apples will be left?

Working: $100 \times 6 = 600$

Answer: 136

$1000 \div 6 = 106 r 2$

$200 \div 6 =$ [illegible] $21 r 2$

$50 \div 6 = 8 r 2$

$6 \div 6 = 1$

$106 + 21 + 8 + 1 = 136$ (127, 9)

$2 + 2 + 2 = 6$

number sense.

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