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ABSTRACT

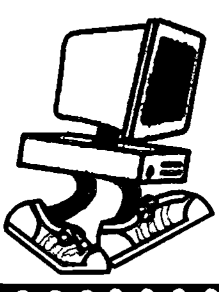
The high levels of anxiety, apprehension, and apathy of students in college algebra courses caused the instructor to create and test a variety of math teaching techniques designed to boost student confidence and enthusiasm in the subject. Overall, this proposal covers several different techniques, which have been evaluated by both students and the instructor. The paper proposes a series of study techniques, which are covered on the first day of class and throughout the course. Armed with practical advice about approaching the algebra course, the students learn by example. The instructor uses examples, such as rules of cricket matches as a way to make the material come to life. Other suggested techniques are making chapter notes available to students, and providing example tests before final exams and additional study sessions. Sometimes students are allowed to work in groups and complete group projects. Within this paper are several examples of the handouts and examples of worksheets given to students. The paper concludes that in addition to succeeding in reducing students' anxiety level so that the instructor could engage in more teaching, the instructor also developed a greater interest and enthusiasm for the topic. (AF)

PAPER FOR INCLUSION IN THE ERIC DATABASE

REDUCING "MATH ANXIETY" IN COLLEGE ALGEBRA COURSES INCLUDING COMPARISONS WITH ELEMENTARY STATISTICS COURSES

by

Mike Bankhead



REDUCING "MATHS ANXIETY" IN COLLEGE ALGEBRA COURSES INCLUDING COMPARISONS WITH ELEMENTARY STATISTICS COURSES

by

Mike Bankhead

The Englishman's Lament

I came from afar to work over here,
Leaving all behind including good beer.
If the words that you see are not as you wish,
Please remember they're spelt in proper English!

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ABSTRACT

REDUCING "MATHS ANXIETY" IN COLLEGE ALGEBRA COURSES INCLUDING COMPARISONS WITH ELEMENTARY STATISTICS COURSES

by

Mike Bankhead

Although I had been teaching for over *twenty-five years* both in **America** and **England**, because of the differences in the education systems, I had never taught a **College Algebra course** at the *100 level* until the ***spring term*** of ***1995***. I was horrified at the apathy, apprehension, and lack of enthusiasm of the students taking the course. They had no confidence in their ability whatsoever with many anticipating failure before the course had started.

During this term I identified what I felt were the underlying problems and then created and tried a large number of different ideas, which I call ***techniques***, on my students. They were designed to help students overcome their fear of **College Algebra**, boost their overall level of mathematical confidence, and give them a positive attitude towards the course.

This proposal discusses **eight techniques** that my students evaluated, eight others that I am currently testing on my students, plus four others that, for one reason or another, were deemed failures. I have also extended these ideas to my **Elementary Statistics Courses** and let those students evaluate the *techniques*. This proposal also compares the results from groups of students in both classes.

CONTENTS

1. INTRODUCTION

Although I had been teaching for over *twenty-five years* both in **America** and **England**, because of the differences in the education systems I had never taught a **College Algebra course** at the *100 level* until the ***spring term*** of ***1995***. It was indeed a jolt to observe the reaction of my students to what is actually very low level Maths. Many in the ***spring class*** of ***1995*** were repeating. Some were on their *third* or *fourth attempt*. Most of them sat around quietly, almost lifeless. Those that were in-groups laughed at their own anticipated incompetence. Those who were first timers were apathetic, apprehensive, and without enthusiasm of any kind. More disturbing was their overall attitude to the subject. They had no confidence in their ability whatsoever. The course was just a joke to many of them, many anticipated failure.

It was a constant battle throughout the term just to keep the students attention and stop them chatting to their neighbors. A number of students brought nothing with them, not even a pencil. They just sat and stared at what I was doing but worse was to come. My evaluations were returned to me. Clearly things had gone far worse than even in the most terrifying nightmares that I had had during and after the course. We use the **IDEA form** at Bellarmine College. ***Question E*** asks "*Overall, I rate this teacher an excellent teacher*". My average had dropped to **3.64**. Clearly I needed to act!

I was also astonished at some of the things they did in tests. The following examples are just two of the many that I encountered while teaching this course.

EXAMPLE 1

On one of my tests, a student had ended up with the expression $\dfrac{x-y}{xy}$.

He proceeded to simplify this expression as follows :

$$\frac{x-y}{xy} = \frac{\cancel{x}-\cancel{y}}{\cancel{x}\cancel{y}} = -$$

This is literally a ***minus sign*** all by itself !

EXAMPLE 2

On another test, a different student had ended up with the expression $\dfrac{\log(1+x^2)}{(1+x^2)}$. She proceeded to simplify this expression as follows :

$$\frac{\log(1+x^2)}{(1+x^2)} = \frac{\log\cancel{(1+x^2)}}{\cancel{(1+x^2)}} = \log$$

While I was horrified at what she had done I could not help being amused at the error. So as a joke I said "Surely that answer is much too big, the answer ought to be **Twig**". I was even more horrified to hear her say, reproachfully, "You have not taught us the **Twig** function yet" !

Both of these examples, together with many others, started me thinking about the underlying problem. Why do they carry out such totally wrong operations? How can I give them confidence in their mathematical ability and stop them treating the subject as one that they must inevitably fail?

It is not unreasonable to expect their negative attitude to extend into any other subject that is mathematically orientated. So possibly finding a way to give them some confidence in their own ability in *College Algebra* may well help them substantially in other courses that involve mathematical concepts and operations.

After teaching the *spring 1995 College Algebra course*, I felt that I needed to

1) reduce the anxiety created by a *College Algebra course*;

2) get my students to relax, talking to me, and to each other in a positive way;

3) create and maintain their interest in the classroom;

4) boost their overall mathematical confidence, and keep this confidence and a positive attitude towards the course going throughout the course;

5) make the tests less worrying;

6) make every student believe from the beginning of the course, right through to the end, that they could not only pass, but also obtain a good grade;

and

7) give my students some way of redeeming themselves if they obtained a bad test mark or a bad assignment mark during the term.

With these objectives in mind, I began creating and trying a large number of different ideas that I refer to as *techniques* (I have never liked this word, but I cannot think of a better one). Some of the *techniques* were designed to address one or more of the above objectives, some to solve a worry expressed by a student, and some to solve a problem I felt existed because of the behavior I observed.

After using these techniques in the fall *1995* and the *spring 1996 College Algebra courses*, I obtained ***5 out of 5*** for **Question E** on the **IDEA form** from every student in both classes. Unfortunately, I had no idea what the students had found helpful and what they had not found helpful. So I chose eight of the techniques I had used and let my students answer some questions and rank them on a form I created. I initially chose just eight because I thought my students would not take a lengthy form seriously. In fact the form I use now is three pages long, and most students clearly do take their time to respond to the questions. I recently found out that if I let them fill out the forms first and then lecture, I get more information than if I do the evaluation last and leave the room without returning. The reason is obvious.

I started using the successful *techniques* in both my *spring 1997 Elementary Statistics Courses*. I used my own evaluation form, which I refer to as the *short form* (in **Appendix A**), with these students so I could compare these courses with the *fall 1996 College Algebra courses*. An analysis of these evaluations compared and contrasted with the *College Algebra* evaluations is in *Section 4*. The numerical data obtained produced some surprises for me. In view of them I have abandoned some of the *techniques*, because they did not achieve their objective, and refined others. The *techniques*, the failures as well, are discussed in the *Section 2*.

I continue to try new ideas and encourage my students to let me know which ideas worked for them and which do not. At the present time I have my students rate some seventeen techniques both numerically and with various questions using a new three page form, which I refer to the *long form* (in **Appendix B**).

For convenience the *techniques* discussed in **Section 2** are summarized below.

1. **CHAPTER NOTES IN LIBRARY (INCLUDES WORKBOOK QUESTIONS)**
2. **WORKBOOK**
3. **TAKE ONE SHEET OF PAPER INTO TEST (INCLUDING FINAL)**
4. **ADDITIONAL STUDY <u>SESSION</u>* DURING FREE PERIOD**
5. **EXAMPLE TEST BEFORE EACH TEST INCLUDING FINAL**

   SOLUTIONS AND MARKING SCHEME IN COMPUTER AREA AFTER TEST
6. **COLOURED CHALK ON BOARD**
7. **GRADED BONUSES ON ALL TESTS (TO ENCOURAGE THEM TO THINK DURING THE TEST)**
8. **GROUP PROJECTS AND WORKING IN GROUPS ON PROBLEMS**
9. **ANY NUMERICAL ANSWER REQUIRES A SENTENCE CONTAINING THE NUMBER IN THE CONTEXT OF THE QUESTION.**
10. **CREATE SPECIAL TECHNIQUES FOR THOSE TOPICS THAT CAUSE DIFFICULTY (KILL FOIL!!!)**
11. **USE THE TEXTBOOK AS A SET OF NOTES TO MINIMISE THE AMOUNT OF WRITING STUDENTS DO IN CLASS.**
12. **MAKE AND USE TRANSPARENCIES OF GRAPHS, RULES, ETC. SO TIME IS NOT WASTED DRAWING OR WRITING THEM ON THE BLACKBOARD.**
13. **ALL QUESTIONS ON TESTS STRAIGHT FROM BOOK**
14. **HAND OUT COMPLETE SOLUTIONS TO SOME TYPICAL PROBLEMS PROPERLY WRITTEN OUT**
15. **USE TOPICAL PROBLEMS IN LECTURE.**
16. **90% OR MORE RULE (4 TO 5 WEEKS BEFORE THE END OF TERM)**
17. **SETTING 8 QUESTIONS ON THE FINAL - STUDENTS SELECT ANY 6 FOR FULL MARKS.**
18. **TWO POINTS FOR EACH LECTURE ATTENDED. FOUR POINTS FOR ATTENDING AN ADDITIONAL STUDY SESSION.**
19. **BEAT THE CLOCK.**
20. **5 MARK ATTENDENCE BONUS.**

<u>MY CATCHPHRASE</u>

On the first day we meet I put an overhead projector slide of the following page on the projector. The purpose is to emphasize the importance of learning the rules of mathematics and practicing them on as many problems as possible. It also seems to break the ice a little. Later in the course when my students and I have got to know each other better, if I start to use the catchphrase, quite often some of my students will chant the rest of it back to me!

MY CATCHPHRASE IS :

MATHS IS FUN

BECAUSE

MATHS IS EASY

IF

YOU KNOW THE

RULES

AND

PRACTICE

To emphasize the "Rules and Practice" part of my catch phrase, I use a number of true stories involving cricket. For example, in 1989 I went with a number of faculty and a group of students to Oxford University, England. We were going to complete some courses over there and sightsee at the same time. Before we went, I knew that students from Ohio State University would be there for their summer school and, at some point, Bellarmine and Ohio State would play cricket against each other.

For some three months before going over, I taught our Bellarmine students the rules of cricket and had them practicing against each other. However, the Ohio State students were all new to the game and did not know that our students had played before. They made all the usual mistakes that Americans make when playing cricket. For example, I had taught my students to make sure the cricket ball, which is about the same size and weight as a baseball, bounced BEFORE it got to the batsman. Americans are not used to this. Unknowingly, the Ohio State students hurled the ball at our batsman as fast as they could at about waist height, BUT a cricket bat is about four inches wide and FLAT. Our Bellarmine students thought Christmas had come early! They amused themselves trying to blast the ball into orbit!!! Bellarmine wiped the floor with Ohio State - it was a massacre!! Why - because our students knew the rules and had practiced! When I tell my students that there is a Maths Test next week and ask them what are they going to do, they know the answer -

KNOW THE RULES AND PRACTICE!

2. THE TECHNIQUES

This section contains three sub-sections called **LIST A**, **LIST B**, and **LIST C**. The first section, **LIST A**, includes all those *techniques* that were initially evaluated. The *Evaluation Analysis* section (*Section 4*) in this proposal, compares and contrasts the *techniques* in **LIST A** between the *College Algebra courses* and the *Elementary Statistics courses* using the *short form* (in **Appendix A**). The second section, **LIST B**, includes those *techniques* currently being evaluated together with those in **LIST A**. The new form (the *long form*), in **Appendix B**, uses an improved evaluation method. Data using the *long form* are still being collected. The final section, **LIST C**, discusses those *techniques* that were tried but deemed failures.

2.1 LIST A

These eight *techniques* were the ones that were initially selected for testing using the *short form*.

1. CHAPTER NOTES (NOW INCLUDES QUESTIONS FOR WORKBOOK)

Originally *Chapter Notes* were intended for my use only. I did not intend to hand them out to my students. When I taught a course for the first time, or a new textbook had been selected for a course I was teaching, I went through it, page by page. I listed definitions, the examples I wanted to go through, together with why they were important, formulae, important exercises, etc. together with the page numbers for everything listed. The objective was to ensure I covered every topic in each chapter that needed to be covered as well as any important examples and exercises. Some of my students noticed them and asked me to make them available to them. Many students have since commented that they have been very helpful. They use them as a checklist i.e. they work through a set of *Chapter Notes* checking to see that they know the meaning of each term. If they do not, the page number in the *Chapter Notes* makes it easy for them to check the forgotten item in the textbook.

Before making the *Chapter Notes* available to my students, I used to write the numbers of the questions for the *workbook* (discussed in the next section) on the board. These exercises are now included in the *Chapter Notes*. I used to put these notes on reserve in the library. Students then took them out to copy them. Now I have my own Web page on the Internet my students can print the *Syllabus*, *Chapter Notes*, *Example Tests*, *Solutions to Problems,* plus any other relevant material on any computer in the college or even from home.

An example of *Chapter Notes* for Chapter 3 of **Barnett and Ziegler's** "*College Algebra*" and Chapter 6 of **Moore and McCabe's** "*Introduction to the Practice of Statistics*" is included in **APPENDIX C**. The format of the *Chapter Notes* various from subject to subject and even chapter to chapter within a textbook. On many occasions other faculty in the department, who are going to teach one of these courses for the first time, have requested copies of my *Chapter Notes*. It is now even easier to oblige them - I just refer them to my Web page!

2. WORKBOOK

The *workbook* was in response to some problems that I observed and to comments from of my students. Students buy a binder and a lot of blank paper. They then complete a collection of questions that I have previously selected from the end of each section of the textbook.

One of the things I noticed the first time I taught College Algebra was that many of my students had the rules of Maths totally muddled up in their minds. So every time we came to a rule, I emphasized it, and introduced them to my new motto "Maths is fun, because Maths is easy, ***IF** you know the rules and practice*". Now in a class when I announce we are going to look at a rule, students will occasionally remark "It's a rule, we had better make sure we know it". The *workbook* was designed to help my students remember the rules by giving them the practice they needed. The 100 bonus points, over and above the 1000 marks for the course, was designed to encourage them to complete it.

A common question from students after a bad first test, was should they drop, or if they did badly in the Final would they fail the entire course. I felt that one bad day should not destroy a student's grade. A student can complete the *workbook* and gain 100 bonus marks (out of 1000). This can certainly help make up for a bad day. Even more important, completing the *workbook* gives a student the necessary practice, which should reduce the chances of having a bad test day.

The important thing about the *workbook* is that it is a **PURE BONUS** added on to their final mark. On a 1000 point scale the *workbook* is worth 100 marks i.e. with 100 marks for the *workbook*, a student could get 1100 marks for a course (at least theoretically). Since this is equivalent to one whole grade, I expect a very high standard. I collect the *workbooks* once per term and mark them. However, this is an *interim mark*. On the day of the Final I collect all of the *workbooks* and mark them from the beginning as if I had not seen them before. This allows a student who did not get a good grade during the term, to go back and correct/improve their work. 15% of the mark is for presentation and 85% for completing all of the questions correctly. The final letter grade is converted into a percentage bonus, which is then added to their total. The percentages for each grade are below.

A+	**:**	**10%**
A	**:**	**9%**
A-	**:**	**8%**
B+	**:**	**7%**
B	**:**	**6%**
B-	**:**	**5%**
C+	**:**	**4%**
C	**:**	**3%**
C-	**:**	**2%**
D	**:**	**1%**
F	**:**	**0**

The task of obtaining a final letter grade for each student is not as daunting as it may seem. I have all the marks stored in an EXCEL spreadsheet. The spreadsheet is programmed to calculate and display their final letter grade. It can also display the letter grade of any student at any time during the course. So if a student wants to know their current standing in the course, I only need to locate a computer to give it to them.

I select questions to be done in the *workbook* from the end of each section of the textbook. For each main topic or concept, I choose one question that is easier than those solved within the text, one about as difficult, and one that is more difficult. I set only odd numbered questions so that they can check there answers in the back of the book. The amount of work involved is substantial, clearly it must be, if I am going to give one whole grade extra for it.

I do not mark every question in a *workbook* (that would be impossible). Instead I select a few questions in each section and check them. My students never know which questions I have selected for checking. Those questions involving a graph(s) are particularly good ones to check because they must draw the graph by hand. This minimizes the possibility of copying either from the answer book or from a past *workbook*. Quite often I am able to check all the questions on a College Algebra course because they are, typically, very simple to check and I have devised a method for speeding up the process.

Since, sadly, it is very easy to cheat with the *workbook*, there are a number of necessary precautions that I take before increasing a students final grade due solely to the *workbook*. These checks are based upon bitter experience!

1) I check that answers have not been copied from the answer book into the *workbook*.

2) I check the working of certain questions that I know take a lot of time to do well.

3) I check that the handwriting in the *workbook* is the same as the handwriting in the Final.

4) I check that the handwriting throughout the *workbook* is the same.

5) To stop students copying from past *workbooks*, I change a few questions. For example, if I required questions 1, 3, 5, 6 in one section last term, then this term I might change it to 1, 2, 4, 7. Then before I increase the grade of a student due purely to the *workbook*, I check to see if the correct questions are present. I am at the moment very fortunate. My fifteen-year-old daughter loves checking that a student has completed all sections and all questions within a section (not the answers of course). She is actually very good at it. I give her a complete checklist and away she goes. Last term it was my daughter, not I, that noticed a *workbook* contained two different handwritings!

The objective of the *workbook* is to provide a student with the necessary practice and help prepare him/her for the tests. This in turn reduces the chances of a student having a bad test day and also, since they are better prepared, reduces worry associated with taking a Maths test. In spite of the amount of work involved, many students have commented on how valuable they have found the *workbook* to be.

3. TAKE ONE SHEET OF PAPER INTO TEST (INCLUDING FINAL)

Each student is allowed to take ***One sheet*** of paper into any test of mine including the final. They can write anything they like on both sides of the sheet. This is to reduce the anxiety over taking a Maths test. From an academic standpoint, in order to decide what to put on the piece of paper, they have to read the book - which is exactly what I want them to do. From the students point of view it would seem that it is comforting to have some information that they have put together with them. Since they can write anything they like on this sheet, cheating is not possible.

I do state clearly that the piece of paper must be the standard size. When I first started doing this, I omitted tell my students the size permitted, and a number of students brought in a huge piece of poster board. Yet another brought in a roll of white wallpaper! It is quite common for students to use a copier to substantially reduce information before pasting it onto their one piece of paper. I have even seen students having to use a magnifying glass to see the print on their one sheet! I have no objection to them doing this.

Clearly, the format of the questions on tests has had to change. I cannot ask for proofs or definitions because some of my students might have brought it with them on their one sheet while some may not. This would give those students who were just lucky enough to put it on their one sheet, an advantage. I have had comments from other faculty that taking a sheet of paper into a test means that students do not have to memorize formulae. I never require my students to memorize formulae. I actually tell students not to waste space on their one sheet with formulae, any formula they need will be writen on the blackboard or at the end of a test. I believe it is worrying for a student, who is already poor and perhaps even a little frightened of Maths, to be forced to memorize a formula. If they were out working in industry and needed to use a particular formula, it is far more important that they know <u>when</u> they need it, and <u>where</u> to find it, than to commit a formula to memory and perhaps not know what it is for. In consequence my test questions overall are far more demanding than they used to be before I started using these techniques.

4. ADDITIONAL STUDY SESSION

During the course of the term, usually before a test, I put on an *additional study session*, which usually lasts from 45 minutes to an hour. Although I am quite happy to stay as long as they want me to stay. They can ask me to do anything from solving a problem to repeating some theory. I have had as many as 80% of a class turn up to one of these sessions. Strangely enough as each student leaves an additional study session, they thank me for it. This never happens after a normal lecture.

These study sessions take an extra 8 to 10 hours of my time during the average term, however, the positive response of my students during a normal class and during a session justifies ever minute of them. I now call *Additional Study Sessions*, *Additional Study Hours* (ASH). This is because on one particular day, without thinking and for speed, I wrote the acronym for *Additional Study Sessions* in big letters on the board and then wondered why my students were laughing! We do not spell it like this in England!

5. EXAMPLE TEST BEFORE EACH TEST INCLUDING FINAL

SOLUTIONS AND MARKING SCHEME ON COMPUTER AFTER TEST

About a week before a test, including the final, I give each student an *example test* to practice on before the real test. I set two questions as assignments from each *example test* as preparation for the real test. I encourage students to complete the remainder, although I do not take their work in and formally mark it. I do tell my students that if they want me to, I am happy to check the extra questions.

None of the questions on an *example test* appear on any real test. **Part A** of an *example test* is a complete past test that was taken by some of my students during a previous term. **Part B** consists of additional questions so that all the concepts that could be on the test are covered. I also put the solutions to the *example test* on my web page after they have had a chance to complete it. I make sure that the questions on the two in-term *example tests* and the *example final* cover the entire syllabus. This is to ensure that my students become familiar with the entire course before the Final. Many students see the advantage of completing these tests before the real tests.

The objective of providing *example tests* is to reduce the anxiety associated with taking a Maths test. It also gives them the necessary practice and shows them the format of one of my tests. Another thing I have found reduces anxiety in the real test is to ensure that there is plenty of time to complete and check the test. Now I always write tests so that my students are not competing against the clock.

6. COLOURED CHALK ON BOARD

I have used colored chalk on the blackboard for many years. Although this *technique* did not do well in the evaluations, due possibly to the ranking approach of the form used, the latest form I use suggests that it does help my students. Also comments I get from students on the latest evaluation form make it quite clear that it is helpful during a lecture.

I use different colors for different things depending on the course. I make a list of the colors I use for different things in any course. Then if I do not teach the course for a while I have a reference list to which I can refer. For example, in a course involving differentiation, I use white chalk for the main working, red chalk for a derivative, yellow chalk for a second derivative, blue chalk for a rule i.e. the product rule etc. In College Algebra, the minus sign before a bracket is in red (for danger!). After I have done this a few times making sure my students are familiar with the danger, I stop doing it and ask my students what is missing. They usually tell me to change the white minus sign to a red minus sign. For a course involving computers, I use white chalk for the characters the computer puts on the screen, orange chalk for characters that must be typed in by the student etc. Students get used to the meaning of a particular color. It breaks up what is on the blackboard and makes it easier to pick off the derivative, a formula etc.

Another idea I found useful is to divide up the blackboard. I might divide it into three parts. For example, the left part for terms using red chalk, the middle portion for main work, the right portion for graphs drawn using different colors. I am frequently teaching a topic that I have taught hundreds times before, having to think about which color to use helps me stay focused.

7. GRADED BONUSES ON ALL TESTS (TO ENCOURAGE THEM TO THINK DURING THE TEST)

The objective of this *technique* is to make my students think throughout a test. This *technique* came in fourth on the student evaluations. The comments I have received and the overall response to this *technique* suggests that it does indeed cause students to think more deeply about the questions that they have to answer on the test.

What I give bonuses for depends on a variety of things. I expect more from my students as the course progresses – hence **graded** bonuses. On the first test, I may give one point bonus for putting a complete title on a graph or the equation of a line alongside the drawn line. On future tests the title and the equation by the line are expected, so there would be no bonus available. In Test 1, if they solve a problem, then carry out a relevant check, I may give a bonus, but checking an answer is expected on future tests. I want them to be constantly thinking about ways to gain bonuses so they constantly think about, not just the questions on the test, but also the underlying concepts and related topics.

For each test I create a complete marking scheme. It will include marks for everything I require to obtain full marks to each question. After marking every student's script, I can then award bonuses based on anything extra that has been included in an answer. To gain a bonus or bonuses anything extra must be relevant to the question. I have had students in the past who were unable to answer the question on the test invent their own question, then proceed to answer it, expecting to gain bonuses. Obviously they gained nothing. I do not give bonuses for an answer that could have been brought in on their one sheet. I am much more interested in an answer that shows they understand the underlying concepts. Something that could have been memorized, perhaps without understanding, would not gain a bonus mark. As a course progresses I find that students improve their understanding of the underlying concepts, so they are better able to gain bonuses during tests.

8. GROUP PROJECTS AND WORKING IN GROUPS ON PROBLEMS

I like to have my students working in groups, partly because this is often what they are likely to encounter out in the real working world, and partly because it promotes student interaction. I create deliberately open-ended projects because it allows students to use their imaginations and creativity. This works well with Statistics courses but not with College Algebra. The problem is that I have been unable to devise projects that are sufficiently open-ended to use as group projects in College Algebra. At the present time I have my College Algebra students completing one workbook for two students with the answers in the workbook alternating between students. This is a very new idea. My theory is that to produce an alternating workbook, they must talk to each other! However, I am currently trying to devise suitable projects for College Algebra because working in groups of three works so well in my statistics classes, that I want to do the same thing in my *College Algebra courses*. I include some examples I use in my Statistics courses to show why I am so keen to introduce open-ended projects into my *College Algebra courses*.

I use the following projects in my statistics classes. They are deliberately very open-ended so that my students can use their imaginations and be creative. For each project they must think of an idea that requires them to measure something. I do not permit students to lift data out of a book or off the Internet. Then they must collect the data, complete a write-up, and give an oral to the class. The oral requires a group to create what I call visuals. These are poster board sized graphs, data, or results, large enough to be seen from the back of the classroom. They are used, by the group member giving the oral, to show that the conclusion reached by the group is correct.

PROJECT 1

Collect **50** data values that you believe will be normally distributed. Plot your observations on a graph together with the associated normal curve. Explain how you determined where to draw the normal curve on your graph. Use the **68-95-99.7 rule**, as shown in **Example 1.18 - P66**, to assess the fit of the normal curve to your data and a **Normal Quantile Plot**. Comment on your results in your write-up and justify your conclusion in your report to the class.

PROJECT 2

Select two variables that you believe can be modeled by a straight line. Collect a minimum of **20** pairs of data. Draw a scatterplot of your data together any other graph you feel appropriate. Compute the least squares regression line and draw it on the graph of your data using the same axis. Compute any other values that you believe are relevant to your study. Explain clearly what you were trying to do, any assumptions you made, and your conclusion.

The instructions for marking this type of work were developed over a period of time. The latest marking scheme, which solve the problems I had originally, are as follows :

Each member of a group will get the ***same*** mark based upon the following *marking scheme*.

Correct Procedure and Analysis	**40 marks**
Written Presentation	**20 marks**
Oral Presentation	**20 marks**
Visuals	**20 marks**
TOTAL MARKS FOR GROUP PROJECT	**100 MARKS**

1. The typed report, one per **group**, should include the original data, any graphs, and an explanation of what you are trying to do in your study. Discuss any factors, which may affect the validity of the study. Do not include the calculations just the results (unless you feel that they are essential). State your conclusion clearly.
2. A representative of each group will present a **BRIEF** report on the topic to the class (it should be between **4** and **5 minutes** long). **DO NOT READ YOUR REPORT**. You should state what the study was about and what you discovered, and the conclusion your group reached.
3. Hand in **TWO** copies of your work. One I keep and one I return to you.

I am frequently amazed at some of the ideas my students come up with and the standard of the work they hand in. Here are three of the many ideas I have received.

The Length of an Airplane Project

This group built **15 model planes** using the same weight of material for each plane but making each plane a different length. The shortest plane was **14 centimeters long**, the next **15 centimeters long**, etc., up to the longest at **28 centimeters long**. The group then flew each plane five times to obtain the average distance traveled. The objective was to determine if a linear model would accurately predict the distance that a model plane of a particular length would travel. This group also carried out a **residual analysis**. Since **residual analysis** is not part of the course, they had to go to the library to find out how to do it.

The Potato Gun

This group built a **potato gun**. It was made out of PVC piping four inches in diameter and about 6 feet long. At one end there was a special section into which hair lacquer could be squirted and ignited. They went to a park and shot 60 potatoes out of it, measuring the distance each one traveled. The objective was to determine if the distance traveled was normally distributed. I was amazed to learn that this gun could shoot a ***softball***-sized potato **400 feet!** They found the instructions for building the gun on the **Internet**. They were very keen to demonstrate the capabilities of their new-found toy. They arrived with the gun on the day of their oral ready to shoot a large potato out of the classroom window and over a nearby building! While I was very interested to see what it could do, I would not let them blast a potato over the top of our new library!

Dropping Eggs from Different Heights

Another group bought **fifteen eggs** and, using a ladder, dropped them from **fifteen different heights**. They then measured the **maximum width** of the **splatter on the ground**. The objective was to determine if a linear model would accurately predict the **maximum splatter** given the **height** from which the egg was dropped. They were careful to reduce the effect of **lurking variables** while obtaining the measurements. For example, one member of the group dropped all the eggs making sure the orientation of each egg was the same before it was dropped, another member of the group measured the width of the splatter, while the third member of the group recorded the measurements. It was amazing to see how well the least-squares line fitted the data on their "*splatterplot*" (their idea not mine!). The **correlation coefficient** was **0.998**.

The standard of the work that I have received has been quite amazing. A number groups came in to do there presentation with props or dressed up in clothing related to the project. Often the graphs in the written work have been completed in color using SPSS. This is a statistical piece of software that I do not teach on this course – group members teach themselves! Just recently groups have started to do their presentations in **Powerpoint**. They have to teach themselves this piece of software because I do not teach it during the course.

Initially marking their written work was difficult because of the variety of topics. To get around this problem I created a general checklist from the textbook and marked each project against it. This made it possible to compare and contrast projects and ultimately come up with a comparative fair grade.

There are frequently problems with some groups. These arise when a group member never shows up for an out of class meeting, or does nothing or very little towards the project. I do not believe it is the student's job to police the projects. It is my job to check that all is working well within the groups. I have created a checklist that I fill out during a group session in class. I go through the project with a group asking who did the various parts of the project and writing the names down alongside the item on my checklist. I have tried many different approaches to this problem. I have found that the checklist idea is the only one that works.

I firmly believe that students who are having fun are also learning something. My students seem to find working in-groups in class more fun than a normal lecture. I assume that this is because they are working together and discussing questions. While they are working in-groups I walk around answering questions. This is fun for me too!

LIST B

These techniques are currently being tested using the *long form*.

9. ANY NUMERICAL ANSWER REQUIRES A SENTENCE CONTAINING IT IN THE CONTEXT OF THE QUESTION.

Some years ago I became unhappy about giving as many as 10 marks for obtaining the correct numerical answer to a question. It occurred to me that I could probably train the average monkey to press the correct keys on a calculator and get the right number. However, the monkey would not know the meaning of the number, and I felt that some of my students did not know either. So I began to insist that students showed me that they knew the meaning of the number they had calculated by including it in a sentence in terms of the question. I soon discovered that many students did not know the meaning of the number they had obtained on their calculator screens. Consider the following example from a past exam, the exam question was :

Dan wants $2,000 now, from a bank, to be repaid 18 months from now. How much will the repayment be if the discount rate is 15%?

The correct numerical answer is $2,580.65. <u>Five years ago, this answer would have earned this student 10 marks.</u> However, the sentence from this student was :

In 18 months time, with a discount rate of 15%, the bank will pay Dan $2,580.65.

(I am still trying to locate this bank : and when I do I won't tell anyone – its mine!!!).

Students who write a correct sentence with the correct numerical answer in it, but with no working to back it up, do not get the marks allocated to the correct answer in a sentence. They must show all of the working to gain credit for their answer (I never forget that they bring one sheet of paper in with them!).

I have asked students for their opinion on writing a sentence for any numerical answer. Last term one of my students said "Having to write a sentence helps me understand the ideas". I was very pleased with this response, although there are certainly students who do not share this view!

I now feel that if a calculator is used in any course, checking that students do understand the meaning of the number that has appeared on the calculator screen is essential.

10. CREATE SPECIAL TECHNIQUES FOR THOSE TOPICS THAT CAUSE DIFFICULTY

I found that many students had considerable difficulty with certain topics which could easily be modified to either simplify the approach or make routine what students had to do to obtain the correct solution(s). I simplified this type of topic by creating what I call a *special technique*. The objective of a *special technique* is to reduce "Maths Anxiety" by making it possible for any student, whatever their mathematical ability, to obtain the solution(s) to a type of problem that causes difficulty using the approach described in most textbooks. Of course, it is important that the *special technique* does not obscure the underlying concept(s). To illustrate what I mean by a *special technique* I include two in this paper.

I created the first *special technique* in the second week of the *Spring term of 1995*. I called it the *Arrow Method*. I needed to teach my students how to multiply out algebraic expressions, a process which some students found very difficult. However, the book used a method called **FOIL**, which I had never encountered before in spite of 25 years of teaching Maths.. This method amazed me. It was indeed a new experience! My students had to learn the meaning of the word **FOIL,** and then but only if there are two terms in each bracket. If the problem was not of this type, **FOIL** was discarded, because it could not be used, and another method was employed. It seemed crazy to me to force students, who are frequently very weak in Maths, to learn a special process that only works in one specific situation, and then discard it and teach another approach for all those situations in which **FOIL** cannot be used. The *Arrow Method* (*Special Technique 1* – on the next page) can be used to multiply out ***any*** algebraic expressions. However, since this database is not reproduced in color, I have used different types of lines and fonts to show the color of the chalk I use in class. The notation is shown before the examples.

The *Arrow Method* uses what I call *arrow sets*. The highlight in ***1)*** covers ***one*** *arrow set*. So ***1)*** has *two* arrow sets, one above the expression and one below, while ***3)*** has *three* arrow sets, one above and two below the expression. I draw each *arrow set* using chalk of a different color and write the resulting terms to the right of the equal sign in the same color as the arrow set, before finally collecting up terms using white chalk. Even though the *arrow sets* below the expression in ***3)*** cross each other, there is no confusion because they are drawn using different colored chalk. I tested various ways of drawing the *arrow sets* and found that those on the next page are the most effective. When teaching the *Arrow Method* I always have a colored transparency of the next page on the overhead projector.

I wanted to include the results of a survey at this point, to show that my students preferred the *arrow method* to **FOIL.** However, in each evaluation *every* student preferred the *arrow method*. Many students include the arrows on their tests and in the Final and some even draw them in color, even completing the problem in color. Is it time to lay **FOIL** to rest forever?

I created the second *special technique* after I noticed that many students had difficulty finding the two linear factors with integer coefficients of a second-degree polynomial (if they exist) i.e. finding the factors of a second-degree polynomial *relative to the integers*. The *Guaranteed Factor Method* (*Special Technique 2* - described after the *Arrow Method*) uses a program that each student inputs into their own **TI-83** calculator. There are two major advantages to this approach. One is that ***any*** student can obtain the correct linear factors with integer coefficients using this technique, and when using the program they can see that the quadratic formula is being used. The other advantage is that they learn the meaning of some of the important programming terms and phrases, such as *input*, *editing a program*, *executing a program*, and *output* from a program. Clearly, this program can also be used if just the roots of a quadratic equation are required.

There are many topics that cause students difficulty, and hence Math Anxiety, that can be modified and simplified. Some of these *special techniques* take a lot of time to create and test. However, I have always found that this extra time is well spent. If there was a *first law of teaching*, I think it would read :

Teacher gives more;
Students get more;
Teacher gives less;
Students get less;
There are no shortcuts!

SPECIAL TECHNIQUE 1 – THE ARROW METHOD

NOTATION

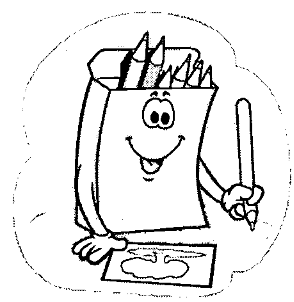
Color of Chalk Used for Arrow Sets (using different styles of lines)

Red Chalk = Yellow Chalk = Green Chalk =

Color of Chalk Used for Characters (using different fonts)

Red Chalk = $10x^2$-$35x$ Yellow Chalk = $4x + 14$ Green Chalk = $21x+14$ White Chalk = $7x+2$

Arrow Set



1) $(5x - 2)(2x - 7) = 10x^2 - 35x - 4x + 14$

$= 10x^2 - 39x + 14$

---

2) $(7x + 2)(x^2 + 3x - 9) = 7x^3 + 21x^2 - 63x + 2x^2 + 6x - 18$

$= 7x^3 + 23x^2 - 57x - 18$

---

3) $(2x^2 - 5x + 7)(3x + 2) = 6x^3 + 4x^2 - 15x^2 - 10x + 21x + 14$

$= 6x^3 - 11x^2 + 11x + 14$

---

4) $2x^3(3y + 2z) = 6x^3y + 4x^3z$

SPECIAL TECHNIQUE 2 – THE GUARANTEED FACTOR METHOD

Many students have difficulty finding the two linear factors with integer coefficients of a second-degree polynomial (if they exist) i.e. finding the factors of a second-degree polynomial *relative to the integers*. The **Guaranteed Factor Method** uses a program, input by a student into the **TI-83** calculator, to find these factors for any second-degree polynomial of the form $\mathbf{Ax^2 + Bx + C}$. For example, if the two linear factors with integer coefficients of the second-degree polynomial $\mathbf{12x^2 + 7x - 10}$ are required, this method will let ***any*** student find the expression $\mathbf{(3x - 2)(4x + 5)}$. The output from the program will make it clear, if the polynomial **cannot** be factored relative to the integers. The following program, called **QUADPROG**, must be entered into the calculator first. If you do not know how to enter a program into the **TI-83**, the full *key sequence* from the *home screen*, is shown after the examples in **Table 1**.

```
PROGRAM:QUADPROG
:Prompt A,B,C
:(-B+√(B² – 4AC))/
(2A)→P
:(-B–√(B² – 4AC))/
(2A)→Q
:Disp "ZEROS ARE
",P▸Frac,Q▸Frac
```

The program can be executed with the *key sequence* **<PRGM> 1 <ENTER>** (assuming **QUADPROG** is program **1** when you press the **<PRGM>** key).

When this program is executed there are ***three*** and ***only three*** possibilities :

Case 1 : The program will display the ***error message*** **ERR:NONREAL ANS**, in which case the second-degree polynomial ***CANNOT*** be factored relative to the integers because the zeros are imaginary.

Case 2 : The program will output **TWO decimal *numbers***, in which case the second-degree polynomial ***CANNOT*** be factored relative to the integers.

Case 3 : The program will output ***two fractions***, **one fraction and one *integer***, or ***two integers***, in which case the second-degree polynomial ***CAN*** be factored *relative to the integers*.

Note

1) The *denominator* of the fraction(s) will be the coefficients of the **x term** in each linear factor (the denominator for any integer is **1**).

2) The *numerator* of each fraction will be the constant in each linear factor.

Summary of the Method

Step 1 : Rewrite the expression if the sign of the $\mathbf{x^2}$ term is not positive.
i.e. rewrite $\mathbf{10 - 7x - 12x^2}$ as $\mathbf{-(12x^2 + 7x - 10)}$

Step 2 : Remove all common factors.
i.e. rewrite $\mathbf{48x^2 + 28x - 40}$ as $\mathbf{4(12x^2 + 7x - 10)}$,

Step 3 : Identify the values of **A**, **B**, and **C**
i.e. compare the given or rewritten expression with $\mathbf{Ax^2 + Bx + C}$

Step 4 : Execute the program **QUADPROG** with the *key sequence* **<PRGM> 1 <ENTER>**. If the two values displayed indicate that the expression does have two linear factors with integer coefficients go to **Step 5**, otherwise **STOP**.

Step 5 : Let one of the values displayed in **Step 4** be **a** and the other be **b**, substitute these values into the expression $\mathbf{(x - a)(x - b)}$. Now move the denominators in the fractions in each bracket in front of the **x** in the same bracket. If there was no changes in **Steps 1** and/or **2**, the two linear factors obtained at this point will have integer coefficients and are the required linear factors of the original second-degree polynomial. If there were changes in **Steps 1** and/or **2** go to **Step 6**, otherwise **STOP**.

Step 6 : Adjust the final expression in accordance with changes made in **Steps 1** and/or **2 – STOP**.

The **Guaranteed Factor Method** is best illustrated by examples for each of the three cases.

Case 1 – Error Message

Example 1

Find the factors, relative to the integers, of the expression $\mathbf{2x^2 + 5x + 7}$.

Solution

Step 1 : *No Change* – the sign of the $\mathbf{x^2}$ term is positive

Step 2 : *No Change* – there are no common factors

Step 3 : Comparing $\mathbf{2x^2 + 5x + 7}$ with $\mathbf{Ax^2 + Bx + C}$ gives $\mathbf{A = 2}$; $\mathbf{B = 5}$; and $\mathbf{C = 7}$

Step 4 : Execute the program with $\mathbf{A = 2}$; $\mathbf{B = 5}$; and $\mathbf{C = 7}$. The calculator displays the following *error message* :

```
ERR:NONREAL ANS
1:Quit
2:Goto
```

In this case the second-degree polynomial $\mathbf{2x^2 + 5x + 7}$ ***CANNOT*** be factored *relative to the integers* because its zeros are imaginary – **STOP** (press **1** to **Quit** and return to the home screen).

Case 2 – Decimal Numbers

Example 2

Find the factors, relative to the integers, of the expression $\mathbf{9x^2 - 5x - 6}$.

Solution

Step 1 : *No Change* – the sign of the $\mathbf{x^2}$ term is positive
Step 2 : *No Change* – there are no common factors
Step 3 : Comparing $\mathbf{9x^2 - 5x - 6}$ with $\mathbf{Ax^2 + Bx + C}$ gives $\mathbf{A = 9}$; $\mathbf{B = -5}$; and $\mathbf{C = -6}$
Step 4 : When the program is executed with these values for **A**, **B**, and **C**, the calculator displays the two *decimal numbers* **1.140231928** and **–.584676372**. In this case the second-degree polynomial $\mathbf{9x^2 - 5x - 6}$ ***cannot*** be factored *relative to the integers* – **STOP**.

Case 3 - Polynomial CAN be Factored Relative to the Integers

Example 3

Find the factors, relative to the integers, of the expression $\mathbf{12x^2 + 7x - 10}$.

Solution

Step 1 : *No Change* – the sign of the $\mathbf{x^2}$ term is positive
Step 2 : *No Change* – there are no common factors
Step 3 : Comparing $\mathbf{12x^2 + 7x - 10}$ with $\mathbf{Ax^2 + Bx + C}$ gives $\mathbf{A = 12}$; $\mathbf{B = 7}$; and $\mathbf{C = -10}$
Step 4 : Execute the program with $\mathbf{A = 12}$; $\mathbf{B = 7}$; and $\mathbf{C = -10}$. The calculator displays the two fractions $\frac{2}{3}$ and $-\frac{5}{4}$. Since the program has displayed two fractions, this expression ***can*** be factored *relative to the integers.*

Step 5 : In the expression $\mathbf{(x - a)(x - b)}$ replace the **a** with $\frac{2}{3}$ and the **b** with $-\frac{5}{4}$ giving

$$\left(x - \frac{2}{3}\right)\left(x - \left(-\frac{5}{4}\right)\right) = \left(x - \frac{2}{3}\right)\left(x + \frac{5}{4}\right)$$

Now move the **3** and **4** in the denominators in the each bracket in front of the **x** in the same bracket, giving :

$$\mathbf{(3x - 2)(4x + 5)}$$

There were no changes in **Steps 1** and **2** so $\mathbf{12x^2 + 7x - 10 = (3x - 2)(4x + 5)}$

Example 4

Find the factors, relative to the integers, of the expression $\mathbf{10 - 7x - 12x^2}$.

Solution

Step 1 : Re-write $\mathbf{10 - 7x - 12x^2}$ as $\mathbf{-(12x^2 + 7x - 10)}$, since the sign of the $\mathbf{x^2}$ term is negative;
Step 2 : *No Change* – there are no common factors;
Step 3 : Comparing $\mathbf{12x^2 + 7x - 10}$ with $\mathbf{Ax^2 + Bx + C}$ gives $\mathbf{A = 12}$; $\mathbf{B = 7}$; and $\mathbf{C = -10}$;

From Steps 4 and 5 in Example 3

$$12x^2 + 7x - 10 = (3x - 2)(4x + 5)$$

Step 6 : There was a change in **Step 1** so

$$10 - 7x - 12x^2 = -(12x^2 + 7x - 10) = -(3x - 2)(4x + 5).$$

This can be written as $\mathbf{(2 - 3x)(4x + 5)}$.

Example 5

Find the factors, relative to the integers, of the expression $\mathbf{48x^2 + 28x - 40}$

Solution

Step 1 : *No Change* – the sign of the $\mathbf{x^2}$ term is positive
Step 2 : Re-write $\mathbf{48x^2 + 28x - 40}$ as $\mathbf{4(12x^2 + 7x - 10)}$,
Step 3 : Comparing $\mathbf{12x^2 + 7x - 10}$ with $\mathbf{Ax^2 + Bx + C}$ gives $\mathbf{A = 12}$; $\mathbf{B = 7}$; and $\mathbf{C = -10}$

From Steps 4 and 5 in Example 3

$$12x^2 + 7x - 10 = (3x - 2)(4x + 5)$$

Step 6 : There was a change in **Step 2** so

$$48x^2 + 28x - 40 = 4(12x^2 + 7x - 10) = 4(3x - 2)(4x + 5)$$

Example 6

Find the factors, relative to the integers, of the expression $2x^2 + 3xy - 2y^2$.

Solution

Since this expression is degree two in both **x** and **y**, this method can still be used. The one change occurs in **Step 5**. The expression into which the fractions are substituted becomes

$$(x - ay)(x - by)$$

to take into account the fact that the expression is also a quadratic in **y**.

Step 1 : *No Change* – the sign of the x^2 term is positive
Step 2 : *No Change* – there are no common factors
Step 3 : Comparing $2x^2 + 3xy - 2y^2$ with $Ax^2 + Bx + C$ gives $A = 2$; $B = 3$; and $C = -2$
Step 4 : Execute the program with $A = 2$; $B = 3$; and $C = -2$. The calculator displays the two fractions ½ and **–2**. Since the program has displayed a fraction and an integer, this expression ***can*** be factored *relative to the integers.*
Step 5 : In the expression $(x - ay)(x - by)$ replace the **a** with ½ and the **b** with **–2** giving

$$(x - \tfrac{1}{2}y)(x - (-2)y) = (x - \tfrac{1}{2}y)(x + 2y)$$

Now move the **2** in the denominator in the first bracket in front of the **x** giving :

$$(2x - y)(x + 2y)$$

There were no changes in **Steps 1** and **2** so

$$2x^2 + 3xy - 2y^2 = (2x - y)(x + 2y)$$

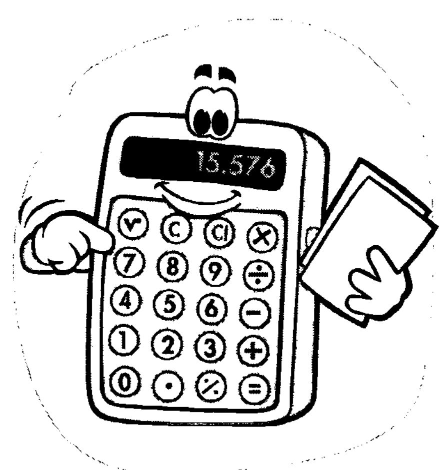


Table 1

NOTE

The symbols < > are called *angle brackets*. They are used, in the table below, on either side of the name of a key on the calculator keyboard. When you see these symbols, find the key with the name on it, and press it. For example, when you see the symbols **< 2nd >**, find and press the *yellow key* marked **2nd** on the calculator (it is the only yellow key on the calculator). The symbol **< → >** represents the *right arrow key*. The integers **0, 1, 2, ⋯ , 9** are ***NOT*** enclosed in *angle brackets* in the table. The *underlined characters* in the **DISPLAY** column of the table below are output to the screen by the calculator

The name of the program you will store is **QUADPROG**. It can be entered into the **TI-83**, from the ***home screen***, by the **10** steps in the table below . The key sequence in **Rows 5** and **7** are divided into separate rows to make it easier to read ***only***. Each line in **Rows 5** and **7** add some additional characters to the display screen. They are shown in the display column of the table. Do ***NOT*** press the **ENTER key** at any time within **Rows 5** and **7**, ***only*** when you specifically see the symbols **<ENTER>**.

✻ The key marked APPS on a *TI-83 Plus*, is marked MATRIX on a *TI-83* ✻

	KEYS TO BE PRESSED	DISPLAY	EFFECT
1	< PRGM > < → > < → > 1	PROGRAM Name=	Select the ***Create New*** option to *create a new program*
2	9 5 <MATH> < x^{-1} > 8 < × > 7 <TAN> <ENTER>	PROGRAM :QUADPROG :	Assign the name **QUADPROG** to the new program
3	< PRGM > < → > 2	PROGRAM :QUADPROG : Prompt	Paste the *Prompt* instruction to the screen
4	<ALPHA> <MATH> < , > <ALPHA> < APPS > < , > <ALPHA> <PRGM > <ENTER>	PROGRAM :QUADPROG : Prompt A,B,C :	Append the variables to be input to the *Prompt* instruction
5	< (> < (-) > <ALPHA> < APPS > < + > < 2nd > < x^2 > <ALPHA> < APPS > < x^2 > < – > 4 <ALPHA> <MATH> <ALPHA> <PRGM> <) > <) > < ÷ > < (> 2 <ALPHA> <MATH> <) >	: (-B : (-B+ √(: (-B+ √(B^2– : (-B+ √(B^2–4AC : (-B+ √(B^2–4AC)) / : (-B+ √(B^2–4AC)) / (2A)	Enter the *Quadratic Formula* for **+** **DO *NOT* PRESS THE ENTER KEY AT ANY TIME DURING THIS KEY SEQUENCE.**
6	< STO > <ALPHA> 8 <ENTER>	: (-B+ √(B^2–4AC)) / (2A)→P :	Assign the First Zero of the Expression to the Variable **P**
7	< (> < (-) > <ALPHA> < APPS > < – > < 2nd > < x^2 > <ALPHA> < APPS > < x^2 > < – > 4 <ALPHA> <MATH> <ALPHA> <PRGM> <) > <) > < ÷ > < (> 2 <ALPHA> <MATH> <) >	: (-B : (-B– √(: (-B– √(B^2– : (-B– √(B^2–4AC : (-B– √(B^2–4AC)) / : (-B– √(B^2–4AC)) / (2A)	Enter the *Quadratic Formula* for **–** **DO *NOT* PRESS THE ENTER KEY AT ANY TIME DURING THIS KEY SEQUENCE.**
8	< STO > <ALPHA> 9 <ENTER>	: (-B– √(B^2–4AC)) / (2A)→Q :	Assign the Second Zero of the Expression to the Variable **Q**
9	<PRGM> < → > 3 < 2nd > <ALPHA> < + > 2 <SIN> < × > 7 < LN > 0 <MATH> < × > <SIN> < + > <ALPHA> < , > <ALPHA> 8 <MATH> 1 < , > <ALPHA> 9 <MATH> 1	: Disp : Disp " : Disp "ZEROS ARE " ",P▸Frac ",P▸Frac,Q▸Frac	Paste **Disp** to the screen *Turn Letters On* " Append "ZEROS ARE" to **Disp** " *Turn Letters Off* Display the Values of **P** and **Q** as *Fractions* (*if possible*)
10	< 2nd > < MODE >	***PROGRAM COMPLETE***	QUIT to the Home Screen

11. USE THE TEXTBOOK AS A SET OF NOTES TO MINIMISE THE AMOUNT OF WRITING STUDENTS DO IN CLASS.

I use the textbook as if it was set of notes using my *Chapter Notes* to ensure that each term, formula, concept, and important example is covered. The objective is to minimize the amount of time my students spend copying material off the blackboard. This approach also allows me more time to ask questions, allowing me to interact as much as possible with my students. The method of teaching I now use is similar to how I taught in England. For most courses in England students do not buy textbooks, instead the instructor writes notes, duplicates them, hands them out and then goes through them with the students. I know from my own experiences as a student that I can copy from the blackboard without thinking about what it means. I do not want my students doing this. This technique is closely related to *Technique 12* below.

12. MAKE AND USE TRANSPARENCIES OF GRAPHS, RULES, ETC. SO TIME IS NOT WASTED DRAWING OR WRITING THEM ON THE BLACKBOARD.

I create transparencies of diagrams, tables, menus from the **TI-83,** sometimes even solutions from the textbook we are using, so that I have more time to ask questions directly to the class and the students do not waste time copying off the blackboard. It saves a lot of time during a lecture. Also students can listen to me and ask questions because they are do not have to take notes off the blackboard. If I need to enhance a pre-prepared slide that is on the overhead projector, I lay a blank slide over it and write onto it with an erasable marker (I clean the blanks later). This leaves the original undamaged. When I am doing this I am still facing the class, which allows better interaction with my students. I also use a laser pointer for some slides. This allows me to continue asking questions about the content of the slide while moving around the classroom.

For some classes I use the **TI-83** with a projector device that sits on top of the overhead projector. I found that after placing a blank transparency on this projector device I could emphasize a graph(s) or some of the data projected onto the screen using colored projector pens. This made the discussion that followed much easier for students to follow.

I found that creating a slide for some of the **TI-83** menus speeded up the learning process. The **TI-83** STAT TEST menu on the next page, when used as a slide, allows me to compare and contrast the ideas in Chapter 6, 7, and 8 in Moore and McCabe's statistics textbook. This shows students the strong links between these three chapters. With similar intentions in mind, I have created slides for the College Algebra course as well.

STAT TESTS MENU (<STAT> ←, P13.9)

EDIT CALC TESTS	Pressing the **STAT** key allows you to select the **EDIT** menu (the *default menu*), the **CALC** menu, or the **TESTS** menu.
1 : Z - Test ...	**Test** for **1** μ , *known* σ *(Sec. 6.2)*
2 : T - Test ...	**Test** for **1** μ , *unknown* σ *(Sec. 7.1)*
3 : 2 - SampZTest ...	**Test** comparing **2** μ**'s** , *known* σ**'s** *(Sec. 7.2)*
4 : 2 - SampTTest ...	**Test** comparing **2** μ **'s**, *unknown* σ**'s** *(Sec. 7.2)*
5 : 1 - PropZTest ...	**Test** for **1** proportion *(Sec. 8.1)*
6 : 2 - PropZTest ...	**Test** comparing **2** proportions *(Sec. 8.2)*
7↓ ZInterval ...	**Confidence Interval** for **1** μ , *known* σ *(Sec. 6.1)*
8 : TInterval ...	**Confidence Interval** for **1** μ , *unknown* σ *(Sec. 7.1)*
9 : 2 - SampZInt ...	**Confidence Interval** for **2** μ**'s** , *known* σ**'s**
0 : 2 - SampTInt ...	**Confidence Interval** for **2** μ**'s** , *unknown* σ**'s** *(Sec. 7.2)*
A : 1 - PropZInt ...	**Confidence Interval** for **1** proportion *(Sec. 8.1)*
B : 2 - PropZInt ...	**Confidence Interval** for Difference of **2** Proportions *(Sec. 8.2)*
C : χ^2- Test ...	**Chi-Square Test** for **2-way** Tables *(Sec. 8.3)*
D : 2 - SampFTest ...	**Test** comparing **2** σ**'s**
E : LinRegTTest ...	**t-Test** for Regression Slope and ρ
F : ANOVA(...	One-Way Analysis of Variance

13. ALL QUESTIONS ON TESTS STRAIGHT FROM TEXTBOOK (SOMETIMES WITH MINOR MODIFICATIONS)

The objective of this technique is to encourage students to solve more questions from the textbook so they get additional practice. If they solve more questions they might complete one or more of the questions that are actually on the test. I do change the odd sign or interchange numbers in a question I intend to put on a test (I warn them about this possibility). I do this because students sometimes bring in the answers and even the working of some questions on their one sheet of paper. This is perfectly permissible, but obviously I do not want a copied answer on the test.

14. HAND OUT COMPLETE SOLUTIONS TO SOME TYPICAL PROBLEMS WRITTEN MATHEMATICALLY CORRECTLY

Mathematics is a language. I want my students to write mathematics correctly. However, when writing on the board I partly say and partly write a solution to a problem. Time rarely permits a full correctly written problem to be put on the blackboard. I cannot expect my students to write out a solution correctly if they do not see a solution written out correctly. In order to remedy this, I write solutions to various problems and put them into my computer area so they can print them.

15. USE TOPICAL PROBLEMS IN THE LECTURE

I use what I call topical problems during a lecture because it seems to increase student's interest and enthusiasm. It is very easy for me, because I just use topics to do with England. An American teacher could use either topics from their home state, if they are teaching in a different one, or from another country that they know well. An example is as follows :

Example 1

The railway station with the longest name in Great Britain is as follows :

LLANFAIRPWLLGWYNGYLLGOGERYCHWYRNDROBWLLLLANTYSILIOGOGOGOCH

Questions associated with the name of this village would vary from subject to subject. Some examples are as follows.

- **a)** Find the set of vowels and the set of consonants.
- **b)** Write the number of letters in this name in completely factored form.
- **c)** What is the probability of selecting a vowel from the letters that form this name?

Clearly there are many other possibilities. If a student cannot do the question or finishes quickly they can amuse themselves trying to say it!! A very useful place for topical problems is the Internet. I can obtain all sorts of news, audio clips, and data from the British Broadcasting Corporation's Web page at ***www.bbc.co.uk***.

16. 90% OR MORE RULE (4 TO 5 WEEKS BEFORE THE END OF TERM)

The 90% percent rule is very simple. About four or five weeks from the end of term (see **Plan 1 or 2** in *Section 3*) I tell my classes that if they obtain 90% or more on the final, I will give them an **A** for the course whatever their current standing. The objective is to give hope to those who are in trouble and to those who may be capable of doing the work but are not applying themselves. Clearly a student having a **D** or **C** is most unlikely to obtain 90% or more on the final. In fact only two students (out of approximately 300) who were holding a **D** or **C** prior to the final have ever obtained 90% or more on the final. However, hope is a wonderful thing, and this rule does have a positive effect on many students.

LIST C

The following techniques were deemed failures - mostly by me!

17. SETTING 8 QUESTIONS ON THE FINAL - STUDENTS SELECT ANY 6 FOR FULL MARKS.

In England it is quite normal for students to have to complete five or six questions out of eight to obtain full marks for an examination. I only tried this twice over here. My students thought it was great, they could avoid topics that they did not know well. However, for me it was horrendous. Many students tried all eight questions, so I had to mark all eight and select the best six. I ended up doing around 33% more marking. Also it did not occur to me beforehand to have each question worth the same amount of marks, so I had to scale all the questions out of 20 before I could select the best six. Never again!

18. TWO POINTS FOR EACH LECTURE ATTENDED. FOUR POINTS FOR ATTENDING AN ADDITIONAL STUDY SESSION

Normally I do not take the register at the beginning of a class, only in the first few weeks and just long enough to get to know my students by name. Since I have no formal attendance policy, some students take advantage of it and do not come as often as they should. I could not help but notice a few terms ago that when the sun came out the golfers were gone! I thought that by offering two marks for attending a normal class and four for attending an *Additional Study Session*, my more reluctant mathematicians would be persuaded to attend more frequently. I scaled the total attendance marks down to 50 out of 1000 (half a grade). I had to start taking attendance each time we met, which wasted time, and this approach did not achieve the objective anyway. The student's who always attended anyway got the marks, the ones who needed to attend still did not attend, and the golfers were gone as soon as the sun appeared. I have abandoned this *technique* forever!

19. BEAT THE CLOCK

I created sets of colored index cards, and at the beginning of a lecture gave one set to each student. Each set consisted of five cards, the colors were blue, white, red, orange, and green. Each color had a single letter on it A, B, C, D, or E. I then created a set of multiple choice questions on a transparency. Each question had five possible answers, A, B, C, D, or E. I covered all questions on the transparency except the first, and told the students to solve it, then hold up the card that represented the answer. I then indicated whether they were right or wrong and waited for other students to finish. I then uncovered the next question and so on. My students seemed to enjoy doing it, but it wasted a lot of time. For example, the first student finished just sat there waiting for others to finish. Except for small classes, where speed is very important, i.e. *Examination 100 of the Society for Actuaries,* where each question must be done in four minutes, I do not do this any more.

20. 5 MARK ATTENDENCE BONUS

This was another attempt to entice the less enthusiastic students to attend. I told my students that I would give five bonus marks (the total for the entire course is always 1000) for solving a problem that I would write on the board. However, I would not tell them when I was going to give out a problem and only students who were present would get the 5 marks (assuming their answer was correct). When I gave out a problem, I had to take the register, so that I would know who was eligible for the 5 marks.

Some typical questions are as follows :

1. The Lead in a Boat Problem

A **man** and a **piece of lead** are in a *boat* in the middle of a totally enclosed lake. On a partially submerged stick in the bank, there is a **mark** showing the *level of the lake* while the **man** and the **lead** are *in the boat*. The man then throws the **lead** into the lake (*no water enters or leaves the lake*). Does the level of the lake go **up**, **down**, or **stay the same**?

2. The Five Hat Problem

There were **three men** in a room. **One** was ***blind*** and the other **two** were ***blindfolded***. On a table in the room there were **five hats**, ***three* gold hats** and ***two* silver hats**. Each man picked a hat off the table and placed it on his own head. ***Clearly, no one knew if his own hat was silver or gold***. *The remaining two hats were removed from the room.*

The **first man** took off his *blindfold* and saw the hats the other two men were wearing but **not** his own. He said :

"I do not know if my hat is silver or gold"

He put his blindfold back on.

The **second man** took off his *blindfold* and saw the hats the other two men were wearing but **not** his own. He said :

"I do not know if my hat is silver or gold"

He put his blindfold back on.

The **blind man** then said :

"I know the color of my own hat"

He then **correctly** identified the color of his own hat.

Was the **blind mans' hat** ***silver*** or ***gold*** and how did he know it's color?

3. The Subtraction Problem

In the problem

$$\begin{array}{r} \mathbf{TWO} \\ -\,\underline{\mathbf{ONE}} \\ \underline{\mathbf{ONE}} \end{array}$$

each letter represents a different integer from **0** to **9**. For example, **T** could be **8**, **W** could be **6**, **E** could be **4** (in which case both **E's** would be **4**), but different letters cannot be the same integer in any one solution.

How ***many*** solutions are there?

4. The Watch Problem

According to the ***Greenwich Mean Time*** time signal, the hour and minute hand of my watch coincide every ***65 minutes exactly***. How long will my watch take to *gain* or *lose* an hour?

P.S. One good idea and you can almost solve this problem in your head!!

The *College Algebra students* found Questions 1, 2, and 3 fun to try. In fact, there is always one student in a group, who goes home, pours out a glass of water, finds a piece of wood, and a stone or sometimes a marble, and proceeds to model Question 1, getting the right answer. Although so far they have not been able to tell me the reason for the effect they saw. Question 4 is too difficult for a College Algebra student. I used it recently on my Differential Equations students. I particularly like this question because with one good idea, it really can be done in one's head!

I am unable to report success or failure at this point. It seems to vary with the class of students. However, it is in the failure section because things do look a little bleak! A few students may have been enticed to come along but it was not the rousing success I had hoped for. I will probably try it a few more times though.

3. COURSE METHODOLOGY, CONTENT, AND COST

In order to allow the *techniques* to be effective, it was necessary to modify my teaching style. I needed more time during a lecture to interact with my students more effectively. Using the textbook as a set of notes, (*Technique 11*) and making transparencies of graphs, solutions to questions from the textbook etc., (*Technique 12*), gave me the necessary additional time by reducing the amount of time I spent writing on the blackboard. I also use a laser pointer which means that I do not have to be close to the overhead projector or the blackboard. I now find that I spend far more time either facing my students or moving around among them while asking questions. This seems to make my students more relaxed and willing to talk. Using *colored chalk on the board,* (*Technique 6*), makes what I am doing on the board clearer. I can emphasize common errors, like a minus sign before a bracketed expression, using red chalk or a rule using yellow chalk etc.

All my lectures are now interactive discussions covering each section of the textbook. I use many transparencies of various parts of the textbook. Throughout every lecture I constantly ask questions about the material I am covering to keep my students involved. *Topical problems*, (*Technique 15*), seem to create interest and additional discussion. I tell my students that some of the data or ideas come from the Internet and sometimes students will go and search the Internet for themselves. The *Additional Study Sessions*, (*Technique 4*), also cause many students to be more relaxed. During or after one of these, they are quite happy to chat about topics other than Maths and this effect spills over into the normal lecture. *Group Projects,* (*Technique 8*), is certainly an effective way of getting students talking to each other. This, therefore, is a must for the *College Algebra course* that starts next term.

I realised fairly early on that one of the major problems was the fact that many students had the rules of Maths muddled up in their minds. This meant that no matter how hard they tried, they could never get the correct answer, this leads to frustration, anxiety, and sometimes just plain anger. The answer was to spend far more time on any rule we encountered, often using transparencies to do with the rules to give me the necessary time. Telling some of them that this was their problem seemed to help. Mathematics ceased to be that vague, incomprehensible subject that they feared. There was a reason why things went wrong, and, more important, they knew how to put it right. The *workbook,* (*Technique 2*), gives them the practice they need with the rules. It is also popular because, if something goes wrong, the bonus marks for the *workbook* can make up for a bad test or final. The *Chapter Notes,* (*Technique 1*), provide a list which can be used as a checklist.

The *one sheet of paper,* (*Technique 3*), *example tests,* (*Technique 5*), *using questions from the textbook,* (*Technique 13*), together with making sure that they were not competing against the clock during a real test, seems to have successfully reduced the worry of taking a Maths test. The *Additional Study Sessions,* (*Technique 4*), also seems to have helped with this as well. The possibility of *gaining bonus points on a test,* (*Technique 7*), is also attractive to many, since it reduces the number of marks needed for a good grade on the Final (every Final is worth 400 marks out of 1000 for the entire course). The *90% or more rule,* (*Technique 16*), creates hope for some and motivates many as well.

Providing *complete solutions to typical problems,* (*Technique 14*), has certainly improved the answers to the assignments and tests. It also shows students what I want, which in turn reduces anxiety. I now find my students demanding more solutions! Even demanding a *sentence for a numerical answer*, (*Technique 9*), which I wanted, because I disliked giving full marks to just a number, and creating a *special technique,* (*Technique 10*), which is fun for me, have both received some very positive comments.

The two plans, on the next two pages, make sure I complete everything on schedule. **Semester Plan 1** is for a course like the *Elementary Statistics course* that includes Group Projects. **Semester Plan 2** is for a course like the *College Algebra course* that does not include Group Projects, instead the College Algebra has Group Workbooks. I have found it helpful to cross out lines on a semester plan when it has been completed, but I do not want to ruin one, so I lay a blank transparency over the top of a plan and write on this. It saves me having to print additional plans.

After using these techniques for sometime now, I have found the atmosphere in the classroom is far more relaxed, students are certainly more confident and willing to talk, and while there is still much to be done, teaching a *College Algebra course* is certainly a far more pleasant experience than the first time I encountered it three years ago.

SEMESTER PLAN 1 : FOR COURSES WITH GROUP PROJECTS

1st WEEK
Handout and Introduce Syllabus
Handout Chapter Notes for Chapter 1 - Advise Other Chapter Notes in Library
Advise Students to Purchase Workbook (i.e. binder plus blank paper)
Show Examples of Past Workbooks, Group Projects, and Visuals
(Examples of other Visuals are on the wall outside my office)

2nd WEEK
Handout Project 1 on the Monday (Advise Due Date)

3rd WEEK
Handout Example Test 1 (or put in Library)
Assign Two Questions for Homework from Example Test 1
Start Other Questions on Example Test 1

4th WEEK
Remind Students that <u>ALL</u> Questions are Straight from the Book *
Advise Students to Start Preparing their One Sheet
Collect Homework from Example Test 1 on the Friday

5th WEEK
Return Marked Homework from Example Test 1 on the Monday
Complete Questions on Example Test 1 by Wednesday
Hold Additional Study Session During Free Period
Collect Write-up for Project 1 on Friday, Read and Prepare Questions for after the Oral
TEST 1 ON THE FRIDAY

6th WEEK
RETURN MARKED TEST 1 ON THE MONDAY
Hand Out Solutions and Marking Scheme to Test 1 (or put in Library)
Project 1 Orals - by One Group Member (**BUT** any Group Member can Answer My Questions)

7th WEEK
Handout Project 2 on the Monday (Advise Due Date)
Return Marked Project 1

8th WEEK
Handout Example Test 2 (or put in Library)
Assign Two Questions for Homework from Example Test 2
Start Other Questions on Example Test 2

9th WEEK
Remind Students that <u>ALL</u> Questions are Straight from the Book *
Advise Students to Start Preparing their One Sheet
Collect Homework from Example Test 2 on the Friday

10th WEEK
Return Marked Homework from Example Test 2 on the Monday
Complete Questions on Example Test 2 by Wednesday
Hold Additional Study Session During Free Period
Collect Write-up for Project 2 on Friday, Read and Prepare Questions for after the Oral
TEST 2 ON THE FRIDAY

11th WEEK
RETURN MARKED TEST 2 ON THE MONDAY
Hand Out Solutions and Marking Scheme to Test 2 (or put in Library)
Handout Project 3 on the Monday (Advise Due Date)
Project 2 Orals - by One Group Member (**BUT** any Group Member can Answer My Questions)

12th WEEK
Advise Students of 90% Rule (earlier if hope appears to be fading!)
Return Marked Project 2

13th WEEK
Handout Example Final (or put in Library)
Assign Two Questions for Homework from Example Final
Start Questions on Example Final

14th WEEK
Remind Students that <u>ALL</u> Questions are Straight from the Book *
Advise Students to Start Preparing their One Sheet
Collect Write-up for Project 3, Read and Prepare Questions for after the Oral
Collect Homework from Example Final on the Friday

15th WEEK
Return Marked Homework from Example Final on the Monday
Complete Questions on Example Final by Friday
Project 3 Orals - by One Group Member (**BUT** any Group Member can Answer My Questions)
Hold Additional Study Session During Free Period

16th WEEK
Return Marked Project 3 at the end of the FINAL
FINAL

> **Throughout the semester remind students which section they need to have completed to be up to date with their workbook.**

SEMESTER PLAN 2 : FOR COLLEGE ALGEBRA – NO GROUP PROJECTS

1st WEEK
Handout and Introduce Syllabus
Handout Chapter Notes for Chapter 1 - Advise Other Chapter Notes in Library
Advise Students to Purchase Workbook (i.e. binder plus blank paper)
Show Examples of Past Workbooks

2nd WEEK

3rd WEEK
Handout Example Test 1 (or put in Library)
Assign Two Questions for Homework from Example Test 1
Start Other Questions on Example Test 1

4th WEEK
Remind Students that <u>ALL</u> Questions are Straight from the Book *
Advise Students to Start Preparing their One Sheet
Collect Homework from Example Test 1 on the Friday

5th WEEK
Return Marked Homework from Example Test 1 on the Monday
Complete Questions on Example Test 1 by Wednesday
Hold Additional Study Session During Free Period
TEST 1 ON THE FRIDAY

6th WEEK
RETURN MARKED TEST 1 ON THE MONDAY
Hand Out Solutions and Marking Scheme to Test 1 (or put in Library)

7th WEEK

8th WEEK
Handout Example Test 2 (or put in Library)
Assign Two Questions for Homework from Example Test 2
Start Other Questions on Example Test 2

9th WEEK
Remind Students that <u>ALL</u> Questions are Straight from the Book *
Advise Students to Start Preparing their One Sheet
Collect Homework from Example Test 2 on the Friday

10th WEEK
Return Marked Homework from Example Test 2 on the Monday
Complete Questions on Example Test 2 by Wednesday
Hold Additional Study Session During Free Period
TEST 2 ON THE FRIDAY

11th WEEK
RETURN MARKED TEST 2 ON THE MONDAY
Hand Out Solutions and Marking Scheme to Test 2 (or put in Library)

12th WEEK
Advise Students of 90% Rule (earlier if hope appears to be fading!)

13th WEEK
Handout Example Final (or put in Library)
Assign Two Questions for Homework from Example Final
Start Questions on Example Final

14th WEEK
Remind Students that <u>ALL</u> Questions are Straight from the Book *
Advise Students to Start Preparing their One Sheet
Collect Homework from Example Final on the Friday

15th WEEK
Return Marked Homework from Example Final on the Monday
Complete Questions on Example Final by Friday
Hold Additional Study Session During Free Period

16th WEEK
FINAL

Throughout the semester remind students which section they need to have completed to be up to date with their workbook.

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3.1 COURSE CONTENT

Both the *College Algebra course* and the *Elementary Statistics Course* are typical. For completeness, the course content for each course is shown below as they appear within my syllabi.

3.1.1 M105 – COLLEGE ALGEBRA SYLLABUS

There are no prerequisites for this course and it assumes no prior knowledge of *Algebra*. We use the following textbook

College Algebra by Barnett and Ziegler
5th Edition, Publisher : McGraw-Hill

The syllabus is typical of a **College Algebra course**. The course covers the following topics :

Algebra and Real Numbers; Polynomials; Rational Expressions; Exponents; Radicals; Linear Equations; Linear Inequalities; Complex Numbers; Quadratic Equations; Polynomial and Rational Inequalities: Graphing Straight Lines and Circles; Functions; Rational Functions; Operations on Functions; Inverse Functions; Synthetic Division; Remainder and factor Theorems; Fundamental Theorem of Algebra; Zeros of a Polynomial; Partial Fractions; Exponential and Logarithmic Functions.

3.1.2 M205 – ELEMENTARY STATISTICS SYLLABUS

This course assumes no prior knowledge of *Statistics*. We use the following textbook.:

Introduction to the Practice of Statistics by *Moore and McCabe* - 2nd Edition,
Publisher : W. H. Freeman and Company.

The syllabus is typical of a first Statistics course. The course covers the following topics :

Displaying and Describing Data; The Normal Distribution; Scatterplots; Regression; Correlation; Design of Experiments; Sampling Design; Statistical Inference; Probability Models; Probability Laws; Discrete and Continuous Random Variables; Sample Counts and Sample Proportions; Sample Means; The Sampling Distribution of the Sample Mean; Inference - Quantitative Variables (σ **known**) : Confidence Intervals and Tests of Significance; Inference - Quantitative Variables (σ **unknown**) : Confidence Intervals and Tests of Significance; Inference for Count Data; Use of the **TI-83 Calculator** throughout course.

3.2 COST

The cost of implementing the *techniques* discussed in this proposal is very low. It involves additional paper used by students for copying the *example tests*, *chapter notes*, etc. and the cost of Overhead Transparency Slides.

I would estimate that the average cost, assuming a new textbook for both copying and slides, is less than **$25** per course. This is substantially reduced if there are multiple sections of the course per term and further reduced if the textbook is retained in later terms. In fact my department does not pay the copying costs anyway. So for us copying the *example tests*, *chapter notes*, etc. is not a problem.

There is no additional equipment required beyond an overhead projector. which is usually in every classroom anyway. Students purchase the **TI-83** calculators individually, and the projector can be obtained free from **Texas Instruments** once the department requires it to be used for a course. I do not use it in my *College Algebra courses* at the moment. Although not essential. I found a laser pointer very useful. Laser pointers are now very cheap often less than **$20** each.

4. EVALUATION ANALYSIS

The *short form* produced some surprising results. I thought beforehand that students would prefer those *techniques* that gave them marks. However, this was not the case.

Table 1 (next page) has the results from the seven additional questions I inserted into the **IDEA form**. The table shows the average mark on a **five-point scale** per class. The averages for each class are then totaled. The *Algebra* and *Statistics* totals are kept separate for comparison purposes. The maximum total per question is ten. The number of the question is shown across the top of the table. The order of preference is shown below. I was very surprised to find that the preferences for both courses were ***identical***.

1 Question 44 - I found that being able to take a sheet of paper into every test very helpful.
2 Question 42 - I found the Example Tests very helpful.
3 Question 46 - Being able to obtain bonus marks on any test made me think about how to obtain them during a test.
4 Question 40 - I found completing the workbook very helpful.
5 Question 41 - I found that the instructors use of colored chalk on the blackboard helped make the work on the blackboard clearer
6 Question 45 - The chapter notes were very helpful
7 Question 43 - I found the additional study session, held usually during the free period, very helpful.

Table 2 (page after next) has the results from the eight *techniques* I selected to be tested. The table shows the average mark on a **seven-point scale** per class. The averages for each class are then totaled. The Algebra and Statistics totals are kept separate for comparison purposes. The maximum total per *technique* is fourteen. The number of the *technique* is shown across the top of the table. It is the ***lowest*** total that is best in this case. The results were as follows :

ALGEBRA

1. Technique 2. - Workbook
2. Technique 5. - Example Test before Each Real Test
3. Technique 3. - Take One Sheet of Paper into a Test
4. Technique 7. - Graded Bonuses on All Tests
5. Technique 1. - Chapter Notes
6. Technique 4. - Additional Study Hour During Free Period
7. Technique 6. - Colored Chalk on Blackboard
8. Technique 8. - NOT APPLICABLE FOR ALGEBRA COURSES

STATISTICS

1. Technique 5. - Example Test before Each Real Test
2. Technique 2. - Workbook
3. Technique 3. - Take One Sheet of Paper into a Test
4. Technique 7. - Graded Bonuses on All Tests
5. Technique 1. - Chapter Notes
6. Technique 8. - Group Projects and Working in Groups
7. Technique 6. - Colored Chalk on Blackboard
8. Technique 4. - Additional Study Hour During Free Period

There were no Group Projects in either of the two Algebra courses. Group Projects came in sixth for the Statistics course, which pushed *Techniques 6* and *4* down one slot. Otherwise with only the top two and the bottom two reversed, the results are remarkably similar. I realized after carrying out the evaluations that ranking may not be the best way of evaluating these *techniques* because a *technique* that students found very helpful, may be forced towards the bottom simply because others are preferred. The comments I have received about using colored chalk on the blackboard have made it clear that this *technique* should be continued. However, it came in fifth on the questions and seventh as a *technique*. The new long form lets students give a number to a question or *technique*. My initial observations place using colored chalk as high as the *workbook* or *one sheet*. Overall the long form should give me a much better idea of my student's preferences.

TABLE 1 : SEVEN EXTRA QUESTIONS ON IDEA FORM

These questions were answered using numbers 40 to 46 on the IDEA form.

40. I found completing the workbook very helpful.
41. I found that the instructors use of colored chalk on the blackboard helped make the work on the blackboard clearer.
42. I found the Example Tests very helpful.
43. I found the additional study session, held usually during the free period, very helpful.
44. I found that being able to take a sheet of paper into every test very helpful.
45. The chapter notes in the library very helpful.
46. Being able to obtain bonus marks on any test made me think about how to obtain them during a test.

1 DEFINITELY FALSE 2 MORE FALSE THAN TRUE 3 IN BETWEEN 4 MORE TRUE THAN FALSE 5 DEFINITELY TRUE

QUESTION #	40	41	42	43	44	45	46
ALGEBRA F96	4.30	4.30	4.60	3.33	4.80	3.90	4.50
ALGEBRA F96	4.83	4.67	5.00	4.50	4.83	4.50	4.67
TOTAL	9.13	8.97	9.60	7.83	9.63	8.40	9.17
POSITION	4	5	2	7	1	6	3
STATISTICS S97	4.55	4.09	4.55	3.27	4.91	3.36	4.36
STATISTICS S97	4.19	4.06	4.63	2.87	4.81	4.44	4.75
TOTAL	8.73	8.15	9.17	6.14	9.72	7.80	9.11
POSITION	4	5	2	7	1	6	3

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TABLE 2 : RESULTS FOR THE EIGHT TECHNIQUES EVALUATED

TECHNIQUES

1. CHAPTER NOTES IN LIBRARY
2. WORKBOOK
3. TAKE ONE SHEET OF PAPER INTO A TEST
4. ADDITIONAL STUDY HOUR DURING FREE PERIOD
5. EXAMPLE TEST BEFORE EACH REAL TEST
6. COLORED CHALK ON BLACKBOARD
7. GRADED BONUSES ON ALL TESTS
8. GROUP PROJECTS AND WORKING IN GROUPS

TECHNIQUE #	1	2	3	4	5	6	7	8
ALGEBRA F96	5.83	2.10	2.80	5.30	2.70	4.90	4.00	NA
ALGEBRA F96	4.57	1.71	4.57	5.52	3.24	7.05	5.33	NA
TOTAL	10.40	3.81	7.37	10.82	5.94	11.95	9.33	NA
POSITION	5	1	3	6	2	7	4	
STATISTICS S97	5.27	2.64	2.82	6.45	2.91	5.82	4.18	5.73
STATISTICS S97	4.19	3.05	3.00	7.38	2.13	6.50	4.00	5.19
TOTAL	9.46	5.70	5.82	13.83	5.03	12.32	8.18	10.91
POSITION	5	2	3	8	1	7	4	6

5. CONCLUSION

One of the objectives of this work was to increase the confidence of my students in their mathematical ability and give them a positive attitude towards the course that would last throughout the course. Using these *techniques* has substantially increased their overall confidence in themselves. The atmosphere in the classroom is far more relaxed and pleasant. Students do not seem to feel as threatened by the subject or the Maths tests as they were the first time I taught *College Algebra*. The *workbook* does give my students the idea that if something goes badly during the course, there is a way for them to redeem themselves.

These *techniques* have helped me as well because I enjoy creating *special techniques* for those ideas that cause my students difficulties. This increases my interest and enthusiasm even for topics I have taught hundreds of times before. I have noticed a substantial difference in the attitude of my students to the course and this has made teaching them far more pleasant. This is particularly good for me.

Writing constantly on the blackboard with your back to the class causing "*teacher isolation*". It builds a wall between the teacher and the class. If your students can only recognize you from the back – it is time to change your teaching style! Using *techniques* 11 and 12 gives me more time to interact with my class while, much of the time, facing them. When I am discussing a transparency, using a laser pen allows me to wander around the room among my students asking questions. I become a person not someone out the front behind an invisible wall.

It is my students who ultimately judge whether these *techniques* are successful or not. During the *fall 1997 term*, I used these *techniques* in my *M210 - Maths for Management course*. This was the first time for this course. The entire class signed up for my *M205 - Elementary Statistics course* for the *spring 1998 term*. My *fall 1999 Elementary Statistics course* was full as of *April 14th, 1999*. The students are voting for these *techniques* with their feet – they sign up for my courses! However, the job is by no means complete. I am still collecting more data with the *long form* (in **Appendix B**), refining some of the *techniques* currently used, and trying more new ideas. Overall the average student grade is up by around half a point on a four-point scale even though the questions on my tests are more demanding than they used to be. Although less important in my view, I have noticed that my own average *Good Instructor* mark is also up by about half a point on a five-point scale. While there is still much to do, I am very encouraged by the results.

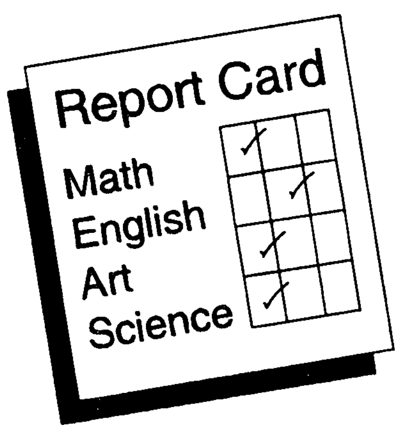
I have long held the belief that at the present time teachers are islands. We do not learn from the mistakes of our predecessors and we should. When I started teaching, I completed a teacher-training course, like so many others. However, it did not prepare me, in any way, for teaching mathematics to students who started out on the course fearful of it. When I walked into the classroom for the first time years ago, I had only my observations as a student to fall back on. I did the best I could, making, I am sure, all the same mistakes my predecessors had made before me. This is not the way it should be. As teachers we should be learning from each other, so that a new teacher has at his/her disposal the combined experience of other mathematics teachers who have gone before them. I am hoping that the work I am doing now and my future plans will stop a new teacher making some of the same mistakes that I made when I first started.

As far as my students are concerned, I realized, from the beginning of this work, that I would never be able to pass on my love of Maths to them. However, a comment like "***I hate Math***, but this course was not as bad as I thought", with the first three words said with some feeling, is certainly more common now, and is probably about as good as it is going to get!

APPENDIX A

THE

SHORT FORM



ADDITIONAL INFORMATION REQUEST

PART 1

To be completed on numbers 40 to 46 on the IDEA form.

40. I found completing the workbook very helpful.

41. I found that the instructors use of colored chalk on the blackboard helped make the work on the blackboard clearer.

42. I found the Example Tests very helpful.

43. I found the additional study session, held usually during the free period, very helpful.

44. I found that being able to take a sheet of paper into every test very helpful.

45. The chapter notes in the library very helpful.

46. Being able to obtain bonus marks on any test made me think about how to obtain them during a test.

PART 2

To be completed in the blank box on the IDEA form.

1. Chapter Notes in the Library
2. Workbook
3. One Sheet into a test
4. Additional Study Hours
5. Example Tests
6. Colored Chalk
7. Bonuses on Tests
8. Group Projects

Example

If you think the **Workbook** was the most help, followed by the **Example Tests**, and then your other choices in order, write :

2, 5, 1, 7, 8, 4, 3, 6

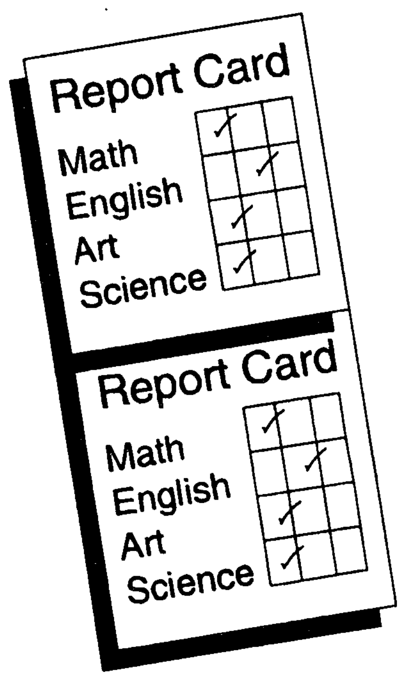
If you think the **Workbook** and the **Example Tests** were ***EQUALLY*** helpful, and the **Additional Study Hours** and the **Group Projects** were ***EQUALLY*** helpful, ***circle them*** (either way round) followed by your other choices in order, write :

2, 5, 1, 7, 8, 4, 3, 6

APPENDIX B

THE

LONG FORM



ADDITIONAL INFORMATION REQUEST

COURSE NUMBER : ____________________

PART 1

0. *NOT APPLICABLE* 1. NO HELP AT ALL 2. A LITTLE HELPFUL
3. HELPFUL 4. VERY HELPFUL 5. INDISPENSABLE

1. Chapter Notes ☐
2. Using Colored Chalk on the Blackboard ☐
3. Workbook ☐
4. Additional Study Session during the Free Period ☐
5. Example Tests ☐
6. Taking One Sheet of Paper into a Test ☐
7. Being able to Obtain Bonuses on a Test ☐
8. Complete Example Questions ☐
9. Test Questions Straight from the Textbook ☐
10. Putting a Numerical Answer into a Sentence ☐
11. 90% or More Rule ☐
12. Group Projects or Working in Groups ☐

PART 2

0. *NOT APPLICABLE*	1. DEFINITELY FALSE	2. MORE FALSE THAN TRUE
3. IN BETWEEN	4. MORE TRUE THAN FALSE	5. DEFINITELY TRUE

1. I found the Chapter Notes very helpful.
2. I found that the instructors use of colored chalk on the blackboard helped make the work on the blackboard clearer.
3. In courses I take I would like my instructor to use colored chalk.
4. I found completing the workbook very helpful.
5. I found the additional study session, held usually during the free period, very helpful.
6. I found the Example Tests very helpful.
7. I found that being able to take a sheet of paper into every test very helpful.
8. Being able to obtain bonus marks on any test made me think about how to obtain them during a test.
9. I liked having all the questions on tests straight from the textbook.
10. The fact that all the questions on tests were straight from the textbook made me complete more questions from the textbook.
11. I enjoyed this course

11. Overall, I rate this INSTRUCTOR an excellent teacher.

12. I am an athelete CIRCLE ONE OF THE FOLLOWING : **YES NO**

PART 3

1. IS THERE ANYTHING I COULD HAVE DONE FOR YOU OR GIVEN YOU THAT WOULD HAVE HELPED YOU MORE? IF SO, WHAT?

2. IS THERE ANYTHING (OR THINGS) THAT YOU DID NOT LIKE ABOUT THIS COURSE?

3. IS THERE ANYTHING (OR THINGS) THAT YOU LIKED ABOUT THIS COURSE?

4. CAN YOU THINK OF ANY WAY OF IMPROVING THIS COURSE?

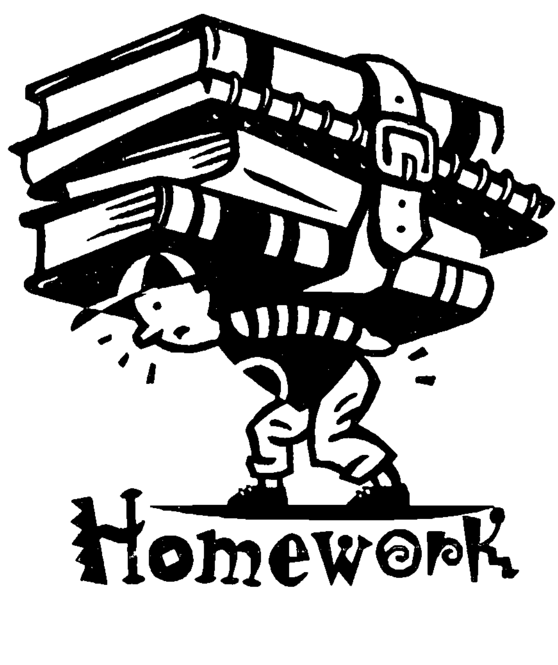
5. DID WORKING IN GROUPS HELP YOU, IF SO HOW OR IF NOT WHY NOT?

APPENDIX C

CHAPTER NOTES

WITH

WORKBOOK PROBLEMS



SECTION 3.1 - Basic Tools : Circles

- **Cartesian Coordinate System – P168**

    Real Plane - P168
    Cartesian or Rectangular Coordinate System - P168
    Horizontal Axis - P168
    Vertical Axis - P168
    Coordinate Axes = Horizontal Axis + Vertical Axis - P168
    x-axis = Horizontal Axis - P168
    y-axis = Vertical Axis - P168
    Quadrants : the Four Parts of the Real Plane - P169
    Coordinates : (a, b) form the Coordinates of P - P169
    Ordered Pair : Example is (a, b) - P169
    Abscissa : the First Coordinate in (a, b) i.e. a - P169
    Ordinate : the Second Coordinate in (a, b) i.e. b - P169
    x coordinate : the First Coordinate in (a, b) i.e. a - P169
    y coordinate : the Second Coordinate in (a, b) i.e. b - P169
    Origin : the Point with Coordinates (0, 0) - P169
    Fundamental Theorem of Analytical Geometry - P169
        There is a one-to-one correspondence between the points in a plane and the elements in the set of all ordered pairs of real numbers.

- **Graphing : Point by Point – P169**

    Solution - P169
    Solution Set - P169
    Plot Real Solutions of an Equation Only - P169
    Graph of an Equation in Two Variables - P170
        This is the Graph of its Solution Set - P170

    DO Example 1 : Graphing an Equation using Point-by-Point Plotting - P170
    DO Problem 1 - P170

- **Symmetry – P170**

    Definition 1 : *Symmetry* - P171
    Theorem 1 : *Tests for Symmetry* - P171

    DO Example 2 : Using Symmetry as an Aid to Graphing - P171
    DO Problem 2 - P175

- **Distance Between Two Points – P175**

    Theorem 1 : *Distance between* $P_1(x_1, y_1)$ *and* $P_2(x_2, y_2)$ - P175

    DO Example 3 : Using the Distance-Between-Two-Points Formula - P175
    DO Problem 3 - P176

- **Circles – P176**
  - **Definition 2** : *Circle* - P176
  - **Theorem 3** : *Standard Equation of a Circle* - P176

  DO Example 4 : Equations an Graphs of Circles - P177
  DO Problem 4 - P177
  DO Example 5 : Finding the Center and Radius of a Circle - P177
  DO Problem 5 - P178

QUESTIONS FOR WORKBOOK (P179) - 1, 3, 5, 9, 13, 17, 21, 23, 25, 47, 73

SECTION 3.2 - Straight Lines

- **Graphs of First-Degree Equations in Two Variables – P181**
  Theorem 1 : *The equation of a Straight Line* - P182
  Standard Form : Ax + By = C; A and B are constants, not both zero

  DO Example 1 : Using Intercepts to Graph a Straight Line - P182
  DO Problem 1 - P182

- **Slope of a Line – P183**
  Definition 1 : *Slope of a Line* - P183
  Table 1 : Geometric Interpretation of Slope - P183

  DO Example 2 : Finding Slopes - P184
  DO Problem 2 - P185

- **Equations of a Line - Special Forms – P185**
  Theorem 2 : *Slope-Intercept Form* - P185

  DO Example 3 : Using the Slope-Intercept Form - P185
  DO Problem 3 - P186

  Theorem 3 : *Point-Slope Form* - P186

  DO Example 4 : Using the Point-Slope Form - P187
  DO Problem 4 - P187
  DO Example 5 : Business Markup Policy - P188
  DO Problem 5 - P188

  Theorem 4 : *Vertical an Horizontal Lines* - P189

  DO Example 6 : Graphing Horizontal and Vertical Lines - P189
  DO Problem 6 - P189

  Table 2 : Equations of a Line - P189

- **Parallel and Perpendicular Lines – P190**
  Theorem 5 : *Parallel and Perpendicular Lines* - P190

  DO Example 7 : Parallel and Perpendicular Lines - P190
  DO Problem 7 - P191

QUESTIONS FOR WORKBOOK (P192) - 1, 3, 5, 7, 11, 21, 25, 31, 39, 43, 45, 81

SECTION 3.3 - Functions

- **Definition of a Function - P196**
  Table 1 - P196
  Table 2 - P196
  Table 3 - P196
  Definition 1 : *Rule Form of the Definition of a Function* - P196
  Definition 2 : *Set Form of the Definition of a Function* - P197

  DO Example 1 : Functions Defined as Sets of Ordered Pairs - P197
  DO Problem 1 - P197

- **Functions Defined by Equations – P198**
  Domain : x values, See Figure 2, P202 (Set of Real Numbers) - P198
  This can be written using Set Notation, Interval Notation, or Inequality Notation
  Range : y values, See Figure 2, P202 (Set of Real Numbers) - P198
  This can be written using Set Notation, Interval Notation, or Inequality Notation
  Independent Variable : x - P198
  Dependent Variable : y - P198
  Functions Defined by Equations - P198

  DO Example 2 Determining if an Equation Defines a Function - P199
  DO Problem 2 - P199

  Theorem 1 : *Vertical Line Test for a Function* - P200
  Agreement on Functions and Ranges - P200

  DO Example 3 Finding the Domain of a Function - P201
  DO Problem 3 - P201

- **Function Notation – P201**
  Rule of Correspondence - P201
  Set of Ordered Pairs - P201
  Function Notation - P201
  Definition 3 : *The Symbol f(x)* - P202

  DO Example 4 : Evaluating Functions - P202
  DO Problem 4 - P203
  DO Example 5 : Finding Domains of Functions - P203

DO Problem 5 - P204

Difference Quotient - P204

DO Example 6 : Evaluating and Simplifying a Difference Quotient - P204
DO Problem 6 - P205

NOTE THE CAUTION - P205

- **Applications – P205**

  DO Example 7 : Construction - P205
  DO Problem 7 - P206

- **A Brief History of Function Notation – P206**

  OMIT

QUESTIONS FOR WORKBOOK (P206) - 1, 3, 5, 7, 11, 13, 15, 17, 19, 25, 31, 45, 49, 55, 73, 81

SECTION 3.4 - Graphing Functions

- **Basic Concepts – P211**
  Graph of the Function f = Graph of the Equation y = f(x) - P211
  x intercept - P211
  y intercept - P211
  Definition 1 : *Increasing, Decreasing, and Constant Functions* - P212

- **Linear Functions – P212**
  Definition 2 : *Linear Function* - P212
  Graph of f(x) = mx + b, m ... 0 - P212
  Constant Function : f(x) = b - P213

  DO Example 1 Graphing a Linear Function - P213
  DO Problem 1 - P213

- **Quadratic Functions – P213**
  Quadratic Function : $f(x) = ax^2 + bx + c$; a ... 0 - P214
  The Domain of a Quadratic Function is the Set of ALL Real Numbers
  Table 1 : Values of a Quadratic Function - P215
  Properties of $f(x) = ax^2 + bx + c$; a ... 0, and its Graph - P214

  DO Example 2 : Graph of a Quadratic Function - P217

DO Problem 2 - P218

- **Piece-Wise Defined Functions – P218**

  Piece-Wise Defined Functions - P218

  Functions whose Definitions involve more than one Formula

  Example : the Absolute Value Function - P218

  DO Example 3 : Rental Charges - P218
  DO Problem 3 - P219
  DO Example 4 : Graphing a Function Involving Absolute Value - P219

  Continuous – P219
  Discontinuous - P220

  DO Problem 4 - P220

- **The Greatest Integer Function – P220**

  <u>OMIT</u>

QUESTIONS FOR WORKBOOK (P223) - 1, 3, 5, 7, 9, 11, 13, 15, 19, 23, 27, 39, 45, 75

<u>SECTION 3.5 - Aids to Graphing Functions</u>

- **Graphs of Basic Functions – P227**

  Definition 1 : *Even and Odd Functions* - P228
  Theorem 1 : *Tests for Even and Odd Functions* - P229

  DO Example 1 : Testing for Even and Odd Functions - P229
  DO Problem 1 - P230

- **Graphing Aids - P230**

  DO Example 2 : Vertical Shift - P230

  Theorem 2 : *Vertical Shifting (Translation)* - P231

  DO Problem 2 - P230

  DO Example 3 : Horizontal Shifts - P231

  Theorem 3 : *Horizontal Shifting (Translation)* - P232

  ## <u>NOTE THE CAUTION - P232</u>

  DO Problem 3 - P232

DO Example 4 : Reflections, Expansions, and Contractions - P232

Theorem 4 : *Reflections, Expansions, and Contractions* - P233

DO Problem 4 - P233

DO Example 5 : Combining Graphing Aids - P234
DO Problem 5 - P235

QUESTIONS FOR WORKBOOK (P236) - 1, 3, 5, 7, 9, 11, 13, 15, 19, 21, 23, 39, 43, 49, 73, 75

SECTION 3.6 - Rational Functions

OMIT

SECTION 3.7 - Operations on Functions ; Composition

OMIT

SECTION 3.8 - Inverse Functions

OMIT

SUMMARY OF CHAPTER 6 from Mike Bankhead

INTRODUCTION

The purpose of statistical inference is to draw conclusions from the data where the data is in the form of a sample. When you use statistical inference you are acting as if the data came from a random sample or a randomized experiment. If this is ***not*** true, your conclusion could be challenged.

Section 6.1 introduces ***confidence intervals*** and Section 6.2 introduces ***tests of significance*** (**ToS**). Both of these are types of inference which are based on the ***sampling distribution of a statistic***.

6.1 ESTIMATING WITH CONFIDENCE

The purpose of this section is to introduce the term **confidence interval**. We use the mean of the sample, $\bar{x}$, to estimate the value of a **population mean** together with an *indication of its accuracy.*

DO Example 6.1 - P435

Notes for Example 6.1

1. μ = **461** because $\bar{x}$ is the ***natural estimator*** of the **unknown population mean** μ.
2. An estimate ***without*** an indication of its variability is of little value.

- ### Statistical Confidence : P435

  3 bulleted points – P435

  DISCUSS Figure 6.1 - P436

  2 numbered points – P437

- ### Confidence Intervals : P437

  estimate $\pm$ **margin of error** – P437

  Margin of Error : P437

  DISCUSS Figure 6.2 - P438

  2 bulleted points – P438

Confidence Level - P438

A *confidence level* gives the probability that the method produces an interval that contains the **parameter**. The letter **C** is used to represent the confidence level. The most common values for **C** are **0.9**, **0.95**, and **0.99** these correspond to a 90% confidence interval, 95% confidence interval, and a 99% confidence interval respectively.

[Confidence Interval] : (*Formal definition*) - P438

The definition of **confidence interval** involves the following variables :

1. **C** is the **confidence level** expressed as a decimal;
2. θ is the general *unknown* **parameter**.

A **confidence interval** has **two parts** (P438) :

1. An **interval** - the **interval** is computed from the data i.e. the sample;
2. A **confidence level** - this gives the probability that the method produces an interval that includes the **parameter**.

- ## **Confidence Interval for a Population Mean** – P439

Critical Value - P434

The number **z*** is called the **upper p critical value** of the **standard normal distribution**. Later you will see that the **test statistic** is compared to the **critical value**.

DISCUSS Figure 6.3 - P439

[Confidence Interval for a Population Mean] – P440

DO Example 6.2 - P440

STUDENTS DO Exercise 6.1 – P447

DO Example 6.3 - P441

- ## **How Confidence Intervals Behave – P441**

DISCUSS Figure 6.4 - P441

3 bulleted points – P442

DO Example 6.4 - P442

STUDENTS DO Exercise 6.4 – P448

- ## **Choosing a Sample Size - P438**

[Sample Size for Desired Margin of Error] - P443

DO Example 6.5 - P443

STUDENTS DO Exercise 6.14 - P450

- **Some Cautions - P444**

  <u>**Students Read**</u>

  1. The data must be an **SRS** from the population.
  2. The formula is incorrect for *multistage* or *stratified samples* **i.e** if the probability sampling design is more complex than an **SRS**.
  3. If the **bias** is of unknown size, there is **no** correct method for **inference**.
  4. The mean is not a **resistant measure of center**, so **outliers** ***can*** have a *significant effect* on the **confidence interval**.
  5. The true **confidence level** will not be **1 -** α if the sample size is small and the population is not **normal**.

     NOTE - The interval relies on only on the distribution of $\bar{x}$ and not the individual observations. Even for small sample sizes (***n* > *15***) $\bar{x}$ is much closer to normal than the individual observations.
  6. **YOU MUST KNOW THE STANDARD DEVIATION** σ **OF THE POPULATION.** (This means that in practice this approach is not useful).

- **Beyond the Basics → The Bootstrap – P445**
  OMIT

PROBLEMS FOR THE QUESTION BOOK - 2, 3, 5, 7, 10, 22, 23, 24

<u>6.2 TESTS OF SIGNIFICANCE</u>

The purpose of this section is to introduce **tests of significance**. They are used to assess the evidence provided by the data in **favor** of some claim about the **population**.

DO Example 6.6 P453

2 bulleted points – P453 - 454

DO Example 6.7 P454

- **Stating Hypotheses**

  DISCUSS Figure 6.4 - P454

  [Null Hypothesis] – P455

STUDENTS DO Example 6.26 P468
STUDENTS DO Example 6.27 P469
STUDENTS DO Example 6.28 P469

Alternative Hypothesis – P455
One-Sided and Two-Sided Alternatives – P455

- **Test Statistics – P456**

2 bulleted points – P456

DO Example 6.8 P456

Test Statistic – P456

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

DO Example 6.8 - P456

- **P-Values – P457**

[P-Value] – P458

DO Example 6.10 - P458

- **Statistical Significance – P458**

Significance Level – P458
[Statistical Significance] – P458
4 bulleted points – P459

- **Test for a Population Mean – P460**

[z Test for a Population Mean] – P461

DO Example 6.11 P462
DO Example 6.12 P463

- **Two-Sided Significance Tests and Confidence Intervals – P463**

DO Example 6.13 - P464
DO Example 6.14 - P465

[Confidence Intervals and Two-Sided Tests] – P466

- **P-Values versus Fixed α**

  DO Example 6.15 - P466

  critical value – P466

  DO Example 6.16 - P466

PROBLEMS FOR THE QUESTION BOOK - 34, 36, 40, 43, 46, 49

6.3 USE AND ABUSE OF TESTS

STUDENTS READ

PROBLEMS FOR THE QUESTION BOOK – 53, 54, 58, 59

6.4 POWER AND INFERENCE AS A DECISION

- **Power**

  [Power] - P484

  DO Example 6.17 - P484

  Two Sentences on Power - P485

- **Two Types of Error – P488**

  [Type I and Type II Errors] - P489

  DO Example 6.20 - P490

  [Significance and Type I Error] - P491

  Power and Type II Error - P491

PROBLEMS FOR THE QUESTION BOOK – NONE FROM BOOK

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