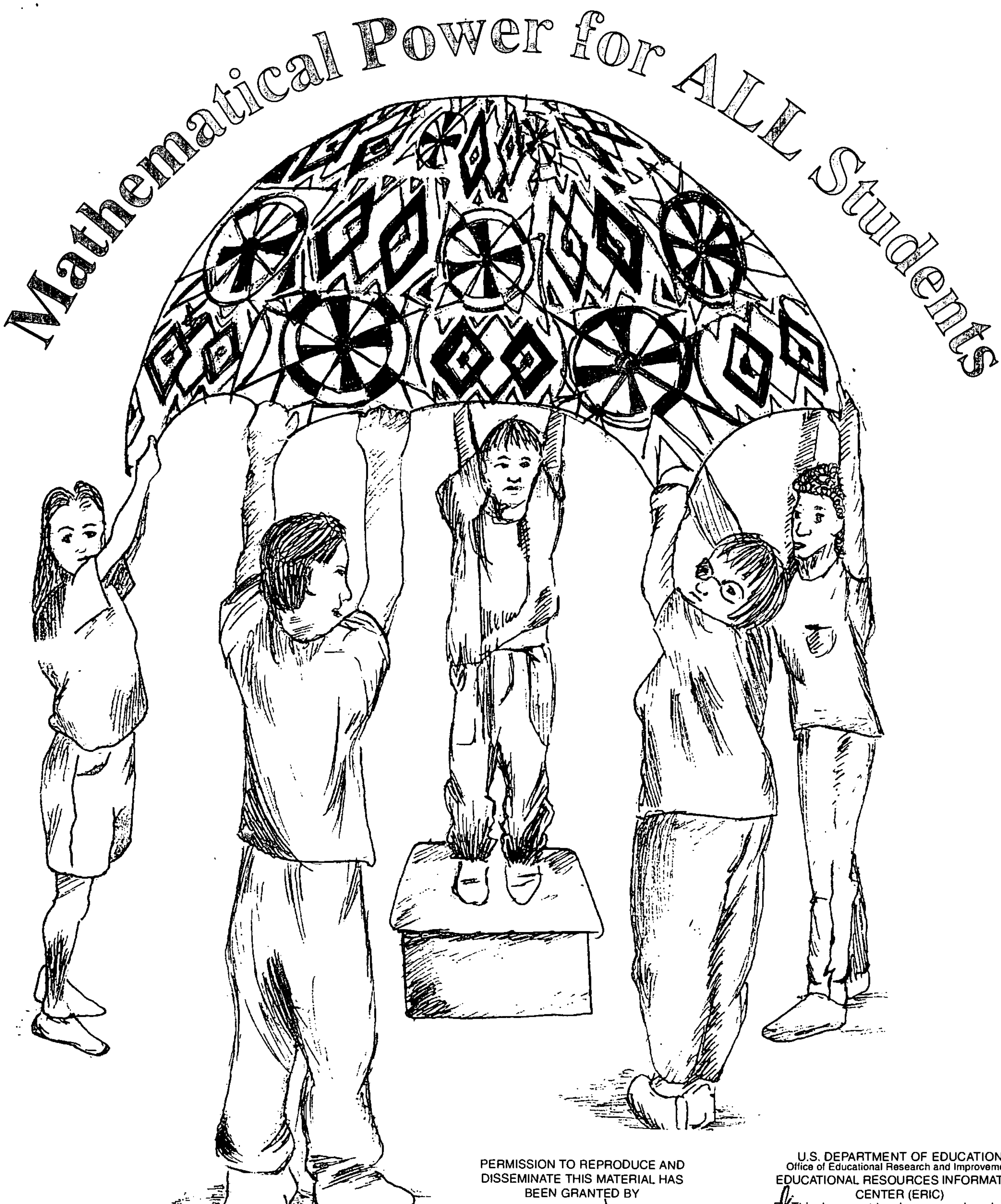
DOCUMENT RESUME

ED 445 887		SE 063 694
TITLE	Mathematical Power for ALL Students: The Rhode Island Mathematics Framework, K-12.	
INSTITUTION	Rhode Island State Dept. of Elementary and Secondary Education, Providence.	
SPONS AGENCY	Department of Education, Washington, DC.; National Science Foundation, Arlington, VA.	
PUB DATE	1995-10-00	
NOTE	121p.	
CONTRACT	R68A20018; TEP-9150056	
AVAILABLE FROM	RI Department of Elementary and Secondary Education, Shepard Building, 255 Westminster Street, Providence, RI 02903-3400.	
PUB TYPE	Guides - Non-Classroom (055) -- Reference Materials - General (130)	
EDRS PRICE	MF01/PC05 Plus Postage.	
DESCRIPTORS	*Academic Standards; *Educational Change; Educational Research; Elementary Secondary Education; *Evaluation; Inclusive Schools; *Mathematics Curriculum; Mathematics Education; *Numeracy; State Curriculum Guides; Teaching Methods	
IDENTIFIERS	*Rhode Island	

ABSTRACT

A mathematically literate citizenry is essential to maintaining democracy and ensuring a competitive position in a global economy. Instruction, curriculum, and assessment related to the teaching and learning of mathematics must be such that all students are given the opportunity to learn high levels of mathematics and encouraged to succeed. This report on the Rhode Island Mathematics Framework K-12 addresses the goals put forth in the state's Common Core of Learning as well as mathematics content and process standards, the teaching and learning of mathematics, and the support needed for implementation. An executive summary and a mathematical framework table are appended. (WRM)



The Rhode Island Mathematics Framework K–12

Published by the
Rhode Island Department of
Elementary and Secondary Education

Mathematical Power for ALL Students

The Rhode Island Mathematics Framework K–12

Published by the
Rhode Island Department of
Elementary and Secondary Education

STATE OF RHODE ISLAND AND PROVIDENCE PLANTATIONS

Lincoln C. Almond, Governor

RHODE ISLAND BOARD OF REGENTS FOR ELEMENTARY AND SECONDARY EDUCATION

RHODE ISLAND DEPARTMENT OF ELEMENTARY AND SECONDARY EDUCATION

Peter McWalters, Commissioner

The Board of Regents does not discriminate on the basis of age, sex, sexual orientation, race, religion, national origin, color, or handicap in accordance with applicable laws and regulations.

This material is based upon work supported by the United States Department of Education under Award No. R 68A20018 and by the National Science Foundation under Cooperative Agreement No. TEP 9150056.

Acknowledgements

This mathematics framework has been coordinated by the Rhode Island Department of Elementary and Secondary Education, and developed by numerous individuals representing teachers, teacher educators, administrators, families, business people, and other citizens.

Albert Alba, RI Training School
Elizabeth Alexander, Flynn School
Joseph Alfano, Johnson & Wales University
Chris Allen, John Horgan Elementary School
Carol Andrews, Gilbert Stuart School
Linda Bello, Oak Lawn School
James Bierden, Rhode Island College
Ida Cabral, Colt-Andrews School
Joyce Camara, Smithfield High School
Sandy Campo, Providence Collaborative
Mary Canole, Newport County Technical
David Capaldi, Toll Gate High School
Sharon Capobianco, Scituate School Dept.
Mary Caswell, Slater Junior High School
Pat Charland, Hope Elementary School
Dennis Cheek, RI Department of Education
Terry Coes, Rocky Hill School
Dan Commisky, The Spaghetti Warehouse
Kathleen Cook, Oak Lawn School
Beth Coughlin, Cumberland Middle School
Henry D'Aloisio, RI Department of Education
Patricia Daniels, East Providence School Dept.
Joan Dambruch, New England Technical
Jean Dawson, Edward Rhodes School
Anne DeFanti, RI Department of Education
Louise Denhem, Calef School
Marilyn DeStefano, N. Kingstown High School
Marie C. DiBiasio, RI Department of Education
Kathy Dickenson, Underwood School
Charlotte Diffendale, RI Department of Education
Marion Digiammo, Barrington Middle School
Judy Dimitri, Cranston East High School
Diane DiSanto, Smithfield Public Schools
Patricia Dulac, West Bay Collaborative
Jean Esposito, Scituate High School
Ann Fagundes, Thompson School
Richard Finlaw, Hope Valley Elementary School
William Fiske, RI Department of Education
Joyce Fitzpatrick, Carl Lauro School
Pamela Fontaine, Mount St. Charles Academy
Debra Gagnon, North Cumberland Middle School
Cheryl Gibbs, Reservoir Avenue School
Joanne Gilbert, Gilbert Stuart School
Vincent Hawkins, Warwick School Department
Mariann Hayward, Peace Dale Elementary School
Ellen Hedlund, RI Department of Education
Vincent Hey, Central Falls High School
Betty Holden, Hampden Meadows School
Diane Horm-Wingered, University of Rhode Island
Arlene Iannazzi, Arlington School
Leslie Improta, Richmond Elementary
Judith Keeley, Northern Rhode Island Collaborative
Jack Keough, RI Department of Education
Richard Ku, Wheeler School
Vincent Lauro, Warwick
Nancy Lavey, Byfield School
Judy Leonard, Southern RI Collaborative
Sue Licciardi, Bain Junior High School
Maria Lindia, RI Department of Education
Helen Litterst, RI School for the Deaf

continued

Acknowledgements *(continued)*

Bernadette Marchak, Coventry Middle School
George Marques, East Providence High School
Terry Martland, Rockwell School
Donna McBurney, North Kingstown High School
Ann McCrave, Westerly High School
Mary McNulty, East Greenwich High School
Meme Meadows, RI School for the Deaf
Chuck Mello, Mary V. Quirk Building
William Meyer, Hoechst Celanese Corp.
Donna Morelle, Sowams Elementary School
Geraldine Moretti, Oak Lawn School
Sandra Moyer, Stadium School
Maria Negro, Portsmouth High School
David Olson, Blackstone Valley Electric
Susan Osberg, North Kingstown High School
Claire Palmer, Chariho School Department
Joanne Paquette, Raymond LaPerche School
Jospeh Pasonelli, Edgewood Highland School
Jane Pasqual, Reynolds School
Mary Ann Phillips, Charlestown Elementary
Claire Pollard, Providence Collaborative
Joanne Rahme, Barrington Middle School
Sandy Rainone, Gilbert Stuart School
Gayle A. Raposa, Ranger School
Jack Riley, Davies Career & Technical High School
Carolyn Saunders, West Broadway School
Diane Schaefer, RI Department of Education
Annette Schofield, Ponaganset Middle School
R.A. Schoonover, Cranston Calvert School
Ginny Schuttert, Washington Oak School
Lois E. Short, Burrillville High School
Jerry Simmons, Newport County Technical
Richard Soroka, East Providence High School
Ed Speedell, Cranston Area Technical School
Janet Stone, Rocky Hill School
Joan Stott, Calef School
Tom Stott, RI Department of Education
Leo Sullivan, Bristol Warren School Committee
Tony Terceira, Providence School Department
Ellen Thompson, St. Luke's School
Claire Thornton, Providence College
Nancy Totten, Washington Oak School
Diane Tucker, Ponaganset Middle School
Maria VanAnglen, Reynolds School
Judy Wilson, East Bay Collaborative
Betty Young, University of Rhode Island

Cover art by Lay Thet, ninth-grade student and Marjorie Gallo, teacher; Bain Junior High School, Cranston.

Table of Contents

Preface

A pillar of the Common Core of Learning must be responsibility. The 21st century will require citizens to take responsibility for themselves, their learning and their society. By gaining an awareness of their responsibilities to themselves as individuals and to society as a whole the youth will be better equipped to meet the challenges of tomorrow."

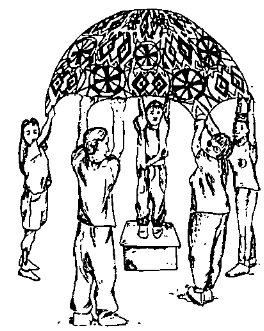
The Rhode Island Common Core of Learning

The discipline of mathematics is changing. Mathematics originally arose from the needs of various cultures to quantify, to communicate, and to solve what initially seemed to be very concrete problems. The abstractness of mathematics, which constitutes much of the mathematics studied today, often had a basis in simpler problems. As a society we must develop an awareness of the history of the development of mathematics, remaining cognizant of its changing nature as a discipline, and refine the mathematics instruction within our schools to meet not only the present, but also the future needs of our children.

Technology is one example of a cultural influence that has changed mathematics in the last twenty years. It has changed the teaching of mathematics and the ways in which it is viewed. Technology changes how we approach problems. It opens up more challenging mathematics to younger students. It redefines content and actually creates new mathematics. Technology can facilitate conceptual understanding in mathematics and create new procedures for solving mathematical problems. An understanding of the role of mathematics in a technological age is essential if we are to understand the present and the future of mathematics.

We must model what it means to take responsibility for our society by developing a framework for mathematics teaching and learning that provides the citizens, educators, and students of Rhode Island with a clear vision of the mathematics students will need in the 21st century. We cannot be content with providing our students with the same mathematics education we received. To do so would prepare them for the past, not for today or for the world that will be their future.

1



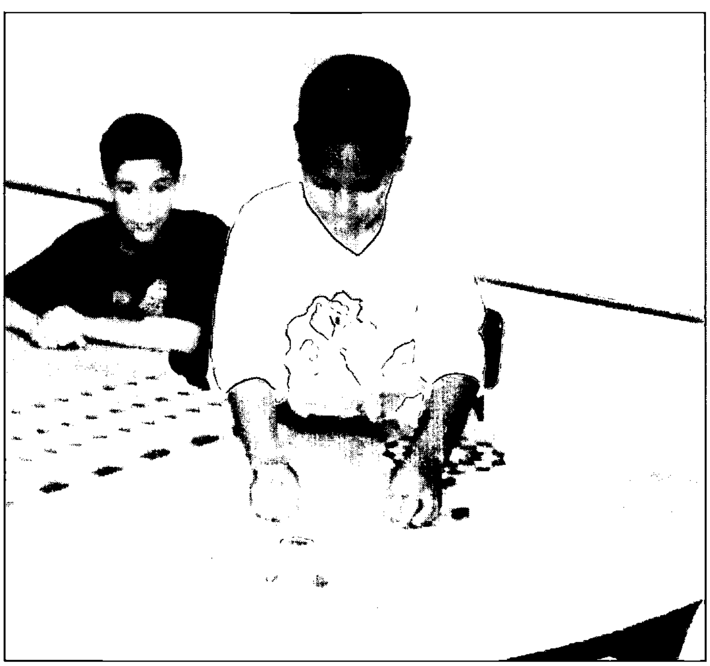
DEVELOPING A FRAMEWORK FOR MATHEMATICS EDUCATION

What Is Mathematics?

Mathematics has been defined in a myriad of ways. Entire books have been written that strive to define the discipline of mathematics. For our purposes in this framework, readers should consider the definition of mathematics to be the study of patterns and relations, with people interacting with each other and the physical world as they explore the process of thought, solve problems, make connections, reason, and communicate ideas.

"Mathematics is the subject that helps you mentally organize your thoughts."

A parent at a Providence focus group



Critical Beliefs of Mathematics

The development of the Rhode Island Mathematics Framework was guided by the following five critical beliefs. Each will be discussed separately in the pages that follow.

- Reform in mathematics education in Rhode Island must be consistent with the direction established by our local schools, our state initiatives, and national efforts to improve the quality of mathematics education.

- Mathematics is for *all* students and the framework must address the mathematics education of every student in Rhode Island's schools.

- The development of a framework for mathematics curriculum is a necessary step in the reform of mathematics education. To be successful, reform must be systemic and include changes in the ways schools are structured, lessons are taught, disciplines are integrated, students are assessed, and teachers are supported in their change efforts. Therefore, development of a framework does not end at the articulation of content standards.

- A framework provides a vision for mathematics programs. It is a document that guides the creation, selection, and implementation of effective programs designed to meet the specific needs of individual districts, schools, classrooms, and students.

- Changes in mathematics classrooms will not happen simply because a framework is published and disseminated by the Department of Education. Meeting the needs of all students will require the support and partnerships of classroom teachers, administrators, families, business people, higher education faculty, policy makers, and other community members.

Reform in mathematics education in Rhode Island must be consistent with the direction established by our local schools, our state initiatives, and national efforts to improve the quality of mathematics education.

A mathematically literate citizenry is essential to maintain our democracy and ensure a competitive position in a global economy. This has been a consistent message in national and local reports published by panels and commissions charged with evaluating the status of education and the economy in preparation for the 21st century.

The business community wants and needs workers who are capable of solving problems individually or as part of a team, capable of using technologies, and capable of transforming knowledge and skills to new situations. Educated consumers are necessary to sustain a healthy economy. Informed decisions about purchases, selections among products, and interpretation of information presented in support of claims require mathematical and technological literacy.

A review of some reports on education, including *A Nation at Risk*, *Everybody Counts*, and the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards*, reveals a common message. That is, we need a population that is mathematically literate in terms of number sense; measurement and geometry; functions, relations, and patterns; and probability and statistics. If we are to compete on a level equal to that of other industrialized nations, then we require a work force that can solve problems alone or in groups, communicate mathematically, and present valid arguments in support of claims.

At the state level, the 21st Century Commission and the Rhode Island Skills Commission each drafted plans for the restructuring of Rhode Island's educational system. Both plans asked the citizens of the state to work together to establish goals and high standards for our students.

Jobs that contribute to this world economy require workers who are ...prepared to absorb new ideas, to adapt to change, to cope with ambiguity, to perceive patterns, and to solve unconventional problems. It is these needs, not just the need for calculation...that make mathematics a prerequisite to so many jobs. More than ever before, Americans need to think for a living; more than ever before, they need to think mathematically.

Everybody Counts
National Research Council

Rhode Island created a Common Core of Learning that addresses four dimensions of education: communication, problem solving, body of knowledge, and responsibility. It suggests that "students in their studies concurrently acquire bodies of knowledge while communicating new learning, solving relevant problems, and taking on responsibilities related to their learning. These goals form the basis for developing more explicit curriculum documents." It was the preliminary discussion of the Common Core that launched this mathematics framework.

The publication of each of these reports has been instrumental in beginning to effect the changes in what mathematics is taught, how it is taught, and the ways in which it is assessed in classrooms in Rhode Island and across the United States. This framework includes key recommendations of the reports that preceded it and builds on the ideas previously offered in support of the reform of mathematics education.

Mathematics is for *all* students and the framework must address the mathematics education of every student in Rhode Island's schools.

Underlying each report calling for reform in mathematics education to meet the demands of a changing world, is the belief that *all* students can learn mathematics and achieve high standards. The National Council of Teachers of Mathematics consistently uses the term "all students" throughout its curriculum and evaluation standards. The selection of this terminology was deliberate and is echoed in the messages set forth by the Rhode Island Board of Regents for Elementary and Secondary Education and publications of the Rhode Island Department of Education.

> *"All students can and must learn. That simple phrase is the goal of the Rhode Island Department of Elementary and Secondary Education. The Department believes that our children, not just some, but all children - must achieve high levels of performance. Making sure that all students reach these high levels of performance should and will be the number one priority when we talk about education in Rhode Island."*

Reaching for High Standards: Student Performance in Rhode Island,
December 1993

Stating that all students can and must learn mathematics is not enough. All students must have access to and be provided with the opportunity to engage in the learning of meaningful mathematics. High expectations and encouragement must be maintained for all students, regardless of their diverse learning needs. Our focus needs to be on recognizing students' experiences and learning styles and helping to nurture students to grow and achieve to their best ability.

Historically, some students have not been encouraged to continue to study mathematics or to explore more complex areas of mathematics. Establishing quality mathematics education would encourage and offer all students the opportunity to learn mathematics. Since students learn best when they are challenged and motivated, mathematics education must provide students with problems that arouse their curiosity and an environment that supports their efforts.

Challenging problems that engage students in doing mathematics are appropriate for all students. By challenging problems we mean those that engage students in observing, analyzing, interpreting, and developing an understanding of mathematical concepts, rather than merely executing rules to find answers to problems. All students need to solve problems, communicate, reason, and use appropriate technologies and resources. We need to assess what students can do and provide them with challenging problems that enable them to think and grow beyond where they are. Although every student has the capability to learn, some students may ultimately progress further in their learning than others, and the rate of learning may vary.

The instructional environment is an important component of learning mathematics. The instructional strategies that support diverse learning needs, such as heterogeneous grouping, make it possible to include all students in mathematical tasks that engage all learners and promote their achieving mathematical power. Cooperative learning in conjunction with individual accountability increases academic achievement, self-esteem, and confidence on the part of the learner. Grouping also enhances social skills and promotes acceptance of the diversity of learning needs among students. To meet the diverse needs of all students, additional accommodations may need to be provided to assist students to achieve. Schools need to provide the most effective learning environment to promote growth for all students.

The development of a framework for mathematics curriculum is a necessary step in the reform of mathematics education. To be successful, reform must be systemic and include changes in the ways schools are structured, lessons are taught, disciplines are integrated, students are assessed, and teachers are supported in their change efforts. Therefore, development of a framework does not end at the articulation of content standards.

The articulation of a mathematics framework that explicates what constitutes mathematics content is a necessary first step in the process of change. However, reports and state frameworks, no matter how well crafted and widely disseminated, will not alone change mathematics education. The reform efforts must go further. There are several paths, any of which can be explored as a means of supporting the development of a new system of school mathematics.

- Create a vision and build consensus around that vision with everyone.
- Identify instructional practices that address the needs of all students.
- Align instructional approaches, curriculum materials, and assessment systems.
- Offer on-going professional development for all.
- Address teacher preparation.
- Use technologies and resources.
- Evaluate working conditions, identifying and eliminating barriers to change.
- Research what other communities, states, and universities are doing and have documented as successful practices.
- Review textbooks and other materials for their applicability to the beliefs set forth in the framework.

State frameworks can guide curriculum development that meets the particular needs of individual schools and communities. Some schools may choose to design courses in which mathematics is taught through its application in other disciplines, such as a social studies course in survey and data as a means of teaching statistics and probability. Others may opt to design courses that integrate algebra, geometry, and other domains.

"The first wave of reform has broken over the nation's public schools, leaving a residue of incremental changes and an outmoded educational structure still firmly in place. The second wave must produce strategic change that restructures the way our schools are organized and operate. We've been tinkering at the margins of the education problem for too long. It's time to get to the heart of the matter."

David T. Kearns, former Deputy Secretary, United States Department of Education

Course materials, including replacement units, teacher resource books, and manipulatives, can support ways of teaching mathematics that are not necessarily textbook based. Technologies must be available and integrated with instruction. All students, including those acquiring English proficiency, must have the opportunity to learn mathematics.

A new vision of mathematics instruction is emerging. The preparation of new teachers, along with the continuous development of today's teachers, will allow them to explore mathematics and experience the kinds of instruction we envision them providing. Schools must support this change by working with teachers to find ways for them to collaborate with colleagues and to reflect thoughtfully on their own teaching. The concepts of curriculum (what students should learn), instruction (how they should be taught), and assessment (how progress should be measured) must be viewed as dependent variables that mutually support powerful learning.

Assessment that focuses on conceptual understanding and emphasizes problem solving, reasoning, connections, and communicating should complement traditional proficiency- or performance-based assessments. Open-ended questions, student self-assessment, and portfolios strengthen the role of assessment as a tool for evaluation.

A framework provides a vision for mathematics programs. It is a document that guides the creation, selection, and implementation of effective programs designed to meet the specific needs of individual districts, schools, classrooms, and students.

Mathematics is first explored in early childhood, developed throughout the school years, and used in lifelong living and learning. Students build knowledge, create meaning, and make sense of the world as they perceive, interpret, verify, assimilate, and consolidate information and experience. As learners grow in sophistication and experience with different concepts, they move from concrete to abstract thinking. This learning model is at the heart of this framework.

This framework articulates the kinds of mathematics instruction, curriculum, and assessment that support learning within the context of what Rhode Island students should know and be able to do by the end of the fourth, eighth, tenth, and twelfth grades. It is not prescriptive. We envision many different mathematical programs, each of which will be consistent with the vision of this framework and responsive to the characters of individual communities and needs of the student populations. We hope the recommendations within this framework begin to elicit the support of communities in the development and nurturing of effective mathematics programs.

Changes in mathematics classrooms will not happen simply because a framework is published and disseminated by the Department of Education. Meeting the needs of all our students will require the support and partnerships of classroom teachers, administrators, families, business people, higher education faculty, policy makers, and other community members.

Change is a difficult process. Implementation of this framework may face a number of barriers. Some teachers still value computational speed over problem-solving ability. Some communities understand scores on standardized tests and may be reluctant to accept assessments that evaluate responses to open ended questions, portfolios, and contextualized computation. Parents and families may want only the mathematics that is familiar to them, with problems similar to those they once had, rather than open-ended problems with multiple solutions. The political environment in which schools exist may present additional challenges.

It's all too easy to agree with the rhetoric of reform, but still maintain long-held beliefs or practices inconsistent with intended reform practices. Likewise it is easy to agree but at the same time claim that 'it won't work here.' We challenge readers to recognize their beliefs and practices and test them against the standards we have proposed.

NCTM Curriculum and Evaluation Standards

Meeting and overcoming barriers require the support and collaboration of individuals beyond just mathematics teachers. This reform effort cannot rest solely on the backs of mathematics teachers. Partnerships among parents; schools; universities; professional organizations; and policy makers in government, business, and industry will enable the reform effort to progress.

School administrators must recruit qualified teachers of mathematics and make assignments based on their qualifications. They must provide a system for beginning and experienced teachers that supports their continuous professional growth. Adequate resources, equipment, and time are needed in support of the teaching and learning of mathematics. Outreach programs that involve families and other members of communities are essential. Partnerships with the business and industry community are vital for effective change in the teaching and learning of mathematics. Colleges and universities must encourage their faculty to spend time in schools, working with teachers and students, collaborating with them to design preservice and continuing education programs. Finally, the support of professional organizations will help to bring about positive change in mathematics education.

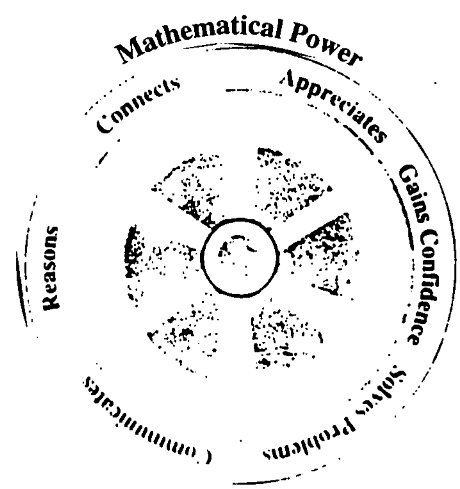
2

A VISION OF MATHEMATICS EDUCATION: POWER, PARTNERSHIPS, PREPARATION

Mathematical Power

The primary goal of mathematics programs is the development of mathematical power by all students. Mathematical power exists when a student...

- Appreciates the value of mathematics;
- Gains confidence in one's own mathematical ability;
- Engages in mathematical problem solving;
- Communicates mathematically;
- Reasons mathematically;
- Connects what is learned in mathematics with other mathematics topics, other disciplines, and daily life.



In this vision, formulating problems and explaining the methods used for arriving at solutions become just as important as finding the answers. Each problem solved builds a foundation for the next challenge. Students are encouraged to communicate and collaborate with each other as a means to increase understanding and enrich both the process and product. Finally, learners grow in understanding by reflecting on and analyzing what they have learned, how they have learned it, and the ways in which it connects with what they already know and the real world.

Lifelong learning and problem solving are not separate from the common body of knowledge that comprises the content of schooling...

Rhode Island Common Core of Learning

If all students are to develop mathematical power, then it is necessary that teachers develop an understanding of:

- How students learn mathematics;
- The effects of students' interests, experiences, and development on learning mathematics;
- The influences of students' linguistic, ethnic, cultural, socioeconomic backgrounds, and gender on learning mathematics;
- Ways to affirm and support full participation and continued study of mathematics by all students.

Partners' Roles and Responsibilities

Quality education of our youth is crucial for the success of our communities. The responsibilities for educating the next generation should be shared by all community members. Everyone—from parents and elected officials to local business owners and industrial leaders, should join with educators to help our students get the best possible education.

As schools and school systems work toward forming these new partnerships, it is helpful if they delineate the roles and responsibilities of the groups who will contribute to their efforts toward providing effective mathematics programs. Here are some of the responsibilities for each role.

Students

- Be active participants in and take responsibility for learning.
- Value mathematics.
- Help others to develop mathematical skills and an appreciation of mathematics.

Teachers

- Create and maintain classroom environments that facilitate cooperation; communication; and hands-on, activity-based instruction.
- Engage students in meaningful and developmentally appropriate tasks or investigations.
- Work cooperatively with other teachers to develop interdisciplinary curriculum.
- Promote learning and growth for all students and demonstrate sensitivity to various learning styles.
- Connect mathematics with other content areas, daily life, and the workplace.
- Work with school officials, parents, and community members to promote mathematics education.

- Participate in appropriate on-going professional development.

Administrators

- Provide support in the development of quality mathematics programs at school and district levels.
- Foster systemic change through partnerships and provide opportunities for collaboration among colleagues.
- Promote a culture that plans, designs, and evaluates professional development as described in this framework.
- Encourage and support professional development for teachers, administrators, families, and others.
- Support, praise, and reward teachers who promote the ideas of this framework and the NCTM standards.

Families

- Create a home environment that fosters high expectations in mathematics achievement.
- Volunteer time and expertise in the schools.
- Become and remain informed about issues in the teaching and learning of mathematics.
- Support funding of mathematics education reform programs.

Business and Industry

- Invite teachers and students to work with businesses and industries to understand first-hand how mathematics is used in the workplace.
- Allow employees to co-teach with mathematics teachers in their classrooms.
- Help organize activities such as career days, field trips, or mathematics fairs to stimulate students' interest in mathematics.
- Assist schools in acquiring technologies and materials that enhance mathematics teaching and learning.

"Our company does a lot with schools at all levels. I give an open invitation to schools—if they ever need someone to speak to a class about the relevance of what they are doing and how it connects to the future of work...I'll be happy to do it...I'm willing to work with students asking the question 'Why study this?'"

Rhode Island focus group participant

Higher Education Faculty

- Establish mathematics teacher education programs consistent with the Rhode Island Mathematics Framework.
- Teach future teachers in the ways that they will be expected to teach.
- Develop with teachers and school systems appropriate in-service courses for professional development.
- Work collaboratively with teachers in the classroom to improve instruction.

- Provide leadership and research that furthers mathematical reform.

Policy Makers

- Be knowledgeable of the mathematics framework and promote mathematics education.
- Provide technical assistance to develop high quality mathematics programs.
- Allow flexibility in state mandates and regulations to encourage the development of high quality educational systems.
- Provide the resources and funding necessary for schools and districts to implement the mathematics framework.

Other Partners

- Produce and disseminate information about reforms in mathematics education.
- Strive to counteract stereotypes and promote positive images of mathematics.
- Participate in activities that foster the importance of learning mathematics to the students and to the community.

Preparation for the 21st Century

"It may be difficult to find jobs if they do not learn mathematics."
Providence focus group participant

When we compare national, state, and local school education goals, we find a common message, which states that if students are to meet the challenges of the twenty-first century, then they must:

- Develop higher order thinking skills to enable them to become problem solvers;
- Be actively engaged in their learning such that they construct their own knowledge and understanding;
- Communicate their findings and understanding to others;
- Work cooperatively with others as part of a team;
- Accept responsibility for their learning.

In order to meet these goals, education needs to address certain issues. As depicted in the Secretary's Commission on Achieving Necessary Skills (SCANS) report, the traditional classroom needs to be transformed into a learning community. The following chart is taken from the report and is intended to help readers see the ways they can encourage and assist students to become workers who will meet the challenges of the twenty-first century.

In order to bring about this systematic change, students, teachers, administrators, families, business, and other community members need to work together, sharing a common vision.

THE TRADITIONAL CLASSROOM	THE LEARNING COMMUNITY
TEACHER KNOWS THE ANSWER.	MORE THAN ONE SOLUTION MAY BE VIABLE AND TEACHER MAY NOT HAVE IT IN ADVANCE.
STUDENTS ROUTINELY WORK ALONE.	STUDENTS ROUTINELY WORK WITH TEACHERS, PEERS, AND COMMUNITY MEMBERS.
TEACHER PLANS ALL ACTIVITIES.	STUDENTS AND TEACHERS PLAN AND NEGOTIATE ACTIVITIES.
TEACHER MAKES ALL ASSESSMENTS.	STUDENTS ROUTINELY ASSESS THEMSELVES.
INFORMATION IS ORGANIZED, EVALUATED, INTERPRETED, AND COMMUNICATED TO STUDENTS BY TEACHER.	INFORMATION IS ACQUIRED, EVALUATED, ORGANIZED, INTERPRETED, AND COMMUNICATED BY STUDENTS TO APPROPRIATE AUDIENCES.
ORGANIZING SYSTEM OF THE CLASSROOM IS SIMPLE: ONE TEACHER TEACHES 30 STUDENTS.	ORGANIZING SYSTEMS ARE COMPLEX: TEACHER AND STUDENTS BOTH REACH OUT BEYOND SCHOOL FOR ADDITIONAL INFORMATION.
READING, WRITING, AND MATH ARE TREATED AS SEPARATE DISCIPLINES; LISTENING AND SPEAKING OFTEN ARE MISSING FROM THE CURRICULUM.	DISCIPLINES NEEDED FOR PROBLEM SOLVING ARE INTEGRATED: LISTENING AND SPEAKING ARE FUNDAMENTAL PARTS OF LEARNING.
THINKING IS USUALLY THEORETICAL AND "ACADEMIC."	THINKING INVOLVES PROBLEM SOLVING, REASONING, AND DECISION-MAKING.
STUDENTS ARE EXPECTED TO CONFORM TO TEACHERS BEHAVIORAL EXPECTATIONS; INTEGRITY AND HONESTY ARE MONITORED BY TEACHER; STUDENT SELF-ESTEEM IS OFTEN POOR.	STUDENTS ARE EXPECTED TO BE RESPONSIBLE, SOCIABLE, SELF-MANAGING, AND RESOURCEFUL; INTEGRITY AND HONESTY ARE MONITORED WITHIN THE SOCIAL CONTEXT OF THE CLASSROOM; STUDENTS' SELF-ESTEEM IS HIGH BECAUSE THEY ARE IN CHARGE OF THEIR OWN LEARNING.

The learning community depicted above correlates what is being taught and learned with what is happening in our world. Additionally, students in this environment understand the relevancy of what they are learning and how it applies to their lives. By engaging in problem solving, communicating, reasoning, and making connections, students understand both the applications and the concepts behind the mathematics they are learning.

3

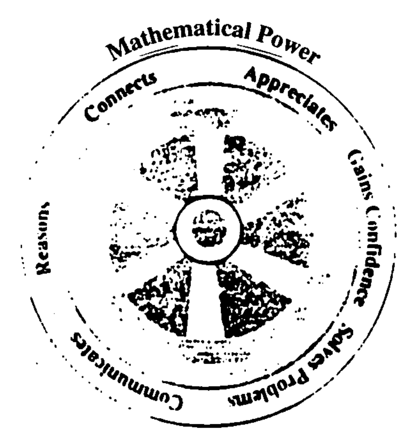
TEACHING AND LEARNING MATHEMATICS

"You can solve the problems in many different ways. Just like life."

Contribution of a parent attending a focus group

Mathematics education reform suggests a reexamination of the teaching and learning that takes place in most Rhode Island classrooms. Inquiry-based, learner-centered explorations need to be the focus of the classroom. Students become actively engaged in doing and learning mathematics. Through hands-on explorations of mathematical ideas, students construct their own understanding of concepts and procedures in a way that connects mathematical ideas with the world. Conjecturing, inventing, and problem solving are valued by students and teachers. In learner-centered classrooms students talk and write about mathematics. Reasoning, logic, and mathematical evidence provide the base for discussions.

If we are to reach our goal of all students' developing mathematical power, then the instruction, curriculum, and assessment must be such that all students are given the opportunity to learn high levels of mathematics and be encouraged to succeed. The image of mathematics teaching suggests elementary and secondary teachers who are proficient in:



- Selecting mathematical tasks that engage students' interests and intellect;
- Providing opportunities to deepen students' understanding of the mathematics being studied and its applications;
- Orchestrating classroom discourse in ways that promote the investigation and growth of mathematical ideas;
- Using technology and helping students to use technology and other tools to pursue mathematical investigations;
- Finding and helping students to seek connections to previously learned concepts;
- Guiding individual, small-group, and whole-class work.

An examination of mathematics teaching focuses on the *tasks* students are expected to perform, the nature of the *discourse* that takes place in classrooms, the *environment* created for learning, and *analyses* of the teaching and learning that takes place. A discussion of each of these lenses follows.

Tasks Promote Ability

"I want my children to enjoy learning mathematics, like any other academic subject, especially in their first years of school. I would like them to learn mathematics by using art as a tool to understand the concept of relations."

A Providence parent's response to "How do you want your children to learn mathematics?"

Authentic, challenging tasks form the core of reform in mathematics. Tasks provide the stimulus for students to think about particular concepts and procedures, their connections with other mathematical ideas, and their applications to real-world contexts. Tasks that are grounded in meaning for the students are usually more complex than those assigned with a discrete-skills approach. Interdisciplinary tasks that cut across disciplines will usually require more time for planning and implementation. Tasks that require students to reason and communicate mathematically will enhance students' abilities to formulate and solve problems and make the valuable connections with what they already know, other disciplines, and daily life. Tasks that engage students in hands-on activities add a dimension of meaning that promotes students' ownership of their knowledge.

These complex tasks involve group work. Members of groups accept responsibility for a facet of an entire project. Working collaboratively sets the stage for students to justify their reasoning and explain what they know. Students can rely upon their individual strengths, as they depend upon the group for bolstering their weaknesses. Collaboration promotes student self-reflection, enhancing the internalization of mathematics.

The following vignette is an example of the kind of mathematical tasks that promote mathematical power.

A new fish pond is under construction at City Park. The city planner visited our school and asked the students to create designs for consideration. Given specified dimensions, students were encouraged to work as independent architects while part of an architectural team, a cooperative group of four. Students used manipulatives, including color tiles, pattern blocks, and Cuisenaire™ rods, along with rulers, dot paper, and graph paper to make scale drawings.

Using the concepts of area and perimeter, pond designs with brick borders were explored. Geoboards were used to help the students visualize some of the possibilities.

After completion of the designs, students were each given a budget of $8,000 for the purchase of bricks, trees, and shrubs. Using calculators, they were asked to review and modify their plans for cost effectiveness, taking into account the cost of bricks per linear foot, the space needed between trees and *shrubs, and ample space for viewing by citizens.*

Next, all of the students conferenced with their architectural team to present and promote their individual designs. A single best design was chosen by each team. Then, working together, each team wrote a proposal defending their choice. Word processors and computers were used to help with the writing process.

We extended the task to include the concept of volume by asking questions such as "How much dirt will be removed in order to make the fish pond?...How much water will be needed to fill the pond?"

At the end of this project, each architectural team gave an oral presentation and defended their proposals at a mock town meeting. All proposals were considered and the students voted for the most comprehensive design of the fish pond exhibit.

Student work was assessed several ways. Each day, students entered a paragraph or two in their math journals, summarizing their learning for that day. Architectural portfolios were kept with original sketches, designs, calculations, and notes. Teacher observations of both individual and group performances were noted. One final assessment was of the accuracy and appearance of the design submitted and the completeness of the presentation at the town meeting.

Linda Bello and Kathleen Cook
Oak Lawn School, Cranston

A Teacher's Guide for Considering Mathematical Tasks

- Does the task address important mathematical ideas, processes, and perspectives?
- How does the task incorporate students' understandings, interests, and experiences?
- Does the task acknowledge the different ways students learn mathematics?
- Does the task engage students' intellects?
- Will the task develop students' mathematical understandings and skills?
- In what ways does the task stimulate students to make connections and develop a coherent framework for mathematical ideas?
- Does the task require problem formulation, problem solving, and mathematical reasoning?
- Does the task reflect experiences that may be encountered in the workplace and real life?
- Will the task promote the development of all students' mathematical confidence, flexibility, willingness to persevere, interest, curiosity, inventiveness, monitoring, and reflection in mathematics?

Discourse Encourages Inquiry

The role of the teacher shifts from that of a dispenser of information to one of facilitator of mathematics learning. The teacher guides the students in a way that promotes inquiry and discovery. Flexibility allows teachers to expand or contract the lessons, activities, and projects as needed, based upon students' understanding. Effective discourse involves teachers listening to students, gathering an understanding of their informal knowledge, and building upon it. It implies multiple strategies for solving problems. The teacher is no longer the source of the right answer for a passive learner. Rather, the teacher is a stimulus that promotes active learning by all students. The ways ideas are exchanged are shaped by the tasks. If students are to construct their own mathematical meanings, then teachers need to resist pleas to solve problems for their students.

A Teacher's Guide for Reflection on Mathematics Discourse

- Do I pose questions and tasks that elicit, engage, and challenge each student's thinking?
- Do I listen carefully to students' ideas?
- Do I encourage invented as well as conventional terms and symbols?
- Do I ask students to clarify and justify their ideas orally and in writing?
- Do I decide what to pursue in depth from among the ideas that students bring up during a discussion?
- Do I attach mathematical notation and language to students' ideas?
- Do I consider carefully when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty?
- Do I monitor students' participation in discussions and encourage each student to participate?

Environment Is Fundamental

"We need to create the environment where in learning mathematics, kids can make mistakes and not put the emphasis on finding the right answer. We need to humanize mathematics more than we have in the past."

A Rhode Island Superintendent of Schools

Just as the role of the mathematics teacher is shifting, so is the emphasis in the mathematics classroom. No longer is the major emphasis on speed, accuracy, and neatness. Rather, the environment teachers create is one wherein students are challenged to justify their solutions, work collaboratively, formulate their own problems, and design alternative approaches for solving problems. While some of this suggests that the environment is derived from the tasks, it is not solely from the tasks that the environment is staged. The environment should be one that welcomes partial as well as complete proposals, analyzes incomplete or incorrect solutions to problems, and praises collaborative efforts. The classroom environment is such that all students are supported in their quest to learn high levels of mathematics and succeed in constructing their own mathematical knowledge.

A Teacher's Guide for Establishing a Mathematics Learning Environment

- Do I provide the time necessary for students to explore mathematics and grapple with mathematical ideas and problems?
- Do I use the physical space and materials in ways that facilitate students' learning of mathematics?
- Am I conveying the notion that mathematics is a subject to be explored, created, and appreciated, both individually and in collaboration with others?
- Do I respect and value students' ideas, ways of thinking, and mathematical dispositions?
- Do I remember to affirm and encourage full participation and continued study of mathematics by *all* students?

Analysis Is Essential

Throughout each year, teachers should continually ask themselves how well the tasks, discourse, and environment are working to foster the development of each student's mathematical power. By exploring and questioning the effects the classroom is having on each student, teachers can modify what and how they teach. Short-term reflection becomes the primary source for long-term planning and instruction. If assessment is the measure of what students have learned, then analysis is the measure of the impact of the tasks, discourse, and environment upon students.

A Teacher's Guide for Ongoing Analysis

- Am I observing, listening to, and gathering information about students to assess what they are learning?
- Do I use a variety of assessment methods to determine students' understanding of mathematics?
- Do I match assessment methods with the developmental level, mathematical maturity, and background of each student?
- Have I aligned assessment methods with what is taught and how it is taught?
- Do I analyze each student's understanding of and disposition toward doing mathematics, so that information about mathematical development can be provided to the students, their parents, and appropriate school personnel?
- Do I base instruction on information obtained from the assessments?
- Do I work with colleagues to help them reflect on their classroom practices?

Technology as a Tool for Learning

The primary reason for using technologies is that they support and enhance student learning. Technologies support the idea that students need to be active learners, engaged in tasks that require comprehension, reasoning, communication, and problem-solving skills. In fact the real measure of the benefits of technologies and technology-related programs are the extent to which these tools promote and support active learning wherein students accept responsibility for their own learning. This suggests that students develop their own thinking and learning strategies, exhibit a desire to learn, and work with others to develop ideas and understanding.

Technology needs to be accessible to all students because it is through the use of technology that all students are given an opportunity to communicate, comprehend, and apply skills to complex tasks. Basic skills are learned in the context of solving challenging tasks that require higher order thinking skills.

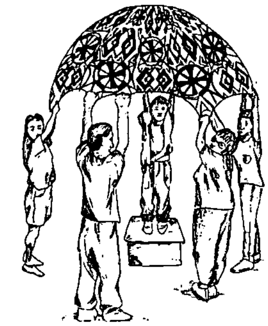
There are four main uses of technology, tutorial, exploratory, application, and communication. The *tutorial* use of technology most resembles a lecture type of format in which students receive information, watch demonstrations, or solve given problems. The *exploratory* use of technology offers students the opportunity to explore the information and control their own learning. Such uses include reference applications, such as CD-ROM encyclopedias and dictionaries. Technology *applications* include word processors, spreadsheets, management programs, and other software programs that give students the tools to aid them with writing, data analysis, and other applications. *Communication* uses allow students and teachers to send and receive information around the world through networks, interactive distance learning, or other means.

The NCTM *Curriculum and Evaluation Standards, Everybody Counts* and other national documents stress the use of calculators, graphing calculators, and computers to solve real-life problems that model what takes place in the real world. Computers and other technologies are necessary tools used in the workplace and in everyday situations. If students are to be prepared for the work force and challenges of the twenty-first century, then they must be skilled in using the tools of the future. If technology is a part of the learning environment, then it is integrated into the curriculum and not treated as an isolated topic to be studied.

A Teacher's Guide for Considering Technology

- Are the students and I using technology to enrich the learning environment?
- Are we using technology that enables students to engage in meaningful learning and collaboration?
- Do teachers and students have access to calculators, computers, and other technology resources?
- Are we using Internet or other networks?
- Do I have support available to assist me when problems or questions arise?
- Are professional development opportunities in using technology available?
- Do I help to promote the use of technology in education among students, teachers, administrators, families, and others?

4



ASSESSMENT

What Is Assessment?

Assessment is a wide variety of techniques used to gather information and evidence of students' knowledge and understanding. Assessment results are most useful when they highlight mathematical thinking and the connections between the curriculum and what students can do with what they have learned. In addition to solutions to problems, the assessment should examine the approaches students use to complete tasks. All mathematics assessment includes four phases—planning the assessment, collecting evidence, interpreting the evidence, and using the results. The following discussion reflects the NCTM assessment standards.

Not everything that counts can be counted and not everything that can be counted counts.

Reportedly posted on Albert Einstein's office wall

Assessment should reflect the mathematics that all students need to know and be able to do. Assessing students' ability to do mathematics includes assessing their skills, procedural knowledge, and factual knowledge. It goes beyond testing students' ability to get a correct answer on computation or word problems. Assessment has to match the vision reflected in this Rhode Island Mathematics Framework and the NCTM *Curriculum and Evaluation Standards for School Mathematics.*

Mr. Corelli has had an eye-opening experience. In looking over last year's materials, he has come across a pile of graded multiple-choice mathematics tests that he had saved for parent conferences. They are marked 72%, 94%, 59% and so on.

This year he realizes that he has a great deal more information to share with parents, as he has been trying out some other forms of assessment with his class. He asked his students to write about how they solved some of the mathematics problems and to share their writing with classmates. Some students have worked together on projects and presented their findings to the class, using graphs and charts. Others have helped tutor classmates. Mr. Corelli still has some quizzes and tests to show parents on conference day, but they have become only part of the rich variety of information he has gathered on his students' *learning in mathematics.*

Ellen Thompson, St. Luke School, Barrington

Assessment should enhance mathematics learning. Since good assessment reflects good instruction, assessments are learning opportunities as well as ways for students to show what they know and can do. Assessment becomes embedded in the learning and becomes part of the instruction. As the classroom becomes student-centered rather than teacher-centered, students routinely reflect upon their learning and accomplishments and are encouraged to use this self-assessment to enhance their understanding.

Assessment should promote equity. All students, including those with special needs or talents, are given the opportunity and are expected to demonstrate mathematical power to their fullest. Since each student brings unique experiences to the classroom, assessors need to be flexible to alternative solutions and assumptions made by students. Students must be given the opportunity to explain their understanding of the problem and defend their solution based on their interpretation.

Assessment should be an open process. How students are assessed should not be a surprise to them. All students should be aware of what they need to know and how they must demonstrate this knowledge. Students' work will improve when students understand the criteria upon which their work will be judged.

Assessment should promote valid inferences about mathematics learning. Since teachers need to make valid inferences or conclusions about students' work or performance, using multiple sources of evidence can improve the quality judgment made on students' learning. Assessment needs to be based on important mathematics, use multiple sources of student work, minimize bias, and support student learning.

Assessment should be a coherent process. Assessment needs to be aligned with instruction and curriculum. Since students' learning and assessment experiences are connected, each must reflect diverse activities to provide multiple opportunities for students to learn and demonstrate their understanding. The use of appropriate mathematics assessment helps students to understand how assessment is connected to the mathematics they are learning.

Assessment Purposes and Beliefs

The four purposes of any assessment as stated in the NCTM Assessment Standards are to:

- Monitor student progress toward learning goals;
- Make instructional decisions based on evidence of students' understanding;
- Evaluate student achievement at various times;
- Evaluate programs.

Similarly, in Rhode Island, statewide assessment information is used to:

- Inform and improve instruction;
- Measure school achievement for accountability purposes;
- Provide information about student knowledge and understanding for students, parents, teachers, and others.

Since good assessment is an integral part of good instruction, assessment should encourage and allow the use of appropriate tools in the teaching and learning of mathematics—calculators, computers, and other technologies, measuring devices, geometric models and other concrete materials that enhance the learning of mathematics.

A Teacher's Guide for Considering Assessments

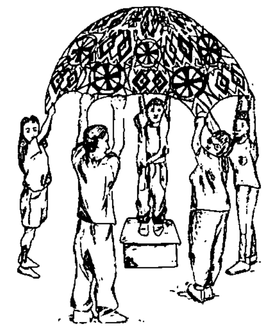
- What mathematics is represented in the assessment?
- Does the assessment provide realistic situations?
- How does the assessment promote students' learning?
- How does the assessment provide opportunities for all students to exhibit what they know and can do?
- How is the assessment being judged to recognize students' backgrounds and experiences?
- How are teachers and students involved in the selecting of activities, establishing criteria, and assessing results?
- How is bias minimized in assessing and reporting results?
- How is the assessment connected to the instruction and curriculum?
- What did we learn that will help us improve instruction?

Types of Assessment

A variety of assessment methods are used by teachers to understand student performance. Each method has its own special features and merits. A combination of assessment techniques like the ones shown below work together to provide multiple sources to assess students' learning.

- Student work and responses to open-ended questions
- Projects and investigations
- Portfolios
- Mathematics journals or other written explanations
- Student-formulated problems and questions
- Presentations and discussions
- Observations, interviews, and conferences
- Paper and pencil tests and quizzes
- Norm referenced and other standardized tests
- Self and peer assessments

5



PROFESSIONAL DEVELOPMENT

Why provide professional development?

The Rhode Island Department of Education has developed Standards for Professional Development that describe professional development as a system of continuous growth and learning that builds the capacity of a school community to respond to the needs of all learners. The goal of professional development is to improve students' learning by enhancing the knowledge and skills of everyone who affect it. This suggests participation by all members of the school community—teachers, school administrators, family members, higher education faculty, local businesses, and community members.

These stakeholders must continuously engage in their own professional development and collaborate with other groups to promote systemic change in which each school focuses on how to educate its students to meet the demands of the future. Since the education of our children is the responsibility of everyone within the learning community, professional development opportunities need to be fostered in risk-free environments that clearly focus on common goals.

"...Professional development has to be sustained. In other words, if people engaged in true curriculum work...across grades and across the district, then you would have a viable product in the end. ...I think it has to be well-planned development, not one quick staff development session...you're talking a program that goes three to five years."

A teacher participant at a Framework Focus Group

The exponential growth in technology that permeates the workplace and home has yet to be reflected in educational systems. In order to prepare students for an ever-changing future, schools must develop and model the capacity to deal with constant change.

A major vehicle for affecting change in the educational environment is that of professional development. Properly designed professional development creates the opportunity to acknowledge and analyze existing changes, to understand the nature and implications of those changes, and to design educational systems that will address the never-ending world of change. Professional development helps teachers, families, administrators, communities, business members, and others to reach a better understanding of the mathematics learning process.

This understanding enables professionals and communities to help students achieve to the best of their abilities.

The kinds of teaching and learning that are reflected in this framework and NCTM's *Curriculum and Evaluation Standards for School Mathematics* and *Professional Standards for Teaching Mathematics* are different from what most adults have experienced in their own learning of mathematics and mathematics education courses. It is essential that teachers become engaged in on-going professional development that will assist them in making the necessary changes in their teaching practices. It is vital that teachers design and participate in professional development that meets their needs and helps them to achieve excellence.

What is quality professional development?

The four principles of good teaching identified in the Rhode Island Standards for Professional Development are the *culture, planning, design,* and *evaluation* of professional development. These serve as the qualifiers of quality professional development.

Culture, which refers to the conditions that support individual and organizational growth and development, sets the atmosphere for success. The attitudes, beliefs, and values of people within the learning community are critical in the designing, building, and sustaining of professional development. It is imperative that the school culture acknowledges and responds to the diversity of its participants when building a supportive community that will focus on student achievement.

Planning, which refers to what and how decisions are made in preparing professional development programs, gives participants opportunities for partaking in the decisions that will affect them. This planning process needs to be consistent with larger systemic and school improvement efforts, for as we have seen from experience, isolated efforts do not create the systemic changes that are needed to bring about excellence in our educational system.

Design, which refers to the content of professional development programming, how it is organized, and how it is delivered, reflects what is already known about adult learning and diverse learning styles. There are many different models of professional development with no one model being appropriate for all learners. If the offerings are cognizant of this fact, then meaningful professional development is more likely. Research has shown that one-day workshops do not effect long-term changes in people's behavior or what they do in classrooms. For professional development to have lasting impact, it needs to be on-going and supportive. Some of the most meaningful professional development takes place on the job, with colleagues sharing ideas, visiting classrooms, and engaging in action research, or participating in study groups. Quality professional development occurs within an atmosphere that encourages people to experiment, permits mistakes, and fosters growth through learning. With this in mind, learning communities are challenged to design comprehensive professional development plans that reflect the school philosophy and culture. Since any design plan will have an effect on aspects of the school day and calendar, systems will require redesigning to accomplish the goals set by the learning community.

Evaluation, which refers to determinations of program effectiveness, incorporates opportunities for periodic input by the participants. Since the ultimate goal of professional development is to effect changes in students' achievement, attitudes, and skills, it is imperative that the effectiveness of the professional development programs are continuously evaluated.

How do these criteria apply to mathematics?

To improve the teaching and learning of mathematics, professional development in mathematics must mirror these four principles. Teachers are the professionals who are primarily responsible for determining what mathematics is taught and how students are engaged in learning mathematics. It is the teachers who need to know how students learn mathematics, recognize and appreciate diversity, and create environments that are sensitive to the learning needs of all students.

Professional development opportunities should engage educators in investigating mathematical concepts, structures, and procedures, including the connections among them. Using a developmental approach to build mathematical knowledge, teachers provide concrete, pictorial, and abstract models for their students. Teachers engage in problem solving, mathematical reasoning, communications, and making connections as they become confident and develop an understanding of teaching and learning mathematics.

A Learning Community's Guide for Professional Development

- Is there an agreed-upon vision for all students, based on high academic standards and adequate designs and plans to implement the vision?
- Does the program focus clearly on academic achievement?
- Have we extended planning to allow for a collaborative program design?
- Does a well-defined organization and management structure exist?
- Have we addressed the entire professional community?
- Is the program sensitive to cultural inclusiveness?
- How are families and the community involved?
- Is there a mechanism for evaluating school and student progress?

(Source: *An Idea Book*, U. S. Department of Education)

It is imperative that schools and school districts invest in professional development for members of their learning communities. Learning how to work together to accomplish the desired outcomes for our students requires support and guidance. By providing this support to those working with students, we show our commitment to helping Rhode Island students achieve excellence.

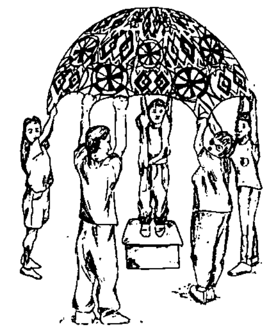
The following list of models is intended to help people when they consider designs for professional development. Rather than work from what may already exist, people are encouraged to identify their specific needs and then create learning opportunities based upon their goals for students. We all need to regard ourselves as lifelong learners and engage in on-going professional development.

Models of Professional Development

from the *Journal of Staff Development*

- **Individually Guided Staff Development** Participants identify goals and select tasks that will help them to accomplish their goals.
- **Observation and Assessment** Participants reflect upon and analyze their roles and contributions in order to improve students' learning.
- **Involvement in a Development and Improvement Process** Participants are involved in solving a problem that addresses issues relating to school improvement or curriculum development.
- **Training** Participants engage in training to acquire knowledge and develop skills in specific areas.
- **Inquiry** Participants, individually or in groups, formulate questions and explore possible solutions to those questions.
- **Case Development** Participants, through the use of case studies, analyze situations, communicate recommendations, and broaden their understanding of pedagogy and knowledge.

6



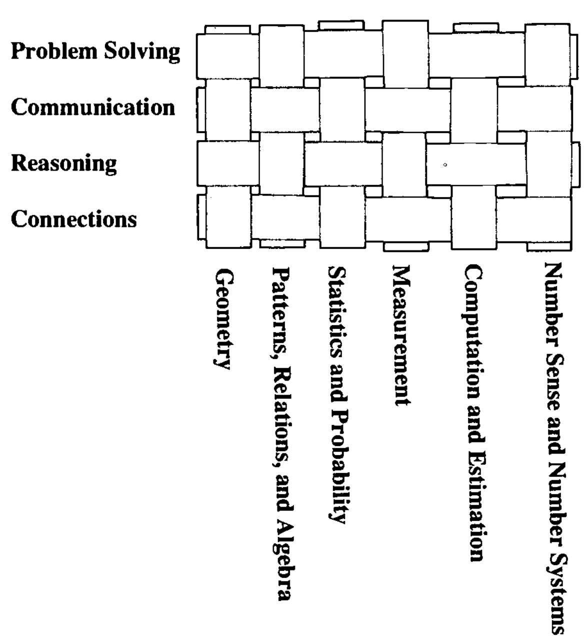
MATERIALS AND RESOURCES

Instructional materials and resources influence and determine the teaching and learning that takes place in most classrooms. Traditionally, the mathematics textbook has dictated the mathematics curriculum in most schools. The intent of this section is to address the need to shift from using one resource, a textbook, to using multiple materials and resources. This shift will enable students and teachers to engage actively in meaningful opportunities that explore more real-world situations and applications that connect with home and work-related experiences. Therefore, in keeping with the goals of the NCTM *Curriculum and Evaluations Standards* and this framework, consideration and selection of instructional materials should address mathematical content, program organization and structure, student experiences and teacher's role, and assessment. Each criterion is discussed separately in the paragraphs that follow.

Mathematical Content

It is essential that all students in grades K–12 engage in mathematics programs that include all of the process and content standards depicted in this framework.

The Weaving of Process and Content Standards



To accomplish this requires the use of exemplary materials that address each of the various standards. The use of multiple resources allows teachers flexibility and enables them to select learning experiences that engage students in real-world problem solving. Together, students and teachers will generate learning situations that are both meaningful and relevant to the learner.

Program Organization and Structure

Since the primary focus of many mathematics programs is that students understand and apply mathematical concepts and skills, we cannot assume that this is best accomplished by teaching two-page daily lessons from a textbook in a traditional 45-minute class period. With this in mind, the selection of materials and resources that allows students sufficient time to engage and explore mathematics in depth is critical. Instructional materials that engage students in tasks and projects that are multi-day and span a period of time allow for and emphasize the connections between mathematical concepts and their applications in daily living.

Students are a diverse population with many different learning styles. Materials that capture students' curiosity and invite them to explore and discover their own methods and solutions help encourage all students to learn mathematics. Programs that advocate the use of calculators and other tools to enhance mathematics learning are sensitive to the diversity of students. Since all students must have the opportunity to engage in the learning of mathematics, it is vital that only those materials and resources that offer a rich and broad range of opportunities be considered for selection.

Student Experiences and the Teacher's Role

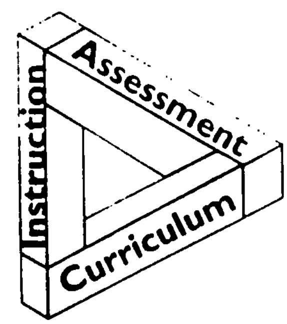
Materials that advocate learning by doing and actively participating in the learning process emphasize the need for students to become active, responsible learners. Students are expected to investigate tasks, discuss findings, make conjectures, test hypotheses, and defend their solutions. If materials do this, then they stress the importance of the learning process as much as finding the correct answer. Students in these programs are expected to construct their own understanding and build upon their prior knowledge.

Students in a recommended program are free to select appropriate technologies and other materials required to conduct investigations. Making decisions about procedures and judgments about their work are other experiences that students need to encounter. Materials that allow students to reflect upon their work and then make adjustments, help students self-assess their performance and gain greater insight into their learning.

Instructional materials also need to help teachers create a learning environment that engages and challenges all students to achieve their best. These materials encourage teachers to accept and respect the thinking of all their students and give suggestions for probing students' thinking and solutions. Recommendable materials offer suggestions for creating a safe, nurturing, stimulating environment in which effective teaching and meaningful learning can occur.

Assessment

Effective selection of mathematical materials and resources is embedded in the perspective that curriculum, what students learn; instruction, how students learn; and assessment, how students are assessed, are integrated and influence each other. These interwoven strands work together to facilitate the teaching and learning of mathematics.



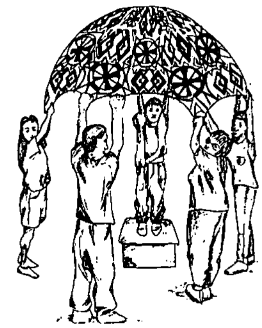
Programs that use multiple ways to assess student's progress look very closely at the underlying goal of assessment—determining what students know and are able to do. Assessment, which can be both formal and informal, is a means of examining how students understand and integrate information, then apply it to new situations, and use their understanding to communicate their ideas and reasoning to other people.

44

A Teacher's Guide for Selecting Instructional Materials and Resources

- What mathematics is presented in the materials?
- Are students engaged in active learning?
- Are students engaged in solving problems, reasoning mathematically, communicating their ideas, and making connections?
- Are students engaged in tasks that require in-depth investigations over a period of time?
- Are students engaged in real-life and work-related experiences?
- Are students given the opportunity to work individually, in small groups, and in large group settings?
- Do students work with technologies and other materials to enhance their learning?
- Is assessment integrated with the curriculum and instruction?
- Are students assessed through multiple methods?
- Are teachers given suggestions for creating and maintaining a safe and challenging learning environment?
- Are teachers given strategies and suggestions for working with families and other partners?

7



K–12 MATHEMATICS STANDARDS

The educational goals that have been identified in Rhode Island's Common Core of Learning reflect the need for all students to become problem solvers and possess mathematical power. To accomplish this task of developing and achieving mathematical literacy, all students must learn to value mathematics and its importance in their everyday lives. All students must have confidence in their ability to do mathematics and believe that they can succeed in learning and applying mathematical skills and concepts to everyday and work-related situations.

"They should possess such qualities as devoting time to school work, patience, persistence, and intelligence."

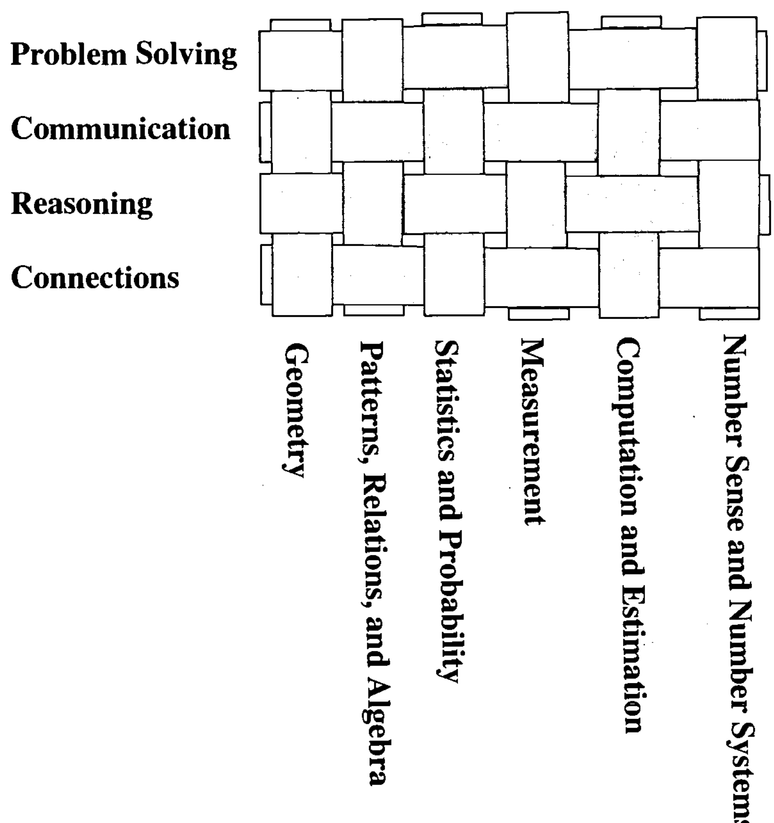
Response of focus group participant to "What skills and qualities do your children need to be productive citizens?"

The development of mathematics came about through people's need to quantify, solve problems, and communicate situations that arose throughout daily life. For students to value mathematics, they have to understand how mathematics affects their lives at home and in the workplace. Since culture, linguistics, and environment have an effect on how people view and learn mathematics, environments must exist that encourage students to explore, take risks, persevere, and reflect upon their own thinking.

With the advances made in technology, the teaching and learning of mathematics is changing to reflect and align with these advancements. Students must value the tools of mathematics, such as technology and models, and use these tools when solving problems, communicating, and understanding the present and future of mathematics.

Mathematics as problem solving, communication, reasoning, and connections can be considered the processes by which one learns mathematics. The content students learn can be clustered into the standards explicated in this framework, number sense and number systems; computation and estimation; measurement; statistics and probability; patterns, relations, and algebra; and geometry. Process or content alone cannot create an environment that will encourage and promote mathematical power for all students. Rather, it is the weaving or meshing of rich mathematical concepts, combined with a freedom to explore, which enable students to build their understanding of mathematics.

The Weaving of Process and Content Standards



Process Standards

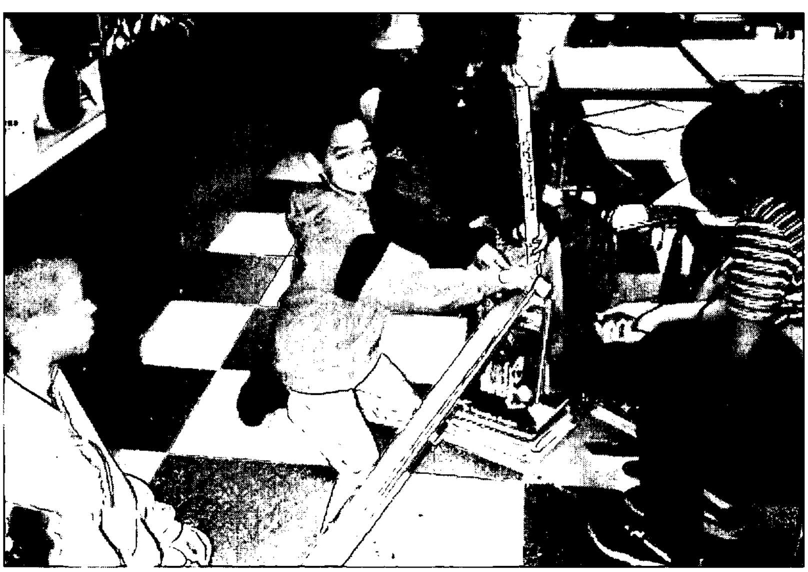
The process by which students learn mathematics is just as important as the mathematics content they learn. For students to develop an understanding of and appreciation for mathematics, it is essential that they learn mathematics through solving problems, using mathematical reasoning, communicating mathematical ideas, and making mathematical connections.

"Quality education and hands-on experience are needed, so are problem solving, analyzing, and logic skills."

Rhode Island focus group participant

These four mathematics process standards encompass the first four curriculum standards created by the National Council of Teachers of Mathematics. The Rhode Island process standards are:

- **Mathematics as Problem Solving**
- **Mathematics as Communication**
- **Mathematics as Reasoning**
- **Mathematics as Connections**



MATHEMATICS as PROBLEM SOLVING is the central focus of mathematics education and should engage students in developmentally appropriate problem-solving situations. Problem solving is a method of inquiry and application and provides opportunities for all students. "Problem solving should be the central focus of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic, but a process that should permeate the entire program and provide the context in which concepts and skills are learned."(NCTM *Curriculum and Evaluation Standards*)

Students will engage in problem solving to construct their own understanding, so that by the end of grade 4 they will:

- Use various approaches to investigate and understand mathematical concepts.
- Formulate and pose problems from everyday situations.
- Develop and apply strategies to solve a wide variety of problems.
- Verify and interpret their results.
- Use models to describe and solve real-world problems.

Students will engage in problem solving to construct their own understanding, so that by the end of grade 8 they will:

- Use problem-solving approaches to investigate, understand, and develop conjectures about mathematical concepts.
- Generalize solutions to apply strategies to new situations.
- Use concrete, pictorial, and symbolic representations to develop and apply strategies to solve multi-step and non-routine problems.
- Apply the process of mathematical modeling to real-world situations.

Students will engage in problem solving to construct their own understanding, so that by the end of grade 10 they will:

- Apply integrated technological and non-technological problem-solving strategies to solve problems.

Students will engage in problem solving to construct their own understanding, so that by the end of grade 12 they will:

- Use sophisticated as well as basic problem-solving approaches to investigate, understand, and develop conjectures about mathematical concepts.

MATHEMATICS as COMMUNICATION helps students construct meaning between their concrete, informal, intuitive understandings and the abstract, symbolic language of mathematics. Mathematics communication refers to the ability to observe, represent, talk, listen, read, and write about mathematics. Students become actively engaged in doing mathematics when they explore, investigate, describe, and explain mathematical ideas. Students learn when they are given the opportunity to think about their ideas; talk with and listen to other students; and share ideas, strategies, and solutions. Reading and writing about mathematics helps students and teachers reflect on their work and clarify ideas and conjectures. Communication gives teachers valuable insight from which to make instructional decisions as well as provide students with opportunities to demonstrate their understanding and knowledge. "Mathematics can be thought of as a language because of its power to represent and communicate ideas concisely." (NCTM *Curriculum and Evaluation Standards*)

Students will engage in communication to construct their own understanding, so that by the end of grade 4 they will:

- Relate physical materials, pictures, and diagrams to mathematical ideas.
- Reflect on, clarify, and articulate their thinking about mathematical ideas and situations.
- Express mathematical ideas orally and in writing.
- Discuss, read, write, and listen to mathematics.
- Relate their everyday language to mathematical language and symbols.

Students will engage in communication to construct their own understanding, so that by the end of grade 8 they will:

- Model situations in mathematics and other disciplines through communication.
- Develop an understanding of mathematics concepts, including the role of mathematical notation and definition.
- Use the skills of language and observation to interpret and evaluate mathematical ideas.
- Use mathematical ideas, language, and symbols to converse with others and to make conjectures and convincing arguments.

Students will engage in communication to construct their own understanding, so that by the end of grade 10 they will:

- Express generalizations discovered through investigations by using mathematical language.
- Communicate mathematical ideas, processes, concepts, and solutions through the use of technology.

Students will engage in communication to construct their own understanding, so that by the end of grade 12 they will:

- Provide clarifying and extending questions related to mathematical conjectures.

MATHEMATICS as REASONING is the justification of solutions, thinking processes, and conjectures in a variety of ways. The ability to reason enables students to solve problems in their lives, to validate their thinking, and to enhance their mathematical confidence. Through the development of logical reasoning, students develop both intellectually and verbally. "Reasoning is fundamental to the knowing and doing of Mathematics." (NCTM *Curriculum and Evaluation Standards*)

Students will engage in reasoning to construct their own understanding, so that by the end of grade 4 they will:

- Use models, known facts, patterns, and relations to explain their thinking.
- Use patterns and relations to analyze mathematical situations.
- Validate their solutions and methods.
- Make logical conclusions about mathematics.

Students will engage in reasoning to construct their own understanding, so that by the end of grade 8 they will:

- Identify and use deductive and inductive reasoning.
- Judge the validity of their thinking.
- Formulate and validate mathematical conjectures.

Students will engage in reasoning to construct their own understanding, so that by the end of grade 10 they will:

- Recognize the power of reasoning as a persuasive tool.
- Develop and validate conjectures.

Students will engage in reasoning to construct their own understanding, so that by the end of grade 12 they will:

- Formulate counter examples to support logical arguments.
- Construct simple valid arguments based on logic.

MATHEMATICS as CONNECTIONS; addresses the need for students to make connections among various topics within mathematics as well as to make connections between mathematics and other disciplines. The ability to explore, recognize, develop, and expand the interconnectedness of topics within mathematics and other disciplines enhances students' understanding of the usefulness of mathematics and how it relates to everyday situations. "Connections help students broaden their perspective, to view mathematics as an integrated whole rather than an isolated set of topics, and to acknowledge its relevance and usefulness both in and out of school." (NCTM *Curriculum and Evaluation Standards*)

Students will engage in making connections to construct their own understanding, so that by the end of grade 4 they will:

- Recognize and use mathematics in their daily lives.
- Explore, recognize, and use relationships among different topics in mathematics.
- Relate various representations of concepts or procedures to one another.
- Connect conceptual and procedural knowledge.

Students will engage in making connections to construct their own understanding, so that by the end of grade 8 they will:

- See how topics and concepts in mathematics are integrated.
- Explore problems and describe results by using multiple representations.
- Build upon a mathematical idea to understand other mathematical ideas.
- Use mathematics to solve problems and model situations in other disciplines.

Students will engage in making connections to construct their own understanding, so that by the end of grade 10 they will:

- Recognize and apply different representations for the same concept.
- Apply procedures used in one problem situation to other similar situations.

Students will engage in making connections to construct their own understanding, so that by the end of grade 12 they will:

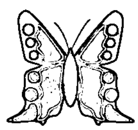
- Use and extend the connections among mathematical topics.
- Use and extend the connections between mathematics and other disciplines.
- Use and extend the connections between mathematics and the world of work.

Content Standards

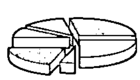
There are six mathematics content standards, which encompass the curriculum standards created by the National Council of Teachers of Mathematics. The Rhode Island Content Standards are:

- **Number Sense and Number Systems**
- **Computation and Estimation**
- **Measurement**
- **Statistics and Probability**
- **Patterns, Relations, and Algebra**
- **Geometry**

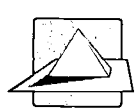
Each content standard features vignettes for



Elementary School



Middle School



High School

NUMBER SENSE and NUMBER SYSTEMS: refer to an intuitive understanding of numbers in the following ways: "[1] understanding the meanings of numbers; [2] having an awareness of multiple relationships among numbers; [3] recognizing the relative magnitude of numbers; [4] knowing the relative effect of operating on numbers; and [5] possessing referents for measures of common objects and situations in the environment." (NCTM *Addenda Series*, Grades 5-8)

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Have an intuitive understanding of whole numbers.
- Develop number meanings by exploring number relationships through counting, comparing, estimating, ordering, grouping, and patterning.
- Demonstrate place value concepts.
- Construct physical models to represent and demonstrate understanding of whole numbers, integers, fractions, and decimals.
- Communicate the reasonableness of possible solutions.
- Create and solve real-world problems to interpret the use of numbers.

Through problem solving situations, all students will construct their own understanding, so that by the end of grade 8 they will:

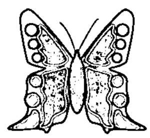
- Have an intuitive understanding of rational numbers.
- Justify the appropriateness of a method of approximation or calculation and verify the reasonableness of a result.
- Represent and use numbers (integers, fractions, decimals, exponents, and scientific notation).
- Represent and apply ratios, proportions, and percents.
- Demonstrate how basic arithmetic operations are related to one another.
- Demonstrate the composition and decomposition of numbers (primes, factors, and multiples).
- Investigate and represent number relations, using various types of graphs.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 10 they will:

- Have an intuitive understanding of the real number system.
- Apply properties of the real number system.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 12 they will:

- Have an intuitive understanding of the concept of limits and infinity.



More or Less—A Place Value Game

When the third grades in our school began using the *Everyday Math* program in September, there were many concepts that, although not new to the students, were expected to be processed in a different way by them. One of these topics was place value. The students were frequently asked to tell the number that was one hundred more or less than a given number. It proved to be much more difficult than it sounded. They knew how to add and subtract; carry and borrow. In fact, they were skilled in algorithms and could describe each step exceptionally well, however, the concept of place value as number sense seemed foreign to them.

While reviewing information that came with the materials kits, I tried a new game, modified it a few times, then played it with my co-teacher. We worked out the remaining bugs and agreed that it was exactly what the students needed to get them headed in the right direction. A student and I modeled the game for the class. While modeling it, I made sure that an example came up where the 9 would be covered with a zero and the next column would need to be changed. At this point children were encouraged to ask questions if they were confused.

More or Less can be played individually, in pairs, or in small groups.

Materials: deck of cards numbered 0 to 9, four of each

Rules:

1. Shuffle the cards and deal five cards to each player.

2. Players place their cards side-by-side, face-up on the table—two cards for a simple game of tens and ones, or all five to designate a number in the ten thousands. The deck is then placed face-down on the table.

3. The first player looks at the number created by the cards and checks her hand to see if she has a card that is one number away from any of those digits. If so, she places her card directly on that one, states the difference in the new number from the previous one (using the words *more* or *less*), reads the new number and takes the top card off the deck to add to her hand. For example, if the number showing on the table is 381 and she has a 2 in her hand, she has the choice of placing the 2 on top of the 3 and saying, *100 less and the new number is 281* or placing it on the 1 and saying, *1 more and the new number is 382.*

4. If a player has no cards that can be played, she passes. If a player passes twice in a row, all her cards are placed on the bottom of the deck and she draws the top five cards.

5. The game ends when no player can place a card on the table or when the last card is played.

If a digit is 9, a player may put a 0 card on it, but only if she also has the card to change the next place to the left. For example, if the number showing is 6,392, she can only put a 0 on the 9 if she also can put a 4 on the 3. Two cards are taken from the deck and added to her hand. Likewise, a 0 may be covered with a 9, if the appropriate card can be produced to cover the card showing to the left.

To assess the students' understanding of place value, I circulated around the classroom observing, facilitating, and answering questions. Students were enthusiastic about the game. It was such a big hit, that many students played it during their free time. I noticed a marked improvement in their ability to answer the same type of questions that had, only a few days before, caused frustration.

Gayle A. Raposa
Ranger School, Tiverton

Twenty Questions

In the fifth grade at our middle school, we encourage our students to work as members of a team. On the first day of school, I challenge the class to a game of *Twenty Questions*. I begin by stating that I am thinking of a number between zero and one. Immediately someone always shouts out, "There aren't any."

"You are down to 19 questions," I respond. "Remember to try to ask questions that will help your team find my number." After they have used up their questions and guesses (usually fractions), I tell my students to keep thinking and we'll play again another time. I make sure they don't guess my number.

Later in the week, we begin our counting strip for the number of days of school. Unlike earlier grades, however, we begin counting at .01. For example, if it is the third day of school, we will add .02 and .03 to the counting strip. Students pose questions. "When will we get to 100?" "How about one?" I also have an enlarged 9" x 14" copy of a dollar bill, to which we attach a paper penny for each day. We discuss the number of pennies needed to equal $1.00 and the representation of one cent as $0.01. Students begin to make connections between money values and the decimal numbers we have discussed.

One of the projects the students work on at this time is finding words that are worth exactly $1.00. Alphabet letters are assigned values from .01 (a) to .26 (z). Students scour the dictionary and practice their estimating and computation skills while trying to locate the most words. This is an excellent activity for calculator practice. I offer a prize of $1.00 to the student who finds the most $1.00 words.

I ask my students to begin collecting examples of decimal numbers. They might cut out batting averages from the newspaper, save weight labels from deli products, use food packaging, and even price tags. When we have saved many samples, the students are divided into groups and assigned a segment of a number line. Groups work together to find the appropriate decimal values and place them in the correct sequence on the number line.

To help the students gain a better understanding of the importance of place value, we play a game using two dice, each labeled .1, .1, .01, .001, and .001. (I purchase blank dice or use wooden cubes and place labels on them.) Two players are opponents. Each tries to be the first player to reach a total of 10 by adding the values of two dice on each roll. It becomes evident that .1 is much more valuable than .01 or .001.

The next time I challenge my class to play *Twenty Questions*, I notice that students' questions and guesses are more accurate. One of the first questions may be, "Is your number a decimal?" (as opposed to a fraction). Student become adept at asking questions regarding tenths, hundredths, or thousandths and using terms such as greater than and less than.

Claire Palmer
Chariho Middle School, Richmond

POPULATION, POLITICS, AND MATHEMATICS

The mathematics and social studies teachers explain to their students that by studying local, national, and world populations, great insight can be gained into the cultural, political, and economic climate of a group of people. While the social studies class researches current philosophers and historians, the mathematics teacher challenges her class to collect, analyze and share population data.

The class is divided into 12 groups of two or three students. The twelve groups are divided into four regions of the world—Africa, North America, Europe, and Asia. Students are encouraged to join a group of interest to them. The teacher instructs one team per region to collect population data for the entire region between the years 1900 and 1990. The second team collects data for one or more countries in a region, while the third group researches contributing factors they feel will shed light on their region, such as famine victims, war losses, disease, migration, and immigration.

Students are provided with graph paper, graphing calculators, almanacs and other reference books, overhead transparencies, computers with spreadsheet and statistics packages, and other materials that will help them with their tasks. While students are working, the teacher circulates to answer questions and facilitate discussions. The teacher also acts as a sounding board and catalyst for student ideas.

Once data are collected, student pairs graph their data in a minimum of two formats—population during the years 1900–1990 and population during 1950–1990. Students explain why these groupings are different. They are encouraged to formulate an equation that best fits their graphed data. Students are also asked to represent data in another format or for another span of years, whichever best shows the population trends in their region. Regions synthesize their findings to answer such questions as:

Is the population in my region growing? Declining?
Is the growth or decline at a steady rate?
What is happening within the region that affects population?
What can society learn from our observations?
What predictions can be made from our data?

On about the fourth or fifth day, students present their findings to the class. They may use an overhead projector, handouts, or other materials in their presentations. Following a whole class discussion of the similarities and differences across the regions, the class assesses the net effect of their findings upon world population.

Student research is placed in binders so that the population materials can easily be used in social studies classes where the political and social trends can be further discussed and synthesized.

Lois E. Short
Burrillville High School

COMPUTATION and ESTIMATION refer to computational and non-computational procedures that deal with numbers that students select and use. They include invented methods and technologies that are applied when conducting real world and other investigations.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Use estimation for quantities, measurement, computation, and problem solving.
- Recognize and demonstrate when the use of estimation is appropriate.
- Communicate the reasonableness of results.
- Develop an understanding of the concepts of addition, subtraction, multiplication, and division, using concrete models.
- Demonstrate reasonable proficiency with basic facts and algorithms.
- Develop proficiency with addition, subtraction, multiplication, and division of whole numbers.
- Use a variety of computation techniques, including mental strategies, paper-and-pencil, and technology, as appropriate.
- Use four function calculators in appropriate computational situations.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 8 they will:

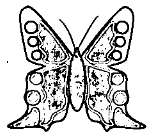
- Develop proficiency with addition, subtraction, multiplication, and division of rational numbers.
- Create, analyze, and communicate procedures for computation and strategies for estimation.
- Develop and communicate an understanding for solving proportions.
- Use estimation to defend the reasonableness of solutions.
- Use scientific calculators and computers in appropriate computational situations.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 10 they will:

- Develop proficiency with addition, subtraction, multiplication, and division of real numbers.
- Apply a variety of computational methods and estimation techniques in finding solutions.
- Demonstrate the ability to select and use appropriate technology (spreadsheets, databases, graphing calculators).

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 12 they will:

- Solve problems by integrating computational methods and estimation techniques with technology.

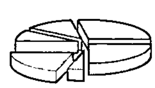


THING-A-MA-JIGS

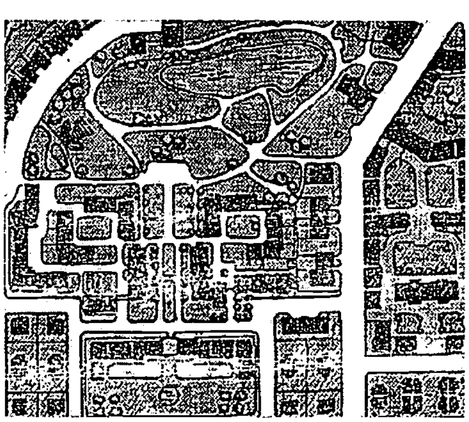
Here is how our class applied what we knew about the value of coins and adding them to make $1.00. Together, we assigned monetary values to geometric shapes. We decided that a triangle would be worth 1¢, a rectangle worth 25¢, a square worth 5¢, and a circle worth 10¢. Students were asked to create on paper something that would be worth exactly $1.00 when finished. While they worked, I moved around the room and made informal assessments. After completing their creations, students used calculators to check their computation and confirm that their creation was worth $1.00.

To extend the activity, we used construction paper and glue to build their creations. We had some very interesting ones, including a tree with a tree-house, a wagon, a dog, a snowman, and several "thing-a-ma-jigs." After our sharing time we displayed our creations on the bulletin board for all to see.

Mary Ann Phillips
Charlestown Elementary



TO BUILD OR NOT TO BUILD...THAT IS THE QUESTION



As part of an interdisciplinary unit on World War II, specifically, rebuilding Europe after the war, this mathematics segment delves into the tasks of brainstorming a city site, prioritizing community needs and wants, accounting for geographical limitations, factoring in climate and weather conditions, and constructing a scaled model city.

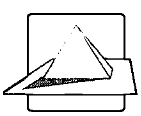
After a mock town meeting, the proposed city is constructed in class by using foam core, exacto knives, and glue. Town boundaries are established with particle board or carpeting, making sure that all natural geographical features have been positioned.

Students explore the aspects of city planning, finding ways that people choose among priorities of needs vs. wants to design a city. Working in pairs, they survey a population to determine the needs for their city. They analyze the data they collect. Next, students draw residences or businesses on graph paper. They cut and build scale models of their designs. The project ranges from 5 to 7 weeks in length.

After buildings are completed, they are placed on carpet remnants or particle boards. Rugs or boards are designed with dimensions previously calculated to contain all natural features, road, and sites for the designated structures, including hospitals, schools, town hall, and housing.

All students with ranges of social and academic skills become engaged in this project. Because of its interdisciplinary nature, teachers have the opportunity for flexible scheduling with other teachers. It is recommended highly as a unit that implements inclusion, since there are relevant tasks for every student.

Marion Digiammo and Joanne Rahme
Barrington Middle School



PATHWAYS AND COMMUNICATION

I challenged my ninth grade class to determine whether a map of the United States could be colored by using only four colors so that bordering states would not be the same color but that states meeting at a corner (for example, Arizona and Colorado) would share a color.

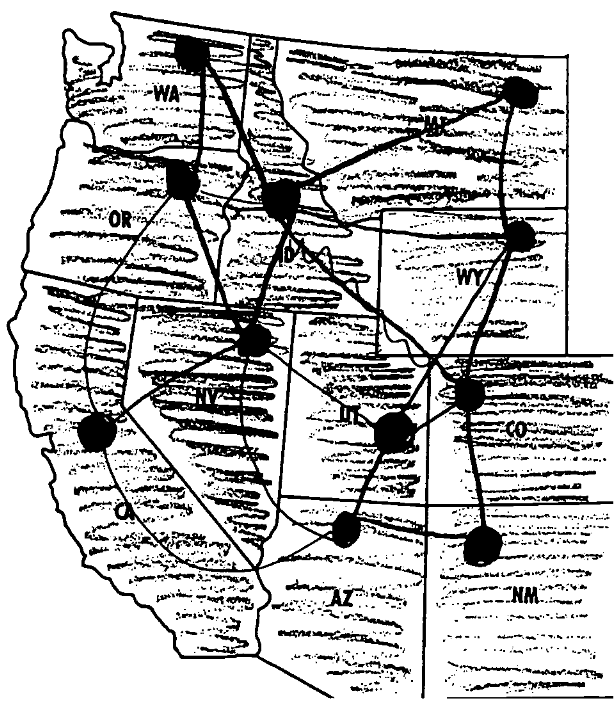
The class worked in pairs, coloring various patterns requiring a minimum number of colors. After coloring a few maps, I explained networks and conflicts, and showed the class how to make a network for groups of states. More maps were colored, first establishing the conflict network to simplify the coloring. At the end of the week, a map of the lower 48 states was given to students to network and color.

While they worked on their maps, one student asked how computer and other communication lines work. From that question, I asked students to estimate the number of different paths that their data could follow if an e-mail message was sent from North Kingstown, Rhode Island to Olympia, Washington. As the students were working on the problem, myself and the resource teacher walked around the room, assisting with the "conflict resolution" and listening to students estimating the number of pathways.

This project, which was enriched by student questions, integrated many mathematics concepts and applications. Students discussed and defended their solutions to the question about the number of different paths. Students went on to discuss how this activity could be used for scheduling meetings, storing materials in warehouses, locating animals in a zoo, and a myriad of sorting and arranging applications in the workplace.

The need for good estimation skills was also discussed. Students saw the importance of being able to determine if their solutions were reasonable and made sense.

Susan Espelin Osberg
North Kingstown High School



BEST COPY AVAILABLE

MEASUREMENT refers to selecting and understanding appropriate attributes, units, and tools needed to estimate, make, and use measurements.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Identify and understand the concepts of length, angle of measure, area, capacity, volume, mass, temperature, time, and money.
- Have an intuitive understanding of the process of measurement.
- Select and use appropriate standard and non-standard units of measure.
- Compare and order objects according to some measurable attribute.
- Estimate measures.
- Take actual measurements and record and explain the results.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 8 they will:

- Develop formulas or procedures to solve problems involving perimeter, area, capacity, angle measure, and weight and mass.
- Extend and apply the process of measurement.
- Select and use appropriate units and tools to measure.
- Develop concepts of rate (miles per hour, feet per second) and other derived and indirect measurements.
- Demonstrate a proficiency in measurement and "measurement sense".
- Have an intuitive understanding of systems of measurement.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 10 they will:

- Select and apply appropriate techniques and tools to measure quantities and recognize relationships among precision, accuracy, and error of measurement.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 12 they will:

- Select and apply trigonometric functions to solve problems.

How Big?

This interdisciplinary activity is one way for students to recognize the need for standard units of measurement, estimate the measurement of a familiar object, and compare and contrast estimations.

Before reading the story *How Big Is A Foot?* by Rolf Myller, pose the following question. "If you were to build a bed, how would you know how big to make it?" Discuss the answers and comments that students contribute.

Tell students before you read the story that they may notice a problem. During the reading, stop briefly to discuss the problem (How big should the bed be?) and how the mistake of the too-small bed could be avoided.

At the end of story, illustrate the problem by tracing a student's foot on cardboard or oak tag. Use this 'foot' to measure off a 'bed' on the floor that is 3 'feet' by 6 'feet'. Use another student's foot and make a different-size bed.

Introduce a ruler and show that it is a standard foot made up of smaller units called inches. In the same manner as above, make a standard bed that measures 3 ft by 6 ft.

Students can work with partners to measure lengths of familiar classroom objects such as pencils, crayons, books, and desk tops.

Geraldine Moretti
Oak Lawn School, Cranston

Measuring a Penguin

Could students create a penguin by using LegoTC LOGO and program it to mimic a realistic behavior? Yes! Our challenge was to build a robotic creature by using Lego™ building blocks and bring it to "life" by using the computer language LOGO to allow it to interact with its natural habitat.

The class of 27 students divided themselves into five groups: researchers, engineers, artists, builders, and programmers. Researchers discovered as much about penguins as they could. They shared their information with the class and in a written report. Engineers designed a penguin on graph paper and explored gears, sensors, motors, and the mechanics of movement. Designing how a penguin would look when constructed with Lego™ blocks and creating a display board to showcase the work of all groups was the task of the artists. Builders created a penguin that was appropriately scaled and symmetrical. Finally, the programmers wrote a computer program that simulated a penguin's behavior as it waddled forward, sensed snow, backed up and returned to the water. The penguin had a light sensor that would react when it saw white (snow). It would then back up to wander about in its environment (water). The teacher's role was to support and direct each group as they worked.

Measurement was a central issue with all groups. Researchers discovered the life cycle, longevity, length of gestation, and the number of eggs penguins were likely to produce in a life span. They found information about penguins' habitat, different types of penguins, and their habits. They used their information to determine its extinction status.

The engineers used the information from the researchers to design a scaled animal that was proportionally correct. They drew their design on graph paper. Students determined what the approximate area for the front of the penguin should be and the length and width of the animal.

Now the builders were ready to begin their task. Using Legos ™ as their measurement tool, they soon discovered that three bumps on a Lego™ approximated one inch. The black body of the penguin needed to be larger than the perimeter of its white belly. Measurements of the length and width of the mechanics of the penguin would determine the area of the body needed to hide the gears.

Showcasing the work of all the groups became the task for the artists. They began to design a backboard that was big enough to display all of the research reports, engineering drawings, a printed copy of the computer program and a pamphlet that they developed about penguins. They began to explore the area of each piece so their showcase would be as visually balanced as possible.

The builders assumed their role, with input from the other groups about how it would work, look, and move. Measuring the speed at which gears turned would help them determine the animal's pace. They discovered that small gears turned larger gears at a slower pace and that large gears turned small gears at a faster pace. They decided that gears of equal size gave them the speed best suited to the movement of their animal. Students began building the mechanics of the animal and finally added the front body, creating the penguin.

Giving it "life" was the job of the programmers. They discovered another factor in pacing the penguin. The rate of speed at which the penguin moved depended on the power setting used in LOGO. More experimentation was necessary. The time it would take the penguin to cross the white area (snow) before it walked into the blue area (water) was measured. They also measured the wait time between the robot's entry and exit into the water. Students needed to measure how long it took the penguin to waddle an inch as it moved through its habitat in order to create a cycle.

Each group collaborated with the other groups in order to make sure they shared the same vision of a penguin. Each group was dependent on the work of at least one other group which made sharing a valuable time saver. Consultations between groups became a daily event. It validated the work they did as important to the project. Cooperation among groups blossomed as members shared their knowledge and expertise.

Besides the measurement skills gained through this project, there were many other positive outcomes. The students created a penguin that looked and acted much like a real penguin. They gained experience in cooperative group work as they worked in small groups and among groups to meet the challenge. Students appreciated the opportunity to do real science and mathematics for a project of their choosing. They were thrilled to write about and discuss their work with anyone who would listen. Students saw that communicating mathematically about their building, designs, and programming made the task easier. They became better problem finders and solvers as they were motivated to revise, fix, or improve their original thoughts and designs.

The classroom penguin was named Junior. He became real as he waddled through the snow until he reached water. He would squawk and back up before continuing to wander about in his habitat, until he once again reached the water. We had successfully completed our challenge.

Mariann Hayward
Peace Dale Elementary School, South Kingstown

Brainstorming, Bystanders, and Measurement

I introduced the new task by saying, "For the next few days we will be exploring ratios and proportions. To begin we need to determine what we know. Take five minutes to record in your journals what you already know about ratios and proportions. Feel free to use diagrams, charts, graphs, text or any reasonable representation for your ideas."

Students began to write, but took less than five minutes to record their ideas.

"Let's pair up and brainstorm lists, combining ideas that you each recorded in your journals." After five minutes of brainstorming with partners, the class as a whole generated the following list of ideas on the blackboard.

2:1
7 to 1
Odds in gambling
2/3 = X/6
a proportion is two ratios
ratios are fractions
ratios have numerators and denominators
proportions are equations

"We have collected some information about ratios. Will you accept as a proportion H/L = H'/L'? May I add this to your list?" The new proportion was added.

"Now I would like you to work in teams of four to brainstorm another list of different ways that you suggest measuring the height of our school building. One person from each group is to be the reporter who will share your list with the rest of the class. You have five minutes to brainstorm ideas." The students took some time to get started, but once a few ideas were recorded they generated a rather creative list of strategies. Some are listed below.

fire truck ladder and hose
utility truck bucket and long rope
unrolling a roll of paper towels from the top of a roof

get the building plans from the town hall
find the average height of one floor and multiply by the number of floors
measure the shadow of the school building at noontime

"For tonight's homework explain what connections you see between the two lists. Part two of your assignment is to bring in an empty bathroom tissue or paper towel roll. It's hands-on mathematics tomorrow and you will each need your own roll!"

The next day the students were grouped in pairs. "Do you have any thoughts about how these two lists may be connected?"

"The number of paper towels is in proportion to the height of the building."

"The shadow is a ratio in relation to the building."

Listening to other ideas led to suggestions for future exploration. We decided to retain our list for reconsideration later in the week.

"Our problem for today is to determine the height of our school building without utility truck buckets, long hoses or other suggestions generated during yesterday's class. To do this I need a volunteer demonstration team."

One team was selected to demonstrate the experiment. A tape measure was taped to the wall of the classroom. Using a meter stick, calculators, and tissue rolls, students were asked to record the length of the tape measure they viewed through the tissue roll standing 100 cm, 200 cm, and 400 cm away from the wall. Partners helped one another.

Once students completed this portion of the experiment, they began to dismantle equipment and set out for the outside of the school. Some students went outside without any measuring instruments but soon returned for the materials. The front of the building had teams of students measuring and viewing.

A passerby stopped and watched, then asked a student what was so interesting at the top of the building. After explaining our problem to him, he participated in our experiment.

After completing this part of the experiment, the students returned to the classroom. We discussed how we could use the experiment in the classroom together with the information we gathered outside to calculate the height of the school. After finding the height of the school, we discussed questions such as, what patterns, if any, did they discover in the two heights? Would this experiment work if different ratios were used?

One student asked if sextants used in navigation worked on the same principle as the tissue rolls. Did the sextant use proportions? The student's questions were added to the revisited list, along with the question: Do shadows help us determine the height of a building?

Extensions to our experiment with ratios and proportions and tissue rolls might include investigating occupations that use measuring instruments or researching other instruments that are used to measure distance.

Judith Keeley
Northern Rhode Island Collaborative, Cumberland

STATISTICS and PROBABILITY refer to collecting, organizing, displaying, and analyzing information, and making and interpreting predictions based on uncertainty.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Collect, organize, and describe data.
- Draw conclusions and make predictions, using collected and recorded data.
- Record and display data in a variety of ways, including the use of appropriate technological tools.
- Recognize and identify the likelihood of the occurrence of an event based on data (Is the event...impossible? not likely? equally likely? more likely? definite?).
- Read and interpret data.
- Formulate and solve problems that involve collecting and analyzing data.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 8 they will:

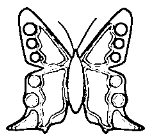
- Construct and interpret tables, graphs, and charts.
- Analyze data to formulate reasonable predictions and convincing arguments based on the analysis of data.
- Recognize and use the measures of central tendency (mean, median, mode, range).
- Use probability and statistics to make informed decisions in real-life situations.
- Design and carry out experiments or simulations to determine probability.
- Communicate conclusions based on analysis of data.
- Use a scientific calculator to explore statistics.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 10 they will:

- Use and interpret sampling techniques.
- Apply concepts of central tendency, variability, and correlation.
- Generate and interpret normally distributed data.
- Apply the laws of chance to predict and communicate probable events.
- Develop convincing arguments and inferences.
- Recognize the importance of statistical claims (hypotheses).

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 12 they will:

- Use curve fitting to predict from data.
- Understand and use experimental and theoretical probability.
- Use a graphing calculator to explore statistics.
- Use technology to generate random variables.
- Have an intuitive understanding of statistics and probability.

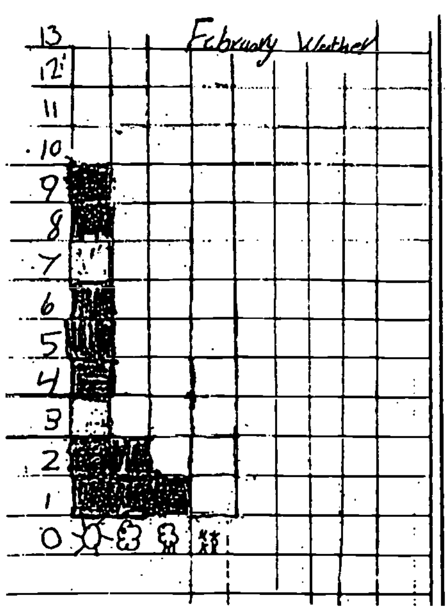


WHAT'S YOUR PREDICTION?

It is a typical day in the Grade 1 English as a Second Language (ESL) classroom. Against the eastern wall there are two computer terminals tied into the information highway. A large student-made calendar is strategically located on the north wall of the classroom, becoming the focal point for students to gather, record, analyze and apply data to be used to determine the agenda for the day.

All students quickly settle into their early morning routine, getting their journals out of their desks and checking the "Classroom Community Workers Chart" for their job as the teacher also goes about her morning responsibilities.

One student, who is the designated meteorologist for the day, approaches the calendar and places a "sun" symbol on the correct location on the graph. He tells the teacher he is ready. The teacher summons the class to attention as he gives his weather forecast:



"Today is Monday, February 13, 1995. Today I predict sunny weather with very cold temperatures. Today we go to art. It will be indoor recess. On Mondays, indoor recess is in the cafeteria....."

Students proceed to plot the weather information on their individual weather graphs. This procedure continues each day until the end of the month.

As a culminating activity, students summarize their daily findings and interpret the data to complete a monthly weather report in oral and written form. Beginning at this point in the year, third grade collaborating peers estimate and confirm the average monthly temperature and add it to the class weather report.

31 JAN 1995 data interpretation

January Weather Report

in January Ther was 12 Sunny days
in January Ther was 2 vanny days
in January Ther was 4 Snowy days
in Jauary Ther was 3 clawdy days
12 Sunny days is $>$ Than 2, Vanny days. 2 Vanny is $<$ Than 3, clawy days

Using Internet, this information is communicated to primary schools in parts of the country where weather and climate are similar and different. Plans are being made to include a videotaped weather report that can be broadcast within our school and, through computer communications, to others.

On the last day of each month, a whole-class "Monthly Weather Meeting" is held to discuss the month's weather report. In addition, plans are made to change and update the calendar to include:

- Name of the upcoming month
- Number of days in the upcoming month
- The day of the week on which the month begins
- Holidays celebrated during the new month

These activities begin on the first day of the school year. Students learn to gather information and record data, beginning with the concrete (color-coded unifix cubes) and moving toward the abstract (graph paper). Monthly weather reports begin as whole-group projects and, as the year progresses, these reports and broadcasts are written and recorded in groups and individually.

Weather data collection, recording, and interpretation are continued until the last day of school, with a major emphasis on patterns of change in weather during the past ten months and four seasons.

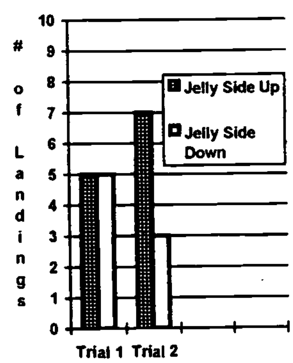
Ida Cabral, Nancy Lavey, Chuck Mello, Maria Van Anglen
Bristol Warren Schools

JELLY SIDE UP? OR DOWN?

"Every time I drop a piece of bread with jelly on it, the bread lands jelly side down on the floor!" Believe it or not, that rueful complaint was the genesis of a classroom lesson on probability.

One Thursday morning in April, Mrs. Moitoza's fifth-grade class arrived at school to find loaves of bread, a jar of jelly, newspapers, and a folded plastic sheet prominently arrayed on the round activity table in their classroom. Students eyed this arrangement with barely concealed curiosity during morning administrative activities that preceded their first-period mathematics class. Finally, it was time to resume their unit on probability, but with a difference.

Dropping Jelly-Covered Bread by Hand



"Today," Mrs. Moitoza announced to her class, "we are going to learn about probability by dropping jelly-covered bread on the floor." That statement caused an understandable stir and some laughter among the students. By way of explanation, the teacher noted that someone once commented that dropped bread tends to land jelly-side down and she thought it would be fun and instructive to test that observation in class. First, however, the class had to design an experiment and decide how to record and display their results.

"What could affect the outcome?" asked Mrs. Moitoza. "How we drop it." "How high it's dropped." "How much jelly is on the bread." The class quickly came up with those factors that could affect results, then discussed ways to minimize their effects.

To be consistent, measuring spoons standardized the amount of jelly to be spread. The students noted that the way the bread was dropped would be the hardest factor to control, and decided it should be done by holding the bread in both hands, then letting go with one before the other, to simulate realistically an accidental dropping. Students would record how many times the bread landed jelly side up or jelly side down, as well as any observations about the conduct of the procedure. They also decided a double bar graph would best show a comparison of the frequency of occurrence for each of the two possible results. Lastly, since the lesson was about probability and being able to predict future results, one trial was needed to calculate the probability a piece of bread would land jelly side down, and a second trial would then test the prediction. Dividing the class in half seemed like a good way to conduct two trials, with one half performing the tests and the other half observing, then switching roles and performing the experiment again.

With all that decided, the class set to work. The plastic sheet was unfolded, spread out on the floor, and then covered with a layer of newspapers. Half the class, 10 students, took turns spreading jelly on their pieces of bread, then dropping them onto the newspapers. The other half of the class observed, taking notes and recording results on the chalkboard. In the first ten drops the bread landed jelly side up five times and jelly side down five times. Those results were displayed as "Trial 1" in a double bar graph on the chalkboard, and the students used their mathematical skills to calculate 50 percent probability that the bread would land jelly side down.

The experiment was then repeated as "Trial 2" by the second half of the class, with the bread landing jelly side up seven times and down only three times. What could have affected the results? Students compared observations and noted that everyone dropped the bread a little differently, that some dropped the bread nearly flat and others literally spun the bread when releasing their slices. From that they concluded that whether the bread spun and, if so, how quickly it spun, would affect the result. The class finished the graphs of both trials and took a moment to think about what they had learned.

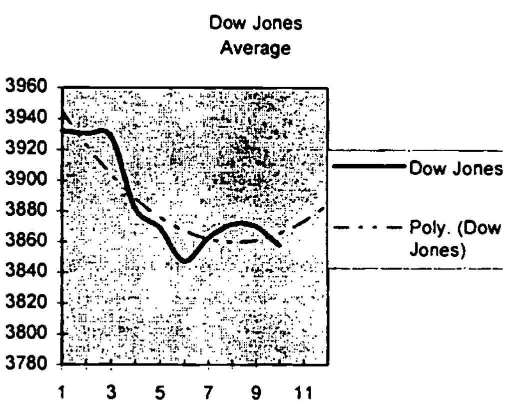
"Can anyone think of a practical application that uses probability?" asked Mrs. Moitoza. One of the boys responded that batting averages could be read as the probability a player would be successful the next time at bat, and a girl added that basketball free throw shooting percentages did the same thing. It was also noted that colleges frequently use a student's grade point average to measure the probability of a student's success in college.

With that, the lesson was over and it was time for the students to clean up the newspapers and plastic sheet. Now, this might seem a messy way to explore probability and statistics, but proper planning made the clean up easy and left the room, students, and teacher none the worse for wear. Besides, if you wanted to see messy, you should have been there when the class dissected ten squid. That, however, is another story.

R. A. Schoonover
Cranston Calvert School, Newport

STOCK MARKET ANALYSIS

After completing a unit of instruction on the use of Excel™ spreadsheets, students apply their learning in a real-life situation, using the program to conduct a stock market analysis project. Each student invests a hypothetical $3000 in stock traded on the New York Stock Exchange. Working in teams, some have researched their investments carefully, using library reference materials; others are speculating, using "gut instinct" as a guide. Students scan newspapers to find their stocks' daily performance along with the latest Dow Jones report. Others use computers, posting information to the spreadsheets they have developed. Still others tap software and are graphing their stock performances and predicting the trend for the two weeks to come.



The groups use telecommunications software to access a market analysis server on Internet. There is always a murmur of active collaboration in the room as students help one another. There is good-natured bantering about who is making a killing on the market, and whose company has gone bust, but the atmosphere is one of cooperation, not competitiveness. Everyone is involved.

Unexpectedly, the project once enabled students to apply their new-found knowledge across the curriculum, to an upcoming science fair. The teacher intended a worthwhile venture, but no one predicted that the students would become so involved that they would come early and leave late. To link this project with the professional community and disseminate it for other schools, I demonstrated it to a group of teachers.

Maria Negro
Portsmouth High School

PATTERNS, RELATIONS, and ALGEBRA refer to studying patterns to make conjectures about relationships, exploring relationships, and developing generalizations.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Identify and describe patterns.
- Use patterns to communicate relations.
- Extend and create patterns.
- Use concrete, pictorial and abstract variables to solve open sentences.
- Use technology to explore patterns.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 8 they will:

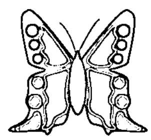
- Identify, analyze, extend, and create patterns in different formats.
- Describe and represent relations through different formats (tables, graphs, verbal rules, open sentences, equations, and geometry).
- Identify and justify an appropriate representation for a given situation.
- Use patterns and functions to represent and solve problems.
- Use concepts of variable, expression, and equation.
- Analyze tables, graphs, rules, equations, and identify relationships.
- Solve linear equations.
- Investigate inequalities.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 10 they will:

- Represent situations that involve variable quantities with expressions, equations, and inequalities.
- Use tables or graphs to interpret expressions, equations, and inequalities.
- Solve equations and inequalities.
- Model real-world situations with a variety of patterns and relations.
- Recognize and model generalities of patterns.
- Have an intuitive understanding of algebraic procedures.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 12 they will:

- Use a variety of techniques to solve linear systems.
- Use tables or graphs to investigate the properties and behaviors of patterns and relations.
- Demonstrate the properties and behaviors of patterns and relations.
- Analyze the effects of change on patterns and relations.



PATTERNS, PATTERNS, PATTERNS

This is a task my class and I do to explore and understand repeating patterns. It integrates children's literature by means of the picture book, *The Hungry Caterpillar* by Eric Carle, which is available in most public and school libraries.

As I read the story, which features a caterpillar eating a variety of foods each day of the week, students choose color cubes to represent the foods. As a class, we brainstorm to create patterns with our blocks that are representative of what the caterpillar ate. The patterns are shown on a flip chart and left in view of the students.

Using squares of construction paper, students select, create, and repeat any of the patterns on long strips of paper.

A related activity is to read a quilt story and display a quilt, if one is available. Lead students to realize that quilts are often made up of repeating patterns of colored fabrics or designed squares.

In pairs or groups of three, help students decide on a paper quilt to make. Each students designs a 12 in. by 12 in. square. Groups decide how to combine their squares to form a pattern. Group members make additional copies of the squares to repeat the pattern and make a paper quilt. Completed quilts can be displayed and the pattern designs can be observed and discussed.

Geraldine Moretti
Oak Lawn School, Cranston

Working to Buy Computers

The students in our class have decided to work to raise funds to buy another computer for our classroom. While looking through the newspaper, we saw ads for student employment. The ads read:

Ad #1: *Hardworking class of students needed for twelve weeks to paint walls at the Community Center. Work Saturdays 8 a.m. to 4 p.m. Salary $100.00 per week! Deal of the Century! Hurry, this job opportunity won't last!*

Ad #2 *Looking for energetic class of students to work on Saturdays with children ages three to five. Must be caring, kind, and love to play. Free lunch and snacks provided with a salary of $1.00 per week, with salary doubling each week. Needed for the next twelve weeks.*

Which job would students choose for our class? How much would we earn working at each job? How would we explain our decision? How would we support our answers with numbers and graphs? Would computer graphing programs help us to develop our graphs?

Working in groups of four, students used calculators and computers to help them resolve this dilemma. Groups presented their findings and defended their decisions with graphs and charts. There were several discussions concerning the nature of the jobs and what each one entailed. Some students felt that what they did at their jobs was as important as how much money was made. Many students felt it was important to choose a job that they enjoyed.

Student groups were assessed in this project by using this rubric:

4 Excellent class presentation
Complete analysis of the problem
Calculations accurate

3 Good class presentation
Complete calculations
Partial analysis of the problem

Patterns and Functions

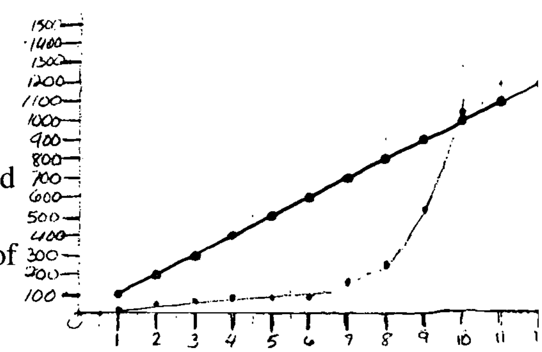
by Robert Russo

Arithmetically:

Ad #2: week 1 1.00
week 2 1.00 x 2 = 2.00
week 3 2.00 x 2 = 4.00
week 4 4.00 x 2 = 8.00
week 5 8.00 x 2 = 16.00
week 6 16.00 x 2 = 32.00
week 7 32.00 x 2 = 64.00
week 8 64.00 x 2 = 128.00
week 9 128.00 x 2 = 256.00
week 10 256.00 x 2 = 512.00
week 11 512.00 x 2 = 1,024.00
week 12 1,024.00 x 2 = 2,048

total: $4,095.00

difference: $2,895.00



Verbally:

I would personally choose the job described in ad 2. In job 1, our class would make a total of $1,200.00. In job 2, our class would make $4,095.00. Job 2 pays $2,895.00 more than job 1. Job 2 would definitely help us more in our quest to raise money for computers.

2 Weak class presentation
Partial calculations
Partial analysis of the problem

1 Student attempts to do the work
Calculations inaccurate
No analysis of the problem

0 No work

As an extension, I led the class in a discussion of the importance of having a job that they would be happy doing. Students realized how it is important to earn money to pay bills and to buy things they wanted, but at the same time they appreciated the need to feel their jobs are satisfying.

Linda Bello
Oak Lawn School, Cranston

Helping the Fire Department

Our class was called upon to assist the Captain of the Cranston Fire Department. Our role was to provide the department with a variety of methods for scoring the physical fitness portion of an examination.

In order to be considered for employment in the fire department, an applicant must pass a written examination and a physical examination. The Captain understood how to grade the written section, but he was having difficulty scoring the fitness section. Although he did not go into specifics of what physical tasks the applicants were asked to perform, he did state that there was a course of events. He wanted any applicant who completed this course in four minutes to receive a score of 100 points and any applicant who required seven minutes or more to receive a score of 0.

Our job was to devise a way of scoring, based on the time taken to complete the physical fitness course, that was fair and provided a grade for each applicant's performance.

Working in pairs, students submitted at least two methods of scoring. Students were asked to explain their methods, using a table of values, a formula, a graph, a chart, or whatever else would help explain their method and its degree of fairness to the class. The class was to decide which methods to forward to the Captain.

The Captain also asked us to provide him with a way of scoring, should he ever decide to change the four minute and the seven minute times. That is, he wanted us to provide a general formula for our methods. This suggested using m to represent the minimum amount of time needed to complete the course (and receive a score of 100 points), and M to represent the Maximum amount of time allowed (and receive a score of 0 points).

As the groups worked on this project, I circulated throughout the classroom, listening carefully to the students' discussions, asking questions, and offering suggestions.

At the beginning of the fourth day, students were called upon to present their methods to the class. The class discussed the fairness and appropriateness of each and decided which proposals to forward to the fire department.

Judith Dimitri
Cranston High School East

GEOMETRY refers to exploring geometric properties and developing spatial sense.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Identify geometric shapes in the real world.
- Describe, draw, and classify geometric shapes in the real world.
- Demonstrate spatial sense by transforming and comparing shapes.
- Visualize and create geometric shapes.
- Relate geometric ideas to number and measurement sense.
- Use geometry as a means of describing the physical world.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 8 they will:

- Identify, describe, classify, and compare two- and three-dimensional geometric figures.
- Explore the transformations of geometric figures (translation, rotation, and reflection).
- Represent and solve problems, using geometric figures.
- Construct models of two- and three-dimensional shapes.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 10 they will:

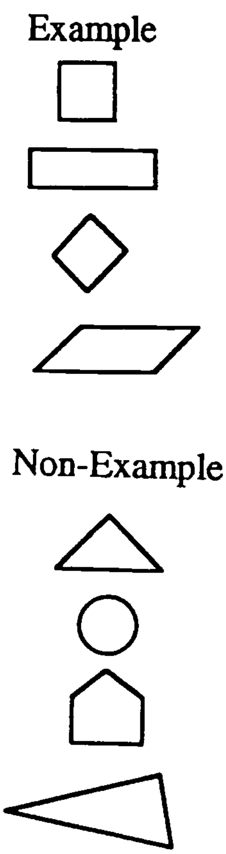
- Identify congruent and similar figures, using transformations.
- Investigate properties of figures, using transformations, coordinates, and vectors.
- Represent problem situations with geometric models.
- Investigate the properties and behaviors of geometric figures through algebraic patterns.

Through problem-solving situations, all students will construct their own understanding, so that by the end of grade 12 they will:

- Deduce properties of, and relationships between figures, given assumptions.
- Extend two-dimensional ideas into spatial sense.
- Investigate and compare various geometries.

QUADRILATERALS AND ATTRIBUTES

Nancy Miller is beginning a unit on geometry, focusing on the properties of geometric shapes. She is concerned with the language aspect of naming shapes in her class where three different languages are spoken. She begins the class with a literature connection, using Tana Hoban's book, *Shapes, Shapes, Shapes*, a picture book wherein the reader is encouraged to associate geometric shapes with everyday objects. As the class looks at the photographs, they describe in their own language the shapes that they see in the book.



After reading the story aloud, Ms. Miller tells the students that she would like to play a guessing game with them. She lays out a sheet of paper labeled with two columns; one titled *Example*, and the other titled *Non-Example*. She tells them, "I am thinking of a mathematical idea. I am going to give you some examples of that idea and some non-examples of that idea. When you think you know what my idea is I would like you to raise your hand and come forward to place a geometric shape on the paper, either under the *Example* column or the *Non-Example* column." Ms. Miller is focusing on the concept of a quadrilateral today. In her box of geometric shapes she has a saltine cracker, a Lego™ block, as well as paper and plastic models of shapes.

Later, Ms. Miller elicits from the students the attributes of the items in the *Example* column. The children respond, "They all have four sides...they all have four corners...they have straight sides that go up and down...they are not crooked." Ms. Miller records these responses and says that we have a special name for the shapes they have placed in the *Example* column—quadrilaterals. Ms. Miller uses the language of the students to define this mathematical concept.

To assess the students' developing understanding of the idea, she distributes pattern blocks and asks students to work in pairs to sort them according to which shapes are quadrilaterals and which are not. As each pair demonstrates how they sorted their shapes, Ms. Miller asks them to explain their reasoning, to tell why they placed the blocks in the category. She also gives them an opportunity to make quadrilaterals on their geoboards and record their solutions on dot paper. From her observational assessment Ms. Miller begins to see who can demonstrate the concept and who needs further concept development. By giving her students an opportunity to demonstrate their knowledge in a non-language environment, Ms. Miller is tapping into their mathematical thinking, without relying solely on vocabulary.

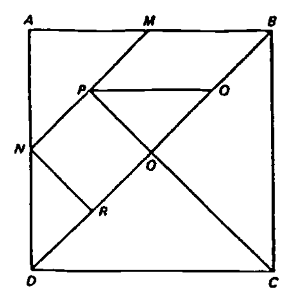
The homework is that students collect and bring objects from their home to include in a quadrilateral museum. They will write why they think the objects belong in the museum,

On subsequent days, Ms. Miller continues to use the concept development model of instruction (example, non-example), to develop the ideas of triangle, hexagon, and other polygons. She continues to use literature connections, including *Color Zoo* and *Color Animals* by Lois Ehlert to introduce new shapes.

ESL Team
East Providence and Bristol Warren

The Art and Poetry of Tangrams

Geometry and measurement provide many opportunities for students to work cooperatively in mathematics labs. Manipulatives such as tangrams can help students use problem-solving strategies as they learn geometric concepts and linear measurement. To introduce tangrams, a teacher might encourage students to observe and discuss the shapes, sizes, and congruence of each piece.

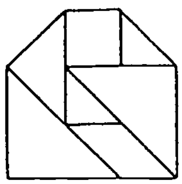


Students may construct their own sets of tangrams, using pieces of oak tag or they may be purchased commercially. Rather than assigning students exercises from mathematics textbooks on finding perimeters and areas of polygons, I tried lab activities with tangrams. We explored estimation, measurement, and comparisons of perimeters of convex polygons. An extension involving an interdisciplinary project is challenging students to create their own tangram designs.

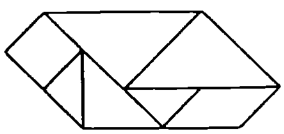
After introducing the concept of perimeter and allowing time for practice measuring with a metric ruler, I introduced the problem. *Find the perimeter of a tangram puzzle and discover if its perimeter is different as you change the shape of the design, using all seven pieces of the tangram set.*

(There are thirteen possible convex polygons, including 1 triangle, 6 quadrilaterals, 2 pentagons and 4 hexagons. Teachers might challenge students to discover all thirteen.)

Accompanying the problem given to students was the following set of instructions.

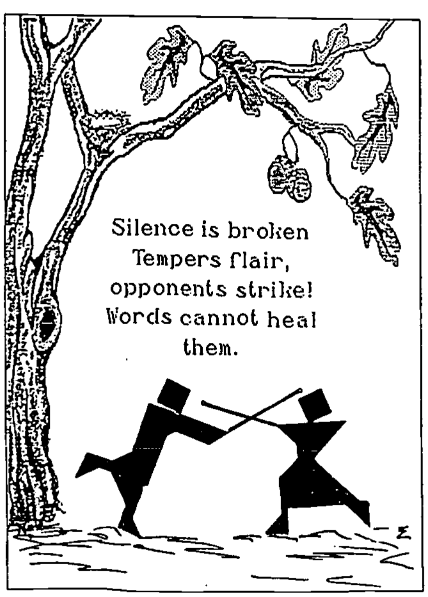


1. Estimate the perimeter of each polygon in centimeters.
2. Use a ruler to measure the perimeter of each polygon in centimeters.
3. Record the difference between your estimate and actual measurement.
4. Use all 7 tangram pieces to form each polygon.
5. Does the perimeter of a tangram puzzle change? Why or why not?



As an extension, I provided pictures of other figures made with tangrams, and challenged students to arrange their tangrams to make the figures. Requirements for this extension portion to the lesson were:

1. Trace the tangram pieces that solve your puzzle on graph paper.
2. Measure and label the perimeter of your puzzle.
3. Make your own set(s) of tangrams on the paper provided in art class.
4. Glue your set(s) of tangram pieces on a poster board 22 in. by 28 in. Add a background and if you wish, include another design of tangrams to your poster.
5. Write a Haiku poem describing the tangram "scene" that you created. Mount it on the poster board.



To assess students' posters, I predetermined categories and assigned point values according to adherence to requirements specified for extensions (25%), creativity (50%), and visual appeal (25%).

Students displayed their excitement, determination, and success in estimating, measuring, and making the shapes provided and those which they chose to create for their interdisciplinary project. This lesson and its extensions afforded me and the students an opportunity to assess more than skills and ability in measurement, estimation, and problem-solving. From the interdisciplinary projects submitted by students, I learned more from my eighth grade students about their motivation for solving problems and their talents for displaying their expressions through art and poetry.

Bernadette Marchak, Coventry Middle School

USA Pavilion for EXPO 2000

I presented the following task to my class. You and your partners are architects submitting a bid for the contract to design the Geodesic Dome for the USA Pavilion at EXPO 2000.

As an introduction to the task, I posed the question to the students, "Have any of you seen a picture of the USA Pavilion that was built for EXPO '67 in Montreal?" After hearing from the students, I asked students to research the pavilion, find pictures, and write about their findings in their math logs.

As part of the preparation for this task, students investigated the concept of symmetry as they worked with mirrors, identifying objects in the classroom and in nature that have symmetry, and explored and built platonic solids. Students discussed the properties of platonic solids and were able to construct tetrahedra, hexahedra, octahedra, dodecahedra, and icosahedra by using string and straws. In their math logs, students made a table showing the number of faces, vertices, and edges of each. Students also discovered Euler's formula, F+V-2=E.

After the research was completed and the students discussed their findings, teams of students began to work on their task of designing and constructing their domes. The task included building a scale model of the pavilion, showing the geodesic dome that students designed and constructed. The models had to be made to scale and meet specifications. The actual pavilion will be about 187 feet high and will require at least 6 million cubic feet of exhibit space. Since the pavilion will be used for exhibits and demonstrations, students needed to construct a dome that was durable, yet offered the maximum space possible.

Students' drawings and research were kept in portfolios and their models were displayed in the school. Teams of students presented analyses of their models to a team of judges. Students were assessed on their models and their analyses of why they constructed them as they did.

Sue Licciardi
Bain Junior High School, Cranston

References

ABE Math Standards Project. *Massachusetts Adult Basic Education Math Standards*. Holyoke, MA: Holyoke Community College, 1994.

Alchele, Douglas B., ed. *Professional Development of Teachers of Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1994.

Alligood, Robert, ed. *Improving Mathematics, Science, and Computer Education in Florida: A Comprehensive Plan*. Tallahassee: Florida Education and Industry Coalition, 1989.

Association of State Supervisors of Mathematics and National Council of Supervisors of Mathematics. *Guidelines for Selecting Instructional Materials for Mathematics*. Fall 1993.

Board of Mathematical Sciences. *Mathematical Sciences: A Unifying and Dynamic Resource*. Washington, DC: National Academy Press, 1986.

Brown, J.S. and R.R. Burton. Diagnostic Models for Procedural Bugs in Basic Mathematical Skills. *Cognitive Science*, 2, 1978.

Charles, Randall I. and Edward A. Silver, eds. *The Teaching and Assessing of Mathematical Problem Solving*. Reston, VA: National Council of Teachers of Mathematics, 1988.

Conference Board of the Mathematical Sciences. *The Mathematical Sciences Curriculum K-12: What Is Still Fundamental and What Is Not*. Report to the NSB Commission on Precollege Education in Mathematics, Science, and Technology. Washington, DC: CBMS, 1983.

Cooney, Thomas J., ed. *Teaching & Learning Mathematics in the 1990s*. Reston, VA: National Council of Teachers of Mathematics, 1990.

Davis, Robert B. *Learning Mathematics: The Cognitive Science Approach to Mathematics Education*. Norwood, NJ: Ablex Publishing, 1984.

Denham, Walter F. and Edward T. O'Malley, eds. *Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve*. Sacramento, CA: California State Department of

Education, 1985.

Dixon, Susan, et. al. *Performance Task Sampler*. Hartford, CT: Connecticut State Department of Education, 1991.

Ginsburg, H.P. *Children's Arithmetic: How They Learn It & How You Teach It*. NY: Van Nostrand Reinhold, 1977.

Grouws, Douglas A., ed. *Handbook of Research on Mathematics Teaching and Learning*. Reston, VA: National Council of Teachers of Mathematics and Macmillan Publishing Company, 1992.

Hiebert, James and Merlyn Behr. *Number Concepts and Operations in the Middle Grades*. Reston, VA: National Council of Teachers of Mathematics, 1988.

Issues of Curriculum Reform in Science, Mathematics and Higher Order Thinking Across the Disciplines. Washington, DC: U.S. Department of Education, 1994.

It All Adds Up. Alexandra, VA: National Association of State Boards of Education, n.d.

Johnston, William B. and Arnold E. Packer, eds. *Workforce 2000: Work and Workers for the Twenty-First Century*. Indianapolis: Hudson Institute, 1987.

Journal of Staff Development. Winter 1995, Vol. 16, No. 1 P. 12

Krulik, Stephen, ed. *Problem Solving in School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1980.

Kuhs, Therese, ed. *Mathematics Assessment: Alternative Approaches*. Reston, VA: National Council of Teachers of Mathematics, 1992.

Learning a Living: A Blueprint for High Performance, A SCANS Report for America 2000. U.S. Department of Labor. April 1992.

Lodhoiz, Richard, ed. *A Change in Emphasis*. Parkway, MO: Parkway School District, 1986.

Mathematical Sciences Education Board, National Research Council. *The Teacher of Mathematics: Issues for Today and Tomorrow*. Washington, DC: National Academy Press, 1987.

——. *Counting on You: Actions Supporting Mathematics Teaching Standards.* Washington DC: National Academy Press, 1991.

——. *Measuring Up: Prototypes for Mathematics Assessment.* Washington, DC: National Academy Press, 1990.

——. *Reshaping School Mathematics: A Philosophy and Framework for Curriculum.* Washington, DC: National Academy Press, 1990.

National Association for the Education of Young Children. *Developmentally Appropriate Practice in Early Childhood Programs Serving Children from Birth Through Age 8.* Washington, DC: National Association for the Education of Young Children, 1987.

National Commission of Excellence in Education. *A Nation at Risk: The Imperative for Educational Reform.* Washington, DC: U.S. Government Printing Office, 1983.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1989.

——. *Professional Standards for Teaching Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1991.

——. *Algebra for the Twenty-First Century, Proceedings of the August 1992 Conference.* Reston, VA: National Council of Teachers of Mathematics, 1993.

——. *Assessment Standards for School Mathematics.* Reston, VA: National Council of Teachers of Mathematics, 1995.

——. *Addenda Series, K-12.* Reston, VA: National Council of Teachers of Mathematics, 1992.

National Research Council. *Moving Beyond the Myths: Revitalizing Undergraduate Mathematics.* Washington, DC: National Research Council, 1991.

——. *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington, DC: National Research Council, 1989.

Perrone, Vito, ed. *Expanding Student Assessment.* Alexandria, VA: Association for Supervision and Curriculum Development, 1991.

Peterson, P.L. and T.L. Carpenter, eds. Learning Through Instruction: The Study of Students' Thinking During Instruction in Mathematics. *Educational Psychologist*, 1989.

Piaget, Jean. *The Construction of Reality in the Child.* NY: Basic Books, 1954.

Resnick, Lauren B. *Education and Learning to Think.* Washington DC: National Academy Press, 1987.

Romberg, Thomas A. *School Mathematics: Options for the 1990s.* Washington, DC: U.S. Government Printing Office, 1984.

Rowan, Thomas A. and Lorna J. Morrow, eds. *Implementing the K-8 Curriculum and Evaluation Standards: Readings from the Arithmetic Teacher.* Reston, VA: National Council of Teachers of Mathematics, 1992.

Schoenfeld, Alan H. *Cognitive Science and Mathematics Education.* Hillsdale, NJ: Lawrence Eribaum Associates, 1987.

Steffe, Leslie, ed. *Transforming Early Childhood Mathematics Education.* Hillsdale, NJ: Lawrence Eribaum Associates, 1989.

Stenmark, Jean K. *Assessment Alternatives in Mathematics: An Overview of Assessment Techniques That Promote Learning.* Berkeley, CA: Lawrence Hall of Science, University of California, 1989.

Stenmark, Jean K., ed. *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.* Reston, VA: National Council of Teachers of Mathematics, 1991.

Stenmark, Jean K., Virginia Thompson, and Ruth Cossey. *Family Math.* Berkeley, CA: Lawrence Hall of Science, University of California, 1986.

Swan, Malcom. *The Language of Functions and Graphs.* Nottingham, England: Shell Centre for Mathematics Education, 1987.

Swetz, Frank and J.S. Hartzier. *Mathematical Modeling in the Secondary School Curriculum: A Resource Guide of Classroom Exercises.* Reston, VA: National Council of Teachers of Mathematics, 1991.

Thornton, Carol A. and Nancy S. Bley. *Windows of Opportunity: Mathematics for Students with Special Needs*. Reston, VA: National Council of Teachers of Mathematics, 1994.

Trafton, Paul R., ed. *New Directions for Elementary School Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1989.

Using Technology to Support Education Reform. Washington, DC: U.S. Department of Education, 1993.

Webb, Norman L. *Assessment in the Mathematics Classroom*. Reston, VA: National Council of Teachers of Mathematics, 1993.

NOTES

96

DEVELOPING A COMMON CORE OF LEARNING

A Report On What We Heard

Revised, Fall 1995

Published by the
Rhode Island Department of
Elementary and Secondary Education

THE OUTCOMES AND ASSESSMENT TEAM

Pat DeVito and Marie C. DiBiasio, Co-Chairs

Joyce Andrade, Wrwk.School Committee
Pam Ardizzone, Westerly Public Schools
Wendy Aronoff, Providence Public Schools
Ron Bate, Barrington, RI
Colleen Bielecki, RI Skills Commission
Cathy Biggins, Scituate Public Schools
Betty Ann Blackmer, S.E.A.C.
David Brell, Rhode Island College
Barbara Brittingham, University of RI
Bob Britto, Providence Public Schools
Bert Brousseau, Central Falls Schools
Susan Bryan, RIEMA Representative
Peter Calvet, Portuguese Cultural Fnd.
Sharon Capobianco, Scituate Schools
Janet Carroll, RIDE
Mary Ann Carroll, Johnston Public Schools
Jerry Carter, Providence Public Schools
Mary Chirico, Johnston Public Schools
Elizabeth Cohen, Newport Public Schools
Joan Colwell, East Greenwich Schools
Kathy Connolly, RI Skills Commission
Ed Costa, RIDE
Roberta Costa, E. Providence Schools
Nancy Cummings, Albion, RI
Henry D'Aloisio, RIDE
William Davis, Narragansett Public Schools
Marge DeAngelis, E. Providence Schools
Rob DeBlois, Urban Collaborative
Charlotte Diffendale, RIDE
Bernardine DiOrio, Exeter-W.G. Schools
Thomas DiPaola, Providence College
Domenic Dougherty, Chariho Schools
Cynthia Duffy, E. Greenwich Public School
Patricia Lewis Dulac, RIOHE
Sandy Dunn, Cumberland, RI
Linda Filomeno, Providence Schools
Ken Fish, RIDE
Bill Fiske, RIDE
Nancy Frausel, RIDE
Jo Eva Gaines, Middletown Public Schools
Paul Gounaris, Providence Public Schools
Barrie Grossi, RI Parent Info Network
Jackie Harrington, RIDE
Vincent Hawkins, Warwick Public Schools
Ellen Hedlund, RIDE
Gloria Howard, RI Skills Commission
Diane Horm-Wingerd, University of RI
Betsey Hyman, E. Providence Schools
Arlene Iannazzi, West Bay Collaborative
Tom Izzo, RIDE
Michael Jolin, RI Children's Crusade
Allegra Jones, RIDE
Susan Kaplan, Providence Public Schools
Jim Karon, RIDE
Josephine Kelleher, Woonsocket Schools
Jack Keough, RIDE
Diane Kohler, N. Smithfield Public Schools
Nina Laurenzo, Providence, RI
Marilyn Levine, Westerly Public Schools
Maria Lindia, RIDE
John Long, University of RI
Judy Mamaras, CCRI
Andrea Mattia, Providence Public Schools
Sondra McBride, North Providence, RI
Cheryl McCarthy, University of RI
Michael Mello, Portsmouth Public Schools
Arlene Militello, Warwick Public Schools
Stephen Mitchell, Burrillville Public Schools
Fran Mossberg, Providence Public Schools
Olga Noguera, Dept. of Health/Human Svc
William Oehlkers, Rhode Island College
Al Olsen, PACSE
Fred Pasquarillo, Johnston Public Schools
Rocco Rainone, RIDE
Tomas Ramirez, Providence Public School
Cornelia Readinger, Gordon School
Jack Riley, Davies Vocational-Technical
Mary Ann Roll, RI PTA
Susan Rotblat-Walker, RIDE
Lynne Ryan, Providence College
Diane Schaefer, RIDE
Marti Schwartz, Smithfield Public Schools
Robert Shapiro, Warwick Public Schools
Susan Skawinski-Lima, RIDE
Helene Skrniarz, East Providence Schools
Mary Ann Snider, RIDE
Joyce Stevos, Providence Public Schools
Stephanie Sullivan, RIMSEC
John Susa, Warwick, RI
Gladys Thomas, RIDE
Beverly Travers, Bristol-Warren Schools
Margaret Vendituoli, Bristol-Warren School
Linda Washington, Pawtucket Schools
Ina Woolman, RIDE
Karen Wyche, Brown University
Robin Yates. Central Falls Schools
Chip Young, RI Skills Commission

WRITING TEAM

*David Brell**

Sharon Capobianco

Pat DeVito

Marie DiBiasio

*Nancy Frausel**

Susan Kaplan

*Diane Kohler**

Nina Laurenzo

Arlene Militello

*William Oehlkers**

*Jack Riley**

*Marti Schwartz**

Bob Shapiro

*Margaret Vendituoli**

Ina Woolman

*Robin Yates**

Jane Arsenault, Consultant

*Members of the Primary Writing Team

DATA ANALYSIS TEAM

Lynne Ryan, Chair

Betty Ann Blackmer

Peter Calvet

Pat DeVito

Marie DiBiasio

Domenic Dougherty

Patricia Lewis Dulac

Jim Karon

Vincent Hawkins

Ellen Hedlund

Gloria Howard

Cheryl McCarthy

COVER ART

Lay Thet, 9th grade student and Marjorie Gallo, teacher
Bain Junior High School, Cranston, Rhode Island

TABLE OF CONTENTS

* * PREAMBLE * *

What the best and wisest parent wants for his own child, that must the community want for all of its children.

--- *John Dewey*

Historically, Rhode Island has expected much from its educational system. In the future we will expect even more. The needs of a changing society place new and greater demands on students and on schools. Both must prepare for a future we cannot truly envision. Schools must prepare students not only to work, but to live, in the 21st century.

It is important to ask how well our current educational system has served Rhode Island, and how it must be improved and changed. Our speculations about life in the future are tested against what sociologists, economists, scientists and artists tell us that we may expect. Each of us filters these ideas through our own cultural, political and family experiences. We question what we will require from our educational institutions. How will we earn our livelihoods? What advances in technology, transportation, health, the arts and recreation will influence the way we live?

Developing A Common Core of Learning - A Report on What We Heard pays its respects to our past at the same time that it represents our promise to the future. ALL KIDS, not just some kids, deserve the best education. With this in mind together we will create an educational system that renews our society and prepares Rhode Island for the future.

INTRODUCTION ...

Three important events provide the foundation of Rhode Island's Common Core of Learning. The nation's Governors drafted "America 2000" goals (now called *Goals 2000*) in 1989. In March 1994, these national educational goals were finally enacted into law. At the state level, the 21st Century Commission and the Rhode Island Skills Commission each drafted plans for the restructuring of Rhode Island's educational system and called for the citizens of the state to establish learner goals and high standards of performance for all of our students.

In 1992, the Board of Regents and the Commissioner of Elementary and Secondary Education convened Rhode Island's Common Core of Learning Team, comprised of over 100 parents, educators, civic, business and corporate leaders. Together they researched the issues and conducted focus groups around the state. This work led to the development of a survey printed in five languages, and 200,000 were distributed throughout the state. It asked, *"What should all young adults in Rhode Island know and be able to do to meet the responsibilities and challenges of the 21st Century?"*

The responses were tabulated. Writers drawn from the larger team distilled the collected thoughts expressed by the respondents into Rhode Island's Common Core of Learning. This document represents the collective thinking of the citizens of our state about the goals of education.

The respondents identified four major goals of education in Rhode Island:

> ***Communication*** - reading, writing, speaking, listening and conversing effectively.
>
> ***Problem Solving*** - viewing learning as a lifelong process in which problem solving complements the body of knowledge by helping students acquire and apply new knowledge.
>
> ***Body of Knowledge*** - acquiring ideas and skills that have been passed on by past generations and that form the base for the future progress of society.
>
> ***Responsibility*** - accepting responsibility for oneself, one's learning, and one's role in society.

These areas are four dimensions of a whole rather than discrete segments that can be selected piecemeal. They balance knowledge of content, skills and attitudes. Thus, students in their studies concurrently acquire bodies of knowledge while communicating new learning, solving current problems and taking on responsibilities related to their learning. These goals form the basis for developing more explicit curriculum documents.

Rhode Island's Common Core of Learning is intended to guide schools and classroom teachers in the design of curriculum and instruction. It has a second and equally important purpose: to provide a forum for discussion among educators and the general public. We must share a common vision and direction for education in our state if it is to serve us all equally well.

COMMUNICATION

"It is imperative that good oral and written communication skills are stressed." (respondent #00361)

"Communication via written words, oral expression or computer transmission will be a necessity in the 21st century." (respondent #02584)

One of the hallmarks of an educated person is the ability to read, write, speak, listen, and converse effectively. People with well-developed communication skills understand others and express themselves well. In addition, they give and receive constructive feedback, adapting their words and actions as reason and circumstances dictate.

Students who have acquired a common core of communication skills will...

Read widely and attentively by ...

- Reading for a variety of purposes: to gain understanding, to appreciate the experience of others, to gather information and to enjoy leisure time.
- Building meaning while reading, determining the relative importance of ideas and connecting what is read to prior knowledge, other sources and their own experiences.
- Reading critically: distinguishing fact from opinion, identifying inconsistencies and recognizing bias.
- Utilizing reference materials, both print and electronic (e.g. dictionaries, encyclopedias, atlases, and original sources via Internet) and features of written and electronic texts (e.g. tables of contents, indices, chapters, headings and electronic search strategies).
- Following written instructions.

Write persuasively and expressively by ...

- Writing for a variety of purposes and audiences.
- Writing as a way of discovering and clarifying ideas.
- Engaging in a process that involves planning, organizing, revising and editing one's own writing.
- Supporting ideas through the use of facts, examples, quotations and arguments.

***Communication* (continued)**

- Gathering information, taking accurate notes and summarizing accurately, noting sources properly.
- Using correct spelling, punctuation, grammar and other language conventions.
- Making use of print and electronic reference tools, such as handbooks and grammar and spelling checking programs, to locate language conventions.
- Using technology and software including text, data, graphics and communication, to produce documents.

Speak, listen and converse intelligently by ...

- Listening and conversing in order to share information, build relationships and promote understanding
- Engaging constructively in oral exchange of ideas.
- Asking and answering questions.
- Delivering oral presentations to a group, using appropriate language, information, gestures and media.
- Conducting and being the subject of an interview.
- Forming, expressing, and defending a point of view.
- Giving, understanding and following spoken instructions.
- Listening carefully and giving constructive feedback.
- Communicating with others using electronic media (e.g., audio, video, Internet).
- Communicating with others in more than one language.
- Communicating with people from various social, occupational and cultural groups.
- Working to clarify misunderstandings and to resolve conflicts peacefully and democratically.
- Understanding the impact of one's language on others.

PROBLEM SOLVING

"They should be able to reason a problem out in a step-by-step fashion until a worthwhile solution is reached." (respondent #00527)

A rapidly changing world requires all people to be lifelong learners and problem solvers. Along with the common body of knowledge and skills needed for a productive life, students need to learn continuously and apply what they have learned critically and creatively to solve real life problems. Lifelong learning and problem solving are not separate from the common body of knowledge that comprises the content of schooling; rather, they complement it by helping students to acquire new knowledge and to apply it in original and effective ways.

Students who have acquired a common core of problem solving skills will ...

Learn continuously by ...

- Setting high standards in developing personal learning goals.
- Accepting and pursuing challenges - stretching personal limits.
- Working cooperatively and/or independently as the situation demands.
- Using learning approaches that are suitable to personal, community, and work related settings.
- Giving, receiving, and evaluating constructive criticism.
- Learning from failure.

Solve problems effectively by ...

- Defining a problem, posing meaningful questions, generating and testing alternative hypotheses, establishing criteria for measuring success, and determining a course of action.
- Planning and organizing a problem solving task requiring multiple steps, sustained concentration and long term commitment.
- Demonstrating flexibility, inventiveness and persistence, revising a problem solving plan as needed.
- Seeking advice and information, presenting data and analyses, persuading others of a particular course of action and modifying plans in light of the input of others.

***Problem Solving* (continued)**

- Using appropriate techniques and technologies in acquiring new knowledge and skills and in researching and solving a problem.
- Observing, selecting and recording meaningful data.
- Accessing, applying, interpreting, recording, evaluating and integrating information from a variety of sources and subject areas.
- Questioning the validity of sources, recognizing fallacies, detecting a writer's or a speaker's point of view and examining how facts and language are being used.
- Considering and weighing diverse perspectives, then defending a decision to accept, reject or modify each.
- Employing a range of strategies, including those which involve the application of technologies.
- Evaluating the quality and success of their own work.

BODY OF KNOWLEDGE

"Successful education can be a blend of teamwork, skills, historical knowledge, scientific application, including the use of technology and multicultural awareness. The elements are not presented piecemeal but are taught in context." *(respondent #01937)*

People today build upon the ideas and skills of the civilization that went before them. This Body of Knowledge forms the base for schooling in the modern world. This schooling relies on a knowledge of human thought and creativity from historical, scientific and social perspectives. The focus here is the application of knowledge in improving the quality of life for the individual in the community, workplace and society. The statements in this section are not intended to be an exhaustive catalog of subjects or precise content standards, but they suggest the general knowledge that all students should acquire and be able to apply.

Students who have acquired this body of knowledge will ...

Know about themselves by ...

- Understanding the workings of the human body and mind in order to maintain personal health.
- Applying information and skills that enable successful functioning in everyday tasks.
- Developing skills necessary for employment.

Know about others by ...

- Understanding the American political system in order to fulfill the duties of citizenship for personal and community purposes.
- Understanding the principles of the American economic system that allows the individual to participate in and benefit from that system.
- Showing an appreciation of their own culture and the culture of others, knowing the influence of cultural differences upon human interaction and having the ability to employ this understanding in improving cross-cultural relations.
- Understanding the influence of religious views and values on past and present society.

Body of Knowlege (continued)

- Developing an understanding of literature and the arts as a reflection of values shaped by social or historical forces.
- Appreciating the major art forms: drama, dance, music and the visual arts.
- Having a basic understanding of the history and structure of the English language.

Know about the natural world by ...

- Demonstrating an understanding of people's relationship to the environment, the influence of the environment on human life and the use of the environment in conserving and improving life.
- Understanding key concepts of mathematics, science and technology; the relationships between and among them; and their strengths and limitations.
- Recognizing the impact of technology on the workplace and society, including implications for the environment.
- Recognizing the importance of mathematics, science and technology in daily life.
- Interacting and communicating confidently with others in using mathematics, science and technology to ask and answer relevant questions.
- Using scientific processes, mathematical reasoning and technology to solve problems and build an understanding of the natural world.

RESPONSIBILITY

"I believe that all young adults should #1 learn how to accept responsiblility for their actions both academically and socially." (respondent #00142)

A pillar of the Common Core of Learning must be responsibility. The 21st Century will require citizens to take responsibility for themselves, their learning and their society. By gaining an awareness of their responsibilities to themselves as individuals and to society as a whole, the youth of Rhode Island will be better equipped to meet the challenges of tomorrow.

All Rhode Islanders, therefore, must be encouraged to take responsibility for their lives and the role each will play in society. As lifelong learners they will act on goals that they set for themselves, develop healthy habits and establish positive relationships, at home, in the workplace and in the community. They will develop personal characteristics that enable them to become good citizens, family members, and parents, as well as productive workers.

Students who exhibit responsible behavior will ...

Accept personal responsibility for the well being of self and society by ...

- Developing habits to ensure physical, emotional and mental health.
- Making informed career and life decisions.
- Developing strategies to manage stress.
- Coping successfully with negative peer pressure and media influences.
- Making and keeping healthy relationships.
- Buying and consuming responsibly.
- Understanding how technology affects human culture, the workplace and the environment.
- Being aware of our interdependence with the environment.

Work responsibly in groups and as an individual by ...

- Working cooperatively with others in achieving a group decision or goal.
- Sharing, delegating, leading, contributing and following through.
- Respecting opposing points of view.
- Carrying through responsibilities and completing tasks.
- Knowing how and when to negotiate or compromise to reach a consensus.
- Using technology appropriately.

Responsibility ***(continued)***

Acquire the necessary skills, competencies and personal qualities to succeed in the workplace by ...

- Understanding the multiple pathways through which one may prepare for various careers.
- Applying the body of knowledge, communication and problem solving approaches appropriately in one's occupations.
- Understanding the value of labor and developing a work ethic.
- Managing time and resources effectively.
- Being flexible in adapting to new situations, analyzing information, and solving problems through the use of existing and emerging technology.
- Setting high personal standards for quality work which satisfies the needs of clients and customers.
- Demonstrating dependability, honesty, productivity, leadership and initiative.

Show tolerance for human diversity by ...

- Learning about differences among people, religions and cultures.
- Understanding the causes of prejudice and its contribution to social injustice.
- Showing courtesy towards others.
- Respecting the rights of all people.

Understand the importance of family and community by ...

- Practicing the duties and responsibilities of citizenship.
- Engaging in meaningful service to the community.
- Understanding the ethical dimensions of citizenship and parenting.

Respond to challenges with integrity, honesty, and courage by ...

- Maintaining high standards of academic honesty.
- Acting in an honest manner when dealing with others.
- Accepting responsibility for personal decisions and actions.
- Setting priorities and accepting responsibilities in the home, family and community.

Display a strong sense of self-worth and personal competence by ...

- Exhibiting self-respect and respect for others.
- Relying on strong interpersonal skills.
- Setting challenging, realistic goals.
- Knowing his or her own heritage.
- Developing and pursuing personal interests and goals.

Mathematical Power for ALL Students

EXECUTIVE SUMMARY

The Rhode Island Mathematics Framework K–12

Published by the
Rhode Island Department of
Elementary and Secondary Education

The Mathematics Framework K–12
Executive Summary

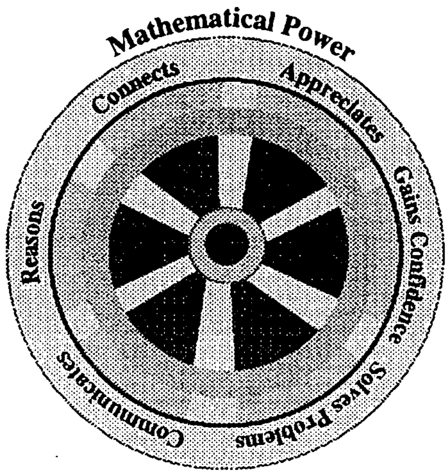
A mathematically literate citizenry is essential to maintain our democracy and to ensure a competitive position in a global economy. This has been a consistent message in national and local reports published by panels and commissions charged with evaluating the status of education and the economy in preparation for the 21st century.

The business community wants and needs workers who are capable of solving problems individually and as part of a team, capable of using technologies, and capable of transferring knowledge and skills to new situations. Educated consumers are necessary to sustain a healthy economy. Informed decisions about purchases, selections among products, and interpretation of information presented in support of claims require mathematical and technological literacy.

The citizens of Rhode Island were asked, "What should all young adults in Rhode Island know and be able to do to meet the responsibilities and challenges of the 21st century?" From this question and the responses received came the development of Rhode Island's Common Core of Learning, which identifies four major goals of education, **communication, problem solving, body of knowledge,** and **responsibility.** The Mathematics Framework K-12 address these goals, as well as the mathematics content and process standards, the teaching and learning of mathematics, and the support needed for implementation.

The question "What is Mathematics?" is one that is frequently asked. In developing the framework, the following definition was used: Mathematics is the study of patterns and relations, with people interacting with one another and the physical world, as they explore the process of thought, solve problems, make connections, reason, and communicate ideas.

The **VISION** of Rhode Island mathematics education is one of *power, partnerships, and preparation.* All Rhode Island students will develop mathematical power, that is students...



- *Appreciate* the value of mathematics;
- *Gain confidence* in their own mathematical ability;
- *Engage* in mathematical problem solving;
- *Communicate* mathematically;
- *Connect* what is learned in mathematics with other mathematics topics, other disciplines, and daily life.

For this to occur, students, teachers, administrators, families, business members, higher education faculty, and policy makers will identify their roles and responsibilities and work together as partners. Finally, to prepare our students to meet the challenges of the future, the mathematics classroom needs to be transformed into a learning community. What is being taught and learned has to correlate with what is happening in our world. Students in this learning environment understand the relevancy of what they are learning and how it applies to their lives.

The CRITICAL BELIEFS of this framework guided its development. The beliefs articulate that reform must be consistent with state initiatives and national efforts; mathematics is for all students; the development of a framework is a necessary step in a systemic process; a framework provides a vision for mathematics programs; and that changes in mathematics classrooms require support and partnerships.

As we look at the **teaching and learning of mathematics**, we see that the instruction, curriculum, and assessment must be such that all students are given the opportunity to learn high levels of mathematics and be encouraged to succeed. The Mathematics Framework K-12 offers reflective questions for consideration and is meant to be used as a guide for school districts as they develop their local mathematics curriculum. The framework is asking us to think about the necessary components needed to prepare students to be successful in the 21st century. The following is one of the many teacher's guides that appears in the framework. These guides are questions that are posed to stimulate thinking, responses, and more questions.

A Teacher's Guide for Considering Mathematical Tasks

- Does the task address important mathematical ideas, processes, and perspectives?

- How does the task incorporate students' understandings, interests, and experiences?

- Does the task acknowledge the different ways students learn mathematics?
- Does it engage students' intellects?

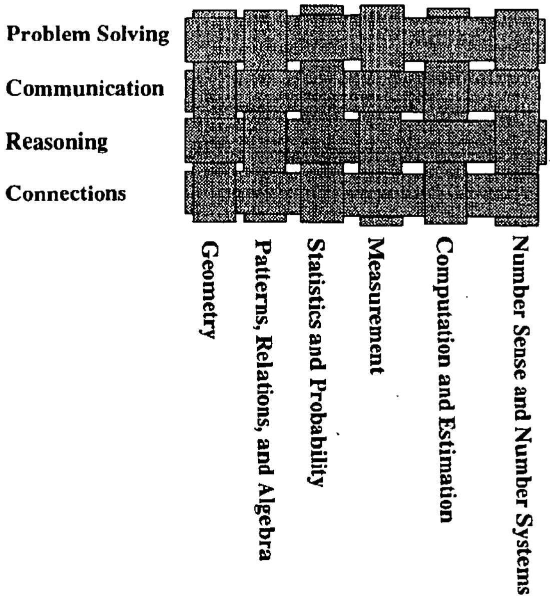
- Will the task develop students' mathematical understandings and skills?

- In what ways does the task stimulate students to make connections and develop a coherent framework for mathematical ideas?

- Does the task require problem formulation, problem solving, and mathematical reasoning?

- Does the task reflect experiences that may be encountered in the workplace and real life?

- Will the task promote the development of all students' mathematical confidence, flexibility, willingness to persevere, interest, curiosity, inventiveness, monitoring and/or reflection in mathematics?



Another important component of the framework is the K-12 MATHEMATICS STANDARDS, which are represented as the weaving of process and content standards. The process standards, *problem solving, communication, reasoning*, and *making connections*, are the means by which students learn mathematics, while the content standards, *number sense and number systems; computation and estimation; measurement; statistics and probability; patterns, relations, and algebra;* and *geometry*, identify the mathematics that students learn. Process or content alone cannot create an environment that encourages and promotes mathematical power for all students. Rather, it is the meshing of rich mathematical concepts, combined with a freedom to explore that enables students to build their understanding of mathematics.

The following are excerpts from the Computation and Estimation standards for elementary grades, identifying **learner goals** with an accompanying **vignette**.

Through problem solving situations, all students will construct their own understanding, so that by the end of grade 4 they will:

- Use estimation for quantities, measurement, computation, and problem solving.
- Recognize and demonstrate when the use of estimation is appropriate
- Communicate the reasonableness of results.
- Develop an understanding of the concepts of addition, subtraction, multiplication, and division using concrete models.
- Demonstrate reasonable proficiency with basic facts and algorithms.
- Develop proficiency with addition, subtraction, multiplication, and division of whole numbers.
- Use a variety of computation techniques, including mental strategies, paper-and-pencil, and technology, as appropriate.
- Use four function calculators in appropriate computational situations.

THING-A-MA-JIGS Vignette

Here is how our class applied what we knew about the value of coins and adding them to make a $1.00. Together, we assigned monetary values to geometric shapes. We decided that a triangle would be worth 1¢, a rectangle worth 25¢, a square worth 5¢, and a circle worth 10¢. Students were asked to create on paper something that would be worth exactly $1.00 when finished. While they worked, I moved around the room and made informal assessments. After completing their creations, students used calculators to check their computation and confirm that their creation was worth $1.00.

To extend the activity, we used construction paper and glue to build their creations. We had some very interesting ones, including a tree with a tree-house, a wagon, a dog, a snowman, and several "thing-a-ma-jigs." After our sharing time we displayed our creations on the bulletin board for all to see.

The development of the Rhode Island Mathematics Framework K-12 was a joint effort of teachers, administrators, parents, business members, higher education faculty, and policy makers. However, we cannot stop now. We have to continue to work together to implement the ideas found within the framework to enhance mathematics education for our students.

For more information or a copy of the complete Rhode Island Mathematics Framework K–12 and Common Core of Learning, please write to:

Rhode Island Department of Elementary and Secondary Education
Office of Instruction
Room B-4
22 Hayes Street
Providence, RI 02908

ratios, ents.	Use estimation to defend the reasonableness of solutions.	Develop concepts of rate (miles per hour, feet per second) and other derived and indirect measurements.	Use probability and statistics to make informed decisions in real-life situations.	Use patterns and functions to represent and solve problems.	Construct models of two- and three-dimensional shapes.
sic arithmetic l to one another.	Use scientific calculators and computers in appropriate computational situations.	Demonstrate a proficiency in measurement and "measurement sense."	Design and carry out experiments or simulations to determine probability.	Use concepts of variable, expression, and equation.	
nposition and mbers (primes, s).		Have an intuitive understanding of systems of measurement.	Communicate conclusions based on analysis of data.	Analyze tables, graphs, rules, equations, and identify relationships.	
sent number ous types of			Use a scientific calculator to explore statistics.	Solve linear equations.	
				Investigate inequalities.	
derstanding of the	Develop proficiency with addition, subtraction, multiplication, and division of real numbers.	Select and apply appropriate techniques and tools to measure quantities and recognize relationships among precision, accuracy, and error of measurement.	Use and interpret sampling techniques.		
he real number	Apply a variety of computational methods and estimation techniques in finding solutions.		Apply concepts of central tendency, variability, and correlation.	Represent situations that involve variable quantities with expressions, equations, and inequalities.	Identify congruent and similar figures, using transformations.
	Demonstrate the ability to select and use appropriate technology (spreadsheets, databases, graphing calculators).		Generate and interpret normally distributed data.	Use tables or graphs to interpret expressions, equations, and inequalities.	Investigate properties of figures, using transformations, coordinates, and vectors.
			Apply the laws of chance to predict and communicate probable events.	Solve equations and inequalities.	Represent problem situations with geometric models.
			Develop convincing arguments and inferences.	Model real-world situations with a variety of patterns and relations.	Investigate the properties and behaviors of geometric figures through algebraic patterns.
			Recognize the importance of statistical claims (hypotheses).	Recognize and model generalities of patterns.	
				Have an intuitive understanding of algebraic procedures.	
lerstanding of the d infinity.	Solve problems by integrating computational methods and estimation techniques with technology.	Select and apply trigonometric functions to solve problems.	Use curve fitting to predict from data.	Use a variety of techniques to solve linear systems.	Deduce properties of, and relationships between figures, given assumptions.
			Understand and use experimental and theoretical probability.	Use tables or graphs to investigate the properties and behaviors of patterns and relations.	Extend two-dimensional ideas into spatial sense.
			Use a graphing calculator to explore statistics.	Demonstrate the properties and behaviors of patterns and relations.	Investigate and compare various geometries.
			Use technology to generate random variables.	Analyze the effects of change on patterns and relations.	
			Have an intuitive understanding of statistics and probability.		

	Apply the process of mathematical modeling to real world situations.	Use mathematical ideas, language, and symbols to converse with others and to make conjectures and convincing arguments.		Use mathematics to solve problems and model situations in other disciplines.	Represent and ap proportions, and p
					Demonstrate how operations are rel
					Demonstrate the c decomposition of factors, and multi
					Investigate and re relations, using va graphs.
Secondary 9-10	Apply integrated technological and non-technological problem-solving strategies to solve problems.	Express generalizations discovered through investigations by using mathematical language.	Recognize the power of reasoning as a pervasive tool.	Recognize and apply different representations for the same concept.	Have an intuitive real number syste
		Communicate mathematical ideas, processes, concepts, and solutions through the use of technology.	Develop and validate conjectures.	Apply procedures used in one problem situation to other similar situations.	Apply properties o system.
Secondary 11-12	Use sophisticated as well as basic problem-solving approaches to investigate, understand, and develop conjectures about mathematical concepts.	Provide clarifying and extending questions related to mathematical conjectures.	Formulate counter examples to support logical arguments.	Use and extend the connections among mathematical topics.	Have an intuitive u concept of limits a
			Construct simple valid arguments based on logic.	Use and extend the connections between mathematics and other disciplines.	
				Use and extend the connections between mathematics and the world of work.	