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ABSTRACT

This publication is a compilation of examples of practical, easily implemented activities to help mathematics, science, and education faculty duplicate efforts by the Rocky Mountain Teacher Education Collaborative (RMTEC) to reform and revise curriculum for preservice educators. Activities are organized by content areas: mathematics; geology, physics, and biology; chemistry; and educational methods. Each activity lists the RMTEC strategies relevant to that activity, the class in which the activity was used, and the instructors. There is some repetition among activities, since each is intended to stand alone. Activity handouts for students are in boxes and may be reproduced. Contact information is listed for participants in the production of this publication. The publication concludes with a description of searching the ERIC Database on science and mathematics topics. (SM)

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Reinventing the Undergraduate Curriculum:

Strategies to Enhance Student Learning in Mathematics and Science

Compiled by Barbara J. Nelson and
Barbara K. Wallner

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This project was funded by the National Science Foundation under the Collaborative for Excellence in Teacher Preparation initiative.

RMTEC

Rocky Mountain Teacher Education Collaborative
April 2000

Colorado State University
Fort Collins, Colorado, USA

Rocky Mountain Teacher Education Collaborative

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Re-inventing the Undergraduate Curriculum

Introduction

As universities are asked to be more and more accountable, many institutions are rethinking the undergraduate experience. This typically entails a review of general education core requirements as well as content delivered within majors; how that content is delivered; evaluation of effective teaching and advising; and most importantly, an analysis of how to move toward a student-centered learning community.

In most professions, unqualified individuals are not allowed to practice. These professions set standards for preparation that individuals must meet for the state to issue a license. However, in teaching this is not always the case. Indeed, many higher education faculty enter academia with a desire to disseminate and discover knowledge, but have not been taught pedagogy or its relationship to learning outcomes. The Rocky Mountain Teacher Education Collaborative (RMTEC), funded by the National Science Foundation, is an endeavor to both improve practice, as well as to inform the knowledge base and professionalize teaching within the academy. This means, of course, that we believe we can improve practice and positively impact our learning community.

As faculty move toward increasing student-centered activities and placing an emphasis on learning, they must take into account the needs of individual students and also reflect on their current practice. As they begin reform efforts, faculty soon recognize the isolation and departmentalization of teaching in higher education. Administrators must realize that innovations take time. They need to encourage experimentation, and recognize the necessity of supporting faculty. If faculty are willing to readjust and rethink their curriculum, they must be bolstered in these endeavors. This means that department heads and deans must encourage a climate that promotes student-centered learning and values teaching and learning. This value should be demonstrated by giving teaching and advising awards, creating other forms of recognition to acknowledge good teachers, and building it into the reward structure.

As we think about it, change efforts currently underway in K-12 cannot succeed without changes in the way higher education does business. The early work of K-12 reform was built largely on standards-based education, developing assessments and designing accountability systems. Now that reform is finally in place, we must be able to show the relationship among standards identified for K-12 learners, what a student knows and is able to do, and the requirements in higher education.

Why should higher education be involved in K-16 reform? First, we must begin to look at some of the issues that face higher education related to K-16 reform. One of the most significant issues is the large number of entering freshmen who need remediation. Another factor is the pressure to reduce or eliminate the use of race in college admissions. The final issue that college leaders must address is the growing recognition that if K-16 reform is to be effective, we must improve our public schools. If we are to address problems facing higher education and society today, we — K-12 and higher education—must work together to make our basic institutions work for all the students they serve. This means that we must reassess our orientation to students, our basic means of assessing student knowledge, and our willingness to rethink how we teach at all levels. It is just as important to value good teaching in our colleges and universities as it is in our elementary schools. As educators, we are committed to preparing our future citizens—an extraordinary responsibility. The RMTEC initiative was designed to assist educators in fulfilling our responsibilities.

— Nancy K. Hartley
Dean, College of Applied Human Sciences, Colorado State University
Principal Investigator, Rocky Mountain Teacher Education Collaborative

RMTEC Background

In 1994, the National Science Foundation awarded the Rocky Mountain Teacher Education Collaborative (RMTEC) one million dollars per year for up to five years. The grant was made to establish a collaborative designed to reform the way in which mathematics and science pre-service teachers were educated, and to work to increase the number of under represented population members who become science and mathematics teachers. RMTEC is designated as a Collaborative for Excellence in Teacher Preparation (CETP).

Six institutions of higher education comprise the collaborative: Colorado State University, Metropolitan State College of Denver, University of Northern Colorado (designated as the primary institutions); Aims Community College, Community College of Denver, and Front Range Community College.

RMTEC goals are:

- To develop collaboration between the primary institutions, community colleges, and local school districts.
- To improve the ways in which mathematics and science pre-service teachers are prepared for careers in teaching, with emphasis upon restructuring, reforming, and/or developing innovative curricula and instructional methods for teaching education, mathematics, and science.
- To recruit and retain those sensitive to issues of women and ethnic minorities into teaching careers in fields of mathematics and science.

Administrators, faculty, and members of other educational institutions, including public schools across the Front Range of Colorado, have collaborated to revise courses and curriculum, conduct research, provide scholarships, train teachers, raise diversity awareness, and disseminate information through workshops, professional products, websites, and other avenues. Expectations of RMTEC faculty members were defined at the outset of the project. Checklists were established to facilitate collection of demographic information, revision of curriculum, collection of evaluative information to determine student perceptions of revised courses, and to assess the effect of reforms on student achievement.

The Instruction vs. Learning Paradigm

RMTEC promotes reforms that support a shift from an instructional paradigm that is teacher centered to a learning paradigm that is student centered. The following highlights were adapted from *From Teaching to Learning – A New Paradigm for Undergraduate Education*, by Robert B. Barr and John Tagg. They serve to help conceptualize what a student centered learning environment should encompass.

The Instruction Paradigm	The Learning Paradigm
Mission and Purposes	
Provide/deliver instruction	Produce learning
Transfer knowledge from faculty to students	Elicit student discovery and construction of knowledge
Offer courses and programs	Create powerful learning environments
Improve the quality of instruction	Improve the quality of learning
Achieve access for diverse students	Achieve the success for diverse students
Criteria for Success	
Inputs, resources	Learning and student-success outcomes
Quality of entering students	Quality of existing students
Curriculum development, expansion	Learning technologies development, expansion
Quantity and quality of resources	Quantity and quality of outcomes
Enrollment, revenue growth	Aggregate learning growth, efficiency
Quality of faculty, instruction	Quality of students, learning
Teaching/Learning Structures	
Atomistic; parts prior to whole	Holistic; whole prior to parts
Time held constant, learning varies	Learning held constant, time varies
50-minute lecture, 3-unit course	Learning environments
Independent disciplines, departments	Cross discipline/department collaboration
Covering material	Specified learning results
End-of-course assessment	Pre/during/post assessments
Grading within classes by instructors	External evaluations of learning

Learning Theory

Knowledge exists "out there"	Knowledge exists in each person's mind and is shaped by individual experience
Knowledge comes in "chunks" and "bits" delivered by instructors	Knowledge is constructed, created, and delivered by instructors
Learning is cumulative and linear	Learning is nesting and interacting of frameworks
Fits the storehouse of knowledge metaphor	Fits learning how to ride a bicycle metaphor
Learning is teacher centered and controlled	Learning is student centered and controlled
"Live" teacher, "live" students required	"Active" learner required, but not "live" teacher
The classroom and learning are competitive and individualistic	Learning environments and learning are cooperative, collaborative, and supportive
Talent and ability are rare	Talent and ability are abundant

Productivity/Funding

Definition of productivity: cost per hour of instruction per student	Definition of productivity: cost per unit of learning per student
Funding for hours of instruction	Funding for learning outcomes

Nature of Roles

Faculty are primarily lecturers	Faculty are primarily designers of learning methods and environments
Faculty and students act independently and in isolation	Faculty and students work in teams with each other and other staff
Teachers classify and sort students	Teachers develop every student's competencies and talents
Any expert can teach	Empowering learning is challenging and complex
Line governance; independent actors	Shared governance; teamwork

Adapted from, *From Teaching to Learning—A New Paradigm for Undergraduate Education*, Robert B. Barr & John Tagg, *Change*, November/December 1995, pp. 16-17. Reprinted with permission of the Helen Dwight Reid Educational Foundation. Published by Heldref Publications, 1319 18th St. NW, Washington, DC 20036-1802. Copyright 1995.

Revising, Restructuring, and Designing Curricula

RMTEC developed the following checklist to guide the construction, development, and reform of courses. As instructors developed reforms, they tested them against this list to determine if their approach would:

- Engage students in inquiry-based, hands-on, minds-on science, mathematics, and education coursework, activities, and field experience.
- Emphasize classroom teaching strategies, such as cooperative learning, that are successful with women and students of color.
- Be consistent with existing and emerging national standards for curriculum, teaching, and assessment in mathematics, science, and education.
- Be learner centered and encourage student initiative and design, and allow students to take responsibility for their own learning.
- Involve a cohort or community of learners.
- Demonstrate a variety of approaches to teaching and learning.
- Use technology to support learning.
- Integrate mathematics, science, and education.
- Involve collaboration among faculty in departments, clusters, and the collaborative.
- Result in varied forms of student projects that demonstrate students' individual attainments of the learning goals.
- Have teacher-, peer-, and self-assessment embedded throughout.
- Involve reading, writing, and communication skills to develop student learning.
- Emphasize the development of skills in teaching specific subject matter content to a variety of students in a variety of contexts.
- Build on research and practice about best models.
- Use input and needs assessments of teachers and students.
- Effectively incorporate the knowledge and skills of experienced teachers.

Meeting the Needs of Underrepresented Populations

RMTEC used another set of guidelines to ensure that revised curriculum and instructional strategies would meet the needs of individuals from underrepresented populations who are entering careers as mathematics and science teachers. When developing curriculum and instructional strategies, instructors asked themselves:

- What insights have you applied about "teacher attitudes" which will promote the achievement of men and women from diverse cultural groups?
- What techniques will you use to build a sense of community among learners in your class?
- What strategies will you use to connect your curriculum to the concerns of women and/or people of diverse cultures?

- What materials, examples, and role models do you use that convey the idea that science and math contributions are global, representing men and women of all cultures?
- What teaching or assessment techniques (other than lecture and "traditional tests") will you use that either promote or reflect significant learning of women and diverse cultural groups?
- What communication skills or interaction patterns have you developed that show enhanced sensitivity to women and diverse cultural groups?
- What kinds of out-of-class campus activities do you participate in to promote retention of women and diverse cultural groups?
- How do you intend to show students they belong in the class and in the teaching fields of science and mathematics?

Teaching Strategies

The following is a list of teaching strategies RMTEC identified that are consistent with the overall goal of the Collaborative—to enhance student learning in mathematics and science. All RMTEC activities were assessed with regard to frequency of use of these teaching strategies.

- Cooperative groups
- Use of technology
- Use/solution of real world applications/problems
- In-class discussion
- Ongoing student self-evaluation techniques
- Activities involving writing
- Alternative assessments
- Consistency with state standards
- Making connections to other fields (science and non-science)
- Constructivist methods
- Solution of complex problems by students
- Attention to creating learning settings sensitive to cultural differences
- Attention to creating learning settings sensitive to gender issues
- Chance for students to ask questions in class

Reinventing the Undergraduate Curriculum: Strategies to Enhance Student Learning in Mathematics and Science

This publication is a compilation of examples of practical, easily implemented activities to help mathematics, science and education faculty duplicate RMTEC efforts to reform and revise curriculum. Activities are organized by content areas — Mathematics; Geology, Physics, and Biology; Chemistry; and Educational Methods. Each activity lists the RMTEC strategies relevant to that activity, the class in which the activity was used, and the instructors. There is some repetition among activities, since each is intended to "stand alone."

Some activities were excerpted in partial form from other publications, so they may refer to other "chapters," etc. that are not relevant to this publication. Activity "handouts" for students are in boxes and may be reproduced. Contact information for participants in this publication is located on the list of contributors. Faculty members would welcome contacts from colleagues regarding their reform efforts, helping them to extend the "reach" of the RMTEC initiative.

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1 Mathematics

3 x 5 Index Card Proofs

Strategies:	**Constructivist methods** **Solution of complex problems by students**
Class:	**Abstract Algebra (Mathematics 321)**
Instructors:	**Richard Grassl** **Tabitha Mingus** **University of Northern Colorado**

Introduction: This class is a reformed abstract algebra class at UNC. The majority of the 24 students are math majors with a secondary teaching emphasis. The students will soon be practicing teachers in sole charge of developing and implementing curriculum change in their own secondary classrooms. Their exposure to a variety of teaching styles plays a dominant influential role in how these changes occur.

Course Characteristics: The class is broken into six cooperative learning groups of about four students, half men—half women in each group, seated at hexagonal shaped tables. Groups are fixed for half of the semester, then rotated for the second half. Networking is encouraged; phone numbers are exchanged; students are urged to consult with one another on homework, course work, and for complete information when absent. A significant portion of each class is devoted to group work. Each period starts with a five to ten minute session. During this time, students consult with one another on homework questions and the reading as the instructors walk from one table to another in a support role, asking probing questions to direct them down more fruitful avenues, but rarely giving, at this time, complete solutions. After allowing sufficient time for struggle, conjectures, and understanding, unanswered questions are dealt with by the instructor on the board. This technique saves time and seems to develop confidence in the students, a better attitude and a deeper sense of understanding the content.

Activity: 3 x 5 Index Card Proofs
Each student selects one card containing a result in abstract algebra from a deck of thirty cards and becomes responsible for preparing and delivering a "perfect" five- minute presentation to the class. Prior to the presentation, the student is required to go over the proof with one of the instructors. Often it takes several tries, but the contact with one of the two instructors is extremely valuable from a mentoring point of view. Language is improved, concepts become clearer, confidence level is soaring, and they are driving each other to greater successes. Research shows that

student-faculty contact is extremely important in student attitudes, willingness to persist, to achieve understanding, and to take risks. A new level of expectation is experienced. Extra cards with additional results are available, allowing students the opportunity for further challenge.

Thirty index cards were prepared, each containing a specific result that required a formal proof.

Example of a 3 x 5 Index Card Proof:

Let H be a subset of an abelian group G that consists of all h such that $h = h^{-1}$. Prove that H is a subgroup of G.

Four Faces of a Function

Strategy:	**Constructivist methods**
Class:	**College Algebra in Context (Mathematics 180)**
Instructors:	**Glenn Bruckhart** **Kate Fellenz** **Ken Klopfenstein** **Colorado State University**

Introduction: The course, *College Algebra in Context*, reflects national recommendations that mathematics curriculum should be context driven, constructivist in approach, include cooperative problem solving, stress communication with and about mathematics, and build connections between different areas of mathematics and with the real world. At Colorado State University, this course and *Euclidean and Non-Euclidean Geometry* provide "bookends, a first impression and a memorable conclusion, that serve as role models for future teachers of mathematics."

College Algebra in Context, an introductory course, serves as a role model to attract students (particularly students from underrepresented groups) into a teaching career, and provides practical classroom experience for advanced undergraduates who are majoring in Mathematics Education. Students work in groups of 4 or 5 in a large classroom. Graduate and advanced undergraduate assistants each have responsibility for working closely with 25 students (usually 5 groups of 5); reading homework and responding to student questions. The classroom design facilitates small group interaction but allows scaling to a large number of students at moderate cost. A typical classroom session may include slightly more than half the class period with students actively engaged in group activities that reinforce concepts using graphing calculators, slightly less than half in whole-class discussion.

The course is designed to reflect the ideals reflected in the Curriculum Standards of the National Council of Teachers of Mathematics and the reform movement at the collegiate level most often identified as reform calculus. In particular, *College Algebra in Context* was designed around the following principles:

Context Driven: New ideas are introduced in the context of real problems.
Constructivist: Students construct their own knowledge.
Cooperative Problem Solving: Students interact with other students to solve problems.
Communication: Students communicate with and about mathematics.
Connections: Students continually reinforce connections between algebraic, numeric, and geometric representations.

The course uses materials from three sources: course materials prepared by the instructors and distributed in class at no charge; *TI-82 Graphics Calculator Guidebook*; and reference textbooks that may be checked out from the Resource Desk in the IPM Center and used in the Tutoring Center. The enrollment is expected to level out at between 300 and 500 students per semester and will directly involve between 12 and 20 Math Education course assistants.

Activity: Four Faces of a Function

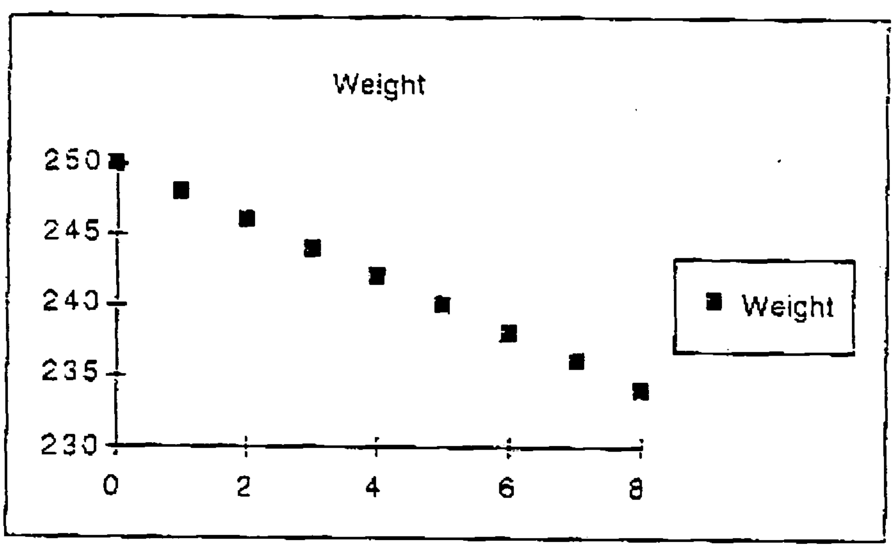
The following example illustrates the four faces of a function, showing vital connections within mathematics and with a context:

1. A Context: *Glen weighs 250 on January 1, decides to change his lifestyle to include a proper diet with plenty of exercise, and loses 2 pounds each week.*

2. A Table of Values:

Week	0	1	2	3	4	5	6	7	8
Weight	250	248	246	244	242	240	238	236	234

3. A Formula: $y = 250 - 2x$
4. A graph:



To Model a Cloud

Strategies:	**Cooperative groups** **Constructivist methods** **Hands-on learning**
Class:	**College Algebra in Context (Mathematics 180)**
Instructors:	**Glenn Bruckhart** **Kate Fellenz** **Ken Klopfenstein** **Colorado State University**

Introduction: The course, *College Algebra in Context*, reflects national recommendations that mathematics curriculum should be context driven, constructivist in approach, include cooperative problem solving, stress communication with and about mathematics, and build connections between different areas of mathematics and with the real world. At Colorado State University, this course and *Euclidean and Non-Euclidean Geometry* provide "bookends, a first impression and a memorable conclusion, that serve as role models for future teachers of mathematics."

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Activity: Sample Plan, Session 8, To model a Cloud*

Time

15 min. 1.Opening Exercise

- A. Sit with your permanent group (Keep chair arrangement tight).
- B. Have the runner come up and get the group's folder.
- C. Complete the Exercise 1.6.3 side of the group sheet.
- D. Each group will have one of the instructors working closely with it. Find out who your assigned instructor is.
- E. Make your group folder distinctive. (Or if you would like to provide a more elaborate folder, plan how that will happen.)
- F. Exchange phone numbers in group.

10 min.(class) 2. Project 1 Discussion

25 min.(group work)

15, 40 min. 3. Expanding the 3-point problem to a cloud of data

- A. The averaging of points activity

20, 60 min.
- B. Divide the cloud into three sections, average, and complete the three point analysis on it.

10, 70 min. 4. In groups, attack homework exercise.

5, 75 min. 5. Closing Exercise

- A. Summary
- B. Place your Session 8 group sheet in your group folder. Write it up nicely! Make sure everyone's name is on it with project 1 tasks. Runners bring to assistants.
- C. Assignment: Complete 1.7.1, 1.7.2, 1.7.3.
- D. Return seats to the original positions.

*Selected handouts from this assignment are included.

Narrative of Implementation of Plan - Session 8, To Model a Cloud

11:00 Opening Exercise on overhead: A. Sit with your permanent group (Keep chair arrangement tight). B. Have the runner come up and get the group folder. C. Complete the Exercise 1.6.3 side of the group sheet. D. Each group will have one of the instructors working closely with it. Find out who your assigned instructor is. E. Make your group folder distinctive. (Or if you would like to provide a more elaborate folder, plan how that will happen.) F. Exchange phone numbers within your group.

Students rearrange chairs and start discussion without prompting. Most groups have chairs tightly grouped, and instructors are able to move around fairly well.

11:05 Students are still engaged. I count 53 students. Several groups have only 2 or three members. 54. Instructors are talking with individual groups.

11:10 Continue in small groups with no whole-class discussion. Only an occasional raised hand. here seem to be plenty of instructors to talk with groups. 55.

11:15 Groups continue to appear to be engaged in quality discussion. Students are beginning to look over today's student packet (Session 8). Instructors are initiating discussion, rather than waiting for raised hands. (How are you coming on 1.6? Make sure you understand...)

11:18 "Ladies and gentlemen!" Glenn discusses project description. Each member is to have assigned duties. Each person has specific responsibilities. Example of such responsibilities are identified. You will need to include graphs; computer generated or comparable quality.

11:20 Glenn continues... Word processed, not hand written. Someone to process data. Researcher; to define the question and connect data you choose to people who might actually be interested in such data. For example, forensic evidence, fabric and clothing. Example of jobs are given, some groups have only 2 people here today, you decide on specific jobs, give expectations for each job. Will use expectations to evaluate project.

11:25 Glenn continues... "Are there any questions?" We will fill in more information each day until projects are due. "Do we have actual data? Or are you giving it to us?" Glenn: "You took this data last time. This is not data we gave you. You use the data you generated." "Explain this assignment." Glenn: "You are putting together a research project. There are lots of questions concerning measurements of the human body. Decide who might be interested in a particular question that you state, and answer it. Talk to someone. If you need other specific information, we can gather it." "Give two examples of suitable questions?" "Is there a linear relationship between two different body measurements?"

11:30 "Will we use just our data, or the whole class data?" "The whole class. A small number of points may not be very reliable. We want a good model, so you will need to use a large cloud of data. Here is a typical cloud of data (Overhead from last time; lots of points with circles around three clumps of data). With real data, you may not get perfectly linear data. We know how to work with 3 points. Our plan is, reduce clouds to three points. Learn how to do an easy problem; then reduce a hard problem to an easy problem.

11:35 What is averaging? Question: Here are two stacks of pennies (on overhead). How can I create two equal stacks? Take some from tall stack and put them on the short stack. Start with 3 stacks. How can I calculate an average? Ans 1: Add em up and divide by 3. Ans 2; Make stack equal. Averages is more than adding up and dividing. You have a sheet that looks like this (graph paper with sets of 2; 3; 4; 5 points). Start with case 1. Decide as a group, if you had to pick one point to represent clump, which point would it be?

11:40 Group activity. Find one point in each case that best represents the collection of points. Integer arithmetic. First two points are included in 3-point problem; 3 points are included in 4-point problem; etc. Glenn, return to whole class discussion. "Here's what I'm seeing. To calculate the average of points, do an average for the x-coordinates, then for the y-coordinates. You should have gotten nice (whole number) answers." Glenn creates a batch of points at random. "I hope they don't come out nice." Return to group activity.

11:45 Glenn puts values on overhead that a few of the groups generated. Seems to be general agreement. "Let's return to the cloud from last time. We know how to do 3 points; we know how to take an average of several points. This example has 10 in each extreme clump, but only 9 in the middle. Will that make much of a difference? Divide them up as evenly as possible, take averages, find a fitting line. Look at 1.7. The 12-point cloud comes out even. Look at the details. Look at the 20-point cloud. Read the section and fill in the details. There are 3 problems at the end. Notice that the problems are related."

11:50 Question: What is special about 3 points? Glenn: Nothing. Could we use a different scheme? Yes. What we are doing is reducing a hard problem to an easy problem that we know how to do. Return to group activity, looking at 12- and 20-point clouds. Groups are actively engaged in discussion.

11:55 Group activity continues. A few questions. One or two (out of 54) appear to be day dreaming. Almost all are on task. Group members are comparing and explaining their work. Most students seem to be doing pencil calculations; a few are actively using their calculators.

12:00 There are 55 students, and a few have already left. Students appear comfortable moving around, or leaving the room for a few minutes to return later. There must have been 57 or 58 at the peak. Students continue group activity. Group interaction varies greatly from group to group. Some work in pairs, some as individuals, some as groups of 5.

12:05 Most students are still engaged in group activity. 12:08 Kate gets everyone's attention. Remember the homework, if your group has missing male/female data, please add that. Before you go, Glenn wants to talk about your exams. One of the questions in forensic science, Are these the bones of a male or of a female?

12:10 Glenn: The tests are for more than just numbers. There will be activities to follow upon the exams. Old exams will become useful for future learning. We hope to get the exams back by next Thursday. Will talk a lot about exams. Students are itching to get out of the room; their bags are packed. Students have returned room to reasonable facsimile of initial conditions.

Activities: To Model a Cloud

1.7 Reduction to the Three-Point Problem

Experiments and measurements of real objects often involve large datasets. The previous section looked only at one-, two-, or three-point datasets. In this section we look at the problem of reducing a cloud of data points into a set of three points, then use the method given in the previous section to fit a line to the three-point dataset.

To analyze a dataset, the first thing to do almost always is to generate a scatter plot of the data. If it looks linear, it probably is. Otherwise, a different model may be more appropriate. Later chapters will look closely at a few of these other models. If there are more than three points, we first reduce the problem to a 3-point problem.

1.7.1 A Twelve-Point Cloud

For example, suppose there are 12 points, given by the following table of values.

x	10	13	16	17	21	23	25	27	31	33	36	40
y	23	30	33	38	45	53	60	59	65	72	73	78

We first generate a scatter plot of the data, as given in Figure 8.

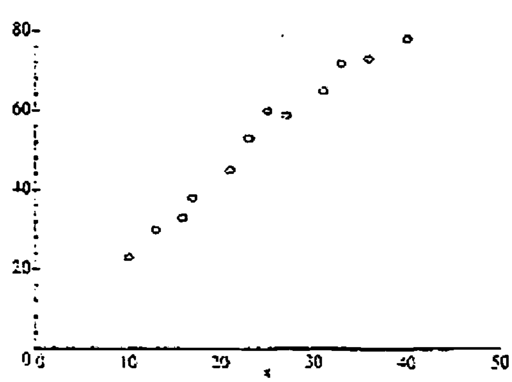


Figure 8

We next clump the points into groups of four, with the first four points drawn as circles, the next four as boxes, and the last clump of four as diamonds in Figure 9.

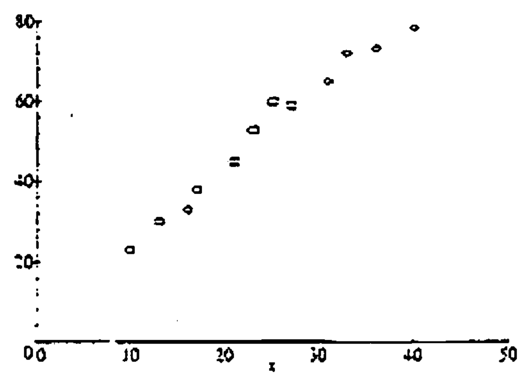


Figure 9

The problem remains of how to collapse four points into a single point.

Geometrically, the average of the two points (1, 5) and (3,1) is the midpoint (2, 3) of the line segment joining the two points, as illustrated in Figure 10. Algebraically, each coordinate of the point (2,3) is the average of the corresponding coordinates of the two points (1, 5) and (3, 1): that is, $2 = \frac{1+3}{2}$ and $3 = \frac{5+1}{2}$

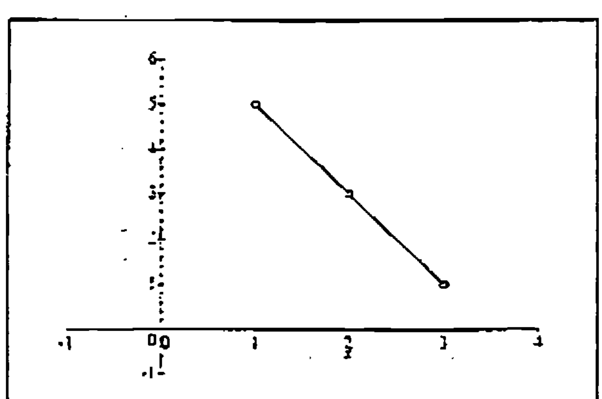


Figure 10

The average of a larger collection of points can be found in a similar manner. For example, given the first four points (10, 23), (13, 30), (16, 33), and (17, 38) in Figure 9, the average point has first coordinate $\frac{10+13+16+17}{4} = 14$ and the second coordinate is $\frac{23+30+33+38}{4} = 31$. In Figure 11, the original four points are drawn with circles, and the average is drawn with a cross.

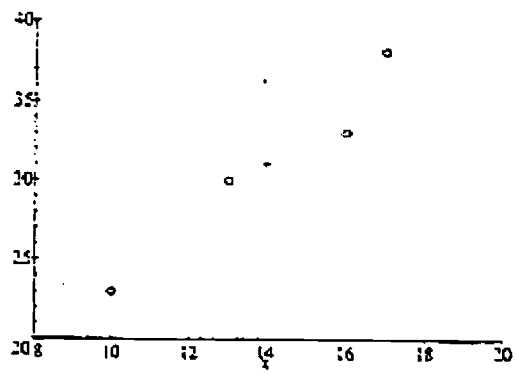


Figure 11

Do the same thing with the middle clump of points to get the average point (24, 54.25) and with the extreme points on the right to get the average point (35, 72). These points are drawn with crosses in Figure 12, first together with the original 12 points, then as a three-point set.

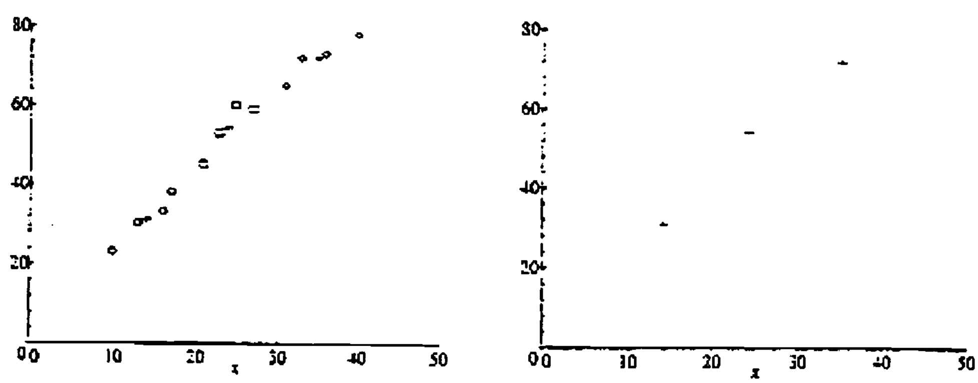


Figure 12

The line through the extreme points (14, 31) and (35, 72) has equation $y = 3.667 + 1.9524x$. The line through the middle point (24, 54.25) with the same slope has equation $y = 7.3929 + 1.9524x$. The desired y-intercept is $b = \frac{2x3.6667+7.3929}{3} = 4.9088$, so the desired line has equation $y = 4.9088 + 1.9524x$. The line is shown with the reduced set of three points in Figure 13a, then with the original cloud of 12 points in Figure 13b.

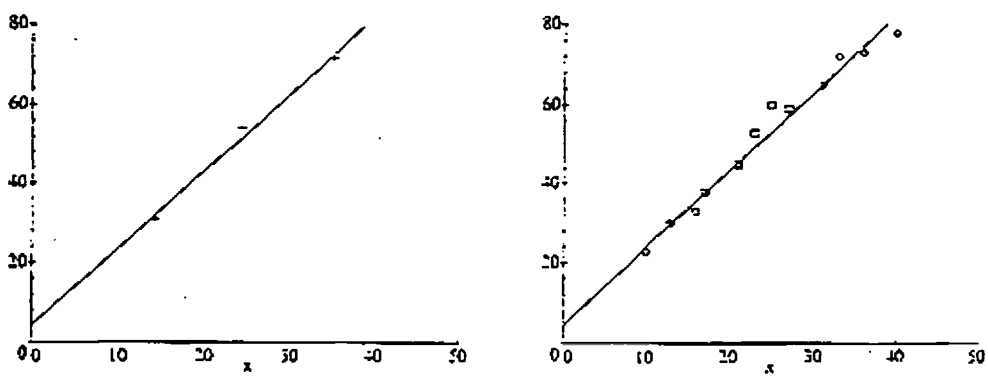


Figure 13ab

1.7.2 A Twenty-Point Cloud

Here is a cloud consisting of 20 data points, collected by a researcher during a controlled experiment.

x	46	44	12	30	15	13	6	33	43	4
y	130	121	55	85	50	49	43	86	109	32
x	26	23	4	45	21	34	3	5	27	2
y	73	76	42	114	65	97	26	42	89	27

Figure 14 shows a scatter plot of these points, broken into 3 clusters. The first cluster is drawn with circles, the middle cluster with boxes, and the third with diamonds. In this case, each of the extreme clusters has 7 points, but only 6 points in the middle cluster. Clusters won't always have exactly the same number of points, but we try to equalize things as much as possible. It won't make much difference which of the clusters has only 6 (instead of 7) points.

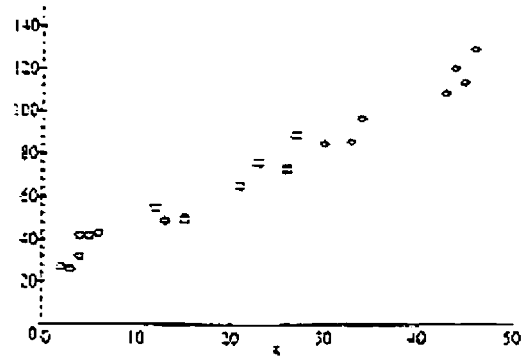


Figure 14

The average of the cluster on the left has first coordinate $\frac{13+6+4+4+3+5+2}{7} = \frac{37}{7} = 5.2857$ and second coordinate $\frac{49+43+32+42+26+42+27}{7} = \frac{261}{7} = 37.286$. The average of the middle cluster has first coordinate $\frac{12+15+26+23+21+27}{6} = \frac{62}{3} = 20.667$ and second coordinate $\frac{55+50+73+76+65+89}{6} = 68$. The cluster on the right has first coordinate $\frac{46+44+30+33+43+45+34}{7} = \frac{275}{7} = 39.286$ and second coordinate $\frac{130+121+85+86+109+114+97}{7} = 106$. The line through the two extreme average points has equation $y = 26.603 + 2.021x$, the line through the average of the middle points has equation $y = 26.232 + 2.021x$, and the line that is twice as close to the extreme points as to the middle point is given by $y = 25.479 + 2.021x$. Figure 15 shows the original 20 points, the reduced set of three points (indicated by crosses) and the approximating line.

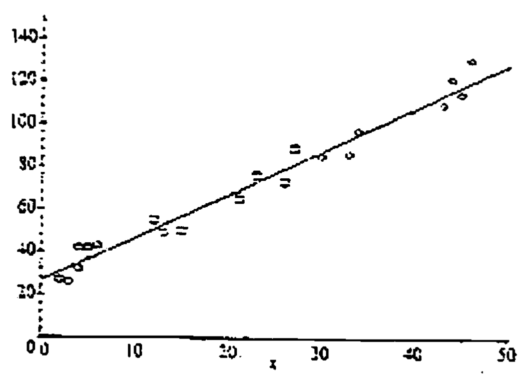


Figure 15

Exercise 1.7.1 Consider the cluster of four points (-3, -5), (3, -1), (-2, 6), and (6, 8).

(a) Plot these four points on some graph paper.
(b) Plot the line segment joining the first two points, and plot the line segment joining the last two points.
(c) Find the midpoint of each of the two line segments.
(d) Plot the line segment joining the two midpoints you found in (c).
(e) Find the midpoint of the line segment you plotted in (d).
(f) From your graph paper, estimate the coordinates of the point you found in (e).

Exercise 1.7.2 Start with the same four points (-3, -5), (3, -1), (-2, 6), and (6, 8).

(a) Calculate the average of the first coordinates -3, 3, -2, and 6.
(b) Calculate the average of the second coordinates -5, -1, 6, and 8.
(c) Compare the point whose first coordinate is calculated in (a) and second coordinate in (b) with the point you found in Part (f) of Exercise 1.

Exercise 1.7.3 Consider the cloud of points

x	-3	3	-2	6	10	15	15	21	19	25	26	35
y	-5	-1	6	8	8	9	14	29	34	33	40	41

(Notice that the first four points are the same as those given in Exercises 1 and 2.)

(a) Make a scatter plot of the 12 points, draw a circle around first one-third of the points, a circle around the middle one-third of the points, and a circle around the last one-third of the points.
(b) Reduce from 12 to three points by replacing the points inside each circle by their average.
(c) Find an equation of a line that fits the three points.
(d) Graph the line from (c) together with the original twelve data points.

Research Paper

Strategies:	**Cooperative groups** **Activities involving writing** **Making connections to other fields** **Constructivist methods** **Hands-on learning**
Class:	**College Algebra in Context (Mathematics 180)**
Instructors:	**Glenn Bruckhart** **Kate Fellenz** **Ken Klopfenstein** **Colorado State University**

Introduction: The course, *College Algebra in Context*, reflects national recommendations that mathematics curriculum should be context driven, constructivist in approach, include cooperative problem solving, stress communication with and about mathematics, and build connections between different areas of mathematics and with the real world. At Colorado State University, this course and *Euclidean and Non-Euclidean Geometry* provide "bookends, a first impression and a memorable conclusion, that serve as role models for future teachers of mathematics."

College Algebra in Context, an introductory course, serves as a role model to attract students (particularly students from underrepresented groups) into a teaching career, and provides practical classroom experience for advanced undergraduates who are majoring in Mathematics Education. Students work in groups of 4 or 5 in a large classroom. Graduate and advanced undergraduate assistants each have responsibility for working closely with 25 students (usually 5 groups of 5); reading homework and responding to student questions. The classroom design facilitates small group interaction but allows scaling to a large number of students at moderate cost. A typical classroom session may include slightly more than half the class period with students actively engaged in group activities that reinforce concepts using graphing calculators, slightly less than half in whole-class discussion.

The course is designed to reflect the ideals reflected in the Curriculum Standards of the National Council of Teachers of Mathematics and the reform movement at the collegiate level most often identified as reform calculus. In particular, *College Algebra in Context* was designed around the following principles:

Context Driven: New ideas are introduced in the context of real problems.
Constructivist: Students construct their own knowledge.
Cooperative Problem Solving: Students interact with other students to solve problems.
Communication: Students communicate with and about mathematics.
Connections: Students continually reinforce connections between algebraic, numeric, and geometric representations.

The course uses materials from three sources: course materials prepared by the instructors and distributed in class at no charge; *TI-82 Graphics Calculator Guidebook*; and reference textbooks that may be checked out from the Resource Desk in the IPM Center and used in the Tutoring Center. The enrollment is expected to level out at between 300 and 500 students per semester and will directly involve between 12 and 20 Math Education course assistants.

Project Description (Example)

Due Date

Each group will be responsible for writing a research paper that looks for a relationship between two of the variables for which data were taken during session X. The research paper will contain at least three distinct sections: an introduction, a body, and a summary.

The introduction should define the question being investigated and indicate the methods and scope of the investigation. This is also an appropriate place to include research on actual use of the relationship between your variables by other external working groups such as forensic specialists or clothing manufacturers. The body of the paper will present the findings (data collected) and the analysis of that data. The analysis needs to include the "Four Faces" of the modeling function. The summary should succinctly state conclusions derived from the investigation, any observation derived from those conclusions, and questions generated by the investigation.

Each member of the group will be asked to assume very specific responsibilities in the preparation of this paper. These responsibilities could go under headings as: production manager, graphics producer, word processor, data analysts, researcher. You are asked to declare your specific responsibility.

Activity: Group Sheet

Group Name: ______________________
Group members first and last names and project 1 responsibilities:
__________________________ will assume the task of ____________________.
In this task, I expect to: (give specific details on which you will be evaluated.)

__________________________ will assume the task of ____________________.
In this task, I expect to: (give specific details on which you will be evaluated.)

__________________________ will assume the task of ____________________.
In this task, I expect to: (give specific details on which you will be evaluated.)

__________________________ will assume the task of ____________________.
In this task, I expect to: (give specific details on which you will be evaluated.)

__________________________ will assume the task of ____________________.
In this task, I expect to: (give specific details on which you will be evaluated.)

Geometric Sketchpad and Grading Rubric

Strategies:	**Use of technology** **Hands-on learning**
Class:	**Euclidean and Non-Euclidean Geometry (Mathematics 470)**
Instructor:	**Glenn Bruckhart** **Nicholas Krier** **Colorado State University**

Introduction: The course, *Euclidean and Non-Euclidean Geometry*, reflects national recommendations that mathematics curriculum should be context driven, constructivist in approach, include cooperative problem solving, stress communication with and about mathematics, and build connections between different areas of mathematics and with the real world. At Colorado State University, this course and *College Algebra in Context*, provide "bookends, a first impression and a memorable conclusion, that serve as role models for future teachers of mathematics."

Euclidean and Non-Euclidean Geometry is a required senior-level course required of all students majoring in Mathematics Education, and is the course students in the past were most worried about as they went out to student teach. The revised course acknowledges the different levels of geometry readiness, and that most high school sophomores are not ready for abstract proofs. This course prepares future teachers to use problem solving and to build many vital connections between geometry and algebra.

In particular, the course acknowledges the Van Hiele Model for the Development of Geometric Thought. This is explicitly treated in class and the discussion peaks an awareness of the different levels through which each geometric concept can be approached. This hierarchy is apparent in the instructional approach throughout the remainder of the course.

Every student is seated at a computer during class and uses Geometric Sketchpad to help understand problems and to suggest solutions. Geometry is often the last mathematics course students take before they student teach. This course provides them a role model for mathematics instruction that they can reflect on as they themselves begin to teach.

Originally, the plan was to use Geometric Sketchpad for some classroom activities and supplement with other types of activities. However, Sketchpad has proven to be so robust that it is being used not only to study straightedge and compass constructions, but also such topics as hyperbolic geometry and fractal geometry. Now, instead of confusion, students say things like, "I understand what the theorem means, and I am convinced that it must be true, but I don't yet see how to prove it." This is clearly an improvement over, "I can repeat the proof given in the book, but I have no idea what it really means or why it is true."

The revised course consolidates student knowledge of Euclidean geometry through problem solving. A list of problems were investigated and posted on the wall, with solutions eventually developed by the students for nearly all. These problems were used to motivate many of the explorations that followed.

The revised course focuses on transformations as a way to look at geometric structures. During our investigations of transformations the potential of Sketchpad became apparent and changed the direction of the course.

Conics are developed and modeled from their geometric properties. We use the Power Mac Graphing Calculator to help in this process as well as the Sketchpad.

Hyperbolic Geometry is introduced from a circle inversion transformation, which led to the Poincare Disk model. This investigation started with the model at the Van Hiele level 0 (Visual) and proceeded toward the formal (rather than the reverse, which has been a standard approach).

Fractal Geometry focuses on five major properties of recursion, self-similarity, complexity, sensitivity, and chaos. This topic leads naturally to the notions of fractional dimension and infinity which, in this context, are new to the students.

The methodology employed assumes that conceptual learning requires time and multiple views for sound development of concepts. This implies that all content be introduced by about week 10 of the 16-week semester. During the remaining time, students consolidate their knowledge through the following activities:

Taking a series of knowledge tests.
Writing a summary paper.
Working on problems in a chosen area of interest.

The assessment process for the course is certainly nonstandard. Most of the assessment comes during the second half of the semester, and students have nontrivial roles in the evaluation process that include putting together a portfolio of accomplishments in the course.

Activity: Sketchpad Investigations

In the folder, "Sample Sketches," there are some very interesting problems. Class today will be centered around your investigation of some of these sketches. This is to lead to your investigations assignment. As a guide for today's explorations, would you please answer the following questions for the indicated investigations.

1. In the Miscellaneous folder, open the sketch, "Odds and Ends." Pick one of the constructed elements in this sketch and drag any of its dragable parts.
Question: How can one construct a diagram like this that maintains its basic properties even as it is moved about?

2. In the Pythagorean folder, open the sketch, "Shear Pythag." Follow the directions listed.
Question: What is a shear transformation?

3. In the Triangle Exercises folder, open the sketch, "SSA." Again follow directions.
Question: How is this sketch constructed to keep the integrity of the two sides and the angle indicated?

4. In the Investigations folder, open the sketch, "Box Volume." Move the designated points and observe all the changes in the sketch prompted by their motion.
Question: How is the length of an independent segment used to define the length of segments that are not visibly connected to that segment?

5. In the Investigations folder, open the sketch, "Inversion." Use this diagram to explore the concept of circle inversion. There are several implied questions in this investigation.
Question: What is this "Inversion" transformation? What are some of the properties of this transformation?

6. Begin devising a Sketchpad investigation you would like to produce. You have seen simple (SSA) to complicated (Box Volume) in the sketches investigated today. It is much more important to pick a problem that is doable rather than fancy.

Grading Rubric M470 Computer Investigation

Name: ______________________________

Investigation Communication: The investigation is clearly defined, questions are posed, it is clear what actions are to be taken.

Rating 5 4 3 2 1 0

Comments

Investigation Format and Function: The investigation works, there are no glitches, it is attractively formatted, uses the capabilities of the Sketchpad to show things not easily seen otherwise.

Rating 5 4 3 2 1 0

Comments

Investigation Quality: The investigation can actually lead to answers to some questions and the formation of others. The investigation "grabs" the investigator, leaves him or her wanting to know. It's open, not closed.

Rating 5 4 3 2 1 0

Comments

Quality of Investigation's Mathematics: There are clear links to good and interesting mathematics.

Rating 5 4 3 2 1 0

Comments

Write-up: The write-up clearly states what the investigation is aimed at accomplishing. It connects the investigation to the mathematics mentioned above.

Rating 5 4 3 2 1 0

Comments

Total Rating ______ /25

Group Work

Strategies:	Cooperative groups In-class discussion Hands-on learning
Class:	Linear Algebra (Mathematics 221)
Instructor:	Jeff Farmer University of Northern Colorado

Introduction: A few excellent students do not feel helped by group work, but on final evaluations most rated the cooperative aspects of this course as high as the lecture and discussion.

Reading, Writing, and Communication: Verbal communication is emphasized in this reform course. Students are required to read the text and take notes on it. We discuss how to read a mathematics text, and model note-taking. A wide variety of note-taking styles is acceptable, as long as students include relevant vocabulary and examples, and generate questions about the reading.

Oral communication, a must for teachers, is practiced in the group setting and in class discussions. Students must express ideas in a way that is clear to their peers in the course. Quizzes and examinations emphasize writing and explaining by asking essay and short answer conceptual questions.

Curriculum Changes: Naturally, with many new elements added to the course, the number of topics covered is reduced. During the first reform semester, the reduction was drastic; it is anticipated that during the second semester this will change somewhat. Nonetheless, there is a reduction. The project faculty noted that our independently arrived at reorganization is consistent with current reforms of linear algebra, notably the recommendations of the Linear Algebra Curriculum Study Group. Since the course at UNC is introductory, and can be taken after only one semester of calculus, it is not surprising that the level is rather elementary. On the other hand, the enhancements of the course add to the traditional curriculum in a way that is consistent with the *Standards* movement and other reform efforts.

Pedagogy: Every effort is made to help students to view each concept from a variety of perspectives; algebraic, geometric, numeric, and verbal representations of ideas and objects are often used. There are many small projects designed to introduce students to the concepts of the course. Students are expected to check consistency of results and to explore connections between various representations. Interactive mini-lectures and class discussions are often driven by student questions.

Technology: A computer program was written by project faculty (Bosch) to help students visualize the effect of some linear transformations on the Euclidean plane. The students get this program on their own disks and have access to it thereafter.

Assessment: The assessment procedures for the reform course have a traditional appearance; they involve exams, quizzes, and homework. The questions on quizzes are as often conceptual in nature as they are procedural, and homework grades include assessment of group activities (including labs). This seems appropriate for the context and has been accepted by students.

Examples of Small Group Activities

Activity: Group Work, *Linear Algebra 221*

Objectives: To demonstrate there are multiple methods for solving systems of equations. To emphasize that different group members may have different ways of solving/approaching problems. To call up prior student knowledge with regard to solving systems of equations. To provide motivation for writing a system of equations as an augmented matrix or as a matrix equation.

Description: The groups are given three systems of linear equations with two unknowns. Each member is to solve the three systems individually and then when finished, compare his or her solutions and methods of solving to those of other members of their group. The group should discuss the advantages and disadvantages for each method that was used to solve the system.

Once the groups have finished solving the systems and also discussing their methods for solving systems, each group will share with the class one method they used to solve the systems. If a method has already been discussed, they must choose another method used until all methods have been exhausted.

Time Required:	Individual work	5 to 7 minutes
	Group discussion	3 to 5 minutes
	Class discussion	10-15 minutes

Remarks: The systems given to the groups to solve were designed to have integer solutions.

Activity: Group Work, *Linear Algebra 221*

Objectives: To illustrate the difference between a homogeneous and inhomogeneous system. To demonstrate that the solutions to an inhomogeneous system can be determined if a single solution is known and the solutions to the homogeneous system are known. To encourage students to look for patterns in a set of data and to explore other relationships between sets of data that may not be obvious.

Description: The groups are given two linear equations. Both equations have the same slope; however, one of the equations has a y-intercept of 0 and the other has a y-intercept of a constant other than 0. The groups are then to make two tables of values for the linear equations. They are then to compare the table of values to each other and to look for regularities within the tables.

Time Required:	Group discussion	5 to 7 minutes
	Class discussion	10-15 minutes

Differentials, Related Rates

Strategies:	Use/solution of real world applications/problems Activities involving writing Constructivist methods Hands-on learning
Class:	Calculus I (Mathematics 201)
Instructor:	Bill Hoard Front Range Community College

Introduction: The assignments were designed as a result of a mini-grant from the Rocky Mountain Teacher Education Collaborative, an NSF funded project. The lessons below are part of a collection of applied learning activities designed by the Mathematics Department members at Front Range Community College. The contact person for the project is Bill Hoard, Chairman.

Activity: Differentials, MAT 201, Calculus I

Introduction: This activity is to investigate applications of differentials.

Exercise:

The equation of a pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$ where T is time in seconds, L is length in feet, and g is the acceleration due to gravity.

1) Assume that $g = 32\ ft/sec^2$. Solve for L:

2) Find a "bob" and use the given string to calculate T for a given length. Mark the string in some way so that the length of the pendulum can be measured later. Use 10 complete back and forth swings. and then estimate the number of seconds for one period:

3) Predict the length of L (don't measure yet):

4) Now consider, dT. What toes dT represent? Estimate its value:

5) Use the given values to find dL. What does this represent?

6) Predict an interval for the length L of your string given the above. Measure L and comment on the result:

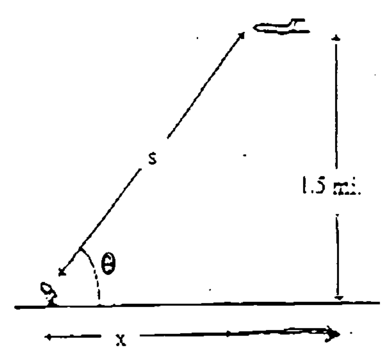
Activity: Related Rates, *MAT 201, Calculus I*

Introduction: This activity is to investigate related rates in real-life situations.

Exercises:

1) An airplane flying 200 miles per hour is on a path that will pass directly over a radar antenna at an altitude of 1.5 miles.

a) Find an algebraic expression that relates x and s, where x is the distance on the around from the radar antenna to the point directly below the plane, and s is the straight line distance from. the antenna to the plane.



b) Explain why $\frac{dx}{dt} = -200$. Explain the meaning of $\frac{ds}{dt}$ in the context of the problem.

c) Write $\frac{ds}{dt}$ as a function of x.

d) Graph the function found in part (c).

e) Explain the features of the graph in the context of the problem.

2) Repeat the steps (a-e) of problem (1), using θ in the place of s.

3) Think of a real-life situation in which the concept of related rates can be applied.

a) Describe the situation and sketch a drawing.

b) Use calculus to determine the related rate.

c) Write a paragraph about the result of part (b), and explain how it relates to the context of the problem.

Collaborative Projects

Strategies:	Cooperative groups Use of technology Use/solution of real world applications/problems Activities involving writing Making connections to other fields Constructivist methods Solution of complex problems by students Attention to creating learning settings sensitive to gender Hands-on learning
Class:	Mathematical Modeling (Mathematics 390)
Instructor:	Thomas Kelley Metropolitan State College of Denver

Introduction: This class was conceived with the intention of having students learn mathematics as they worked through extended mathematical modeling problems in a group setting. Following are project assignments along with some remarks from the instructor on the process and outcomes. For Project #2, an example is also included.

Note that the statement of the project is short, requiring the students to spend a considerable amount of time talking among themselves and with their instructor to clarify just what the problem is. This problem construction facet of the modeling process proves to be indispensable in all areas involving problem solving, not just in mathematics.

Project #1 How to Read a Textbook

Activity: Build a mathematical model of how you plan to read the textbook. First build an individual model and then get together with the other two group members to build the group's model.

Instructor Comments: This activity introduced students to the ideas involved in math modeling and disrupted their usual notions of what went on in a "math class." This activity also demonstrated to students that mathematical models could be developed in "non-mathematical" contexts. Finally, students were introduced to the interaction and negotiation that must go on in constructing a group consensus.

Project #2 How to Measure Heights

Activity: Develop and explain a method to measure the height of an inaccessible object. This includes the construction of a device from readily available materials in order to measure distances and/or angles. Use that method to measure the height of two assigned objects (one of which was a roller coaster under construction nearby). The written report should enable a person, inexperienced in trigonometry, to obtain the height by following your instructions.

Instructor Comments: The groups quickly realized that this was "just a trig problem" that they had seen before. The purpose of this project was to force the students to notice that just knowing the mathematics involved did not mean that the problem was solved. As a matter of fact, the groups' self-assessment comments often expressed amazement as to how much time the problem took after they had solved the mathematics. Each group learned just how difficult it was to actually use the mathematics, to obtain accurate measurements, and to account for errors in measurement that gave erroneous results. In fact, this was a good exercise in spotting when a result was reasonable or not. The writing of clear instructions for a "novice user" to implement their methods was also a difficult task. When running this model in future classes, I plan to have the students spend more time on the error analysis and the ways in which errors can be adjusted for and ultimately controlled.

Activity: Project #2 Model

Problem – Develop a quick accurate procedure for measuring the height of a tall object, the base of which may not be accessible, using materials at hand.

Model – This model requires a person to estimate two lengths and one angle. The values found can then be entered into the following equation and the height of an object can be estimated with a tolerance of plus or minus 10 percent.

Equation: $H1 = ((\text{Tan } A)(L3)(H2))/(L2)$

Where:

- L3 = Length of Perpendicular line.
- H1 = Height of object being estimated.
- L2 = Distance from eye to ruler used to measure H2 (observed height of object)
- L1 = Distance from observation point to base of object.

- H2 = Observed height of object at observation Point.
- A = Angle formed between the observation point and the object from a 3rd point that is located some distance from the observation point on a line that is perpendicular to the line formed from the observation point to the object.

Assumptions – The ground where the measurements are being taken is fairly level; the top of the object being measured is observable from at least two places; the estimation of the height of the object is from the top of the object to the surface of the ground surrounding the object.

Requirements – The person who uses this model needs to have a ruler and a protractor or a compass to make the measurements required for this model. A scientific calculator is useful, as well.

Methodology

1. Find L2 – Hold your arm straight out and measure its length, record the length in inches.
2. Find H2 – With a ruler in your hand hold your arm straight out and measure the perceived height of the object from the top to the ground. Record the length in inches.
3. Find L3 – Mark the spot that you made your observation in step 2. Take your compass or protractor and sight a perpendicular from the line formed from the observation point to the base of the object being measured. Move a distance along the perpendicular and record that distance in feet to a second point.
4. Find A - From the second point use your compass or protractor to measure the angle between the base of the object and the original observation point. Record this angle in degrees.
5. Calculate the height:

   X = ____ Length of arm in inches (L2)
   Y = ____ Length of perpendicular in feet (L3)
   Z = ____ Measured height of object from observation point in inches (H2)
   A = ____ Angle between object and first point in degrees (A)

Plug into: Height = ((Tan A)(12Y)(Z)/(X))12 to get the height in feet

Notes to Optimize Model

- The stride of a person averages around 5 feet (stride = 2 steps), this fact makes measuring the perpendicular easier.
- If you use a compass avoid metal objects.
- When measuring the perpendicular find a distant object from your observation point that falls on the perpendicular. It is easier to stay on line when you have something to shoot at.
- A scientific calculator makes the number crunching a lot easier.

Project #3 How to Determine Best Coffee Container

Activity: You are a retailer of coffee. Determine the best container for serving your product.

Instructor Comments: The mathematical intent of this project was to have the students experimentally determine Newton's Law of Cooling. Along the way, the groups learned about defining the problem so that they know just what they had to model. Initially, the groups were upset that the problem was "so vague," but they were mollified somewhat when told that part of the purpose of doing this course was to disrupt traditional notions about math courses.

Each group developed its own definition of "best" through interviews with local businesses that served coffee. They discovered that many factors were considered (and not considered) by the companies that made and used the various coffee containers in the area. They also learned about data collection and curve fitting as well as customer surveys and what constituted a good question on a survey.

There were 9 students, 6 males and 3 females in the course. One significant choice I made as instructor was to not have a team with only 1 female student, which more often than not leads to the isolation of the single female student. With this in mind, the groups for the projects were composed of three students and one group was all-female. The "3 groups of 3" structure held throughout the course.

Project #4 How to Define and Construct Conic Sections

Activity: Each group was assigned a different conic and required to present a definition, at least one method of construction, the equations for the group's conic, and a demonstration of a physical manifestation of the conic section.

Instructor Comments: This project differed from the others in that it started with the mathematics and went in search of the application. The intent here was to have the students take a topic that they were accustomed to dealing with (equations and graphing) and view it on a more concrete level. They were required to go back to the definition and applications of the conics so that they could see beyond the abstractions.

Each team did a very good job of showing that they understood the definition through hands-on construction of the particular conic, complete with detailed instructions that a person who was just learning about conics could follow. The teams also did physical demonstrations which showed that they had a grasp of the reflection properties of each of the conics.

Project #5 How to Analyze Chances of Winning

Activity: For a game of chance (craps, roulette, coin toss onto a grid), do an analysis of your chances of winning if you are "the house." What if you are the "customer?" Your analysis should include both physical trials (e.g., actual plays of the game) and a computer simulation. The presentation will be a "Casino Day" where the other teams will try to beat the game that you have "rigged" as "the house."

Instructor Comments: This was a foray into probability. Students learned about analyzing games of chance through analysis, experimentation, and computer simulation. Simulation was a new experience for most of the students and getting a computer or programmable calculator to simulate playing a game helped with writing skills. The different ways in which the students could approach the problem helped them to see that mathematics gives one flexibility in solving problems. They also gained an appreciation for the gaming industry and what is involved in setting up a "salable" game. Having the presentations in the context of a "Casino Day" resulted in a very lively class. (There were no reports of students visiting Las Vegas and "winning big" following this project!)

Project #6 How to Relate Mathematics and Music

Activity: What relationships exist between mathematics and music? Use a monochord to find out what the harmonic sequence sounds like. What do musical sounds look like mathematically? How do you decide where to place frets on a guitar and why does a grand piano have the shape it does? Along the way you will find out about the Fibonaccl numbers and obtain physical objects that are natural occurrences of Fibonaccl numbers. What is the Golden Ration? How do you construct it with numbers—and does it occur in nature?

Instructor Comments: Starting with simple notes, each group proceeded to explore the relationship between mathematics and music. This led them through discussions of rational and irrational numbers, exponentials, and logarithms, Fibonaccl numbers, and the Golden Ratio. Each group had the use of a monochord with moveable frets and rules to see which fractions "sounded" better. The group project was to complete a series of worksheets taken from the book *Algebra in Everyday Life*. This set-up was different in that the main "product" was not a group report but rather a collection of worksheets. In the future, I plan to use the worksheets to construct a project that would "hang together" better than it did this time. Many of the student comments mentioned that the material, though interesting, seemed to be disjointed.

Introduction to Data Collection and Conjecturing

Strategies:	Activities involving writing Constructivist methods Hands-on learning
Class:	Discrete Mathematics—Module 1
Instructors:	Richard Grassl University of Northern Colorado Tabitha Mingus Western Michigan University

Source: From *Data Collection and Conjecturing with Discrete Mathematics*, An Interactive Workbook Designed to Accompany Most Discrete or Problem-Solving Texts at the Secondary and College Levels (Richard Grassl and Tabitha Mingus).

Activity: Introduction to Data Collection and Conjecturing

Motivating Problem

Suppose you have a pile of 1-by-1 tiles of a single color and 1-by-2 tiles of a single color. How many different paths of height 1 and length 20 can you build? You might imagine these paths as tiles lining a garden or a driveway.

Suppose, after drawing lots of pictures and experimenting, you came up with the following data for paths of length 1, 2, 3, 4, 5, 6:

1, 2, 3, 5, 8, 13

Question 1: Can you guess at the next three items?

Question 2: Can you determine any one term in general?

Objective: To be able to determine the next few terms in a given sequence and to determine a formula for a general term

In order to accomplish this objective, you will learn about data collection, forming sequences, finite differences, and using a calculator to "fit" a polynomial to data. You will also gain experience with two special types of sequences: arithmetic and geometric.

Message to the Student

The idea that mathematics is a bunch of number crunching procedures that spit out a single, unmistakably correct answer is false. It is true that mathematics involves numbers and procedures (called algorithms), but mathematicians are like explorers. They search out patterns and try to explain what they see to others. You can explore mathematics just like a mathematician does; you already have all the skills you need. You know what numbers are (counting numbers, integers, real numbers) and how they can represent different things. You understand fractions and decimals. You know how to perform the basic arithmetic operations of addition, subtraction, multiplication, and division. But most importantly, you can observe and experiment. Most people expect to do experiments in learning or exploring science, but they don't think of mathematics as a field that you would need to play with in order to understand. . . Mathematics is not as serious as most people try to make it out to be.

Let's start by finding and describing patterns in sequences of numbers. First, try to determine the next three entries in the sequence of numbers. Then write a sentence that would explain to your friends who are absent from class today, how they could reconstruct the sequence, given a few beginning terms and a "rule" for constructing the next term.

Example sequence: 1, 3, 5, 7, _a_, _b_, _c_, . . .

We have just represented the next three terms in the sequence by *a, b,* and *c.* You could have used just about any symbol, like a triangle, square, circle, or no symbol at all. Somehow, you would have to indicate to your teacher or friend where the numbers you write down appear in the sequence and how you know. The three dots at the end of the sequence, . . . , are called an ellipsis. They signal that the sequence keeps on going even though we aren't writing any more of the terms down. You will find that mathematicians use many shorthand techniques, like the ellipsis, to introduce symbols to make communicating with other mathematicians and people easier.

Answer: For this sequence a = 9, b = 11, and c = 13. **Rule:** You can always find the next term in the sequence by adding two to the previous term.

How Many Squares in a Checkerboard?

Strategy:	**Use/solution of real world applications/problems**
Class:	**Discrete Mathematics**
Instructors:	**Richard Grassl** **University of Northern Colorado** **Tabitha Mingus** **Western Michigan University**

Source: From *Data Collection and Conjecturing with Discrete Mathematics*, An Interactive Workbook Designed to Accompany Most Discrete or Problem-Solving Texts at the Secondary and College Levels (Richard Grassl and Tabitha Mingus).

Activity: How many squares on a checkerboard?

Problem: How many squares of all sizes are there in an 8-by-8 checkerboard?

First take a 3-by-3 checkerboard. How many 1-by-1 squares are there? How many 2-by-2? 3-by-3? Draw pictures and fill in the chart.

Size	1-by-1	2-by-2	3-by-3
Count			

Fill in the chart for a 4-by-4 "checkerboard."

Size	1-by-1	2-by-2	3-by-3	4-by-4
Count				

How many squares are there in an 8-by-8 checkerboard? _______

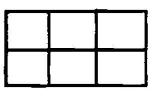
Counting Rectangles in a Rectangle

Strategy:	Solution of complex problems by students
Class:	Discrete Mathematics
Instructors:	Richard Grassl University of Northern Colorado Tabitha Mingus Western Michigan University

Source: *Data Collection and Conjecturing with Discrete Mathematics.* An Interactive Workbook Designed to Accompany Most Discrete or Problem-Solving Texts at the Secondary and College Levels (Richard Grassl and Tabitha Mingus).

Activity: Counting Rectangles in a Rectangle

How many rectangles of all sizes are there in a 2-by-3 subdivided rectangle? Remember that a square is a rectangle. Let's count them by size. Shape makes a difference: a 1 x 2 is different from a 2 x 1.



There are: __6__ 1 X 1, ______ 1 X 2, ______ 1 X 3

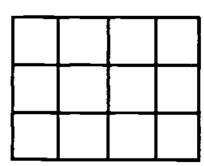
______ 2 X 1, ______ 2 X 2, ______ 2 X 3

The total is 6 + ______ + ______ + ______ + ______ + ______.

Factor strategically and express your answer as a product.

How many rectangles in a 3-by-4 rectangle? Complete the table.

SIZE	COUNT
1 x 1	12
1 x 2	
1 x 3	
1 x 4	
2 x 1	
2 x 2	
2 x 3	
2 x 4	
3 x 1	
3 x 2	
3 x 3	
3 x 4	



SUM = (12 + _____ + _____ + _____) + ( ) + ( )

= ( ) ( )

How many rectangles in a 4-by-5 rectangle? Complete the data table.

Size	Count

Size	Count

What is (1 + 2 + 3 + 4)?

1 + 2 + 3 + 4 + 5 =

Can you express these sums?

Conjecture a formula for the number of rectangles in an *n*-by-*n* subdivided rectangle:

The Game in General

Strategies:	Constructivist methods Solution of complex problems by students
Class:	College Algebra (Mathematics 321)
Instructors:	Jim Loats Kenn Amdahl Metropolitan State College of Denver

Introduction: According to the book *Algebra Unplugged*, by Ken Amdahl and Jim Loats (Clearwater Publishing, 1995), "Algebra is like a game. Like many games, it has game pieces, moves, strategies, goals, and its own vocabulary. . . . Most of us love games, but aren't crazy about mathematics. We're bored by math, frightened by it, and made to feel stupid by it. We haven't loved it. . . . But algebra is different. While it can, indeed be useful in real life, much of it is simply a game that has no direct relationship to anything we can touch or count. That's fine, in a game" (p. 3).

"There are side benefits to many games . . . . A side benefit of the algebra game is that it may allow you to become a chemist or an electrical engineer," etc. (p. 4).

Excerpted in the following pages from the book, are the authors' creative analogies and perceptions of algebra as a game, and the relationship of all of the parts. The book discusses algebra in a non-threatening, innovative, fun way.

The Game in General

Algebra is a tool kit for solving mysteries. Using whatever clues we find, we translate the mystery into numbers and symbols. Then, by employing specific strategies, we extract information from the numbers and symbols. After we've manipulated the numbers in accordance with the rules, we translate the result back into real-world terms. That's algebra.

Notice the three discrete steps:

1. Translating the real-world situation into numbers and symbols is one step.
2. Manipulating these is a second, completely different activity. This activity is the game of algebra.
3. In the final step, we translate the solution back into the real world and see if it makes sense.

There are two objects of this game. Much of the time, the goal is to pull the mask off an "unknown." Other times, the goal is to translate a repeating pattern into math. Algebra is the language of patterns.

Practical problems begin with a situation that can be described in words and sentences. Often your Real Algebra Book will skip this part altogether, in the interest of saving space, and proceed directly to the manipulation of the numbers and symbols. In the real world, algebra problems won't appear ready-made. They'll be disguised as mysteries, confusions, complications and dilemmas. It will be up to you to translate them.

First we must identify the mystery, which we call "the unknown."

"Subtract some number from 20 and you'll get 15."

"Some number" is the unknown. Traditionally, letters near the end of the alphabet indicate unknowns. The most common letter to use is "x." In our example, we replace "some number" with "x."

"Subtract x from 20 and you'll get 15."

We have identified the unknown.

At this point we employ a subtle shift away from our normal thought process. Usually, results come at the end of a series of logical steps. If I plant the seed, water the sprout, and pull the weeds, then I'll be able to harvest my watermelons at the end of the summer. If I follow the map, I'll reach my destination. Arithmetic behaves like that. We know all the numbers we're going to add up. The unknown is the answer at the end.

In algebra, the unknown often springs up in the middle of events: "If I plant the seed, water the sprout, *and do something else that I've forgotten,* I'll be able to harvest my watermelons at the end of the summer." Or: "If I follow the map, *including the part that's been torn off,* I'll reach my destination."

If we fill in that missing part, it's a true statement. The true statements of algebra are "equations." An equation is a math problem with stuff on both sides of an equal sign. It doesn't matter that some of the items on each side might be unknowns. We arrange the game pieces into a formula that will be true if we substitute the correct answer for x:

$$20-x=15$$

This step is often eliminated from algebra books, probably because it makes it possible for you to use algebra in real life.

Next, we play with the equation, being careful not to alter its truth. We twist and pull until all its numbers sit on one side, and its unknown sits alone on the other side.

Then we complete the arithmetic on the number side to learn the unknown. The unknown has become known. It's our answer.

In step three, we use the answer in the real world to build a bridge, or cut up some pizza, or sew a dress. Your Real Algebra Book will skip this part as well. They rarely let you build the bridge or eat the pizza. Too bad. That's the reason you're playing the game in the first place.

Solving Equations

Years ago people determined how much something weighed by putting it on one end of a scale. In those days, a scale was like a teeter-totter, a long board balanced in the center. They kept various weights handy, each one labeled. If a one-pound weight balanced your sack of peanuts exactly, Mr. Grocer charged you for one pound of peanuts.

An equation is like a balanced scale. The stuff on the left side of the equal sign weighs exactly the same as the stuff on the right side. To solve an equation, we have to figure out what's inside any unmarked boxes.

Whatever tricks we try, we are guided by one rule: we can't let the scale tip away from perfect balance. That's the game. Keep the scale dead flat. The left side must equal the right side at all times. You can do anything you want, if you obey this rule. Tap dance on the scale, drive your car over it, send the whole works down Niagara Falls. Just don't ever let the scale tip away from equality.

Our first instinct is to suspect there's not much we can do to an equation that won't send it tumbling. But there are two things we can do to either a scale or an equation: We can replace one item with an equivalent item. And we can do the same thing to both sides without tipping the balance.

Replace numbers with their equivalent

We can describe any number in many ways. The number "16" for example can be described as 4 times 4, or 16/1, or 32/2, or 64/4. It can be described as (15 + 1), (40 - 24) or 16,000,000 ÷ 1,000,000. One strategy for solving equations is to replace a number with another version of the same number. If we have a one pound weight, and a one pound sack of peanuts, it doesn't matter which one we use on the scale. So, if we see

$$(15+1) + 2/3 = x$$

we can replace (15 + 1) with 16 and the scale won't even quiver. They're simply two different ways to say the same thing. Unfortunately, some substitutions don't do us much good, either. We could replace 2/3 with 4/6 and we will not have affected the truth of the statement. But does it help us? Maybe, but maybe not. Obviously, it's going to take a little practice to decide which of the many variations of any number might do us some good. This is where an algebra class will help you. Any good teacher will give you many opportunities to flounder around helplessly in search of experience. Just remember this: you can always replace a number with something that is exactly equal to it.

Do the same thing to both sides

If the scale is balanced to begin with, and you add a penny to each side, it will still be balanced. If you subtract a one-pound sack of peanuts from each side, it will still be balanced. In fact, you can do anything you want to one side of an equation as long as you do the same thing to the other side. Multiply, divide, add, subtract; it doesn't matter. Double the weight on both sides of the scale and it will still be balanced. That is, multiply both sides of the equation by 2 and you will not have impeached the truth of the statement. Divide both sides by 2 and you'll be back where you started, and still balanced.

You won't affect the balance if you do the same thing to both sides of an equation.

The Goal

There are two common ways to win at algebra. One is to identify the unknown. When we can say, "x equals 43 compassionate Republican senators," we have solved the equation, although in this case we might want to check our arithmetic.

The other is to identify an equation that represents a pattern. Perhaps we've been given a list of some sort, in which one column relates to the others in some common way. When we can say "The number of new social programs each year equals the number of Democratic representatives squared," we have reduced the list to a formula and saved a lot of paper. A formula expresses this more efficiently than a chart with several hundred possibilities.

You need to know which result you're aiming at. If you're trying to solve an equation, you hope to arrive at a simple answer. But sometimes, it won't be possible. In those cases you win when you recognize the nature of the problem.

Some equations are true for all numbers. Doesn't matter what you plug in as the unknown, it'll always work.

$$x - x = 0$$

You can replace x with any positive or negative number, any fraction, any dairy product, any small appliance, and most small children. It will always be correct. Not very useful, but correct.

Some equations are never true, no matter what number you try:

$$x + 1 - x = 0$$

You are done with this problem when you recognize that it can't be solved.

And some unknowns resist a simple definition. The closest you may get is a simpler equation. This is not a failure on your part. Sometimes you get a touchdown, sometimes you settle for a field goal.

Our optimistic assumption is always that, by being persistent, we will discover the identity of the unknown. With that cheerful attitude, we begin manipulating our equations, using the tricks and strategies we've learned. But, until we focus on our goal, we're simply conducting arithmetic exercises, madly squaring numbers and multiplying fractions with no clue as to why anyone would want to spend a pleasant Saturday afternoon this way.

The big strategy is obvious, if you think about it for a moment. You want to get *x* isolated on one side of the equation. You want to be able to say "*x* equals . . . whatever's on the other side of the equation."

To solve equations, we do identical things to each side of the scale, and substitute equivalent numbers. That's the whole game. It seems more complicated than that because it's not always easy to translate a problem from "life" into math. It requires many words and concepts to describe all the possibilities. To survive an algebra course, you're going to have to learn the language.

Often it helps to hear two different explanations for a word or concept. We're here to provide that second explanation. Ours may not be an improvement on your Real Algebra Book's explanation, or on your teacher's. But it will probably be different.

There is no perfect "order" to learn these words and concepts. Like any language, it's hard to explain one word without using other words that may also be new. Some won't make much sense to you until you use them. Don't worry too much about the order of concepts. We're just going to whimsically wander through the landscape, tossing out an occasional pretty word, in whatever order they occur to us. There is no "right" way to do this. There is only "the way it's always been done," and, now, our way.

The Game Pieces

Many games come with a fixed number of pieces. We use 52 cards to play most card games, 32 chess men for chess, and 4 horseshoes make a set. Two guitars, a bass, a drum, and a lot of depression are required to play the "alternative rock band" game.

Depending on our situation, we play algebra with different sets of numbers.

There are many kinds of numbers. For example there are fractions, decimal numbers, and whole numbers. Some numbers fit into several of the categories below. This list is arranged in a specific order. Usually one learns about the numbers in the order below. But more importantly, each of these sets of numbers includes all the numbers in the sets listed above them.

Sesame Street numbers: Prior to 1985, these were the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12. You remember "The Count." By 1985, the Count had learned how to go all the way up to 20. (Note: Mathematicians do not use this classification.)

Counting numbers: As you might guess, these are the numbers you count with. 1, 2, 3, 4, 5, 6, 7, 8, 9, ... (those little dots mean that the list is too long to type out.) Some books will refer to these as the **natural numbers.**

Whole numbers: These are the same as the counting numbers except that zero is also included. So this list goes like this: 0, 1, 2, 3, 4, ... The word "whole" is used to exclude the fractional numbers. This is the kind of numbers you would use to describe how many complete banana cream pies you can to eat, or how many roller coaster rides you enjoyed last night. Some books will refer to *these* as the **natural numbers.** There is some disagreement among text book writers as to whether zero is a natural number or not.

Integers: The integers include all the whole numbers and all of their negatives too. Here's the list:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

Another way to list them is to use the plus or minus sign, ($\pm$):

$$0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5, \ldots$$

Note, for example, the number 5 is a counting number, a whole number, and an integer.

Rational Numbers: These are the fractions. Some examples of rational numbers are:

$$\frac{2}{3} \qquad \frac{34}{9} \qquad \frac{-4}{3} \qquad \text{and} \qquad \frac{21}{-4}$$

The official definition say these are exactly the numbers that can be expressed as the ratio of two integers where the bottom number (the denominator) can't be zero. The word rational refers to "ratio." This is a complicated bunch of numbers. Even deciding when two of them are equal involves some work. A **mixed number** contains both a whole number and a fraction, like:

$$4\tfrac{2}{3}$$

Because this number is equal to

$$\frac{14}{3}$$

it is a rational number. When rational numbers are represented in decimal form, the portion of the number to the right of the decimal point either stops, like:

$$\frac{3}{4} = 0.75$$

or it repeats in blocks, like:

$$\frac{35}{11} = 3.181818\ldots$$

Real numbers: A real number is any number that can be represented as a decimal number, positive or negative, repeating or non-repeating. The real numbers can be used to label every point of a line. In fact, when you draw a number line, it is customary to think of each point as representing a number and vise versa. All the numbers we have discussed so far are included in the real numbers. Some real numbers are not rational. They are called irrational numbers and one example is π. The "square root" of two is another. They resemble rational numbers, except that when you express them in decimal form, those digits go on and on and on, never repeating, and never falling into a repeating block of digits.

Complex numbers: Complex numbers are a combination of real and imaginary numbers. That may take a bit of explaining.

Rather than stop when things actually made sense, mathematicians got carried away and created an even bigger set of numbers.

You can't multiply anything times itself ("square it") and get a result that's a negative number. So, the negative numbers don't have "square roots." It would be handy if this were not the case. So, they invented a kind of number that would satisfy this requirement. Appropriately, they call them "imaginary numbers," represented by a small "i."

A math teacher might explain that a complex number is a number of the form $a + bi$, where a and b are real numbers and i can be thought of as the square root of -1. In other words, complex numbers have a real component and an imaginary component.

The good news is that the craziness stops with the complex number system. If you study college algebra, you may encounter these numbers. They are rarely used outside of scientific settings.

If you like different kinds of numbers, a fascinating literature awaits you. You can discover numbers that are perfect, amicable, palindromic, figurate (like square and triangular, etc.), abundant, etc. You will be following in the footsteps of folks who began looking at these things as early as the Egyptian and Babylonian civilizations. Enjoy yourself.

The Moves

Checkers can move forward diagonally to the left or right, or can jump an opponent. In poker, you raise, call, fold, draw, etc. In football, you run into people. Every game has "moves" of some sort.

There are four basic "moves" in algebra: addition, subtraction, multiplication, and division. These "moves" occur after the words have been translated into numbers and symbols during the manipulation of equations. The "rules" of the game tell us what order these must be done in.

The "moves" are called "operations." An operation is an activity, like adding, or multiplying. The "order of operations" is the rule that instructs you which to do next in a problem.

This may be a welcome surprise to you. There are no new multiplication tables to memorize or anything as difficult as long division. Algebra is a way to use your old skills. You aren't starting over. You're moving on.

The Strategies

There are ten basic strategies in algebra:

1) Your overall strategy is to manipulate the equation, without affecting its truth, until you have a single unknown on one side of the equation and an arithmetic problem on the other. Once you complete the arithmetic, you'll have the answer.
2) You can do the same thing to both sides of an equation without affecting its truth. If you add 30 to the left side, and 30 to the right side, you have not affected the balance.
3) Reversibility. If you double a number, you can find your original number by halving the new number. If you add 10, you can reverse that by subtracting 10. If you square something, you can reverse it by taking its square root. This is a lot handier than it sounds.
4) It doesn't matter what order you add things, if that's all you're doing to them. So, sometimes you can change the order of adding things to gain a strategic advantage.
5) It doesn't matter what order you multiply things, if that's all you're doing to them. It may be useful to change the order in which things are multiplied.
6) When you need to multiply a number times the sum of several other numbers, it doesn't matter if you add them up first, then multiply, or if you multiply each of the individuals first, then add the results.
7) Rule #6 also applies to division. If you are dividing the sum of a bunch of numbers by one number, you can complete either the addition first, or the division.
8) Multiplying anything times "one" doesn't change it. But "one" can be written many different ways, and the results of the various ways will each look different, although they're not.
9) Dividing anything by "one" doesn't change it.
10) If the answer to a multiplication problem is zero, one of the numbers in your multiplication problem must be zero. This is also a lot handier than it sounds. Sometimes, rearranging an equation into a multiplication problem which equals zero simplifies your life. This is called "factoring."

2 Geology, Physics, Biology

Geology Lab Projects

Strategies:	Use of technology Use/solution of real world applications/problems Making connections to other fields Constructivist methods Solution of complex problems by students Hands-on learning
Class:	General Geology (Geology 100)
Instructor:	Rita Leafgren University of Northern Colorado

Introduction: This *General Geology* course serves the dual purposes of providing general education laboratory experiences as well as being an entry-level course for earth science.

Instructional staff were concerned about the very heavy laboratory emphasis on map reading skills and identification of rocks and minerals in labs, feeling that this focus might not be appropriate for the 21st century student. Having students memorize and regurgitate dozens of mineral names for no particular purpose was as bothersome as the fact that students rarely worked collaboratively on lab work. Therefore, the staff decided that the laboratories should be revised. Included in subsequent pages are activities from four labs that illustrate the revised approach to teaching geology, as well as a report on RMTEC-supported *Geology 100* course activities describing reform efforts.

Briefs on Four Labs

Partial materials for the following lab projects are included on the following pages in order to display lab revisions:

1) Where Are We? (Global Positioning System)

This lab starts with a standard description of Earth's latitude and longitude grid, but students use Global Positioning System (GPS) units to accomplish a variety of navigational tasks.

2) World Tour

A computer program allows students to investigate world topography and possible relationships to plate tectonic provinces. This lab affords students an opportunity to critically evaluate correlations between data sets.

3) Fluvial Processes: Big Thompson River

This lab focuses on fluvial processes that shape a canyon where students go on a field trip. The lab analysis which the students do supplements what students observe and question in the field.

4) Seismology: Father Downey and the Case of the Denver Temblors

This lab is an investigation of multiple working hypotheses wherein students assemble and evaluate data in order to find the explanation best supported by the evidence.

Activity/Lab #1 Where are We? Global Positioning System

Today we are really spoiled. With a GPS unit you can simply push buttons and find out your latitude, longitude, elevation and other location data. You can be told precisely where to go to end up where you want to be. Your GPS will even tell you the correct time, just in case your watch is broken. Currently you can get all of this for as little as $200, and no doubt the cost will go down in the future.

The Global Positioning System was developed by the U.S. Department of Defense to help submarines know where they were if they needed to deploy a nuclear weapon. The System involves 24 satellites that constantly circle the earth at high altitude in almost polar orbit (i.e., they go around the earth from north to south.) A GPS unit locks on to high frequency radio signals from some of the 24 satellites and calculates, through triangulation, its exact location. (It's interesting to note that the DOD deliberately introduced an error of 100 feet to make the system less useful for anyone but the military, but the error could be eliminated by anybody "in the know" by use of two GPS units and a computer. So, why bother?) When GPS technology was made available for civilian use it quickly became a favorite with scientists, navigators. and outdoor enthusiasts in general.

What does your GPS do for you?

Depending on the accuracy of your GPS unit – (translates to "how expensive was it") - you can locate yourself anywhere on earth to within 100 feet, or even just a few feet. (If you're really into perfection, use three units and a computer and you can locate yourself within one centimeter!)

Your unit will tell you your elevation above sea level. It will tell you the date and the exact time of day.

You probably are familiar with the relation between distance, speed and time:

distance = rate x time

Since your GPS calculates your position constantly (at least every three seconds – on some models every second), and it always knows the time, if you are moving (covering distance) it will tell you how fast (the rate) you are moving.

You can press a button to locate yourself (called entering a waypoint travel somewhere), and your GPS will tell you how far you have come and how to get back (including the compass course to follow and the distance/time to destination.) .

If you were a big spender, your unit might be capable of entering 1000 waypoints, printing them on a map, and interfacing with a computer that will plot the shape and area of the route you took.

Trucking companies currently use GPSs to keep track of their trucks as they travel around the country. In May, 1996 Yellow Cab taxi company in Denver became the first in the country to make use of GPS technology. They equipped 230 cabs with GPS units that are accurate to within 40 feet. It probably won't be long until cars have built in GPS units, each with a computer screen on the dashboard that will display a map of where you are, and an icon representing your car will move around on it as you drive along. You couldn't get lost if you wanted to.

In exchange for all of the above, what do you have to do for your GPS?

Occasionally give it fresh batteries (just plain old AA on many models). What a bargain!

Using a GPS

(The following specific directions may be revised slightly – and will be added to – by your lab teacher in order to fit the particular model GPS unit you will be using.)

1. GO OUTSIDE.
   The GPS must take fixes on satellites; it needs to be outside to do this.
   Turn GPS to the P 0 S position. In about 5 seconds a data screen will appear.
   Record the following information:

   Date: ____________________ Time: ________________________

   Latitude: __________________ Longitude: ______________________

   Elevation: ________________

2. The dial position S T S (satellites) tells you how many satellites are visible.
You need at least three to get a fix.
Number of satellites being tracked at the moment?

3. Work with waypoints. (Specific directions will be given by lab instructor.)
Record data below.

4. Last - but not least - turn the GPS OFF.

Activity/Lab: 2 World Tour

Introduction: Despite the fact there's lots of ocean, for most of mankind's history it has remained a mysterious realm. History does tell us of the Vikings and Polynesians and Magellan and Columbus etc. etc. who sailed its surface, but until World War II next to nothing was known about what's in the ocean or on its bottom. WWII brought new things to the oceans - submarines, equipment (like radar, sonar, and fathometers,) and scientists who were drafted into the service, became acquainted and fascinated with the ocean, and pursued ocean studies after the war ended. Though we still don't "know it all," our data-base about earth facts increased significantly and acted as final supporting evidence for the revolutionary geology theory, plate tectonics.

Cruising the oceans for the purpose of scientific study is not an option available to most of us, but use of a computer to gather information, to model situations, and subsequently to draw conclusions about science questions is a necessary and expected skill for today's science students. **The following lab consists of an assortment of exercises designed to let you practice these computer skills, or more specifically to:**

a. simply let you look at the earth – its topography and bathymetry ....
(Topography means the ups and downs of earth's land surface; bathymetry means the topography of the ocean floor.)
b. have you think about how topography/bathymetry influences some human concerns - like telling time, or figuring distances. or getting drowned ....
c. see how topography/bathymetry data does support plate tectonics ideas ....
d. play with some plate tectonics graphics ....
e. have practice in using computer software to investigate science questions.

Is this real science?

Admittedly many of the lab activities are rather limited, superficial approaches to some pretty profound science questions. You'll be asked to create a few cross sections, look at a few ocean trenches, check out the mid-ocean ridge in few places ....even your thinking will be purposely directed so you can see how the data can support established science thought. Please be aware that a true scientific study would gather all possible data (and keep gathering more data and re-examining its meaning) - would match up all sorts of data, looking for patterns and for all possible logical answers - would control situations (experiment) so that hypotheses/theories could be tested - and would remain open-minded, accepting if necessary a time when new data or more sophisticated interpretation would prove an established idea was wrong. But - we just don't have the time; this is after all a one-semester course. So you'll have to trust the work of others a lot, accept some thought directing, but hopefully get a little of the feel of scientific investigation.

Activity/Lab #3 Fluvial Processes: The Big Thompson River of the South Platte Drainage System

The earth's hydrosphere, of water supply, is distributed on earth in the following fashion: oceans contain 97.2 percent of the water, leaving 2.8 percent divided among glaciers (2.15 percent), groundwater (0.62 percent), the atmosphere (0.001 percent), and surface water (0.029 percent). This laboratory exercise focuses on a very small, but very important fraction (0.0001 percent) of the surface water—stream channel flow. Rivers and streams are the natural arteries that convey water and sediment downstream (eventually to the sea), and shape the land through their erosion and transportation of solid and dissolved materials.

The objectives of this lab exercise are:

To gain an understanding of the **structure and flow regime of the Big Thompson River.**

To learn about **how flow volume in a river is measured** over time, and how it relates to water depth in a channel.

To learn about the **relationships between discharge and stream velocity, width, and depth.**

To identify the **drainage pattern types of the Big Thompson and South Platte rivers**, and to identify the order of each stream in the upper Big Thompson watershed.

To identify the **changing characteristics of the Big Thompson** as it flows from its headwaters into the South Platte River.

To tie the **annual flow regime** to the influences of snowmelt and precipitation events, and to describe how flow in the canyon section of the Big Thompson reacts to such influences compared to the plains section of the river.

To describe the basic **annual flow regime of the Big Thompson and South Platte rivers** (in the study area) based on **interpretation of annual hydrographs** from 5 gage stations in the region.

To **construct a flood-frequency curve** to estimate the recurrence interval of various-sized floods at the top of Big Thompson Canyon.

The United States consists of several major drainage basins, or watersheds. Within a drainage basin, stream valleys from drainage networks that eventually join to form a single large river. Northern Colorado is part of a South Platte River Watershed which extends to the eastern border of Nebraska where it joins with the Missouri River, which in turn joins the Mississippi River at the border between Missouri and Illinois and flows southward to the Gulf of Mexico. Drainage basins can be further subdivided into sub-basins. For example, the subject of this exercise is the basin that drains the Estes Park area. Figure 1 is a diagram of the South Platte River Basin. In the northwest part of the basin, a "sub-basin" (the upper Big Thompson watershed) is outlined. Figure 2 is a detailed diagram of the upper Big Thompson watershed. We will be using data from 2 stations in this watershed, and from 3 other stations along the stream at lower elevations. Basic information on these stations is proved in Table 1 (page 4). (Note: Figures are not included.)

Activity/Lab #4 Seismology Lab Exercise: Father Downey and the Case of the Denver Temblors

Colorado has experienced relatively mild seismic activity during its recorded history and is considered to have only minor potential for future damaging earthquakes. The largest recorded earthquake in Colorado occurred in 1882 and had an epicenter in what is now Rocky Mountain National Park. It had an estimated Richter magnitude of 6.5—close to that of the damaging Northridge, California earthquake that occurred in 1994. Other significant earthquakes that have shaken Colorado include:

Year	Epicenter	Richter Magnitude
1962	Northeast Denver	3.7
1967	Thornton	5.3
1981	Northglenn	4.5
1994	Castle Rock	4.0

The Denver area experienced over 800 earthquakes between 1962 and 1967. Prior to 1962, Denver residents had not felt an earthquake in 80 years. Beginning in 1962, they began to experience between 10 and 100 sensible earthquakes per month. What was the cause of this sudden increase in seismic activity? At the center of this inquiry was a mild-mannered priest/seismologist/professor who, "secretive as a mouse...carried on his research in the dark chamber under the stairs of the old administration building where the machine [a seismograph] was kept" (Rocky Mountain News, January 21, 1968).

Father Joseph Downey of Regis College in Denver had been operating a set of three seismographs at Regis. Two of the machines recorded horizontal earthquake vibrations at right angles to each other (north/south and east/west) and the third machine measured vertical vibrations. Prior to 1962, the strongest nearby shock waves had come from "the hourly change of classes in the Regis High School building" (RMN, January 21, 1968), so Father Downey's seismographs were built to "pick up the long, gentle swells from distant earthquakes" (RMN, January 21, 1968). When the area began to experience significant tremors in the summer of 1962, Father Downey quickly "rushed around trying to get new equipment to measure these nearby quakes," and was able to procure the needed equipment from "a Government man with three used seismographs" (RMN, January 21, 1968). Between 1962 and 1965, Father Downey's seismographs recorded nearfiy 2,000 earthquakes with epicenters in the Denver area, many of which were felt throughout the metropolitan area and nearby towns.

A geologic mystery of shocking proportions had emerged. The question on everyone's mind was:

What was causing the Denver Tremblors ?

Several hypotheses emerged to explain the sudden onset of the Denver earthquakes. These were:

1. **Resurgence of tectonic/orogenic activity in the Rocky Mountains.** "Perhaps a fifth period of mountain building has begun and the Rockies are cutting a new ridge of teeth along their eastern front. If a new era of mountain building has begun, Denver may be in for the same kind of quakes as California and Yellowstone, both areas where mountains are still growing." (RMN, January 21, 1968)

2. **Disposal of waste chemical fluids by injection into a deep well near Denver.** "... an old fault was put under pressure when they started pumping. They put it under a lot more pressure than nature had, forcing water into area where there hadn't been any water for a long, long time, maybe never. The water lubricated areas that had been static." (Father Downey, January 21, 1968, RMN)
   - The Rocky Mountain Arsenal, a chemical weapons manufacturing site located just northeast of Denver, had begun injecting waste fluids into a disposal well that was 12,045 feet deep in March of 1962. The lowermost 75 feet cut into Precambrian Gneiss.

3. **An increase or threshold-crossing of sediment loading on floodplains in Denver.** (Pressure from weight of the surface sediments fractured the underlying rocks.) Earthquakes can be triggered by "stress from river deposits - the moving of dirt from one area to another... As part of the erosion process, rivers are taking great quantities of dirt from our Rockies and dumping it in the Gulf of Mexico." (Father Downey, Sept. 4, 1964, RMN)

Report on RMTEC Supported *GEOL 100* Course Activities

Spring 1996, planned which course and personnel would undergo RMTEC revisions.

Summer 1996, hired doctoral student to develop several revised *Geology 100* labs.

Activities:

At the start of the semester we met with RMTEC leaders and with chemistry faculty who had already done RMTEC-backed course development.

We began fall semester by developing a "rational/over-all logic statement" of the philosophy behind the changes we'd like to implement in the course.

The first TA meeting (for all graduate students who would actually be teaching the course) was used to acquaint them with these ideas, before we even approached the lab content.

We held weekly TA meetings to preview upcoming labs and to do post-lab critiques. The previews presented concepts we expected to develop and possible teaching strategies, as well as obvious discussion of lab content. The TAs were expected to offer feedback on how the labs were received, offer their opinions on strengths and weaknesses of the labs, and make suggestions for future changes.

We held a semester-end dinner meeting for the class lecturer and TAs to summarize all of the above. We produced a lab-by-lab critique and a list of changes and/or additions to the program for spring semester.

We began fall with 20-minute personal interviews with volunteers from fall geology class to gain some student viewpoints on the lab program.

We attended a meeting at GSA in Boulder November 1996 to exchange ideas with others doing similar things at CSU and CU.

Products Produced for Lab Revision during Fall, 1996

Labs:

The following list of *General Geology* labs was produced. Some were totally new to the program; others were revised labs that existed and had been used previously.

They are intended to fill a 14-week schedule, one 2-hour lab per week.

Geology, Physics, Biology

All were drawn up or revised to better fit the following criteria:

- Correlate labs with lecture
- Use as much local geology as possible
- Structure activities so the students do some real science ... investigate, explore, apply geologic ideas to solving real problems
- Make field trips more important by integrating lecture/lab information and concepts with the trip itself
- Incorporate math at an appropriate level for the students involved
- Have students use computers for a simple geologic investigation
- Cause students to go beyond just mastering of the tools of basic geologic learning (i.e., such things as rock and mineral identification and map-reading skills) and have them apply these skills to realistic geologic questions - many of them of local interest/importance

List of Labs:

- Where Are We?
  Globes; maps; latitude/longitude; time zones
  use of a sextant
  *use of GPS unit (global positioning system unit)
  *new equipment purchased for this investigation and for future field trip activities
- Minerals
  Identification plus "Mineral Trivia" - including practical uses of several specific minerals and some interesting and unique characteristics of others
- Igneous Rocks
  Identification plus special sections on
  Volcanics, Igneous Rock Development, Identifying characteristics of Ign. Rxs & Well Known Locations of Igneous Activity in the Western United States
- Sedimentary Rocks
  Identification plus special sections on
  Local rock layers (formations and local stratigraphic section)
  Development of the Earth and Life On It (Fossils)
  Economic Uses of Sedimentary Rocks (Energy and other uses)
- Metamorphic Rocks
  Identification plus special sections on
  Economic uses of metamorphic rocks and Thin Sections

- Historical Geology
  Geologic development of the local area and Telling Time, Geologically
- Fluvial Lab
  Interpretation of real stream-gauge data from local streams to explain past stream behavior in our local watershed and predict future possibilities
- Topographic Maps
  How to interpret topographic maps using Greeley maps and field trip area maps

Assignments:

- Computer Lab
  Use of "World Geophysical" software to investigate earth's surface and under ocean features and make some plate-tectonics associations
- How Slow It Goes
  Simple math used to measure rates of motion of plates, glaciers, etc.
- Geologic Time
  Exercise to practice basics of relative dating and absolute dating
- Geologic Structure
  Exercise to practice basics of folds and faults

Current Activities:

An earthquake lab is being written for spring. It involves an introductory video, the task of analyzing data to determine the reason for Denver's series of quakes in the 1960s, and an exercise in locating a quake focus using real data.

A new lab is being written to combine the following:

Field Trip Preview:

This will be based on a slide program of the area (being assembled now) and a computer disk of these slides available to students for both preview and review purposes.

Geomorphology of the Area as the result of ...

fluvial factors, local rock layers, local geologic history, and sedimentary environments

The map labs are being extensively reworked to include some geologic structure activities and more geomorphology interpretation. The schedule has been reworked to allow more time for the map lab work.

Two new assignments are being considered, one having to do with a computer modeling of the interior of the earth and the other with a lecture-based paper/pencil activity (to be done concurrently with lecture) on population growth.

Main goals for this semester are to produce camera-ready copy for a lab manual for next fall and to create/gather together extensive graphics/photos/actual materials for the lab program in future semesters. We all agree that science involves creativity and freedom to investigate real problems, but we are faced with the reality of large and ever-growing numbers of students, many of whom have little science background, and little class time. We feel strongly that a quality manual - hopefully one that allows flexibility but that does require practice in basic procedures and leads into real problem-solving – is necessary to supply structure/order needed for the large program we have and proper continuity semester to semester, despite turnover of lab instructors.

Map and Resultant Vector Worksheet

Strategies:	Cooperative Groups Use/solution of real world applications/problems In-class discussion
Class:	College Physics I
Instructors:	Richard Krantz Steve Iona Metropolitan State College of Denver

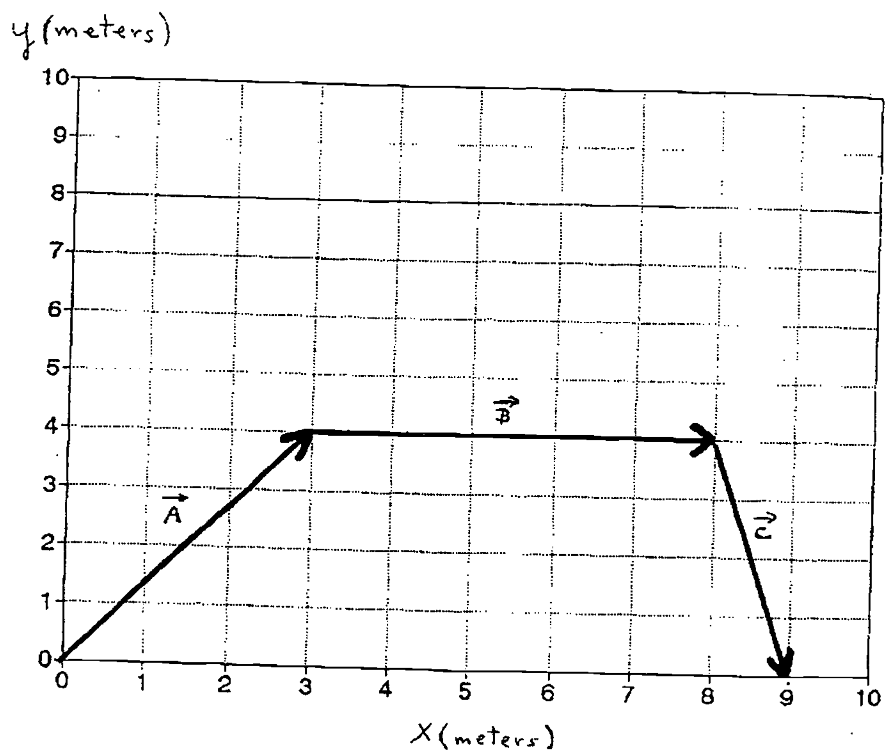
Introduction: This is the first course in a survey of physics. The course concentrates on basic mechanics, an introduction to waves, and an introduction to thermal physics. In lecture the concentration is on the BIG IDEAS in mechanics, waves, and thermal physics. Therefore, students are expected to figure out material on their own or in groups. Students are encouraged to form study groups, to come to office/recitation hours for help, and to ask questions in class. As an aid to organizing students' studying, lists of materials and lecture subjects are provided in advance. Students are expected to ask questions during lecture, thus transforming lecture into lecture-discussion. Familiar subjects, topics, or everyday life experiences are incorporated to enhance learning, as shown in the sample worksheet that follows.

Activity: Ph201 - Worksheet

1) Refer to the figure provided:
   a. Fill-in the following table:

	x-component	y-component	
A			
B			
C			

   b. Show the resultant vector (on the figure).

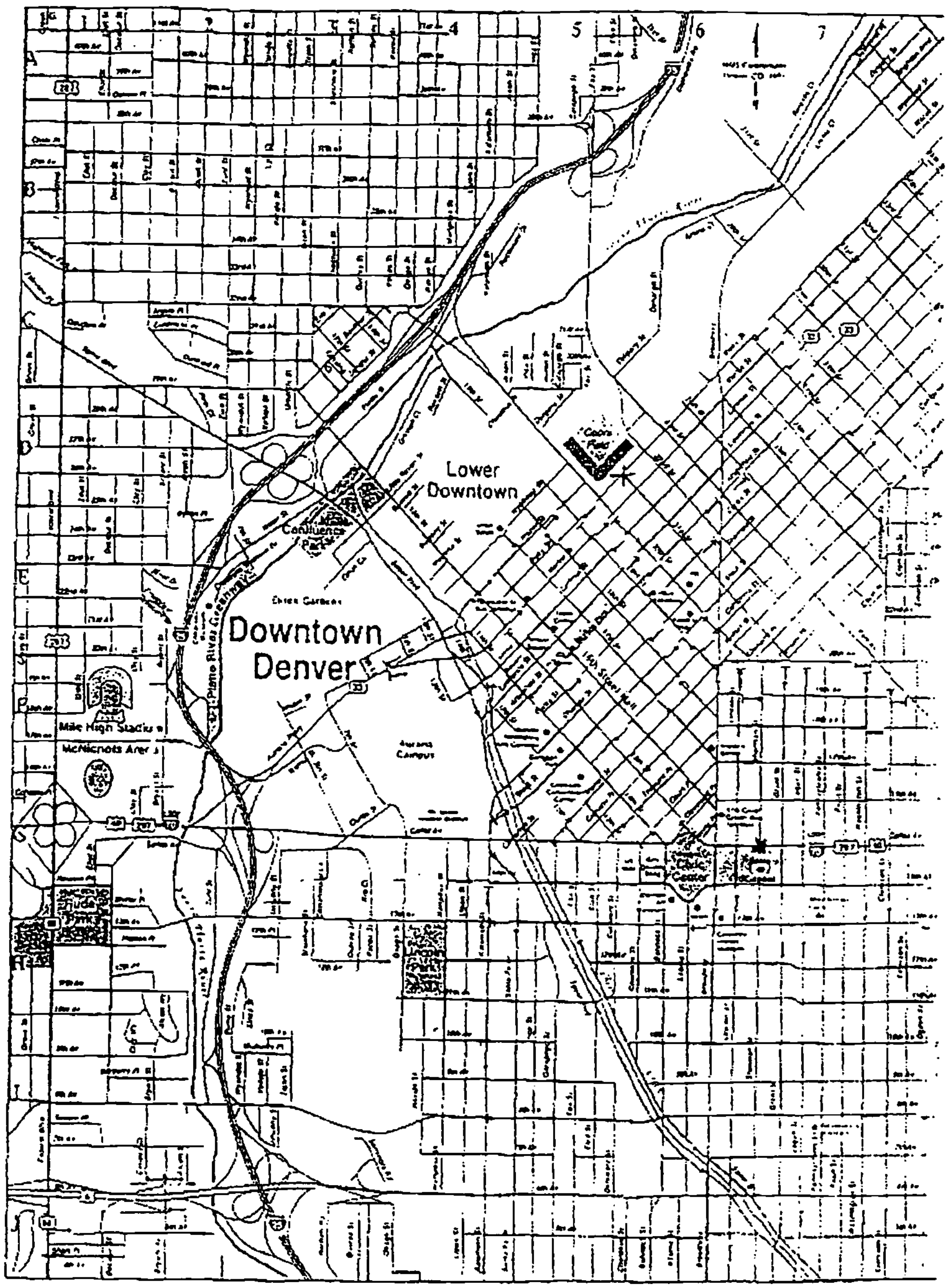


2) Using the map of downtown Denver and surroundings:

a. Describe (in words and by drawing the path on the map) two different ways to get from the front entrance of the State Capitol to the front entrance of Coors Field.

b. Describe the directions to get between these two places "as the crow flies."

Downtown Denver Street Map



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Impulse/Momentum Worksheet & Quiz

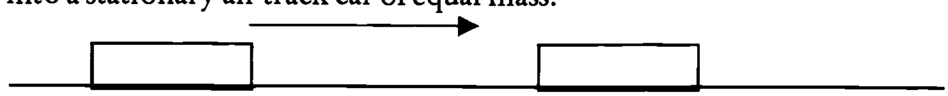
Strategies:	**Constructivist methods** **Solution of complex problems by students**
Class:	**College Physics I**
Instructors:	**Richard Krantz** **Steve Iona** **Metropolitan State College of Denver**

Introduction: This course incorporated a variety of teaching methods and materials that would be used as part of regular instruction in an effort to improve the learning of physics. Information collected from students about the delivery of the class was used to assess the effectiveness of techniques and to make curriculum and instructional decisions.

Following are a sample worksheet and quiz that incorporate teaching methods and materials believed to improve student learning of physics.

Activity: Impulse/Momentum Worksheet

1. An air track car, m1, is traveling to the right, on an air track, at a constant speed. It runs into a stationary air track car of equal mass.



a. Draw the motion map for both masses before they collide.

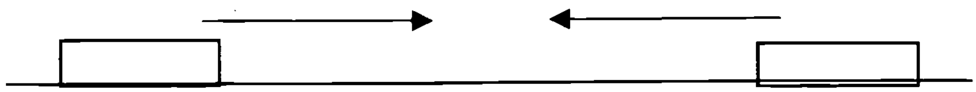
b. If one of the air track cars has an elastic bumper on it, draw the motion map for both air track cars after they collide.

2. An air track car, m1, is traveling to the right, on an air track, at a constant speed. An air track car, m2, is traveling to the left at a constant speed. The masses are the same and the speeds of the cars are the same.

a. Draw the motion map for both masses before they collide.

b. If one of the air track cars has an elastic bumper on it, draw the motion map for both air track cars after they collide.

3. An air track car, m1, is traveling to the right, on an air track, at a constant speed. An air track car, m2, is traveling to the left at a constant speed. The masses are the same and the speeds of the cars are the same.



a. Draw the motion map for both masses before they collide.

b. If one of the air track cars has a sticky bumper, so that the cars stick together, draw the motion map for both air track cars after they collide.

4. An air track car, m1, is traveling to the right, on an air track, at a constant speed. An air track car, m2, is traveling to the left at a constant speed. The masses are different and the speeds of the cars are different.

a. Draw the motion map for both masses before they collide.

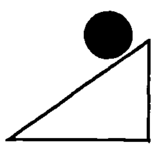
b. If one of the air track cars has a sticky bumper, so that the cars stick together, draw the motion map for both air track cars after they collide.

Activity: Ph 201 - Quiz 17

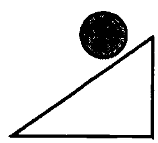
Name ____________________ Student Number ________

1) In the following 9 diagrams, a solid sphere, a solid cylinder, or a hollow cylinder starts from rest at the top of the same incline and rolls to the bottom, without slipping.

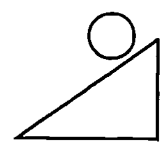
A
sphere
R = 3 cm
M = 500 gm



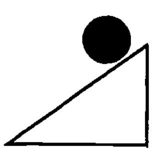
B
solid cylinder
R = 3 cm
M = 500 gm



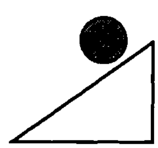
C
hollow cylinder
R = 3 cm
M = 500 gm



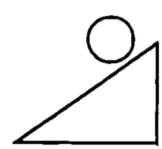
D
sphere
R = 6 cm
M = 500 gm



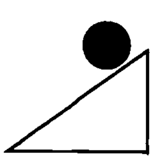
E
solid cylinder
R = 6 cm
M = 500 gm



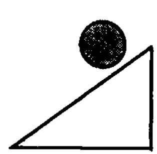
F
hollow cylinder
R = 6 cm
M = 500 gm



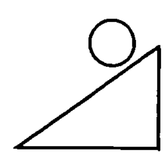
G
sphere
R = 3 cm
M = 700 gm



H
solid cylinder
R = 3 cm
M = 700 gm



I
hollow cylinder
R = 3 cm
M = 700 gm



2) Rank the diagrams, from shortest to longest, according to the time it takes for the rolling object to reach the bottom. Ties are possible, identify them!

shortest time ... longest time

1___ 2___ 3___ 4___ 5___ 6___ 7___ 8___ 9___

Briefly, explain the reason for your ranking: ______________________________

____

____

Biology-based Physics Laboratories

Strategies:	**Use of technology** **Use/solution of real-world applications/problems** **Constructivist methods** **Making connections to other fields** **Solution of complex problems by students** **Hands-on learning**
Course:	**General Physics**
Instructor:	**Brian Jones** **Colorado State University**

Introduction: The *General Physics* course these labs were developed to accompany is one that is taken primarily by students in the biological sciences. Developing exercises in their fields of interest was conceived primarily as a method of engaging student interest, however the instructors have observed many other benefits. The primary one is that by having students make measurements on biological systems, they are introduced to a level of complexity that really challenges them to extend their skills. In addition, the problems that students solve are truly real-world: they make measurements on their own bodies. Students also have the chance to work with technological tools that they may well use at some point in the future. Using the tools from a physics perspective gives them a much better idea of how the tools actually work.

The following are briefs of 3 labs:

- **LAB # 1 Muscles and Bones**

Students make physical measurements on their own bodies of forces and torques involved in the movements of joints in their bodies. In addition, they develop a technique to measure the position of their center of mass, and compare results among students of different body shape to see how this varies. They also explore applications of forces and torques to practical problems such as setting ski binding tension to avoid injury.

- **LAB # 2 Biological Sciences**

Students make measurements of the voltage signals from their hearts and use an optical probe to measure blood flow. The voltages are measured with a very simple ECG system. Students are encouraged to experiment with electrode placement, and can actually determine the orientation of their heart in their chest cavity, which they find very interesting - and difficult. They must contend with signals from other muscles and the fact that the electrical signal from the heart is very complicated and hard to decipher. Students also use the optical probe to measure blood flow in extremities, and are encouraged to use this probe along with the ECG device to design and perform experiments. For example, students have looked at:

- Measurement of time for blood flow from heart to fingers as a function of individual and recent exercise.
- Measurement of volume of blood flow as a function of recent exercise.
- Measurement of volume of blood flow to extremities as affected by nicotine.

Students have been very engaged and creative in identifying experiments to try with this equipment.

■ LAB # 3 Optics and the Eye

Students work with simple optical tools to measure properties of their own eyes. They explore the focusing ability of their eyes, the response of their irises to light, reaction of their retinas to light of different colors and patterns, and means of correcting vision. Students are encouraged to hypothesize and explore, and have come up with a number of novel experiments to try, including:

- Afterimages of color: differences between individuals with normal vision and those with some impairment of color vision.
- Using shadows on the retina to show structures in the eye, including surgical scars.
- Measuring astigmatism with very simple techniques.

Group Poster Project

Strategies:	**Use of small groups in large lecture courses** **Cooperative groups** **Alternative assessments** **Constructivist methods**
Class:	**Microbiology**
Instructors:	**Erica Suchman** **Ralph Smith** **Colorado State University**

Introduction: In Fall 1997, the instructors began experimenting with the use of small groups in their large (100-200 students) introductory microbiology course. They wanted to encourage students to take a more active role in their own education and encourage the development of teamwork skills that would be required in the work force. Students formed 5-person groups early in the semester, worked on projects in groups throughout the semester, and submitted a poster describing a disease of their choice. Students learned a great deal from the experiment, and the instructors have modified/improved the process greatly over the last five semesters.

From Passive to Active Learning: One of the most difficult tasks the instructors encountered when using group projects was luring students away from the comfort of passive learning. Many students were quite resistant to the idea, and some were even resentful. They expected a course in which they were told what to learn and how to learn it, and they did not wish to explore learning on their own. The instructors discussed the problems they encountered, and the solutions they developed to overcome these problems.

Problems and Solutions: Working with other students is a new experience for many students, so it is important to make them feel comfortable with their groups. For this reason, the instructors asked students to choose their own groups. Furthermore, the instructors found there was less complaining about group members if the students were allowed to "try a group on for size." The instructors therefore allowed students to change groups one time if they were unhappy with their initial group. Although this did not completely eliminate poorly functioning groups, it allowed students to leave groups with which they felt they could not work. Another tactic the instructors found useful was to form the permanent groups after the drop period was over, eliminating fragmented groups that resulted from students dropping the course.

The instructors caution that there is always going to be the problem of differing commitment levels by group members. Many students complain that a few students are doing all of the work, but all of the students get the same grade. Although the instructors see this as a problem, they believe that it reflects a real issue students encounter when they enter the work force. Therefore, exposing the students to this circumstance helps prepare them to enter the work force. The instructors addressed this issue by asking students to evaluate and assign grades to the other members of their group. Therefore, if students were truly unhappy with the commitment level of other members of their group, their dissatisfaction was registered. The knowledge that the other members of the group would be determining a portion of their grade also helped motivate students who might be tempted to allow other members to do all of the work. Each member of the group was given the average of the numerical grade assigned to them by their group members. Furthermore, questions from group projects were included on all standard examinations, ensuring that students who participated in their groups would perform better on these examinations.

During first semester of the course, the instructors incorporated group work called "in-class group projects." They found that students came to class unprepared to perform the duties asked of them in the allotted time. Therefore, the instructors subsequently distributed projects a week before the designated class with the understanding that students were to complete these assignments outside of class, and come to class prepared to discuss them with their group to compile the best possible answers.

The instructors were still disappointed with the students' level of preparation on the day the group project was due, and they encountered a great deal of complaining about the lack of time to adequately answer the questions. The next semester, the name of the activity was changed from "group projects" to "in-class group examinations," and project assignments were handed out a week in advance. Students were told that these were take home exams to complete, and in a week the group would meet to formulate a collective answer during an in-class group exam. The level of preparation went up dramatically, and complaints about time were almost completely eliminated.

The instructors advise that when creating group projects, it is important to structure questions very carefully to achieve desired goals. Students have become very good at looking for answers in books and reciting "what the book says." Students resisted sharing what they thought about a subject. The first group project was very disappointing. Instructors asked students to draw pictures of two types of cells and discuss differences between the two cell types. The majority of group work was simply figures from the book copied verbatim. This caused the instructors to re-evaluate the group projects; however, when they examined their first project, it was obvious that the question elicited what the book was already saying. Therefore, it was important to decide what the goals were, to try to create projects to meet those goals, and then to evaluate whether those goals were met.

In-class group examinations served as essential preparation for the course's capstone poster project, the final and most difficult group project. Assigned toward the end of the semester, posters were assembled by students, giving them the opportunity to assume greater responsibility for their own learning. The instructors gave up significant lecture time for this activity, and feel that greater learning occurred as a result. With assistance from their Teacher in Residence (through the Center for Science, Math, and Technology Education), the instructors developed a system to distribute the posters to junior and senior high schools and Departments of Health in the region. The poster activity is described in the following pages.

Activity: Group Poster Project

Criteria for "Disease Education" Posters Spring 2000

Your group has been hired by the health department to create a poster designed to educate the general "non-science" public about your chosen disease. Because the target audience is not well educated about microbes your poster must be interesting to look at, as well as informative!

Important Dates: Poster Project

3/20 Provide instructor with the names of five partners. Select the disease topic the group wishes to work on. The instructor wants each poster to be about a different disease, so it will be "first come, first serve" for each topic.

4/17 Rough draft of poster components must be exchanged within each group. Each person in the group **will submit 2 copies of a typed rough draft of the section for which he or she is responsible**. The member who submitted it for correction will retain one copy, the other will be turned in for 3 points of credit. The name of the etiologic agent and the disease(s) caused by this organism must be identified. Each group will submit proof that this exchange has taken place (for credit).

4/24 Poster is due on this date and it is to be handed in to the Microbiology Departmental Office. Late submission of posters will be penalized by point reduction.

4/26 Evaluations of other members of your group by the end of Lecture. No late evaluations will be accepted. If you want input on the other members' grades, you must turn in your evaluations. Evaluations will not be accepted until after the posters are turned in.

5/1-5/3 Poster observation days. Remember to pick up poster questions!

The health department has requested that **all** of the following information be displayed on each poster:

Etiologic agent

1. Name (common name)
2. Genus and species (for both eucaryotes and procaryotes)
3. Family name
4. Eucaryotic or procaryotic
5. Kingdom classification
6. Type of microbe (for example, virus, protozoan, bacterium, algae, and fungus)
7. For viruses, specify the virion shape, and type of nucleic acids that makes up the genome. For bacteria Gram stain, (Gram positive, Gram Negative, or Gram Variable, if Gram Variable Acid fast + or -)
8. Size of the microbe (or closely related microbes if you can not find the specific microbe)
9. Appearance of the microbe: please create a labeled three-dimensional model of the microbe (Note: all important components of your microbe must be labeled on your model). Model should have sufficient detail such that a person can determine microscopic appearance; must include internal structures, etc.)
10. Copies of micrographs or electronmicrographs of your microbe (or very closely related microbes if micrographs of your specific microbe cannot be obtained)

Disease

1. Name of the disease
2. Major host for the disease
3. Clinical description of the disease in **layman's terms**, including symptoms
4. Pathogenesis of the disease (How does the microbe cause disease in the host? i.e. Hepatitis causes jaundice due to liver damage)

5. Are virulence factors involved in disease formation? (What about Hepatitis-caused liver damage i.e., is it toxins, invasive enzymes etc.?) If this is a viral infection, is the host damaged by the virus, or the immune system?
6. Picture of disease, or of a diseased organ or tissue

Epidemiology of the disease

1. Distribution of the disease. Maps are required (you may need to make your own, if you can not find one).
2. Are particular demographic groups more susceptible than others? Charts, or graphs are required (you may need to make your own, if you can not find it in a chart or graph form).
3. Mechanism of spread of the disease (i.e., does the disease have a vector?).
4. If there is a vector or vectors, pictures of any vectors should be included.
5. Is there a reservoir host? If so what are they? Provide photos of the reservoir hosts.
6. If other species than the major host can carry or be infected with this disease name them and provide photos.

Diagnosis of the disease

1. Please list the methods of detection for the organism.
2. Briefly describe the most commonly used method of diagnosis, **in layman's terms**.
3. Provide photos of either the test, machinery used in the testing, or someone getting tested.
4. Host sources from which the microbe is recovered (if you are testing for infections with this microbe, what tissues will you test? i.e., HIV you test the blood).
5. How reliable are the detection methods?
6. Does diagnosis depend on high-tech equipment and training?
7. Could these diagnoses be readily carried out in poor countries with limited health care budgets?

Prevention and treatment of the disease

1. Are one or more vaccines available?
2. If so, are they required in the U.S. or for travel?
3. If a vaccine is available is it an attenuated, killed, subunit, toxoid, or recombinant vaccine?
4. Briefly, **in layman's terms,** describe how this vaccine is produced.
5. If a vaccine is available, what percentage of population will become infected if not vaccinated? If vaccinated?
6. Does the vaccine have any side effects?

7. In what percentage of the population do these side effects show up?
8. Are antibiotics effective? If so which ones. How much is taken, and for how long?
9. Have antibiotic resistant strains developed? If so describe them.
10. Are there non-antibiotic drug therapies? If so what are they? How much is taken, and for how long?
11. Have resistant strains developed to non-antibiotic therapies?
12. Are there other methods (such as better sanitation, vector control programs, etc.) available for control of the disease? If so, what are they?
13. How effective is disease control?
14. What percentage of the population will die if untreated?

Each student will write about his or her section of the group's poster. The write up should not be in a paragraph form, but in bullet form. This is much easier for the reader to follow. Keep answers short and to the point, and written in a way that non-microbiology students can understand, but don't leave out important information. This section will count toward each individual's grade for the course. Any facts mentioned or pictures, charts, graphs, used in each section should be properly referenced in the text as shown below. Note: References should be listed at the end of each student's section. Each student must have at least three references, at least one of which must be a library reference other than your textbook, and not from the web.

Example:

Detection: Bacteria are cultured from the throat, grown on Blood agar plates and analyzed for the ability to produce hemolysis. The appearance of b-hemolytic colonies, which can completely lyse red blood cells, indicates infection with *Streptococci*[1]

1. Johnson, J., and Brown, T., (1996) Diagnosis of Streptococcal infections. *Journal of Bacteriology*, 66: 22-35.

If you use information obtained from the web, reference the URL of the site. However, be careful, information from the web is not peer reviewed and may not be accurate! The information on your poster must be correct. You will lose points for incorrect information on your poster. Therefore, be sure your web sources are reliable!

Poster format

Standard posterboard may be purchased from the CSU Bookstore. A variety of colors are usually available. Please do not use the thick Styrofoam backed boards as they are difficult to hang! Be sure you indicate somewhere on the poster who the 'presenters' are, and which sections are provided by each member! Please also provide your group and section number.

A suggested format is provided on the following page.

Group Number

Section Number

Refs.
Student name

MODEL
Student Name

Refs.
Student name

Photos

label, Ref.
Student name

Refs.
Student name

Refs.
Student name

Refs.
Student name

Grading

The Poster Prep Group Meeting counts for 5 points. This is a mandatory class session. By this time in the semester, you should have materials prepared for the group to use in the final poster. You will only receive credit if you bring material appropriate for the group to use at this stage of poster preparation. At this group meeting, members of the group will exchange typed materials and make suggestions. **You should bring two copies of your typed materials, one for you to keep with changes your group would like you to make, and one that you will turn in for 3 points of credit.**

Poster presentation:

The group grade counts for up to 10 points. Each student is awarded the group grade. This grade will be assigned by the instructors and will be based on the content, correctness, and appearance of the poster. The group grade will also depend on how well the group has "proof-read" each member's part of the presentation.

Each individual within the group can earn up to 45 points for his or her part of the poster presentation. Ten of these points will be earned from an assessment performed by the other members of the group on a scale of 1-10. Each student will receive the average of the scores assessed by the other members of their group. The instructors will also award up to 35 points based on each individual's contribution to the poster.

Summary of poster grading:

Poster prep meeting		=	5
Poster:	Group	=	10
	Individual	=	45 (35 assigned by Instructor, 10 by group members evaluations)
Total			60

On the next page is the list of diseases. Once you have formed a group of **five** (no more and no less) you may come to the front of the class to pick your group's disease. Each group within a section must do a different disease, therefore, if when you come forward for selection your disease has already been chosen, you must choose another disease. If your group has an interest in a disease not found on the list, please see Dr. Suchman or Dr. Smith to decide if you may do a poster on that disease.

Important note: do not for any reason call the hospital to obtain information! All of this information is available in the library. Most information about microorganisms will be found in MEDLINE OVID, which you can access at the CSU library on any computer. If you do not know how to use MEDLINE, come during office hours and Dr. Smith or Dr. Suchman will be happy to teach you. You may also talk to the information desk at the library.

Viral Diseases:
Chickenpox/Shingles
Hantavirus pulmonary syndrome
Hepatitis B
Hepatitis C
Herpes simplex – oral
Herpes simplex – genital
Influenza
Warts
HIV/AIDS
HIV/Kaposi's Sarcoma
Measles
Mumps
Ebola
Lassa Fever
Hepatitis A
Yellow Fever
Western Equine Encephalitis
Dengue
Infectious Mononucleosis
Rabies

Bacterial Diseases:
Anthrax
Diphtheria
Toxigenic *Escherichia coli*
Group A Strep sore throat (*Streptococcus pyogenes*)
Group A Strep necrotizing fasciitis
Legionnaire's disease
Lyme disease
Meningococcal meningitis
Pertussis
Plague
Salmonellosis
Typhoid fever
Staphylococcal boils
Staphylococcal surgical wound infections
Tetanus
Tuberculosis
Leprosy
Gonorrhea
Syphilis
Gastric ulcers and cancer (*Helicobacter pylori*)

Fungal Diseases:
Ringworm
Blastomycosis
Coccidiomycosis
Histoplasmosis
Pneumocystis pneumonia
Candidiasis

Protozoal Diseases:
Giardiasis
Malaria
Toxoplamsosis
Cryptosporidiosis

Prion Diseases:
Creutzfeldt-Jacob Disease
Chronic Wasting Disease Mad Cow Disease
Scrapie
Kuru

Plant Diseases:
Dutch Elm Disease
Potato Late Blight (*Phytophthora infestans)*

Animal Diseases: or others you can think of
Vesicular stomatitis
Distemper
Brucellosis
Parvo virus
Feline leukemia virus
FIV feline immuno-deficiency virus

Food- and Water-Borne Diseases:
Botulism
Staphylococcal food poisoning
Food infections caused by Salmonella
Polio
Listeris
Shigellosis

Fish Diseases:
Pfiesteria piscicida
Whirling disease

Web CT Discussions

Strategies:	**Breaking down the large lecture** **Use of technology**
Class:	**Biology 103**
Instructors:	**Greg Florant** **June Medford** **Colorado State University**

Introduction: The instructors incorporated WebCT (Web Course Tools) as a special resource to break down the large lecture session. The web site allows students to check their grade and provides an online access to a variety of resources (e.g., discussion groups, old exams, syllabus, etc.). Students are given their private passwords for this resource at the first lab meeting. To encourage the use of this resource, all students visiting the site at least two times were awarded 10 extra points toward their final grade.

The teaching innovation used to break the barrier imposed by large classrooms involved a computerized, Web-based system called Web Course Tools (WebCT). WebCT essentially provides a course-specific firewall so that materials can be made available to students enrolled in a specific course, without allowing full access to the worldwide web users. The class, BY103, is a second semester introductory biology class for life science majors. It typically has large enrollments (e.g., Spring 2000, 462 students) and as such students are reluctant to contact the instructor. Further, because greater than 95 percent of the students are freshmen they are unaware of various resources available to them.

The instructors established a WebCT site for the first time this semester as a result of their work with RMTEC and the Center for Science, Math, and Technology Education (CSMATE). The teacher in residence, Vicky Jordan, was heavily involved with logistics of getting the WebCT site up and running. The WebCT page provides a variety of materials specific for the course (syllabus, outline of notes, old exams, individual student grades with histograms, links to textbook web page) and communication links (bulletin boards, private e-mails, links to teaching assistant and other student e-mails). The instructors also added links to scientific information about "hot topics" discussed in class (e.g., genetically modified foods). These topics were also part of a discussion between students and the instructor via the "bulletin board." At a midpoint in the semester, the 462 students had accessed the WebCT page 5955 times. This means, on average, each student had examined the web page approximately 13 times. Because of the emphasis on computerized technology to enhance, but not replace, the class the students are gaining computer skills essential for their futures. In addition, we have found the students much more willing to bring questions to the instructor (both via the computer and in class) and students feel more a sense of somehow being "linked" to others.

In summary, we have found the information and contacts from RMTEC an important part of biological science education at Colorado State University.

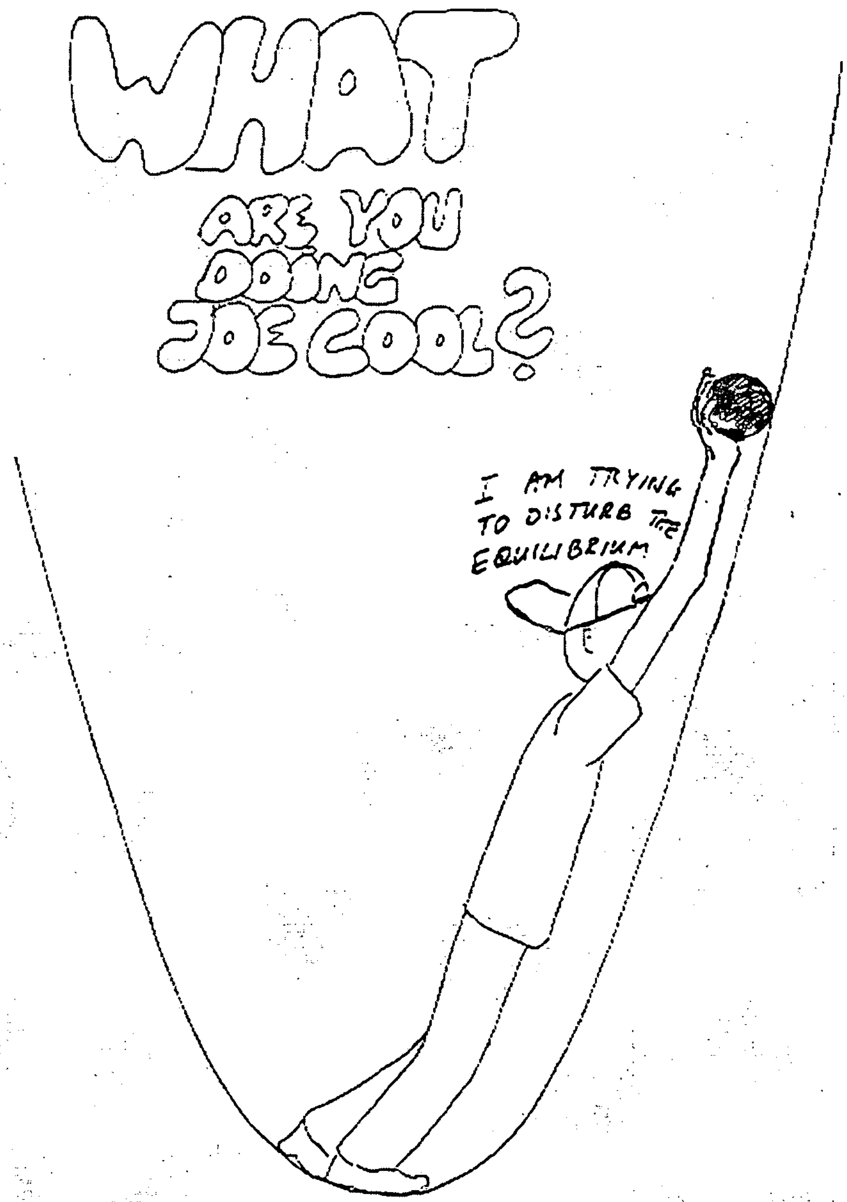
3 Chemistry

Joe Cool's Modules

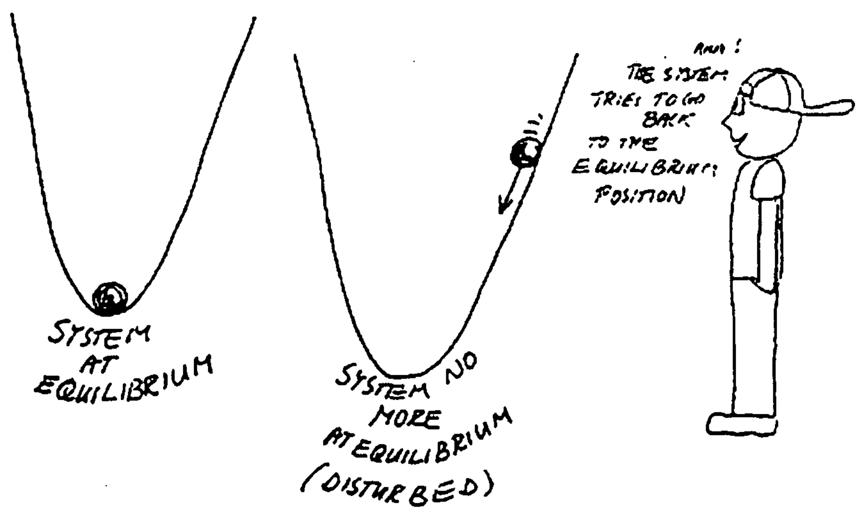
Activities:	**Joe Cool's Modules** **Definition Department** **Equilibrium** **Balancing Chemical Reactions** **Acids and Bases** **The Chemical Naming Band**
Strategies:	**Constructivist methods** **Hands-on learning**
Class:	**General Chemistry**
Instructor:	**Gerhard Lind** **Metropolitan State College of Denver**

Introduction: *Joe Cool's Chemistry Modules* and compact disk set (© by Gerhard Lind 1999) evolved from lecture notes for *General Chemistry*. It began when the instructor made his own lecture notes available to students taking his class. He found that students were willing to pay for his handwritten notes—mostly written in English with a few German sentences now and then. In 1994, Gerhard was asked to become involved with RMTEC to develop new ways to teach *General Chemistry*. Gerhard developed the "cartoon style" modules to be used instead of the usual textbook, which was often untouched by student hands.

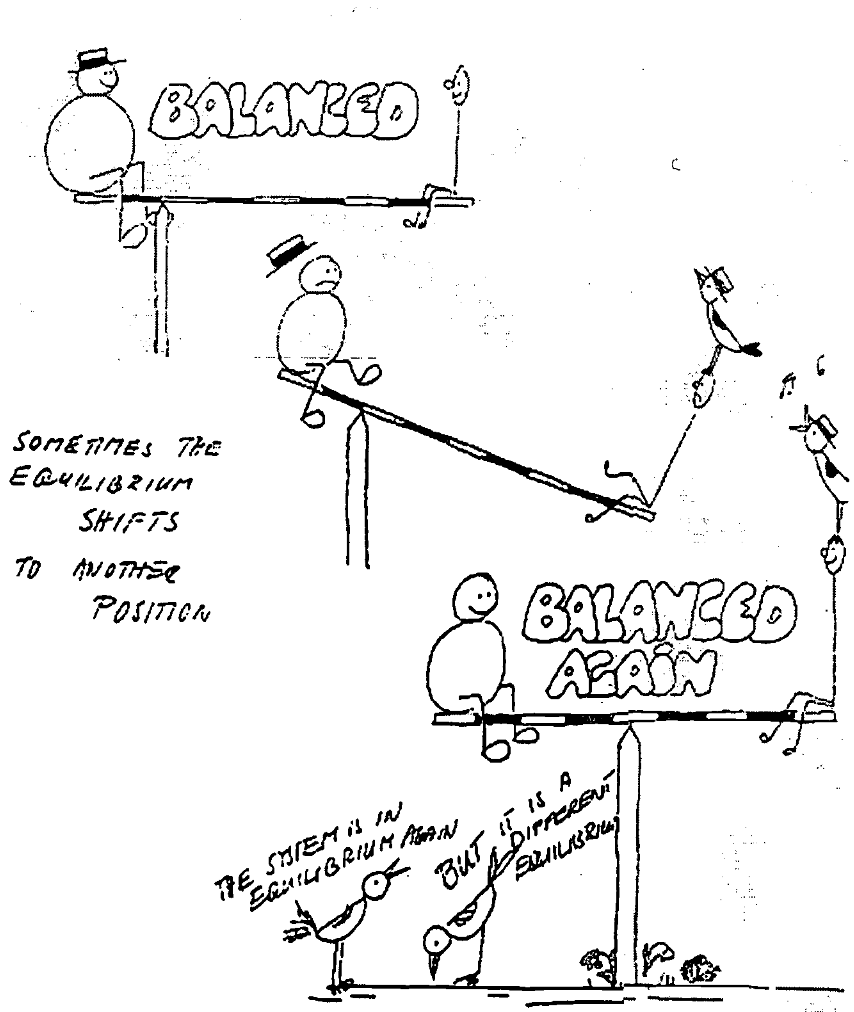
Activity: Equilibrium



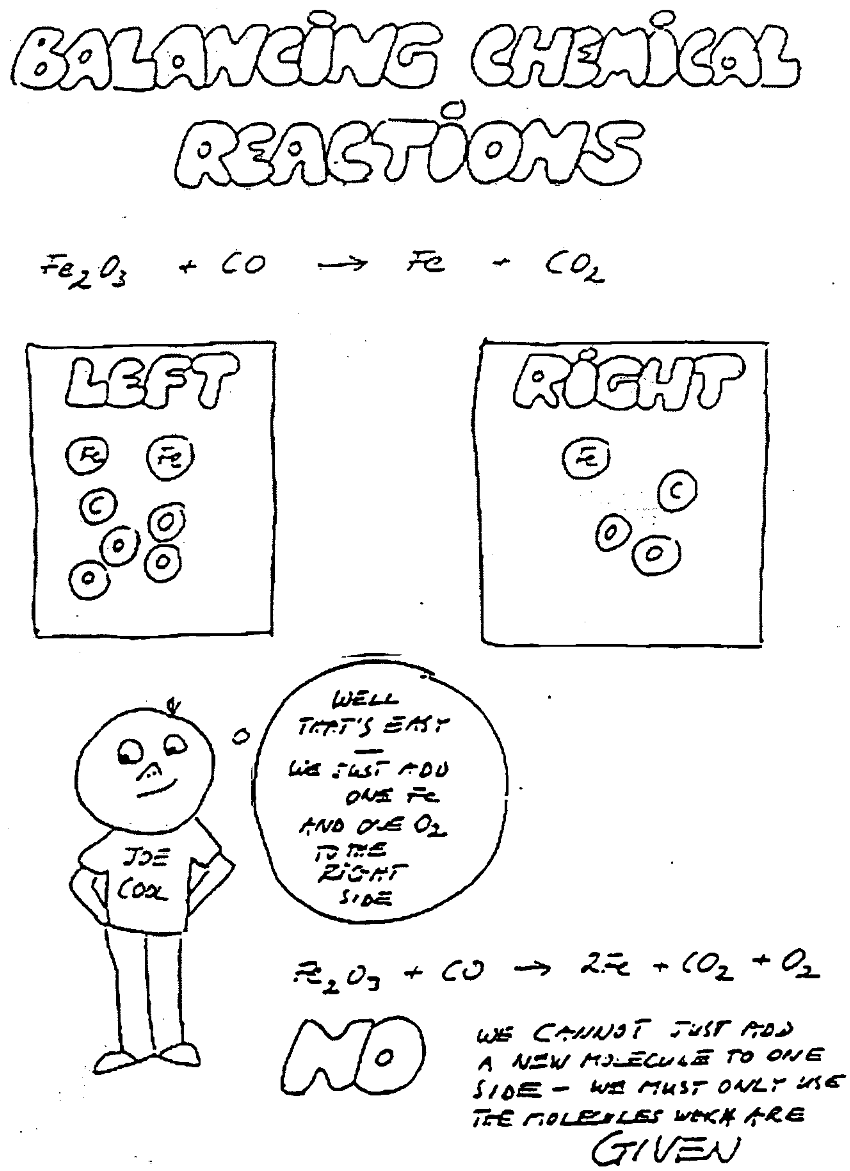
Activity: Equilibrium, Cont'd.



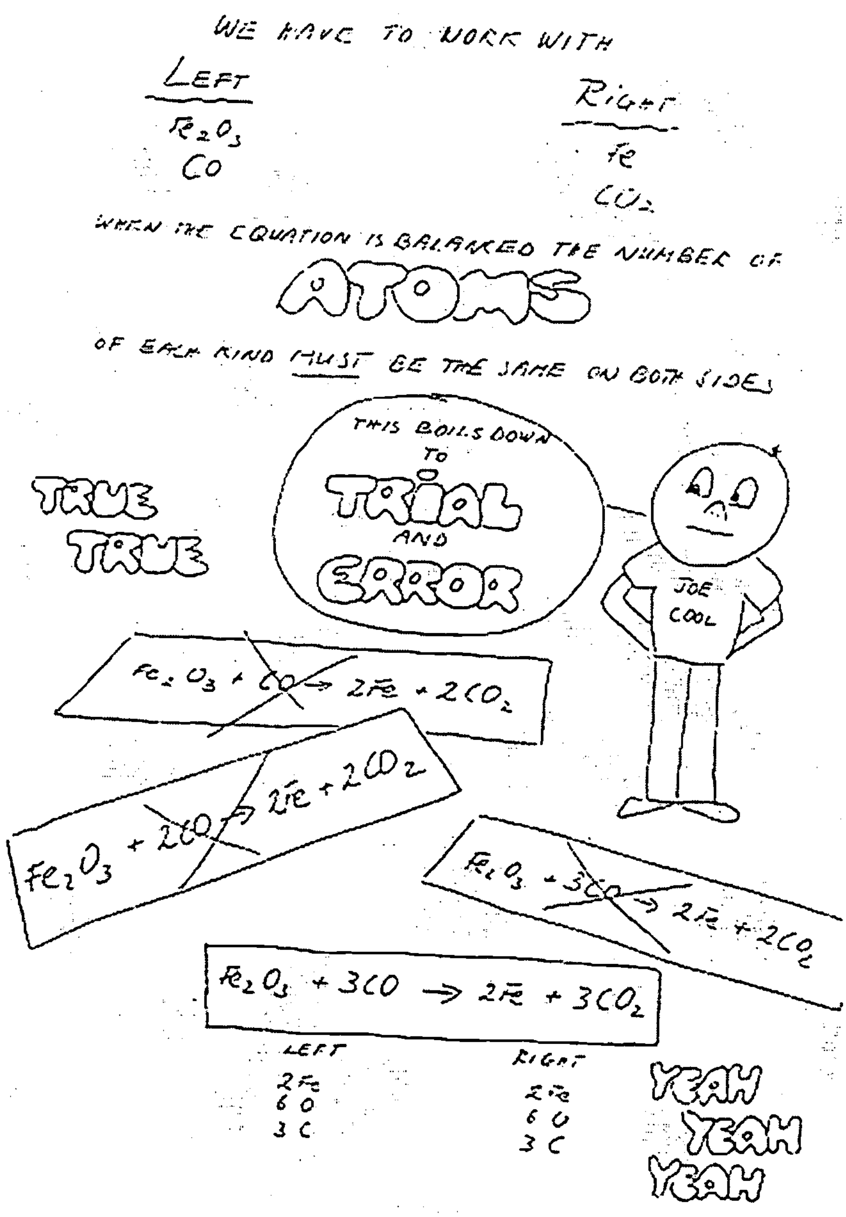
Activity: Equilibrium, Cont'd.



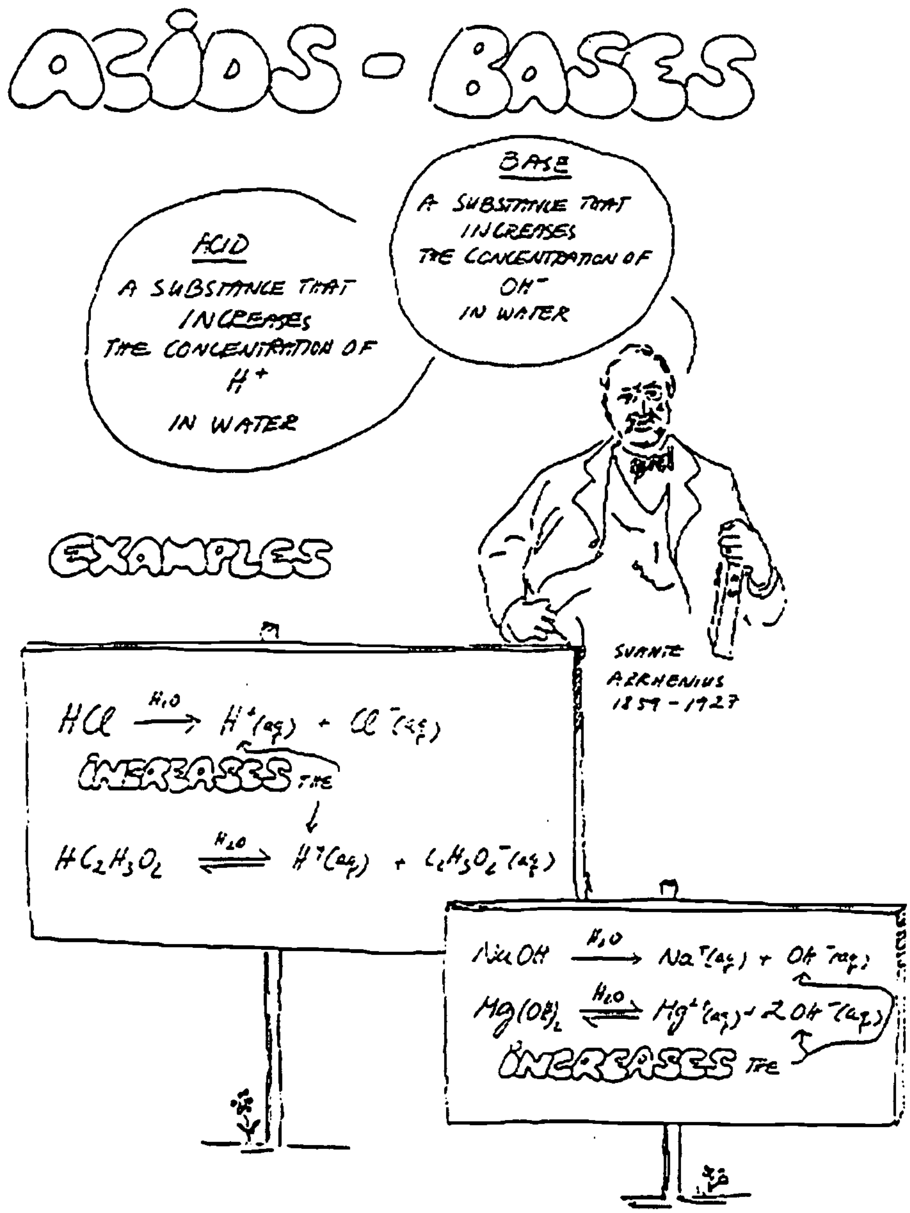
Activity: Balancing Chemical Reactions



Activity: Balancing Chemical Reactions, Cont'd.



Activity: Acids-Bases



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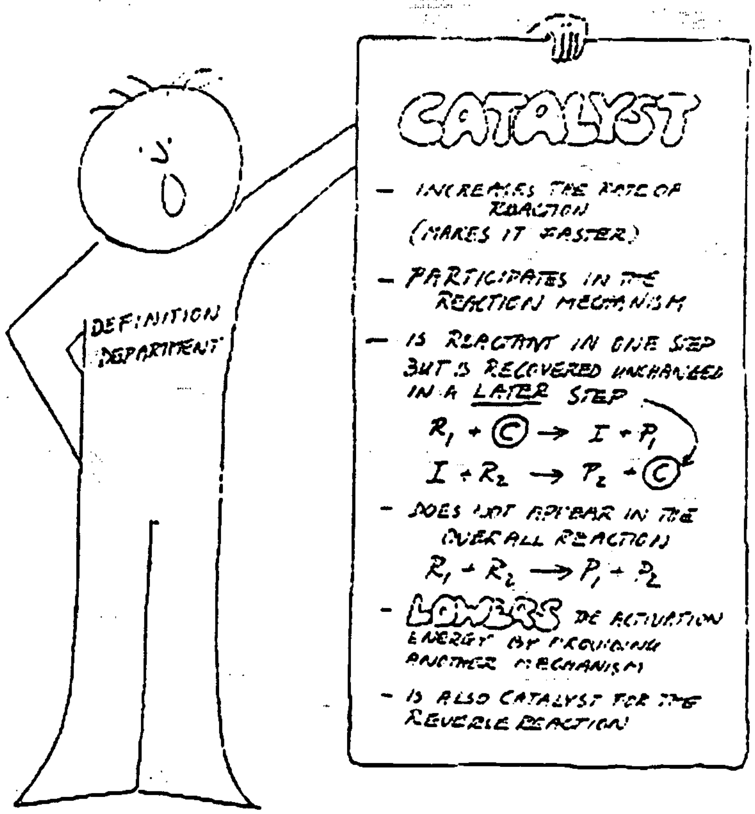
Activity: The Chemical Naming Band

Activity: Definition Department



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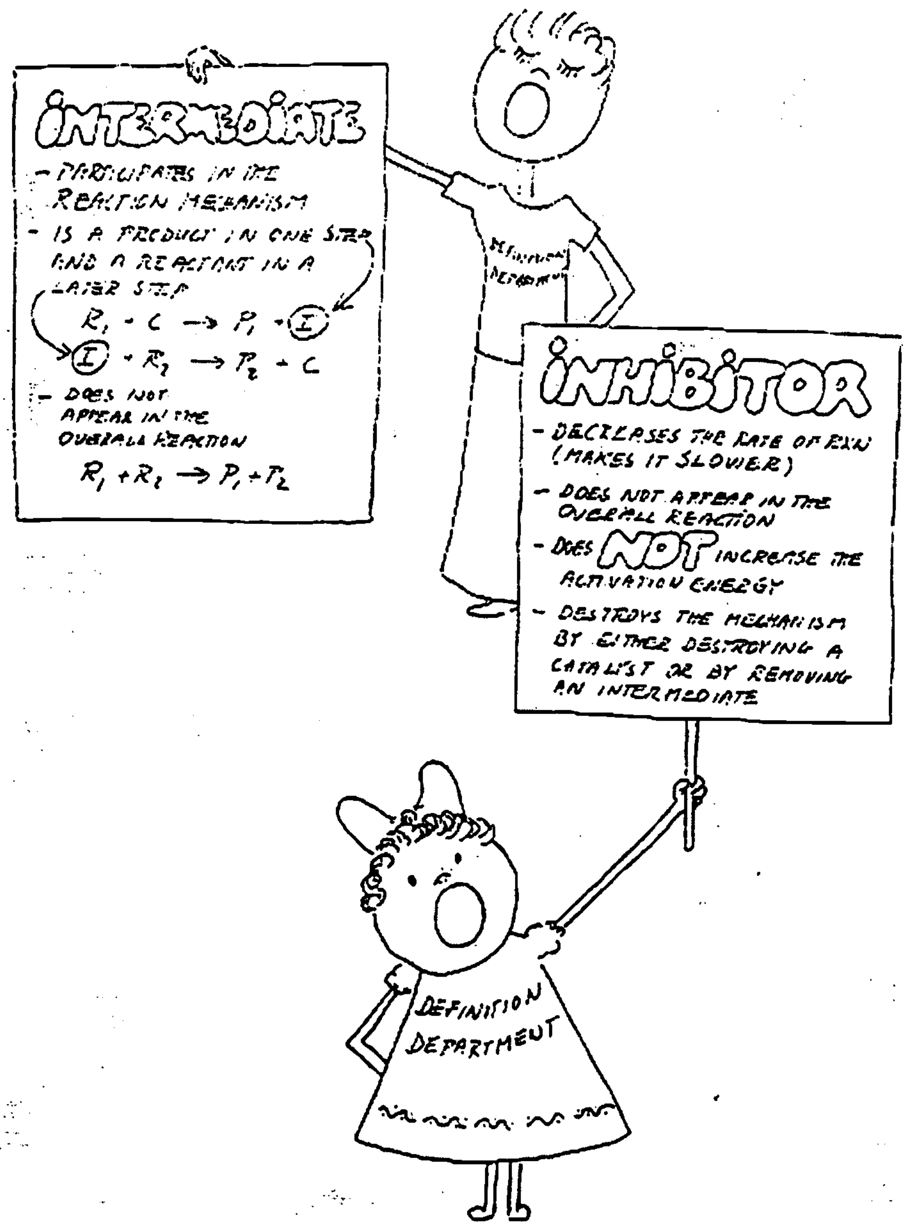
Activity: Definition Department, Cont'd.



Cooperative Learning Teams

Strategies:	**Cooperative groups** **Solution of complex problems by students**
Class:	**General Chemistry**
Instructor:	**Loretta Jones** **University of Northern Colorado**

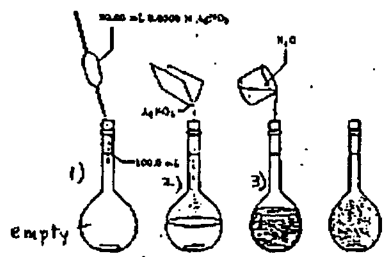
Introduction: The students in this class were organized into Cooperative Learning Teams. Following are some of the cooperative learning tasks assigned to student groups.

Activity 1: Standard Solutions

Task 1. Create an analogy for empirical and molecular formula using common objects (other than bicycles), in which the molecular formula differs from the empirical formula.

Task 2. Your group is conducting a study of coral reefs in the Caribbean. You are based on a research vessel and collect samples to analyze. Among the difficulties you face (manta rays, sharks, sunburn, and falling off the boat) is the need for accurate standard solutions. One day at sea you need 100.0 ml, of 0.0750 M aqueous $AgNO_3$ solution. You have a 100.0 mL volumetric flask, but you have only about 60 mL of a stock solution of 0.0500 M $AgNO_3$ and some solid $AgNO_3$. Since it takes weeks to replenish supplies, you do not want to waste any stock solution. Thus, you decide to:

1) Pipet 50.00 tnL of the 0.0500 M ($AgNO_3$) solution into the 100.0 ml, flask.
2) Add enough pure solid $AgNO_3$ to make up the difference.
3) Dilute the resulting solution to 100.0 mL.



Describe below how you calculated the mass of solid $AgNO_3$ (in grams) you need to add. Include each step and an explanation. You may use additional sheets of paper.

Activity 2: Shapes of Molecules

In this activity you will use gumdrops and toothpicks to explore the shapes of molecules. For these "generic" molecules, design a three-dimensional structure that arranges the valence shell electron pairs about the central atom so that the electron pairs are kept as far away from one another as possible, thus minimizing electron-pair repulsions. Assume that all the valence electrons are used to form bonds; there are no nonbonding or long pairs around the central atom. A = central atom; X = atom attached to A. Draw each structure and in your sketch indicate the X-A-X bond angles. Use the gumdrops, toothpicks and protractor to help you design each generic molecule. Hint: create a first try and look at the bond angles. Is there a small angle that could be made larger? Remember, you want the best overall separation of the bonded atoms. Start with AX_2, then add another atom. Think how the other bonds are being affected by the new one and move them away from the new bond to compensate. Continue this process with each new atom.

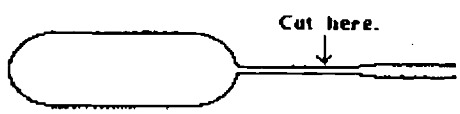
AX_2 AX_3 AX_4

AX_5 AX_6

Activity 3: Calibration of Microburet

One of the most often used tools in your inventory of equipment is your microburet. It will be used to deliver prescribed quantities of chemicals for reactions, often one drop at a time. It is therefore important to know the volume of a drop so the total volume delivered can be determined. Your first assignment is to determine the volume of drops delivered by your microburet. Each microburet and each person's technique differ slightly so your microburet will have to be custom calibrated by you.

Prepare the microburet by wrapping the flexible stem of a dropping buret around your finger and pulling until a narrow section about 1 cm long appears near the bulb. Cut off any of the wider diameter stem remaining, as shown in the diagram:



You and your laboratory group need to determine an appropriate procedure to calibrate your pipet. Once the best procedure is formulated, check out your ideas with those of other groups and then have the TA approve your final plan. Once the plan is approved, each student will calibrate his or her own microburet.

Questions for your group to consider: Would it be more accurate to deliver a prescribed number of drops and then measure the total volume? What advantages would this have? Would it be more accurate to measure the volume or the mass of the liquid delivered (consider the equipment available to you in the lab)? If the mass is more accurate, how is the volume determined from the mass? How many times should the experiment be repeated to assure good results? How should the data be recorded? How should the data be summarized? Once the drop size is determined, where will it be recorded?

2. Describe your procedure in the space below. Record all measurements. NOTE: You will use your microburet in later laboratory experiments, so be sure to keep a record of the calibration data and save your buret.

Small-Scale Chemistry Research Project

Strategies:	**Activities involving writing** **Hands-on learning** **Alternative assessment**
Class:	**General Chemistry**
Instructors:	**Stephen Thompson** **Ken Watkins** **Sreedevi Bringi, Teacher-in-Residence** **Colorado State University**

Source: Integrated Lab-Lecture Freshman Chemistry Course developed by Dr. Stephen Thompson, Colorado State University (Lab Text: *Chemtrek* by Dr. Stephen Thompson)

Introduction: As a requirement in the lab-lecture integrated freshman chemistry course, students are asked to plan and conduct an independent research project during the second semester. Students deliver a formal presentaiton for about 30 minutes in class at the end of the year and submit a written project report. Students benefit from support and mentoring all through the research process. Based on student needs and planning meetings with Dr. Stephen Thompson and Dr. Ken Watkins in 1995-96, Sree Bringi, Teacher-in-Residence, wrote these project guidelines and the scoring rubric for the project report.

Activity: Small-Scale Chemistry Research Project by Freshmen Students

Step 1: Literature Review

Do a literature review of 3-5 research journal articles dealing with a chemistry topic of interest to you. The topic can be significant extensions of lab experiments, other experiments from the text, text readings, or outside scientific journal reading. Your literature review should be about 2-3 double-spaced pages as part of your project report.

Step 2: Proposal

Propose an experimental research project connected to your reading. Write and submit an initial half-to-one-page proposal addressing the following major points.

What are you investigating? Why does it interest you?
How will you design the experiment?
What do you hope to learn from doing the experiment?

In your proposal, outline your major research question/inquiry/investigation along with methods/procedures to be used. Include a list of supplies you will need at the lab and what you will bring in from outside. Start talking to the instructors about what is definitely available in the lab and what is practical to do.

Step 3: Writing Up Your Research Project Report

Using your proposal, literature review, and written log, write an official 5-9 page project report. Include goals or hypotheses, literature review, materials used, experimental approach, brief procedures, data, tables/graphs, diagrams, observations, discussion and conclusion. You can also include recommendations for further research.

Sample Research Projects

- enzyme kinetics—effect of temperature on starch digestion
- determination of the pH of common household products
- inhibition of amylase activity by common beverages
- digestion of starch in various legumes by amylase
- a comparative study of amylase on cooked and uncooked rice
- colorimetric determination of the pH of common beverages
- comparing strengths of different antacids using small-scale titrations
- determining whether Juicy Fruit chewing gum contains fructose or other reducing sugars
- determination of the starch content of common beverages using amylase
- comparison of Group I and Group II hydroxides on the oxidation of fructose using methylene blue
- determination of pH of different kinds of soils from different sources
- exploration of reverse osmosis and the effects that temperature, pressure, and concentrations have on the process
- small-scale field kit for science experiments on the trail
- extraction of copper metal from copper-malachite schist
- small-scale ester synthesis
- fluorescence colorimetric determination of the pH of residue fluid
- a small-scale cloud chamber
- high altitude snow scavenging of acidic pollutants
- experiments with a small-scale lead acid battery

Guidelines for Scoring
Freshman Chemistry Research Project Report

Format of Document:
Project report contains following components:
5-6 double-spaced pages total length, with goals or hypothesis, 2-3 page literature review of 3-5 journal articles, list of materials used, experimental design and procedure, data/observations, discussion/conclusion. ________________ 10 pt.

Scientific Method:
Appropriateness of experimental design to the goals; clarity of hypothesis/goal and suitability of chosen materials and procedures; scientific justification; use of small-scale approach wherever feasible. ________________ 10 pt.

Content:
Discussion of the basic chemistry concepts involved in the literature review and experiment; analysis of observations and results based on this freshman chemistry course background. ________________ 10 pt.

Creativity:
Evidence of student's creative thinking in scientific questioning, experimental goals and design; scientific attempts to go beyond the level of the course. ________________ 10 pt.

Expression:
Acceptable writing style and use of language; clarity in discussion of literature review and research; logical analysis evident in interpretation of results. ________________ 10 pt.

Total: 50 possible points

Group Problem

Strategies:	**Cooperative groups** **Hands-on learning**
Class:	**General Chemistry**
Instructor:	**Loretta Jones** **University of Northern Colorado**

Activity: Calculating pH

First designate the members of your group as persons A, B, and C. Data for blank spaces in each question below is supplied in the following table. Data for each group person is in the table below.

Question	Person A	Person B	Person C
1.	2.3×10^{-2}	1.7×10^{-3}	5.9×10^{-2}
2.	1.9×10^{-3}	9.3×10^{-2}	3.1×10^{-3}
3.	12.95	13.25	12.45
4.	2.75×10^{-6}	1.44×10^{-5}	8.37×10^{-6}
5.	2.65	2.44	2.58

Before working the problem individually do the following:

Read the problem together.

Discuss the strategy for solving the problem.

Agree on ball-park guesses for an answer to each member's problem.

Problems

1. Calculate the pH of a ___________ M HNO_3 solution.
2. Calculate the pH of a ___________ M NaOH solution.
3. Calculate the molarity of a NaOH solution whose pH is ___________.
4. Calculate the pH of a 0.20 M HA solution. The K_a of HA is ___________.
5. Calculate the concentration of an acetic acid solution whose pH is ___________. The K_a for acetic acid is 1.8×10^{-5}.

When each member of your group has completed all problems, your group should sign and turn in their problems. When finished, your group may leave or choose to help others with their problems.

Minute Paper Group-Share

Strategies:	Cooperative groups Hands-on learning
Class:	General Chemistry
Instructor:	Loretta Jones University of Northern Colorado

Activity: Minute Paper Group-Share

1. Assign each person in your group an acetic acid solution with one of the following concentrations. (Be sure that everyone has a different concentration.)

   1.00×10^{-4} molar

   1.00×10^{-2} molar

   1.00 molar

   2.00 molar

2. Calculate the pH and percent ionization for your solution.

3. Join your group and plot percent ionization versus concentration on the graph paper provided. Discuss your results.

4. Hand in both your individual work, your group's plot, and a summary of your group's discussion.

Traveling Problem

Strategies:	**Cooperative groups** **Hands-on learning**
Class:	**General Chemistry**
Instructor:	**Loretta Jones** **University of Northern Colorado**

Activity: Traveling Problem

1. Pick two half reactions. Pass these on to an individual in your neighboring group.
2. Using the half cells provided, design a voltaic (or galvanic) cell.
3. Write the half reactions and the overall reaction for the cell.
4. Calculate the emf (E) for the cell. Pass this back to the originator of this mess.
5. Check over the calculations and then write the correct cell notation for this cell.

Group Problem-Types of Matter

Strategy:	**Hands-on learning**
Class:	**General Chemistry**
Instructors:	**Lynn Geiger** **Belia Straushein** **Loretta Jones** **University of Northern Colorado**

Source: *Student Companinon: Tools and Techniques for Individual and Group Learning for Chemistry – Molecules, Matter, and Change* and *Instructor's Manual for the Student Companion for Chemistry – Molecules and Change, Fourth Edition,* by Lynn Geiger, Belia Straushein, and Loretta Jones (W.H. Freeman and Co., New York).

Activity: Group Problem—Types of Matter

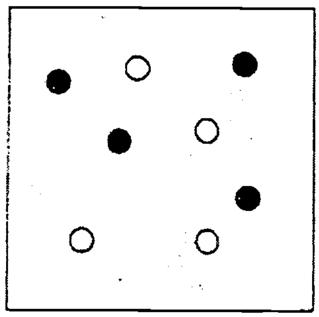
(Note: Answers are on final page of this activity.)

1. What is a pure substance? Define these terms and explain the differences and similarities between mixtures and pure substances.

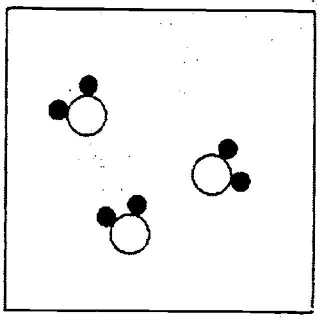
2. Explain the difference between a compound and an element.

3. Consider the gas state molecular pictures drawn below. Which picture is the best representation for a mixture? For an element? For a compound? Discuss your choices with the rest of the group.

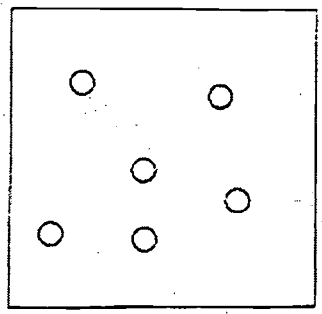
a.



b.



c.



4. What are the three states of matter?

In the three boxes shown below, draw a molecular picture of water in each of its three states.

a. water vapor

b. liquid water

c. ice

Answers

1. A mixture is composed of two or more pure substances. Mixtures have varying composition and can be separated into their components using physical methods. Pure substances have a constant composition. Some pure substance (compounds) can be broken down using chemical reactions to elements.

2. Elements are the basic building blocks of matter in chemistry. They cannot be broken down any further by chemical reactions. Compounds are composed of two or more different elements in a definite proportion by mass. Compounds can be broken down into elements using chemical reactions.

3. a = mixture
b = compound
c = element

4. gas, liquid, solid

a. water vapor—particles that are random in arrangement and far apart
b. liquid water—particles that are random in arrangement and far apart
c. ice—particles that are ordered in arrangement and close together

Cooperative Learning Tools

Strategies:	Cooperative groups Constructivist methods Hands-on learning
Class:	General Chemistry
Instructors:	Lynn Geiger Belia Straushein Loretta Jones University of Northern Colorado

Source: *Student Companion: Tools and Techniques for Individual and Group Learning for Chemistry – Molecules, Matter, and Change* and *Instructor's Manual for the Student Companion for Chemistry – Molecules and Change, Fourth Edition*, by Lynn Geiger, Belia Straushein, and Loretta Jones (W.H. Freeman and Co., New York).

Activity: Cooperative Learning Tools

Introduction

The *Student Companion* contains a variety of activities that use an inquiry-based approach to stimulate critical thinking. These activities can be used in a lecture hall setting or in recitation sections. For each textbook chapter there are the following activities: worksheets, guided readings, in-class group activities, and activities using the CD-ROMs. Worksheets are longer activities that are designed to be used primarily as homework assignments. However, parts of these activities could also be used in class as a quiz or group activity. Guided readings are divided into two parts: an individual assignment to be completed as homework, and an in-class group assignment. The in-class group portion of most guided reading activities is short and can be completed in 10 to 15 minutes. The group activities are designed to be worked in class in cooperative learning groups. These activities vary in length depending on the way you use the activity and the time that you allow for discussion. Approximate times are listed under a description of each type discussed later in this introduction. Monitor the students and stop discussion when you see that students have completed an activity or are getting off task. Keeping track of the time various activities take in the classroom will help you learn to predict how much class time to allow. Many of the group activities can be used as individual homework assignments if you do not wish to use class time.

The *Student Companion* also contains activities that use the CD-ROMs as either primary or supplementary sources of tools and information.

Cooperative Learning

Cooperative learning uses highly structured groups and activities to achieve the educational goals. *Active Learning: Cooperation in the College Classroom* (Johnson, Johnson, & Smith, Interaction Book Company, Edina MN, 1991), is a good introduction to cooperative learning. Some of the goals of cooperative learning are: to increase student achievement by encouraging active learning, to teach the students to assume a greater responsibility for their own learning, to develop group processing skills and provide opportunities for positive interactions among students from diverse backgrounds, and to build a sense of classroom community.

We have found cooperative learning to be a valuable teaching tool. Research conducted in our general chemistry classrooms at the University of Northern Colorado has shown that the use of cooperative learning increases student achievement and improves student attitudes. Key components of a cooperative classroom are:

- Heterogeneous groups of three to four students who work together to solve a problem.
- Positive interdependence and cooperation among group members.
- Individual responsibility and accountability from each student in a group.
- Opportunities to engage in social skill development within the cooperative learning groups.

I. Getting Started

Groups should be set up as soon as possible after the final drop-add date. Setting up the groups takes approximately 20 minutes of class time. Be sure to explain the value of cooperative learning in your course outline or in a letter to the students. Take some time to go over the goals of cooperative learning with your class before you set up groups.

Bring to class group folders with instructions inside each folder (or on an overhead) for getting acquainted in the groups.

Preparing Student Lists for Groups (Assume four students per group.)

Assign the students to groups so that the groups are diverse.

Method 1 **By major.** For example, place one student from each of the following major areas in each group: chemistry, math, and physics; pre-med and health; biology and earth science; miscellaneous and undeclared majors. In a large lecture section, it works well if you put the student's names organized into groups on an overhead. Then ask the students to go to the section of the classroom shown on the overhead to meet their group members.

Method 2 **By self-determined differences.** Ask students to break up into four sections (go to the four corners of the room): by birthdates, majors, etc. Have four people, one from each corner, form a group in which the members do not know each other and are more different than they are alike.

Method 3 **By ability.** Put students in groups that are heterogeneous by student ability, using placement test scores, SAT or ACT scores, or results of your own pretest.

Setting Up Group Folders-All-in-One Folders

Each group is given a folder, referred to as the "all-in-one folder," which is then used to organize all instructor-student written communication. Homework is collected and exams returned in this folder, all group assignments are transmitted and collected this way, and student questions and suggestions can be collected in the folders. Because students pick up their folders at the beginning of each class, the amount of class time needed for paperwork distribution and collections become negligible. Inside the folder students find initial instructions for getting acquainted and for finding value in cooperative groups.

1. Ask each group to assign roles: a group leader, a recorder (who writes out group responses), a reporter (who reports to the entire classroom the group's response), and an encourager (who takes notes for members who miss class and keeps up group moral). Switch group roles several times so that each student in a group has at least one chance to assume each role.

2. Ask a representative from each group to pick up a group folder.

3. Place instructions for getting acquainted inside the folder (or on an overhead). Suggestions for initial activities:

a. List your fears concerning this course.
b. Why are you taking chemistry?
c. List three ways in which you are alike and in which you are different.
d. Name your group.
e. Record your group name and number on the outside of your folder.
f. Develop a "contract" explaining how your group is going to work together to achieve the goals of cooperative learning. Have each member of your group sign the group contract.

4. Bring folders to class every day and use them to pass out papers, return assignments, and communicate with the students.

II. Cooperative Learning Tools-Ideas for Types of Activities

This approach to cooperative learning uses carefully designed classroom cooperative learning tools. Each cooperative learning tool is a framework that can be implemented in "traditional" chemistry classrooms along with the instructor's current test, assignments, and evaluation procedures. The tool approach to cooperative learning provides chemistry instructors with a repertoire of educational strategies and helps faculty and students adopt active learning pedagogies in chemistry classrooms, while simultaneously continuing to rely on some traditional methods of chemistry instruction.

Cooperative learning tools in the chemistry classroom are highly structured tools that allow instructors to monitor a student's time on task, individual accountability, and other problems traditionally associated with group work in the classroom.

The following cooperative learning tools are well suited to the chemistry classroom. Each tool facilitates active learning within the classroom and advances students' academic achievement. These cooperative learning tools offer chemistry students opportunities to clarify understanding, verbalize new concepts, think critically about academic ideas, personalize the lesson, and evaluate their learning. These tools are engaging and provide chemistry students with meaningful lessons on social skill development, group processing, and team building.

Each tool can be implemented within the lecture-discussion format of the traditional chemistry classroom to help college students master their learning. Chemistry instructors can decide which tools best facilitate their academic goals. We have provided examples of our applications of the cooperative learning tools to illustrate their usefulness in the chemistry classroom. If you are trying cooperative groups for the first time, you can start by devoting one day a week to group work, and assigning credit for participation.

1. **Good news:** Ask students to bring some news to share with the class. Select several students at random to share their news with the entire class. You can also ask the individual students to share their news with their fellow group members. This type of activity can be done in a few minutes at the beginning of a class period.

Example	Ask students to bring a news item to class that is related to chemistry. Share news items within the groups and with the entire class.
Assessment	Collect the individual student papers for credit.

2. **Bookends:** Tie together a class discussion by asking the students to observe a lecture demonstration at the beginning of class, or to record questions about the day's topic at the beginning of class. Ask several of the groups to share their questions with the class. At the end of class have the students return to their groups and explain the lecture demonstration or answer their questions using what was discussed in class. This activity can be done quickly at the beginning of a class period. It is up to you how much time is spent in discussion at the end of class.

Example	At the beginning of a class on the law of conservation of mass, perform a lecture demonstration with a sealed flask containing an aqueous solution of silver nitrate. Inside the flask is a test tube containing an aqueous solution of sodium chloride. The mass of this system is determined initially and then again after the flask is turned upside down allowing the two solutions to mix. Students are asked to record their observations at the beginning of class, then at the end of class, to explain why the mass remained constant.
Assessment	Have students turn in their observations and/or questions, along with their group explanation, in their all-in-one folder.

3. **Think-pair-share:** This is a very easy tool to use and it takes only a few minutes of class time. Stop when presenting material in the classroom and ask the students a question. Give students a few minutes to talk with their neighbors and arrive at a solution. Ask volunteer pairs to share their solution with the class.

Examples This tool can be used during discussion and as a way to get the students to formulate strategies for solving problems. A problem can be presented to the class on the board or overhead (or using one of the *Student Companion* activities). After the students think of a strategy and compare with their neighbors, they share with the class different ways of solving the problem. The teacher can then record the strategies and solutions on the chalkboard or overhead.

Assessment Normally the results of this activity are not collected.

4. **Minute paper pair-share:** Assign a problem at the end of class for students to take home and work individually as homework. Each student brings their solution or their part of the solution to the next class meeting. Groups then correct/modify their individual papers or formulate a group response. This tool works well for both thought/ discussion problems and numerical problems. Most group problems in the *Student Companion* can be used as minute paper pair-shares. Allow five or ten minutes at the beginning of class for student discussion.

Examples Explode a hydrogen balloon at the end of class and ask students to go home and explain what happened. Present a problem that summarizes the day's lesson for students to work at home. Assign each student in the group a problem to work that will be tied together the next day in a group activity. Each student brings a solution and the group uses the individual responses to answer a more complicated problem together. (For example, see Activity 14.3 in the *Student Companion.)*

Assessment Either group responses or both the individual responses and the group response can be graded. Group members who do not bring their individual assignments to class forfeit credit. You can also pick one individual paper to stand for the entire group's grade.

5. **Prairie fire:** Write short-answer questions on an overhead transparency, each with the group number. Ask each group to take a few minutes to come up with an answer to their question. Call out the group numbers, and have each group report their answer. Record answers and ask the class to check for errors. This activity takes five to ten minutes for a class of about 100 students. Allow more time if your class is larger.

Examples — This works well in chemistry for short answer questions, such as naming elements and compounds, definitions of terms, and predicting the products of chemical reactions.

Assessment — The students self-correct their answers.

6. **Magic moment quiz:** Allow students some time to discuss the answer to a graded in class assignment with their group members. Stop the discussion after a few minutes and have the students write individual answers. This activity varies in length depending on the problem selected. Group problems and challenge problems found in the *Student Companion* make good magic moment quizzes. Ask students to discuss a problem-solving strategy with their group members, and to solve the problem individually.

Examples — When the class does poorly on an exam allow them to buy their curve. One way to do this is to bring to class a problem similar to one that most students had trouble with on the exam and give it as a quiz to add up to 10 percent to their exam score. The students have 3-4 minutes to discuss a solution with their group. After their discussions, they write individual solutions. Any group that does not stop discussion when they are told loses the opportunity to gain points on their exam.

Assessment — Grade the individual responses.

7. **Consensus testing:** Give quizzes or portions of exams as group projects. Challenge problems take an entire 50-minute class period to complete. Use them either as quizzes or portions of exams or as assignments for a problem-solving day. Shorter quizzes can also be given in groups.

Examples — Give a complicated problem (challenge problem as found in the *Student Companion)* that ties together several units from the semester as a group exercise to account for 25 percent of the final exam score. Announce to the student which units (chapters) the question will cover before the problem is given. Suggest to the student that they assign each group member one of the topics on which to become the group expert. Hand out the problem on the last day of class and give the class the entire time to work the problem. Collect the group solutions in the all-in-one folders.

Assessment

If the score on a student's individual section of the final exam would be lowered by including the result on the group problem, give the student the score on the individual section. If the group problem will help the student, add the two scores together. Set a maximum amount that a student's final exam score can be raised by the group problem, such as 5-10 percent.

8. **Team concept map or web:** Give each team a piece paper and have them create a team concept map on a topic for which you have given them a central concept, such as thermochemistry, bonding, or acids. These concept maps are called "webs" because they are jointly created. See "Concept Maps in Chemistry Education" (A. Regis & P. Albertazzi, *Journal of Chemical Education, Vol. 73* No. 11, November 1996, pp. 1084-1088). Concept maps that are used to review a major topic or concept can be longer activities taking 15-20 minutes of class. Concept maps over simple concepts or topics, or concept maps that are expanded each day, take five or fewer minutes of class time.

Examples

You can distribute colored pens so that each team member can use a different color, allowing you to identify individual contributions. The concept web is a good capstone for a topic or it can be an ongoing one, with students adding to it during each class.

Assessment

The concept webs are collected and evaluated for comprehensives and logical connectivity.

9. **Traveling problem:** Distribute "files" or folders – each containing one, two, or three problems to the teams. Each team discusses and prepares a written answer to the question, which they add to the file. The file is then exchanged with another team that had a different question. The second team may read the first team's answer, but must also create their own response. After teams have responded two or three times, the questions can be discussed by the class as a whole. Traveling problems are longer activities. They take between 15 and 30 minutes of class time, depending on the difficulty of the problems and the time that you allow for discussion.

Examples — Teams phrase a question about some concept they are struggling with learning. The file containing the question is exchanged with another team. The second team attempts to answer the question. The file is then sent to a third team, which checks the answer, correcting it if necessary. The file is then returned to the first team, which must indicate whether or not they feel their question has been answered.

Assessment — Members of each team sign their contributions, which can then be graded.

10. **Three-step interview:** Teams of four split into two, and one person of each pair interviews the other on a topic in the course. Then they switch roles. In the third step, the team members take turns sharing what they learned with the others on their team or with the entire class. This type of activity can be completed quickly in 5 or 10 minutes for each topic reviewed.

Examples — This technique works well after a demonstration, when students share their interpretations of their observations. It can also be used to allow students to share what they have learned in laboratory, computer, or homework assignments.

Assessment — Students can be assessed by each other.

11. **End-of-meeting evaluation:** Give each student a 3 x 5 card on which they write their names. At the end of class, the students within a team place their cards together face down and shuffle them. Students each pick a card, then write honest and constructive feedback for the team member whose name appears on the reverse. Team members return the cards to each other and engage in a discussion about their progress, needs, and successes. A good time to use this activity is when students switch roles in their groups. This activity can be done in a few minutes at the end of a class period.

Examples — Use end-of-meeting evaluations after the teams have worked together a while. This activity provides a wonderful opportunity for students to reward one another for their contributions and to discuss better approaches to working together. Comments include remarks such as "You have a lot to offer, but need to speak up more often;" "Your cheerful attitude helps the team run more smoothly." The activity is a poweful builder of community and team responsibility. You can use several times, for reinforcement.

Assessment

This activity is a form of peer assessment. The instructor need never know what was written, although in a small class students may enjoy sharing their feedback with everyone.

III. Grading Group Assignments

When students earn points for completing group assignments collected in the folders, attendance increases significantly. Students tend to sit with group members and develop a commitment to the group. Students can be given points for contributing to a group assignment or one paper can be selected at random from each folder for grading. The score achieved is then assigned to all group members. In this latter approach, students learn to make sure all group members can understand and complete an assignment correctly.

IV. Some Additional Hints

1. When you are setting up your groups, some students will ask to be grouped together. We usually tell them that we will honor their request as long as it does not destroy the diversity of the group.

2. Sometimes you will have dysfunctional groups. Try to get them to work things out on their own. If they cannot, you may have to meet with them to help out.

3. Some students will argue that they do not like to work in groups. They will claim that they are doing all of the work and that they prefer to work alone. This response is common among pre-med students. Explain to them the benefits of working together, and how much more they will learn by teaching and helping their fellow group members.

4. Closely monitor the time students spend solving problems in their groups or they will tend to get off task. You can time them with a stopwatch. Circulate around the classroom and monitor what they are doing to get a feel for how much time they need to do a problem. Offer encouragement and hints rather than answers to help students build confidence in their own judgment.

5. Use expert teams to help you in large lecture sections. Pick groups to act as expert teams the day before, and give them the problem that they will be helping with. Have the group come in and talk to you about their solution before class. Rotate the role of expert team so that all groups get a chance to help in this way. Give the expert team a perfect score on the project as long as they did a good job.

We have found the use of cooperative learning groups to be very rewarding. We get to know our students better, and have more fun teaching. Students interact with each other, make friends in the classroom, learn more chemistry, and develop a positive attitude toward learning. Cooperative learning has changed our classrooms from a quiet, teacher-centered environment to an energetic, student-centered environment. We encourage you to try cooperative learning, and to be creative in using the ideas presented in this introduction to tailor the activities in the student companion to your own classroom needs.

Good Luck!

Lynn Geiger
Belia Straushein
Loretta Jones

4 Educational Methods

Portfolio

Strategy:	**Alternative assessment**
Class:	**Secondary Mathematics Methods**
Instructor:	**Lew Romagnano** **Metropolitan State College of Denver**

Introduction: Portfolio artists compile portfolios containing work that illustrates their technical strengths, their range, their knowledge of styles, and their abilities to communicate ideas and emotions. Teaching has been described, in part, as a "practical art." Students in this class compile portfolios of their work in the class.

Activity: Portfolio

Compiling a portfolio is a process of "reviewing, selecting, and reflecting" on the term's work. Your goals are to show your mathematical competence; to illustrate what you know about technology and how to use it as a tool to do, learn, and teach mathematics; to highlight educational issues that you have found important throughout the semester, what you think about these issues, and the evidence that supports your thoughts; to communicate to a prospective employer why you should be hired as a mathematics teacher.

Your portfolio should contain seven items. For each item, include a cover sheet introducing the reader to the item and describing why you chose this item and how it contributed to your growth as a teacher. The portfolio should include:

1. your favorite piece of written work;
2. the piece of written work you found most difficult or challenging;
3. the piece of written work that best demonstrates your knowledge of technology as a tool for learning mathematics;
4. the piece of written work that best demonstrates your knowledge of mathematics;
5. two other pieces of written work of your own choosing; and
6. a set of readings from the course. The readings you choose should be tied together by a theme or issue, and your reflections should briefly summarize the articles and focus on this theme or issue.

Your portfolio will be assessed using the following criteria:

- completeness,
- evidence of the mathematics knowledge necessary to teach,
- evidence of knowledge of technology and how it can be used in mathematics classrooms,
- evidence that you are aware of the issues raised during the course and how you supported positions on these issues, and
- evidence that you have taken seriously the task of reviewing and reflecting on the semester's work.

Grades will not be affected by fancy typefaces, lamination, or other surface features. A folder containing seven stapled packets of readable information is sufficient.

Workshop on Color and Light

Strategies:	**Use of technology** **Constructivist methods** **Hands-on learning**
Class:	**Seminar in Teaching Physics in Secondary Schools (Physics 475)**
Instructors:	**Courtney Willis** **David Pinkerton** **University of Northern Colorado**

Introduction: The primary purpose of this course was to have students become familiar with the professional responsibilities of teaching physics in the school setting. Common problems and methodologies of teaching physics to secondary school students are discussed. The course also provides entry level operational knowledge of the instructional strategies, techniques, and materials available for teaching physics.

Course objectives: By the end of the course, the student is expected to:

A. Gain and demonstrate knowledge of how students, particularly in secondary schools, learn physics.
B. Become familiar with various physics curriculum developments of the past 30 years.
C. Become familiar with a number of common labs and activities appropriate to the secondary physics classroom.
D. Be able to integrate effective demonstrations and activities into physics lessons.
E. Become familiar with the role of modern technologies in the teaching of physics and the various instructional techniques associated with them.
F. Become familiar with evaluation methods to (a) evaluate student achievement and (b) evaluate instructional effectiveness.

Activity: A Special Activity

In the fall, the *Seminar in Teaching Physics* class presented a workshop exploring activities for elementary teachers at the Colorado Science Convention. Each student in the class presented at least two activities. The program summary for the session follows.

Workshop on Color and Light for Elementary Teachers

This workshop will concentrate on the concepts involving color and light which are appropriate for elementary grades. It will feature hands-on activities and some make-and-take activities. It is sponsored by the Physics Department at UNC and will be presented by the students in the *Seminar in Teaching Physics* class.

In addition to presenting activities, the students also had to write up an explanation of their activities. These demonstrations were compiled into a booklet and distributed to attendees. Below are two examples of the activities.

Spinning Disks

Part A: Most students know that it is possible to mix colors to get other colors. However, most students (and teachers) do not really know the primary colors of light. If we are working with light, the primary colors are **red, blue,** and **green***. Combinations of these three colors can make all the other colors we see. If you hold a magnifying glass up to a color TV, you will see only small dots of red, blue, and green. By coloring a cardboard disk these three colors and spinning it quickly, you can produce white, which is a combination of all colors.

(*In art classes, students usually do not work with colors of light but pigments of paint. Pigments absorb colors rather than emit colors. The primary pigments which can be mixed together to make other pigments are **red, blue,** and **yellow.**)

Part B: Because of the way our eyes detect light, we can trick them into thinking they are seeing colors. Because the persistence of vision is different for each of the three kinds of cones in our eyes, when we spin a black and white disk with different patterns, we can begin to see some variation in color.

Ultraviolet Beads

The light that we see is not the only kind of light that exists. Light that has a longer wavelength than red light is called infrared, while light that has a shorter wavelength than violet is called ultraviolet. Our eyes cannot detect these wavelengths, but they can be detected by other means. Some photographic films are exposed by these wavelengths. It is the ultraviolet light in sunlight that gives people sunburns. The string of beads you have been given turns to different colors only when the beads are exposed to ultraviolet light. Indoors, the beads are white; but in the direct sunlight, they turn colors.

Roundtable Discussions

Strategy:	Constructivist methods
Class:	Education
Instructor:	Susan Hobson-Panico Front Range Community College Colorado State University

Introduction: Participants in roundtable discussions may be teacher candidates, education faculty, and/or mathematics and science faculty. Individuals are grouped at tables with the following questions about teaching and learning displayed on 3 x 5 cards (folded to make tents). The questions provide focal points for discussion and stimulate lively conversation.

Activity: Roundtable Discussion Questions

The following are questions about teaching and learning that were used as focal points for discussion between RMTEC Scholars and Front Range Community College faculty during a joint roundtable. They created lively discussion and responses reflected both student and teacher perspectives.

How do you think that math/science teaching will change over the next 10 years?

Does a noisy classroom mean that learning is not taking place?

How can you promote the profession of math/science teaching to your students?

Describe the differences between the "Instructional paradigm" and the "Learning paradigm."

How do you use "research" in your classroom?

What does "constructivist" teaching mean?

What does "committed to teaching reform" mean to you?

Express your beliefs about how students learn.

As a science teacher, are you primarily a scientist or an educator? Should you be both?

Describe the most innovative teaching strategy you have ever seen.

Describe something that you have learned about working with minorities/women in your classroom.

Name one thing you do in the classroom (or have done) that supports women and minorities.

Besides tests, quizzes and exams, how can you tell if a student has learned something?

Describe the most unusual assessment tool you have used (or seen used) in the classroom.

Describe a "painful" lesson you've learned about bad teaching/learning.

Why do some students say math/science is boring?

What qualities should effective math/science faculty possess for teaching in the 21st century?

What strategies could you use to connect your curriculum to the concerns of women and/or people of color?

How much science/math should elementary schools teach?

How do you create a "community of learners" in your classroom?

What techniques do you use (or could use) to build a sense of community in the classroom?

Why is cooperative learning generally successful with minority students?

How do you know that you are teaching the most current information in your discipline?

What resources do you use to keep current?

Procedure Writing Workshop

Strategies:	Cooperative groups Activities involving writing Constructivist methods
Class:	Science Methods
Instructors:	Elizabeth Friot Erin Mann Metropolitan State College of Denver

Introduction: This activity is designed to help students develop skill in communicating laboratory procedures and results. Students first do their own write-up; then group members listen to it being read and help the writer to communicate more effectively.

Activity: Procedure Writing Workshop

Writers:

Your workshop group is to spend a minimum of ten minutes per writer.

- Select one person to be the timer in your group. This person's job is to make sure that no one in your group is cheated out of time.
- Read your own paper aloud – slowly. No one else can read another person's paper orally.
- As your group members make comments, take notes on your rough draft. If you don't understand one of their points, ask them. Make sure they are clear and text-specific.

Responders:

When you respond to someone's work:

- Start all your sentences with "I." For example:
  - I understood step ___ because ...
  - I was confused on step ___ ...
  - I think you need to he more specific on step ___ ...
- Be text specific. That means you should only refer to the words in the lab.
- Focus on the criteria below as you listen to the procedure being read.
- Jot down notes regarding the following criteria:
  - Materials
  - –reference for building ecocolumns
  - –directions for addition of contents
  - –identification of control
  - –safety procedures
  - –description of constants
  - –instruction for independent variables
    - time
    - pollution - amount, frequency, application
  - –instruction for dependent variables
    - minimum of four
    - method for making observations
    - frequency for observations
  - –reference to data table

♦ After the procedure has been read, each person must make a response as indicated:
1. Refer to a step that was worded specifically and clearly.
2. Refer to the sequence (order) of instructions.
3. Refer to a step where you were confused.
4. Refer to a missing component.

♦ As a group, discuss the following questions:
1. What did you visualize as the reader read the procedure?
2. Could you repeat the same lab with this set of directions?
3. Are there specific places where more detail is needed?
4. Are the data tables set up so that they are easy to use and the data is clearly presented?
5. Is it apparent from the data tables when observations are being made and what units are used?

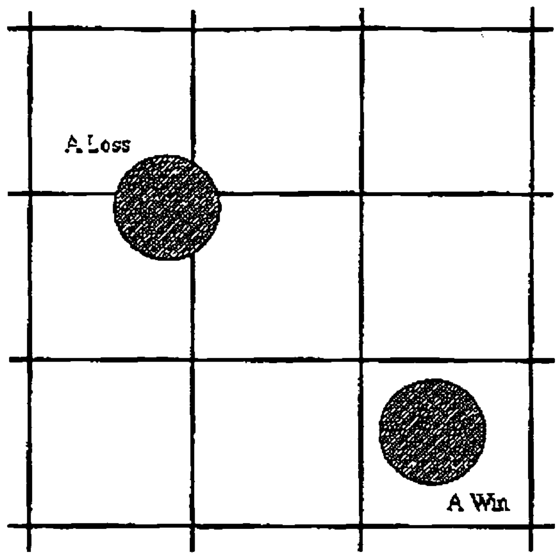
Instructional Analysis

Strategy:	Constructivist methods
Class:	Mathematics Methods
Instructor:	Lewis Romagnano Metropolitan State College of Denver

Introduction: In mathematics methods courses it is critical to get prospective mathematics teachers to integrate content and pedagogy. The following activities from a mathematics methods course are designed to get the teacher candidates to begin "thinking like teachers."

Activity: Analysis of Carnival Coin Toss Game

1. During the first class, we played the carnival coin toss game:



The dark lines on our game board were five graph paper grid lines apart, and the pennies we tossed had a diameter of three of these grid lines.

a) What was our empirical probability estimate, and how did we determine it?

b) If we were to repeat the data collection procedure we followed in class, but with each of you tossing the coin 300 times instead of 30 times, how would the distribution of results compare with the distribution of 30-toss trials we compiled in class? How would you explain this?

c) What is the theoretical probability of winning this game? Explain briefly how we determined this.

d) How many graph paper grid lines should you leave between the dark lines in order for there to be a 50-50 chance of winning this penny toss game?

2. Explain why you think we asked you each of the questions in problem 1.

Activity: Analysis of Mathematics Instruction

1. In this question, we would like you to reflect thoughtfully on our first three classes. Approach this task from two perspectives: that of student of mathematics in those classes; and that of prospective teacher who observed our actions over three class periods. In addition to any overall comments you might have, please attend to each of the following questions:

a. What mathematical concepts did you encounter during the three classes?

b. What did you do mathematically during the three classes and during the time between classes? What did you learn during the three classes and as a result of doing the mathematics questions on the first two homework assignments?

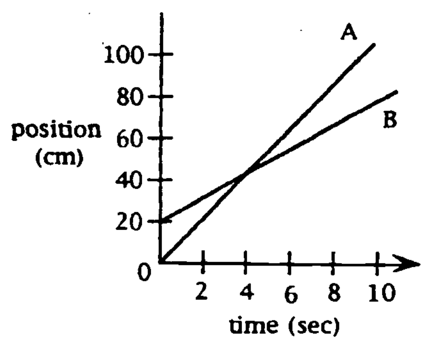
c. For what ages/grade levels would the various parts of these three lessons be appropriate, and why?

Activity: Analysis of Quiz on Slope

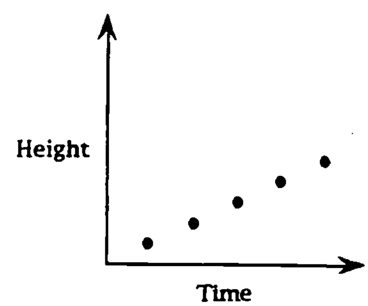
You have taught your lessons on the concept of slope, and you think they went well students were engaged in a good give-and-take during the classes, and they seemed to understand what you taught them. Now you have to determine who learned what.

1. You give your students the following three-question quiz, supplied to you by one of your experienced teacher-colleagues.

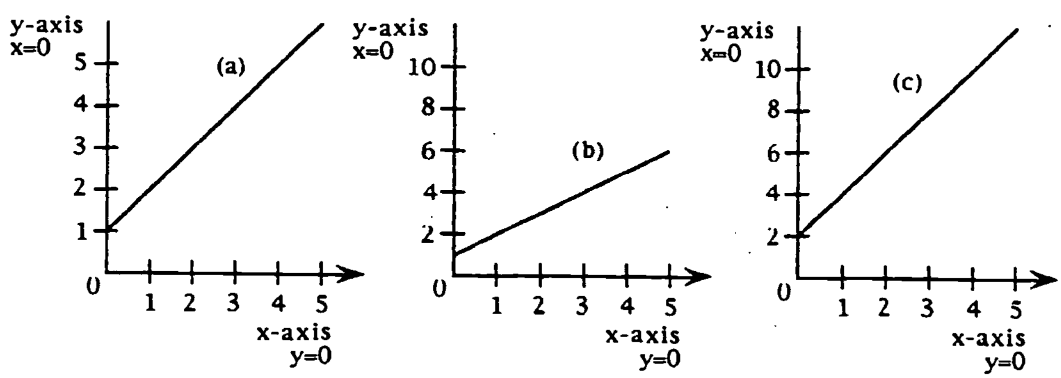
1. At the time t = 2 sec, which object is moving faster, object A or object B?



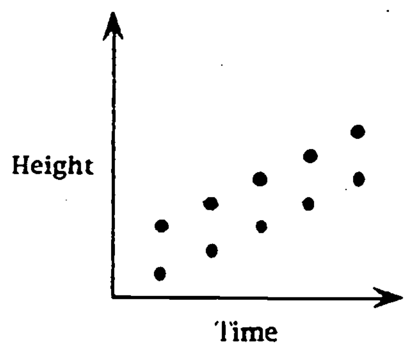
2. The graph below represents a wide jar being filled with water. On these axes, draw a graph representing a narrower jar being filled with water the same way.

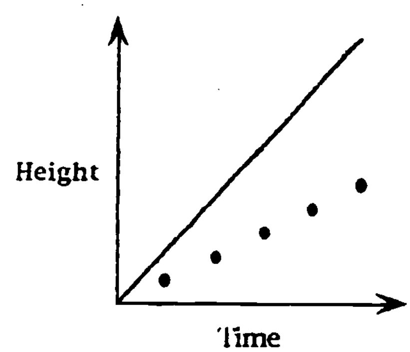


3. Which two graphs below show the same information?



On problem 1, half of your students answer "A" and the other half answer "B". On problem 2, three-quarters of your students give the answer at left below, while the other one quarter give the answer at right. On problem 3, two-thirds of your students answer "(a) and (b)" and one-third answer "(a) and (c)." For each of these questions, what would you conclude about the knowledge of the students who answered the question correctly, and those who answered incorrectly.





2. Write two additional quiz questions to assess your students' knowledge of the concept of slope. For each of these questions, describe the range of responses you would expect students to give.

Activity: Analysis of Mathematics Problems

Part I: Answer each of the following questions:

1. a) A student takes a small mirror out to the plaza in front of Youngfolks HS and places it flat on the ground 15 feet from the base of the flagpole located at the center of the plaza. She backs away from the mirror until she can see the top of the flagpole reflected in it. Can you determine the height of the flagpole? If so, what is it? If not, what else do you need to know?

b) If you had a meter stick but no mirror, and you were unwilling to climb the pole to measure it directly, how could you measure the height of the flagpole?

c) What other experiments could you perform in order to determine the height of the flagpole indirectly?

2. If a 12-inch pizza will feed four kids, how many could you feed with an 18-inch pizza?

3. Describe the process you would have to follow in order to answer this question:

If you were to make a scale drawing of this room – as viewed from above – to fit on your paper, what would be the dimensions of the table at which you are working?

Part II: Answer each of these questions, which refer to the problems you solved in Part I.

4. What are the mathematical concepts that underlie all of the above problems?

5. Imagine you teach a class of geometry students at Youngfolks HS, You begin a class by having your students work in groups of two on problem 1(a) above as a warm-up. Describe how you would conduct a five-minute follow-up whole-class discussion of this problem. You plan to take your class outside to do this experiment as soon as the discussion is over.

6. How would you respond to a student in your eighth-grade mathematics class whose answer to problem 2 is "six kids?"

7. If problem 3 above were to be a quiz question, for what level students do you think it would be appropriate? What characteristics must you see in one of those students' responses in order for you to deem that response "acceptable?"

A Curriculum Change Model

Strategies:	Cooperative Groups Use of technology Use/solution of real world applications/problems In-class discussion Ongoing student self-evaluation techniques Activities involving writing Alternative assessments Consistency with state standards Making connections to other fields (science and non-science) Constructivist methods Solution of complex problems by students Attention to creating learning settings sensitive to cultural differences Attention to creating learning settings sensitive to gender issues Chance for students to ask questions in class
Class:	Science and Mathematics Methods and Evaluation
Instructors:	Barbara J. Nelson Glenn Bruckhardt Colorado State University

Introduction: A group of faculty members at Colorado State University faced the task of designing and implementing an integrated methods course for teacher candidates in science and mathematics. The task was to integrate "what to teach" with "how to teach" — mathematics and science content with pedagogy. The faculty members were Sree Bringi, science educator; Duane Clow, mathematics educator; and Barbara Nelson, general educator.

While these faculty members were in agreement about developing an integrated methods course, each entered the process with different "maps of change" based on their past experiences and beliefs. Glenn Bruckhart, Teacher-in-Residence and a long-time high school mathematics teacher, offered a **Curriculum Planning Model** to coalesce the thoughts, ideas, and perceptions of the methods team. Figure 1 presents the six phases of this planning model.

Figure 1
Curriculum Planning Model

Phase 1	Examining Underlying Beliefs
Phase 2	Defining the Content
Phase 3	Specifying Topics, Activities, Experiences
Phase 4	Establishing Themes
Phase 5	Determining Assessment & Grading Strategies
Phase 6	Developing a Management Plan

In **Phase 1, Examining Underlying Beliefs** about teaching and learning in mathematics and science, the planning team worked toward the development of a unified set of beliefs about how to best prepare prospective teachers. With the luxury of a year for planning, the four individuals committed to bi-weekly meetings where beliefs were shared and differences aired in a safe environment. This time together proved to be critical; the chance to get to know one another paid off in the subsequent planning and implementation. While the education, mathematics, and science faculty members had taught "side-by-side" for a number of years, they had only cursory knowledge of one another's curricula. The typical suspicions between content faculty and education faculty lurked in the background – who was doing what and how well. However, faculty were committed to working together to positively impact the educational experiences of teacher candidates.

As a starting point, the group examined the existing curricula. During the "methods semester" immediately prior to student teaching, science and mathematics teacher candidates completed a common four-credit methods course, *Instructional Methods and Assessment*, and a one-credit field experience, both taught by the education faculty members, and a four-credit special methods course, either *Methods and Materials in Teaching Science* (taught by the science methods faculty member) or *Methods and Materials in Teaching Mathematics* (taught by the mathematics methods faculty member). The instructional methods course focused on general methods applicable to all subject areas whereas the science and mathematics methods explored curricula and methods specific to the disciplines. Working with the eight credits designated for methods courses in the regular program and adding an additional credit of field experience, the goal was to develop a year-long methods experience with four credits of Science and Mathematics Methods and Evaluation and one credit of Practicum each semester. The planning team had an additional goal – to remove the barriers which so typically exist between general methods courses and content-specific methods courses. Also, the group felt it would be vital to build bridges between mathematics and science teaching, since it was not uncommon for science teachers to be hired for mathematics positions and vice versa.

In **Phase 2, Defining the Content,** major units of study were identified. Because the separate courses followed the Colorado Department of Education (CDE) and the National Council for the Accreditation of Teacher Education (NCATE) standards, this process was relatively easy with the team quickly identifying the major units of study as: planning, methodology, assessment, classroom management, and content-specific curricula and methodology.

Armed with agreement on the major content areas, the group proceeded to **Phase 3, Specifying the Topics, Activities, and Experiences** to be addressed in the methods courses. To that end, each team member prepared a stack of 3" X 5" cards (one topic/activity/experience per card), documenting what each member felt was important to be included in the course. At the subsequent team meeting, members worked to get the "big picture," using an entire wall to display their work. First they created five columns, one for each of the agreed-upon units of study; then team members placed their cards in the appropriate columns. In this process, there was extensive discussion and clarification about what should go where. The information on this wall represented all of the topics, activities, and experiences which one or more of the team members felt were important to include in the new course sequence. Shared visions began to emerge. This curriculum activity not only helped us to quickly conceptualize the specific content, but it also moved the planning team from abstract ideas to concrete experiences in a short period of time. The task of "getting together" no longer seemed so formidable. All members ideas were honored, and each member felt she or he had something to offer to the experience.

In **Phase 4, Establishing Themes,** team members participated in an inductive activity, examining the series of cards under each topic area, grouping together those that were related. Some of these themes emerged easily and others took longer to identify as the planning team wrestled with the ideas. The themes that emerged were: problem solving, communication, people, technology, and professionalism. Combining the units and themes, a five-by-five grid (Figure 2) provided cells into which specific topics could be placed.

Figure 2
Integrated Methods Grid

UNITS

Themes	Methods	Planning	Classroom Management	Assessment	Math/Science Curricula & Methods
Problem Solving					
Communication					
People					
Technology					
Professionalism					

In **Phase 5, Determining Assessment and Grading Strategies,** the calls for alternative assessment reflected in the CDE and NCATE standards proved to be consistent with the beliefs and values of the participating faculty members. The assessments that emerged were planning (daily, unit, long-range, and interdisciplinary), microteaching (using traditional and learning cycle formats), assessment (performance assessments, rubrics, tests), journaling (reflections related to course content and practicum experiences), activities development (mathematics- and science-specific experiences), technology (computer-based learning, Hyperstudio, grade book programs, computer-based laboratories), exams (collaborative tests focusing on conceptual knowledge), valued learning behaviors (responsible attendance, active participation, and collaboration), and a culminating portfolio assessment addressing the five CDE standards – teaching and learning, diversity, assessment, democracy, and communication.

Phase 6, Developing a Management Plan, addressed schedules, course materials, procedures, forms, deadlines, record-keeping, etc. The group recognized that the best of plans would be sabotaged without effective management, and coordinating among four individuals would prove to be a challenge. While some of the management details could be anticipated, many emerged based on feedback from students. This curriculum planning process enabled the methods team to quickly coalesce and integrate their different "maps of change."

The plans for the integrated methods course became reality in 1996. A four-credit fall course entitled *Science and Mathematics Methods and Evaluation I* and a four-credit spring course, *Science and Mathematics Methods and Evaluation II* were each accompanied by a one-credit practicum experience. Three instructors and a Teacher-in-Residence were responsible for all instruction. Team planning and team teaching were prevalent, and grades were determined jointly. Class time shifted between meeting together to address common issues, meeting in separate mathematics and science groups to address unique topics, and field-based experiences in the schools to ground the methods experience in reality. The faculty were united in their commitment to new approaches to teaching and learning in mathematics and the sciences – constructivist teaching strategies, active student involvement, interactive learning environments, problem-based instruction, collaborative learning, hands-on inquiry, contextual learning, and culture-sensitive teaching. The faculty established genuine collegial relationships and friendships that translated into more integrated and consistent instruction for the teacher candidates.

Cassette Tape Project

Strategy:	Constructivist methods Use/solution of real world applications/problems
Class:	Mathematics Methods
Instructor:	Bill Blubaugh University of Northern Colorado

Introduction: The following activity represents two kinds of activities that facilitate the growth of prospective teachers: 1) connecting with other teachers/programs via the internet and 2) first-hand experience with a teaching strategy (as a student).

First, this activity was retrieved from an internet site: http:/www.esu10.k12.ne.us/~loupcity/. The internet provides teachers with a wealth of resources – information, activities, and connections to primary data. This particular activity is also interactive in that the originator of this activity asks teachers who use it to share data generated.

Second, if teachers are to truly understand the potential of mathematics activities – the "ins-and-outs" and "pros-and-cons" – it is important that they have first-hand experience as a student, learning by using the particular strategy. Consequently, the focus of mathematics methods is to provide opportunities for these experiences.

Activity: Cassette Tape Project (Data Analysis)

Submitted by: Edwin McCartney, Math Instructor, Loup City High School, Loup City, NE 68853

Date: April 19, 1996

Grade Level: 9th Grade and higher

Introduction

This project can be used when students are studying or after they have studied quadratic functions, however, other concepts are involved. The instructor can decide whether to cover these concepts prior to the project or along with the project.

Note: A more complete graphical explanation of this project can be found through the Loup City Public Schools home page located at http://www.esu10.k12.ne.us/~loupcity/. Choose Academic Activities from the home page, then Mathematics. This project along with others are listed on the Mathematics page.

I designed the project to integrate various areas of mathematics.

Topics Addressed In Project

Modeling
Function Graphing
Quadratic and Linear Relations
Data Collect and Analysis
Best-fit Lines
Analysis and Interpretation of Graphs
Optimization

Description of Project

To capture the student's attention on the day I introduce the project, I have a song playing as the students walk into the room. (I usually use "Joy To The World" by Three Dog Night.) I pose the following problem to the students:

> A new company is producing a cassette tape of a popular group and wishes to determine the selling price that would result in maximum profit for the company.

Procedure

We generally have a discussion about why a higher selling price does not necessarily result in higher profit. Students are then asked to suppose that their favorite recording group had just released a new tape. To collect data, I ask how many of them would be willing to pay specific amounts for the tape. I keep increasing the amount until all students reject the price. The following table shows a sample of the data I have collected thus far. Projected purchases will be explained below. The example on this page is based on this data. When I do the project with a new group, I add the new data to the data collected with previous groups.

COST/TAPE	PURCHASES	PROJECTED PURCHASES
$0.00	82	200000
$5.00	81	197561
$6.95	80	195122
$7.45	78	190244
$7.95	77	187805
$8.25	66	160976
$8.75	61	148780
$9.00	60	146341
$9.25	50	121951
$9.50	47	114634
$10.00	35	85366
$10.50	30	73171
$11.00	24	58537
$12.00	17	41463
$13.00	11	26829
$13.50	9	21951
$14.00	1	2439
$15.00	1	2439
$16.00	0	0

Depending on the background of the students and the technology available, students can make a scatter plot of the data on paper or enter the data on a graphing calculator or into a data analysis program on a computer to create a scatter plot. The purpose of the scatter plot is to determine the equation for a best-fit line.

On paper, the students can either "eye-ball" the line or learn to determine the median-median best fit. Some graphing calculators and computer packages will determine the equation for the students, either using median-median or least-squares. The linear expression from the best-fit equation will represent the approximate number of tapes that could sold based on the selling price of a tape.

As in all modeling situations, certain constraints and parameters must be considered. For this example we assumed a setup and marketing cost of $20,000, a per tape production cost of $1.50, and a projected market population of 200,000 people. Students modify the best-fit equation to reflect the ratio of the number of people in the study to a market of 200,000. Students use the linear expression from the modified best-fit equation to create functions for cost, income, and profit dependent on the per-tape selling price. As with the scatter plot, various methods can be used to graph and analyze the functions. Students can graph the functions by hand and use algebraic methods to determine points for the analysis or they can use graphing calculators or function plotting software on a computer.

The cost function is defined as the per tape production cost ($1.50) times the number of tapes sold (linear expression) plus setup and marketing cost ($20,000).

The income function is defined as the per tape selling price (independent variable) times the number of tapes sold (linear expression).

The profit function is defined as the income function minus the cost function.

The coordinates of the vertex of the parabola representing the profit will represent the optimum solution with the dependent value indicating the maximum profit and the independent variable indicating the selling price that will produce the maximum profit. In addition to the meaning of the vertex, I ask the students to analyze the graphs by interpreting, in relation to this particular problem, points where the graphs intersect the vertical axis and the horizontal axis, the meaning of the slope of the cost function and the best-fit line, etc. Other questions often arise, such as, "If the per tape production cost increases, should the selling price increase by the same amount to maintain maximum profit?", "Are the assumed parameters realistic?", "Should other constraints and/or parameters be considered?". These and other questions make good discussion topics for the students.

Undated Data

The data used in the example above was collected from students at Loup City High School, the Upward Bound Program at Briar Cliff College, Sioux City, Iowa, and the Math and Science Regional Center, Northwest Missouri State University, Maryville, Missouri.

One aspect of this project that keeps it interesting is that the results change each time new data is added to the previous data. I would like to keep my data updated with groups from various areas. If you choose to use this project, please send me the specific data from your class including the number of students in your study, the number willing to buy the tapes at the various selling prices, the name of your school, etc. If you need to include prices not currently listed, that is certainly acceptable.

I will update the data list as more data is sent to me and will display the updated list on the Cassette Tape Project page at the Loup City Public School homepage web site (address above). I will indicate schools and classes that have contributed data. E-mail data to: emccartn@genie.esu10.k12.ne.us.

Questions for Discovery and Discussion

Strategy:	**Constructivist methods**
Class:	**Mathematics Methods**
Instructor:	**Bill Blubaugh** **University of Northern Colorado**

Introduction: Questioning can be critical to an interactive mathematics class. The following list provides some helpful hints on how to enhance the dialogue in a mathematics classroom.

Activity: The Art of Questioning

A "try-to" list.

1. Try to pause after asking a question.
2. Try to avoid frequent questions which require only a yes or no answer.
3. Try to avoid answering my own question.
4. Try to follow up student responses with the question "why?"
5. Try to limit the use of questions that rely almost completely on memory.
6. Try to avoid directing a question to a student for disciplinary reasons.
7. Try to avoid repeating a student's answer.
8. Try to follow up a student's response by fielding it to the class or to another student for a reaction.
9. Try to insist on attentiveness during question periods.
10. Try to avoid giveaway facial expressions to student responses.
11. Try to make it easy for students to ask a question at any time.
12. Try to avoid asking questions that contain the answer.
13. Try never to call on a particular student before asking the question.
14. Also try not to call on a particular student immediately after asking a question.
15. Try to ask questions that are open-ended.
16. Try not to label the degree of difficulty of a question.
17. Try to leave an occasional question unanswered at the end of the period.
18. Try to replace lectures with a set of appropriate questions.
19. Try to avoid asking for verbal group responses.
20. Try to keep the students actively involved in the learning process.

Questions to seldom ask

1. How many of you understood that?
2. Everybody see that?
3. You want to go over that again?
4. Did I go too fast for you?
5. This is a right triangle, isn't it?
6. Right?
7. Do you have any questions?

(from *Every Minute Counts: Making Your Math Class Work*)

Activity: Questions for discovery and discussion

Here are some of my favorite questions for encouraging student discussion. Try them, or others like them, as you work on "the art of questioning."

1. Which is biggest: π, 3.14, or 22/7?

2. Which is greater, x or $-x$?

3. When is $\frac{1}{x} > x$? When is $\sqrt{x} > x$?

4. When is $|x| = -x$?

5. When is $(x - 7)^2$ less than zero?

6. Is $(x^2 + 1)$ ever equal to zero?

7. If $a^2 = b^2$, does $a = b$?

8. Name a real number between 3 and a if $3 < a < 4$.

9. On what side of zero is $(-a)$ if (a) stands for a real number?

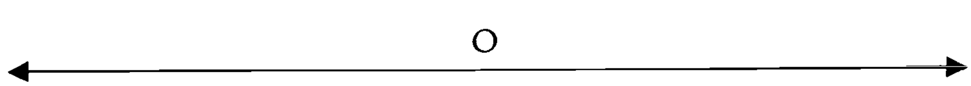
10. When is $(x^2 + 4x + 4)$ negative?

11. Draw number lines and picture:

    a. the set of all real numbers less than their opposites,

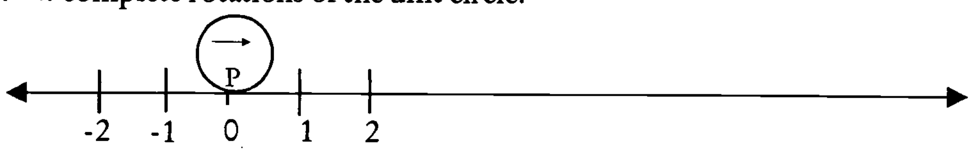
    b. the set of all real numbers less than their reciprocals.

    c. the set of all real numbers less than their square roots.



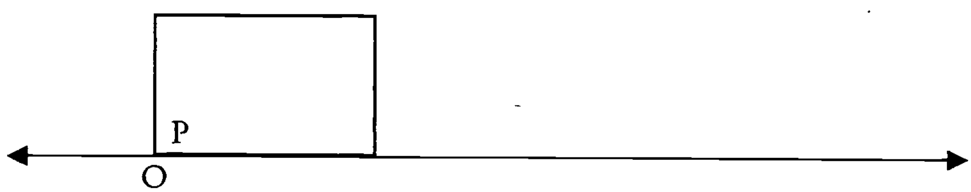
12. *P* is a point on the circumference of a unit circle. Suppose p starts at zero. What real number coordinate is *p* at after:

a. 2 complete rotations of the unit circle?
b. *n* complete rotations of the unit circle?



13. Point *p* is a vertex of a 2 cm x 2 cm square. Suppose *p* starts at zero. What real number coordinate is *p* at after:

a. 2 complete rotations of the square?
b. *n* complete rotations of the square?



14. $(1 + \sqrt{2})$

a. Is this a positive or negative number?
b. Is it rational or irrational?
c. Give the reciprocal. Is it less or greater than zero? Why?

15. True or false: If a number is between 0 and 1, then the number is less than its square. (Justify your answer.)

Concept Mapping and Science Standards

Strategies:	**Cooperative groups** **Constructivist methods** **Alternative assessment**
Class:	**Science Methods**
Instructor:	**Sreedevi Bringi** **Colorado State University**

Introduction: Pre-service science teacher candidates enrolled in science methods courses benefit from learning and using powerful cognitive concept mapping strategies as planning and teaching tools. They also need ongoing immersion in science standards throughout the course. The following activity combines both goals and is useful both for in-class work as well as for take-home projects in training pre-service science teachers. Unit and semester planning can also be done modifying these strategies. (Simple concept mapping strategies can be taught using familiar topics before doing this cognitive mapping activity with standards.)

Activity: Concept Map Activity Using State Science Content Standards

The aim of this activity is to familiarize you with state science content standards in planning your science curriculum. Concept mapping strategies are valuable tools in the overall process of learning, planning and teaching.

Overall Task: You are charged with developing a poster presentation on standards at your school's back-to-school-night. Parents and school board members want to know about the overall principles that guide the new standards-based science curriculum at your middle school or high school.

Task Analysis:

1. Select and write out the component objectives of any one science content standard sub-level among standards 2, 3, or 4 (such as 3.1).

(Colorado Model Content Standards: Science: Standard 2: Physical Science: Students know and understand common properties, forms, and changes in matter and energy. Standard 3: Life Science: Students know and understand the characteristics and structure of living things, the processes of life, and how living things interact with each other and their environment. Standard 4: Earth and Space Science: Students know and understand the processes and interactions of Earth's systems and the structure and dynamics of Earth and other objects in space.)

2. Identify grade level 5-8 or 9-12 for this activity.

3. Create a colored concept map to be used for unit and/or science curriculum planning. Use concept mapping techniques and the rubric on the following page to guide you in this process. Include a brief legend/guide for symbols or color schemes used.

4. Integrate four examples of science activities in the concept map.

Rubric for Concept Map of State Standards

Content of standards accurately represented (paraphrasing/interpretation of scientific concepts)	0	1	2
Conceptual level appropriate to developmental level of age group/grade level chosen (highly appropriate/somewhat/not really)	0	1	2
Hierarchy of concepts at several levels (from more general to more specific)	0	1	2
Link words show relevant connections (all/some/none)	0	1	2
Appropriate cross-links connecting different strands (very relevant/somewhat/not really)	0	1	2
Relevant examples of science activities included (very relevant/somewhat/not really)	0	1	2
Aesthetics and readability of concept map (organization of concepts, format, color scheme)	0	1	2

Total: 14 possible points

Introduction: As part of the training in planning and conducting science activities in the classroom, pre-service science methods students are asked to share a science activity with their peers in the science methods course before trying the same activity in school settings. The following format was developed to guide students in planning and writing out their activities to share with their peers. The grading criteria for scoring the science activity presented in class is on a separate page. Both instructor scoring and peer scoring were used for feedback.

Activity: Science Methods Class Assignment for Pre-Service Teacher Training

Suggested Format for Science Activity Handout

Your handout for your classmates should be between 2-3 pages using the suggested format below. Bring copies to class on the assigned day for your presentation. Present a mini-lesson on your science activity for about 20 minutes in class.

Your Name:
Catchy Title:
Paste a Graphic/Visual Cue/Diagram:
Grade Level(s):
Estimated Time:
Topic/Unit:

Vocabulary: List key terms/concepts.

Content Background: Include a paragraph on the science concepts underlying this activity. Include suggested use and placement in a typical science unit.

Relationship to National/State Science Standards: List the content and process standards which this activity targets.

Real World/Career/Multicultural Applications: Outline key connections to the real world of your student population. Connect to historical relevance /cultural practices /ways of doing science. List career implications.

Materials: List needed supplies in one or two columns. Use readily available materials as much as possible. Provide ordering details for special items.

Advance Preparation: List as separate steps 1, 2, etc., for teacher planning.

Procedures: List as separate steps 1, 2, etc., for a teacher to lead the science activity in the classroom for class participation or demonstration. Include equity considerations: gender, special needs, etc. Safety considerations are important!

Class Discussion Pointers: Emphasize key science concepts and any common misconceptions clarified by this activity.

Student Evaluation: Make suggestions for student evaluation. Include **sample** questions/ guidelines.

Extensions/Follow-up: Where can this activity lead? Further explorations? Interdisciplinary connections?

Introduction: The following criteria were found useful for both instructor scoring and peer scoring of science activities presented in class by pre-service science methods students. Two students were asked to provide feedback on each activity presented.

Activity: Science Methods Class Assignment: Grading Criteria for Science Activity

Science Activity Presentation (25 possible points)

Presenter:______________________________
Title of Activity: ________________________Grade Level:__________

The following criteria need to be met for each science activity that you bring and present in class. Along with a copy of the original science activity for each classmate, you may wish to add additional information relating to content standards addressed by the activity, how you would use it, availability of materials, etc.

Content 5 pts	*Scientific objective clearly stated, linked to state science standards *Relevant science concepts accurately presented in the activity
Materials 3 pts	*Inexpensive, readily available resources; sources cited *Preparation details clear and reproducible *Safety and handling issues adequately addressed
Effectiveness 10 pts	Activity illustrates the science concept(s) adequately *Activity appropriate to student level specified *Clear and sufficient directions given; modeling as needed *High degree of student involvement/interaction *Timing of activity and oral/hands-on presentation skills
Handout 5 pt	*Written copy of activity provided with source cited *Additional information provided as needed to meet criteria *Good clarity/diagrams and appropriate language
Application 2 pts	Presenter clearly showed relevance of the activity to: *students' lives/cultural context *careers/scientific literacy/interdisciplinary connections

Overall Comments/Suggestions for Presenter:

Searching the ERIC Database on Science and Mathematics Topics

by Niqui Beckrum, excerpted from *The ERIC Review*

The ERIC database is the world's largest education database. It is an excellent resource for anyone seeking information on teaching and learning in science and mathematics. ERIC features abstracts of nearly 1 million research reports, curriculum and teaching guides, conference papers, and journal articles dating from 1966 to the present. You can search the ERIC database online at http://www. accesseric.org or through print indexes and CD-ROMs at hundreds of libraries, colleges and universities, and state and local education offices.

The result of an ERIC search is an annotated bibliography of document and journal literature on the topic that you have specified. You can then review the bibliography to determine which listings are most helpful to you and select those citations to get an abstract of each document. To get the full text of a journal article (shown as EJ followed by six digits), you can visit a university library or a large public library, or you can contact a journal article reprint service, such as The Uncover Company (1-800-787-7979) or the Institute for Scientific Information (1-800-336-4474).

To get the full text of a document (shown as ED followed by six digits), you can visit one of the more than 1,000 libraries around the world that maintain an ERIC microfiche collection. There you can view or print the document. You can also order a print copy from the ERIC Document Reproduction Service (1-800-443-3742) or from the document's distributor. Many documents published after 1992 may be ordered and delivered via the Internet at http://edrs.com.

All entries in the ERIC database have been indexed with key words called *descriptors.* These words describe the most important concepts contained in a journal article or document. Although it is certainly possible to search the database using common words and phrases, your search will be far more effective if you use ERIC terminology.

When searching the database for information on science education, begin with the following descriptors:

- *Science Education*
- *Science Instruction*
- *Science Activities*
- *Sciences*

When searching the database for information on mathematics education, begin with the following descriptors:

- *Mathematics Education*
- *Mathematics Instruction*
- *Mathematics Achievement*
- *Mathematics*

To perform an effective ERIC search, use the descriptor that is most specific to your topic. For example, if you are looking for information on teaching mathematics, use the descriptor *Mathematics Instruction* instead of the broader term *Teaching Methods.* If you are looking for documents on the specific subject of biology, use the descriptor Biology instead of the broader term *Sciences.*

If you require assistance in searching the ERIC database on a science or mathematics topic, call the ERIC Clearinghouse for Science, Mathematics, and Environmental Education at 1-800-276-0462. If you find yourself doing extensive searching, you may find the *Thesaurus of ERIC Descriptors* helpful. The *Thesaurus* is available from Oryx Press (1-800-279-6799) and at most places that offer access to the ERIC database. For general information about accessing the database or for a free copy of the brochure *All About ERIC,* call ACCESS ERIC at 1-800-LET-ERIC (538-3742).

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