DOCUMENT RESUME

ED 443 694 SE 063 846

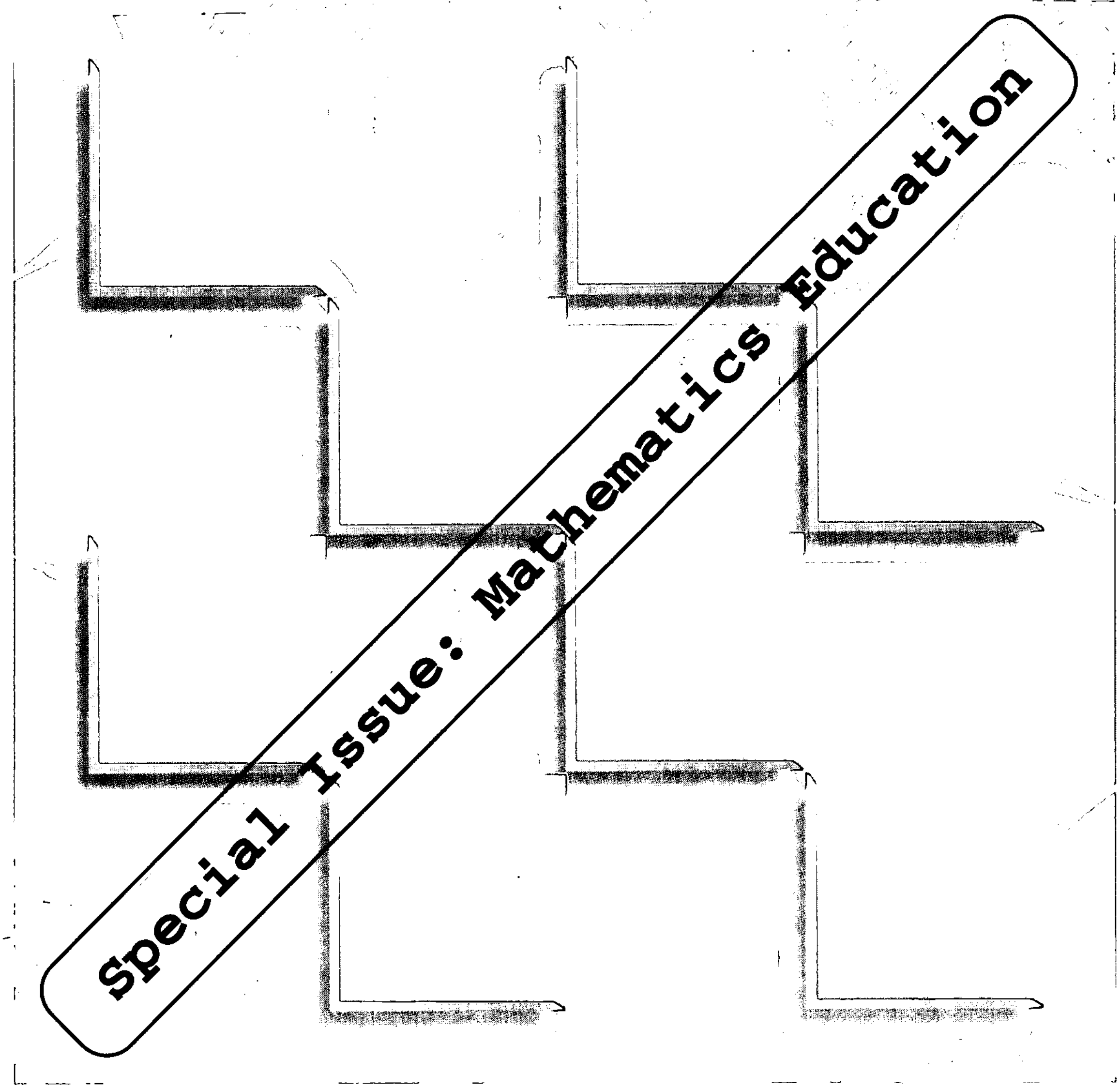
AUTHOR	Langbort, Carol, Ed.; Curtis, Deborah, Ed.
TITLE	Mathematics Education.
INSTITUTION	San Francisco State Univ., CA. School of Education.
PUB DATE	2000-00-00
NOTE	108p.; Published annually.
AVAILABLE FROM	San Francisco State University, College of Education, 1600 Holloway Avenue, San Francisco, CA 94132.
PUB TYPE	Collected Works - Serials (022)
JOURNAL CIT	College of Education Review; v11 spec iss Spr 2000
EDRS PRICE	MF01/PC05 Plus Postage.
DESCRIPTORS	Elementary Education; *Evaluation; Higher Education; *Mathematics Curriculum; *Mathematics Instruction; Middle Schools; *Problem Solving; Teacher Attitudes

ABSTRACT

The focus of this special issue is mathematics education. All articles were written by graduates of the new masters Degree program in which students earn a Master of Arts degree in Education with a concentration in Mathematics Education at San Francisco State University. Articles include: (1) "Developing Teacher-Leaders in a Masters Degree Program in Mathematics Education" (Carol Langbort); (2) "The Impact of a Mathematics Summer Camp Program on Girls' Spatial Visualization Skills and Career Aspirations in Mathematics and Engineering" (Lorraine L. Hayes and Deborah A. Curtis); (3) "The Effects of a Student-Generated Rubric on Third Graders' Math Portfolio Selection Criteria" (Thais Akilah Kagiso); (4) "Observation and Analysis of the Mathematical Problem Solving Behaviors of Six African American Fourth and Fifth Graders" (Norman S. Mattox IV); (5) "Teaching Students To Write about Solving Non-Routine Mathematical Problems" (Delia Levine and Ann Gordon); (6) "The Use of Rubrics by Middle School Students To Score Open-Ended Mathematics Problems" (Ford Long, Jr.); (7) "A Survey of Teacher Beliefs about Pre-Requisite Experiences for Student Success in Algebra" (Audrey Adams); (8) "Fifth Grade Teachers' Attitudes towards Implementing a New Mathematics Curriculum" (Lorene B. Holmes and Deborah A. Curtis); and (9) "Ethnicity as a Factor in Teachers' Perceptions of the Mathematical Competence of Elementary School Students" (Kathlan Latimer). (ASK)

College Of Education *Review*

Volume 11



San Francisco State University
Spring 2000

College Of Education

Review

Volume 11

Special Issue: Mathematics Education

San Francisco State University

Spring 2000

San Francisco State University College of Education *Review*

IN THIS ISSUE . . .

FEATURES

We are pleased to bring you this eleventh issue of the College of Education Review, the official journal of the College of Education at San Francisco State University. As is our custom, the articles in this issue of the Review present a variety of ideas and viewpoints on a topic of interest to the educational community today. Every so often we produce a special issue of the Review. This is one of those special issues. A few years ago, for example, we organized an issue dealing with the topic of science education. In that issue, we discussed a number of state-of-art topics having to do with the teaching of science in elementary and secondary schools. The issue was well-received by members of both the educational and scientific communities, and we received many complements on it.

In this issue, we present a number of articles that deal with the topic of mathematics education. Guest editors Carol Langbort and Deborah Curtis, both professors of education here at San Francisco State University, have compiled a variety of articles in which the authors discuss new and innovative ways to teach mathematics. All of the articles in this issue were written by graduates of San Francisco State's new Master of Art's Degree in Education, with a special concentration in Mathematics Education. This program, under the leadership and direction of Professor Langbort, has been funded since 1993 by a grant from the Eisenhower Mathematics and Science Education State Grant Program. All of the articles are based on work completed by the first cohort of graduates from the program. We think you will find interesting what they have to say, and (we hope) you will gain several new ideas about how to teach mathematics.

We hope that you will be pleased with this issue, and we welcome any reactions you may have to the ideas expressed herein. We will publish your responses in a future issue of the journal.

Lastly, a word of special thanks to my assistant, Michele Larkey, for her very good work and calmness under pressure.

Jack R. Fraenkel
Spring, 2000

Introduction

We are extremely pleased to bring you this special issue of the *College of Education Review,* the official journal of the College of Education at San Francisco State University. The focus of this special issue is Mathematics Education, and all the articles were written by graduates of our new Masters Degree program, in which students earn a Master of Arts degree in Education with a Concentration in Mathematics Education.

The introductory article presents details about the development of this program, which has been funded by the Dwight D. Eisenhower Mathematics and Science Education State Grant Program since 1993. The remaining articles are based on the theses and field studies completed by the first cohort of masters students, who received their degrees in May 1997. There are a variety of topics addressed in these papers, examining both teachers and students in elementary and middle grades. Several articles focus on teachers: teacher beliefs about prerequisites needed for learning algebra, teacher attitudes about implementing new curriculum materials, and teacher perceptions of mathematical competence. Other articles focus on student assessment: third graders developing portfolios, and middle school students using rubrics to score open-ended mathematics problems.

Two papers focus on mathematical problem-solving: one looks at teaching students to write about their mathematical thinking, while the other describes the observation and analysis of African American children's problem-solving behaviors. Lastly, one looks at gender issues, and evaluates the impact of a summer camp math program for girls.

We wish to give special thanks to Linda-Barton White, manager of Academic Programs of the Eisenhower State Grant Program, who had the foresight to fund projects such as this. We also wish to thank the project co-director, professor emeritus José Gutiérrez, and the other faculty who worked with these students during this time: Drs. Barbara Ford, Cecelia Wambach, Anne Gordon and Pamela Tyson, San Francisco State University, and Lise Dworkin and Theresa Hernandez-Heinz from the San Francisco Unified School District. All of these faculty members had enormous impact on the development of the culminating projects of these graduate students--their masters thesis or field study.

We hope that you will be pleased with this special issue. We welcome any reactions you may have to the ideas expressed. This issue is very special to us and we hope you enjoy it as much as we enjoyed putting it together.

Carol Langbort
Deborah Curtis
April 2000

Developing Teacher-Leaders in a Masters Degree Program in Mathematics Education[1]

Carol Langbort

This article describes the development and implementation of this interdepartmental masters degree program in mathematics education for K-8 teachers, beginning with the award of a planning grant in the year 1993. Twenty students graduated in May, 1997, and 19 will graduate in May, 2000. The emphases of the program are described in some detail: increasing the mathematical knowledge of the teachers, discussions of the current issues in mathematics education and the development of teacher-leaders. The development of new courses and descriptions of final projects are also included.

COE *Review*, pp. 5-9

I am a mathematics educator in the Department of Elementary Education at San Francisco State University. Since the 1970s I have been personal friends and colleagues with mathematics educators in the Mathematics Department. Over the years we have worked together in a variety of ways including professional development grants for elementary and middle school teachers and serving in professional organizations. We have been an important means of collegial support for each other. It was no accident then, in 1993, when we received a Request for Proposals (RFP) from the Dwight D. Eisenhower Mathematics and Science Education State Grant Program to develop a master's degree program in mathematics education for minority teachers, that we were ready and willing to take on this task.

José Gutiérrez, a professor in the Mathematics Department, and I welcomed this opportunity to write a planning grant for this purpose. José has spent most of his career teaching mathematics education courses for undergraduates planning to become elementary teachers. For several years, we had been discussing the possibilities for a joint masters degree program, but it was the RFP that gave us the impetus to move forward.

We had been working with K-8 classroom teachers through the San Francisco Math Leadership Project (one of the California Mathematics Projects) since 1984 and knew of the plight of our teachers.

Carol Langbort is a Professor of Elementary Education at San Francisco State University, and Co-Director of the Mathematics Education Masters Degree Project.

[1]This article was originally published in the Fall, 1998 *Newsletter of the Mathematicians and Education Reform Forum.* Reprinted with permission.

They had become interested in mathematics education; they realized that they really could learn and understand mathematics, and indeed, problem solve. Overall, they had a renewed interest and enthusiasm about mathematics learning and teaching. Unfortunately, there was no place for them to go to continue studying in this area. These teachers need to be in an environment where they can work with others, feel free to make mistakes and learn from them; and they need challenging content, which, at the same time, is related to school mathematics. Typically, mathematics departments do not offer the kinds of courses that would be appropriate for these teachers. Another problem is that few courses are offered at hours that classroom teachers can fit into their busy schedules.

Planning and Beginning the Degree Program

We wanted to design a mathematics education degree program would validate the efforts of teachers to improve the mathematics programs in their classrooms and in their schools, and would also give the teachers official recognition of their work in this field. It would encourage them to continue in their studies and add a depth to their knowledge in both the content and the pedagogical issues of the current mathematics reform movement. And so this degree was developed between the Department of Elementary Education and the Mathematics Department with courses from both departments. We utilized some existing courses and designed a few new courses, in particular: *Analyzing Cases of Math Teaching (Ed), Developing Leadership in Mathematics Education (Ed), Assessing Mathematical Thinking (Ed), and Mathematical Investigations: Dissection and Integration of Topics (Math)*. Requirements for the degree would include four courses from the Mathematics Department, four Elementary Education courses, a course in *Research Methods in Education*, and writing a thesis or conducting a field study.

Following the planning grant, we received the full grant to implement this two and one-half year program. As the grant included stipends for teachers, we expected that the recruitment process would be successful. However, we were not prepared for the many, many teachers who attended our information meeting about the program, and had to quickly change the room to an auditorium to hold the more than 160 teachers who attended. We received 80 completed applications for our first cohort, and selected an ethnically diverse group of 30 students. Going through the sequence of courses as a learning community, the teachers provided an enormous amount of support for one another. The first cohort graduated in May, 1997; the second cohort of 19 students will graduate in May, 2000.

Mathematics, Current Educational Issues, and Developing Teacher-Leaders

One focus of this program is to increase the mathematical knowledge of the teachers. There are four courses in the Mathematics Department: *Geometry, Measurement and Probability; Curriculum and Instruction in Mathematics; Computers and Elementary Mathematics*, and *Mathematical Investigations*. Completing these courses gives the teachers the opportunity to apply for a supplementary authorization in mathematics for middle school teaching.

Another focus of the program is current issues in mathematics education. With the many changes occurring in mathematics education, these teachers need to be on the cutting edge of the current reform movement.

The third focus of the program, development of teacher-leaders, is the most unusual aspect of the program. We need more teacher-leaders, and we need more teacher-leaders of color. We've learned that leadership can take a great variety of forms. Presenting workshops at the school, district, state, and national level is one type of leadership. Writing articles and developing curriculum materials is another type of leadership. Planning conferences for teachers and teachers-to-be, taking leadership positions in math organizations, state councils, and statewide committees, and working with preservice teachers are additional examples of roles that our graduates have taken. Through this program, teachers have the opportunity to strengthen their leadership skills in these areas and are encouraged and supported in developing strengths in new areas.

Leadership Development in Mathematics Education

Although the entire program contributes to developing leadership, the course *Leadership Development in Mathematics Education* was designed specifically for the program to focus on leadership issues. It is offered early in the program so that from the beginning the teachers see developing leadership skills as an expectation of the program. The course focuses on three topics: examination of major issues in mathematics education; review and analysis of major documents of the reform movement in mathematics education; and examination of leadership and dissemination practices.

We first focused on issues in mathematics education. Class discussions were built around articles from National Council of Teachers of Mathematics (NCTM) Yearbooks on *Multicultural and Gender Equity in the Mathematics Classroom* (1997) and *Teaching and Learning Mathematics in the 90s* (1990). The teachers then selected issues to pursue in greater depth and were expected to present different perspectives of the issue to the class. The teachers also wrote individual papers on a second issue of their choice.

Although the teachers were familiar with some of the major documents of the reform movement, and even had copies of some documents, they had not really looked at them in depth. Some time was spent examining more closely the *1992 California Mathematics Framework*, NCTMs *Curriculum and Evaluation Standards for School Mathematics* (1989), NCTM's *Professional Standards for Teaching Mathematics* (1991), and *Everybody Counts - A Report to the Nation on the Future of Mathematics Education* (National Research Council, 1989).

We also spent considerable time practicing leadership. Selected articles from the NCTM 1994 Yearbook on *Professional Development for Teachers of Mathematics* were assigned as useful background reading: "Changing Mathematics Teaching Means Changing Ourselves: Implications for Professional Development," by Julian Weissglass and "Ten Key Principles from Research for the Professional Development of Mathematics Teachers," by Doug Clarke. One of the class activities was to develop and get feedback on a workshop to be presented at the teachers' schools. Some students had never before presented a workshop and the support of the group was invaluable. In addition, the grant provided funds for all the teachers to attend the California Math Council's Annual Conference as well as the annual conference of NCTM. We hoped they could envision themselves presenting at these conferences in the future.

Real Life Math Education Activities and Final Project

There was a great deal of mathematics education activity taking place in California during the Fall of 1997, the very semester that the leadership course was taught. The students had an opportunity to hear William Schmidt, Executive Director of the Third International Math and Science Study (TIMSS), talk about the TIMSS results. They also had a chance to attend hearings on the new math standards being developed by the State of California. In fact, a few teachers actually testified at these hearings. For one class assignment the teachers were to submit responses to the draft of the new *Math Framework for California Schools*, which was being developed during the same semester. These real life opportunities for leadership contributed greatly to the teachers' development as teacher-leaders.

The final project, either a field study or thesis, plays an important role in the development of teacher leadership. Several teachers from the first cohort have had opportunities to present their work at local and national conferences. The issues chosen for study by the teachers represent real world questions that have emerged for them in the course of the program, but also reflect their concerns as full-time teachers. A sampling of titles illustrates the variety of topics: *A Second Grade Spanish Curriculum Unit for Developing Mathematical Language; Ethnicity as a Factor in Assigning Mathematical Competence to Elementary Level Students; Teaching Students to Write About Mathematical Problems; Observation and Analysis of 4th Grade Latino Students' Problem Solving Behaviors Using a videotape Protocol; The Development, Implementation and Evaluation of Family Volunteer Workshops that Promote Math Literacy Young Children; Using Hyperstudio as an Assessment Tool; A Survey of Teachers' Beliefs About Pre-Requisite Experiences For Student Success in Algebra.*

Benefits for Faculty

Having a grant allowed us to recruit faculty from outside the university, as well as enlist faculty from several departments. There are a total of nine instructors, including five university professors and four faculty from outside the university, two of whom are from the San Francisco Unified School District. A faculty member teaches courses, works with the teachers on their final projects, or both. The nine faculty members serve on the project Advisory Board as well as attend regular meetings to discuss the program and the progress of the students.

Two weekend retreats helped develop the collegiality among faculty and teachers. The first retreat, held at the start of the program, is an opportunity (1) for participants to get to know each other, (2) to prepare the teachers for the program, and (3) to give introductory sessions to the courses for the following semester. At the second retreat, held at the beginning of the intense work period for the final project, teachers write, meet with faculty, and have an opportunity to develop and expand their ideas with a variety of people.

During the Fall, 1998 semester, we tried something new--three all-day Saturday meetings for the teachers and faculty as a kick-off to the research methods course to be taught in the spring. Teachers presented their beginning ideas for their final projects and the faculty made suggestions and elaborated on the teachers' ideas. The faculty benefited greatly from this exchange.

One of these Saturday meetings was devoted to a discussion of qualitative research methods. The discussion, based on the transcript of an earlier teacher discussion about a case study, allowed the teachers to study themselves as subjects. The interest level was extremely high and both teachers and faculty learned a great deal about the challenges of doing careful qualitative research.

Any undertaking in designing and running a new program yields unexpected outcomes. The benefits to the faculty in this project is one of those pleasant surprises.

The Impact of a Mathematics Summer Camp Program on Girls' Spatial Visualization Skills and Career Aspirations in Mathematics and Engineering

Lorraine L. Hayes
Deborah A. Curtis

The purpose of this study was to design, implement, and evaluate a summer program for sixth and seventh grade girls designed to increase their spatial visualization skills and their interest in engineering careers. A significant change from pretest to posttest was found for one of the two measures of spatial visualization. There was a significant change in the career interests of the girls from pretest to posttest. In particular, at the end of summer camp, nearly half of the girls indicated that they would be interested in engineering as a career. Student evaluations of the effectiveness of the summer program were overwhelmingly favorable. Results are discussed in terms of the value of intensive summer programs for middle school girls.

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By the year 2005, the engineering field will increase by about 200,000 jobs, with civil engineering jobs increasing by 25,000 positions and electrical engineering positions increasing by about 60,000 jobs (Krantz, 1994; U.S. Department of Labor, 1994). Current information indicates that only 17 percent of engineering graduates are women (Basta, 1989). Researchers have found that there are several reasons why girls are not entering into the engineering field. These include (a) lack of spatial visualization skills, and (b) lack of exposure to appropriate female role models. Spatial visualization skills are used when solving three-dimensional problems. Mastery of these skills is needed to be successful in higher level math and science classes leading to engineering degrees (Mistry, Colton, Rossi & Berger, 1994).

One type of spatial visualization skill is the ability to visualize three-dimensional shapes in various positions by rotating the shape or flipping the shape over to see the reflection of that shape as it would be seen in a mirror. An electri-

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cal engineer uses this skill to write commands to a robot so it moves its arm to remove a bomb safely from the ground. Another spatial skill is the ability to see embedded figures in two- dimensional drawings. A civil engineer uses this skill when visualizing the hidden shapes inside a tunnel to decide which direction they should dig to be able to get through a mountain. Studies have found that females do not perform as well as males on these tasks (Gallagher & DeLisi, 1994; Hyde, Fennema & Lamon, 1990).

Some researchers consider spatial visualization skills innate. There are other researchers, however, who have found that training can improve the way students are able to perform spatial visualization tasks. For example, in two studies which required students to build models with blocks and to practice rotations of shapes through varying degrees, students improved their ability to perform spatial tasks (Ferrini-Mundy, 1987; Vandenburg, 1975).

Koballa (1988) and Smith and Erb (1986) have also shown the positive impact that female adult role models can make on math and science course selections of teens. Studies have shown that exposure to female role models in science careers can encourage girls to enter male-dominated careers (see, for example, McNamara & Scherrei, 1982; Swindell & Phelps, 1991). However, there is little evidence that a spatial visualization training program combined with exposure to female role models in math and science will inspire more girls to consider higher level math and science programs and future careers in these fields.

The purpose of this study is to evaluate a summer program for sixth and seventh grade girls which was designed to increase (a) their spatial visualization skills and (b) their interest in math and engineering careers. The first component of the summer program consisted of a collection of activities that taught spatial visualization skills. These activities provided opportunities for the girls to learn how to rotate and reflect shapes by using blocks and computers. There were also games and tasks that helped the students to see hidden geometric figures in a given shape.

The second component consisted of bringing guest speakers to class. These speakers were women, of various ethnicities, who have been working in a variety of engineering fields including civil engineering and electrical engineering.

REVIEW OF THE LITERATURE

The Standards of the National Council of Teachers of Mathematics (NCTM) emphasize the importance of providing equal opportunities for all students to develop math power. This emphasis is summarized in the policy statement that says "Every student can and should learn to reason and solve problems. By every child we mean specifically students who are female as well as those who are male" (NCTM, 1991, pp. 21-22). However, there are a number of studies that have shown that these NCTM standards are often not met for female students. In particular, research has indicated that girls are not given the same opportunities as boys to develop problem solving skills, specifically those skills needed to perform spatial visualization tasks (Bohlin, 1994; Hyde, Fennema, &

Lamon, 1990).

There are a number of issues that need to be addressed in discussing the development of spatial visualization skills. Although some studies have shown that there are differences in the ability of boys and girls to perform spatial visualization tasks (e.g., Johnson and Meade, 1987; Linn & Petersen, 1985), researchers disagree on the age at which these gender differences appear and the reasons for these differences (Johnson & Meade, 1987; Sherman, 1980). Some researchers have argued that the differences are biologically based (see, for example, McGuiness, 1976; Petersen, 1976; Sanders & Soares, 1986). Other researchers have contended that the differences are primarily based on life experiences (Boekaerts, Seegers & Vermeer, 1995; Bohlin, 1994; Hyde, Fennema & Lamon, 1990; Jakobsdottir, Krey and Sales, 1994; Kaiser-Messmer, 1993; Kiesler, Sproull & Eccles, 1983; Serbin, Conner, Burchardt & Citron, 1979; Skolnick, Langbort & Day, 1982).

Having underdeveloped visual spatial skills has been shown to have a negative effect for girls' overall performance on college entrance exams. These exams have historically had spatial skills questions weighted heavily in the math component (Gallagher & DeLisi, 1994; Hyde, Fennema & Lamon, 1990; Mistry, Colton, Rossi, & Berger, 1994). There is likelihood that lower scores of girls on these tests could be keeping girls from entering math/science-related majors. These lower scores could also be limiting the girls' choice of colleges.

Some studies have suggested that if boys and girls are given the same opportunities to learn spatial tasks, the skills can be equally developed (Ferrini-Mundy, 1987; Flores, 1990; Vanderburg, 1975). Research has also indicated that exposure to successful female role models can encourage girls to enroll in male-dominated courses which would then provide opportunities to develop spatial skills (Koballa, 1988; McNamara & Scherrei, 1982; Smith & Erb, 1986; Swindell & Phelps, 1991).

Developmental Differences Between Boys and Girls' Spatial Skills

Using tasks that measured spatial skills, researchers have found that girls and boys develop these spatial abilities differently. For example, Linn and Petersen (1985) reported that spatial perception tasks (horizontality) were more difficult for females than for males. Furthermore, they found that spatial rotation, which is the task of abstractly doing several degrees of shape rotation, was more difficult for females.

Spatial visualization tasks, such as paper folding and finding geometric shapes in a unidimensional representation, were found to be equally difficult for boys and girls. However, researchers have found that as children get older, males perform consistently better than females on spatial visualization tasks (Johnson & Meade, 1987). It has been argued that hormones can explain these developmental differences between the sexes. In particular, McGuiness (1976) argued that spatial skill development is hormonally based, with a positive relationship between spatial visualization skills and puberty or the presence of hormones, specifically, testosterone.

Learning styles have also been discussed as another possible innate factor that may affect spatial visualization skills. It has been reported that males and females come to the classroom with different learning styles, which, in turn, can affect their ability to approach the problem-solving strategies needed in spatial visualization skills. Bohlin (1994), for example, studied differences in styles of learning. In her research, Bohlin classified learners into two types, namely serialistic learners and holistic learners, and found serialistic learners tended to be female, and holistic learners tended to be male. The serialistic learners were found to have "rules without reason," and to prefer problem-solving strategies that used a step-by-step approach. Serialistic learners were found to be uncomfortable with explorative-type problem-solving strategies, which are the type of strategies that are used in spatial skill development. Holistic learners, by contrast, were found to need to see the whole picture before filling in the details. They preferred to explore solutions to problems, such as spatial visualization problems, and did not start with algorithms to try and find the answer. Bohlin stated that this holistic learning style, which was more common for boys, leads to the most success in solving unfamiliar problems.

Effects of Lack of Training and Experience on Girls' Spatial Visualization Skills

Some researchers have argued that the lower level of spatial skills in girls is due to a lack of experience and training. Recreational activities have been shown to limit female exposure to spatial tasks. Serbin, Conner, Burchardt and Citron (1979) found that by age 3, children already influence each other to play with sex-appropriate toys. In addition, parents tended to buy gender-oriented toys. These toys, along with other life experiences at this age, may explain unequal spatial development. The game of pool, for example, provides an excellent opportunity to develop angle and speed concepts, and yet the pool hall is considered "male territory." The present-day pool hall is video arcades. Kiesler, Sproull, and Eccles (1983) did an extensive study on video games. They found that most video games showed a definite gender bias in terms of their content, which, in turn, could adversely effect females who might be interested in trying the games. In particular, the researchers found that the themes of most video games are war, violence, and male sports like baseball, basketball and football. According to these researchers, these games provide excellent opportunities to develop spatial skills as asteroids and battle cruisers, for example, need to be shot down or speed and ball direction, as a second example, need to be assessed to win the games. However, the researchers noted that girls were not seen playing these games, and concluded that this was because they did not include topics of interest to most females.

When children enter school, the educational environment has an impact on spatial skill experiences. Researchers have reported that teachers do not take into account learning styles when providing experiences for developing spatial skills. Bohlin (1994), for example, argued that classroom teachers tend to use the designed lesson, rather than the explorative lesson—meaning that teachers tend to teach the idea and the algorithm, then test what was taught. While this method allows serialistic, often female, problem solvers to perform well on a class math test, it does not provide this same group with the higher level cognitive processing activities needed to develop problem-solving skills used to perform spatial visualization tasks.

This lack of training continued as girls make subject choices in high school. Studies have indicated that more boys take higher level math and science courses. Sherman (1980) did a study which covered grades 8 to 11. She found that even though the math backgrounds and math performances of boys and girls were not different in 8th grade, there was a significant decline in female math performance and female math participation in the 11th grade. The two areas cited by Sherman as the reasons for the change were girls' attitudes and their lower spatial visualization performance.

Hyde, Fennema and Lamon (1990) reported on a study of California high schools which found that girls only made up 38 percent of the physics classes, 34 percent of the advanced physics classes and 42 percent of the chemistry classes. These higher level math and science classes provide the advanced problem-solving and spatial visualization skills needed to take college level math and engineering classes, yet these courses are taken by only a minority of female students.

Effect of Spatial Visualization Training on Future Educational Goals

Studies have shown that the lack of exposure to problem-solving strategies and spatial visualization activities could be affecting the choices that girls have in their future educational goals. Problem solving and spatial skills are the skills needed to score well on the SAT-Math College entrance examinations (Mistry, Colton, Rossi, & Berger, 1994). The scores on these tests could affect a student's major and college choices.

Gallagher and DeLisi (1994) found that overall, girls scored about 50 points lower than boys on the SAT-Mathematics. Girls, however, performed better in the areas of computational problems and algebra. The researchers argued that this was due to the girls' ability to perform better at tasks that use conventional strategies or tasks that can be answered through memorization. However, girls scored significantly lower in word problems and geometry tasks that required problem solving and spatial skills. As Hyde, Fennema and Lamon (1990) reported, these are the skills that have been taught to a minority of girls who enrolled in the higher-level math classes. Hyde, Fennema and Lamon also argued that because the SAT-Math test is used as a criteria for college admission and scholarship grants, girls' lower scores influence their college and career choices as these tests become the filter for math-related fields like engineering and physics.

Effect of Spatial Visualization Training on Spatial Skills

Studies have shown that even if students have had fewer spatial task experiences, they can be trained to develop these spatial skills. Vandenburg (1975) found that sixth grade girls improved their mental rotation skills after receiving spatial training on building models with blocks. Ferrini-Mundy (1987) studied the successful training of high school students. By practicing spatial tasks, these women were able to visualize the rotation of solid objects while doing revolution-type problems, a skill seen as essential to the study of engineering.

<u>Effect of Female Role Models on Engineering Career Choices</u>

Researchers have found that girls have limited their own exposure to higher level math skills, including spatial visualization skills, by not entering the higher level math courses that provided opportunities to develop these skills (Hyde, Fennema, & Lamon, 1990). Kaiser-Messmer studied the reasons that female students do not take these classes. One reason given by the girls in this study was the feeling that there was no need for these courses in their future goals. Other researchers have found that girls recognize the value of these courses when female role models are introduced who showed the girls how they successfully used the skills taught in the higher level math classes (McNamara & Scherrei, 1982; Smith & Erb, 1986; Swindell & Phelps, 1991).

As women enter into male-dominated careers, researchers have found that same-sex role models continued to influence career choices. In a study of 30 geographically and ethnically diverse women pursuing careers in science, mathematics, and engineering (McNamara & Scherrei, 1982), women were interviewed to determine the influential factors that effected their career choices. The study found that as children and teenagers, the women recalled looking to teachers and older female students for encouragement. As they got older, they said they looked for visible female professionals in their field who they could use as models for their careers and personal lives.

<u>The Current Study</u>

For the current study, a summer math camp was developed for sixth and seventh grade girls. The program was based on spatial visualization activities requiring problem-solving strategies. The girls were also given the opportunity to meet and talk to female engineers. It was hypothesized that the spatial visualization training and an exposure to female engineers would lead to increased spatial visualization skills and increased interests in engineering careers .

METHODS

<u>Participants</u>

The study took place in an urban middle school in the San Francisco Bay Area. All the participants were enrolled in the middle school's Chapter One program, which funded the summer camp. To select the participants, six middle-school teachers were asked to identify two groups of girls as candidates for the camp: (a) girls who were considered to be high performing math students, and (b) those ranked as low performing math students. The teachers were instructed to exclude students receiving special services in the school resource programs (i.e. students with IEPs), students who were identified as limited English proficient, and students who were receiving special services in the gifted and talented program. Based on this procedure, 21 girls were admitted to the program, and 15 of these girls completed the program.

In terms of demographics, the girls'

ages ranged from 10 to 12. In terms of ethnicity, over half the group was Latino, and the remainder were from European American, African American, and East Indian backgrounds.

The Summer Math Camp Program

The summer math camp was held at the same middle school that the students attended during the regular school year. The middle school principal and parents of the students gave permission to undertake the study. Testing was conducted in the classroom the first and last days of the camp. The camp was held for one week.

A program was developed that would expose adolescent females to the two areas of concern for this study (a) spatial visualization tasks and (b) engineering career opportunities. Several organizations were contacted in order to find female engineers who would speak to the group. Activities were then developed which could then be incorporated with the speakers in order to develop the spatial visualization skills.

The program was a five-day program meeting from 8:30 a.m. until 12:30 p.m. Each day had a theme. Monday was Exploration Day. In terms of spatial visualization training, students were given two different activities to do: (a) pentomino puzzles and (b) a pentomino computer game. Students were then allowed to freely explore any of the other games that were provided. In terms of career interests, the students revealed their current career aspirations on the pretest questionnaires. They also completed a questionnaire of skills needed to be an engineer using Equal's *Spaces* unit (Fraser, 1982).

Tuesday was Civil Engineering day. A female guest speaker brought slides and discussed her job as a county civil engineer. In terms of spatial visualization training, the students did civil engineering activities by trying to create a cube, first from 3-foot dowels and rubberbands. The students then constructed the cubes from skewers and rubber bands. They ended the day by making accurate representations of their constructions on isometric paper.

Wednesday was Aeronautical Engineering day. The guest speaker was a female electrical engineer who was involved in designing the newest Concorde with NASA. She brought in a sample piece of the wind tunnel and explained how airplanes are able to fly. The students were then given two paper-folding activities: the first was on following directions to create origami structures, and the second one was following computer instructions for creating paper airplanes. The students then had a contest to see how far their planes could fly.

Thursday was Writing Directions Day. The guest speaker was a female electrical engineer who designed robots with Lawrence Livermore Laboratories. She explained that designing robots consisted of writing directions so that a robot could follow them. She brought a robot for the students to use. The robot moved its arm when students gave it directional cues with a remote control. The students were asked to pour water from one cup to another using directional cues. Spatial visualization activities for the day included working with and playing games with the Tangram puzzle.

Friday was Overview Day. The guest speaker was a female quality control engineer from the NUMMI automotive plant. She explained how she found defects in car production and how effective an assembly line could be in production. For further spatial visualization practice, students were asked to make a block building from a picture and then draw the entire structure on isometric paper. The students were also given posttests and questionnaires that were used to evaluate changes in spatial visualization skills, knowledge of engineering as a career and new career interests.

Instruments

Students were given several pretest and posttest assessments. Two different types of spatial visualization tests, which will be described in the next section, were given. The pretest and posttest results were used to evaluate spatial visualization skills. A Career Interest Questionnaire was also given before and after the camp to assess changes in the girls' career choices.

Spatial Visualization Tests

Developmental Test of Visual Motor Integrations (VMI). The students were given two spatial visualization tests on a pretest-posttest basis. The VMI is the Developmental Test of Visual Motor Integrations (Beery and Buktenica, 1989) for ages 3 through 18. This test measures perceptual and motor skills. More specifically, it evaluates the students' ability to look at a picture and then use small motor skills to reproduce the picture accurately. In this test, students are asked to copy 24 geometric shapes into a test booklet. The designs are arranged in order of increasing difficulty. The score is determined by a point system that evaluates the accuracy of the student's reproduction of the design. The number of points attained is then converted using an age and sex norm table to determine the final score.

Spatial Ability Test. The Spatial Ability Test (Barrett & Williams, 1992) is typically given by career counselors to help determine spatial visualization perception skills. This particular test is one component of a battery of tests used to evaluate aptitudes that might indicate success in different careers. The Spatial Ability Test determines the test-taker's ability to visualize a solid three-dimensional object when only given limited two-dimensional information. The seventy-five-point test has nineteen designs, which are analyzed in four ways. Test-takers are asked to examine the original design and then pick out, from a series of four choices, the resulting shapes derived from the original design.

The test consists of two different types of tasks, and each task measures a particular kind of spatial visualization perception skill. In the first type of task, students are asked to evaluate which three-dimensional shape is a result of folding a two-dimensional design. In the second type of task, students are given two shapes and are asked to determine the resulting shape if the smaller piece is removed from the larger shape. Both types of skills are important in applications such as understanding technical drawings and the relationships between objects in space.

Career Interest Questionnaire

For this study, a Career Interest Questionnaire was developed and piloted on 36 males and 52 female

TABLE ONE

Pretest and Posttest Spatial Visualization Test Results (n=15)

	Pretest		Post test		Gain		
Test	Mean	SD	Mean	SD	Mean	SD	T
Spatial Ability	41.33	5.39	49.67	6.16	8.33	5.02	6.42*
VMI	38.33	17.67	45.27	19.67	6.93	13.81	1.94

*Statistically Significant (p<. 05)

students to evaluate its reliability and face validity. After minor revisions, this questionnaire was then given on a pretest and posttest basis. The results were then analyzed in two ways: (a) questions regarding the girls' career interests were compared from pretest to posttest to assess changes in the girls' career aspirations; and (b) open-ended questions regarding knowledge of engineering careers were compared from pretest to posttest.

RESULTS

Spatial Visualization Skills

The spatial visualization tests were analyzed for possible growth in spatial visualization skills. In particular, pretest and posttest means on the two tests were compared using matched pair t-tests at a .05 significant level. As shown on Table One, a quantitative analysis of the two spatial visualization tests gave inconsistent results. The difference between the VMI pretest and posttest between the Spatial Ability Test pretest and post test means was statistically significant ($t(14) = 6.42$, $p < .05$), indicating a significant improvement in spatial visualization perception skills on this test. Students gained an average of 8.33 points on this latter test.

Career Interest Questionnaire

Students were asked the open-ended question, "When you are 30 years old, what type of job do you think that you will have?" on a pretest and post test basis. They were given space to write two possible career choices. The responses were tallied for specific occupational choices. In Table Two, the results are displayed for the percentage of responses falling into "technical and professional" occupation. There was a substantial increase in students choosing engineering careers (+42.9 percent) with a marked decline in students choosing teaching careers (-14.3 percent).

Knowledge about Engineering

At the end of the summer camp, students' were asked, "Did you learn anything about engineering from this program?" Of the fourteen students who responded, all indicated that they had learned something. The students were then asked "What did you learn?" An inductive qualitative analysis was undertaken on students' written responses to this question. The themes which arose with regard to this question are presented in Table Three. (Note that students' original spelling was unchanged for this table). The answers fell into

TABLE TWO

Pretest and Posttest Responses to the Question:
"When you are 30 years old, what type of job do you think you will have?"
(n=30)

Career Choice	Pretest	Posttest	Change
Engineer	0.0%	42.9%	+42.9%
Computer Programmer	0.0%	3.6%	+3.6%
Doctor	14.3%	14.3%	0.0%
Lawyer	14.3%	3.6%	-10.7%
Teacher	17.9	3.6	-14.3

TABLE THREE

Summary of Responses to the Open-Ended Question:
"What did you learn about engineering from the program?"
(n=14)

Theme 1: Engineering as it Relates to Mathematics

- *I learned that you should know your math and you will use it in life and it will be easy if you know it.*
- *I learned that girls can do most anything and that math is important.*
- *That they you need math for all the jobes.*
- *Engineering is cool math.*

Theme 2: Women in Engineering

- *I learned that very fuew wemon ar engineers*
- *I learned that a lot of women are not in engineering.*
- *That there are very few women in engineering.*

Theme 3: Feelings about Engineering as a Career

- *I learned that being an engineer is hard work. But all I need is practice.*
- *I learned that having that job could be fun.*
- *I learned that being an engineer is a lot of fun.*
- *I learned that women have 2 work very hard to be a good engineer because some pig-headed people think that it's a "man's" job. They have 2 work extra harder to be accepted in that type of workforce.*

three thematic areas. In terms of the first theme, several girls indicated that they learned about the connection between engineering and math (N=4 students). An example of this is the response from a student who said "Engineering is cool math." In terms of the second theme, several girls said that they learned that there are very few women in engineering jobs (N = 3 students). For example, one girl said "I learned that very fuew wemon ar (sic) engineers." In terms of the third theme, several girls indicated that they came to recognize the field of engineering as a career choice (N = 4 students). Here, one of the girls said "I learned that having that job could be fun."

DISCUSSION

Overall, the summer camp program was a success both in terms of changes in visual perception skills and changes in girls' career goals. In particular, a statistically significant increase was found in terms of girls' spatial perception skills. Moreover, the female speakers appeared to have a positive influence on the girls, as reflected in their changes in career goals.

Spatial Perception Skills

By providing a variety of activities for all the students, the girls were able to benefit and develop the repertoire of strategies that Linn and Petersen (1985) suggested students need in order to do well on these spatial visualization tasks. The activities were structured in a manner where learning about spatial tasks was acquired through exploration, not just "rules without reason" as described by Bohlin (1994). These experiential activities allowed the students to develop the problem-solving strategies necessary for the spatial visualization test.

Visual Motor Skills

There was no significant change in girls' visual motor skills, as measured by the VMI. This may be because the VMI measures skills that were not emphasized in the summer camp. In particular, because these skills are measured by recreating pictures with a pen, and because no erasing is permitted on this test, an additional factor of fine motor drawing proficiency might have entered into this task. In addition, one week may not have been a sufficient training period for this type of skill. An additional factor that could explain the nonsignificant results could be that for some students, drawing is associated with artistic ability. Thus, if a student has little confidence in her drawing ability, her lack of confidence could negatively effect her performance.

Career Choices

There was a notable change in career choices after the program. The change of career interest to engineering was most likely based on two factors. The first factor was an initial lack of exposure to engineering careers. In the pre-camp questionnaire, students were asked the question "Do you know anyone who has one of these jobs?" Two of the choices were civil engineer and electrical engineer. Only one student responded that she knew a civil engineer (her father). The second factor, which may have affected the career choice changes, was the exposure to female role models in these positions. This exposure prompted students to not only see how women are successful in these careers, but the students were also able to hear how women have developed their math skills in male dominated math/science classes.

Limitations of the Current Study

The current research has several limitations. The findings, therefore, need to be viewed as tentative. First, in terms of internal validity, there was no control group. Therefore, the changes found from pretest to posttest could have been the result of confounding variables such as maturation or testing. Second, in terms of external validity, the research is based upon a small sample size, and the study took place in one particular locale in California. Thus, generalizability is limited.

Conclusion

This study suggests that spatial visualization skills can be developed in a summer math camp setting. These skills, in turn, might then help girls enter higher-level math programs. By providing opportunities to develop these skills, we gave girls one more tool that they can use in their future careers. "How Schools Shortchange Girls," a report by the AAUW (1992), recommended that "girls must be educated and encouraged to understand that mathematics and the sciences are important and relevant to their lives." This program showed the value of girls learning about and speaking with successful female role models in non-traditional female careers like engineering or science. It helped them to see that these careers could be possible for girls to attain.

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The Effects of a Student-Generated Rubric on Third Graders' Math Portfolio Selection Criteria

Thais Akilah Kagiso

The purpose of this study was to test the hypothesis that a student-generated mathematics rubric would help third grade students to apply meaningful, objective criteria when selecting pieces of problem solving work for inclusion in their mathematics portfolios. This 12-week study was conducted in a self-contained classroom. Students were required to provide written justification for their portfolio selections. Student justifications and selections were analyzed before and after the development of the student-generated rubric. The results were generally consistent with the hypothesis. Although the students did not totally abandon their own criteria for evaluating their work, a significant number of the students were able to integrate the standards of the rubric into their selection criteria.

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The aim of this study was to test the hypothesis that a student generated rubric would help students to apply meaningful, objective criteria when selecting pieces of problem solving work for inclusion in their math portfolios. This 12-week study was conducted in a self-contained, third grade classroom. Students were required to provide written justification for their portfolio selections. Student justifications and selections were analyzed before and after the development of the rubric. The results presented here show support for the hypothesis. Although they did not totally abandon their own criteria, a large number of students were able to integrate the standards of the rubric into their repertoire of selection criteria.

One of the objectives of the recent mathematics reform movement is to actively involve students in the assessment of their own learning (National Council of Teachers of Mathematics [NCTM], 1995). By involving students in the assessment process, teachers can assist students in coming to an understanding of the goals of instruction and can learn about the extent to which students are meeting those goals. Some teachers seeking to enter balanced assessment partnerships with their students have involved those students in portfolio assessment systems (Stenmark, 1991). In such partnerships, teachers and students have come together to compile collections of work that provide evidence of students' abilities to think and communicate mathematically. Research indicates that when teachers and students work together in this way, they can both benefit (Kenney & Silver, 1993). The potential benefit to teachers is having an assessment instrument that evaluates performance over time. The potential benefit to students is becoming

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informed evaluators of *their* learning.

Evidence is mounting in support of the value of student self-assessment practices in upper elementary or higher grades (Kenney & Silver, 1993; Warloe, 1993). Research has yet to provide a clear answer as to whether or not such practices can be successfully implemented in primary elementary grades. Questions remain as to whether or not primary grade students are developmentally capable of evaluating their own work. There are indications that the problem may not be in the developmental level of the students, but in the design of the assessment systems (Schunk, 1994; Stipek, 1981). Therefore, the challenge for primary grade classroom teachers is to find developmentally appropriate ways to support their students as they engage in authentic self-assessment activities that yield meaningful results.

The purpose this study was to examine whether modification of the student self-assessment component of a third grade mathematics portfolio system had an effect on the quality of student participation in that system. On the basis of research in support of self-regulated learning, as well as research which raised concerns regarding self-assessment by primary grade students, I hypothesized that in order for third grade students to fully participate in selecting pieces of work for their math portfolios, they would need support in understanding appropriate criteria for such selections. I set out to evaluate whether participation in the development of a scoring rubric, which established standards of performance, would provide the support these third graders needed. If, as indicated by research on self-assessment, students in the third grade are capable of objective self-assessment, I hypothesized that the use of a rubric they helped to develop would provide the students in the study with enough support to make performance standards-based selections for their mathematics portfolios. To that end, the following question was investigated in this study: What effect does the use of a student-generated rubric have on the criteria third grade students use to select pieces of work for their math portfolios? The following sub-questions were considered:

1. In what ways do student portfolio selection comments reflect the standards of the rubric and how do those comments change after introducing the rubric?

2. In what ways do student portfolio selections reflect the standards of the rubric?

3. In what ways are the comments that students make on their work supported by the work itself?

4. In what ways do student comments regarding the purpose of a portfolio change after introducing the rubric?

METHODS

The Sample

This 12-week project was conducted in a self-contained, third grade classroom, at a K-6 year-round elementary school, in a North San Francisco Bay Area school district. The student population of this class was diverse: 47 percent of the students were of European descent, 22 percent were of African descent, 16 percent were of Filipino descent, 9 percent were of Chinese descent, 3 percent were of Middle Eastern descent, and 3 percent were Biracial.

The Design.

Information was collected on one component of the class' mathematics program, namely problem-solving sessions. The design of the study called for students to engage in problem solving sessions once a week. Every third week, students chose one of the three problem solving work products for inclusion in their mathematics portfolio. The resulting four cycles of data collection were referred to as the first, second, third, and fourth data collection periods.

The Rubric

The student-generated rubric was developed during discussion sessions that occurred during the period between the first portfolio selection and second portfolio selections. During these sessions, students participated in discussions which analyzed solutions to problems they had solved during the first three weekly problem solving sessions. During rubric discussion sessions, the students identified elements of student work which provided evidence of successful problem solving. For example, students identified the use of a table as an effective way to organize information. I used the student comments to write a rubric with descriptors of four performance levels. The language of the rubric was designed to be developmentally appropriate.

Instrumentation

Information was collected from students using three instruments, namely (a) a math questionnaire, (b) problem solving worksheets, and (c) portfolio selection comment sheets. The math questionnaire was given to students one week before they engaged in their first problem solving session. The problem solving worksheets were completed by students during each of the 12 weekly problem-solving sessions. (See Appendix 1). Students completed the portfolio selection comment sheets every third week of the study, during portfolio selection sessions. (See Appendix 2). Students completed a post-study math questionnaire, identical to the initial math questionnaire, after the final portfolio selection session.

Portfolio selection comment sheets. The student comments on the portfolio selection comment sheets were analyzed for evidence of the language and standards of the rubric. Student comments were coded in one of three categories: (a) Rubric Only, (b) Non-Rubric Only, and (c) Combination of Rubric and Non-Rubric. Responses in which all comments reflected the language and standards of the rubric were placed in the Rubric Only category. Responses in which all comments had no relation to the language and standards of the rubric were placed in the Non-Rubric Only category. Responses that contained both rubric and non-rubric comments were placed in the Combination category.

Problem solving worksheets. The problem solving worksheets selected by the students for inclusion in their portfolios were analyzed, section by section, for evidence of the standards of the rubric. Part 2 (Descriptions), in which students stated what they thought they needed to do to solve the problem, was analyzed for three rubric standards: (a) Clarity,

(b) Completeness, and (c) Accuracy. A clear description was one in which the student used language understandable to the reader. A complete description was one that included the major elements of the problem. An accurate description was one that recorded the student's correct interpretation of the problem.

Part 3 (Solutions), in which students recorded their solutions to the problem, was analyzed for four rubric standards: (a) Clarity, (b) Correctness, (c) Recordings, and (d) Multiple Representations. Clarity in this section referred to readability, as well as identification of the solution. Correctness referred to the accuracy of the solutions or partial solutions. Recordings referred to methods used to record the solutions such as equations, charts, diagrams, and tables. Multiple representations referred to any attempt to solve a problem in more than one way. This standard did not apply to the many combinations found when solving multiple solution problems.

Part 4 (Justifications), in which students justified their solutions in relation to the elements of the problem, was analyzed for three standards: (a) Clarity, (b) Proof, and (c) Prior Learning. Clarity in this section was demonstrated through the student's use of lucid language to explain his or her thinking. Proof was demonstrated by the use of supportive references to elements of the student's solution. Prior learning was demonstrated by any references to prior problems or mathematical experiences that aided the student in obtaining his or her solution.

Student portfolio selections and portfolio selection comments were analyzed for evidence of consistency between the comments students made about the quality of their portfolio selections and the actual quality of the selected work. This analysis consisted of two categories: (a) Accurately Supported and (b) Inaccurately Supported.

Math questionnaires. The pretest and posttest math questionnaires were analyzed for evidence of change in the understanding of the purposes of math portfolios. This analysis was limited to the responses to two of the four questionnaire items: (a) question 3 (If you had a math portfolio, how would you decide what to put in it?) and (b) question 4 (What could people learn about how you do Math from looking at your Math Portfolio?). Student responses to Questions 3 and 4 were placed in five categories: (a) Evaluative, (b) Process, (c) General Mathematics, (d) Off Topic, and (e) No response/Incomplete.

RESULTS

Before examining the specific results in detail, it is important to address the impact of the vacation break on the work completed during the fourth collection period. Results indicated that there was a significant increase in student selection criteria after the introduction of the rubric (i.e. from the first to the second data collection period). This increase was maintained from the second to the third data collection period. But from the third to the fourth data collection period, there was a clear decline. The time delay between these two periods appeared to have impacted the students' abilities to produce and evaluate the quality of their work. Therefore, the data from the fourth collection period is not included here.

Portfolio Selection Comments

First Selection Comments

Descriptive statistics for the portfolio selection comments are presented in Table One. Students made their first portfolio selections before developing the rubric. None of the students made Rubric-Only comments during this selection session. Non-Rubric Only comments were made by 85 percent of the students. The remaining 15 percent of the students made Combination comments, some of which reflected standards that would appear later in the rubric.

Second Selection Comments

Students made their second portfolio selections the week after developing the rubric. At total of 56 percent of the responses included comments reflecting the standards of that rubric. Rubric-Only comments were made by 30 percent of the students, and Combination comments were made by 26 percent. Less than half the students, 44 percent, responded with Non-rubric only comments.

Third Selection Comments

The third portfolio selection was made three weeks after the second selection. Rubric-Only comments peaked during this selection period, with 41 percent of the student responses fitting into this category. A total of 28 percent of the students made Combination comments.

Change in Portfolio Selection Comments

The data indicate that the development and availability of the rubric did effect the comments that students made when justifying their portfolio selections. The strongest effect was observed during the third selection period, when 69 percent of the students included the standards of the rubric in their selection comments. It is significant to note that the majority of the students did not completely abandon their own selection criteria. On average, approximately 38 percent of the students integrated the rubric standards with their existing selection criteria, whereas approximately 31 percent of the students replaced their own selection criteria with the standards of the rubric. (This 31 percent is equal to the average number of students who did not use the rubric standards in addition to or in place or their own criteria.)

Portfolio Selections

Each student selection was examined for evidence of rubric standards in the work. The purpose of this analysis was to determine whether or not there was any change in the quality of student work produced after the introduction of the rubric. To support the validity of the analysis, a qualified second reader analyzed about 25 percent of the selections. Inter-rater agreement was 91 percent.

The student selections did not show a significant amount of evidence of the work that met the Solutions/Multiple Representations or Explanations/Prior Learning standards. Therefore, the analysis of these two standards is not presented here.

First Portfolio Selections

Descriptive statistics for the portfolio selections are presented in Table Two. The pieces of work selected during this first pre-rubric collection period had the

TABLE ONE Percentage of Students Making Each Type of Selection Comment (Based Upon Student Responses to the Portfolio Selection Comment Sheets)			
	Data Collection Period		
Type of Selection Comment	First	Second	Third
Rubric	0%	30%	41%
Non-Rubric	85%	44%	31%
Combination Rubric/non Rubric	15%	26%	28%

TABLE TWO Rubric Standards Evident in Student Selection Pieces			
	Data Collection Period		
	First	Second	Third
Description Standards (Students state what they think they need to do to solve problem)			
Clarity	52%	70%	67%
Completeness	4%	40%	18%
Accuracy	36%	74%	67%
Solution Standards (Students record their solutions to the problems)			
Clarity	72%	96%	81%
Completeness	56%	89%	52%
Accuracy	76%	100%	96%
Explanation Standards (Students justify their solutions in relation to the elements of the problem)			
Clarity	68%	74%	63%
Proof	72%	56%	33%

lowest correlation of student work and rubric standards. The standard most represented in the work was the Solutions/Recorded standard. Over 75 percent of these selections met this standard.

Second Portfolio Selection

The highest correlation between student work and rubric standards was evident in the pieces of work selected during the second data collection period. Description standards were met in an average of 61 percent of this work. Evidence in support of the Solution standards was found in an average of 95 percent of this work. The Explanation standards were met in an average of 65 percent of the work.

Third Portfolio Selection

There was a decrease in the average number of rubric standards from the second data collection period to the

TABLE THREE Student Portfolio Selection Comments Supported by Evidence in the Selected Pieces of Work			
	Data Collection Period		
	First	Second	Third
Rubric-related comments	12%	64%	70%
a. Accurately supported	12%	51%	54%
b. Inaccurately supported	0%	13%	16%
Non-rubric related comments	88%	36%	30%
Total number of comments	32	55	57

third data collection period. This may be because the rubric development discussion sessions occurred during the time students were solving problems which would be considered for the second portfolio selection. The rubric was available to the student when they worked on the remaining six problems, but it was not discussed after the development period ended.

Student Comments Supported in the Student Work

All rubric-related comments were analyzed for evidence of support in the student work. For example, if a student stated that one of the reasons he or she selected a particular piece of work was the use of a chart, that work was examined for the presence of a chart. Descriptive statistics for this analysis are presented in Table Three.

First Portfolio Selection Comments

The students made a total of 32 selection comments to justify their first portfolio selection pieces. Of the four rubric-related comments made by students during the first selection session, all were supported by evidence in the student work. These 4 comments constituted 12 percent of all selection comments made during the first session.

Second Portfolio Selection Comments

Evidence of support was found for 28 of the 35 rubric-related comments made during the second portfolio selection session. This meant that 51 percent of the selection comments made during this session were accurately supported by the students in relation to the appropriate rubric standard. This was an increase of 31 percent over the first portfolio selection comments. Students attempted to apply the standards of the rubric in an additional 20 percent of the comments made during this selection period. Those comments, however, were not supported in the student work.

Third Portfolio Selection Comments

Students made the most rubric-related comments when justifying their work choices during the third selection session. Fifty-four percent of the comments made in this session accurately identified standards of the rubric present in the work. Only 16 percent of these comments referred to rubric standards that were inaccurately attributed to the work.

Changes in the Math Questionnaire Responses

The changes in the responses to the math questionnaires provided clear evidence of the benefit of involving students in an on-going assessment system. By the end of the study, more than 50 percent of the students were able to identify portfolios as tools for evaluation. Another 20 percent or more knew that portfolios involved some sort of selection, which in this case, was related to math performance.

DISCUSSION AND RECOMMENDATIONS

The results of this study were encouraging. Many students selected quality work for inclusion in their portfolios. However, the students did not consistently apply the appropriate criteria of the rubric to their portfolio selections. In other words, they recognized good work when they saw it, but did not identify what made that work good.

The questions answered and, more importantly, left unanswered by this study point the way to areas of future research. Central to these questions are issues of the language. By language, I mean both the wording of the rubric, and the mode of student expression. It is important to determine what effect the language of the rubric has on the students' willingness and/or ability to incorporate the standards and language of the rubric into their repertoire of selection criteria. It is also important to develop effective ways to support the ability of intermediate grade students to express themselves clearly when writing. A study comparing the oral and written selection comments of students could help determine if it is reasonable to expect third graders to accurately express their selection criteria in written form.

I encourage teachers and researchers to continue to explore ways of making assessment activities meaningful to primary and intermediate grade students. The results of this study indicate that self-assessment, when appropriately supported, is within the cognitive and developmental reach of third grade students.

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APPENDIXES

APPENDIX #1: Problem Solving Worksheet

Problem Solving#: ____________

Name: ______________________________ Date: ______________________

Partners: ____

1. Read the problem.

2. Write what you think you have to do or find out.

3. Solve the problem.

4. Write how you know your answer fits the problem.

APPENDIX #2: Math Portfolio Selection Comment Sheet

Today's Date ______________________________

Dear Teacher,

I have decided to put the following piece of work in my math portfolio.

Date: __________________

Number: ________________

Title ____

I chose this piece because:

____

____

Signed, ________________________

APPENDIX #3: Scoring Rubric

Level 1

A. You are not sure if the student knew anything about the problem because his or her description in #2:
- was not understandable, or
- did not make sense, or
- gave information that had nothing to do with the problem, or
- did not provide any information.

B. You are not sure how the student figured out the problem because in #3 he or she did not show much, or any, of his or her work. The student did not correctly use any of the following:
- tables,
- diagrams,
- lists,
- equations,
- charts,
- pictures,

The student also:
- did not obtain a solution or obtained an incorrect solution,
- did not identify his or her solution

C. The student's explanation in #4 was:
- incomplete or totally missing,
- said that the student didn't know the solution or how to find it,
- had nothing to do with the problem and/or the student's solution.

Level 2

A. You are not sure if the student knew something about the problem because his or her description in #2:
- was confusing, or
- did not make sense, or
- gave some incorrect, or
- did not provide enough information.

B. You are not sure how the student figured out the problem because in #3 he or she did not show much of his or her work. The student did not correctly use one of the following:
- tables,
- diagrams,
- lists,
- equations,
- charts,
- pictures

The student also:
- obtained an incorrect solution,
- did not clearly identify his or her solution

C. You are not sure if the student knew his or her solution was incorrect because in #4 the student's explanation:
- was confusing,
- gave reasons for proof that did not fit the problem and/or the solution.

Level 3

A. You can figure out how the student solved the problem because in #3 he or she showed his or her work. The student may have used one of these ways:
- tables,
- diagrams,
- lists,
- equations,
- charts,
- pictures

The student also:
- obtained a correct solution,
- identified his or her solution

B. You believe that the student knew the solution was correct because in #4 the student's explanation:
- was fairly clear,
- gave some proof from the problem and the solution.

Level 4

A. You can tell that the student knew what the problem was because the description in #2:
- was clear,
- made sense,
- gave the correct information,
- gave all of the information.

B. You can clearly tell out the student solved the problem because in #3 he or she showed all of his or her work. The student may have used one or more of these ways:
- tables,
- diagrams,
- lists,
- equations,
- charts,
- pictures

The student also:
- obtained at least one correct solution,

- identified his or her solution,
- showed his or her solution in more than one way.

C. You know that the student knew why the solution was correct because in #4 the student's explanation:
- was very clear,
- gave proof from the problem and the solution,
- wrote about other problems like this one.

D. You can figure out how the student solved the problem because in #3 he or she showed his or her work. The student may have used one of these ways:
- tables,
- diagrams,
- lists,
- equations,
- charts,
- pictures

The student also:
- obtained a correct solution,
- identified his or her solution,

E. You believe that the student knew the solution was correct because in #4 the student's explanation:
- was fairly clear,
- gave some proof from the problem and the solution.

Observation and Analysis of the Mathematical Solving Behaviors of Six African-American Fourth and Fifth Graders

Norman S. Mattox IV

The purpose of this research was to observe and analyze selected mathematical problem-solving behaviors of six 4th and 5th grade African American students. The behaviors that were observed consisted of mathematical fluency, originality, elegance, communication, collaboration, and persistence. Pairs of African American students were videotaped during two mathematical problem-solving sessions. The performance of the three pairs of students, two high-performing, two average- performing, and two low- performing, was observed and scored by a panel of researchers. A 4-point observational rubric (1 = beginning, 2 = developing, 3 = maturing, and 4 = accomplished) was used to score the pairs of students. The pair of high performing students received scores in the 'accomplished' level of the rubric. The average-performing pair of students received scores in the 'maturing' level of the rubric, while the low performing pair achieved scores in the 'developing' level of the observational rubric.

COE Review, pp. 37-49

Traditionally, African American students have not been the focus of mathematics education research. There is very little research that has been done to examine the mathematical problem solving abilities of African American students at the elementary school level. In the past, most of the emphasis in elementary schools was on mastering basic computational skills. However, in order for students to be successful with the mathematical content of the *Curriculum and Evaluation Standards for School Mathematics*, the National Council of Teachers of Mathematics (NCTM) stated that "problem solving should be the central focus of the mathematics curriculum" (1989, p. 23). Problem solving has become the network and vehicle that helps the learner navigate the various paths to mathematical understanding.

Current research (Kneidek, 1995; Matthews, 1984; Rech, 1994; Tate, 1995) has indicated that many factors have to be considered in order for African American students to be successful in the field of mathematics. The cultural heritage and social structure of the African American child and their family must be taken into consideration. The mathematics teacher must present situations that are relevant to the child's experience in order for the children to be motivated to participate in their own learning. Also, the mathematics teacher

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should pose problems that motivate the African American child to develop and use computational skills in authentic situations (Bell, 1994; Delpit, 1995; Foster, 1994; Hart, 1984; Kunjufu, 1984; Kuykendall, 1989; Shade, 1994; Stiff & Harvey, 1988).

Most mathematics curricula in our public schools has been developed by and for the analytical, logical-mathematical thinker, who is of European-American descent, and who speaks standard English. School mathematics has been described as the expression of how European cultures view the world (Stiff & Harvey, 1988). The learning style practiced by the European and European-American cultures values analytical thinking and systematic approaches to problem solving. These learners tend to be more interested in the details and the understanding of abstract ideas for the sake of having that information accessible to them (Kuykendall, 1989; Stiff & Harvey, 1988).

African-American children bring their own unique and distinctive perspective that is in contrast to the logical-mathematical approach validated in textbooks and most classrooms. Unfortunately, one of the barriers to making mathematical problem solving accessible to the African-American learner is the reliance on textbook word problems with contrived situations that are not relevant to the student's experience (Garaway, 1994; Kunjufu, 1984; Shade, 1994). When the context of the problem situation is familiar to the student, it becomes easier for the student to recognize what the problem is, and identify the necessary mathematical skills that are in use in every day life (Garaway, 1994; Hale-Benson, 1986; Ladson-Billings, 1994).

In addition to the lack of attention paid to cultural issues, efforts in curriculum development have failed to acknowledge the special linguistic differences between standard and nonstandard English. Garaway (1994), for example, considered the issue of language as an influence on mathematical learning. According to Garaway, many African American students need to recognize the differences between standard English and nonstandard English usage in the academic environment.

Garaway further argued that African-American students who are of elementary age also have to translate the mathematical concepts learned in the classroom into terms that are familiar in their experience. He argues that researchers have supported the idea of using the student's own language and the practical problems that come from the student's own society as a foundation for the teaching of culturally relevant mathematics. Because the African-American student is more likely to manage concept development differently than other children, often it is the African-American child's cultural learning style that is being assessed and not his or her mathematical abilities (Bell, 1994; Kuykendall, 1989; Shade, 1994; Stiff & Harvey, 1988).

For the current study, this researcher sought to gain a better understanding of the mathematical abilities of African American students by observing and analyzing the mathematical problem solving behaviors of six African American students enrolled in an urban public elementary school. Specifically, this research attempted to describe the characteristics of fluency, originality, elegance, communication, collaboration, and persistence that are observable during a mathematical problem solving session.

METHODS

The purpose of this research was to observe and analyze selected problem solving behaviors of two high, two average and two low performing fourth and fifth grade African American students. The students worked with a partner at the same ability level. Each pair of students was given two problems to solve. Each problem was to be solved within a one-hour time period. Fifteen minutes of each hour-long videotape were analyzed.

A Description of the Problem Solving Sessions

For each session, the process consisted of administering a warm-up problem, followed by the problem solving session that was videotaped and monitored for the targeted problem solving characteristics, such as fluency, originality, elegance, communication, collaboration and persistence, for each pair of students. The content of these videotaped sessions provided the material that was analyzed using a 4-point observational rubric. In addition to studying these videotapes, each student's teacher was interviewed to understand the student's disposition in his or her regular learning environment.

Analyzing the Problem Solving Sessions

The videotaped problem-solving sessions were observed and analyzed with special attention given to the problem-solving characteristics. The researcher selected a fifteen-minute portion of each videotaped problem that indicated the students' understanding of the mathematical problem and their engagement in the solving of that problem. These portions of the videotaped problem solving sessions were observed and scored by a panel of researchers involved in studying the mathematical problem solving processes of culturally diverse students.

This panel of researchers observed and discussed the level of proficiency of the targeted problem solving characteristics. In particular, the students' work from the problem solving sessions was reviewed and rated by the panel. Based on these ratings, a consensus score was recorded for each characteristic. This was done, not only to neutralize the subjective bias of the researcher, but more importantly, to validate the score that the individual student received for the targeted problem-solving characteristic. One of the problem-solving characteristics, persistence, was rated exclusively by the researcher because of his participation during the entire videotaped problem solving session and access to repeated observations of the videotape. This helped the researcher to identify the nonverbal communications between the partners, non-verbal communications between the individual student and the researcher, and the nonverbal communication with the problem-solving task.

The Participants

The participants were 4th and 5th grade students from a small, ethnically diverse public school in San Francisco. African American and Spanish-speaking students make up the majority of the school population. There were several criteria for student selection: the teacher's assessment of the student's abilities during mathematics activities and instruction; the teacher's experience with the student during mathematics instruction; and the student's practice of mathematical problem solving. For the purposes of this research, mathematical problem solving was defined as the use of mathematical thinking to understand a problem, select an appropriate strategy, apply the strategy to achieve a solution and to reflect on the solution process. Students were rank-ordered according to their problem solving abilities. Specifically, students who exhibited problem-solving abilities in their classroom practice were ranked high on the list. Those students who did not perform well during mathematics instruction while exhibiting problem solving abilities were ranked lower on the list. Those students who did not perform well during mathematics instruction and showed little mathematical problem solving ability were ranked lowest the list. From this list of the fourth and fifth grade African American students, the participants were selected by alternate ranking.

Mathematical Problems to Solve

The two problems that were selected for the students to solve both involved the use of money. These problems were chosen under the assumption that fourth and fifth grade students have a practical familiarity with mathematical concepts using money.

Problem 1. The first problem presented to the students was as follows:

> Slim has $4.00 to spend on hamburgers and colas. Hamburgers cost 80 cents (80¢) apiece. Colas cost 40 cents (40¢) apiece. List all the possible ways Slim could spend his money on hamburgers and colas.

The key mathematical concept to recognize in solving this problem is the two-for-one relationship between the price of a hamburger (80 cents) and the price of a cola (40 cents). In one approach to listing the possible ways that Slim could spend his money, if one hamburger were eliminated, two colas could be substituted in its place. This approach to solving the problem would result in six possible ways for Slim to spend his money on hamburgers and colas. Another approach to listing the possible ways that Slim could spend his money takes into consideration the possibility that Slim did not have to spend all of his money on a combination of hamburgers and colas. If Slim only purchased one hamburger without buying anything else, then his change would have to account for the rest of the four dollars. This approach to solving this problem results in thirty-five possible ways for Slim to spend his four dollars on hamburgers and/or colas.

This first problem that the pair of students were asked to solve involved the search for the possible combinations that would add up to a total amount of four dollars. Of the two problems, this problem was the more traditional money problem which required facility with counting in multiples of four (or forty) and eight (or eighty).

Problem 2. The second problem given to the students consisted of presenting each pair with a pile of coins, and then asking:

> Can you get exactly one dollar's worth of change from the pile by starting at any coin along the edge, moving from coin to touching coin, and ending on another edge coin? No coin may be crossed over more than once.

This second problem challenged the students to follow a path of coins to achieve a total of one dollar within specific guidelines. This problem required the students to stay within these guidelines while keeping track of an accumulating total of coin values.

Pilot Study

In order to develop the parameters for the observation, researcher-student interaction, and videotaping protocols, a pilot problem solving session was conducted. A money problem served as a warm-up problem to the 'actual' problem solving session. During this warm-up, the researcher modeled thinking aloud and sharing one's problem solving processes with a partner. The researcher participated as a partner with each of the three pairs of students during this initial stage of project development. Not only did this give the students a sense of what was expected of them during the problem solving sessions, it also established a level of comfort between the researcher and the students.

After the warm-up problem, another money problem was administered. This second problem served as the 'actual' problem solving session that was video taped. The pair of students was situated at a desk so that they were facing the camera as they worked on the problems. The pair was provided with the problem, written out on one sheet of paper, so they would be encouraged to work together. Both student/participants were given their own pencil to provide equal access to recording during the problem solving session. The students were given materials that would help them solve the problem. These materials consisted of coins, scrap paper, and calculators. In the case of the second problem, different-colored markers were provided to differentiate 'solution' paths. With the aid of these resources, the student pairs were expected to share their strategies and ideas, both verbally and mathematically. These pilot sessions were instrumental in developing the parameters for researcher-student interactions. The pilot sessions also provided information on what types of communication dynamics could be expected between the pair of students.

The Researcher's Role During the Problem Solving Sessions

The teaching actions for problem solving (Charles, Lester, & O'Daffer, 1987) were used as a guide for interactions between the researcher and the students. Before proceeding with the videotaped session, the researcher and the students read the problem aloud and discussed it to clarify any necessary words or phrases. Once the problem situation was understood, the students were encouraged to proceed on their own, and the videotaped observation began.

The researcher intervened during the problem solving session only when either of the students had a question that would assist them both in understanding the problem situation more clearly. If the students proceeded in their problem solving paths for a period of time without speaking with one another, the researcher reminded the students to "talk about what you are doing now." If one of the students was proceeding along a solution path without acknowledging his or her partner, the researcher reminded the students to, "solve it together." If the researcher saw that there were repeated solutions, then the students were advised of the repetitions. If both of the partners concluded that they had found all of the possibilities for a particular solution, and there were, in fact, more possibilities, then the researcher would say, "there are more." These interactions occurred during the problem solving session.

When the students believed that they had encountered all the possible solutions and would not proceed any further, the video camera was turned off and the 'after' problem solving interactions started. This was a period when the researcher became the mathematical problem-solving instructor. The students were encouraged to keep track of their work. The student pairs received direct instruction in how to organize their work so that they might be able to keep better track of their work, check over their work and to recognize patterns that would help lead to a solution.

The Observational Rubric

The content of the observational rubric evolved from a synthesis of descriptions found in several sources that evaluated mathematical problem solving habits and mathematical communication skills. The language used to describe the problem solving behaviors and the communication skills was developed from (a) a problem solving assessment that was prepared by Sawada (1996), (b) a rubric that is part of a mathematics assessment used by the California Department of Education, and (c) a mathematics problem solving scoring guide used by the Oregon Department of Education.

A description of the targeted problem solving characteristics was developed from these various sources and from discussions with other colleagues interested in mathematical problem solving. Consider these characteristics in more detail. *Fluency* is concerned with the level of understanding of mathematical concepts and applying that understanding to the context of the mathematical problem. It also involves the ability to internalize this understanding, and then engage in productive problem solving. *Originality* is the development of a unique idea, or, making an insightful observation about the mathematical task at hand. *Elegance* is concerned with the expression of the individual's thinking in mathematical terms. *Elegance* is also concerned with the converse of this relationship. Expressing mathematical concepts as metaphors related to the individual's experience is also considered elegance. *Communication* is the individual's ability to convey mathematical thinking and reasoning through written and spoken words, and actions. *Collaboration* is the willingness of the individual to share and accept ideas and materials with a partner. Persistence is the ability to continue to focus on a problem in order to find a solution and the willingness to pursue other solution paths, even when the process becomes stalled.

Each of the targeted problem solving characteristics, namely fluency, originality, elegance, communication, collaboration and persistence, were rated on a 4-point scale, as follows: (1) Beginning; (2) Developing; (3) Maturing; (4) Accomplished.

The researcher selected a 15-minute portion of the videotaped problem solving session to be viewed by the observers that indicated the students' understanding and engagement of the problem. During this period, the observers monitored and scored the pair of students. The rationale for each observer's scoring was discussed based on the observational rubric. The scores that the observers gave to each pair of students were recorded.

After the observation period was over, the observers discussed how they scored each of the characteristics. This discussion helped the researcher to calculate the consensus score that would be recorded. This consensus scoring was an average of the scores that were discussed among the observers.

RESULTS AND DISCUSSION

The results of the analysis of the videotape using the observational rubric were summarized for each of the problem solving behaviors for each of the two problems. These data, presented in Table One, indicate the consensus score for each of the students. Recall that the pairs of students were designated as a high-performing pair, average performing, and low performing.

During the search for solutions in both problem situations, the pair of high performing students—Kev and Joan--exhibited an 'accomplished' level of proficiency in the communication of their mathematical thinking and in their collaboration with each other. There was frequent interaction between the students at this level. They spoke with each other in unfinished phrases that sounded like the audible portions of a mental dialogue between the two.

Because of Joan's persistence, the pair of high performing students were able to find all the possible solutions to the first problem. Kev was unsure if there were anymore possibilities when they found the penultimate solution to the first problem. In the second problem, when there was a growing sense of frustration in both students, Joan seemed to be the cheerleader for both partners. She reminded Kev of what they both answered in their student attitude surveys about not 'giving up.' "Try until we die, 'member?" was her motivating statement to Kev.

The pair of average performing students--Lenard and Ray--were proficient in that they both received consensus scores in the 'developing' and 'maturing' levels of the observational rubric. During the first problem solving session both partners chose to use coins to help them list the possible ways of spending the four dollars. It was Lenard's decision to list ways that included getting change back, instead of spending the whole four dollars. This approach was not organ-

TABLE ONE
Student Performance on the Observational Rubric for Problems 1 and 2

	Low Performing		Average Performing		High Performing	
	El	Barry	Lenard	Ray	Kev	Joan
Scores for Problem #1						
Fluency	3	3	3	3	4	4
Originality	2	2	3	2	3	3
Elegance	2	2	3	2	4	4
Communication	3	2	2	3	4	4
Collaboration	3	2	2	3	4	4
Persistence	3	2	3	2	3	4
Scores for Problem #2						
Fluency	2	3	3	3	4	3
Originality	2	3	3	2	3	4
Elegance	2	3	3	3	3	4
Communication	2	3	3	2	3	4
Collaboration	2	3	3	3	4	4
Persistence	2	3	3	3	3	4

ized, so it did not facilitate finding patterns. While Lenard did not communicate his random selection of possible solutions, Ray spent most of the time looking over Lenard's shoulder and moving coins that he assumed Lenard was counting. This caused confusion between the two. A lot of time was spent comparing the quantity of the other's pile of coins and it seemed to put a drain on their motivation. While they were able to communicate their mathematical thinking to the researcher, they did not share their insights with each other, unless they were prompted.

The second problem was one that challenged the spatial abilities of both partners. Though Ray received a consensus score of two for his 'developing' level of originality, he made a remark that was not witnessed by the observers. Ray noted that in "...other problems we could keep track... chart it out and we could come up with the (solution)." When he was asked to clarify what he meant, Ray continued, "This one has the answer in it. There's a dollar inside these coins. It's like an adventure. We gotta find clues so we don't lose track." These remarks reflected Ray's persistence and his effort to make a connection to other experiences outside of mathematics.

The low performing pair-- Barry and El--received consensus scores of two, almost exclusively. This indicated that their mathematical problem solving behaviors were in the 'developing' level of proficiency. Overall, both El and Barry were limited in their understanding of the mathematical concepts involved with solving these problems. They also demonstrated limitations in their facility with some of the basic mathematical skills that would help them to progress through the problem solving process. Most of the calculations were done counting by ones and on fingers, the only strategy these two students used.

Even when they requested the use of coins to help them solve the money problems, both El and Barry counted by ones and on their fingers to keep track of the coin values. At times, if El took a risk, and totaled the addition of one coin to a quantity that was already added up, he would call out the amount, and refer to the researcher for a signal as to whether he was correct or not. Other times, if Barry was voicing his rationale for an answer, he looked to the researcher to validate his response. Because of their general insecurity with the mathematics concepts and applications, both Barry and El were very dependent on the approval of the researcher for how they were proceeding during the problem solving sessions.

Though Barry and El may have been lacking in their experiences with mathematical problem solving strategies, they seemed to recognize each other's limitations. They were supportive of each other's efforts and tried to encourage one another when frustration started to show. This mutual encouragement helped motivate them through some unsuccessful attempts and propelled them to continue the problem solving process. Towards the end of the second problem solving session, and beyond the short portion of the videotape observed by the panel of mathematics colleagues, both Barry and El started to build in their confidence that they were getting close to a solution. When their totals were near one dollar, Barry stated, "I think I'm gonna get this. I'm fittin' to find this!" And as El started to feel Barry's confidence growing, he added, "I'm starting to think it can be done because we're getting so close." They did make more unsuccessful probes towards finding a solution path and they were willing to continue the problem solving process until the end of the allotted time.

The success the high performing pair of students shared was primarily due to their competence and facility with the basic mathematical operations. This facility gave them the confidence to manipulate the numbers through the various problem-solving strategies they attempted. This confidence in their number sense was a reminder that they could find all of the possible answers, and inspired the high performing pair of students to persist until all the solutions were found.

The results for the average performing pairs of students seemed to be indicative of their inexperience with working in collaboration with a partner. Both students were competent with their basic mathematical skills. But their reluctance to communicate or plan a mathematical problem solving strategy together led to incomplete solutions to the given problems. Though the students persisted in their attempts to find a solution during the allotted time, their individual efforts were mostly unsuccessful.

The low performing pair of students were not competent in their basic mathematical operations. Even though the two students communicated openly during the problem solving session and needed little prompting to work together towards solving the given problems, their apparent lack of fluency with numbers diminished their confidence and inhibited any risk taking during the problem solving process. The low performing pair of students had a difficult time defining their problem solving approach and they spent most of their time using the 'Guess and Check' strategy.

Limitations of the Study

There were two main limitations to the study. First, there was a problem with the fact that the panelists only viewed a portion of the problem solving sessions. Specifically, it had been decided that the panelists' observations would begin once the students understood the problem and had articulated the problem in their own words. Even though a consensus was reached in how to score the problem solving behaviors and what to observe during the problem solving sessions, there were actions and comments that were made beyond the portion that was viewed that could have promoted changes in the individual scoring. The researcher was present during the entire problem solving session and had access to the videotape of those sessions. Comments made during dialogue between the problem solving partners and other subtleties of nonverbal communication were not available to the other members of the scoring panel.

The second limitation concerns the cultural backgrounds of the panelists and students. In spite of any background information the researcher may have given to that panel of observers, all the minutiae of communication that occurs between members of the same cultural group can be difficult to articulate. This may have created assumptions that biased the results of the scoring based on such a limited demonstration of problem solving behaviors. By being part of same cultural group as the students, the researcher was aware of the familiarity with cultural communication cues that created an understanding which was distinct from the other members of the panel.

During the discussions that followed the observations of the videotaped problem solving sessions, this familiarity may not have been conveyed so that the other observers could get a sense of what was happening during the 'real time' of the problem solving session. This might account for the "developing" scores that El was given for the portion of the videotape that was viewed by the observers. In terms of persistence, El was not engaged by this problem initially. Ultimately, he and his problem-solving partner, Barry, worked through to the end of the videotaping period. El remarked that he was starting to believe that there was a solution to the problem because they were getting closer and closer to the goal of a one-dollar path. A resolution to this issue might be for the panel of observers to view the entire videotaped problem solving session. This could give the observers a better sense about the problem-solving environment created by the students and the researcher.

Future Considerations

If a teacher presents a problem-solving task to a selected a heterogeneous group of students in his or her class and observed their performance, these observations would bear fruit for the teacher as well as for the students. Initially, the students' performance would give the teacher a sense of how the rest of the class would fare under similar circumstances. With the aid of the 'problem solving behaviors' rubric, the teacher would be able to orient his or her focus to evaluate what and how students understand a given mathematical concept and its application. The teacher could also share the problem solving behaviors rubric with the students so that the expected level of competence, the mindset, and the attitude towards mathematical problem solving would be more explicit. Once the students are aware of the teacher's expectations, they can become more critical of their own performance during the mathematical problem solving periods in their class.

It is extremely important for students to realize that mathematics is much more than just knowing facts and arithmetic operations. Students can have more success in mathematics if they are cognizant of the variety of problem solving strategies, are flexible in applying those problem solving strategies, and persist beyond the first unsuccessful attempt at solving a particular mathematical problem. If teachers are willing to facilitate, students can present an explanation of their solutions to an audience of their peers. Then, students will be able to demonstrate their mathematical reasoning and hone their communication skills. This activity is one that would give the teacher an opportunity to take advantage of the different student abilities within the classroom community.

An area that needs to be explored further is the discrepancy that exists between the teacher's perception of the student's demeanor within a whole class population compared with the student's self-perception as a student within the whole class population. Whereas the teacher might perceive the student as unmotivated and disinterested in learning, the student might be under the influence of other distresses, i.e. separation from family members, transient living arrangements, or inappropriate responsibilities for elementary aged children, that distract them from performing adequately in their academics. This element of resilience, where the student performs successfully in spite of negative conditions, needs to be explored further. Those individuals need to be validated and appreciated for overcoming their difficulties outside of school and doing well in their academic pursuits. By documenting the routes these willful and successful students take to accomplish their tasks, the teachers can become informed of the students' problem solving abilities in the classroom community.

Another consideration for future research might be to look at mathematical problem solving behaviors when the problem situations are set in the context of the student's life. A mathematical problem situation can be tailored to reflect any cultural experience. As we begin to recognize the variety of ways in which children demonstrate their understanding of a problem situation and the solution strategies that satisfy the parameters of the problem, it becomes imperative that the instructor become knowledgeable, if not 'expert,' in the curricular goals and objectives of the mathematics content area. This will enable the instructor to take advantage of the mathematical connections that exist in the students' every day life.

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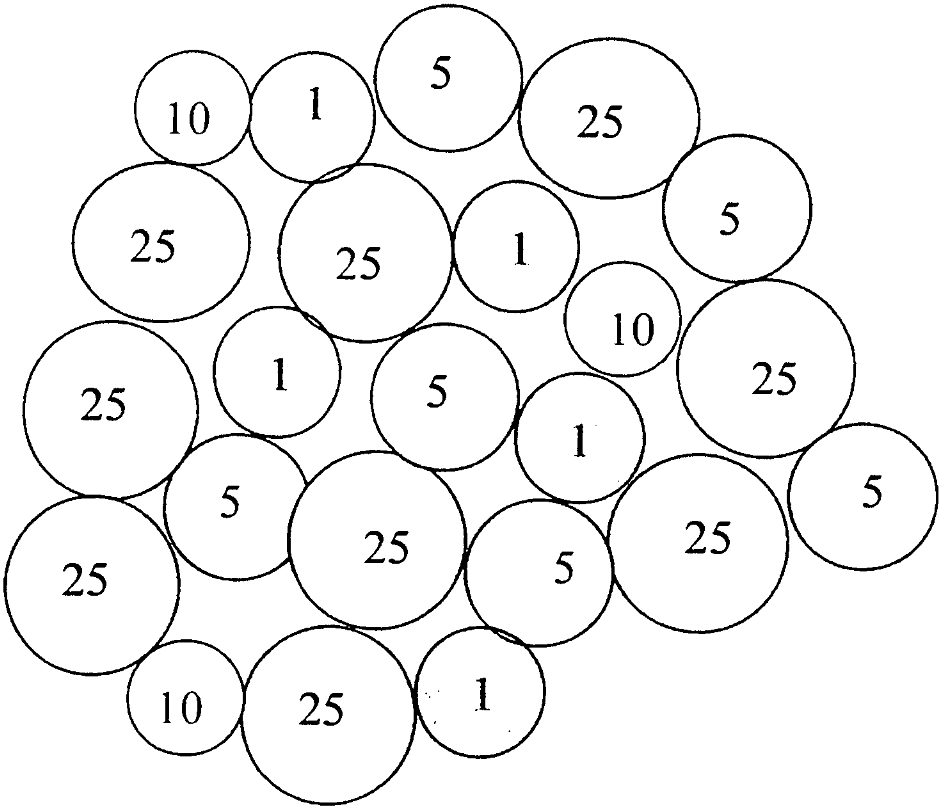
APPENDIX ONE

Can you get exactly one dollar's worth of change from the pile of coins by starting at any coin along the edge, moving from coin to touching coin, and ending on another edge coin? No coin may be crossed over more than once.



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Teaching Students to Write about Solving Non-Routine Mathematical Problems

Delia Levine
Ann Gordon

A two-week writing intervention was developed and used with sixth grade students. Pre-tests and post-tests were administered to measure changes in use of problem solving strategies, mathematical understanding, persistence and use of appropriate representations. Results indicate that mathematical proficiency improved and written explanations became longer, displaying increased understanding of non-routine mathematics problems. Teaching students to write about their mathematical thinking impacted their understanding of mathematical concepts as demonstrated in students' written explanations.

COE *Review*, pp. 50-63

Researchers and educators in the domain of language arts have long argued that having an opportunity to write deepens students' understanding and helps them grasp abstract concepts more completely, because the act of writing requires students to re-organize, and therefore reprocess, the ideas they are writing about (Archambeault, 1991; Fortescue, 1994; Miller, 1991). A number of individuals have extended these findings into the field of mathematics (Burns, 1995; Countryman, 1992; Miller, 1991; NCTM, 1989, 1991; Steward & Chance, 1995). In fact, writing has come to be seen as an important part of problem-solving instruction in many classrooms. For example, Szetela and Nicol (1992) claim that although it is often difficult for students to communicate their thinking to others, it is through communication that we can truly assess our students' problem-solving abilities. Bell and Bell (1985) also found writing to be an effective tool for teaching math problem-solving.

However, many teachers find that it is not easy to get students to write about their mathematical thinking. Students often have had little practice doing this kind of writing and they often don't know how to proceed or what, exactly, is being asked of them. Teachers have few strategies available to them for helping students overcome these difficulties. In fact, little attention has been paid to developing this kind of student

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writing, or how to develop an expository writing program in a math class. In an effort to address this problem, the current study was conducted. The purpose of the study was to evaluate the effectiveness of an instructional model inspired by the Bay Area Writing Project to teach sixth grade students how to write about their mathematical problem solving.

METHODS

Thirty-five percent of the students in the school where this study was carried out scored below the 40th percentile in either reading or mathematics on a state standardized test. Following an introduction and a week of pretests, the project began with a two week writing intervention and continued for 14 weeks, during which time students solved non-routine problems in collaborative groups, were instructed in how to write explanations, and wrote explanations individually. Pre-tests and post-tests were administered to measure changes in the use of problem-solving strategies, mathematical understanding, persistence and the use of appropriate representations over time. In addition to these measures, data consisted of student computational, pictorial and written work.

Four problem types were used in this study: (a) two-constraint problems, (b) pattern problems, (c) grouping problems, and (d) number-at-the-beginning problems. These types were chosen because they were believed rich enough to provide students with opportunities to use multiple solution strategies and expanded explanations of their thinking. The first three of these problem types were used throughout the study and the last was used as a measure of transfer. Each student had a chance to work on two examples of each type by the end of the study, while the transfer type only appeared on the pre- and post-tests.

In the two-constraint problem, two constraints are given and the student's solution to the problem must satisfy both constraints. For example:

> Ted is at the race track. There are lots of horses and jockeys running around the track. There are 82 feet and 26 heads. How many horses are there? How many jockeys?

A pattern problem asks a question in which a particular pattern or sequence needs to be discovered in order to successfully solve the problem.

> Jeremy keeps pennies in a jar. He adds 1 cent for the first week, 2 cents the second week, 4 cents the third week, 8 cents the fourth week, and so on. How many pennies will he add in the twelfth week?

Grouping problems require that a multiple set of criteria be met in order to successfully solve the problem. A solution needs to be found that satisfies all the clues given in the problem.

> Marvin is counting his marble collection. He counts more than 40, but less than 70. When he puts the marbles in groups of 5, he has 1 left over. When he puts them in groups of 4, he has 1 left over. When he puts them in groups of 3, he has 1 left over.

How many marbles does Marvin have?

And finally, number-at-the-beginning problems include information about the situation being described and the end state, but do not give information about the initial state. In order to solve the problem the initial state needs to be figured out, using the information given.

Dad is doing his weekly grocery shopping. The first store he enters is the meat market. At the meat market, he spends half his money and then spends $10 more. The second store he enters is the bakery. Here he spends half of his remaining money and then spends $10 more. He now has no money left. He is totally broke. How much money did he have when he started at the meat market?

During the two week training period, following a structure similar to that used in the Bay Area Writers Project, students were taught how to write a) a *problem statement,* or explanation of the problem in their own words that includes both the facts and the question, b) a *process paragraph* in which they were expected to describe the solution or solutions obtained, as well as their thinking as they did the problem, c) a *solution paragraph,* in which they were expected to give a short explanation proving their solution(s), and d) an *evaluation paragraph,* in which they were expected to give their opinion about the relative difficulty of the problem. A class discussion followed each problem solving session, in which strategies were shared. Written explanations were first done as a whole class activity, modeled by the teacher, then as a group activity among the students, and finally, as an individual independent activity.

The primary question of concern was whether the model used would result in better student-generated explanations of individual problem-solving activity. In order to answer this question, student work collected over the course of this study was evaluated along four dimensions: Did the student use an appropriate strategy or strategies for solving the problem? Did the mathematics and/or written work evidence understanding of the problem? Did the student do more than was necessary to solve the problem, or did he or she continue to explore features of the problem after a solution had already been reached? And finally, did the student make appropriate use of any drawings, charts, graphs, diagrams, or tables needed to solve the problem? The overall quality of a particular piece of work was then rated using criteria to identify high, medium, and low performance. The criteria used for rating each task were as follows:

High Performance Rating

- Reached a correct solution.
- All mathematics work is clearly shown.
- Written explanation is clear and explains all the mathematics.
- If appropriate, mathematics and written explanation shows what was done before a solution was reached.
- Other strategies or other solutions might be mentioned.

Medium Performance Rating

- Reached a correct solution.
- Might have reached an incorrect solution but is on the right track (might have made a careless arithmetic error).
- Mathematics is clear but written explanation may be confusing or visa versa.
- Might only explain the solution and not the other mathematics or what was done before a solution was reached.
- Written explanation is short - things are left out.
- Shows understanding of the problem either through the mathematics or the written explanation but not necessarily both.

Low Performance Rating

- Solution is unrelated to the problem (unrealistic solution) or no solution found.
- Mathematics work doesn't connect to the clues of the problem.
- No written explanation.
- Might have reached a correct solution but no support for it (no mathematics or written explanation).

DATA ANALYSIS

Results indicate that mathematical proficiency improved, students' written explanations became longer, and explanations displayed increased understanding of non-routine mathematics problems that the students were asked to solve. Consider the quantitative data summarized in Table One. Each of the 29 students in the study were given a set of four pretest problems, four posttest problems, and four one-month delayed post-test problems. For each time period, one problem was a two-constraint problem, one was a patterns problem, one was a grouping problem, and one was a transfer problem. Students' written responses to each of these twelve problems was rated as either "high," "medium," or "low," using the rating system described previously. Table One summarizes the changes in student performance that were found over the course of this study. These results were obtained by analyzing individual student work quantitatively and what follows is an analysis of what these changes looked like in more qualitative terms, using the two-constraint problem type as a case in point.

Two-Constraint Problem

As previously indicated, in a two-constraint problem two pieces of information are given. The student's solution to the problem must satisfy both of these constraints. The two-constraint problem type was probably the most familiar of the four problem types. Two of the three practice problems, used the first three days of school, were two-constraint-type problems. This type of problem was also used for three of the four problems used for the intervention. For example, the following problem was given as a pre-test item during Week 2 of the study:

> Ted is at the racetrack. There are lots of horses and jockeys running around the track. There are 82 feet and 26 heads. How many horses are there? How many jockeys are there?

TABLE ONE Problem Solving Performance across Time (n=29)						
Problem Type	**Perfor-mance Rating**	**Pretest**	**Posttest**	**One Month Delayed Posttest**	**Change Pre-to Posttest**	**Change Post-to Delayed Posttest**
1.Two Constraints	H	0%	28%	38%	+28%	+10%
	M	38%	38%	28%	0%	-10%
	L	62%	35%	35%	-27%	0%
2 Patterns	H	3%	24%	38%	+21%	+14%
	M	45%	48%	52%	+3%	+4%
	L	52%	28%	10%	-24%	-18%
3. Groupings	H	3%	21%	14%	+18%	-7%
	M	45%	66%	66%	+21%	0%
	L	52%	14%	21%	-38%	+7%
4.Number at beginning (Transfer Problem)	H	0%	7%	31%	+7%	+24%
	M	34%	31%	41%	-3%	+10%
	L	66%	62%	28%	-4%	-34%

Since the problem says that there were 26 heads, the number of horses plus the number of jockeys needed to be 26. In order to answer the question successfully, furthermore, students need to multiply the number of horses times the number of feet on each horse and add that to the number of jockeys times the number of feet on each jockey, to get a total of 82 feet on the horses and the jockeys together.

As reported in Table One, 62 percent of the responses to this question on the pre-test only met the criteria for a low rating, while none of the responses on this question met the criteria for a high rating. Many of the responses received a low rating because they gave an answer that did not fit either constraint of the problem. One student said, for example, that there were "19 horses and 18 riders" *[S-I, pre]* which would mean that there was a total of 37 heads and 112 feet. Another responded that "The amount of horses were 15 and 22 people" *[S-N, pre]* which would mean that there were 37 heads and 104 feet. Some students who received a low rating on this problem did so because they only dealt with one of the two constraints. One student said there were "13 jockeys and 13 horses" *[S-A, pre]* which was the correct amount of heads, but would yield 78 feet; another said there were "4 men and 22 horses" *[S-M, pre; S-R, pre]*, which would also result in the correct number of heads (26), but would yield a total of 96 feet, which is more than allowed by the problem.

A low rating was also given if the response could not be related to the problem being asked. Students wrote things like, "42 people" *[S-E, pre]*, which was accompanied by a picture that did not appear related to the current problem, or, "I counted them up and my anser came up to be 41 feet and heads . . ." *[S-D, pre]*, which was not the answer to the question that was asked.

However, by the time of the posttest, only 35 percent of the responses to a similar problem were like those described above, while 28 percent now met the criteria for a high rating. Furthermore, by the time of the delayed post-test, which took place a month after the end of the study, 38 percent of the responses met the criteria for a high rating. The following are the two-constraint problems used on these tests:

Post-test problem

> Trisha is visiting her grandpa's farm. She notices that he raises only hens and hogs. She counts 38 heads and 100 feet in the barnyard. How many hens and how many hogs does her grandpa have in the barnyard?

Delayed post-test problem

> The Creepy Critters Scary Tunnel has two types of cars for the children to ride. One type seats 4 and one type seats 6. There are 24 cars that seat a total of 114 children. How many cars seat 6 children and how many cars seat 4 children?

The following patterns emerged in the high responses: (a) most of the responses included lots of written explanation, (b) they included a process paragraph which explained the step-by-step procedures that they used to solve the problem, (c) a problem statement which retold what the problem was about, and (d) a solution paragraph which gave proof according to the clues that the solution was correct. For example:

> First I made two charts. One chart is for the hens and one is for the hogs. For the hen's chart. On the right side I put the mutiples of 2 for the hens leg because hens have 2 legs. And on the right side I put the numbers of heads by the mutiples of 1 because each hen has 1 head. For the hogs I put the same thing except that I put the multiples of four for the legs because hogs have 4 legs. For the hen's I stoped at 37 heads and 74 legs because the problem said that there are 38 heads all together and that there are hens and hogs. For the hogs. I stopped at 25 heads and 98 legs because the problem said there are 100 legs and if I go farther there will be 102 legs. Then I matched the heads from the hogs to the heads of the hens to see if it is equal to 38 because the problem said there are 38 heads. And if it is equal to 38. I matched the hen's legs to the hog's legs and if it is equal to 100 legs that means the answer is

> right because the problem said there are 100 legs all together. And I have only one answer and it is 26 hens and 12 hogs *[S-L, post Process paragraph].*

> I know my answer of 26 hens and 12 hogs is right because there are 38 heads and 100 legs all together. And there are hens and *hogs [S-L, post Solution paragraph].*

Some of the responses in the high group indicated that the student had found his/her solution by using the strategy of making a systematic list and from that list was able to figure out that only one correct solution was possible. All of the strategies described in the high responses led to an efficient solution of the problem. If guess and check was used, the guesses were not random. Since there were 24 cars, all the guesses of six- seated and four-seated cars equaled a total of 24 cars, which was one of the constraints of the problem. One such guess was eight six-seated cars and 16 four-seated cars. When they found out the number of children that was seated, for example 112, they stated that they knew that they were close to the answer so the next guess was nine six-seated cards and 15 four-seated cars, which ended up being the correct solution.

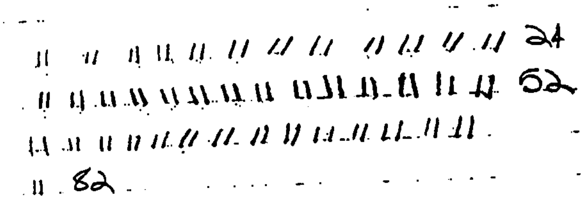
Another efficient strategy used by the students who were given a high rating was making a list of the multiples of four and six and then finding a number from each list that would equal 114 children. They also had to make sure that their two numbers kept the constraint of 24 cars. Yet another efficient strategy was using up all of the children from one constraint and then altering those amounts to fit the other constraint. For example, making 24 circles to represent the cars and first putting four children in each car allows room for 96 children. They then realized that they needed to make room for 18 more children. If they added two more children to nine cars, they would end up with 15 four-seated cars and nine six-seated cars that can seat 114 children.

<u>Problem-Solving Strategies</u>

On the pre-test, many children were unable to solve the problem correctly; although most gave an answer of some kind, they did not indicate that they did not know what to do. It was difficult to describe the strategies used by these students, because many of them carried out activities that appeared entirely unrelated to the problem. For example:

> I found the anser buy Drawing 82 lines the I pretended were feet
> and I counted them up and my anser came up to be 41 feet and heads
> It was quite simple and you don't need a calculator to figer it out

Hears a picture of what I did



[S-D, pre].

> I put them into groups of one and 4 beans under each chip. Then I counted that there were 26 chips and 82 beans. I found out that

there were 19 horses and 18 riders *[S-I, pre].*

By the post-test, however, many students were using strategies and explaining their choices. For example:

> For my work to find out my answer I used all guess and check. For my 1st guess I guessed 15 - 6 seats and 9 - 4 seats. That was wrong because when you times 6 seats to 15 because my guess was 15 you get 90. Then when you times 9 to 4 you get 36. 90 + 36 = 126. I know that is wrong because the total of children in a seat was 114 just like the problem says. Next I guessed 12 - 6 seats and 12 - 4 seats. That was wrong because when you times 12 to six you get 72 and when you times 12 to 4 you get 48. When you add those together you get 120. I know that is also wrong because the problem said that 114 children is the total. All the guessed number have to equal 24 because that is the number of cars so 15 - 6 seats and 9 - 4 seats is the number of cars and seats. I also guessed some more numbers till I got my answer *[S-X, delayed Process paragraph].*

> I know my answer which is 6 seats in 9 cars and 4 seats in 15 cars is correct because when you add 6 seats in 9 cars you get 54. When you add 4 seats 15 times you get 60 - 60 + 54 = 114. just like the problem says there should be 114 children *[S-X, delayed Solution paragraph].*

> First I drew 24 circles because in the problem it says that there are 24 cars. Then I drew 6 dots in each circle for 12 circles and 4 dots in each circle for 12 circles because in the problem they said that there are 2 types of seats one is 4 and one is 6 children. the circles represent the 24 cars and the dots are the children. Then I x 6 x 12 because there are 6 children in one seat for 12 cars to find out how many children are in thos 12 cars and I x 4 x 2 to find out how many children in the cars. There are 72 children in the 6 children seats and there are 48 children in the 4 children seats. I add those 2 numbers (48 + 72) because I want to see if it equaled to 114 because in the problem it said that there are a total of 114 children *[S-AA, delayed Process paragraph].*

Now they have a language to talk about what strategies they are choosing to use; they name their strategies, explain which ones didn't work, and say what they are going to do next.

<u>Mathematics</u>

On the pretest, most papers didn't show any mathematical work at all. Students either just gave a solution with no work shown and no explanation given, or they explained the mathematics in the written explanation, but there was no mathematics visible. In other cases, the mathematical explanations were confusing. For example:

> There are 20 horses and there are 27 Jockeys. The way I got this answer is as soon as I knew that

some of the feet could be human

feet I knew the answer. See 20 x 4 = 80 and there are two remaining feet. Then I added the 1 feet to the 26 heads and I got 27 *[S-Y, pre]*.

First I get 82 feet and 26 heads. Then I divided them into 4 and 3. After that I found that there are 4 heads and 22 feets. Then I get 4 heads and 22 feets. Then I seperate the 4 heads and I divided the 22 feets into 5 & 6. After I did I found there are 2 horses and 2 jockeys. This is how I found out by acting it out, makes a model and guess and check *[S- P, pre]*.

First I took 82 chips out. Then I took 3 chips and put them in one group. When I was finish I added all the heads up then I added all the feets up. After that I counted all of the groups and I turned out having 50 bodys. So then "I" used the 50 and took away 26 and have 24. So al together there ar 24 people *[S-Q, pre]*.

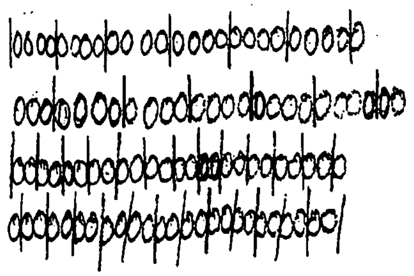
By the time of the posttest, many of the students wrote out the mathematics they were using and clearly explained the mathematics that was shown. In addition, the mathematics work also showed what was tried prior to reaching a correct solution. For example:

For this POW I use Guess and Check. I thought of 19 hens and 19 hogs which make 38 heads. So first I multipled 19 by 2 (because hen have 2 feet) and I got 38 legs. Next I multiplied 19 by 4 (because hogs have 4 legs) and I got 76 legs. Then I added 76 with 38, but I got 104 legs. the problem said that there are 100 legs, that mean 19 hens and 19 hogs is incorrect. Next I tried 18 hens and 20 hogs, it adds up to 38 heads. 18 hens and 20 hogs is incorrect because when I added up the total legs of the hens and hogs together, I had 116 legs instead of 100 legs. I decided to draw a picture next. I drew 100 circles for legs and divided all of them into groups of 4. Out of the 100 legs, there are 25 groups of 4. So if I subtracted 38 by 25, there would be 13 left. So I put 1 line in each groups 13 times to make some groups of 2. I had an answer of 12 hogs and 26 hens. I did some multiple mathworks to make sure it's right and it is. I did some more multiple mathworks to find another anwer, but I couldn't find another answer.

P.O.W. Post test #1

19 + 19 = 38 heads
x2 x4
38 + 76 = 104 legs

18 + 20 = 38 heads
x2 x4
36 + 80 = 116 legs



12 hogs and 26 hens

26 + 12 = 38 heads
×2 ×4
52 + 48 = 100 legs

28 + 11 = 39 heads
×2 ×4
56 + 44 = 100 legs

30 + 10 = 40 heads
×2 ×4
60 + 40 = 100 legs

24 + 13 = 37 heads
×2 ×4
48 + 52 = 100 legs

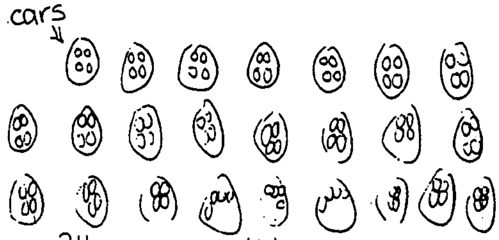
22 + 14 = 36 heads
×2 ×4
44 + 56 = 100 legs

[S-C, post Process paragraph]

First I drew 24 circles. The 24 circles represent the cars. There are total of 114 children. So in each 24 circles I put 4 children because the problem says how many cars seat 4 children. After that I did 24 x 4 to see how many children there are total. When I multiply the answer is 96 children which is wrong because the problem says there are 114 children, so 114 - 96 to see how much more children I need. The answer is I need 18 more. So I add 2 more children in each 9 circles because 9 x 2 = 18. After that I found that it is right because 9 x 6 = 54 which means the place were I add 2 children in each of the 9 cars. The answer is 54. 15 x 4 represent that there are 15 cars which have 4 children. The answer is 60 so 54 + 60 = 114. Which is the correct answer. Because the answer is 114 children just the problem says. So there are 9 cars seat with 6 children and 15 cars seat with 4 children.

Post Test #1

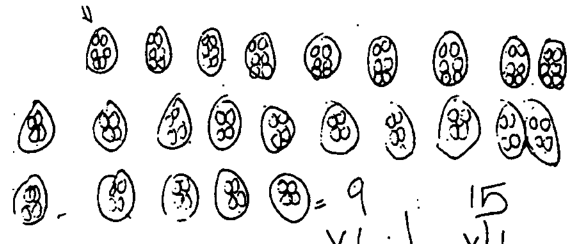
cars



= 24
x 4
96 children

114 children total
− 96 children
18 more we need
so....
again...

cars..



= 9
x 6
54

15
x 4
60
+ 54
114

[S-P, delayed Process paragraph].

Persistence

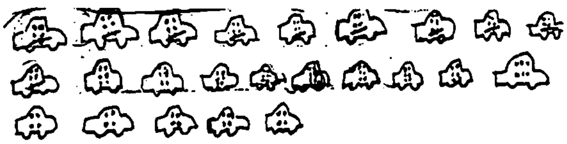
On the pre-test, most of the responses were short or incomplete. Students either just gave solutions, some correct, some incorrect, or wrote a very short written explanation. For example:

> Therer are 13 jockeys and 13 horses Frist I used chip and didn't Work so I deived and it worked *[S-A, pre].*
>
> 42 people *[S-E, pre].*

> There is 52 horses & 2 Jockeys. I use some red circles & Beans to sub. it for getting how many horses. And I use some colored chips to Sub. for getting the answers *[S-K, pre].*

On the other hand, by the end of the study, responses were longer in length and usually had the mathematics and written explanation on two separate pages. The student work also shows mathematics work attempted after a correct solution was reached:

> First I made 24 cars because the paper said that there were 24 cars. I filled 19 cars with 6 because 19 times 6 make 114 children. Next I started to filled the 5 blank cars, and to do that I must take two children from each cars until I fill the left over cars. I took out 2 children from 10 different cars because 10 divided by 2 equal 5. Then I count the cars that have 4 children and those that have 6. There ar 15 cars that have 4 and 9 that have 6, then I add 15 and 9 (to make sure) and I got 24 cars. I also add up the total children in all of the cars of 4 and the total in all of the cars of 6 and I got 114 children. To find another answer, I took 2 cars of 6 which make 3 cars of 4. The took children is till 114, but the are 25 cars and there is 24 cars in the problem. If I keep taking 2 cars of 6 to make 3 cars of 4, the cars will add up. Next I took 3 cars of 4 to make 2 cars of 6, the amount of children is the same, but the number of cars is 23, not 24. If keep taking 3 cars of 4 to make 2 cars of 6, the amount of cars will grow less.



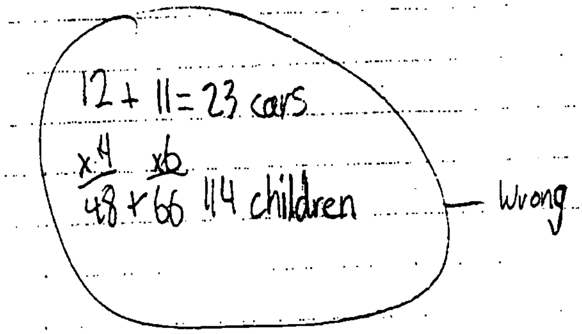
15 cars seat 4
9 cars seat 6

15 + 9 = 24 cars
x4 x6
60 + 54 = 114 children

Right

18 + 7 = 25 cars
x4 x6
72 + 42 = 114 children

Wrong

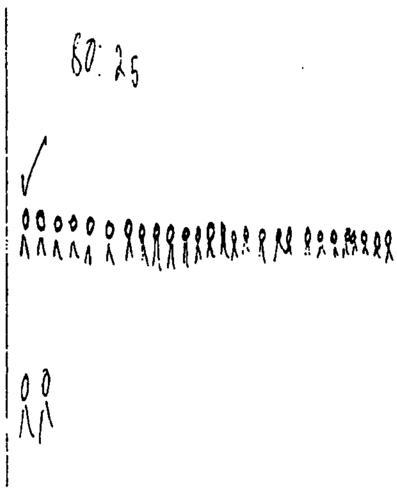


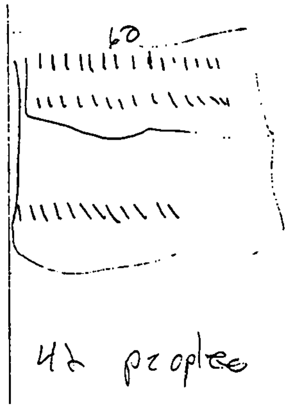
[S-C, delayed Process paragraph].

Another indication of persistence was demonstrated by the work of those students who took the time to construct systematic lists of all the possibilities for a given problem. In those cases, there was no need to continue looking for other possible solutions.

Representations

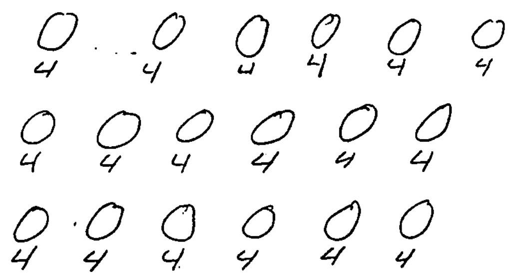
Many of the responses on the pre-test either did not show any representations or used representations that were inappropriate to the problem. These responses sometime included tally marks or circles to represent the 82 feet or 26 heads, but the representations didn't match the clues.





[S-E, pre].

I put them in to groups of one and 4 beans under each chip. Then I counted that there were 26 chips and 82 beans. I found out that there were 19 horses and 18 riders



[S-I, pre].

On the post-test however, the representations were much more sophisticated. Many students made two charts, one for hogs and one for hens. They continued their list until they reached the limit of either constraint, either the 38 heads or 100 feet. They then looked for amounts from each chart that when added together would give both constraints asked for in the problem.

CONCLUSION

Similar analyses as the one just described were carried out for each of the four problem types and the results of these analyses, as already suggested, strongly support the conclusion that students who participated in this project did, in fact, improve in their mathematical proficiency and became better able to explain their mathematical thinking. Using a model inspired by the Bay Area Writing Project to teach students to write about their mathematical problem-solving resulted in students' being able to write clearly about their mathematical thinking. When students were given examples of how to explain their mathematical thinking they had a better idea of what was expected of them. However, not all students benefited equally from this intervention, suggesting that case studies of individual children with different learning profiles might be useful. Most importantly, faced with difficulties getting students to write about their mathematical thinking, this approach might be relatively easy for teachers to implement, particularly if they have already had experience with using similar approaches in their language arts programs.

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The Use of Rubrics by Middle School Students to Score Open-Ended Mathematics Problems

Ford Long Jr.

Student work was analyzed to investigate whether students who became involved in the assessment process would improve their performance on open-ended mathematics questions. Twelve seventh grade students were selected from 150 students to participate in this study. The pretest and posttest consisted of open-ended questions adapted from those used by the Balanced Assessment and New Standards Projects. Students solved a Problem of the Week for six weeks and then scored them using a teacher-designed rubric based on Polya's model. Results indicate that the lowest students benefited the most by being involved in the assessment process in this way.

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Historically, students in traditional math classes have been taught specific procedures for solving math problems and the primary focus has been on learning computational skills (National Council of Teachers of Mathematics (NCTM), 1991). Since the purpose of testing has been simply to evaluate students' computational skills, standardized tests consisting of multiple-choice items have historically been used. These tests have been popular because they are easy to score, but it has been argued that they do not measure students' mathematical thinking. New assessment techniques have emerged, including ways of assessing student work on open-ended tasks, observing students at work, and keeping student portfolios (Branca, 1994; California Mathematics Framework, 1992). Over the years the word 'testing' has been replaced by the term 'assessment' to emphasize the broader scope that has been encompassed.

Recent educational research findings indicate that learning occurs when students actively assimilate new information and experiences and construct their own meanings (NCTM, 1991, p.2). This research has generated a move away from the traditional classroom practice of rote memorization of facts and has led to new teaching practices as well as to new content that includes more than arithmetic. New standards and frameworks for teaching mathematics have been developed (NCTM, 1989; California Mathematics Framework, 1992). These new guidelines for curriculum materials have created a need for new assessment tools (Resnick, 1994; National Research Council, 1989).

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New assessment programs have been funded to monitor the effectiveness of instructional strategies that emphasize thinking and reasoning in mathematics (Katims, Nash, & Tocci, 1993; Lane, 1993; Resnick, 1994). A key component of these programs is open-ended questions that provide opportunities for students to demonstrate their mathematical understanding as opposed to multiple choice questions which were designed to measure computational skills.

The purpose of this study is to examine students' work to determine if their performance on open-ended questions would improve after the students are involved in the assessment process. In this study, students revised a scoring rubric based on George Polya's four-step problem solving model and used this rubric to assess a series of Problems of the Week (POWs) for six weeks. A modified New Standards Project (NSP) rubric was also used by the students to score a pretest and posttest of open-ended questions adapted from the New Standards Project and the Balanced Assessment Project (BAP). These tests were used to evaluate improvement.

It was hypothesized that students' performance on open-ended problems would improve after they were involved in the assessment process. One expected outcome was that, after being taught assessment techniques, students would be better prepared to explain and justify their mathematical thinking by using more precise mathematics vocabulary and multiple representations to illustrate their solutions. In addition to the pretests and posttests, the effectiveness of involving the students in the assessment process was evaluated by comparing the students' assessments to the teacher's assessments. The research questions for this study are:

1. Will using Polya's model as a rubric help students to set standards for quality work and improve their performance on open-ended mathematics problems?

2. How do student scores compare to teacher scores on open-ended mathematics problems?

REVIEW OF THE LITERATURE

For the past century traditional classroom pedagogy and assessment systems have been based on the behavioral theory of learning which stresses that "content can be broken down into small segments to be mastered by the learner in a linear sequential fashion" (Romberg, 1995, p.5). Historically, these traditional teaching practices were accompanied by standardized tests. More recently, researchers have found that standardized tests can not measure mathematical thinking or mathematical literacy and encouraged development of alternative assessment systems (Kulm, 1994; Romberg, 1995; NCTM, 1995).

Constructivist research indicates that learning occurs when students use prior knowledge to actively assimilate new information and experiences and construct their own meanings (Resnick, 1987). In other words, learning is a process where the student gathers, discovers, or creates knowledge in the course of performing a purposeful activity. Katims, et al. (1993) cautioned that when the students are first introduced to complex nonroutine problem solving, their initial reaction is frustration and they are at a complete loss as to how to begin. As the students become familiar with problem solving activities, the period of confusion and frustration at the start of each activity shortens considerably and their planning time increases. She explains that performance assessment can not be successful without some retraining of the students.

Some studies support the need for involving students in the assessment process (Stiggins, 1994; Higgins, Harris, & Kuehn, 1994), and some researchers have developed instructional programs to assist students in learning how to evaluate their own responses and to revise and improve their own work (Katims, et al. 1993; Lane, 1993; Lappan & Ferrini-Mundy, 1993; Resnick, 1994; Baker, O'Neil, & Linn, 1994). A major component of each of these programs has been open-ended questions that are evaluated by using alternative assessments.

Several studies have found that peer interaction enhanced the problem solving process and helped students perform better on open-ended questions (Hart, 1993; Larson, 1996). Schoenfeld (1989) also mentioned how Mason (1982) used the ideas generated by his students to help them develop standards by which their arguments would be considered. In this study, the students worked with their peers to revise a teacher-designed rubric, which they then used to set standards for quality work. They also worked with their peers to score their work.

Scoring rubrics provides a structure to assist students in their problem solving efforts. Pate, Homestead, and McGinnis (1993) described a rubric as a scaled set of criteria that defines for the student and teacher what a range of acceptable and unacceptable performance looks like. They stated that rubrics could be used to evaluate process as well as content, and can guide students in self-assessment. They believed that the "rubrics should have such detail that no one (students, teachers, parents) has questions as to how well the activity was done" (p. 27). Rubrics have been used to provide descriptions of each level of performance in terms of what students are able to do and assigns values to these levels (Herman, Aschbacher, & Winters, 1992). The rubrics are meant to assist the students in developing the self-reflection or metacognitive skills used by expert problem solvers.

According to Wilson, Fernandez, and Hadaway (1985), most formulations of a problem solving framework in U. S. textbooks attribute some relationship to Polya's stages; however, the use of linear models used in these textbooks does not promote the spirit of Polya's stages and his goal of teaching students to think. They described the following defects in the traditional models.

- They depict problem solving as a linear process.
- They present problem solving as a series of steps.
- They imply that solving mathematics problems is a procedure to be memorized, practiced, and habituated.
- They lead to an emphasis on answer getting.

These linear formations are not consistent with the genuine problem solving focus which should continually cycle between the four phases: understand, plan, try, check.

Szetela and Nicol (1992) stated that "effective assessment of problem solving in math requires more than a look at the answers students give. Teachers need to analyze their processes and get students to communicate their thinking" (p.42). Charles, Lester, and O'Daffer (1987) described scales that focus more attention on solution procedures. The California Assessment Program (Pandy, 1991) included comprehensive descriptions of various levels of performance for specific problems. This was appropriate for large-scale assessment programs. However, the classroom teacher has little time to construct scales for individual problems. Teachers need assessment procedures and scales that they can modify or use intact for a wide range of problems (Szetela & Nicol, 1992). Because of this shift in focus, teachers must be provided with tools to evaluate their students' problem solving skills. This study attempts to develop ways of using alternative assessments in a classroom setting.

The uniqueness of this study is that the students designed their rubric based on the questions Polya considers useful to the problem solver who works by himself (Polya, 1957). They also revised a modified New Standards Project rubric that was used to score their tests and solutions on previously scored anchor papers, which serve as examples of particular scores.

METHODS

The students in the study attended a middle school located about 15 miles south of San Francisco. The student population is approximately 63 percent Caucasian and 27 percent other ethnic groups. Students are heterogeneously grouped and randomly assigned to their classes. All students in the five 7th grade math classes were given a pretest, a Problem of the Week (POW) each week for six weeks, and a posttest.

The instruments used for the assessments were open-ended questions designed for middle school students from the New Standards Project 1994 Reference Exam and the Balanced Assessment Project. The pretest consisted of one short-item from NSP and one long-item from BAP. The posttest consisted of a short-item and a modified long-item both from the NSP. One of the short items asked the students to determine the better airline deal based on the specials that they were offering. One of the long items involved a situation where three schools were planning a science fair. There were four parts to this problem in which the students were asked to allot space and dollars to each school based on its population.

The students scored their pretest and posttest using a modified rubric, which they designed, based on the criteria established by the NSP. The students also wrote justifications for their score. These scores and justifications were the self-assessments that were later compared to the teacher assessments. As part of the training in scoring, all the students scored anchor papers with the official scores removed. They did this two times, once at the beginning and once at the end of the study. The anchor papers presented two solutions for each short-item question from the tests. After scoring the papers, the students then worked in pairs to revise and improve the solutions.

A subset of twelve seventh-grade students were selected from the 150 students in the five classes using a stratified sample. Two female and two male students, each from the top, middle, and bottom third of these lists, were selected using a table of random numbers. Eighty-five additional students' scores of the anchor solutions were also compared.

The purpose of this study was to investigate whether students who became involved in the assessment process would be better able to explain and justify their mathematical thinking. The researcher developed a rubric based on the four-part model suggested by Polya: understand, plan, try, check (See Appendix 1). Each part had a 4-point scale that the students used to score their POW's. Each week the emphasis was put on one part of this model and the students, with guidance from the teacher, refined the language of the rubric. The students then used the revised rubrics to score the following week's POW. This self-assessment process continued for six weeks. It was hypothesized that using the rubric to assess their own and other students' work would better prepare them to establish criteria for quality work.

The students also used a modified NSP rubric in the beginning and at the end of the study to score their tests and to score a range of anchor solutions with the scores removed. The teacher and the students discussed the meaning of the scoring criteria for each score point and revised the language of the rubric. The students used this revised rubric to score anchor solutions and their own posttests. The process of revising both rubrics was designed to help the students identify criteria for quality work and help them to better express their mathematical thinking. The responses on the open-ended questions from NSP and BAP were analyzed to determine improvement.

RESULTS

To determine if using a scoring rubric based on Polya's model would help students set standards for quality work and improve their performance on open-ended questions, scores of 12 students on a two-item pretest and posttest were compared. The teacher scores for the twelve students are reported in Table One. On the long item, five students improved on the posttest; seven students' scores remained the same. Furthermore, four of the five increased scores were for students who scored a "1" on the pretest. This indicates that low scoring students might have benefited more from this process. These students had partial success in answering the question and were able to demonstrate some understanding of the mathematics required. The scores of all the students either stayed the same or improved. However, on the short-item the posttest scores of four students had decreased and the scores of three students had increased. One possible explanation for these lower posttest scores is that the single question gave the students less opportunity to demonstrate their understanding of mathematics.

A comparison of student assessments to teacher assessments is reported in Tables Two and Three. In cases where the student scores matched the teacher scores the difference was zero. If a stu-

TABLE ONE Teacher Scores on Long and Short Items from Pretests and Posttests					
Student ID	**Long Term**		**Student ID**	**Short Term**	
	Pretest	**Posttest**		**Pretest**	**Posttest**
1132	3	4	1132	3	2
1137	3	3	1137	2	2
1139	2	2	1139	4	2
1153	3	3	1153	4	4
1229	2	2	1229	2	3
1238	1	1	1238	2	1
1241	1	2	1241	2	2
1253	3	3	1253	3	2
1307	1	2	1307	1	1
1308	1	2	1308	1	1
1321	3	3	1321	1	3
1355	1	2	1355	3	4

dent score is higher than the teacher score, the score was positive. None of the students scored their tests lower than the teacher on the pretest or posttest on either item. One explanation for the students' high scores could be that most of the students assumed their solutions and justifications were correct, therefore they scored their work a 3 or 4.

Table Two compares the student and teacher scores on the long-items. Table Three compares the student and teacher scores on the short items. Four student's scores matched the teacher scores on the long-item from the pretest; only one matched on the posttest. However, 8 out of 12 students (scores highlighted in tables) scored their work within one point of the teacher scores on the pretests. This indicates a high level of agreement on the pretests between the student and teacher on the scoring of their work. There was less agreement on the posttests. On the long-item, seven students' scores were within one point of the teacher's scores. On the short-item there were five. One reason for less agreement on the posttests was that almost all the students scored their work a 4. This makes the comparison of the scores on the posttests less meaningful.

The teacher and student scores were also compared by having all seventh grade students score short item anchor papers at the beginning and end of the study. Table Four shows the scores of the 85 students who scored both sets of papers, which included both a strong and a weak solution for each item. In each case the anchor scores were unknown to the students. For the weak students seemed to score the work a 3 or 4 because the answer was correct, even if the mathematics to support the answer was incorrect or if the explanation or justification was inaccurate or unclear. For the strong anchor papers, most of the students concur with the

TABLE TWO
Student and Teacher Scores on Long and Short Items
from Pretest and Posttest

Pretest				Posttest			
Student ID	Student Score	Teacher Score	Difference	Student ID	Student Score	Teacher Score	Difference
1132	3	3	0	1132	4	4	0
1137	3	3	0	1137	4	3	+1
1139	4	2	+2	1139	4	2	+2
1153	3	3	0	1153	4	3	+1
1229	4	2	+2	1229	4	2	+2
1238	2	1	+1	1238	2	1	+1
1241	3	1	+2	1241	4	2	+2
1253	4	3	+1	1253	4	3	+1
1307	3	1	+2	1307	4	2	+2
1308	2	1	+1	1308	4	2	+2
1321	4	3	+1	1321	4	3	+1
1355	1	1	0	1355	3	2	+1

TABLE THREE
Student and Teacher Scores on Long and Short Items
from Pretest and Posttest

Pretest				Posttest			
Student ID	Student Score	Teacher Score	Difference	Student ID	Student Score	Teacher Score	Difference
1132	3	3	0	1132	4	2	+2
1137	4	2	+2	1137	4	2	+2
1139	4	4	0	1139	4	2	+2
1153	4	4	0	1153	4	4	0
1229	3	2	+1	1229	4	3	+1
1238	3/2	2	+1	1238	4	1	+3
1241	3	2	+1	1241	4	2	+2
1253	4	3	+1	1253	3	2	+1
1307	4	1	+3	1307	3	1	+2
1308	3	1	+2	1308	4	1	+3
1321	3	1	+2	1321	4	3	+1
1355	3	3	0	1355	4	4	0

TABLE FOUR
Students' Scores Compared to Anchored Scores on Short Item

	Beginning of Study		End of Study	
Score given by students	Anchor Score 1	Anchor Score 4	Anchor Score 1	Anchor Score 3
4	4	79	11	24
3	46	5	41	56
2	31	1	31	5
1	4	0	2	0
n	85	85	85	85

score, either the 3 or the 4. This agreement may occur because the students seem to be generous in their scoring overall but the less generous scoring on the weak anchor papers indicates that the students were able to distinguish that the work was not quite right. Most students stated they scored the paper a 3 instead of 4 because the individual did not show their work for the calculations.

DISCUSSION

In class discussions students were consistently prompted to represent the mathematics in different forms using charts, diagrams, or different numeric forms. It was hoped that this would lead them to check their work and help to ensure a mathematically correct solution. The intensity of the focus on the process seems to have directed students away from checking their answer. The students acted like the college students in Schoenfeld's study who continued down a dead end path and never stopped to check their first results. This differs from the mathematician who would check to see if his solution made sense, make adjustments in his thinking, and then would proceed down a new path (Schoenfeld, 1989).

The assignment of a score from 1 to 4 based on the rubric also took on a life of its own for the students. The majority of students assigned themselves a 4 on the posttests because they believed that their work met the criteria of a 4 paper:

- mathematically accurate and explained well,
- each step of the solution is shown,
- all numbers are labeled with words or symbols,
- you showed and explained how you checked your work,
- if appropriate, a diagram or illustration is included.

If a student showed all their work and explained their solution, they felt comfortable assigning their work a 4 because they were unable to consider whether or not their mathematical reasoning was correct. The students were not able to stop and check to see if their solution was appropriate for the question being asked.

Most students represented their solution in more than one way and gave a written justification for their solution. However, their multiple representations were not sufficiently independent to provide a reasonable check for the answer; they reported the same incorrect solution in a different form. Although most students were able to identify the carefully structured criteria of a quality paper, they were not able to identify an incorrect solution or justification.

The six weeks of the study did not provide enough opportunity for the students to make significant changes. Future researchers should provide more time and training for students to focus more on checking the correctness of their answers. It is also necessary to have a variety of rich tasks where several different approaches to solve the problem are readily demonstrated. They need to open up the problem solving process of students so that a range of strategies are available and even required for rich and engrossing problems.

From classroom experience, the researcher noticed that many students who are not successful at mathematics tend to shut down and give up on problem-solving. It was hoped that this model would provide these students with some of the problem solving strategies used by students who are successful problem solvers. One weakness with the model presented was that students used the questions in it as a checklist to simply answer yes or no. For example, one of the questions on the 'Devise A Plan' part of the model was, "Can you solve a related or simpler problem?" Many students wrote, "Yes, I can solve a simpler or related problem." They didn't actually look for a subproblem or simpler problem that could be solved as a first step. Many students were not able to do this on their own.

The students suggested rewriting the questions in the first model presented as statements. The intention of these questions, later statements, was that they could be used as a guide for the students who became bogged down in the problem solving process. The model was meant to illustrate the cyclical nature of Polya's problem solving phases and also to provide the students with a linear structure or outline for the final write-up and presentation of their solution. Many of the low achieving students continued to use the model as a checklist even after the questions were changed to statements.

Inexperienced problem solvers need more instruction on how to use Polya's model. The most important phase of this model is to understand the problem. Students need more training in how to move back and forth between the phases as they gain more information in order to check the reasonableness of their solution path. They can then move on to the 'carry out your plan' phase. With this approach they can determine if their strategies make sense and improve their overall understanding of the problem.

The students' revisions of the wording of each phase each week so that the model made more sense to them did not appear to help them identify when they were heading down an inappropriate solution path. Also, asking the same question in different phases did not appear to help the students check the appropriateness of their solution path. For example the question, "Did you use all the data?" appears twice, in the 'devise a plan' and 'look back' phase. The students did not see the need to answer it again and left out an important stimulus for the cyclic process. More examples need to be provided where the students gain new information and then go back to the original question to determine if they are using all the available data in their solution. Students also need to be presented with more problems that have the correct answers, but incorrect mathematics to support their answers or unclear explanations. It might be more beneficial to solve several problems as a class completing every phase of the write-up together instead of breaking the Polya model into separate phases that are introduced one per week. Consolidating the process in this way might help the students to constantly move back and forth between the understand, plan, try, and check phases.

CONCLUSION

Kulm (1994, p. 25) stated that, "In recent work on mathematical thinking, attention has been given to the notion that people have strategies that guide their choice of what skills to use or what knowledge to draw upon during the course of problem solving, investigation, or verification of a discovery." It is clear that middle school students need to be taught these strategies. This study was intended to help students further develop their problem-solving strategies. The process of scoring student work each week related to Polya's model was utilized so that the students would be involved in the assessment process and they would have examples of student work to revise and improve. It was hoped that by working with their peers to improve student work, they would be better able to apply the strategies they learned to improve their own solutions to open-ended questions. The results indicate that perhaps because of the design of the study, which developed Polya's steps, one at a time, the final step, check, did not get internalized.

Most middle school students have not developed strategies to help them plan and carry out the steps in solving problems. Kulm (1994, p. 26) stated that this strategic knowledge must be explicitly taught, modeled, and practiced. This study reinforces and further supports this conclusion. Students need assistance in developing lines of reasoning and decision making strategies used by successful problem solvers.

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APPENDIX ONE
Rubric Used by the Students to Score Their 'POWS' Each Week

NAME ________________ ________DATE ____________ PERIOD __________

PROBLEM OF THE WEEK = POWerful Mathematics

GEORGE POLYA SAYS: Understand, Plan, Try and Check

As you solve the problem check a bracket for each line you completed. If you checked four brackets give yourself a '4'; three brackets a '3', two brackets a '2', one bracket a '1.'

I. UNDERSTAND THE PROBLEM

( ) Rewrite the main question in your own words.
( ) Underline the questions, circle the word that will describe the answwer (cows, $, feet, etc.).
( ) List the data. What can you conclude from the data?
( ) Make a prediction or estimate the answer.
Circle one of the following: 4 3 2 1 0

II. DEVISE A PLAN

( ) List three problem-solving strategies to solve the problem *(Guess & Check; Work Backwards; Lists; Equations: etc.)*.
( ) Explain how you are going to use at least one strategy.
( ) Make a sketch, illustration, or chart of the problem.
( ) Explain how you will use all of the relevant data.

Circle one of the following: 4 3 2 1 0

III. CARRY OUT THE PLAN

( ) Show all your work.
( ) Use math vocabulary and/or math symbols to explain your answer.
( ) Label all numbers with words or symbols.
( ) Prove each step is correct by showing your solution more than one way.

Circle one of the following: 4 3 2 1 0

IV. LOOK BACK

( ) Explain and show how you checked the result.
( ) Explain and show how you checked all of the relevant data.
( ) Show how you used your answer in a different way. (*Graph, equation, Used implied information*).
( ) Make up a similar problem of your own.

Circle one of the following: 4 3 2 1 0

Overall rating from above:

14-16 pts = 4 12-13 pts = 3 8-11 pts = 2 1-7 pts = 1

Circle your overall score below:

Span		**Grade Scale**	
4	Scholar	Well Done/Complete Work	18--20
3	Practitioner	Acceptable/ Minor Revision	15--17
2	Apprentice	Revision Needed	6--14
1	Novice	Restart / Incomplete	1--- 5

Limit your write up to one page. **ANSWER IN OUTLINE FORM.** Work 15 minutes each night. Write at least 2 sentences that explain what you did to solve the problem. Attach all scratch work.

A Survey of Teacher Beliefs about Pre-Requisite Experiences for Student Success in Algebra

Audrey Adams

In this study, algebra teachers responded to survey questions about their beliefs about what mathematical skills or experiences students need to be successful in their algebra classes and about how frequently they used different teaching strategies. Teachers were grouped according to their self-identification of their teaching strategies as traditional, non-traditional, or a mix of both. Results showed the three groups to be similar in terms of their ratings of necessary skills and experiences, but dissimilar in types of teaching strategies used in their classroom

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For those students who either do not take algebra, or who take it and fail the class, algebra serves as a gatekeeper which bars them from further educational and, often, economic opportunity (NCTM, 1994). The fact that more than half of the students who fail algebra never take another math course further emphasizes the importance of algebra to students' mathematics future (Mullins, Dossey, Owens, & Phillips, 1991). Students must be successful in algebra to continue in more advanced mathematics courses, which are generally a prerequisite for many professional career paths.

The responsibility for preparing students in algebra cannot be left to algebra teachers alone. The entire kindergarten through eighth grade mathematics curriculum needs to prepare students to develop the abstract concepts necessary for algebra. All elementary school and middle school teachers have a role to play in this preparation, but because algebra is typically offered beginning in eighth and ninth grades, sixth and seventh grade mathematics teachers play a particularly critical role.

The way in which an algebra class is taught can affect the algebra teacher's expectations of the prerequisite skills and experiences needed for student success. Because of the mathematics reform efforts of the past decade, algebra is now taught in a variety of ways. Non-traditional algebra teachers teach from a problem-solving perspective and emphasize conceptual understanding and real-world applications, whereas more traditional teachers tend to emphasize memorization and the manipulation of symbolic representations.

It is clear that success in algebra is needed for higher-level mathematics. Therefore, all teachers, but particularly

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middle school mathematics teachers, need to know what preparation will help ensure that success. The purpose of this study is to investigate the beliefs of algebra teachers regarding prerequisite skills and knowledge necessary for student success in algebra classes. In particular, the major research questions for this study are as follows: (a) What prior experiences do algebra teachers think are necessary for student success in algebra?, and (b) Do traditional and nontraditional teachers differ in their views about what those experiences should be?

REVIEW OF THE LITERATURE

The mathematics achievement of students in the United States does not compare favorably with students from many other countries. The Third International Mathematics and Science Study (TIMSS) compared the mathematics achievement of eighth grade students in 41 countries and found that the United States ranked in 28th place (TIMSS, 1996). Another troubling statistic is that only half of U.S. students take more than two years of mathematics in high school (*Everybody Counts*, 1989). The low mathematics achievement for eighth grade students in the United States, according to the conclusions of TIMSS, is due, at least in part, to the lack of focus in the U.S. mathematics curriculum. Teachers are expected to cover too many topics, resulting in a fragmented, unconnected curriculum that does not allow students enough time to study any one topic in depth.

How students are taught is another cause for concern. TIMMS researchers compared lessons taped in real classrooms in the United States and abroad. They found that in comparison to other countries, U.S. students spend more of their time practicing computational skills rather than analyzing relationships, solving problems, and developing deeper conceptual understanding.

The mathematics reform movement in the United States is largely based upon the idea that children learn best when they construct their own meaning (Resnick, 1987). Many of the problems children experience in learning mathematics are thought to result from trying to use formal procedures and algorithms without having conceptual understanding. Kamii and Lewis (1991), for example, found that second grade students who were taught in a constructivist program demonstrated greater ability to use higher-order thinking skills than similar students taught in a more traditional program which emphasized memorization of algorithms.

The research about how children learn mathematics have led some mathematics educators to reconsider what constitutes meaningful mathematics content and effective instructional strategies for algebra classes. The NCTM *Curriculum and Evaluation Standards* (1989) reflect this new understanding. In particular, the NCTM standards describe effective algebra instruction as presenting many varied experiences in inquiry and application that actively involve students in those processes and include opportunities to reflect upon and express mathematical ideas.

Since algebra classes are changing, it is necessary to delineate the basic ideas and concepts which should be taught prior to enrollment in a formal algebra class, especially if algebra is to be made accessible for all students (NCTM, 1994; Moses, Kamii, McAllister, Swap, & Howard, 1989). In preparation for algebra, students need to learn computation in a problem-solving setting, understand the vocabulary of mathematics in order to understand the symbols, be able to recognize patterns, have lots of experiences collecting, graphing and analyzing data, and develop number sense (Howden, 1990). They must personally construct symbolic representations in order to create mathematics with context and meaning (Moses et. al., 1989). In terms of specific arithmetic concepts and specific skills essential for success in algebra, children need be able to solve equations such as the meaning of the equals sign and the order of operations (Herscovics & Linchevski, 1994). In addition, it has been argued that the understanding of proportionality is a bridge between arithmetic and abstract algebra (Post, Behr, & Lesh, 1988).

The view of what algebra classes should be like is changing because of the implications of research as well as the introduction of technology. This means that the prerequisite skills and knowledge necessary for success in algebra are changing and, therefore, the preparation of students for success in algebra must also change.

METHODS

Based on the review of the literature, a three-part questionnaire was designed to elicit information from algebra teachers. The first part contained questions about the teacher's view of the student skills necessary for success in his or her classroom. The second part contained questions about (a) whether the teacher characterized himself or herself as traditional, non-traditional or a mix of both, and (b) the teaching strategies and/or manipulatives, textbooks and curricula used by the teacher. The third part included demographic questions about gender, age, ethnicity, teaching experience and educational background.

The questionnaire was sent to all teachers of eighth and ninth grade algebra in the San Francisco Unified School District. Of the 102 teachers to whom surveys were sent, 51 (50%) teachers responded.

RESULTS

The survey respondents consisted of 31 males and 25 female algebra teachers. European-Americans comprised the largest ethnic group (N = 29), followed by Chinese-Americans (N = 7), Latinos (N = 4), African-Americans (N = 3), Filipinos (N = 2) and all other (N = 6). The ages were fairly evenly distributed, with slightly more teachers in the 51 to 60 year-old category. The number of years of teaching experience ranged from six months to 36 years.

Teachers were asked to identify their classroom teaching strategies as traditional, non-traditional or a mixture of both. Two-thirds of the 56 teachers identified themselves as a mixture of both traditional and non-traditional (N = 37 teachers). Of the remaining third, 11 identified themselves as traditional and 8 identified themselves as non-traditional.

A comparison was made between the three groups of teachers--traditional, mixed, and non-traditional--in terms of their number of years of teaching experience. It was hypothesized that newer teachers would tend to identify themselves as non-traditional. Consider the teachers who had been teaching between two and five years, namely the newer teachers. Of these newer teachers, only 18 percent (2 of 11) consider themselves traditional and 50percent (4 of 8) non-traditional. At the same time, when considering the teachers who had been teaching 20 years or more, 55 percent (6 of 11) considered themselves traditional and only 25 percent (2 of 8) non-traditional.

The teachers were asked how frequently they used certain instructional strategies during a typical week. The nine strategies were separated by this researcher into three groups of three strategies each: traditional (multiple algebraic exercises, memorizing procedures, and whole class instruction), non-traditional (the use of manipulatives, use of cooperative groups, and writing about mathematics) and those which did not fit easily into either of the first two categories (pencil and paper graphs, real-world problems, and graphing calculators).

A scale was then created to compare the self-identification of each teacher with the strategies used in his or her classroom. The strategies in each category listed above were assigned a value that was then multiplied by the number of times per week the teacher used that strategy. A score was then calculated for each teacher. A low score meant that the teacher frequently used non-traditional strategies; conversely, a high score meant that a teacher frequently used more traditional classroom strategies. The teachers who identified themselves as traditional all scored in a range from 35 to 43. Those teachers labeling themselves as non-traditional had scores ranging from 19 to 34. The remaining group of mixed designation teachers had scores ranging from a low of 22 to a high of 40. Thus, the range for teachers in the mixed group was greater than for either of the other groups and was slightly higher than for the lowest score of non-traditional teachers and slightly lower than for the most traditional teacher. The designations chosen by teachers and their use of particular classroom practices seem to be consistent with what the research says are traditional and non-traditional teaching methods.

Teachers were also asked to respond to a series of eleven 5-point Likert scale questions scale that asked them to evaluate specific mathematics skills and mathematical experiences in terms of their importance as pre-requisites for success in algebra. A rating of "1" indicated that a skill or experience was considered to be highly important and a rating of "5" indicated that a skill or experience was considered least important. The skills in Table One are ordered from most traditional to least traditional.

A comparison of the mean ratings of each of the three teacher groups (traditional, non-traditional, and mixed) indicated agreement, for the most part, about the relative importance of these skills. But there were some notable exceptions when comparing the three groups. First, on average, the skill of

TABLE ONE
Teacher Ratings of the Relative Importance of Specific Skills and Experiences as Prerequisites to Algebra

Ability to:	Traditional		Mixed		Non-Traditional	
	Mean	SD	Mean	SD	Mean	SD
1. Master operations with whole numbers	1.55	0.69	1.62	0.68	2.00	0.93
2. Master operations with fractions	2.09	1.3	1.95	0.97	2.50	1.07
3. Master operations with integers	1.55	0.69	1.76	0.83	2.38	1.19
4. Memorize and use formulas	2.73	1.19	3.30	1.05	4.25	0.87
5. Understand order of operations	1.46	0.69	1.84	0.83	2.25	1.04
6. Solve equations with an unknown	1.36	0.51	2.16	1.14	3.25	1.39
7.Conceptually understand proportional reasoning	2.10	0.88	1.97	0.93	2.13	0.84
8. Recognize patterns and make generalizations	2.36	1.5	1.95	0.88	1.75	0.46
9. Problem-solve	1.55	0.69	1.62	0.76	1.63	0.74
10. Communicate math ideas in writing	3.18	1.25	2.11	1.09	1.86	0.35
11. Work cooperatively in groups	3.73	1.01	2.35	1.06	2.25	0.71

memorizing and using formulas (Skill 4) was rated as more important by the traditional and mixed teachers than the nontraditional teachers. (Overall, across the groups, the skill of memorizing and using formulas was rated as less important, on average, than the other skills). Second, the skills of writing about mathematics (Skill 10) and working cooperatively in groups (Skill 11) were rated as more important by the nontraditional and mixed teachers than the traditional teachers.

DISCUSSION

The number of teachers who participated in this study constituted more than half of the algebra teachers in a fairly large urban school district. Two-thirds of the teachers identified their teaching strategies as a mixture of both traditional and non-traditional. Most teachers in the survey indicated that they sought a balance between the practice of routine skills and the development of conceptual understandings, using what they believed was valuable to create an effective algebra program.

When comparing the teachers' self-identification of strategies used and their years of teaching experience, the general presumption that newer teachers tend to be more non-traditional and older teachers tend to be more traditional was supported in this case. When the self-designations of teachers were compared with their actual classroom practices, the traditional and non-traditional teachers identified themselves in ways that are consistent with the literature, thereby validating these self-reported data.

All three of the teacher groups believed that students need to have a strong foundation in the traditional, basic operations using whole numbers, integers and fractions. There was also agreement among the three groups about the importance of certain skills or experiences typically identified as non-traditional. In particular, there was some agreement among the three teacher groups about the importance of the skills of proportional reasoning, recognizing patterns and making generalizations, and problem-solving. The responses of the teachers in this study clearly indicate that there is more agreement than disagreement about what skills and experiences are necessary for student success in algebra.

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Fifth Grade Teachers' Attitudes towards Implementing a New Mathematics Curriculum

Lorene B. Holmes
Deborah A. Curtis

MathLand is a relatively new mathematics curriculum program that has been adopted by a number of school districts in the San Francisco Bay Area. This curriculum is a product of the mathematics reform movement, and requires a major shift in mathematical content and pedagogy. The purpose of this study was to survey fifth grade teachers in one particular school district that was in its second year of implementing MathLand. In particular, the goal of this study was to assess fifth-grade teachers' opinions about the MathLand curriculum as well as their opinions about the current mathematics reform movement in general. Although the teachers tended to agree with many of the current mathematics reform principles, they were generally dissatisfied with the MathLand curriculum. A number of teachers cited the lack of practice with basic skills as a limitation of the new curriculum. Results are discussed in terms of implications for classroom practice.

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A major concern of educators in many California school districts is that the mathematics education reform movement has entered into the classrooms by way of new mathematics textbook series adoptions. In California, the 1994 adoption of instructional materials in mathematics was a milestone in the state's ten-year effort to advance mathematics education reform (California Basic Instructional Materials in Mathematics Adoption Recommendations, 1994). This adoption of mathematics instructional materials reflected the influence of the 1992 California Mathematics Framework, which called for instruction to develop "mathematically powerful" students, including both instruction in basic computational skills as well as the opportunity to develop these skills through real-world problem solving and investigations.

In the San Francisco Bay Area, the adoption of the *MathLand* curriculum in 1995-96 by one of the large, urban school districts in the area was a major shift toward math reform for that district. This adoption was more than a movement from an old mathematics textbook series to a new one. The *MathLand* curriculum includes a major change in pedagogy and mathematics content. Consequently, in-service training for teachers was an essential part of the district's implementation process. During the first year of implementation (1995-96), all the kindergarten through

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fifth grade teachers in the school district were required to attend three professional development days on the *MathLand* curriculum. This in-service covered three *MathLand* units, namely number strategies, geometry, and logic, which were to be taught by all teachers during the 1995-96 school year.

Teachers' attitudes and beliefs about the mathematics education reform movement will influence the way in which a new mathematics curriculum is implemented in their classrooms. Consequently, a number of questions arose related to the adoption of this curriculum. Does the adoption of *MathLand* promote movement toward math reform in the district? How do teachers view the new curriculum? How does it affect their teaching and beliefs about mathematics teaching and learning? What support do teachers need to fully implement new curriculum materials?

The purpose of this study is to examine fifth-grade teachers' views and attitudes toward the *MathLand* program and the influence of the new curriculum on their teaching of mathematics; their involvement in math education reform; and their fifth-grade teaching experience with the new curriculum. Fifth grade teachers were chosen because fifth grade is perceived of as being critical to the transition from elementary school to middle school.

REVIEW OF THE LITERATURE

The *MathLand* series contains many of the current mathematics reform movement's goals and defining characteristics. The current call for reform in mathematics education (National Council of Teachers of Mathematics, 1989, 1991; *California Mathematics Framework*, 1985, 1992) has been shaped by a constructivist approach to teaching and learning mathematics. How does the implementation of new curriculum support the mathematics reform process? A review of the literature suggests that the answer to this question is very complex.

Mathematics education in the United States has been in a state of reform for more than 30 years. Ever since the Russians launched Sputnik into space in 1957, Americans have worried that our children are not being taught enough mathematics and science to compete with other nations.

During the 1950s through the 1970s, the mathematics reform movement sprang from many roots and took on many different (and sometimes opposing) forms. Hence, the curricular reforms undertaken during those years under the slogans of "modern mathematics" or "new math" left a mixed legacy to American mathematics education (*Everybody Counts*, 1989). Jack Price (1995), president of the NCTM, wrote that the "new math" reform of the 1950s to 1970s failed for several reasons, including: (a) top-down attempts at development and implementation; (b) a lack of consensus within the mathematics community as to it value; and (c) a lack of understanding of the goals on the part of teachers, administrators, boards of education, and other policy makers.

The "new math" era was followed by a "back to basics" emphasizing mathematics as a core of rules, procedures and facts during the mid-1970s. Then, in 1980, the National Council of

Teachers of Mathematics (NCTM) published *An Agenda for Action: Recommendations for School Mathematics of the 1980s*, which called for the implementation of a 10-year reform program. One goal of this agenda was to move the focus of school mathematics curriculum beyond basic skills objectives to a more problem-solving conception of mathematics content and instruction (NCTM, 1989).

The NCTM Standards, unlike previous mathematics reform attempts, have been supported by many professional organizations, colleges and universities, large segments of the private sector, and the general public (Hatfield & Price, 1992; Price, 1995). For instance, in 1988, the National Council of Supervisors of Mathematics (NCSM, 1988) updated its 1977 basic skills position statement to describe twelve essential mathematical competencies that citizens would need to begin adulthood in the next millennium.

Textbooks drive what is taught in schools (Flanders, 1987). In fact, 70 to 90 percent of classroom curricular decisions made by teachers are based on the textbooks adopted by their schools or districts. Chandler and Brosnan (1994) found that mathematics textbooks published after 1989 tended to be longer and that there was a shift in emphasis among the five content areas of arithmetic, measurement, geometry, data analysis and algebra. These researchers also found that there were changes in the amount of content development, drill, word problems and problem solving, and there were increases in the use of estimation and calculators. According to the Third International Mathematics and Science Study (TIMMS Report), (Schmidt, McKnight, & Raizen, 1996) most American mathematics textbooks are unfocused in a number of ways, including the tendency to cover too many topics with a "breadth rather than depth approach."

The *MathLand* program has no textbooks for the students, but does provide a teacher's *Guidebook* for each grade level that covers an entire year of curriculum. Each *Guidebook* consists of ten units, each taking from two to five weeks to teach. It is the lack of textbooks, along with the activity-based lessons and the use of manipulatives in this program that has brought a flood of criticism from educators, parents, and the public. Debra Saunders (1996), one of the most vocal critics of the mathematics reform curricula (which she calls the "New-New Math"), wrote that programs like *MathLand* are causing test scores to drop because these programs lack basic computational skills, use confusing time-intensive "fuzzy math" activities and do not prepare students for standardized tests.

However, the *MathLand* authors (Randolph & Charles, 1996) argued in their position paper that in order for students to have real math power, they must memorize basic arithmetic facts. Thus, the formal memorization of facts begins in second grade in the *MathLand* program. They also argued that *MathLand* provides several different vehicles for students to practice basic skills including games in the *Guidebook*, and practice in the *Daily Tune-Ups* and *Arithmetwist* booklets. The *MathLand* authors strongly believe their program will develop students into mathematical thinkers. The authors' goal is that students using *MathLand* will become confident problem solvers.

Does the adoption of a new mathematics curriculum promote movement toward mathematics education reform? According to some researchers (e.g. Cohen &, Ball 1990; Cronin-Jones, 1991) the answer is no. However, other studies (e.g. Yore, 1991) have shown that textbooks are the main influence in teaching mathematics, since most teachers adhere closely to the strategies and examples used in textbooks for planning their lessons. In general, teachers do not always implement a curriculum in their classrooms in same way the curriculum was designed to be implemented (Cohen & Ball, 1990; D'Ambrosio, 1991; Cronin-Jones, 1991).

Researchers have begun to acknowledge the powerful influence teachers have on the curriculum implementation process (Cohen & Ball, 1990; Good, Grouws, & Mason, 1990; Cronin-Jones, 1991; Brosnan, 1994). Even if teachers are provided with all the tools (all the materials, in-service training and support), the mathematics reform movement might still be hindered by teachers' beliefs, if these beliefs are not supportive of the newly recommended style of instruction. Consequently, it is the attitudes of the teachers and their beliefs about mathematics education reform issues that will influence what is implemented.

METHODS AND RESULTS

The purpose of this research was to study fifth grade teachers' attitudes toward the new mathematics curriculum program, and their opinions about mathematics education in general. A six-page questionnaire was the primary tool used to assess their beliefs and attitudes about the new mathematics curriculum. In particular, this questionnaire addressed: (a) teachers' attitudes toward the new curriculum; (b) the implementation of the new curriculum; (c) the support teachers' believed they needed to adequately implement the new curriculum; and (d) their opinions about the mathematics reform methods. There were two open-ended items that asked for the main strength of *MathLand* and the main weakness of *MathLand*. The teachers were chosen by cluster random sampling, where a subset of 30 elementary schools in the district were randomly selected, and then all of the fifth grade teachers in the selected schools were surveyed. Seventy-three fifth-grade teachers were sent a survey packet. Thirty-eight teachers (52 percent) returned the questionnaires. A summary of their opinions is presented in Table One.

The teachers' responses were for the most part consistent with ideas of the NCTM *Standards* and the *California Mathematics Framework*. For example, teachers tended to agree with the statement "In teaching mathematics, one of my primary goals is to help students develop their abilities to solve problems and think mathematically." Also teachers tended to agree that students learn mathematics best when they construct their understanding from a variety of experiences.

One of the goals of mathematics reform is to make mathematics learning more meaningful to students. Teachers

TABLE ONE
Teachers' Opinions about Teaching Mathematics in General

	Mean	SD	n
Students learn mathematics best in groups with students of similar abilities.	2.64	1.02	36
Some students are inherently more capable in mathematics than others.	3.28	1.21	36
Students learn mathematics best when they construct their understanding from a variety of experiences.	4.26	0.92	35
Students should learn computational skills within the context of problem solving	3.43	1.17	35
Students need to master basic computational facts and skills before they can engage effectively in mathematical problem solving.	3.33	1.43	36
In teaching mathematics, one of my primary goals is to help students develop their abilities to solve problems and think mathematically.	4.39	1.08	36
In teaching mathematics, one of my primary goals is to help students master basic computational skills	3.94	1.24	36

Note: Opinion Scale: strongly disagree=1 to strongly agree=5

were asked to rate the importance of certain teaching methods for effective instruction in mathematics. A summary of their responses is presented in Table Two.

Again, teachers' responses were for the most part consistent with ideas of the mathematics reform movement. For example, most teachers (72 percent) thought it was "very important" to introduce applications of mathematics in daily life. But less than half the teachers thought it was "very important" to use a variety of assessment strategies.

The teachers were asked to report onhow much time was spent on certain instructional strategies since the adoption of *MathLand*. These data are presented in Table Three.

More than half the teachers reported that they were spending more time in having the students discuss different ways to solve problems and having the students use manipulative materials or drawings to solve problems. None of the teachers reported spending more time having the students work alone on assignments.

The teachers were asked to indicate which *MathLand* units they taught during the first year of implementation of the new curriculum (Year 1) as well as which units they planned to teach during the second year (Year 2). A summary of these data are presented in Table Four.

TABLE TWO
Teachers' Ratings of the Importance of Certain Teaching Methods for Effective Mathematics Instruction (n = 36)

	Percentages			
	Not Important	Somewhat Important	Fairly Important	Very Important
a. Introducing applications of mathematics in daily life.	2.8	2.8	22.2	72.2
b. Using a variety of assessment strategies.	5.6	11.1	36.1	47.2
c. Engaging students in open-ended investigations.	5.6	8.3	30.6	55.6
d. Making connections between mathematics and other disciplines.	2.8	5.6	38.9	52.8
e. Having students explain their thinking with other students.	5.6	5.6	33.3	55.6

TABLE THREE
Teachers' Responses to the Statement:
"Since the adoption of *MathLand*, the amount of class time spent on the following is:"

	Percentage of Math Time			
	More Time	Less Time	Same Amount	n
a. Students practice multiplication facts.	8.3	36.1	55.6	36
b. Students solve story problems or other problems that don't have obvious solutions.	45.7	14.3	40.0	35
c. Students discuss different ways that they solve particular problems.	58.8	11.8	29.4	34
d. Students discuss mathematical ideas, as a class or in small groups.	42.9	11.4	45.7	35
e. Students practice or drill on computational skills.	5.7	48.6	45.7	35
f. Students use manipulative materials or drawings to solve problems.	57.1	8.5	34.3	35
g. Students work alone on assignments.	0.0	51.4	48.6	35
h. Students work on self-assessments.	24.2	12.2	63.6	33
I. Students work in cooperative groups.	42.9	5.7	51.4	35
j. Students work on long term projects.	37.1	17.1	45.7	35
k. Students reflect on their own math learning.	38.9	8.3	52.8	36
l. You explain concepts or computational procedures.	22.2	30.6	47.2	36

TABLE FOUR Percent of Teachers Who Taught Specific *MathLand* Units during Year 1 and Their Plans for Year 2		
	Percentages	
	Year 1	Year 2
a. **Unit 1: All About Us** Collecting, Communicating, and Interpreting Data	63.9	77.8
b. **Unit 2: Using Predictions** Exploring Patterns as a Problem-Solving Tool	55.6	64.9
c. **Unit 3: Strategies** Building a Network of Number Relationships	41.7	55.6
d. **Unit 4: Organizations** Creating Attribute Sets and Exploring Logical Operators	52.8	44.4
e. **Unit 5: Millions** Explorations with Everyday Objects, Models, and Equations	16.7	25.0
f. **Unit 6: Inside/Outside** Explorations in Volume and Capacity	16.7	61.1
g. **Unit 7: Equivalence Thinking** Exploring the Relationships between Whole and Fractional Numbers	36.1	69.4
h. **Unit 8: The Triangle Angle** Exploring Angles and Triangles	38.9	63.9
i. **Unit 9: System Thinking** Analyzing and Interpreting Number Systems	16.7	19.4
j. **Unit 10: Confidence** Developing Probability Sense through Sampling and Possible Outcomes	8.3	27.8

In the first year, the district provided inservice training on three *MathLand* units, namely Unit 3, Unit 4, and Unit 8, and teachers were expected to have implemented these units in their classes during Year 1. Roughly 40 to 50 percent of the teachers reported using these three units in Year 1 of implementation. Generally, the teachers reported that they planned to use more of the *MathLand* units during the second year.

Finally, teachers were asked to rate their overall satisfaction with *MathLand*. Results are presented in Table Five.

The teachers tended to disagree with the statement "I am very satisfied with *MathLand*" (Item g). Furthermore, they tended to disagree that *MathLand* had everything they needed to teach their students (Item b). Additionally, they did not think it prepared their students for middle school mathematics (Item f). However, they tended to agree with the statement "*MathLand's* explanations of the Key Mathematical Ideas are helpful" (Item d).

Twenty-nine of the 36 teachers wrote comments for the main strength of the

TABLE FIVE Teachers' Responses to Satisfaction with *MathLand*			
	Mean	SD	n
a. MathLand is a well-balanced mathematics curriculum.	2.61	1.18	36
b. MathLand has everything I need to teach my students fifth grade mathematics.	2.12	1.27	35
c. MathLand's layout is easy to follow.	2.85	1.33	34
d. MathLand's explanations of the Key Mathematical Ideas are helpful.	3.18	1.03	34
e. The assessment piece in MathLand is an effective means to look at growth and understanding of math concepts.	2.74	1.21	34
f. MathLand prepares my students for middle school mathematics.	2.50	1.14	30
g. I am very satisfied with MathLand.	2.65	1.32	34

Note. Satisfaction Scale: strongly disagree=1 to strongly agree=5

MathLand program. Four different themes arose from the written comments: (a) the usefulness of using manipulatives, (b) the strength of using cooperative groups, (c) the worthwhile problem-solving and critical thinking activities, and (d) the benefits of writing. Thirty-two teachers wrote comments on the main weakness of the *MathLand* program. These comments were divided into four themes: (a) lack of sufficient practice in computational skills; (b) lack of addressing items on the CTBS test; (c) too much time needed for preparation and for teaching topics; and (d) the need to supplement *MathLand* with additional materials.

DISCUSSION

The results indicated that these fifth-grade teachers' opinions about teaching mathematics and using certain teaching methods for effective mathematics instruction were compatible with the underlying philosophy of the mathematics reform movement. However, the results of this study also revealed that the attitudes that these teachers have toward *MathLand* itself were mixed, with a number of teachers indicating their overall dissatisfaction with the curriculum.

Consider first teachers' beliefs about mathematics instruction in general. The teachers agreed that students learn mathematics best when they construct their understanding from a variety of experiences. Also, the teachers tended to agree that it was very important to introduce their students to applications of mathematics in daily life. Moreover, the teachers believed that one of their primary goals is to help their students develop their abilities to solve problems and think mathematically. These three results reflect some of the recommended practices in the 1992 *California Framework* in order to empower students mathematically.

These teachers' opinions were compatible with the 1992 *California Framework* and the *NCTM Standards,* which would suggest that they were ready to make changes in their teaching, provided that the opinions they reported were actually internalized into their belief systems. These teachers were aware that mathematics can be taught in ways other than simply memorizing facts, rules, and procedures, and were aware that mathematics can be taught in ways that will empower students mathematically. Thus, the teachers should have been in a good position to implement the new *MathLand* curriculum.

Now consider the results regarding the *MathLand* curriculum itself. The results also indicated that the teachers might be going through some of the conflicts and dilemmas similar to other teachers described in the research literature (Putnam, Heaton, Prawat & Remillard, 1992). In terms of the quantitative data, roughly half of the teachers did not implement the three units they were expected to implement in Year 1. In addition, their overall satisfaction with *MathLand* was mixed, and the teachers tended to disagree that *MathLand* is a well-balanced curriculum. These conflicts with the new curriculum are further highlighted in the teachers' responses to the open-ended questions about the main strength and main weakness of the *MathLand* Program. Although teachers cited the problem-solving activities, cooperative learning, and the manipulatives as the main strengths of *MathLand,* one of most frequent main weaknesses of *MathLand* reported by the teachers was lack of computation practice of basic skills. This may be an indication that these teachers still believe that mathematics is a subject of rules and procedures, to be learned through practice in mastering the basic skills.

According to Wood, Cobb, and Yackel (1991), struggling with conflicts and dilemmas are important in the change process for teachers. Teachers must experience some elements of disequilibrium so that they can question their own understandings about mathematics education reform and their own teaching practices (Ball, 1996). The conflicts these teachers have with the new curriculum and math reform could provide an opportunity for them to reflect, debate and challenge their understandings about mathematics reform education. This would be a good opportunity to shift staff development toward a format where critical discussions could take place about the rationale and theoretical perspectives underlying the mathematics reform movement. Then ideally, teachers can make informed decisions about mathematical content and instructional strategies when implementing a new curriculum.

This study looked only at one grade level of teachers during the implementation of a new mathematics program. The conclusions drawn from the results are limited to this particular group, and therefore may not be applicable to the beliefs and attitudes of the teachers of other grade levels in the district. Further research is needed with teachers of different grade levels.

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Ethnicity as a Factor in Teachers' Perceptions of the Mathematical Competence of Elementary School Students

Kathlan Latimer

The purpose of this study was to evaluate the extent to which teachers use ethnicity in attributing mathematical competence to their students. A group of elementary teachers who were teaching an ethnically diverse group of students were asked to describe a "mathematically competent" student as well as to describe the behaviors and characteristics of their most and least competent students. Although some differences were found in teachers' perceptions of student competence based on student ethnicity, greater differences were found based on teachers' perception of the level of student competence. The most common descriptors of students judged as most competent were (a) having a positive attitude and disposition and (b) being studious; the most common descriptors of students judged as least competent students were (a) having poor work habits and (b) exhibiting negative behavior.

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Effective teaching and learning are facilitated by the teacher's awareness of a student's knowledge and capabilities. Although a teacher may have a general impression of a student's abilities and have access to the student's standardized test results, it is difficult at best for the teacher to determine exactly what a student does know and how well he or she can perform. One important source of information about student knowledge and performance is teacher judgment.

This study looks at the factors teachers use to make judgments about the mathematical competence of students. The way in which teachers define mathematical competence as well as their expectations for mathematics achievement are also examined.

BACKGROUND

Currently, people of color comprise approximately 25 percent of the total U.S. population. In California, people of color represent 44 percent of the population (U.S. Dept. of Commerce, 1993), whereas 87 percent of the teaching force is European American (National Center for Education Statistics, 1995b). California's student enrollment is 5,341,025—59 percent are students of color. By the year 2000, it is anticipated that the proportion of students of color enrolled in California public schools will grow to 65 percent (*EdSource*, 1995). Who will these children? If current trends continue, these students will continue to be taught by teachers whose ethnicity and/or culture is different from their own.

In this study, I attempt to examine

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the impact, if any, of the cultural and ethnic mismatches between teachers and students. As stated by Viadero (1996), schooling does not occur in a cultural vacuum; school is a cultural enterprise. Culture, according to Anderson (1988), is built upon a philosophical worldview or conceptual system of patterns of beliefs and values that are transmitted to members of a cultural group through socialization. It has been suggested that these values and worldviews may be accompanied by differing cognitive styles and patterns of communication. Therefore, differences in culture or ethnicity may call for modifications of the pedagogical strategies utilized by teachers and a re-examination of teachers' perceptions of student ability. This study begins work in this area by examining teachers and their cultural lenses.

Teacher judgment of a student's capacity to learn, i.e. teachers' perceptions of student competence, is the focus of this study. This study examines judgments of mathematical competence that teachers make based on teachers' perceptions of their students. The use of race and ethnicity as a basis for teachers' differential expectations of students for mathematical achievement is considered. Specifically, the purpose of this study is to investigate teachers' use of ethnicity or racial identification as a factor in assigning mathematical competence to elementary level students. Of particular interest is the situation in which the ethnicity of the student does not match that of the teacher.

The research questions to be investigated are:

1. What factors do teachers ascribe to a mathematically competent student?
2. Do the factors considered vary with the ethnicity of the student?
3. Do these factors vary with the ethnicity of the teacher?

In this study, these factors included, but were not limited to, social class, gender, reputation, ethnicity/culture, physical attractiveness, verbal skills, attentiveness, maturity, and family characteristics.

METHODS

Sampling. Fourteen elementary school teachers of ethnically diverse students participated in this study. A total of 519 students were taught by these teachers. Most grade levels as well as multi-age classes were represented.

Instrumentation. A questionnaire was developed which was designed to elicit information on teachers' conceptions of mathematical competence. The questionnaire consisted of twelve items. Ten of these items called for open-ended responses; the remaining two items asked for demographic information. Specifically, each teacher who participated in this study was asked to select, in terms of mathematical competence, the top three and bottom three students in his or her class. The teachers were asked to describe the attributes and behaviors of these selected students.

Data analysis. Teacher responses to this questionnaire were analyzed to understand the bases upon which the teachers made assignments of competence. Responses to the open-ended questions were categorized according to similarity of response. These categories generated themes, which were then labeled for further analysis.

TABLE ONE Characteristics of Students Selected by Teachers (n = 84 Students)			
Ethnicity	**Most Competent**	**Least Competent**	**Total**
European-American	17	14	31
Hispanic	7	12	19
African-American	7	11	18
Asian-American	9	3	12
Filipino	3	1	4
Southeast Asian	1	0	1
Eastern Indian	3	0	3
Chinese	1	0	1
American Indian	0	0	0
Other (Biracial)	2	2	4
Bilingual Classification			
English-Only	32	33	65
Fluent English Proficiency	7	5	12
English Language Learner	3	4	7
Gender			
Male	25	20	45
Female	17	22	39

RESULTS

Demographics

Consider first teacher demographics, which are present in Table One. The 14 teachers in this study were predominantly female and, reflecting the overall district ethnicity proportions, European American. Of the 84 students selected by the teachers (42 of whom were identified as "most competent" and 42 of whom were identified as "least competent"), the largest group was European American, and most of these students were male. Interestingly, although teachers selected students from a variety of ethnic backgrounds, more European Americans students were selected as both "most competent" and "least competent" even though the classrooms of these teachers had a majority of students of color.

Actions and Observable Behaviors of Most and Least Competent Students

Teachers were asked to make two lists—one of the observable actions and behaviors of the mathematically most competent students in their classes, and one of the observable actions and behaviors of the least competent students in their classes. More specifically, for the most competent students, teachers were asked "What do these students *do* (actions, observable behaviors; e.g. eagerly share ideas in class) that distinguishes them from the rest of the class?" Teachers were again asked this question for the least competent students.

Teachers' descriptions of the actions and behaviors of mathematically "most competent" students. An analysis of teachers' responses is presented in Table Two. The descriptions of mathematically competent students fell into six main categories. A student's positive attitude/disposition was the characteristic listed most often for mathematically "competent" students. In particular, teachers listed traits such as a student's eagerness to learn, interest in work and persistence. In order of decreasing frequency of response, the other five characteristics of mathematically competent students identified by the teachers were: communication of mathematical thinking, facility with procedures and skills, understanding of mathematical concepts, reasoning ability and problem solving ability.

When asked to list observable actions and behaviors of mathematically

TABLE TWO
Teachers' Descriptions of the Actions and Observable Behaviors of Most Competent and Least Competent Students by Student Ethnicity (n = 84 Students)

Most Competent Students	European American n=17	African-American n=7	Hispanic n=7	Asian American n=9	Biracial n=2	Total 42
Attitude	17	5	9	10	2	43
Communication	6	1	2	4	1	14
Skills & Procedures	5	3	1	1	2	12
Concepts	5	4	0	1	1	11
Reasoning	2	2	2	3	1	10
Problem Solving	4	1	2	2	0	9
Least Competent Students	**European American n=14**	**African-American n=11**	**Hispanic n=12**	**Asian American n=3**	**Biracial n=2**	**Total 42**
Work Habits	13	10	5	3	0	31
Behavior	7	8	4	0	0	19
Attention	9	2	3	2	0	16
Attitude	4	5	5	1	1	16
Concepts & Skills	2	4	4	0	4	14

competent students, some variations of responses became apparent, but overall factors used were fairly evenly applied across groups. In examining what highly competent students do, responses relating to attitude appeared most often across all ethnic groups. This was reported by a large margin for all groups except for African American students. For this group, responses relating to concepts (e.g., sense of number, grasps new ideas) rivaled those relating to attitude. Other categories of response that emerged for this group were attitude or disposition, communication of mathematical thinking, reasoning, conceptual understanding, problem solving ability, and knowledge of skills and procedures.

Teachers' descriptions of the actions and behaviors mathematically "least competent" students. Teachers were also asked to list the observable actions and behaviors of mathematically "least competent" students. When looking at students identified by teachers as least competent, not only did different categories emerge, but also an emphasis on deficits rather than on actual performance was noted. Responses relating more to work habits and student behavior, particularly negative attention-seeking behavior predominated. Poor work habits were most often mentioned, but even more so, in terms of frequency, for students of color. Other factors ascribed to the least competent students were negative attitude, lack of attention or focus, inappropriate behavior, and lack of knowledge of concepts and skills.

Characteristics that Set Selected Students Apart from Other Students

Teachers were asked to make two lists—distinguishing characteristics of the mathematically most competent students, and one of the distinguishing characteristics and least competent students in their classes. More specifically, for the most competent students, teachers were asked "Are there characteristics (personality traits, etc.) which set them apart from the other students. " Teachers were again asked this question for the least competent students.

Results are presented in Table Three. In terms of the most competent students, studiousness emerged as the top characteristic for all ethnic groups of students except Hispanic/Latino students; for this group quietness was listed more than any other response. In terms of the least competent students, different characteristics emerged; for these students the most frequent category listed was inappropriate attitude. However, behavior was identified most frequently for the least competent European Americans and introversion was identified most frequently for Hispanics. It is of further interest that yet again entirely different categories emerged for students seen as least competent rather than the negation of those listed for highly competent students, thus questioning if and how the perception of ability impacts the mathematical experiences of these students.

TABLE THREE
Teachers' Descriptions of Characteristics that Set Selected Students apart from Other Students (n= 84 Students)

Most Competent Students						
	European American (n=17)	African-American (n=7)	Hispanic (n=12)	Asian American (n=9)	Biracial (n=2)	Total 42
Studiousness	7	6	2	7	1	23
Popularity	6	2	0	2	1	11
Confidence	5	1	2	1	0	9
Quietness	1	2	4	1	0	8
Curiosity	4	1	1	0	1	7
Sense of Humor	2	1	0	2	0	5
Leadership	1	1	1	1	0	4
Least Competent Students						
	European American (n=14)	African-American (n=11)	Hispanic (n=12)	Asian American (n=3)	Biracial (n=2)	Total 42
Attitude	5	5	4	2	3	19
Behavior	7	3	2	2	0	14
Work Habits	4	3	3	1	1	12
Introversion	0	4	5	0	1	10
Attentiveness	3	1	3	0	1	8
Attendance	0	3	2	0	0	5
Home Support	0	3	1	0	0	4
Maturity	2	0	0	0	1	3

Differences in Perceptions Based Upon Teacher Ethnicity

Descriptive statistics for this analysis are presented in Table Four. First consider European American teachers' perceptions. In terms of teacher ethnicity, European American teachers applied factors evenly except in the case of positive disposition or attitude when judging the most competent African American students. For African American students, positive disposition and attitude was noted by European American teachers equally as often as knowledge of mathematical concepts. For all other groups of students, positive disposition /attitude was clearly listed by European American teachers most often. When considering the least competent students, European American teachers listed factors that were fairly representative across all groups. Work habits were mentioned most often by European American teachers for all student

TABLE FOUR
Ethnicity of Students Judged Most Competent and Least Competent by Ethnicity of the Teacher (n = 14 Teachers)

Most Competent Students					
	Teacher Ethnicity				
Student Ethnicity	European America) (n=10)	African-American (n=2)	Hispanic (n=1)	Asian American (n=1)	Total =42
European-Amer	12	3	0	2	17
African-Amer	5	1	0	1	7
Hispanic	4	0	3	0	7
Asian-Amer	9	0	0	0	9
Biracial	0	2	0	0	2
Least Competent Students					
Student Ethnicity	European America (n=10)	African-American (n=2)	Hispanic (n=1)	Asian American (n=1)	Total
European-Amer	10	2	0	2	14
African-Amer	9	1	0	1	11
Hispanic	6	3	3	0	12
Asian-Amer	3	0	0	0	3
Biracial	2	0	0	0	2

ethnic groups except Hispanic students for whom attitude was chosen as often and for biracial students for whom knowledge of concepts and skills was a clear choice.

There were too few teachers of color to make general statements about specific teacher ethnic groups. Therefore the responses of all teachers of color were combined for analysis. No teacher of color selected Asian American students; all teachers of color selected European American students when such students were a part of the class. Similar to European American teachers, teachers of color also selected more students of color as least competent than most competent. Attitude and disposition figured prominently in their responses for most competent students; factors applied to least competent students were scattered across categories.

DISCUSSION

Based on the top ranked responses, a profile of teachers' perceptions of the characteristics of a mathematically competent student emerged from the data. Such students are curious, excited, and motivated to learn mathematics. Their attitude towards mathematics is positive and they exude confidence. According to these teachers, these students have good concepts of number and a highly developed, as developmentally appropriate, number sense. They are problem solvers; they are able to reason logically and use critical thinking skills. They are persistent, take risks, and learn from their mistakes. Mathematically competent students understand what mathematics is about as a discipline and can make connections within the discipline and to the world around them. For the majority of teachers surveyed these attributes set the stage for successful mathematical achievement.

The attributes of mathematical competence described by these teacher parallel themes detailed in the goals for students in the assessment of mathematical power in the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989). In particular, the *Standards* describe the importance of learning to value mathematics, becoming confident in one's own ability, becoming a mathematical problem solver, learning to communicate mathematically, and learning to reason mathematically. Additionally, the description of mathematical competence based on teacher responses is strikingly similar to the NCTM notion of mathematical power:

> This term denotes an individual's abilities to explore, conjecture, and reason logically, as well as the ability to use a variety of mathematical methods effectively to solve non-routine problems. This notion is based on the recognition of mathematics as more than a collection of concepts and skills to be mastered; it includes methods of investigating and reasoning, means of communication, and notions of context. In addition, for each individual, mathematical power involves the development of personal self-confidence. (p. 5)

In examining descriptions of highly competent students, factors relating to attitude appeared most often across all ethnic groups. Although most factors that teachers reported related closely to those detailed in the NCTM's description of mathematical power, a few teacher comments related to other factors such as attentiveness and study skills. When looking at students judged as least competent, by comparison, responses that related more to student behavior, particularly negative attention-seeking behavior, and work habits predominated. This suggests that the management of student behavior may become the focus for those students who are not seen as competent. This may lead to a downward spiral--students do not attend to mathematics instruction or experiences, exhibit acting out or apathetic behavior, and miss out on more mathematics. And so the cycle continues, enabling the downward spiraling, in terms of mathematics achievement, of these students.

In terms of characteristics of the most competent students, being hardworking or studious were factors used most often to describe these students across all groups, whereas negative or inappropriate attitude was seen most often for the least competent students. Thus entirely different categories again emerged for students seen as least competent, thus raising the question of how teachers' perceptions of ability impact the mathematical experiences of these students.

Even in diverse classrooms, European American students were selected more often—in both the "most" and "least" competent student categories—relative to their proportion of classroom enrollment. Bilingual students were judged most competent at about the same rate as they were judged least competent. Yet on the whole, factors were applied evenly across all ethnic groups; the major differences in responses were based on perceived level of competence.

Based on this study, the ethnicity of the student does not appear to be a primary factor in assigning competence. Likewise, the ethnicity of the teacher does not appear to be a primary factor in assigning competence. Yet teachers' comments on culture and ethnicity were quite interesting. Many teachers embraced the importance of culture, but eschewed the idea of differential treatment of individual students based on culture. However, according to Anderson (1988):

> Whereas it was once fashionable and sometimes academically rewarding to deny the existence of cultural assets and variation among non-white populations, social scientists and researchers now recognize that such traditional approaches have become anachronistic ... A different set of understandings about the way diverse populations communicate, behave, and think needs to be developed by educators. (p. 8)

Emphasis on culture or ethnicity can be construed negatively, as indicated by Knapp and Turnbull (1990):

> First, stereotypic ideas about the capabilities of a child who is poor or belongs to an ethnic minority will detract from an accurate assessment of the child's real educational problems and potential. By focusing on family deficiencies, the conventional wisdom misses the strength of the cultures from which many disadvantaged students come. (p. 5)

The influence of culture and ethnicity can be positive, but these advantages are overlooked. This may be due to the historically held, but currently untenable, goal of a colorblind society. Although it may be prudent to be objective in terms of assessing students' ability, overlooking the impact of culture creates a missed opportunity. This missed opportunity results when teachers fail to look to culture and ethnicity as sources of valuable, pertinent information. Given that the culture and ethnicity of teachers and students will match less often as population demographics change, this is a critical area. As noted by Anderson (1988):

> At the superficial level, cross-racial, cross ethnic teaching, learning and/or service delivery occurs when the persons interacting are of different racial or ethnic identities. When one adds to this equation the differences in degrees of acculturation and type of cognitive/learning style, the examination and explanation of these differences becomes more complicated and the urgency to identify the critical dimensions associated with them more pronounced. (p.8)

Thus the mismatching may be even broader than demographics alone would suggest.

In conclusion, more differences were seen based on perceived level of ability rather than on any other factor. Much of this may be attributable, however, to the inability on the part of teachers to allow for and/or acknowledge the impact and importance of culture and its role in their own assignment of competence. Yet this, the consideration of culture, is the first step in moving from mere acknowledgement to the affirmation of culture and ethnicity as well as a step towards taking advantage of what up to now has been a missed opportunity, the opportunity to embrace culture as an asset rather than a liability.

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San Francisco State University College of Education *Review* Manuscript Submission and Review Process

The COE Review is published annually by the Research and Development Center of the College of Education and San Francisco State University. It is designed to stimulate thinking about a variety of educational issues and to foster the creation and exchange of ideas and research findings in a manner that will expand knowledge about the purposes, conditions and effects of schooling. The focus of the journal is on best practice, supported by research, theory, or experience. Manuscripts on a variety of tropics are welcome, as long as they are related in some way to educational issues and concerns:

- conceptual pieces;
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All manuscripts submitted for publication must conform to the style of the *Publication Manual of the American Psychological Association* (3rd ed.). Manuscripts should be typed on 8.5 x 11 inch paper, upper and lowercase, double-spaced, with 1.5 inch margins on all sides. Subheads should be at reasonable intervals to break the monotony of lengthy texts.

Four copies of each manuscript (one original and three copies) must be submitted. All manuscripts are sent (names removed) to three outside reviewers. As such, the complete title of the manuscript and author(s) name(s) should be listed on a separate sheet of paper to assure anonymity in the review process, and the first text of the article should have the complete title of the manuscript, but no list of author(s). In addition, please send a 3.5" disk containing your manuscript (including any tables and graphics), formatted in Microsoft Word for the Macintosh. Once reviewed, manuscripts are accepted, accepted pending revisions, or rejected.

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