DOCUMENT RESUME

ED 444 854 SE 063 978

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TITLE	Preparing School Mathematics Teachers To Meet the Challenges of Reform.
SPONS AGENCY	Michigan State Dept. of Education, Lansing.
PUB DATE	1997-02-14
NOTE	51p.; Paper presented at the Annual Meeting of the Association of Mathematics Teacher Educators (Washington, DC, February 14-15, 1997).
PUB TYPE	Reports - Descriptive (141) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC03 Plus Postage.
DESCRIPTORS	*Course Descriptions; *Educational Change; Higher Education; *Mathematics Education; Middle Schools; *Pedagogical Content Knowledge; *Preservice Teacher Education; Secondary Education

ABSTRACT

This paper focuses upon three recently revised preservice, content-specific methods courses: "Teaching of Middle School Mathematics," "Computing Technology in Secondary School Mathematics," and "Teaching of Secondary School Mathematics." Changes in these methods courses better address and build upon preservice teacher beliefs about, and knowledge of, school mathematics, the teaching of mathematics, the assessment of learned mathematics, and the student as learner of mathematics. The authors believe these modifications increase the likelihood of graduates successfully implementing mathematics reform curricula in middle and secondary school classrooms. Three appendixes contain sample materials used in academic courses in mathematics and computing technology (Author/ASK)

PREPARING SCHOOL MATHEMATICS TEACHERS TO MEET THE CHALLENGES OF REFORM

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Abstract

This paper focuses upon three recently revised preservice, content-specific methods courses: "Teaching of Middle School Mathematics," "Computing Technology in Secondary School Mathematics," and "Teaching of Secondary School Mathematics." Changes in these methods courses better address and build upon preservice teacher beliefs about, and knowledge of, school mathematics, the teaching of mathematics, the assessment of learned mathematics, and the student as learner of mathematics. The authors believe these modifications increase the likelihood of graduates successfully implementing mathematics reform curricula in middle and secondary school classrooms.

Materials described in this paper were written and compiled for the Secondary School Mathematics Teacher Preparation Improvement Project co-funded by the Dwight D. Eisenhower Higher Education Professional Development Grant Program through the Michigan Department of Education and by Western Michigan University. Opinions expressed are those of the authors and not necessarily those of the funding agencies.

Paper Presented at the First Annual AMTE Conference, Washington, D. C., February 14-15, 1997

Preparing School Mathematics Teachers To Meet the Challenges of Reform

The movement to reform school mathematics, as called for in several documents including the National Council of Teacher of Mathematics *Curriculum and Evaluation Standards for School Mathematics* (1989), the Mathematical Sciences Education Board documents *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (National Research Council, 1989) and *Reshaping School Mathematics: A Philosophy and Framework for Curriculum* (National Research Council, 1990), and the Michigan State Board of Education's *Academic Core Curriculum Content Standards* (1994), is well underway. One major vision of this reform movement is that each student gain *mathematical power*:

> "[Mathematical power] denotes an individual's ability to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve nonroutine problems. This notion is based on the recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual, mathematical power involves the development of personal self-confidence." (NCTM, 1989, p. 5)

As acceptance of this broadened view of mathematics grows, traditional instructional patterns and roles of both students and teachers are changing. In reform-minded classrooms, emphasis is shifting from a curriculum dominated by memorization and paper-and-pencil skills to one that emphasizes conceptual understanding, multiple representations, mathematical modeling, and problem solving. Instructional emphasis is shifting away from teacher-dominated lecture and demonstration techniques toward small-group work, individual exploration, and discussions in which the role of the teacher is that of moderator, facilitator, and assessor rather than that of dispenser of knowledge. Assessment techniques are shifting from the dominant use of objective measures to include alternative means such as open-ended questioning, oral and written reporting, projects, interviews, and portfolios. The National Science Foundation has funded several projects to develop and publish curriculum materials to support these changes in mathematics content, classroom instruction, and assessment. These project materials are being designed to help teachers address the needs of all learners and to provide those learners with experiences that take advantage of pedagogical tactics that fit their learning styles.

Successful implementation of these changes requires that classroom teachers adopt reform-minded beliefs both toward the nature of school mathematics and toward the nature of the learner of mathematics. Since most teachers have had little to no direct experience with reform-oriented instruction, carefully planned and implemented programs for both inservice and preservice mathematics teachers are required. The Michigan Statewide Systemic Initiative (MSSI), with the overall goal to "... transform the way that science and mathematics are learned, taught, assessed and perceived in our state for the expressed purpose of enabling all students to achieve scientific literacy and mathematical power" (MSSI Mid-Point Review), has provided a leadership role in supporting such programs. In particular, the MSSI component on Teacher Education and Reform (MSSI-TER) has focused resources and efforts on the reform of mathematics and science teacher preparation in the State. This paper focuses upon the efforts of a project to plan and implement reform-minded changes to the mathematics methods content in the secondary mathematics teacher preparation program at Western Michigan University (WMU). The WMU project, initially formulated through work with the MSSI-TER, is co-funded by the Dwight D. Eisenhower Higher Education Professional Development Grant Program through the Michigan Department of Education and by Western Michigan University. Before discussing this project in more detail, we provide a brief overview of the mathematics teacher preparation program at WMU.

The WMU Secondary School Mathematics Teacher Preservice Program.

Western Michigan University offers both a Secondary School Mathematics Teaching major and minor for college students who elect to prepare themselves for careers in middle school or high school mathematics teaching. The faculty who design and teach courses in these programs are committed to the realization of the standards for the professional development of teachers of mathematics as stated in the *Professional Standards for Teaching Mathematics* (NCTM, 1991). In brief, our goal is to offer a mathematics teacher preparation program where students experience good mathematics teaching, acquire a strong and varied mathematics background and an understanding of school mathematics, gain an understanding of the precollege learner, and acquire a knowledge of a variety of pedagogical techniques appropriate for facilitating mathematical learning at the middle and secondary school levels. The program is designed around the assumption that students need varied experiences over a long period of time in order to mature both as a student of mathematics and as a student of the teaching of mathematics

The Secondary School Mathematics Teaching major is offered through the Department of Mathematics & Statistics in the College of Arts & Sciences in cooperation with the College of Education. The major program has components in mathematics content, general methodologies for teaching and learning, and special methodologies for teaching and learning mathematics. Mathematics courses have been selected to provide the major with a broad range of experiences that include exposure to calculus, discrete mathematics, statistics, geometry, modern algebra, and computing technologies. Students who have completed the mathematics requirements of this major have a mathematics background equivalent to that taken by non-teaching majors. As such, our students gain important understandings of the concepts, skills, procedures, and methods of inquiry in the discipline of mathematics.

Three mathematics methods courses are also required in the major and focus upon both middle school and high school teaching. The second course in the sequence focuses exclusively upon the uses of computing technologies in the teaching and learning of middle and secondary school mathematics. These courses force students to confront their existing beliefs about mathematics teaching and learning, to question traditional roles of the teacher and student in the mathematics classroom, to reflect upon alternatives for organizing classroom instruction, and to consider alternative assessments of student understanding. All of the content and specialty methods courses in this 40-semester hour major are currently taught by faculty in the Department of Mathematics & Statistics. By sequencing coursework in these areas over the entire four-year program, students have multiple experiences that both enhance their understanding of mathematics and allow them to develop into competent and reflective teacher practitioners. Other general pedagogical and field experience coursework in support of this program, totaling a minimum of 31 semester hours, are currently offered through the College of Education. This mathematics teaching major option enrolls approximately 190 college students with 40 to 50 students completing the program each year.

Redesign of the WMU Mathematics Teacher Preparation Program.

During 1996, and continuing into the first half of 1997, the authors have collaborated in an Eisenhower Higher Education Professional Development funded project to study and redesign the methods coursework required in the WMU preservice program. Although the project, as a whole had a wider emphasis, this paper focuses upon the work to redesign and coordinate the content and assessments in the three required methods courses. (Note: all three courses are required of program majors; minors in the program are required to take only the first two courses.) An abbreviated catalog description of each of the three courses is given as follows:

Math 350 Teaching of Middle School Mathematics (3 hours)

> This course considers curriculum issues and trends in middle school mathematics focusing on methods and materials for teaching mathematics effectively to middle school students. Activity and laboratory approaches for teaching mathematics are emphasized.

Math 351 Computing Technology in Secondary School Mathematics (3 hours)

> This course introduces uses of computing technology to enhance and extend the learning of mathematical topics in grades 7-12. Emphasis is placed on the use of technology in problem solving and concept development.

Math 450 Teaching of Secondary School Mathematics (3 hours)

> This course considers curriculum issues and trends in secondary school mathematics focusing on methods and materials for teaching mathematics effectively to secondary school students.

To describe the three courses in more detail, we focus upon four goal areas of importance in the preparation of mathematics teachers. These areas, adapted from the Professional Standards for Teaching Mathematics, are the preservice teacher's knowledge of school mathematics, knowledge of the teaching of mathematics, knowledge of the assessment of learned mathematics, and knowledge of the student as learner of mathematics. A brief overview of the attention paid to each of the four goal areas is provided for each course along with appendices that provide samples of the activities, assignments, assessments, etc. used in these courses.

Math 350: Teaching of Middle School Mathematics.

This course focuses upon the teaching and learning of mathematics at the middle school level. Class time is spent in whole class or small group discussions, viewing video tapes of peer presentations and/or elementary/middle school classrooms, group or individual presentations, group work on activities, etc.. For the majority of students, this is the first time they have seen manipulatives used for the presentation and understanding of mathematical concepts, thought about using calculators as tools for learning vs. checking computational answers, or developed lessons focusing on student activities vs. teacher monologues. An intent of the variety in class structure/methodology, vis a vis the traditional mathematics class structure of lecture and listen, is to provide opportunities for the preservice teachers to confront their beliefs on the teaching and learning of mathematics. What follows below is a brief description of a sampling of activities/opportunities, sorted into the "Knowledge of..." categories.

<u>Knowledge of School Mathematics</u>

"To me, mathematics at the middle school level looks like a lot of repetition, which is essential." "Personally, I hate calculators."
"It is difficult to describe what a middle school mathematics looks like (sic). I believe it similar (sic) to giving a speech on a subject you know about to a bunch of people who have a little bit of knowledge of that subject."

These are student quotes provided at the onset of the course relative to middle school mathematics. To confront the implicit and explicit beliefs expressed in these quotes, we examine reform-minded curriculum projects and supplements such as *The Connected Math Project*, *Mathematics in the Mind's Eye*, NCTM's *Addenda Series*, and *Seeing and Thinking Mathematically*. These materials provide students little drill of computational procedures. Concepts and procedures are reviewed in new contexts that make the revisiting of ideas more interesting. Examples of more meaningful use of calculators in the reform materials supplement the brief in-class examples using ideas from a variety of resources such as Calculators and Mathematics Project - Los Angeles (CAMP-LA), Explorations with Calculators from the University of Houston and the Alief Independent School District, and Teachers Teaching with Technology Middle School Workshop.

Students are required to complete a group project that involves role playing teaching a middle school mathematics lesson. The content standards are divided among the class for the lesson focus and the reform curriculum materials are available for their examination. After the lesson, the group summarizes key ideas of the curriculum materials for the class (see Checklist for Assessing Group Presentations for details of task, Appendix A).

To allow for connections between problem-solving, rational number concepts, and the middle school curriculum, the students create problems to submit to the Menu Problem Department of NCTM's *Mathematics Teaching in the Middle School* (see Menu of Problems Protorubric, Appendix A). A first hurdle for most students as they create the problems is to hone their definitions of what is an exercise vs. what is a problem. Several class sessions have been devoted to solving non-routine problems, discussing what is a problem, using a variety of problem-solving strategies but the creation of problems requires a different level of thought. Generally a first draft of the problems provides a series of exercises. Although students find this task difficult, many commented that it was a good learning experience for them to develop interesting problems for middle school students.

"After sitting through three years of what has essentially been lecture-based, "high-level" material, it's difficult to think about guiding students who are still in what is very much a mathematical discovery stage. Just divorcing myself from that type of thinking has been, and will continue to be a huge challenge." (Student quote at end of course.)

Knowledge of the Teaching of Mathematics

"I need to remind myself that what I think is fairly easy to comprehend, the students may not, therefore I'll need to go slow. I plan to lecture with notes, then examples on the board, some problems for their notes and an occasional activity."

Throughout the semester, students are exposed to alternative approaches to presenting mathematical ideas other than lecture. Many lessons are started with activities to introduce new manipulatives to the students, e.g. pattern blocks, tangrams, attribute blocks, Cuisenaire rods, base-10 blocks, geoboards, Miras, etc. Small group discussions begin centered on questions from the previous reading assignments or problem-solving tasks. Students either record comments on sheets of notebook paper or a larger sheet is distributed to collect all the groups comments. These larger sheets are posted in the room and are edited as the semester progresses to accommodate new thoughts. Individual presentations are given on material that not everyone in the class had read, thus the preservice teacher is contributing new information and insights to her peers, not just the instructor (see Presentation of Info from Math 350 Readings Protorubric, Appendix A). Case studies are read and reacted to, providing glimpses into many diverse classrooms and a variety of teachers thoughts on teaching mathematics. Middle-school classroom visitations also take place during the semester in a local city school district, providing for more experience with diverse student audiences. Examples and nonexamples of reform-minded teaching of mathematics are provided and allow for further discussion of how one can effectively teach mathematics. As mentioned previously, group presentations are given that simulate a middle-school classroom lesson. Here is where the preservice teacher has some practice in planning, leading, guiding, encouraging, questioning, etc. if only for a brief time.

"In order to break the chain of mediocrity, teachers need to push their students to reach higher and farther by challenging their minds and providing them with informational building blocks with which they can construct further knowledge, rather than simply showing them a finished house and asking them to describe it again later. Teachers need to create wonder in their students and show them that math is more than just magical formulas and correctly calculated answers. That's what a teacher has to do: facilitate wonder, learning, and inquiry."

Knowledge of the Assessment of Learned Mathematics

Throughout the course, students have been exposed to a variety of assessment tasks along with corresponding rubrics. Students also evaluate peer problem-solving work using rubrics designed by the Oregon Department of Education. Additionally, they are asked to design their own rubric for evaluating class work that is submitted to the instructor. There is mixed reaction about the use of rubrics, especially when applied to their own work. Some feel constrained by the use of the guidelines, minimizing their creativity, while others feel that the guidelines provide structure and expectations for the task thus minimizing guess work on what they should do.

When students develop lesson plans, an assessment component is required that includes questions and anticipated responses, in- and/or out-of-class tasks, or intended observations (see Math 350 Lesson Plan Evaluation, Appendix A).

Knowledge of the Student as Learner of Mathematics

"I think they will only learn new math methods by practice."

The two chapters, "Students' Thinking: Middle Grades Mathematics" from *NCTM's Research Ideas for the Classroom* (1993) and "Developing Understanding in Mathematics" from Van de Walle's *Elementary School Mathematics: Teaching Developmentally* (1994) provide a theoretical backdrop for how students learn mathematics. In class, discussions focus on summarizing their thoughts on the learning theories with an emphasis on how to help children construct their own mathematics. These readings and discussions are then followed up by 6 classroom visitations where the preservice teachers observe and listen to students as they learn mathematics. The preservice teachers are to work with the students during the class whenever possible, asking questions about what the students are thinking and doing. All Math 350 students have commented that this is a valuable component of the course. It provides a brief glimpse into what is happening in the middle school classrooms and provides a means of making sense of theory. *"However, I would have liked to have seen this strategy (constructivism) in motion in a real .. classroom."* What most of our preservice teachers observed is traditional practice. A workshop is planned for June 1997 for the participating middle school teachers, providing them an opportunity to reflect on their practice based on the preservice teachers' comments of what they observed.

Math 351: Computing Technology in Secondary School Mathematics.

This course can best be described as a laboratory/discussion course in which students are either working in pairs on mathematical situations that have applicability to middle and secondary school mathematics curricula or conducting whole class discussions on the teaching issues raised by such situations. All class meetings take place in a Macintosh computer lab and each student is loaned a programmable/graphing calculator (currently a TI-82) for the term. Although this course places a greater emphasis on mathematical content than on teaching methodologies, attention is given to each of the four identified areas of concern. A general description of how each of the four areas are addressed is given followed by a sampling from the unit assignments, projects, assessment items, and evaluation forms used in the course.

Knowledge of School Mathematics

An important goal of this course, and a difficult one to reach, is to have preservice teachers seriously reflect upon their beliefs about what mathematics is and what topics belong in the school curriculum given the availability of advanced computing technologies for classroom use. The course provides a "review" of school mathematics from a computing technology perspective with an emphasis on multiple representations, applications, and alternative solution methods. Topics are currently selected from five mathematical content areas: discrete processes and algorithms, interpreting graphs of functions and relations, statistical modeling, models of randomness, and topics in geometry. Expectations are to expand the set of content areas to include symbolic manipulation and graph theory applications. Problems taken or adapted from recent middle school and high school mathematics reform curricula are used throughout the course. Students are asked to reflect upon the affect that technology has on the mathematics curriculum by considering what content is made less important than before, what content is made more important than before, and what content becomes newly available for study at the school level.

Knowledge of the Teaching of Mathematics

A strong emphasis is placed on questioning & discussion, small group and pair investigations, use of manipulatives and technology to development of key concepts and problem solving strategies, interpretation of the approximate nature of results from the use of technology, and other pedagogical issues related to the use of technology. A common approach in this class is to present students with some mathematical situation or problem to solve on their own or with a partner, follow this with a discussion of the various approaches used and any difficulties encountered, and then finish with a discussion of the implications of their experiences for the teaching of school mathematics. The opportunity to share with peers always proves to be a valuable mechanism for our students to reflect upon and test their own beliefs about teaching. In addition, our preservice teachers gain confidence in the use of technological tools including graphing calculators, calculator-computer links, Calculator Based Laboratory equipment and data collection probes, various software (including a function grapher, a statistical analyzer, and a dynamic geometry package), and the use of the World Wide Web as a resource for teaching mathematics.

Knowledge of the Assessment of Learned Mathematics

Instruction for the course models the expectations of a reform classroom with specific emphasis given to ongoing assessment. Observation, questioning, written and oral reporting, and the weaving of whole class/small group interaction characterize the learning environment. Although many projects and reports are done by groups of 2 or 3 students, individual accountability is maintained through the use of in-class midterm and final examinations. Although this course does not focus on requiring students to create assessment forms and rubrics, the reasons for using multiple and varied assessments are discussed and their use reinforces the more formal study of assessment techniques provided in the other two classes. Since many of the course projects are done in small groups, students evaluate the contributions of their peers and these evaluations are used in determining individual grades for the group projects.

Knowledge of the Student as Learner of Mathematics

Given the heavy mathematical content, less attention is given in this course to a discussion of the mathematical learner than in the other two courses. However, case studies from the Harvard Mathematics Case Development Project focusing on the use of computing technologies in mathematics instruction are woven into course discussions as time permits. One of these case studies, "How High Can You Go?", involves participants in a discussion on the use of manipulatives and scientific calculators in a high school geometry class to solve a problem involving exponential growth, on strategies for using small groups, and on dealing with unmotivated students. Such case studies bring an element of realism to the class and help our preservice teachers reflect upon the roles of the teacher and the learner when using technology in instruction. In addition, the studies help to highlight some of the difficulties students may experience in their learning of mathematics as well as in their use of technology.

Sample Print Materials Used in Math 351

Appendix B provides a sampling of the materials that have been developed for use in Math 351 and an abstract page describing a National Science Foundation, Division of Undergraduate Education funded project to formally develop, field test, and disseminate materials for a combined methods and content course for preservice teachers focusing on the use of computer and calculator technology in secondary school mathematics.

Math 450: Teaching of Secondary School Mathematics.

Math 450 serves as a capstone to the work begun in Math 350 and continued in Math 351. The emphasis in the course is on the active participation of students in developing their own understanding of the teaching of secondary school mathematics. Approaches taken in the course include discussion of articles relating to the teaching and learning of mathematics, doing mathematics, reading and analyzing cases of mathematics instruction, and interacting with high school students and teachers. Students become familiar with current curriculum problems and trends in secondary school mathematics, investigate ways to assess student understanding of important mathematical concepts and processes, and construct unit and lesson plans for selected mathematical topics. As is true of the entire sequence, the focus is on *teaching as a profession*.

Reflection on class activities and field experiences (done concurrently through College of Education coursework) is a crucial component of the course. A unifying theme of the course is the importance of *dialogue*. Throughout the course, students are provided with opportunities for engaging in informed conversation about mathematics teaching and learning with colleagues. These opportunities range from weekly participation in an electronic conference to making curricular decisions with a team of peers.

Within the course we address knowledges mentioned earlier: school mathematics, the teaching of mathematics, the assessment of learned mathematics, and the learner of mathematics. Keeping in mind that a single activity, such as a case discussion, often encompasses multiple areas of knowledge, a description of representative activities reflective of the four areas is provided below.

Knowledge of School Mathematics

The 7-12 section of the *Curriculum and Evaluation Standards for School Mathematics* is addressed both directly and implicitly. One of the first assignments of the course is to read carefully the first 35 pages of this section (introduction through the process standards) and become familiar with the content standards. In class we revisit the four process standards (a review from Math 350) and focus on the lists of topics/teaching methods that are to be increased and decreased at the secondary level. One activity that we use to do this is a card sort. Each item in the increase and decrease list is placed on an index card. Groups of four or five students are each given a stack with the instructions to correctly sort them into the categories of increasing and decreasing attention. Groups that finish early are challenged to further sort their stacks into the correct content standard categories. This provides a review of the reading as well as a springboard for discussion. After the groups have finished their sorting they are asked if there were any items on the increased/decreased list that surprised or confused them. One pair of items that typically puzzles some students is the increase in "functions that are constructed as models of real-world problems" and the decrease in "formulas given as models of real-world problems." They often include both items in the increase stack. As a class, we discuss the differences between these two statements and the reasons for increasing the first while the second is decreased. The students are encouraged to think about what the proposed changes mean in the context of a high school mathematics classroom. We make to point to discuss how "decreased attention" is not the same as "no attention" and other ways that the *Standards* have been misinterpreted.

The major project for the course (and also a major assessment) is the development of a unit plan. In small groups, the students select a topic from the 7-12 *Curriculum Standards*. Their task is to put together a unified group of lessons around this topic that reflects the ideas presented in the *Standards*. They are to include enough detail (in the form of teacher notes) so that a competent teacher would be able to use the materials in the way intended by the developers. Students are encouraged to research available activities and assessments and to create their own when appropriate materials are not available. The unit is to include both formative and summative assessment that is also reflective of current reforms.

Knowledge of the Teaching of Mathematics

The students gain knowledge in this area from, among other things, class discussions of relevant readings, case discussions, and interacting with experienced mathematics teachers. Perhaps most importantly, every attempt is made to model good teaching practices within the course. Although the focus is not always explicit, our belief is that these experiences will provide the basis for our students to provide similar experiences to their students. We also feel that it is important to ground the discussions of teaching within a context to which our students can make connections. To that end we have made a point of layering instructional episodes. Typically, the first layer involves students completing a mathematical task, the second involves them reflecting upon their work and the thinking processes they used, and the third focuses on discussing issues involved with teaching and learning the content and processes imbedded within the task.

One example that has provided successful opportunities for these layering episodes is the Raven Experiment. The central focus of this series of lessons is a case study developed by the Harvard Case Development Project, "Marble Line, Why Can't You be True?" In this case, an algebra teacher uses a laboratory experiment to illustrate linear functions. To better understand her students' learning as a result of the activity, the teacher reflects upon a videotape of the lesson. Two pedagogical issues that come up in the case are the use of hands-on experiments and effective questioning techniques. Mathematical issues raised revolve around the practical interpretation of the slope and vertical intercept in the equation of a line. The case includes examples of student work.

We begin by doing the Raven Experiment, a laboratory activity based on the legend about the thirsty raven who added stones to a well until the water was at a level he could reach with his beak. Students are provided with a beaker, measuring tools, water, and marbles. They measure the change in the height of water in the beaker as additional marbles are added. The Math 450 students are put in the same situation as the students in the case and asked to complete the same tasks. They record their results on the same information forms as the students. Afterwards, they discuss their learning and we reflect upon the mathematics that is involved in the experiment. Then the students are given the case to prepare for the next class period. Preparing for the case discussion involves reading the case at least twice, examining and reflecting upon the high school student work in relation to what is described in the case, and responding to the questions listed at the end of the case.

The case discussion focuses attention on the teaching and learning of the mathematics. The case, combined with the experiences the preservice teachers have had with completing the identical experiment involved in the case study, provides a rich context for the discussion of teaching and learning. The conversation is able to focus on the theoretical in the context of the practical. The Math 450 students walk away from the class with experiences to which they may attach their new knowledge and vocabulary. They are better able to follow the conversation and actions of the students in the case because they had experienced the same context. Sometimes the discussions and actions were very similar and other times the Math 450 students were surprised by the differences. This surprise took on multiple forms - surprise because the students in the case didn't understand something that the Math 450 students took for granted as well as surprise because the students in the case took a simple and elegant approach to solving a problem that the Math 450 students thought required advanced mathematics.

Knowledge of the Assessment of Learned Mathematics

Along with classroom activities and discussion that focuses on assessment, the course itself contains a variety of assessments. The semester begins with the students developing a rubric to be used for assessing their class contribution. In this context, we discuss the purpose of rubrics and ways in which they can be developed. The students are asked to reflect upon what it would look like if a person was making a positive contribution to the class and how that should be documented. With whole-class input, the instructor then pulls together the class ideas to develop a useable instrument.

The majority of the assessments, including those covering the reading, are performance-based. For example, after reading an initial series of articles discussing current reforms in mathematics education, the students are given the following question:

> *You are in a job interview for a high school mathematics position. Sitting around the table with you are the principal, two math teachers, and an English teacher. The principal looks at his watch and says: "For the next fifteen minutes tell us what you know about the current calls for reform in mathematics education." You have with you a notebook with some of your notes from Math 450 class and can refer to them during the discussion. You say:*

Another assessment, given after we have studied innovative curricular materials, asks them to respond to this question:

> *On the way to a curriculum meeting with your mentor teacher you casually mention that you studied innovative curricular materials in your math methods class. She says, "Great! The committee really hasn't had time to investigate the different options so you can tell us what's being developed and point out their differences and similarities." Always being prepared, you have your notes from Math 450 with you. In the following space, write out what you would say to the committee:*

Using this type of question helps the students put what they are learning in a context and models the nature of the assessments they are being asked to use with their students. It has been very rewarding to have students return from intern teaching and interviewing and say that these types of things actually do happen to them and they are grateful that they had practice organizing their thoughts and applying what they learned outside of the college classroom.

Another form of assessment in this course is a take-home midterm. As many of the students in our program have friends and roommates slightly ahead or behind them we've been working on developing a rotating collection of problems on which to base the midterm. The key commonalties among the problems are that they involve significant mathematics and they can be solved a number of equally successful ways. The first step is to solve the problem and provide a complete but concise write-up of the solution. If applicable, a scale model is to be provided as well. The second step is to identify and briefly explain as many approaches to solving the problem as possible (without having to actually carry out the complete solution). The third and final step is to critique the problem in relation to the NCTM *Standards* - the process standards and any relevant content standards. After the midterms have been collected and graded the students share their solutions with their classmates. Often they are amazed at the variety of solutions and the depth and breadth of mathematics that can be involved in a single problem.

Knowledge of the Learner of Mathematics

One of the ways that Math 450 increases the students' understanding of the learner of mathematics is incorporated into a lesson focused around Core-Plus Mathematics, one of the NSF-sponsored reform curriculums. Consistent with the multi-layered approach we have taken through out the course, the lesson begins by involving the Math 450 students in a mathematics lesson from the Core-Plus textbook. The instruction models what would be done with high school students. Next, we watch a videotape of a high school classroom where the same lesson is being taught. This allows the Math 450 students to observe what high school students do with the mathematics tasks that the Math 450 students have just completed.

The Math 450 students also have the opportunity to view and discuss samples of high school students' work. This, in combination with the video, provides them with an understanding of how high school students approach mathematics and what they are capable of doing. Having realistic expectations for high school students is something that the prospective teachers find difficult. They often base their expectations on their memories of what they did in high school. This generalizes to the feeling that "if I did it, all high school students must be able to" and "if I didn't do it, it must be too hard." Given that not many of our students have yet been through a reform curriculum, topics such as discrete mathematics and activities such as writing and presenting mathematical arguments are perceived as difficult, while a high level of computational skill is expected. Seeing high school students and their work helps our students to gain a more realistic perspective.

The lesson ends with a guest visit by a classroom teacher who has recently taught the Core-Plus lesson we've been studying. This allows the Math 450 students to engage in dialogue with someone who is currently experiencing high school students and can address how the students respond to innovative materials, what is reasonable to expect of a high school student, and other related issues.

As preparation for their unit plan, we are now also asking our students to develop a pre-assessment instrument and pair up with a high school student to pilot the instrument. As the unit plan groups usually contain four students, this should provide a range of results. The Math 450 students will be asked to look for commonalties among the student responses, as well as differences, and to keep these factors in mind when developing their unit plan. Our intend is for this to provide the Math 450 students with a context as they develop their unit plan and to help them develop a deeper understanding of the learner of mathematics.

Appendix A

Sample Materials Used in

Math 350 - The Teaching of Middle School Mathematics

Checklist for Assessing Group Presentations
Math 350

Criteria	Not Met	Minimally Met	Adequately Met	Extremely Well Met
1. presentation well organized, clear and professionally done				
2. equivalent participation of all members of group				
3. activity well-planned and grade-level appropriate				
4. activity clearly involves the first four NCTM standards: mathematics as communication, problem solving, reasoning, or connections				
5. purpose of activity and place in the curriculum discussed				
6. students were introduced to content of the lesson in a way that motivates involvement				
7. activity attended to the corresponding NCTM content standard in one or more ways				
8. activity made relevant use of manipulatives, technology or innovative teacher made materials				
9. activity presents mathematics in a constructivist fashion				
10. learning was induced from experience rather than through teacher-telling				
11. activity emphasized teacher-student and student-student interaction and collaboration				
12. group members facilitated classroom discourse				
13. appropriate mathematical language and notation (where appropriate) used				
14. on-going assessment of students' understanding evident through questioning, observation, tasks, etc.				
15. overview of remainder of resources well organized and informative				
16. big ideas of curriculum project materials brought out in discussion				
17. approaches used by curriculum projects to teach big ideas summarized				
18. discussion of how well project materials meet relevant NCTM curriculum standards (both process & content) given				
Total points earned (50 points maximum):				

Rubric for Presentation Evaluation

Exemplary (A: 47-50 pts)	The presentation was very well done. All of the criteria were met extremely well or most were met well with 2 or 3 criteria attended to adequately.
Very Good (B: 41-46 pts)	The presentation was well done. Most of the criteria were met well with 2 or 3 criteria met adequately and/or minimally.
Acceptable (C: 36-40 pts)	The presentation was adequately done. All or most of the criteria were met adequately with 2 or 3 met minimally and perhaps 1 or 2 met well.
Inadequate/Unacceptable (D/E: ≤ 35 pts)	The presentation was not done well. Most of the criteria were met minimally with 2 or 3 not attended to at all.

"Menu of Problems" ProtoRubric

You are to create one problem for each "entree" on the Menu: an Appetizer, a Main Course, and a Dessert. In addition to the problems, you need to provide an answer key similar to those presented in the *Mathematics Teaching in the Middle School* journal.
The solutions to the Appetizers may be brief and to the point. Those provided for the Main Course and the Dessert need to provide more detail and thought and perhaps provide further questions to be explored.

Criteria	Exemplary	Adequate	Needs Work
Problems created	The problems created meet the level of expectations for each category; they are well written and are not confusing; they are related to rational number concepts; they focus on several key areas of rational number concept development; they have students make connections within mathematics; they promote reasoning. 11-12 pts	The problems created "almost" meet the level of expectations for each category; they are adequately written and not confusing; they are related to rational number concepts; they focus on one or two key areas of rational number concept development; they have students make some connections within mathematics; they promote reasoning. 8-10 pts	The problems created somewhat meet the level of expectations for each category; writing of problems needs work; some problems are confusing to read; they are related to rational number concepts; they focus on only one key area of rational number concept development; they have students make minimal connections within mathematics; they promote some reasoning. 0-6 pts
Mathematics Involved	The rational number concepts and procedures involved in the problems are presented appropriately; problems do not mislead the students mathematically; the mathematical reasoning required becomes more challenging as students progress through the menu; there is a good variety of mathematical procedures utilized in solving the problems. 9-10 pts	The rational number concepts and procedures involved in the problems are presented appropriately; problems do not mislead the students mathematically; the mathematical reasoning required becomes more challenging as students progress through the menu. 6-8 pts	The rational number concepts and procedures involved in the problems are not always presented appropriately; some problems tend to mislead the students mathematically; the mathematical reasoning required does not becomes more challenging as students progress through the menu. 0-5 pts

Presentation	The work is presented in a neat and organized fashion; it is word processed with graphics when appropriate; the writing is clear and concise with minimal (max 2) spelling and grammatical errors; it is submitted on time; this puppy is ready for publication. 7-8 pts	The work is presented in a neat and organized fashion; it is word processed; graphics are done by hand; the writing is clear and concise with few (max 4) spelling and grammatical errors; it is submitted on time (or is a rewrite); this puppy is basically ready but could be tweaked a bit. 5-6.5 pts	Organization and neatness were given only some thought; it is word processed; no graphics are provided; the writing needs work; there are several (>4) spelling and grammatical errors; it is submitted late; the editors don't want to see this one. 0-4 pts
Solution Key	The key is written in a clear and concise manner; the solutions are correct; the solutions become more detailed and thoughtful as one progresses through the menu. 9-10 pts	The key is written in a fairly clear manner; the solutions are correct; the solutions become more detailed as one progresses through the menu. 6-8 pts	The key is not written in a clear and concise manner; the solutions are not always correct; the solutions do not become more detailed and thoughtful as one progresses through the menu. 0-5 pts
Creativity	The problems are essentially original and creative; they are interesting to read and to solve. 5 pts	The problems are modified versions of problems found in other resources; they are presented in an interesting manner. 4 pts	The problems are essentially those found in other resources with minimal modifications; they are very traditional in format; not to exciting to read. 2.5 pts.
In General	The category descriptions are written to give an idea of the expectations for each level. Yet the work will not exactly meet the category descriptions. This is where the evaluator's professional judgment comes into play. Most likely, each work will have a sampling of characteristics from each level. Hence, the work will be placed in the category that most adequately describes it with possibly a modification of points.		

Presentation of Info from M350 Readings ProtoRubric

You are to carefully read and highlight your assigned article/paper and present the important ideas and points made in the article. You will have 5 minutes for your presentation with 2 minutes for questions from your colleagues. You should have copies of your key points available for the class to facilitate your presentation.

Criteria	Exemplary	Adequate	Below Par
Key points	The 5-7 key points/main ideas presented were substantive and central to the author's focus. These points were not sentences or phrases lifted from the article but were ideas taken from the article thoughtfully rewritten by the preservice teacher. Connections to previous readings and/or experiences (in or out of class) were made when appropriate. Handouts provided in class were word-processed and clearly presented key points with minimal (max 2) spelling and grammatical errors. 12 pts	The 5 key points/main ideas presented were central to the author's focus. One or two points were sentences or phrases lifted from the article with the other ideas being thoughtfully rewritten by the preservice teacher. Connections made to previous readings were given but were not always clearly presented. Handouts provided in class were neatly written and clearly presented key points with few (max 3) spelling and grammatical errors. 10 pts	The 5 or fewer key points/main ideas presented were mostly minor and somewhat central to the author's focus. The points focused chiefly on details from the article and were mostly sentences or phrases lifted from the article. Very little thought was put into summarizing and synthesizing the reading. Few connections were made to previous readings or experiences. Those connections that were provided were weak. Handouts were provided but were not written up clearly. 8 pts
Delivery	Presentation was completed within 5 min. The preservice teacher spoke clearly and was audible to all in the room. Responses to questions were done well and professionally. 3 pts	Presentation was completed within 5-7 min. The preservice teacher spoke clearly and was audible most of the time. Responses to questions were adequate but it appeared that not all questions were taken seriously. 2 pts	Presentation took longer than 7 minutes with no time for questioning. It was difficult to understand the preservice teacher at all times. 1 pt
In General	(If there is a mixture of Exemplar and Adequate characteristics, the work will be placed in the category with the most characteristics circled with a modification on points)	(If there is a mixture of Adequate and Below Par characteristics, the work will be placed in the category with the most characteristics circled with a modification on points)	

Math 350 - Lesson Plan Evaluation

The following checklist offers one possible format for evaluation of lesson plans.

Component	Quality of Component 0 => Criteria not met. 1=> Criteria met minimally. 2 => Criteria met adequately. 3 => Criteria met extremely well.	Evaluator Comments
Preliminaries		
Objectives stated clearly in terms of student expectations.	0 1 2 3	
Needed materials listed.	0 1 2 3	
Lesson Procedures		
"Good start" used to motivate lesson.	0 1 2 3	
Guiding/lead questions provided.	0 1 2 3	
Anticipated/desired responses to lead questions included.	0 1 2 3	
Classroom organization noted.	0 1 2 3	
Activities well described with useful directions stated.	0 1 2 3	
Activities are motivating.	0 1 2 3	
Activities involve students in a meaningful way.	0 1 2 3	
Learning is based upon experiences rather than teacher telling.	0 1 2 3	
Emphasis is on classroom discourse rather than teacher talk.	0 1 2 3	
Time frame for activities noted and reasonable.	0 1 2 3	
Assessment procedures well defined and appropriate.	0 1 2 3	
Use of manipulatives and/or calculators or computers is included as appropriate.	0 1 2 3	
Connections to other subjects (science, literature, etc.) have been made whenever possible.	0 1 2 3	
Closure to the lesson is planned.	0 1 2 3	
Post lesson follow-up is provided (in-class or homework, parent involvement, etc.)	0 1 2 3	
Resource list provided.	0 1 2 3	

Appendix B

Sample Materials Used in

Math 351: Computing Technology in Secondary School Mathematics

Math 351
Project 1: The World Wide Web

Most of you have had some experience navigating the World Wide Web (aka The Web). The Web contains an enormous amount of information and it is often difficult to separate useful information from that which is not so useful without spending inordinate amounts of time "surfing" different sites. There are a number of websites focusing on mathematics and mathematics teaching and you should have some familiarity with such sites as a resource for your teaching. The objective of this project is for you to develop such a familiarity.

Attached is a short article from *The Mathematics Teacher* describing the Web and listing some mathematics related sites along with their web addresses. Unfortunately, changes occur so quickly that some of these sites may no longer exist and other, more useful, sites may have been developed. Following the article is a list of several other websites and their addresses. Some of these sites have connections to huge databases of information; others are more specific and focus on a smaller area of emphasis. One particularly useful site, the Forum, maintains many databases. A Forum Quick Reference page, listing all of the areas available at the Forum, is also attached.

For this project, you are to complete the following activities:

- Read the MT article *Navigating the Web.*
- I will assign you a specific website from the list *A Sampling of Web Sites Related to Mathematics & Mathematics Teaching* to investigate using Netscape (software available on many of the computers across campus including most of those in our Macintosh lab). You are to search through the information provided on your assigned site with the intention of determining what that site has to offer a teacher of mathematics. If you so desire, you may work with a partner in preparing and giving your reports. In such arrangements, the pair will be responsible for searching and reporting on two different websites.
- Prepare a short 2-4 page word-processed report detailing your findings and submit that report to me. Copies of your report will be distributed to the class and you will give a short 5-7 minute oral presentation overviewing your findings to the class. Your reports should provide general information concerning the types of information available at the site, some specific examples to elaborate and clarify the general information, and your evaluation of the usefulness of the site for a middle school or secondary school mathematics teacher (that is, where and how could it be used). Your goal is to convince others of the usefulness (or lack of usefulness) of the site for them as a mathematics teacher. Your article can include printed copies of website pages but these should be supported with your interpretive and evaluative comments. It is possible to set up a projection device so that you can connect to the Web during your oral presentation and display pages from your selected website. If you desire to do this, please tell me well in advance of your scheduled presentation.

All written reports will be due on Monday, February 10 and oral reports will be scheduled during class sessions the following two weeks (prior to mid-semester break) with approximately 4 presentations per session.

Math 351
Project 2: Calculator Linking/Recursion

These activities are designed to acquaint you with the linking features of TI calculators and the computer software "TI-Graph Link (82)." Although you will use a TI-82 for all of these activities, similar software and procedures can be used to link TI-83, TI-85, and TI-92 calculator models.

1. Task: Send and receive a program from one calculator to another.

   Connect the cable: The TI-82 LINK port is located at the center of the bottom edge of the calculator. Insert either end of the black link cable into the port very firmly (this cable comes with every calculator). Repeat with the other TI-82.

   Prepare the receiving unit: Press [2nd] [LINK] and highlight RECEIVE (use the arrow keys). Press [ENTER] . (The message "Waiting ..." is displayed.) The receiving calculator must be "waiting" when the sending calculator is given the command to transmit.

   Prepare the sending unit: Press [2nd] [LINK] to display the SEND menu. Press 2 to select : SelectAll– and to display the SELECT screen. Press the down arrow key until the cursor is on the line with any stored information you wish to transmit. Press [ENTER] and a square dot will appear to the left of the item indicating selection for transmission. (Multiple items can be selected and deselected in this way for transmission.)

   Transmit: Highlight the word TRANSMIT at the top of the display (use the arrow keys) on the sending unit. Press [ENTER] to begin transmission. Information on the item(s) transmitted will be displayed on both units.

   (If procedure is unsuccessful, check that cable connections are tight on both units and repeat the above process.)

2. Task: Send/Receive files between calculator memory and computer disk via the Graph Link.

   Procedure: Make certain a gray Graph Link cable is attached to the Macintosh you are using. (This cable and its associated software are purchased separately from the calculator.) Attached the calculator end to the link port in the TI-82. On the Macintosh, find the TI-82 Graph Link program and open it (double click on the icon). Set the Graph Link to receive individual files by pulling down the "Receive" menu and highlighting "to Individual Files." Select the file(s) to be transmitted just as you did in the first activity. Transmit the files from the calculator to the computer. You will be asked to specify where you wish the file stored. Save the file on the computer disk provided you. The program will be given an 82p extension, for example, "FACTOR.82p" for a program called FACTOR in the TI-82 memory.

   Once transfer is completed, delete the program from your calculator's memory. Transmit the program file back to the calculator from the computer via the Graph Link. (Any program with the .82p extension can be downloaded to a TI-82. Just select the "program" option from the "Send" menu and select the file(s) you wish to transmit.)

3. Task: Print a copy of a TI-82 program file on the laser printer.

   Procedure: Pull down the "File" menu and highlight "Print Info" and "Program" in the submenus. You can then select files for printing.

4. Task: Obtain an LCD (liquid crystal display) screen from the calculator and print it on the laser printer.

   Procedure: Graph a function on your TI-82. Pull down the "Receive" menu and select the "Get LCD from TI-82" option. Print the LCD screen.

5. Task: Copy and paste TI-82 screen images into a word processing document and print that document.

   Procedure: Open any word processing software available on the Macintosh you are using. Type your name and any descriptive information at the top of a new document. Move to the Graph Link software. Graph a function and send a screen image of the graph to the computer. Copy this screen image and paste it into your document. Change the TI-82 screen to show the window dimensions (range) for the graph. Send this window image to the computer and copy and paste it into your document. Print the document from the word processing software. You can also save screen images to your 3.5" disk and then retrieve them at a later time.

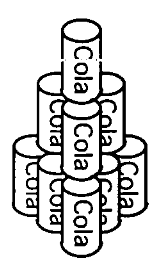
6. The Graph Link can also be used to create, edit, and save program files. This is sometimes more convenient than trying to edit a program on the calculator itself. Note, however, you can not graph functions or run programs using the Graph Link.

7. Submit your work on the following:

   You are to use calculator technology to determine a solution to the aluminum can pyramid problem as shown below. Explain your plan of attack in solving the problem, describe your method(s) of solution, include a statement of the recursive function you create to model this situation, and provide a statement of the problem solution. In formulating your explanations, provide screen images (via the TI-Graph Link (82)) of each of the following calculator windows as appropriate: Y=, WINDOWS, TblSet, TABLE, and GRAPH (with a trace at or near the solution).

   Type up your solutions using any available word processing package in the lab. Your screen images should be pasted into the document (not attached as a separate sheet).

   Aluminum Can Pyramid Problem: During a city-wide cleanup activity one weekend, some students collected litter from city parks and playgrounds and accumulated 3000 aluminum beverage cans. To call attention to their efforts, they decided to use the cans to build a pyramid-shaped tower in the community center lobby (figure below). To build the pyramid they needed to know the number of levels in the largest triangular pyramid they could construct with 3000 cans, and the number of cans in the bottom level. (Note that a 3-level pyramid uses 10 cans: 1 can in the top level, 3 cans in the second level, and 6 cans on the bottom level.) Determine this information.



A 3-level pyramid.

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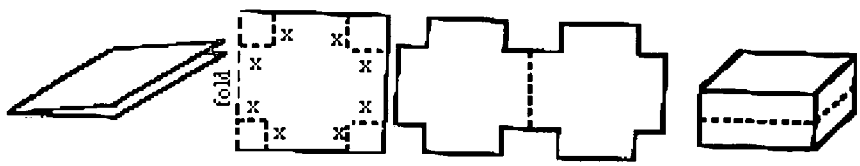
Math 351
Project 4: Developing Mathematical Models

In this project you will collect data for two separate experiments and use curve fitting techniques to develop mathematical models (functions) relating the variables under study. Your write up is to include a brief description of the procedures used, a tabular listing of the data collected, a description of the various mathematical functions you considered as fitting the data, a rationale for your selected "best fit" function, and an interpretation of the reasonableness of the function for modeling the given physical situation.

Experiment 1: All Boxed In

Consider a 20-by-40 inch piece of material. If this material is folded in half and congruent squares are cut from each corner, the resulting shape can be opened and refolded to form a "suitcase". (Refer to the diagram below.)



Begin by constructing a table of ordered pairs for x and V for several of the possible values of x. Use curve fitting techniques to determine a polynomial function that describes the volume V of the suitcase in terms of the length of the cut x used to form the congruent square cutouts.

x								
V								

Once you have determined a polynomial fit for the (x, V) data, use an algebraic analysis of the problem to determine a polynomial relating x and V. How does the "best-fit" polynomial compare with the one found through algebraic analysis?

Experiment 2: EliM&Mination

Count out at least 150 M&M's and place them into a container. (Make certain each of your M&M's has one face with the *m* printed on it. You may eat any "defects.")

Toss (gently pour) the M&M's onto a clean surface and remove the ones that land ***m*** side up. Count how many remain. Record the number of the toss *t* and the number of remaining M&M's *N* in a table.

Repeat the tossing, removing, counting, and recording routine until all of the M&M's are removed or until 8 tosses have been attempted, whichever occurs first.

t	1	2	3	4	5	6	7	8
N								

Enter your data into your calculator and determine a function of "best-fit" describing the relationship between t and N.

Math 351
Project 7: Monte Carlo Simulations

1. When a Monte Carlo simulation is run to estimate the answer to a problem, a random sample from the population of possible outcomes is generated. [For example, assume 10 trials of a simulation for the six-pens collection problem are run and an average of 17 purchased boxes (rounded up) is found. The 17 is one outcome selected from a population that could range from as small as 6 to as large as possible (even though extreme values are very unlikely to occur).] In general, the population of possible outcomes has some distribution that we may not know but would like to approximate. One way to approximate this information is to obtain several sample values and to examine their distribution. The descriptive statistics for the population are approximated by those of the samples collected and, in general, the more samples studied, the closer the approximation will be. (For those with some statistics background, this is an application of the Law of Large Numbers.) An examination of the distribution of sample statistics provides a means to place reliability statements on our estimates. The following problem focuses on this process.

   During the regular 1990-91 NBA season, Jon Paxson of the Chicago Bulls had a field goal shooting average of 0.548 (i.e., if he had attempted 1000 shots, he would have made 548 of them). During the five championship games with the LA Lakers, Paxson made 32 of 49 attempted shots for a championship shooting average of $\frac{32}{49}$ or 0.653. Was Paxson's championship game performance phenomenal, or could one reasonably expect a 0.548 shooter to make 32 of 49 attempted shots?

   Analysis: Define a trial for this situation as one 49 shot "game" in which we count a field goal as made (a basket) if a randomly generated number is less than or equal to 0.548. Take a sample of 99 (the capacity of the TI-82 lists) such games storing the number of baskets made for each game in a list.

   [Note: Begin the program by clearing out the list used to store results from each trial. A program simulating 99 trials will take around 4 minutes to run so debug with a fewer number of trials before attempting to run all 99.]

   Examine the summary statistics of the list and a distribution of the results to complete the following statements:

   a. A 0.548 field goal shooter can be expected to make 32 or more shots out of 49 attempts ____________ % of the time.

   b. 50% of the time, a 0.548 field goal shooter can be expected to make between ____________ and ____________ shots out of 49 attempts. (Center this around the median.)

   c. 90% of the time, a 0.548 field goal shooter can be expected to make between ____________ and ____________ shots out of 49 attempts. (Center this around the median.)

   Provide a listing of your program, the summary statistics for your data, a box-plot of the data (along with the viewing window), and a histograph of the data (along with the viewing window). Answer the three questions and manually mark your histograph to show the "regions of solution" for each.

2. The Monte Carlo procedures you have used to approximate areas of regions bounded by curves can be generalized to approximate volumes of regions enclosed by surfaces. In this 3-dimensional dart tossing experiment, a "dart board" is actually a right rectangular prism surrounding the space whose volume is to be approximated. Develop and carryout a Monte Carlo simulation to approximate the volume of the region bounded by the surface defined by $x^2 + y^2 + z^2 + 8x - 8y - 2z = 16$. (Note: modifying this equation by "completing the square" would be a good first step.) Provide a copy of your program listing, a brief analysis of the program statements, and the results from a sample run.

Sample Exam Questions used in Math 351

These sample items have been selected from more than one exam and are representative of the types of questions ask on midterm and final exams in this course. The actual exams have a fewer number of items. and workspace is provided for students to record their thoughts and problem solving strategies.

General directions to the students: Your responses to items on this exam will be evaluated on the basis of your demonstrating an understanding of key concepts, the details and facts you use to support statements, and the clarity and organization of your written comments.

1. Kenny Pay needs to borrow $5000. He reads that his local credit union is offering 3-year loans at 9% annual interest (payable on the unpaid balance at the beginning of every month). Kenny has sufficient income to make monthly payments of no more than $150? Is the $150/month payment sufficient for repayment of the loan over 36 months? (Note: interest is charged at the 1st of each month beginning one month after the initial loan is made and Kenny would make his monthly payment immediately after this interest was added to the loan balance.)

   You are to solve this problem using the sequence generating (recursion) capabilities of the TI-82. In the space below, provide critical information regarding your entries for the following: the Y= menu, the WINDOW dimensions, the TblSet window, at least two lines from the TABLE window showing the information needed for determining a solution, and a rough sketch of the sequence graph of the loan balance each month over the 3-years of the loan.

   Un= ____________
   Vn= ____________

   WINDOW FORMAT
   UnStart= ______
   VnStart= ______
   nStart= ______
   nMin= ______
   nMax= ______
   Xmin= ______
   Xmax= ______
   Xscl= ______
   Ymin= ______
   Ymax= ______
   Yscl= ______

   TABLE SETUP
   TblMin= ______
   ΔTbl= ______
   Indpnt: Auto Ask
   Depend: Auto Ask

n	Un

   n=

   Explain your reasoning for the selection of the window parameters.

   Explain your solution to the problem using information from the table or graph to support your conclusions.

2. A scatterplot and regression equation are found for the data at the right (you are not required to find either of these). Determine the Xmin and Xmax for a "friendly window" that will allow the TRACE cursor to move on the regression equation at one year intervals. Recall that the TI-82 has 95 columns and 63 rows of pixels.

   Women's 800 m run

Year	Time
1925	144.0
1945	132.0
1965	118.0
1985	113.3

   Xmin = __________ Xmax = __________

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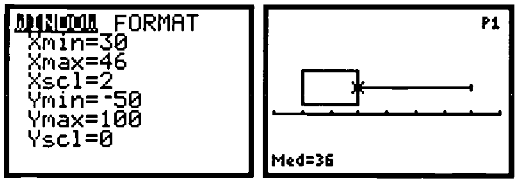
3. Give a set of 10 data values that would produce the given box-and-whisker plot on the selected window:

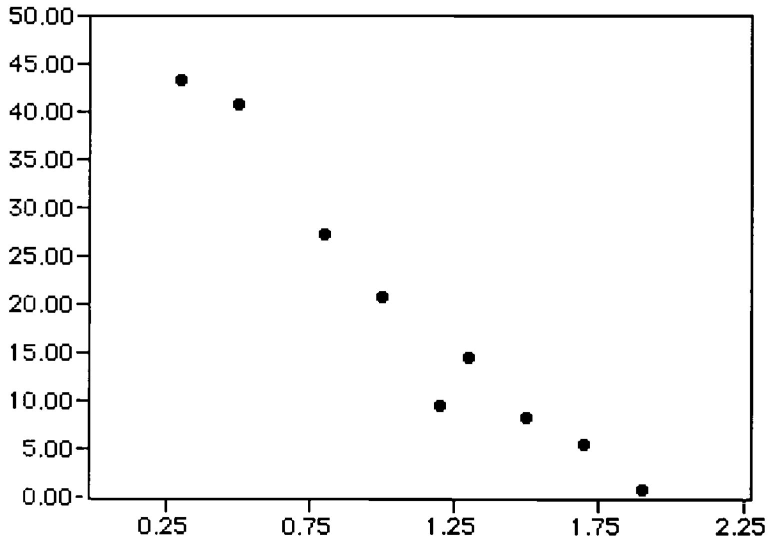


____ , ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____ , ____

4. The data in the table below shows position of an object from a reference point with respect to time.

time (sec)	0.3	0.5	0.8	1.0	1.2	1.3	1.5	1.7	1.9
position (cm)	43.1	40.6	27.2	20.6	9.5	14.3	8.1	5.3	0.6

A scatterplot of the data as produced by the computer software *StatExplorer* is shown below.



Assume that time serves as the independent variable and that position serves as the dependent variable. List the coordinates of the three points used in determining a median-median line of fit for this data by hand (that is, without the use of the TI-82).

(_____, _____)   (_____, _____)   (_____, _____)

Sketch the median-median line of fit on the scatterplot provided. (You are not required to determine the equation of this line nor to use the TI-82 for this task.)

5. The following data set shows the reaction times of a group of eight people administered various amounts of a drug.

person	dosage (mg)	react time (sec)
1	40	0.3
2	52	0.5
3	64	0.8
4	76	0.9
5	82	1.0
6	94	1.1
7	100	1.0
8	112	0.9

a. Enter the data into your calculator and run a linear regression. Write the regression equation below using D for dosage in milligrams and R for reaction time in seconds.

____

b. Is the linear model calculated above a good fit for the data presented or would some other model be more appropriate? Explain your reasons for either acceptance or rejection.

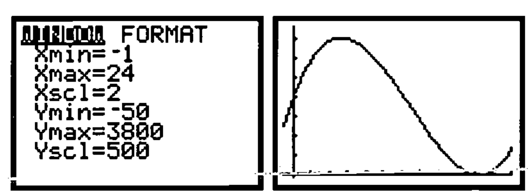
6. If a set of data points (x, y) are related by the exponential function $y = a \cdot b^x$, a logarithmic transformation of the data can be performed to linearize the data (that is, the transformed data will lie on a line).

Apply a logarithmic transformation to the function $y = a \cdot b^x$ and derive the equation of the line fitting the transformed data (show your work). Express the slope of the line and its y-intercept in terms of the parameters a and b found in the original function.

7. A biologist treated several colonies of bacteria with a slow-acting poison designed to kill the bacteria. Over a period of 16 hours, she counted the number of live bacteria at various times. She then used her data to derive the following model for approximating the number of bacteria B(t), measured in thousands, living in the colony t hours after treatment: $B(t) = 2t^3 - 77t^2 + 642t + 1968$.

a. One of your students is trying to obtain a view of the graph of this function that shows the size of the colony over the day following treatment with the poison. He knows that the interval for time should span from 0 to 24 hours but is having difficulty with the appropriate interval to use for representing the colony size. How would you suggest he determine an appropriate Ymin and Ymax for the initial viewing window? Be specific.

b. One pair of your students produces the view of the function shown at right. The students are arguing whether the bacteria colony dies out or not. One says yes because the graph touches the t-axes after several hours. Another says no because the right end of the graph is increasing. Which student, if either, is reasoning correctly? Explain.



c. Reproduce the graph shown above on a TI-82. Using TRACE only, what can be said about the minimum size of the bacteria colony as a result of information collected?

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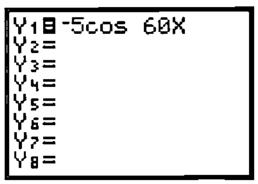
8. The **Trace** feature of an automatic grapher has been used on a view of the graph of a function $y = f(x)$ to obtain the following coordinates read from consecutive pixels (standard forms of the y-coordinates have been provided for ease in interpretation of this information):

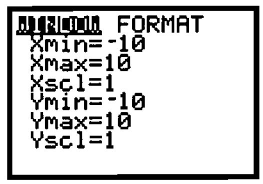
$x = 92.931915$	$y = 5.2804E{-}8$	(0.000000052804)
$x = 92.965957$	$y = 1.02027E{-}6$	(0.00000102027)
$x = 93$	$y = 1.3899E{-}6$	(0.0000013899)
$x = 93.034043$	$y = 6.1401E{-}7$	(0.00000061401)

What information do these pixel coordinates convey about the theoretical value of the x-coordinate of a relative extrema of this function? State your response using as much precision as is justified.

What information do these pixel coordinates convey about the theoretical value of the y-coordinate of a relative extrema of this function? State your response using as much precision as is justified.

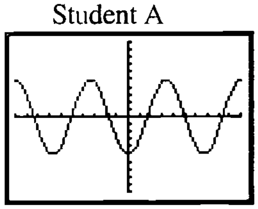
9. Two students studying the graphs of trig functions use separate TI-82's to get a view of the graph of $y = -5\cos 60x$. Each correctly enters the function and sets the window dimensions to the default viewing screen. (See below).



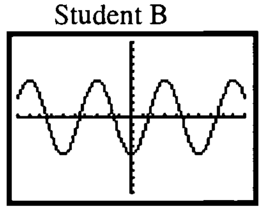


The two students get the following graphs and are concerned about the slight differences in their results. Both graphs seem to indicate a period between 6 and 7 (and 2π is between 6 and 7) and both show an amplitude of 5 (which each student agrees they should).

Student A



Student B



What reasons could account for the different graphs? Are both views "accurate" representations of the function? Comment on their reasoning concerning the period.

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10.Consider the function $y = 5\sqrt{1 + x^3}$

a. Assume that you plan to use the Monte Carlo technique of "dart throwing" to approximate the area of the region bounded by this function, the x-axis, and the vertical lines $x = {}^-1$ and $x = 2$. Use your graphing calculator to obtain a graph of the curve $y = 5\sqrt{1 + x^3}$ containing the x-interval $[{}^-1, 2]$. Draw a sketch of the defined region along with the rectangle you would use as the "dartboard" for the Monte Carlo simulation. Label your sketch so that the dimensions of the dartboard are obvious.

b. Complete the following program so that it can be used to conduct n trials (n input by the program user) of the dart throwing simulation for the function $y = 5\sqrt{1 + x^3}$ over the x-interval $[{}^-1, 2]$. The program should output the approximate area of the region based upon all trials. (You are not expected to enter and run the program.)

```
ClrHome
0→H
Prompt N
For(____________________)
____________________→X
____________________→Y
If____________________
H+1→H
End
__________________→A
Disp "APPROX AREA",A
```

11.The following situation is to be simulated using a Monte Carlo model:

Pat is playing basketball and has just been fouled. She gets two free-throw shots at the foul line. Her past performance indicates that she makes 65% of her free-throw attempts and that consecutive shots are independent of each other. What is the probability that she will make at least one of her shots?

a. Write a program for your calculator to simulate 250 trials of this situation. Have the program output an approximation to the desired probability after every 50 trials of the simulation. Recall that the value of T is a multiple of 50 whenever `T/50=int (T/50)` is a true statement. Write down the lines of your program in the space provided.

b. Enter and run your program. Enter `.5→rand` prior to running your program **or** place this command as the first line of your program.) Record the information output from your program in the following table.

Number of Trials	Approximate Probability
50	
100	
150	
200	
250	

12.A program designed to carry out the bisection algorithm to approximate the zeroes of a function displays the information shown at the right. What information does this convey about the theoretical value of the x-coordinate of a zero of this function? State your response using as much precision as is justified including place-value accuracy and correctness when rounded.

```
ROOT IN INTERVAL:
      3.600265503
      3.600273132
MIDPOINT
      3.600269318
Y1(MIDPOINT)
    -1.40278402E-7
```

13.Consider the system $\begin{cases} f(x) = 3\cos 2x \\ g(x) = (x-4)^4 \end{cases}$, where x is assumed to be in radian measure. Describe how one could use a program designed to carry out the bisection algorithm for approximating a zero of a single function to solve this system. Use such a technique to actually solve the system with accuracy on the x-coordinate of any solution to the nearest millionth. (You should have such a program in your calculator. If not, have me transfer one to you.)

Learning to Teach School Mathematics with Technology (LTSMT)

NSF Proposal ID No.: DUE9652810

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PROJECT ABSTRACT

Learning to Teach School Mathematics with Technology (LTSMT) is a project to develop a combined methods and content course for preservice teachers focusing on the use of computer and calculator technology in secondary school mathematics. Goals of the LTSMT project for the preservice teacher are a) to develop confidence and ability in the use of commonly available mathematics software; b) to build an understanding of the importance of providing accessibility to software for students as a matter of equity, of developing meaningful conceptual understanding, and of promoting mathematical power in all; and c) to develop a reflective attitude that brings about a serious examination, in light of available software, of beliefs about what mathematics is, what topics belong in the curriculum, and what roles the teacher and learner should play in classroom interaction. LTSMT materials will contain a rich mix of mathematics content and methods consistent with the NCTM Standards and other reform recommendations. Six modules organized by content area will involve students in individual and cooperative-group investigations. Students will use a function grapher, a data analysis package, a dynamic geometry program, a programming language, and a symbolic manipulator. In order for LTSMT students to be exposed to the latest in mathematics content and reform-oriented curriculum projects, problems and investigations from recent NSF-funded middle and high school curriculum development projects will be incorporated into the materials, coordinated with a systematic examination of their pedagogical implications. The three-year project will create, pilot, and evaluate these materials, with the goal of publishing the results for use in preservice programs throughout the U.S. The flexible, modular design of the materials will allow teacher educators to customize their use in a variety of course formats, hardware and software configurations, and personal inclinations toward mathematics content.

Appendix C

Sample Materials Used in

Math 450 - The Teaching of Secondary School Mathematics

Generic Rubric for In-Class Writings
Math 450

Exemplary (A) 9.5-10	• Demonstrates a clear understanding of the key issues involved, communicates them in an organized and effective manner, and provides support from relevant literature.
Good (B-BA) 8.5-9	• Demonstrates a clear understanding of the key issues involved and communicates them in an organized and effective manner but does not provide support from relevant literature. • Demonstrates a clear understanding of many of the key issues involved and communicates them in an organized and effective manner with support from relevant literature. • Demonstrates a clear understanding of the key issues involved and communicates them in a coherent manner with support from relevant literature.
Acceptable (C-CB) 7.5-8	• Demonstrates a clear understanding of many of the key issues involved and communicates them in a coherent way but does not provide support from the literature. • Demonstrates an understanding of many of the key issues involved and communicates them in a less-than-coherent manner with support from the literature. • Demonstrates a clear understanding of some of the key issues involved and communicates them in a coherent way with support from the literature.
Substandard (D-DC) 6-7	• Demonstrates a clear understanding of some of the key issues involved and communicates them in a coherent way but does not provide support from the literature • Demonstrates a clear understanding of some of the key issues involved and communicates them in a less-than-coherent manner with support from the literature. • Demonstrates a less-than-clear understanding of many of the key issues and communicates them in a less-than-coherent manner with support from the literature.
Unacceptable (E) 0-5.5	• Demonstrates a less-than-clear understanding of some of the key issues and communicates them in a less-than-coherent manner without providing support from the literature.

Math 450 Readings

Background

Equity

Effective Teaching

Discrete Math

Problem Solving

Technology

Assessment

Cooperative Learning

Writing

Curriculas

Miscellaneous

Homework

Bulletin Boards

References

Professional Development Requirement

Two primary ways for continuing to develop as a professional mathematics teacher are belonging to professional organizations and attending professional conferences. To introduce you to some of the options available for professional development as a mathematics teacher, Math 450 requires the following:

A.Student membership in the National Council of Teachers of Mathematics

For Math 450 the *Mathematics Teacher* is the most relevant journal but if you plan to teach in a middle school or junior high *Mathematics Teaching in the Middle School* is an acceptable alternative. Membership must be applied for **no later than Friday, January 12, 1996**. Make sure that you write down your membership number as soon as you get it, the membership cards can take awhile to arrive. The membership application form follows.

B. Attendance at a mathematics education conference

There are two conferences this semester that are in our geographical area: The University of Michigan *Mathematics Education Conference* in Ann Arbor on Saturday, February 3, 1996 and the Michigan Council of Teachers of Mathematics regional conference *Mathematics in Action* at Grand Valley State University on Friday, February 23, 1996. Both are small conferences that will provide you with the opportunity to network with area teachers (i.e. get a jump on the job search) as well as get some great ideas from sessions. Information and the registration form for the University of Michigan conference follows. Information for the Grand Valley conference should be available soon. It is a smaller conference than Michigan's but also includes lunch in a registration of $10 or less. After attending a conference, collect your thoughts by writing a brief (1-2 page) summary of the sessions you attended. **Turn this summary in within one week of the conference you attend**.

If for some reason (weather, emergency) you are unable to attend either of these conferences, the following alternative assignment will satisfy the conference attendance requirement: Find three teachers who are able to provide you with a Standards-based activity that they have successfully used in their classrooms. Interview each teacher about their activity and the impact it had on their students' learning. Turn in a copy of each activity accompanied by a brief (1-2 page) summary of the corresponding interview.

Take-Home Midterm
Math 450 Winter 1996

For the problem on the back of this page, do the following:

1. Solve the problem and provide a complete but concise write-up of the solution and a scale-model of the dipstick. (45 pts.)
2. Identify and briefly explain as many approaches to solving the problem as possible. (15 pts.)
3. Critique this problem in relation to the NCTM Standards. (40 pts.)

Total: 100 pts.

The midterm may be completed in pairs (2) or individually. If completed in pairs, one paper will be turned in for the pair with both members receiving the same grade.

Due at 2 p.m. on Friday, February 9, 1996.

~Rubrics~

Part 2: Approaches to Solving the Problem

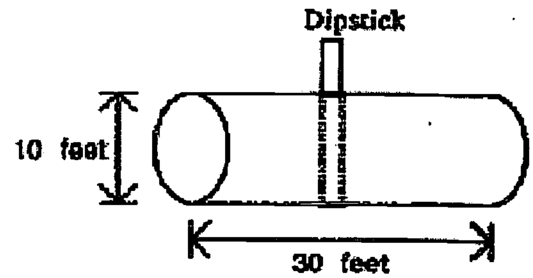
Score	Criteria
14-15	A number of clearly explained approaches
12-13	A few approaches clearly explained or a number of sketchy approaches
10-11	A few sketchy approaches or one strong approach
0-9	Inadequate response to no response at all

Part 3: NCTM Standards Critique of the Problem

Score	Criteria
37-40	All standards addressed in an exceptionally clear and relevant manner
35-36	All standards addressed in a clear and relevant manner
31-34	Most standards addressed in a clear and relevant manner or all standards addressed in a mostly clear and relevant manner
26-30	Some standards addressed in a manner that is less than clear and relevant
24-25	Few standards addressed in a manner that is less than clear and relevant
0-23	Inadequate response to no response at all

Dipstick Problem

Johnie's aunt and uncle, Myrtle and Larry, live in the Upper Peninsula of Michigan. They have a right cylindrical fuel tank buried on their property. Standing upright, the tank has diameter 10 feet and height 30 feet. The tank, however, lays on its side, and thus has length 30 feet and height 10 feet, as shown here.



Last week, Johnie got this letter:

Dear Johnie,

We need your help again! When you're here for Thanksgiving, could you make us a dipstick that we can use to measure the proportion of fuel in that oil tank out back? We want the dipstick to be graduated to show increments of 10% ranging from 0% full to 100% full.

We'd like the dipstick to be 18 feet long. We figured 10 feet for the tank, 5 feet for the ground depth on top of the tank, and an extra a feet sticking up above ground. So maybe before you come up, you could create a model of the part of the dipstick that goes into the tank, and bring it with you. Why don't you use a scale of 1 inch = 1 foot. So, Johnie, please bring a model dipstick 10 inches long, with the graduations we described in the first paragraph.

We know you can do it, Johnie, but to help us understand what you did, on the model dipstick please include the height, to the nearest thousandth of a foot, at which each graduation is marked. Something like this: 40% ⇒ 2.073 feet. (No, this isn't right, just an example Larry suggested!). Thanks - see you soon!

Love, Aunt Myrtle

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