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ABSTRACT

This paper describes mathematics as a social practice and bases this idea on recent developments in diverse domains of research related to mathematical cognition. In the context of theoretical development, this paper is concerned with the question originally raised by Socrates but rephrased from the perspective of social practice theory: What kind of human being do members of a mathematics community value? How do they support the development of that kind of human being in their teaching practice? and How does a learner get socially transformed into that kind of human being when learning mathematics? This study reports on the ethnographic research of communication in a university mathematics department. A case study of a professor who taught mathematics for approximately 30 years at the university is presented. This routines shared among mathematicians in mathematical communication, particularly sequential ordering of a mathematical argument, are investigate. (Contains 47 references.) (ASK)

Communicative Routines in Mathematics Class

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1. Introduction

> "The subjects best calculated to turn about the souls of future rulers toward the true day of being are arithmetic, plane and solid geometry, astronomy, and harmonics, if studies not merely for utilitarian purposes but as compelling the student to pass from sense phenomena to abstract science, and ultimately to "the idea of the good"" (From the Dialogue of Plato).

In this citation from the Republic, Socrates describes his philosophy of education, that is, how to educate a man with a righteous mind, and he chose diverse subjects in mathematics as fundamental to the purpose. So for him, learning mathematics was considered as a process in which a learner is becoming a human being with educated virtues – i.e., a man who is able to see and to practice the good for his people. In this way, Socrates provided a philosophical foundation to introduce mathematics in western school.

However, even though Socrates brought up an important aspect of mathematics education that is often forgotten in mathematics class, it is necessary to note that the notion of "the good" is not absolute but defined in a specific cultural setting. In general, the citation reflects a set of dichotomies such as absolute and context-specific, academic and utilitarian, abstract and sense, and ultimately mind and body. In fact, the mind and body split has constituted the two poles of the twentieth century philosophies, that is, phenomenological voluntarism and structuralist determinism, and learning theories of mathematics are most often the offspring of the marriage between them.

For instance, a learner of mathematics – often called as a mathematical problem solver -- has been characterized as a rational individualistic self-interested being who is seeking for the maximal efficiency. The problem solver is depicted as an active being with full choices in asserting claims, pursuing goals, selecting strategies, and so on. So mathematics learning has been described as a process happening in the mind of a learner isolated from any sociocultural context of a society. However, in spite of the infinite amount of freedom, a learner of mathematics never fails to follow the ONE passage of a mathematical development (Piaget, 1950). Piaget's theory of a mathematical development is a good example. His theory wonderfully explained the process in which a learner's cognitive schema is getting transformed to go through the developmental states from the most primitive to the most abstract and in that way, provided a powerful perspective on how to support a constructive learning. However, his theory does not explain how an individual learner's experience is integrated into that specific kind of a developmental trajectory.

In general, the relationship between a mathematical system and an individual learner is left unexplained in most learning theories of mathematics and the pendulum has been swinging between the two extremes. However, such shadows in our understanding of learning should be cleared because such mystification ultimately becomes a matter of symbolic violence in our lives. If we regard a certain kind of developmental passage in learning mathematics as NATURAL, we will exclude other kinds as improper to be replaced. In fact, a decision-making in problem solving is predetermined by a system of mathematics that a problem solver is engaged with so that the system defines the proper developmental passage of an individual mathematics learner. Thus, the unexplained relationship implicitly assumes the superiority of the system over an individual actor who is ultimately regarded as passively reproducing the system, even when s/he seems to be "actively" involved with the construction of mathematics. This notion of learning mathematics based on the distinction between synchrony and diachrony is what we can find in Platonic Idealism of mathematics. However, as we will discuss later in this paper, it is merely a matter of unexamined faith and scholars in diverse disciplinary areas have developed theoretical perspectives to dissolve the wall between the two spheres and to explain the connection between them.

Another aspect of learning mathematics that this research intends to reconsider is the impact of experience of learning mathematics. For instance, Piaget's theory of mathematical development identifies the developmental stages that a learner proceeds as learning mathematics. Then, what is the meaning of such transgression from the most primitive sensory motor stage to the abstract operation stage? The theory lists a set of skills to characterize each stage in terms of what a kid can do and still cannot do. What is the meaning of the change in the level of mathematical cognition, particularly in relation to "becoming a certain kind of human being"?

In order to consider the questions, I applied social practice theory as an analytic unit. What a social practice theory intends to explain is the change of form and meaning of a sociocultural system by the human actor and vice versa (Ortner, 1984). From the theoretical point of view, the system confines possible conducts by an actor. At the same time, the actor renegotiates the form and the meaning of the system through engagement. A learner as a human actor is engaged with a communal practice as a virtual sociocultural system situated in a specific time and place, and as a consequence of the engagement, both the system and the actor get transformed. Thus, learning goes beyond short-term tactical moves in the vacuum of sociocultural meaning. It is a process of social transformation that is involved with far-reaching impact on a learner's state of being in the world (Giddens, 1979; Lave & Wenger, 1991; Ortner, 1984). From the perspective, this paper is concerned with the description of social transformation in learning mathematics.

2. Learning Mathematics as a Situated Practice

In the western rationalist tradition, mathematics has been considered as a set of a prior, transcendental, universal, abstract and immanent truths. People can reach the realm of the truths through pure reason and it is the reason that characterizes them as human. Thus, mathematics has constituted the core of the rationalist tradition and has been treated as an exceptional kind of knowledge, even when sociology of knowledge has been developing to broaden our notion of knowledge (Mannheim, 1936).

However, this does not mean that mathematics was completely excluded from the efforts. In fact, there have been lots of mathematicians arguing that mathematics is the product of human experience in the world (e.g., Freudental, 1973; 1983; Klein, 1980; Mac Lane, 1981):

> "Mathematics consists in the discovery of successive stages of the formal structures underlying the world and human activities in that world, with emphasis on those structures of broad applicability and those reflecting deeper aspects of the word" (Mac Lane, 1981, p. 471).

Moreover, recently it has been shown that mathematics is not independent of the way that human beings are but embodied. By being embodied, it means that human beings become to have a certain kind of mathematics due to the way our body and brain function and also due to the sociocultural matrix that our experience is organized.

First of all, mathematics is possible because of the way that our body and brain function. Specifically, it has shown that infants are born with primitive arithmetic capabilities. The innate arithmetic capabilities are extended through metaphorical reasoning which is bridging between the domain of concrete sensory motor experience in the physical world and the domain of an abstract mathematical concept. In general, mathematical abilities are based on the same cognitive apparatus – such as image schemas and conceptual metaphor -- used for ordinary everyday thought (Gelman & Meck, 1992; Lakoff & Nunez, 1997; in press).

While the neurobiological and cognitive psychological theories of embodied mathematics explain why mathematics is universal and stable across cultures, ethnographic studies have explained how a socially shared mathematics is getting diversified in a specific context of the practice of mathematics. In the perspective, even though the kind of mathematics is based on the function of human body and brain, a system of mathematics is a specific kind of sociocultural and historical product of a society and an individual actor's mathematical reason is fundamentally formed within the cultural matrix of a practice of mathematics (de la Rocha, 1986; Lave, 1988; Murtaugh, 1985; Nunes, et. als., 1993; Saxe, 1981; 1982; 1985; Schliemann & Acioly, 1989; Scribner, 1986; Vygotsky, 1997).

In this perspective, it is argued that the rationalist tradition in mathematics originated from the value system of ancient Greek society and its practice of mathematics (Kline, 1980; Mac Lane, 1981). In ancient Greece, it was people from the privileged class who were involved with the production of mathematics. They belonged to the leisured class and were not involved themselves in manual labor. The social context of Greece produced a belief system that distinguished mental from manual, that is, mind from body. When the first version of mathematics was produced in ancient Egypt and Babylon, mathematics was purely for practical purposes such as taxation and construction. It was the Greeks who transformed Babylonian mathematics by introducing Greek style of reasoning, i.e., deductive logic. Thus, the transformation reflected and in turn, consolidated the social value system. Furthermore, cognitive scientists argue that

Platonic idealism is a product of the folk theory of kinds and the metaphors of essence that are the product of our brains and that has played a major role in Western philosophy (Lakoff, 1987; Lakoff & Johnson, 1999; Lakoff & Nunez, 1997). Therefore, all these arguments together reveal the self-contradictory aspect of rationalism in mathematics: that is, the embodied disembodiment.

So far, I have discussed that mathematics is a social practice based the recent development in diverse domains of research related to mathematical cognition. In this context of the theoretical development, the purpose of this paper is concerned with the question originally raised by Socrates, but rephrased from the perspective of social practice theory: What kind of human being do members of a mathematics community value? How do they support the development of that kind of human being in their teaching practice? How is a learner getting socially transformed to that kind of human being as learning mathematics?

3. Research Method and Data Sources

In order to describe the social transformation in mathematics education, I conducted an ethnographic research of communication at a university mathematics department in the US. Ethnography of communication is based on the well-know historical debates on the relationship of language to culture and thought, which was framed as "the principle of linguistic relativity" by Whorf -- so often called the Sapir-Whorf hypothesis -- claming:

> "Users of markedly different grammars are pointed by their grammars toward different types of observations and different evaluations of externally similar acts of observation, and hence are not equivalent as observers, but must arrive at somewhat different views of the world" (1956, p. 221).

Whorf shared with his precursors, particularly Sapir and Boas, the awareness that the grammatical structure of a language contains an implicit theory of the structure of the universe held by the group of speakers of the language, which becomes evident when one examines language. However, Whorf advanced the argument as he shed light on the importance of linguistic patterns to reference and meaning. He concluded that large-scale linguistic patterns form the habitual thought characteristic of ordinary speakers of a language, which he defined as "the microcosm that each man carries about within himself, by which he measures and understands what he can of the macrocosm" (Whorf, 1956, p. 147).

Although the Sapir-Whorf hypothesis has been criticized in various ways, the theoretical reconsideration has extended the range of phenomena investigated and reconstituted methodology to approach the relationship of language to culture instead of simply rejecting the principle. Particularly, increasing attention has been given to meaning and discourse, in other words, how interpretive differences are rooted as much in the systematic uses of language within a sociocultural context as in its structure. From the point of view, ethnography of communication is a research methodology to describe the worldview of a cultural group by entering into a web of meaning actualized in authentic discourse by culturally and linguistically competent speakers (Gegeo & Watson-Gegeo, 1999; Gumperz, 1972; Gumperz & Levinson, 1996; Hill & Mannheim, 1992; Hymes, 1966; 1974; 1974; Sherzer, 1987).

In particular, ethnography of communication reflects the tendency in linguistics way from the study of sociocultural form and content as products toward their study as processes. It emphasizes the influence of language on "present-day" members of the society over the content of the language as "historical products." Thus, more emphasis is placed on speech than on code, on function than on structure, on context than on message, and on the appropriate to the arbitrary. However, this does not suggest a dichotomous perspective on function and structure. Rather, communicative form and function have to be studied in integral relation to each other. In other words, instead of making an inference about the behavior of a language speaker based on identified linguistic patterns, a researcher has to go out to the field to discover what the cultural meaning of linguistic patterns is to people speaking the language in their lived world (Hymes, 1972; 1974).

From the perspective, I investigated classroom discourse of an expert mathematics teacher teaching at a university mathematics department in the US. First of all, it is necessary to note that this choice of the research setting does not mean that the university mathematics department is the only place that people practice mathematics. Indeed, ethnographic research has shown that mathematics is so pervasive in human conduct and that each cultural group has developed a kind of mathematics practice meaningful to its cultural organization of social life. However, a university mathematics department was considered as a unique place because people come and practice mathematics. For them, mathematics is what they do everyday and this research is concerned with the question of how mathematics as their daily conduct is related to who they are.

This research was a case study of a mathematics professor who had taught mathematics for about thirty years at a university. He was highly esteemed as a mathematics teacher and researcher by his colleague faculty and students in the mathematics department. I observed his classes including introductory calculus class, upper division level and a graduate level seminar of mathematics during 1998-9 academic year. Several sessions were video recorded and transcribed for detailed discourse analysis.

In addition to class observation, I interviewed 40 mathematicians at the department of mathematics: 5 faculty members including the mathematics professor (4 male and 1 female) and 35 students (25 male and 10 female). The student interviewees include 24 mathematics majors (5 undergraduate students, 1 MA student, and 18 Ph.D. students) and 11 from outside of the mathematics department (6 freshmen, 2 seniors, and 3 Ph.D. students). The demography of the interviewees reflects the ethnical diversity of the university. There were 12 Anglo-Americans, 1 African American, 1 Latin American, 10 Europeans, 9 Asian Americans, 2 Russians, 3 Asians, 1 Brazilian and 1 from India. 5 interviewees were involved with the classes by the mathematics professor (3 GSIs and 2 readers). 25 interviewees participated in at least one of the mathematics classes I observed (7 in the introductory calculus class, 10 in the upper-division course, and 10 in the graduate course). All the interviews were audio recorded and transcribed for analysis.

Often the question is raised for a case study whether a research subject is a good representative of the group under inquiry. In the regard, the interview data were used to cross verify the findings from class observation. One of the purposes of the interview was to probe the notions of mathematics shared among the practitioners of mathematics in the mathematics community. However, from the perspective of social practice theory, every understanding is partial because it reflects a practitioner's specific way to engage with the social practice. So what matters in social practice theory is a meaningful understanding of a social life rather than a universal statement for prediction. In this context, the interview basically became a source of data to contextualize the mathematics professor's practice of mathematics within a broader community of mathematics.

4. Communicative Routines in Mathematical Conversations

As ethnography of communication, this research has particularly focused on communicative routines in mathematics classroom. Communicative routines are defined as "a sequence of utterances or behaviors which is regular and procedural, and communicates as much by its form as by its content" (Watson, 1975, p.53). Recently, mathematics classroom is considered as a community of mathematics where participants negotiate mathematical meanings and norms of doing mathematics and they share a repertoire of communicative routines for the purpose of negotiation (Cobb, Wood, & Yackel, 1996; Voigt, 1985; 1989a;1989b).

As a social reality is not independent of a subject's interpretation, so is a classroom situation. Each participant interprets a situation based on one's own definition of the situation which depends on a subject's previous experience, on the subject's present impressions, and on the subject's expectations. It is natural for a teacher and a student to lead to different interpretations of a situation. Thus, there is a principal tension between their definitions. In that context, routines facilitate mathematical communications among participants by helping them cope with the complexity and the ambiguity of a classroom situation and function to minimize risks by producing mutual presumptions and expectation to support stability of communication (Cobb, Wood, & Yackel, 1996; Voigt, 1985; 1989a;1989b).

Moreover, communicative routines are adapted to symbolize the cultural values attached to a specific context of cultural life. The association between linguistic forms and social meanings is part of a community's rules of speaking. Hence, the analysis of linguistic forms provides clues to understand the cultural meaning underlying the conversations based on the routines (Albert, 1972; Gumperz, 1972; Hymes, 1972; Schegloff, 1972; Watson, 1975).

From the viewpoints, I investigated routines shared among mathematicians in mathematical communication, particularly sequential ordering of a mathematical argument. There observed a routine of a sequencing in the construction of a mathematical argument, as we can see in the following example of a mathematical discourse:[1]

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[1] Symbols for discourse transcription

T	The teacher
Ss	The students in the class
..	A short pause
/	A quick rephrasing
:	Lengthened syllable
\	A falling tone
V	Fall-rise tone
=	No interval between adjacent utterances

T: for that function ((f(x)=1)) what's the derivative\
Ss: zero
T: the derivative is zero
we know that
because the graph of the function is everywhere horizontal
((moves his left hand horizontally))
so the slope is zero
or if we take the rate of change of something that's always equal to 1
it's zero
because it is not changing
((stretches out his right hand))

This short episode of a mathematical communication illustrates the routine to organize a mathematical argument. The conversation was beginning with the professor's question to seek for a fact about the derivative of a constant function, which was immediately followed by students' response. The conversation was completed by confirmation and justification ~~followed the confirmation~~.

This short illustration of the routine suggests that in mathematics class, even when participants were dealing with a well-known mathematical fact, they were responsible for providing a reasonable justification. In fact, analysis of the routine suggests that a proof is an essential part of the communicative routine and furthermore, it is a way to mediate an individual mathematician's imagination to a socially shared mathematical fact. In that regard, the routine is involved with constructing a mathematical fact out of a mathematical idea. Since the above episode is from the beginning part of the lecture where the professor was giving a brief review to move to a new topic, the conversation is rather directly dealing with a fact about the derivative. However, the professor still justified the fact the derivative of the constant function is zero by provoking

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(()) A description of a speaker's gestures mathematical imaginations. For instance, he used two kinds of imaginative gestures. The first gesture moving his hand horizontally connects the slope of the flat line – i.e., zero -- to the derivative of the constant function. The second gesture of stretching out his hand represents the preservation of the characteristic to all x to claim that the derivative function f'(x)=0 for all real number x.

The above short transcription effectively illustrates the routine and the function of the routine to connect between a mathematical imagination to a mathematical fact. However, as noted, since it was from a review part, the participants in the class had already developed a shared perspective enough for an efficient communication. As a result, the routine was used in a very direct and condensed way. Then, how is the routine used differently in a setting that participants communicate a rather noble mathematical idea? In order to answer the question, it is necessary to look at an entire mathematical communication to observe the interaction between a mathematical idea and a mathematical fact in the use of the routine.

1.	T:	I wanna start with two basic functions
2.		first of all the function f(x) equals 1
3.		we can't get much more..
4.		well there is a function f(x)=0
5.		but we can't build much out of that
6.		so I will leave f(x) equals 1
7.		for that function what's the derivative\
8.	Ss:	zero
9.	T:	the derivative is zero
10.		we know that
11.		because the graph of the function is everywhere horizontal
12.		so the slope is zero
13.		or if we take a rate of change of something that's always equal to 1
14.		it's ZERO
15.		because it is not changing.

CAPS Emphatic stress

16. the other basic function is f(x)=x.
17. the graph looks like that
18. ((drawing a straight line passing through the origin of the x-y plane))
19. In this case, what's the derivative\
20. ((Pause to wait for students' responses))
21. derivative i:s
22. Ss: one
23. T: =one
24. so the derivative of this function ((pointing f(x)=x)) i:s
25. this function ((pointing f(x)=1)) up here
26. so so far we gonna start with these two basic functions and now I am
27. gonna tell you some operations on functions and I will explain what
28. happens to the derivative of functions when you apply these operations
29. so the first operation i:s multiplication by a constant
30. we've already seen ((stops talking to write "multiplication by a constant"))
31. we've seen this operation/this is/this is why I am saying that uhm..takes us
32. back to chapter one
33. this corresponds to vertical stretching squeezing flipping\/
34. namely if g(x) is c times f(x)
35. I will try to be fairly consistent in using the letter c to denote a constant
36. or it could/if I explain it more I could mean something else
37. then the derivative of g is c times the derivative of f
38. so this is/this is one of the/this is the first what we call differentiation
39. rules
40. the rule that says if we know how to differentiate some function f..
41. however we know it
42. and if we take/build a new function by multiplying f by a constant
43. that means stretching or squeezing the graph vertically or flipping it over
44. and stretching or squeezing if c is negative
45. then the derivative of g is c times the derivative of f
46. so this is very simple
47. um and..I wanna say something about the differentiation rules in general
48. now
49. there are different approaches to teaching about these rules
50. in certain math courses the approach is to give a proof of these rules
51. and in some/I mean thats the..
52. thats the way mathematicians would like to..do a:ll the time.
53. and the proof/it is important to know that there is a proof of this rule.
54. this is something thats absolute without any qualification
55. if the function f has a derivative..
56. so that would be a little bit of caviar
57. if f has a derivative..at x
58. then the function you've get by multiplying by c also has a derivative
59. and the derivative of g is c times the derivative of f.
60. it's something you can depend on.
61. the proof is not very difficult.

it is based on the theory of limits
but in this course we're taking the attitude that you should know what the
rules are you know why they are reasonable and you should know how to
use them.
and if any of you are interested in seeing the proofs I am happy to show
you or to show you references where they are worked out
so for this/to understand this particular differentiation rule
I want to think about what multiplication by a function might mean and to
see why it is reasonable that this happens to the derivative.
so in this case I wanna think about the derivative is/uhm a rate of
change
so uhm we might/let's think of an example where the gra/where the
function f takes some values
uh let's say here's x and here's f(x)
and we might have the values 1/((erasing 1))let's say 2 the kind of
calculation we did 2.1, 2.01 so on.
so f here I might take the value 5, and then 2.1 may takes the value 5.021
and here it takes the value 5.0201 something like that.
if you are asked to calculate the derivative using these data,
you'll do things like you'll take 5.21 minus 5 and divide it by .1
that'll be .21 divided by .1 which is 2.1
then if you use next pair of data
(pointing 2 and drawing a circle around 2.01)
you compute 5.0201 minus 5
this time you divided by .01
because thats how far you moved in independent variable to get to 2.01
and this ((poiting the numerator of the difference quotient) would give you
2.01
and if things follow this pattern,
you probably conclude that the derivative is equal to 2.
now suppose instead of f(x) I will use the function g(x) which is 3 times
f(x).
so if I use the function three times as big,
then the values up here will be 15, 15.63, 15.0603
and now I start calculating difference quotients
well let's just go on this one here ((pointing 15 and 15.0603))
what's the difference quotient corresponding to these values?
it's 15.0603 minus 15 divided by .01
which is .0603 divided by 0.\01
which is 6.03
and assuming again that this is indicating whats going to happen for
smaller and smaller (difference: inaudible) in x
we will probably conclude that the derivative of this function at x equals
to 2 is equals to 6
which is 3 times the derivative of the original function
so what's REALLY going on here is

that in this numerator here both of the numbers that are being subtracted
are THREE times the numbers that we used up here
(pointing 3 in the numerator of the different quotient of the function g)
and when you subtract two numbers that are three times as large as two
other numbers,
the difference is three times as large.
when you divide by some/the same thing,
the quotients are gonna be three times as large.
so A:LL the difference quotients that approach the derivative are three
times as large when a function is multiplied by 3
and hence the derivative gets three times as large as well.
so that's one way of thinking about what was going on| in terms of
numbers

In this episode, the mathematics professor was teaching a basic rule of differentiation: the constant multiple rule. He began with a brief review of the derivatives of some basic functions: a constant function y=1 and a linear function y=x (Line 1-25). He extended the topic of the mathematical communication by reminding of basic operations to construct complex functions out of the two basic functions, which the class had covered in the beginning of the semester. Then, he raised a question about how to differentiate a function in relation to the basic operations on functions (Line 26-28). Indeed, it is important to note that his way of posing a question reflects his perspective on the topic that he had developed through the engagement with mathematics: i.e., differentiation as a linear operator on the vector field generated by {1, x, x^2, …, x^n}.[2]

In line 29, the professor narrowed the question specific to a differentiation rule with respect to a certain type of operation, that is, multiplication by a constant. He summarized the properties of that operation (Line 30-33) and presented a mathematical sentence: " if g(x)=cf(x) where c is a constant, then g'(x)=cf'(x)" (Line 34-37). He

[2] ^ represents a repeated multiplication. For instance, x^2 is x squared.

declared it as "a rule" (Line 38-39) and interpreted it in connection to the property of the basic operation to give an intuitive graphical understanding of the rule (Line 40-45).

Even though he already said that it is "a rule", he was transferring to prove the rule. However, instead of directly moving to a proof, he negotiated the degree of a rigor rather at length in terms of the tradition of a mathematics community, the goal of the course and the needs of the students in the class (Line 49-60). He ended the negotiation by explicitly stating the meaning of giving a proof:

> "To *understand* this particular rule, I want to think about what multiplication by a function might *mean* and to see why it is *reasonable* that this happens to the derivative."[3]

In this remark, the professor connected "knowing a fact" to "knowing a meaning" to introduce the cultural notion of "knowing" shared among the practitioners of mathematics. In other words, even if the professor presented the rule on the board and even if the participants grasped the mechanical use of the rule, that does not mean that they understand the rule until they see the subtlety behind it. And the function of the routine is to combine the two kinds of knowing to produce a culturally meaningful kind of knowing.

In order to prove the rule, the professor explored the behavior of the rate of change. He drew a two-row table with values of x and of f(x) and computed difference quotients (Line 73-89). Based on the computations, he concluded that the derivative of f at x=2 is 2 by evoking an imagination of the limit of the sequence by saying "if things follow this pattern" (Line 90-91). He added the third row for the function g(x) which was three times f(x) and computed the difference quotient for a pair of points. Because the participants had experienced the imagination of a convergence in term of a tendency, the class rather quickly led to get the derivative of g at x=2. They tried only one pair of numbers and regarded the same tendency. And the assumption is part of the rule which the professor pointed out in the interpretation of the rule (Line 57-58): if f has a derivative at a point, the new function c times f also has a derivative at the point. The class got the derivative of g and the professor decomposed the derivative into 2 and 3 to claim that the derivative preserves the ratio 3 between the two functions f and g (Line 92-106).

In order to explain the meaning of the ratio 3, the professor closely looked at the difference quotient and analyzed where the number 3 came from in the given quotient. He identified the factor 3 in the numerator using the distributive law (Line 107-113). Then he generalized the relationship to all difference quotients based on shared understanding of numbers (Line 111-117) and finally concluded the fact about the derivative of the function g (Line 118-9). He finished his argument with another negotiation concerning the level of rigor.

5. Routines as a Channel of Social Transformation

The above mathematical conversation illustrated a shared communicative routine in a mathematical conversation to develop a mathematical fact: that is, asserting a fact → examining the fact → declaring the fact as a theorem. In the illustration, another purpose

[3] The author added Italic for emphasis

was to observe the interaction between a mathematical fact and a mathematical idea in the process of the social construction of a mathematical fact. In that context, it is important to note that a cultural meaning of a fact is varying through the conversation. In other words, a fact in the earlier part of the communication is closer to a mathematical idea of an individual mathematician and gradually transformed into a socially shared fact, often called a theorem or a rule.

Indeed, in the episode, before the professor presented the rule to the students, he told a story connecting the topic of the conversation to mathematical topics that the class had covered: a story about differentiation as a linear operator on the vector space of polynomials (Line 26-39). In that context of the conversation, the rule had just come out from the professor's imagination, even if the students perceived it as a final product. However, it is necessary to point out that the professor's imagination is not entirely personal. As observed, the professor based his argument on the review of shared mathematical facts. Furthermore, his story about differentiation rules is based on his mathematical insight produced through his engagement with the practice of mathematics. Thus, his/her practice of mathematics is historically contingent to the communal practice of mathematics. In other words, while it reflects his/her creative perspectives on the communal practice, the creativity has developed through the engagement with the social practice of mathematics.

While the routine was a shared way of speaking mathematics, the use of the routine was not identical. For instance, there was a difference in the duration of a switch between a mathematical fact and a mathematical idea in the use of the routine. In the introductory calculus course, the duration was rather short. For instance, in the transcript, right after the professor presented his idea of what to do (Line 26-28), he gave the name of the rule (Line 29). Then he explains the meaning of "multiplication by a constant" intuitively (Line33) as reminding the students that the idea was already established as a shared fact (Line 32) and gave a symbolic representation of the idea (Line 34) and of the rule (Line 37), and so on. In general, the professor switches back and forth to interweave his ideas to shared facts in short terms in the calculus course. However, in the upper division course and the graduate course, the duration – particularly the part of a mathematical idea -- was longer. His story included more of his insights and interpretations, generally speaking his personal understandings of mathematics, developed through his long period engagement with mathematics.

In addition to the difference in duration, there was a difference in the extent of negotiation in the use of the routine. In the calculus class, the professor rather explicitly negotiated with students to transgress to a next step of the routine. For instance, in the transcript, he explained the cultural meaning of a proof (Line 68-70) before he proved the rule, while he moved to a proof without any transition in upper division mathematics classes. In general, the professor was more careful with the routine and tried to be explicit in every step in the calculus class, while it was used in an implicit way in the conversations with old-timers. In fact, in the introductory calculus class, it was often observed that the professor encouraged students to participate in a mathematical communication following the routine. Such activities usually followed question-answer procedure by turn taking. Sometimes the procedure is used to seek for a fact or to check the degree of acquaintance. In the cases, the professor usually provided a justification.

Or he asked back "why?" Again, explicit support was needed in the calculus class, while the routine was spontaneously used in the upper division and graduate classes.

In addition, there was a difference in how students reacted to a mathematical argument following the routine. Old-timers had a shared understanding of the meaning of the routine. For them, the routine is a channel through which a personal mathematical idea is getting socially transformed to a socially shared mathematical fact. Without the process, an idea is still remained as an idea – i.e., a statement lack of socially established reliability and validity. So asking "why?" is the important part of everyday conversations among mathematicians, while it sometimes turns out to be a nuisance to a novice mathematician because "it is obvious." For newcomers, the social transformation through the routine is less obvious. They don't see the subtle change in the social status of a statement. So, for them, sometimes a conversation following the routine looks like a redundant rephrasing (Student interviews).

So far, I ~~have compared the use of the routine in different settings. Particularly,~~ I have compared the use and the interpretations of the communicative routines in different setting of the practice of mathematics. Then, is there any commonality behind the differences? In the comparison, I suggested that the efficiency of the routine was depending on the level of mathematical expertise which is often defined in terms of the amount of mathematical facts possessed. Indeed, students in the calculus course are at the lowest level of mathematical expertise. Even the rules of differentiation, which are basic to the old-timers, are noble to them, in other words, not shared yet. So more of a teacher's utterances are perceived as his/her personal mathematical imagination that requires more effort to make sense than a shared fact does. Because of the lack of a shared ground for a communication, the professor needed to switch more often back and forth between two different domains – i.e., personal vs. shared – for a meaningful communication of mathematics. On the contrary, in the upper division and graduate classes, participants have engaged with the practice of mathematics for a longer period of time and developed more shared understandings. As a consequence, sharing a personal imagination, which is argued as fundamentally social, is much easier because the participants can see the social aspect of the practice of mathematics in the imagination more easily.

However, I have argued that a mathematical fact and a mathematical idea are dialectically constituting one another. Furthermore, when considering mathematics as a social practice, sharing – i.e., a kind of possession but of what is socially relevant -- is more essential than possession. In that light, a mathematical fact is defined as a fact with respect to a practitioner's position in the practice of mathematics. As learners, we are on the edge of the world from our own point of views and the boundary of our world defines what is a fact or an idea. Thus, the boundary between a fact and an idea is temporary and sheer. In that regard, the quantitative definition of mathematical expertise misses the relativity and unanimously imposes a certain group's position on other's practice of mathematics.

The comparison suggests that learning mathematics is concerned with more than acquisition of skills. It is true that a teacher and a student deal with skills in mathematics class and the routine is one of what a mathematician needs to learn. But they are necessary but not sufficient. Then, what is missing when a skill is dealt with as "simply a skill"?

As the profession remarked (Line 68-70), knowing is not a simple possession of a mathematical fact as a thing. In fact, a mathematical fact is not a value-free product. What this technocratic notion of mathematics misses is the fact that even a very trivial techniques such as walking or laughing is the culmination of a historical consciousness of a cultural group. Even if these skills seem to be naturally acquired, they are earned as a consequence of long period discipline which delivers cultural values and norms. Learning mathematics is not exceptional. A novice mathematician practices how to count, how to use algorithms and theorems in problem solving, etc. Through the process, not only the amount of mathematical skills but also the depth of meaning increases.

Moreover, even the exchange of facts is not done in the vacuum of meaning since a fact reflects the cultural and historical consciousness of a community of mathematics. In addition, as a fact is interwoven into cultural communicative routines, it becomes a vehicle to deliver a worldview of a mathematics community. In general, the meaning of mathematics is not discovered at once but is growing up through participating in culturally legitimate activities including communication following the culturally legitimate routine. In that regard, learning mathematics is a process to develop a culturally legitimate kind of competence historically constructed within a community of mathematics.

6. Learning Mathematics as Social Transformation

Mathematics is a social practice of a community of mathematics, even when a mathematician practices mathematics in his/her own office. Problem posing and problem solving require the creativity of a mathematician. But the creative decisions are largely social in a sense that most part of the reason why s/he is doing that problem not any others and of the way to approach the problem are determined by the social system of mathematics. This social aspect of a mathematics practice explains why a learner moves through a linear kind of trajectory in the development of mathematical cognition. Even though an individual practitioner of mathematics tactically creates his/her own space in the practice of mathematics, his/her practice is fundamentally "developing in and through a constant reciprocal appropriation of the objective social practice subjectively" (Hall, 1973).

From the perspective, I identified a routine shared among mathematicians to communicate a mathematical idea, or put differently, a shared communicative to routine for a social construction of a mathematical fact. The routine reflects the psychological process of mathematical reasoning of an individual mathematician. When a mathematician encounters a noble situation, s/he ponders over it and gets an idea which is a seed of a new understanding of the world. However, the psychological process is not entirely subjective but shaped within the matrix of culture of a mathematics community. Specifically, the routine is a sociocultural artifact which embodies the cultural meaning of mathematics as a communal practice, in other words, the shared understanding of what is mathematics about and what is legitimate way to do mathematics with respect to that notion of mathematics. From this point of view, the psychological process based on the routine is the reification of a culturally valued human disposition in a community of mathematics and learning mathematics is a process of social transformation according to the worldview of the community.

When mathematics is understood as a social practice of a certain cultural group, learning mathematics is not merely concerned with a set of knowledgeable skills. As has shown, there is culturally legitimate kind of representation, exploration, topics, questions, etc. These are social in the nature because they are the products of an individual mathematician's creativity which is historically and culturally contingent to the communal practice of mathematics. This implies that through learning mathematics, a person is changing into a certain kind of human being with a certain way of speaking, seeing, in general a certain kind of a worldview. In the beginning, a learner may be involved with minor things such as how to use skills. However, the meaning of a skill is changing through daily engagement and eventually learning is the process to reorganize the way of seeing the world in terms of the skill. Also, for a novice mathematician, the use of the communicative routine may be simply a matter of a style: how to write a proof. However, finally the routine becomes a meaningful way to explore the world. This suggests that a learner is changing to become a certain kind of a person as sharing more cultural meaning of the social practice of mathematics.

If we describe the process of a social transformation from a perspective of a practice community, a learner is moving toward the center of the community from the periphery and his/her position is largely defined by the extent of the sharedness. This model of "Legitimate and Peripheral Participation" (Lave & Wenger, 1991) in mathematics class gives a new analytic perspective to understand difficulties in learning mathematics. Since mathematics has been the product of the western rationalism, failure in mathematics learning has been regarded exclusively a matter of personal inability.

However, based on developed in this paper, a mathematical competence is not personal but cultural, hence, learning difficulty is socially constructed.

Mathematics had been introduced in western schools due to its power to form a person. However, at the turn of the twentieth century, schools were converted to a mainstream social institution for industrial education from humanist liberal education. Since then, mathematics has become a subject of a future productivity (Stanic, 1986). Furthermore, as the economy of industrial society is recently undergoing a transformation, knowledge becomes a new force of production to take the place of traditional capital such as labor and property. In the context of the transformation, mathematics takes the most prestigious position because of its future productivity and the discourse of productivity has involved with creating a technocratic model of school mathematics education (Apple, 1992; Stehr, 1994). In the model, only efficiency is what matters. A teacher's worldview formed through the engagement with mathematics is treated as something personal and gets marginalized in education.

In this context, this research intended to broaden the schooling discourse by closely looking at the educational practice of the practitioners of mathematics. Particularly, this research implies that a mathematics teacher is a representative of a mathematics community. S/he has been transformed according to a worldview of a mathematics community through his/her engagement with the communal practice. As teaching mathematics, s/he delivers the socially and historically constructed way of being in the world and supports a newcomer to be transformed as a practitioner. Therefore, teaching and learning mathematics is involved with developing a cultural competence to bridge the gulf between "the everyday actions of individuals and the historically new form of the societal activity that can be collectively generated as a solution to be double bind potentially embedded in everyday actions (Engestrom, 1987, p. 174). Since difficulty in learning mathematics is socioculturally constructed, a teacher's critical effort to make sense of the culture of mathematics is of essence to support a learner to become a culturally competent member of a mathematics community.

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