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ABSTRACT

The National Assessment of Educational Progress (NAEP) is mandated by the United States Congress to survey the educational accomplishments of U.S. students and monitor changes in those accomplishments. For more than 25 years, NAEP has assessed the educational achievement of 4th-, 8th-, and 12th-grade students in selected subject areas, making it the only nationally representative and continuing assessment of what U.S. students know and can do. NAEP assessments are based on content frameworks and specifications developed through a national consensus process involving teachers, curriculum experts, parents, and members of the general public. The frameworks are designed to reflect a balance among the emphases suggested by current instructional efforts, curriculum reform, contemporary research, and desirable levels of achievement. In 1996, NAEP assessed the abilities of students in grades 4, 8, and 12 in the subjects of mathematics and science. The first release of results from the mathematics assessment appeared in the "NAEP 1996 Mathematics Report Card", a report designed to provide policymakers and the public with a broad view of student achievement. The current report, which provides a more detailed perspective on mathematics achievement and practices in 1996, is primarily for teachers, curriculum specialists, and school administrators. To illustrate what students know and can do, the report presents examples of student work in five different content strands of mathematics. Information on current instruction in mathematics classes, as reported by students and teachers, is also included. This report presents three types of information derived from the NAEP 1996 mathematics assessment: (1) information on what students know and can do in mathematics; (2) information on course-taking patterns and current classroom practices in this subject area; and (3) information on student attitudes toward mathematics. The first portion of this information is derived from an analysis of student performance on the actual assessment exercises. The latter two portions draw upon the questionnaires completed by the students who participated in the assessment and their mathematics teachers. The chapters on student work are organized around the five content strands

assessed by NAEP: (1) Number Sense, Properties, and Operations; (2) Measurement; (3) Geometry and Spatial Sense; (4) Data Analysis, Statistics, and Probability; and (5) Algebra and Functions. Discussion within these chapters also highlights students' proficiency on a number of cognitive skills that cut across the different content areas. These include conceptual understanding, procedural knowledge, and problem solving, as well as the ability to reason in mathematical situations, to communicate perceptions and conclusions drawn from a mathematical context, and to connect the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation. (ASK)

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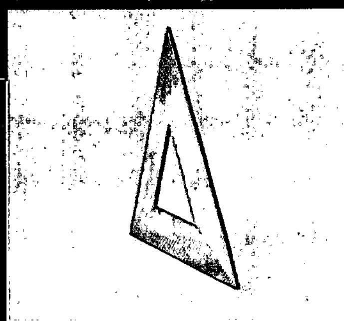

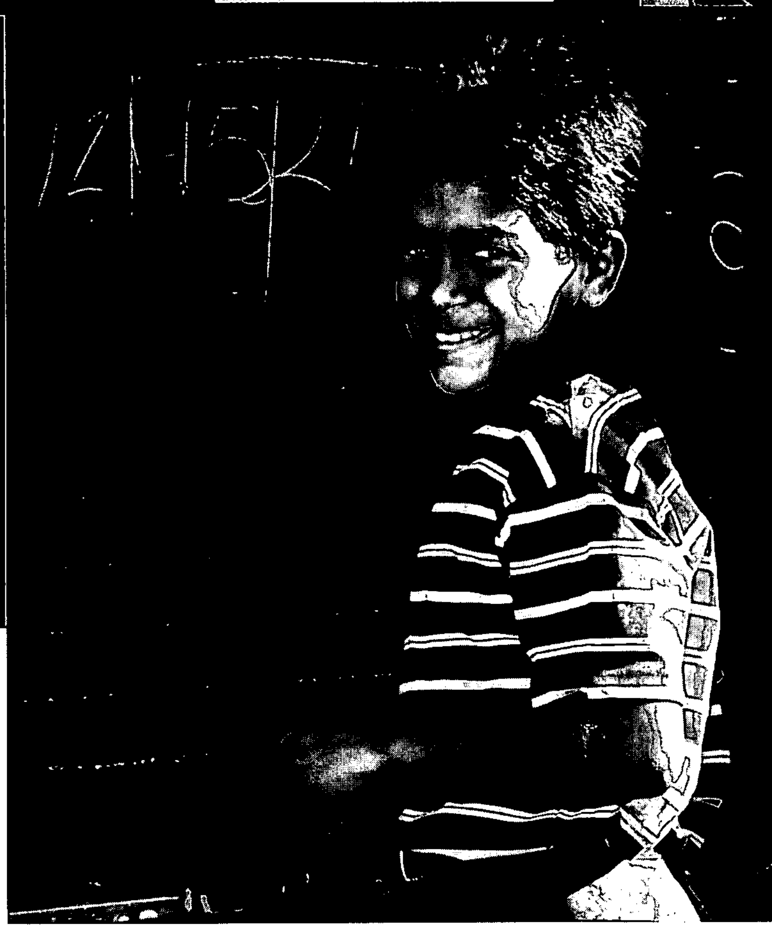

THE NATION'S REPORT CARD

U.S. Department of Education
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NCES 1999-453

What is The Nation's Report Card?

THE NATION'S REPORT CARD, the National Assessment of Educational Progress (NAEP), is the only nationally representative and continuing assessment of what America's students know and can do in various subject areas. Since 1969, assessments have been conducted periodically in reading, mathematics, science, writing, history, geography, and other fields. By making objective information on student performance available to policymakers at the national, state, and local levels, NAEP is an integral part of our nation's evaluation of the condition and progress of education. Only information related to academic achievement is collected under this program. NAEP guarantees the privacy of individual students and their families.

NAEP is a congressionally mandated project of the National Center for Education Statistics, the U.S. Department of Education. The Commissioner of Education Statistics is responsible, by law, for carrying out the NAEP project through competitive awards to qualified organizations. NAEP reports directly to the Commissioner, who is also responsible for providing continuing reviews, including validation studies and solicitation of public comment, on NAEP's conduct and usefulness.

In 1988, Congress established the National Assessment Governing Board (NAGB) to formulate policy guidelines for NAEP. The Board is responsible for selecting the subject areas to be assessed from among those included in the National Education Goals; for setting appropriate student performance levels; for developing assessment objectives and test specifications through a national consensus approach; for designing the assessment methodology; for developing guidelines for reporting and disseminating NAEP results; for developing standards and procedures for interstate, regional, and national comparisons; for determining the appropriateness of test items and ensuring they are free from bias; and for taking actions to improve the form and use of the National Assessment.

The National Assessment Governing Board

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Student Work and Teacher Practices in Mathematics

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Chapter 1

Introduction

The National Assessment of Educational Progress (NAEP) is mandated by the United States Congress to survey the educational accomplishments of U.S. students and monitor changes in those accomplishments. For more than 25 years, NAEP has assessed the educational achievement of fourth-, eighth-, and twelfth-grade students in selected subject areas, making it the only nationally representative and continuing assessment of what U.S. students know and can do. NAEP assessments are based on content frameworks and specifications developed through a national consensus process involving teachers, curriculum experts, parents, and members of the general public. The frameworks are designed to reflect a balance among the emphases suggested by current instructional efforts, curriculum reform, contemporary research, and desirable levels of achievement.

Purpose and Audience for the Report

In 1996, NAEP assessed the abilities of students at grades 4, 8, and 12 in the subjects of mathematics and science. The first release of results from the mathematics assessment appeared in the *NAEP 1996 Mathematics Report Card*,[1] a report designed to provide policy makers and the public with a broad view of student achievement.

The current report, which provides a more detailed perspective on mathematics achievement and practices in 1996, is primarily for teachers, curriculum specialists, and school administrators. To illustrate what students know and can do, the report presents examples of student work in five different content strands of mathematics. Information on current instruction in mathematics classes, as reported by students and teachers, also is included.

A companion report, *School Policies and Practices Affecting Instruction in Mathematics*,[2] provides information on school policies and other practices affecting mathematics education.

[1] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states.* Washington, DC: National Center for Education Statistics.

[2] Hawkins, E. F., Stancavage, F., & Dossey, J. A. (1998). *School policies and practices affecting instruction in mathematics.* Washington, DC: National Center for Education Statistics.

1996 Mathematics Framework

The design of the NAEP 1996 mathematics assessment was guided by a framework that was closely aligned with the frameworks used in 1990 and 1992.[3] This framework was influenced by the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*[4] and was updated prior to use in 1996 to better reflect contemporary curricular emphases and objectives. However, a connection with the 1990 and 1992 assessments was maintained in order to measure trends in student performance.

Content strands

The framework for the 1996 mathematics assessment included a broad content domain consisting of five content strands: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. For descriptions of the content covered in these strands, see Chapters 3–7, which describe student performance in each content strand. Table 1.1 presents the percentage distribution of questions prescribed by the framework across the content strands for each grade level and shows the changes from 1990 and 1992 to 1996. Separate subscales were produced for the five content strands that summarize the results for each strand. Questions that tap content from more than one strand were grouped according to their primary content classification.

Table 1.1 Percentage Distribution of Questions by Content Strand and Grade

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	Grade 4			Grade 8			Grade 12		
Content Area	1990	1992	1996	1990	1992	1996	1990	1992	1996
Number Sense, Properties, & Operations[a]	45	45	40	30	30	25	25	25	20
Measurement	20	20	20	15	15	15	15	15	15
Geometry & Spatial Sense[b]	15	15	15	20	20	20	20	20	20
Data Analysis, Statistics, & Probability	10	10	10	15	15	15	15	15	20
Algebra & Functions	10	10	15	20	20	25	25	25	25

[a] Approximately half the questions in 1996 at each grade level involved some aspect of estimation.
[b] At grade 12 in 1996, approximately 25 percent of the geometry questions involved topics in coordinate geometry.
SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

[3] National Assessment Governing Board (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress*. Washington, DC: Author.

[4] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Mathematical abilities

A domain of general abilities associated with doing mathematics also was included in the framework. These mathematical abilities — conceptual understanding, procedural knowledge, and problem solving — describe the nature of the knowledge or processes involved in successfully completing the types of mathematical tasks that students are expected to master. For example, conceptual understanding can be viewed as a student's knowing "about" something, while procedural knowledge can be viewed as a student's knowing "how to do" something. These two abilities combined provide a base for problem solving, that is, for recognizing and understanding a problem, formulating a plan or strategy, arriving at a solution, and reflecting upon or evaluating the solution.

Mathematical power

The third domain included in the framework is mathematical power. Mathematical power is defined as a student's ability to reason in mathematical situations; to communicate perceptions and conclusions drawn from a mathematical context; and to connect the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation. The cognitive skills of reasoning, communicating, and connecting lie at the foundation of each of the content strands and each of the mathematical abilities.

Assessment questions were classified according to mathematical ability and mathematical power, as well as content. Figure 1.1 shows how the content strands, mathematical abilities, and mathematical power combine to form the framework for the NAEP 1996 mathematics assessment.

Figure 1.1 ***Mathematics Framework for the 1996 Assessment***

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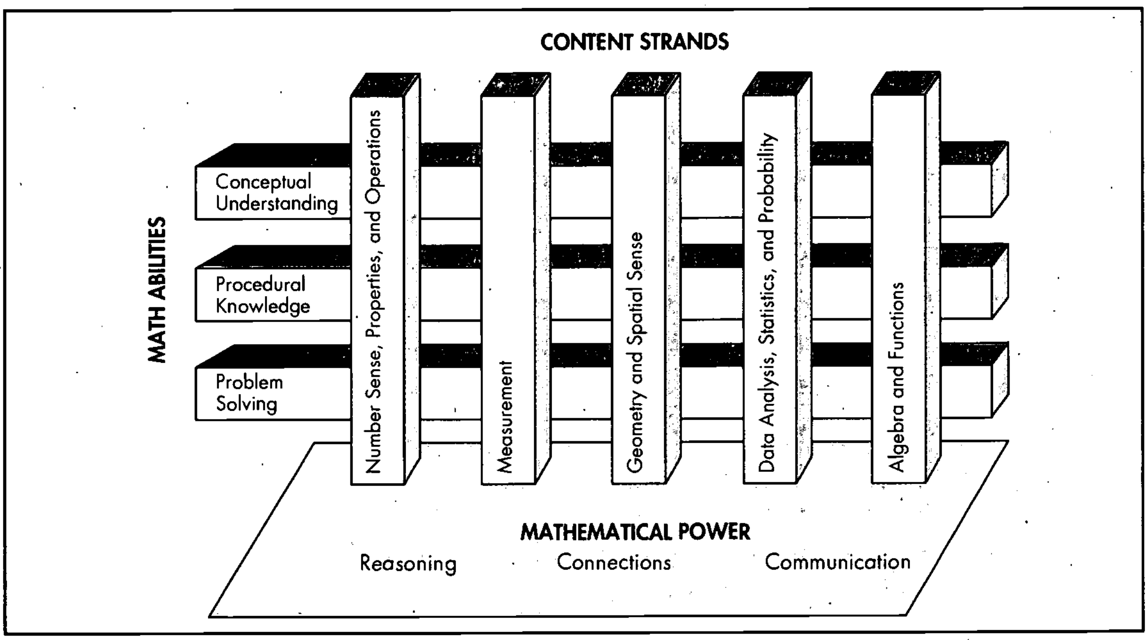

SOURCE: National Assessment Governing Board, *Mathematics Framework for the 1996 National Assessment of Educational Progress.*

Question types

The NAEP mathematics framework also prescribed a mix of question types: multiple-choice, short constructed-response, and extended constructed-response. Multiple-choice questions require students to select the answer that best expresses what they believe is correct. Short constructed-response questions require students to provide a brief response, which might be a numerical result, the correct name or classification for a group of mathematical objects, a drawn example of a given concept, or perhaps a brief written explanation for a given result. Extended constructed-response questions require students to consider a situation that demands more than a numerical response or a short written explanation. The response mode requires that students provide evidence of their work on some aspect of the solution and communicate their decision-making steps in the context of the problem.

Table 1.2 shows the distribution of questions by type for the 1990, 1992, and 1996 assessments. As the table shows, the 1996 assessment continued a shift begun in 1992 toward the use of more constructed-response questions. Current recommendations call for the use of constructed-response questions as a way to assess students' abilities to reason and to communicate mathematically. They provide an added dimension to the information that can be gleaned from multiple-choice questions.

The framework also called for the assessment to incorporate the use of calculators, rulers, protractors (grades 8 and 12 only), and manipulatives (including geometric shapes, three-dimensional models, and spinners).

Table 1.2 ***Distribution of Questions by Type*** THE NATION'S REPORT CARD

	Grade 4			Grade 8			Grade 12		
Question Type	1990	1992	1996	1990	1992	1996	1990	1992	1996
Multiple-Choice	102	99	81	149	118	102	156	115	99
Short Constructed-Response[a]	41	59	64	42	65	69	47	64	74
Extended Constructed-Response[b]	–	5	13	–	6	12	–	6	11
Total	143	163	158	191	189	183	203	185	184

[a] Short constructed-response questions previously used in the 1990 and 1992 assessments were scored dichotomously (right/wrong). New short constructed-response questions included in the 1996 assessment were scored to allow for partial credit.

[b] No extended constructed-response questions were included in the 1990 assessment.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

Estimating Mathematics Achievement

Information from both the multiple-choice and constructed-response questions was combined in order to estimate mathematics achievement in the five content strands and overall. Constructed-response questions were first scored by trained readers using criteria that distinguished among two to five levels of performance.[5] When the questions were anchored to the NAEP scale and used in the estimation of students' mathematics achievement, each of the scoring levels was anchored separately. However, for a few of the questions, adjacent score categories were collapsed because the responses lacked sufficient structure to maintain statistically the distinctions implied by the hand scoring. These instances will be noted in the text.

In addition, because of the broad content domain covered by the assessment and the need to reduce the burden on individual schools and students, no student who participated in the NAEP mathematics assessment answered all of the questions. Rather, each student was administered a portion of the assessment, and then data across students were combined to provide estimates of the achievement of fourth-, eighth-, and twelfth-grade students overall and within important subgroups, such as those defined by gender or race/ethnicity. No individual student scores were derived. Further details on scoring and other technical aspects of the assessment are provided in Appendix A.

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[5] Each NAEP assessment contains questions that were used before (for trend analysis), as well as new questions. Short constructed-response questions that had previously appeared in the 1990 and 1992 assessments were scored right or wrong in 1996, as they had been in the earlier assessments. All other constructed-response questions were scored using more complex, partial-credit guidelines.

Reporting NAEP Results

Student performance on NAEP assessments has been reported using a variety of measures. Results for the main NAEP mathematics assessment are reported using the NAEP composite mathematics scale, which summarizes performance across five separate subscales — one for each of the five content strands. Achievement levels, authorized by the NAEP legislation and adopted by the National Assessment Governing Board (NAGB), help to make these scaled results meaningful and interpretable. The achievement levels are defined by broadly representative panels of teachers, education specialists, and members of the general public, and they therefore represent collective judgments about what students should know and be able to do relative to the content reflected in the NAEP frameworks. Brief policy descriptions of the levels are provided in Figure 1.2.[6]

Figure 1.2 ***Policy Definitions of NAEP Achievement Levels*** THE NATION'S REPORT CARD

Basic	This level denotes partial mastery of prerequisite knowledge and skills that are fundamental for proficient work at each grade.
Proficient	This level represents solid academic performance for each grade assessed. Students reaching this level have demonstrated competence in challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Advanced	This level signifies superior performance.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

It should be noted that setting achievement levels is a relatively new process for NAEP, and it is still in transition. Some evaluations have concluded that the percentage of students at certain levels may be underestimated.[7] On the other hand, critiques of those evaluations have asserted that the weight of the empirical evidence does not support such conclusions.[8] A further review is currently being conducted by the National Academy of Sciences.

[6] Further information about NAEP scale construction and about the achievement levels can be found in Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.

[7] United States General Accounting Office Report to Congressional Requestors (1993). *Education achievement standards: NAGB's approach yields misleading interpretations.* Washington, DC: United States General Accounting Office.

[8] Cizek, G. (1993). *Reactions to National Academy of Education report.* Washington, DC: National Assessment Governing Board.

The student achievement levels in this report have been developed carefully and responsibly, and the procedures used have been refined and revised as new technologies have become available. Upon review of the available information, the Commissioner of Education Statistics has judged that the achievement levels are in a developmental status. However, the Commissioner and the Governing Board also believe that the achievement levels are useful and valuable for reporting on the educational achievement of students in the United States.[9]

Organization of the Report

The body of this report is divided into two main sections: Section I — a report on student work, and Section II — a report on classroom instruction. Some readers may prefer to read Section I before reading Section II; however, others may prefer to read about classroom instruction (Section II) before reading about student performance (Section I).

Section I (Chapters 2–7) provides information on trends in achievement since 1990, as well as examples of student performance in each of the five mathematics content strands. Chapter 2 presents summaries of performance for 1990, 1992, and 1996. Results for each of the five content strands are presented by average scale score for all students and separately for some of the more common demographic and education groupings, such as gender, race/ethnicity,[10] and, at grades 8 and 12, the kinds of courses taken. Chapters 3–7 each consider one content strand. Each begins with a brief discussion of the expected knowledge and skills that students are asked to demonstrate in that content strand. Each chapter then presents an item map (a visual representation of the NAEP mathematics scale) for the content strand, with selected questions from the content strand mapped onto the 0 to 500 scale. Finally, sample questions from different points on the map are presented, along with a discussion of student performance on these questions. For constructed-response questions, actual student responses are included to provide the reader with illustrations of partial- and full-credit responses.

Section II includes Chapters 8–10. Chapter 8 describes the mathematics course-taking patterns of eighth- and twelfth-grade students. Chapter 9 discusses classroom activities, including instructional emphases and approaches, assessment activities, and calculator use. Chapter 10 reports on student attitudes about mathematics.

Finally, Chapter 11 presents an overall summary of the report. The chapter summarizes what students know and can do in the five content strands of the NAEP 1996 mathematics assessment, course-taking patterns and classroom practices in mathematics, and student attitudes toward mathematics.

The report also contains two appendices that support the results presented. Appendix A contains an overview of the procedures used for the NAEP 1996 mathematics assessment. Appendix B presents standard errors for the performance data presented in the body of the report.

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9 For fourth-grade students, 0–213 is defined by the National Assessment Governing Board as below *Basic*, 214–248 is *Basic*, 249–281 is *Proficient*, and 282–500 is *Advanced*; for eighth-grade students, 0–261 is below *Basic*, 262–298 is *Basic*, 299–332 is *Proficient*, and 333–500 is *Advanced*; and for twelfth-grade students, 0–287 is below *Basic*, 288–335 is *Basic*, 336–366 is *Proficient* and 367–500 is *Advanced.*

10 In designations of race/ethnicity, White is defined as White non-Hispanic, and Black is defined as Black non-Hispanic. See Appendix A for more detail.

Chapter 2

General Results — Summaries of Performance in Mathematics Content Strands

In this chapter, student performance is examined as it relates to proficiency in the five content strands of the NAEP 1996 mathematics assessment. Summaries of overall performance, as well as performance in the five mathematics content strands, are presented for 1996, 1992, and 1990. Results are presented by average scale score and are shown for demographic and education groupings, such as gender, race/ethnicity, and, for grades 8 and 12, by the types of mathematics courses taken. The five content strands are Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. A brief description of each of the five strands is given in Figure 2.1.

Figure 2.1 ***Descriptions of the Five NAEP Mathematics Content Strands***

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Number Sense, Properties, and Operations
This content strand focuses on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, and complex numbers), operations, and estimation, and their application to real-world situations. At grade 4, this strand emphasizes the development of number sense through connecting various models to their numerical representations and an understanding of the meaning of addition, subtraction, multiplication, and division. At grade 8, number sense is extended to include positive and negative numbers, and the strand addresses properties and operations involving whole numbers, fractions, decimals, integers, and rational numbers. At grade 12, this strand includes real and complex numbers and allows students to demonstrate competency up to the pre-calculus or calculus level.
Measurement
This content strand focuses on an understanding of the process of measurement and the use of numbers and measures to describe and compare mathematical and real-world objects. Students are asked to identify attributes, select appropriate units and tools, apply measurement concepts, and communicate measurement-related ideas. At grade 4, the strand focuses on time, money, temperature, length, perimeter, area, capacity, weight/mass, and angle measure. At grades 8 and 12, the strand includes these measurement concepts, but the focus shifts to more complex measurement problems that involve volume or surface area or that require students to combine shapes and to translate and apply measures. Eighth- and twelfth-grade students also solve problems involving proportional thinking (such as scale drawing or map reading) and do applications that involve the use of complex measurement formulas.
Geometry and Spatial Sense
This content strand is designed to extend beyond low-level identification of geometric shapes to include transformations and combinations of those shapes. Informal constructions and demonstrations (including drawing representations), along with their justifications, take precedence over more traditional types of compass-and-straightedge constructions and proofs. At grade 4, students are asked to model properties of shapes under simple combinations and transformations, and they are asked to use mathematical communication skills to draw figures from verbal descriptions. At grade 8, students are asked to expand their understanding to include properties of angles and polygons. They are also asked to apply reasoning skills to make and validate conjectures about transformations and combinations of shapes. At grade 12, students are asked to demonstrate an understanding of transformational geometry and to apply concepts of proportional thinking to various geometric situations.

Descriptions of the Five NAEP Mathematics Content Strands

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Data Analysis, Statistics, and Probability

This content strand emphasizes the appropriate methods for gathering data, the visual exploration of data, various ways of representing data, and the development and evaluation of arguments based on data analysis. At grade 4, students are asked to apply their understanding of numbers and quantities by solving problems that involve data. Fourth graders are asked to interact with a variety of graphs, to make predictions from data and explain their reasoning, to deal informally with measures of central tendency, and to use the basic concepts of chance in meaningful contexts. At grade 8, students are asked to analyze statistical claims and to design experiments, and they are asked to use simulations to model real-world situations. This strand focuses on eighth graders' basic understanding of sampling, their ability to make predictions based on experiments or data, and their ability to use some formal terminology related to probability, data analysis, and statistics. At grade 12, the strand focuses on the ability to apply the concepts of probability and to use formulas and more formal terminology to describe a variety of situations. For twelfth graders, the strand also emphasizes a basic understanding of how to use mathematical equations and graphs to interpret data.

Algebra and Functions

This content strand extends from work with simple patterns at grade 4 to basic algebra concepts at grade 8 to sophisticated analysis at grade 12. It involves not only algebra, but also pre-calculus and some topics from discrete mathematics. Students are expected to use algebraic notation and thinking in meaningful contexts to solve mathematical and real-world problems, specifically addressing an increasing understanding of the use of functions (including algebraic and geometric) as representational tools. The grade 4 assessment involves informal demonstration of students' abilities to generalize from patterns, including the justification of their generalizations. Students are expected to translate between mathematical representations, to use simple equations, and to do basic graphing. At grade 8, the assessment includes more algebraic notation, stressing the meaning of variables and an informal understanding of the use of symbolic representations in problem-solving contexts. Students are asked to use variables to represent a rule underlying a pattern. Eighth graders are asked to demonstrate a beginning understanding of equations and functions and the ability to solve simple equations and inequalities. By grade 12, students are asked about basic algebraic notation and terminology as they relate to representations of mathematical and real-world situations. Twelfth graders are asked to use functions as a way of representing and describing relationships.

SOURCE: *NAEP 1996 Mathematics Report Card for the Nation and the States.*

Interpretation of the Data

In general, mathematics results for all content strands and all groups of students have been improving since 1990. However, not all of these changes were statistically significant, where "statistical significance" means that there is a high level of certainty that the results would not have occurred by chance. Factors that affect statistical significance include the magnitude of the difference between the group averages (e.g., between average performance in 1992 and average performance in 1996), the amount of variability of performance *within* each group, and even the size of the groups surveyed. Thus, for example, improved performance by a specific amount in one of the content strands might be found to be statistically significant for White students, but improvement by the same amount in that strand might not be statistically significant for students of other racial/ethnic backgrounds. Statistically significant differences are noted in the figures and text that follow. All comparisons discussed in this report are statistically significant unless otherwise noted. It is important not to focus on apparent differences that are not statistically significant because these differences might be a result of sampling error. In some cases where differences among groups appear large, but, in fact, are not significant, it is noted in the text that the group differences are not "statistically significant," or there were no "significant differences."[1]

Figure 2.2 presents information on the average proficiency in each content strand for all students in grades 4, 8, and 12 for 1996, 1992, and 1990. The average proficiency on the NAEP composite mathematics scale also is shown. Table 2.1 disaggregates this information by gender, and Figures 2.3–2.8 break it out by race/ethnicity. Average proficiencies for eighth-grade Asian/Pacific Islander students are not included in the figures, however, due to concerns regarding the quality and credibility of the results obtained for this group. Data from the NAEP state assessment program in mathematics also conducted in 1996 provided an independent data source to aid in evaluating the accuracy of the national grade 8 NAEP results for Asian/Pacific Islander students as well as for other subgroups. These results suggested that the 1996 national results may substantially underestimate the actual achievement of the Asian/Pacific Islander group. In view of the potential to misinform, it was decided to omit the national grade 8 Asian/Pacific Islander results from the body of the report.[2] Appendix A includes average scale scores on the national assessment for this group along with a description of the findings that led to this decision.

A brief discussion of observed trends and significant results follows. The discussion refers to Figures 2.2–2.8 and Table 2.1.

[1] See Appendix A Guidelines for Analysis and Reporting for further discussion of determining statistical significance.

[2] Asian/Pacific Islander students are included, however, in performance data for all students.

Trends

Comparisons with 1990

In 1992, significant gains over performance in 1990 were observed in mathematics performance on the composite scale and in each content strand for the general population at all three grade levels. Considerable gains also were evident in most cases for the female, male, and White subgroups of students. The exceptions were 1) males in the Data Analysis, Statistics, and Probability strand at all three grades and in both the Geometry and Spatial Sense content strand and the Measurement content strand at grade 8; and 2) White students in Data Analysis, Statistics, and Probability at grade 12.

Other improvements from 1990 to 1992 were observed for Black students in overall mathematics performance at grade 12, as well as in Geometry and Spatial Sense at grades 4 and 12. Hispanic students improved between 1990 and 1992 in Geometry and Spatial Sense at grades 4 and 12.

The same trends were noted when comparing 1996 performance with 1990 performance. Additionally, males and White students showed improvement in the areas noted as exceptions for 1992. In 1996, Black students showed additional gains in performance relative to 1990 in overall mathematics performance at grade 4, as well as in Number Sense, Properties, and Operations at grade 4 and in Measurement and in Algebra and Functions at grade 12. Hispanic students showed additional gains in Algebra and Functions at all grades, as well as in Geometry and Spatial Sense at grade 4.

Comparisons with 1992

In 1996, improvement over 1992 performance was noted in overall mathematics performance for the general population, males, females, and White students at all grades, with the exception of eighth-grade White students and eighth-grade male students. Improvement also was noted for the general population in Geometry and Spatial Sense and in Algebra and Functions at all grades; and in Number Sense, Properties, and Operations and in Data Analysis, Statistics, and Probability at fourth grade; as well as in Measurement and in Data Analysis, Statistics, and Probability at twelfth grade. Thus, there appears to be continued improvement to 1996 in the content strands of Geometry and Spatial Sense and Algebra and Functions. This is less true for the other content strands, where improvement for at least some grades appears to have leveled out after 1992.

Trends for male, female, and White students were similar to those observed for all students, with the following exceptions: 1) males did not show significant improvement in Geometry and Spatial Sense at fourth grade or in Algebra and Functions at twelfth grade, and 2) females did not show significant improvement in Data Analysis, Statistics, and Probability at fourth grade but did show improvement in this content strand at eighth grade. Additionally, Black and Hispanic students showed improvement in 1996 in Algebra and Functions relative to their performance in 1992 in grades 4 and 8. The same was true for the performance of American Indian students in grade 8.

Subgroups

Gender

As can be seen in Table 2.1, in 1996, gender differences in performance favoring males were observed for grade 4 overall proficiency and for three content strands: Number Sense, Properties, and Operations; Measurement; and Algebra and Functions. At grade 12, gender differences, also favoring males, were observed for two content strands: Measurement, and Geometry and Spatial Sense. There were no significant differences in performance between males and females at grade 8.

Race/ethnicity

Figures 2.3–2.8 show that in 1996, White and Asian/Pacific Islander students at grades 4 and 12 performed better than other ethnic groups overall and in each of the content strands of mathematics. White students at grade 8 also outperformed Black, Hispanic, and American Indian students in terms of overall proficiency and in each of the five content strands. At grade 4, Hispanic students performed better than Black students in Geometry and Spatial Sense, and American Indian students performed better than Black and Hispanic students in all strands. At grade 8, Hispanic students outperformed Black students in Measurement and in Geometry and Spatial Sense. At grade 12, Asian/Pacific Islander students performed better than White students in Algebra and Functions, and Hispanic students outperformed Black students in Measurement and in Data Analysis, Statistics, and Probability.

Figure 2.2

Average Proficiency in Mathematics Content Strands, Grades 4, 8, and 12

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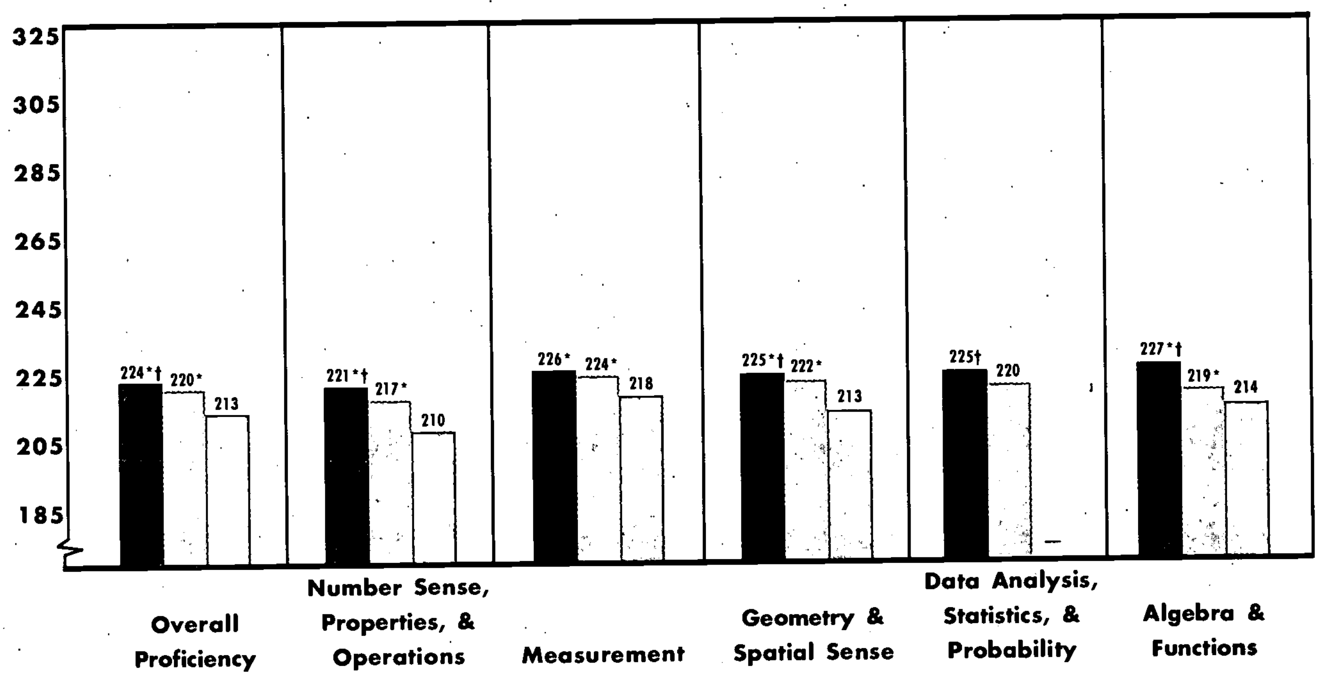
Grade 4



Grade 8

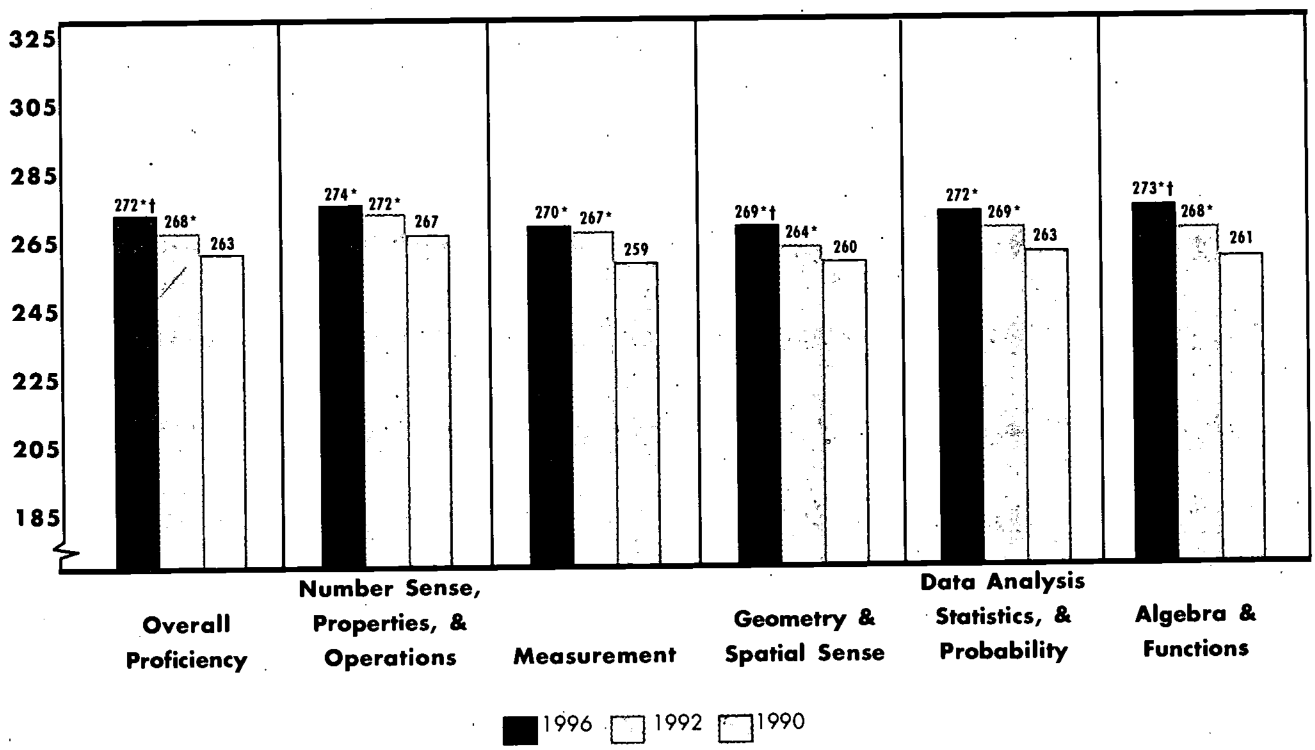
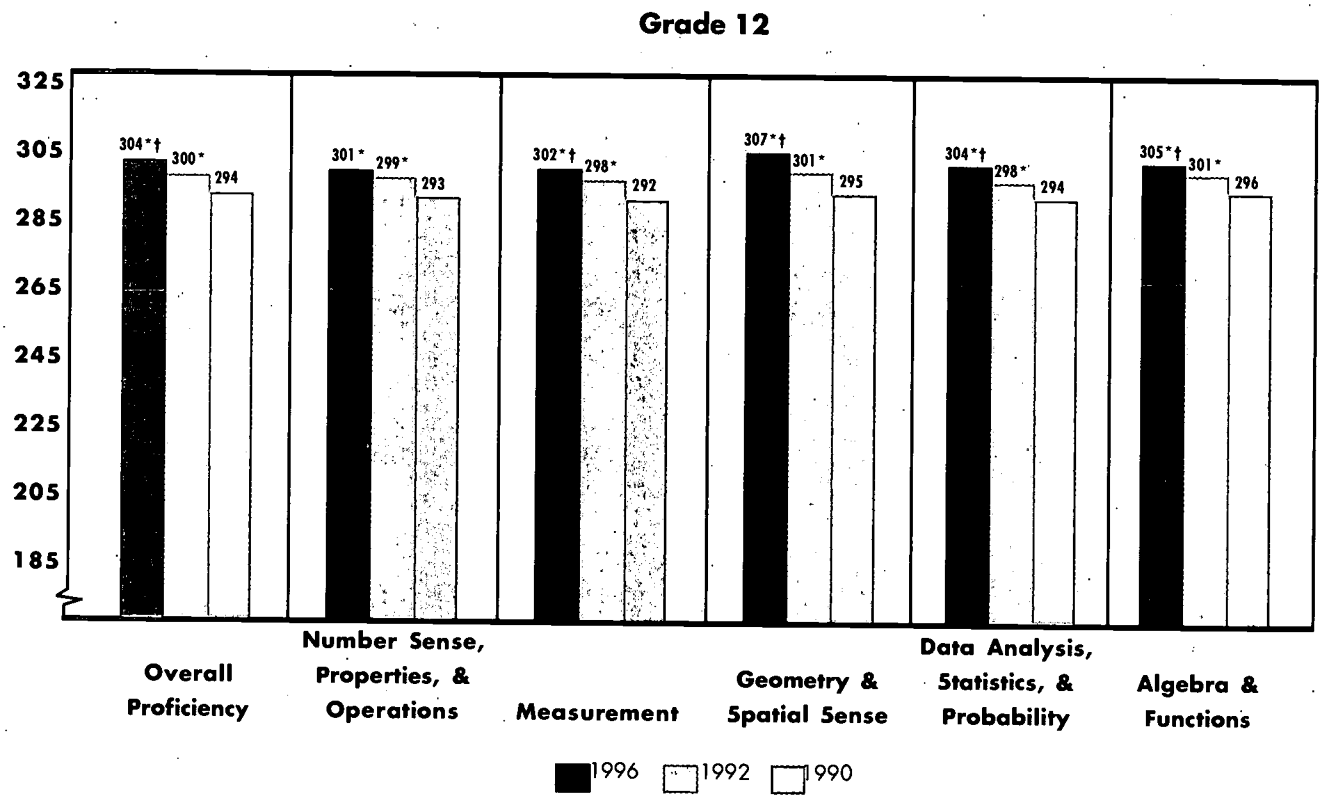


Figure 2.2 (cont)

Average Proficiency in Mathematics Content Strands, Grades 4, 8, and 12

THE NATION'S REPORT CARD



* Significant difference from 1990.

† Significant difference from 1992.

— 1990 data are not available.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Table 2.1 Average Proficiency in Mathematics Content Strands by Gender, Grades 4, 8, and 12

	1996			1992			1990		
	All Students	Male	Female	All Students	Male	Female	All Students	Male	Female
Grade 4									
Overall Proficiency	224*†	226*†	222*†	220*	221*	218*	213	214	212
Number Sense, Properties, & Operations	221*†	223*†	220*†	217*	218*	216*	210	210	210
Measurement	226*	228*	223*	224*	226*	223*	218	221	216
Geometry & Spatial Sense	225*†	225*	225*†	222*	223*	221*	213	213	213
Data Analysis, Statistics, & Probability	225†	226†	223	220	221	220	—	—	—
Algebra & Functions	227*†	230*†	225*†	219*	218*	219*	214	214	214
Grade 8									
Overall Proficiency	272*†	272*	272*†	268*	268*	269*	263	263	262
Number Sense, Properties, & Operations	274*	274*	274*	272*	272*	273*	267	266	267
Measurement	270*	271*	268*	267*	269*	264*	259	263	255
Geometry & Spatial Sense	269*†	269*†	270*†	264*	264	264*	260	261	259
Data Analysis, Statistics, & Probability	272*	271*	274*†	269*	268	269*	263	264	263
Algebra & Functions	273*†	273*†	273*†	268*	266*	270*	261	261	262
Grade 12									
Overall Proficiency	304*†	305*†	303*†	300*	301*	298*	294	297	292
Number Sense, Properties, & Operations	301*	303*	300*	299*	300*	298*	293	296	290
Measurement	302*†	306*†	299*†	298*	302	295*	292	298	288
Geometry & Spatial Sense	307*†	309*†	305*†	301*	304*	299*	295	298	293
Data Analysis, Statistics, & Probability	304*†	304*†	304*†	298*	299	297*	294	297	292
Algebra & Functions	305*†	305*	305*†	301*	301*	300*	296	297	295

* Significant difference from 1990.
† Significant difference from 1992.
— 1990 data are not available.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.3 Average Mathematics Proficiency, Composite Scale by Race/Ethnicity, Grades 4, 8, and 12

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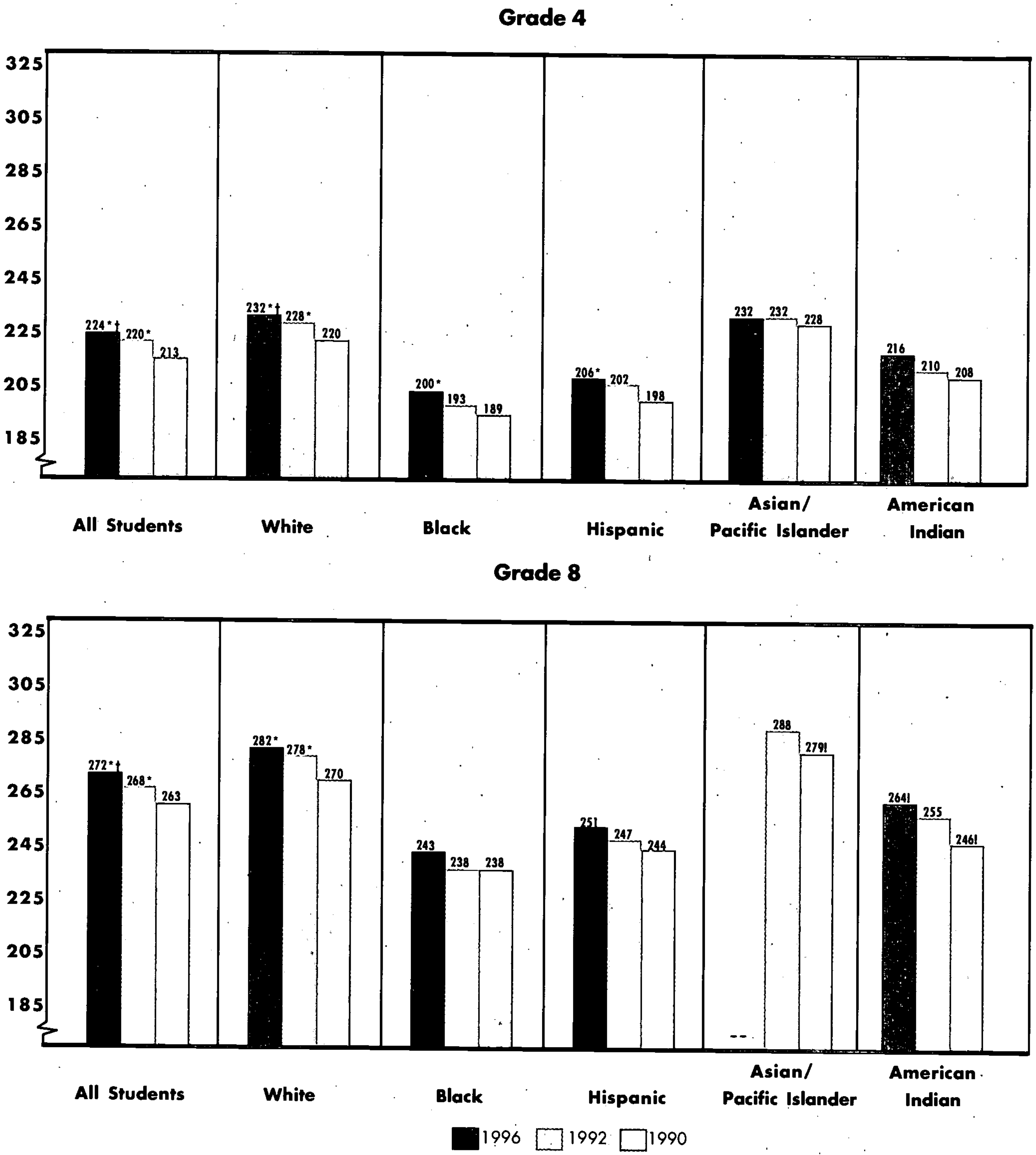

Figure 2.3 (cont)

Average Mathematics Proficiency, Composite Scale by Race/Ethnicity, Grades 4, 8, and 12

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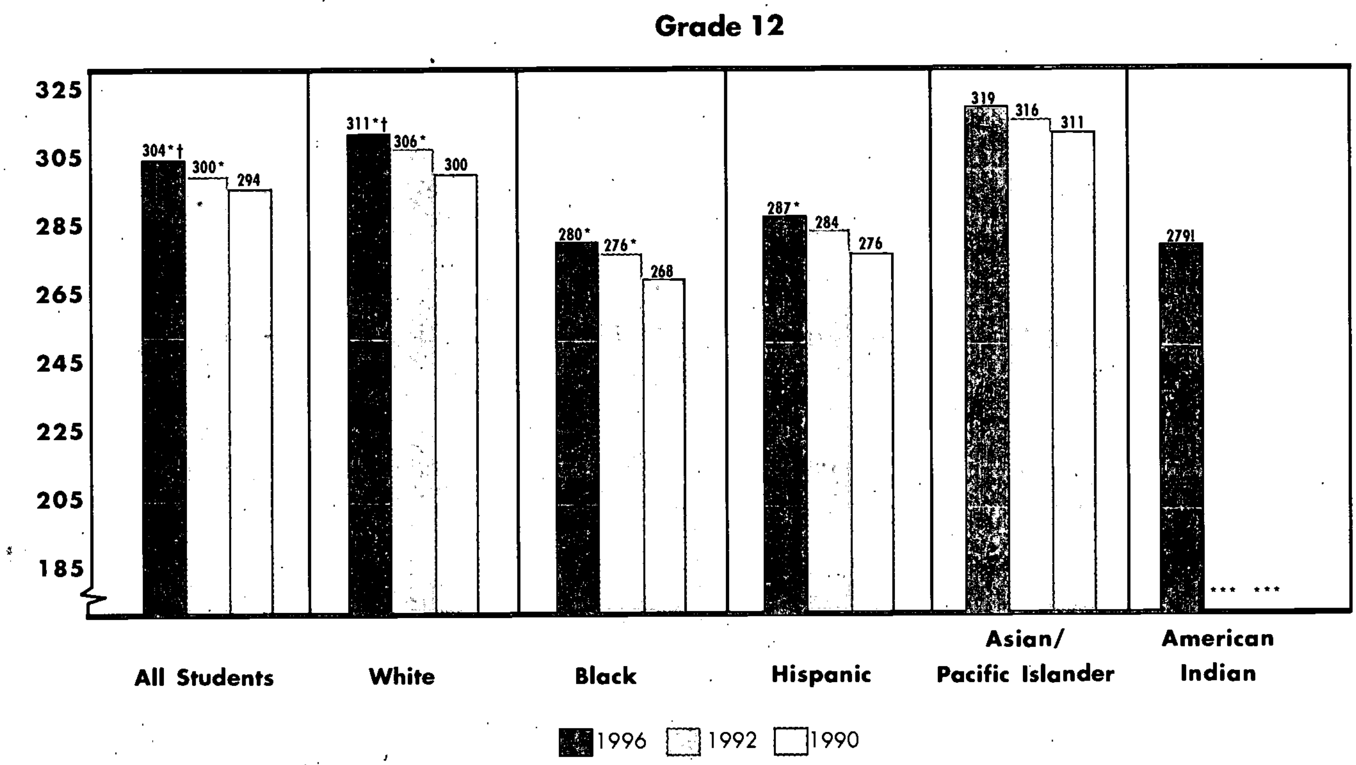

* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.4

Average Proficiency in Number Sense, Properties, and Operations by Race/Ethnicity, Grades 4, 8, and 12

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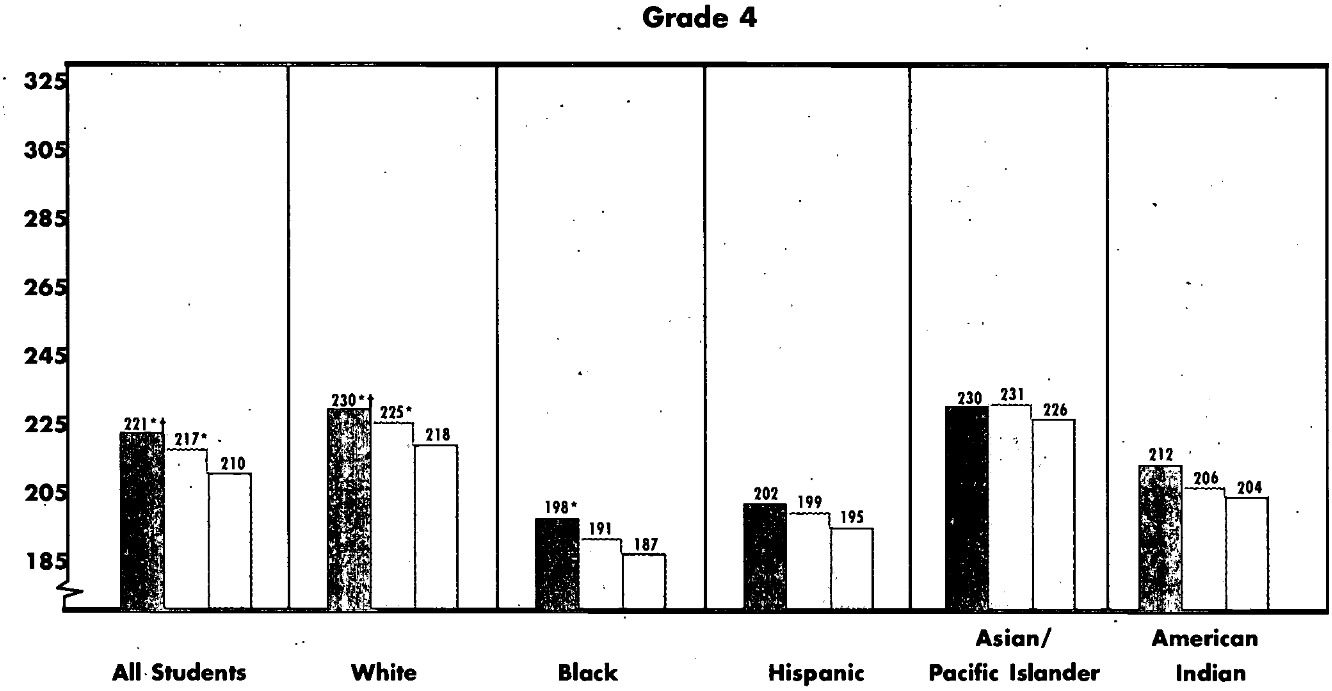

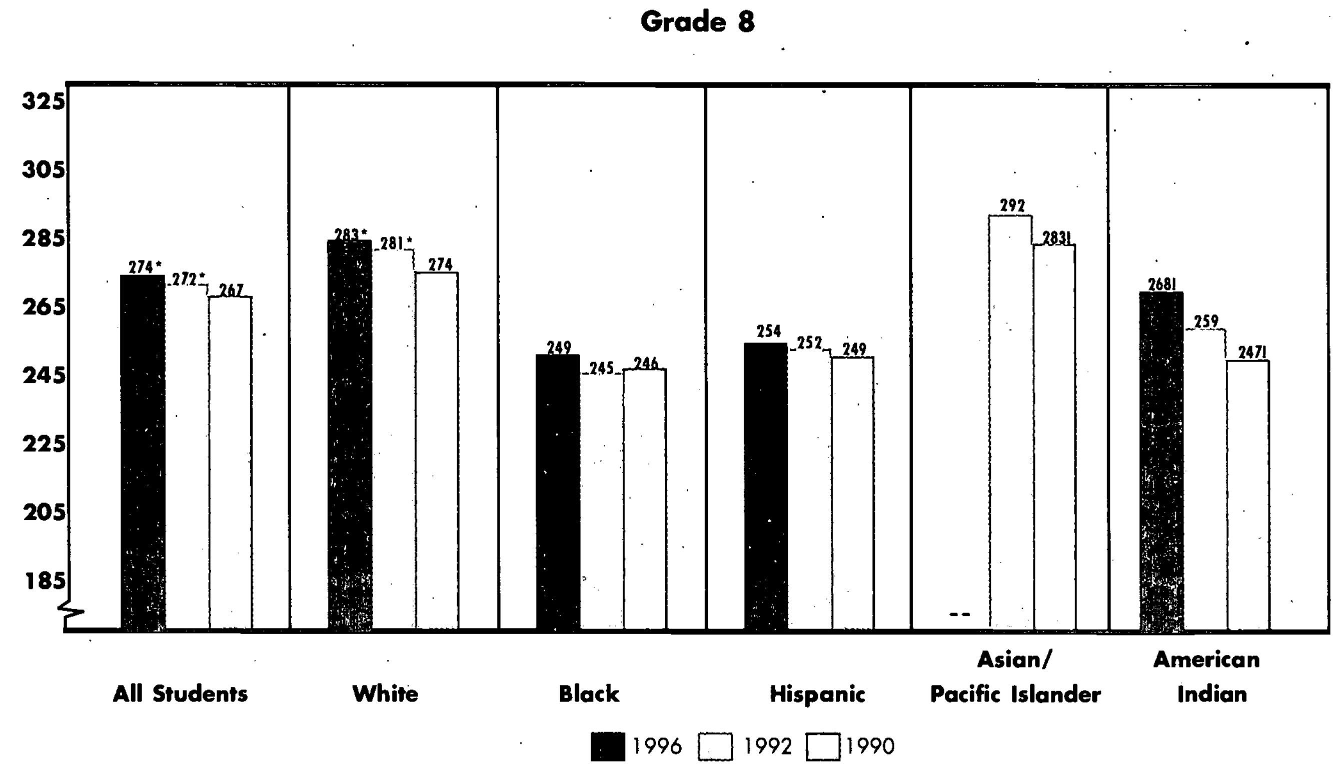

Figure 2.4 (cont) Average Proficiency in Number Sense, Properties, and Operations by Race/Ethnicity, Grades 4, 8, and 12

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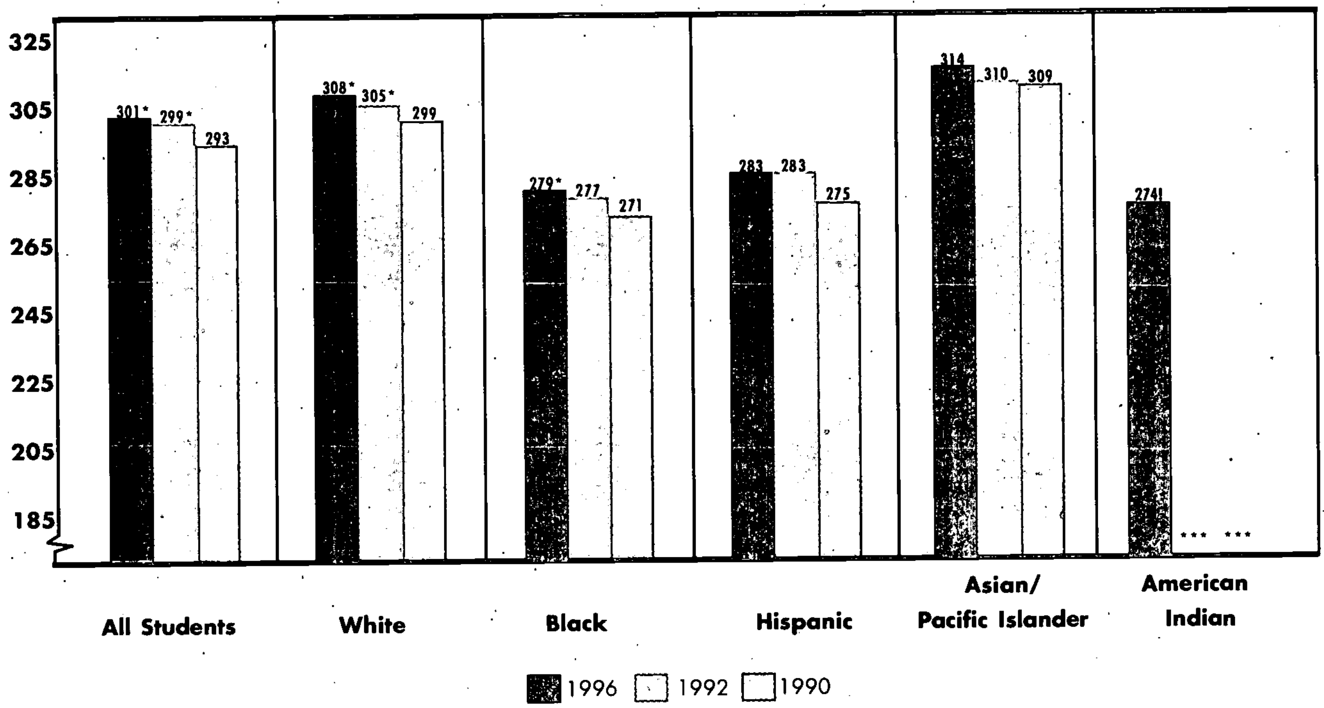
Grade 12



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure 2.5 *Average Proficiency in Measurement by Race/Ethnicity, Grades 4, 8, and 12*

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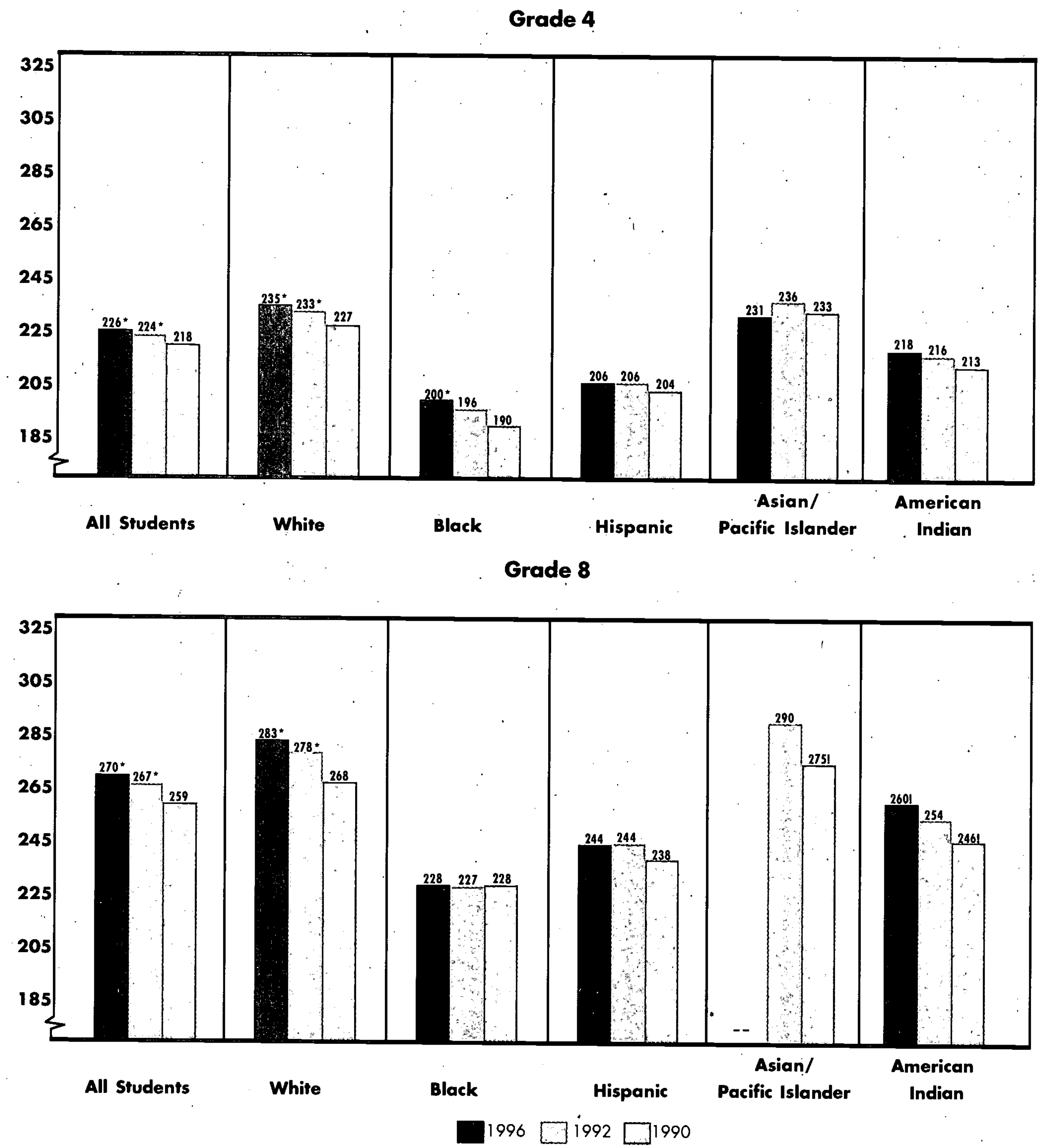
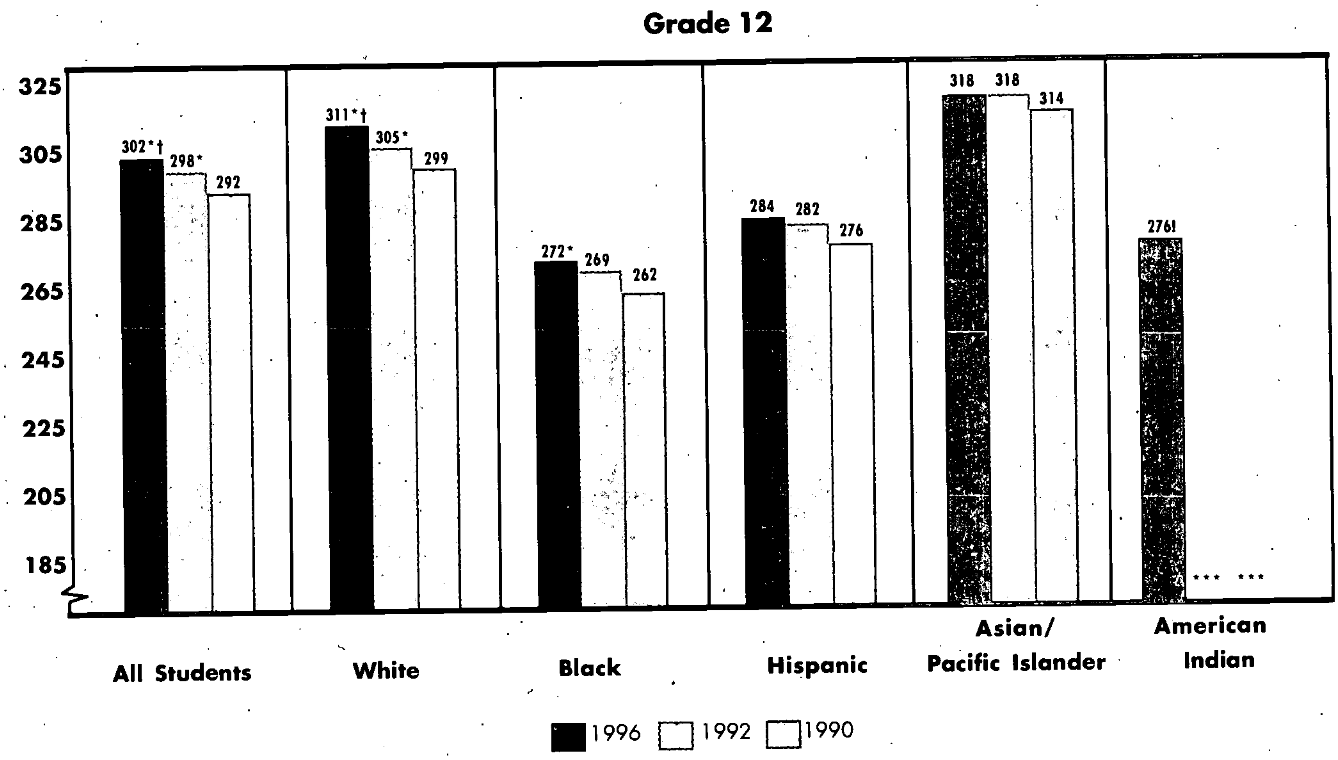


Figure 2.5 (cont) Average Proficiency in Measurement by Race/Ethnicity, Grades 4, 8, and 12



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Figure 2.6 *Average Proficiency in Geometry and Spatial Sense by Race/Ethnicity, Grades 4, 8, and 12*

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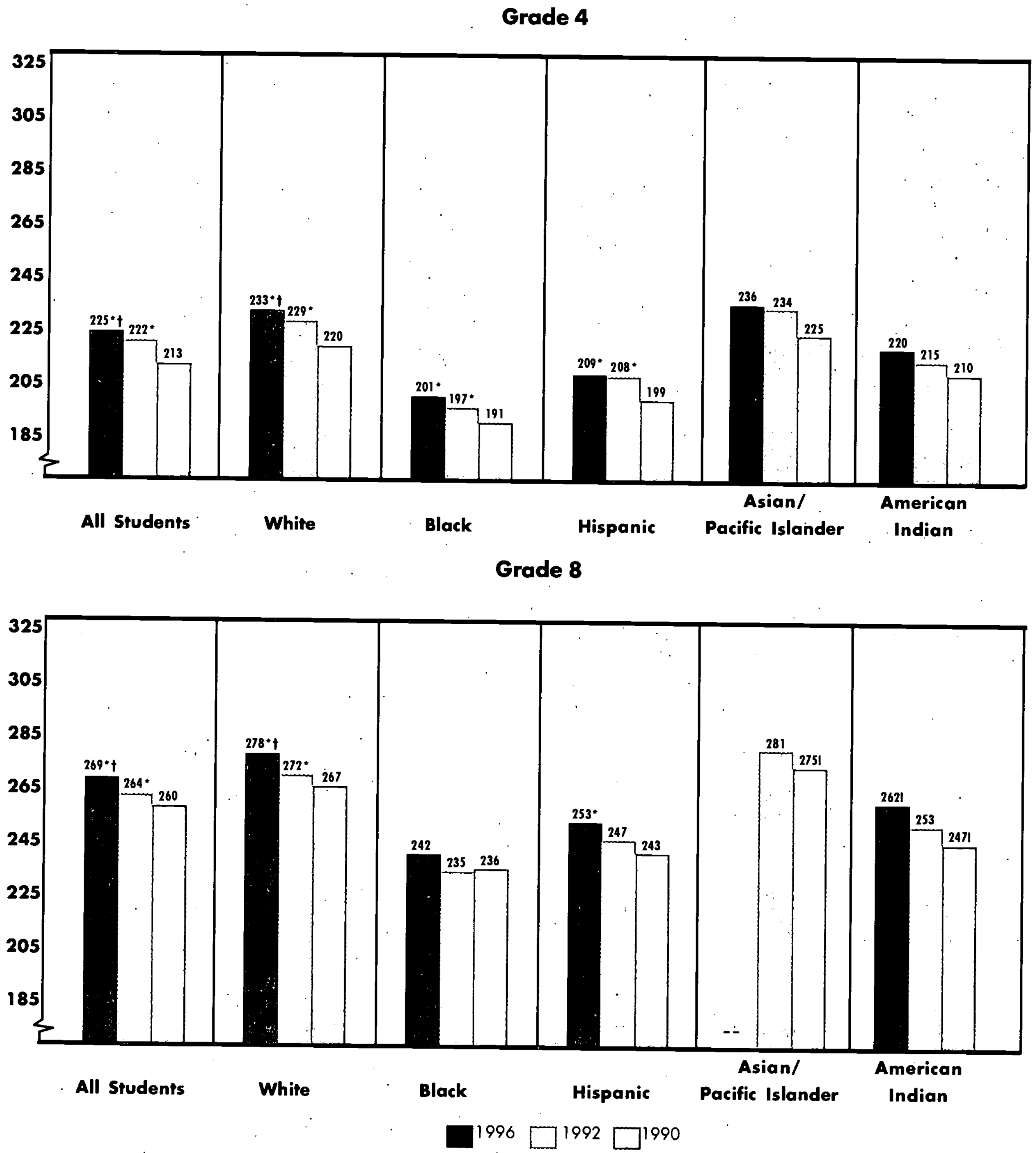

39

Figure 2.6 (cont) Average Proficiency in Geometry and Spatial Sense by Race/Ethnicity, Grades 4, 8, and 12

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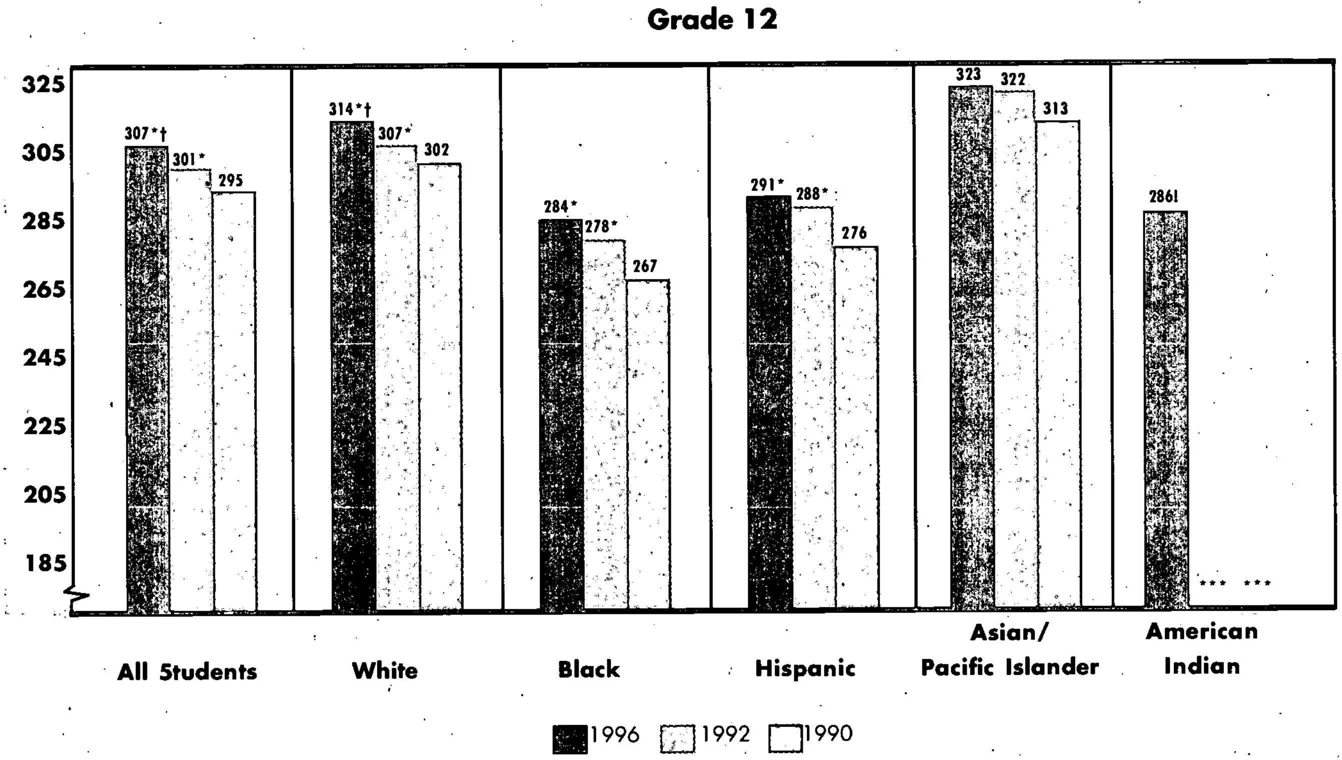

* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

I Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Figure 2.7 ***Average Proficiency in Data Analysis, Statistics, and Probability by Race/Ethnicity, Grades 4, 8, and 12***

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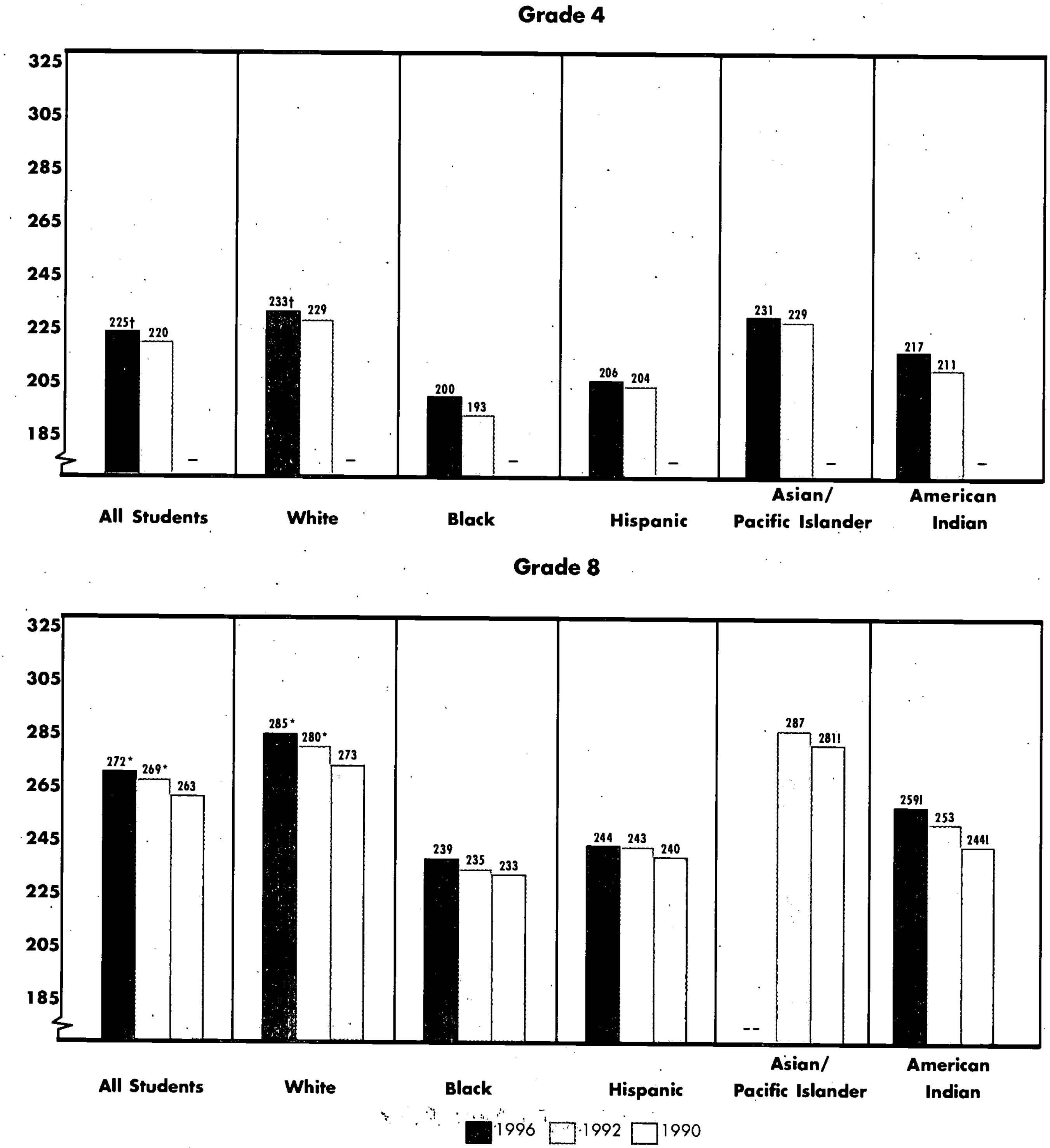

41

Figure 2.7 (cont)

Average Proficiency in Data Analysis, Statistics, and Probability by Race/Ethnicity, Grades 4, 8, and 12

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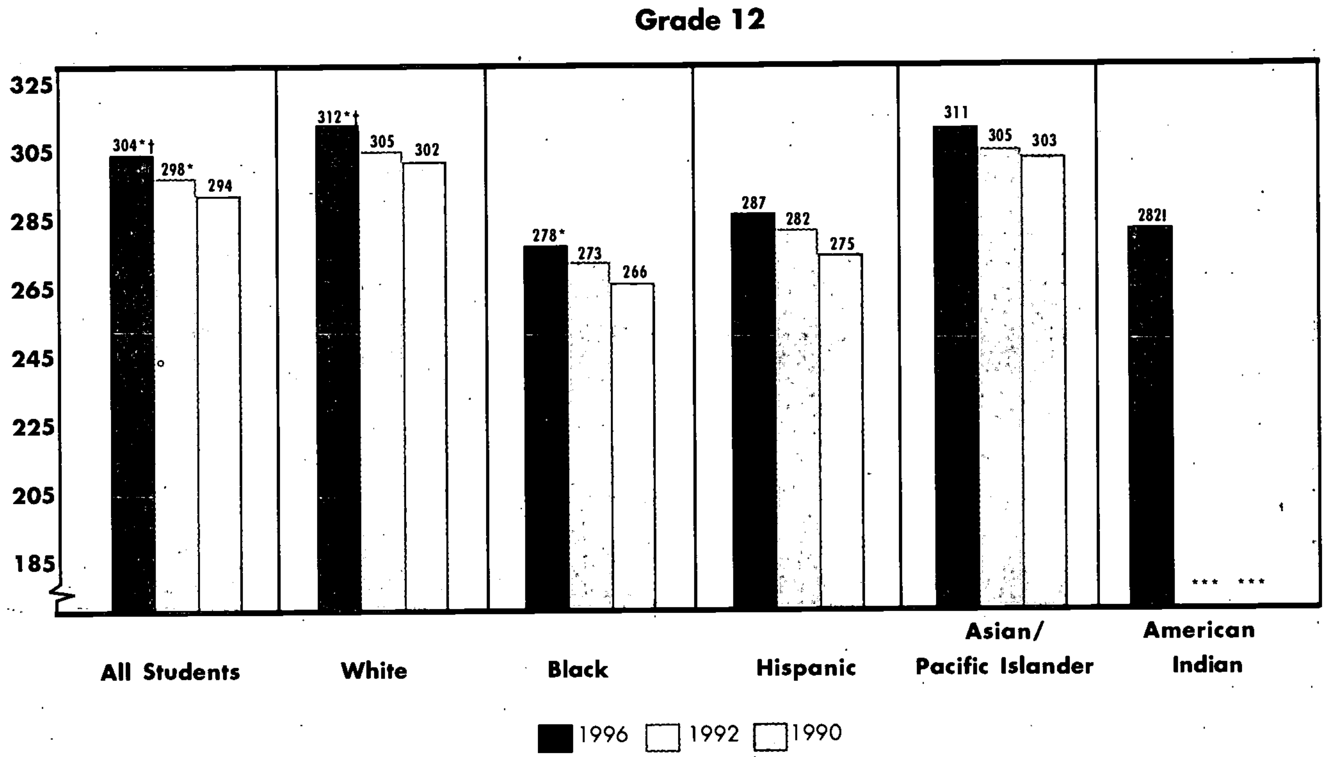

* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

— 1990 data are not available.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Figure 2.8 ***Average Proficiency in Algebra and Functions by Race/Ethnicity, Grades 4, 8, and 12***

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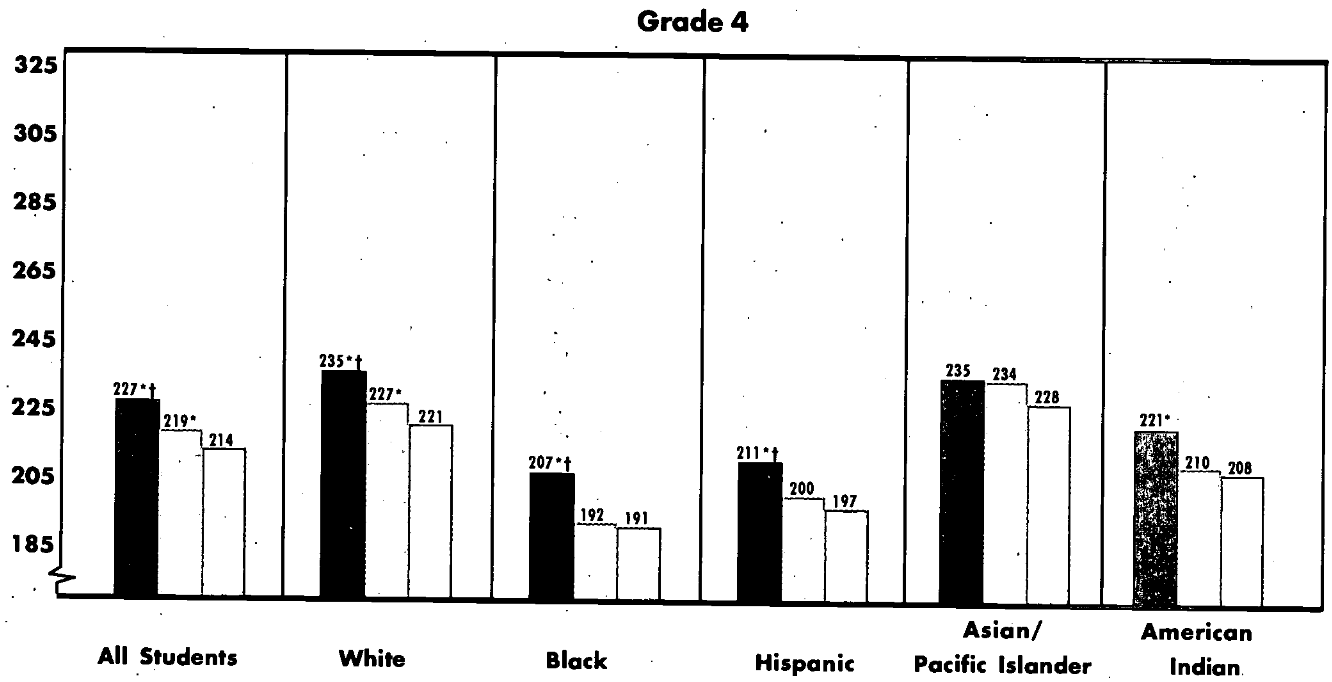


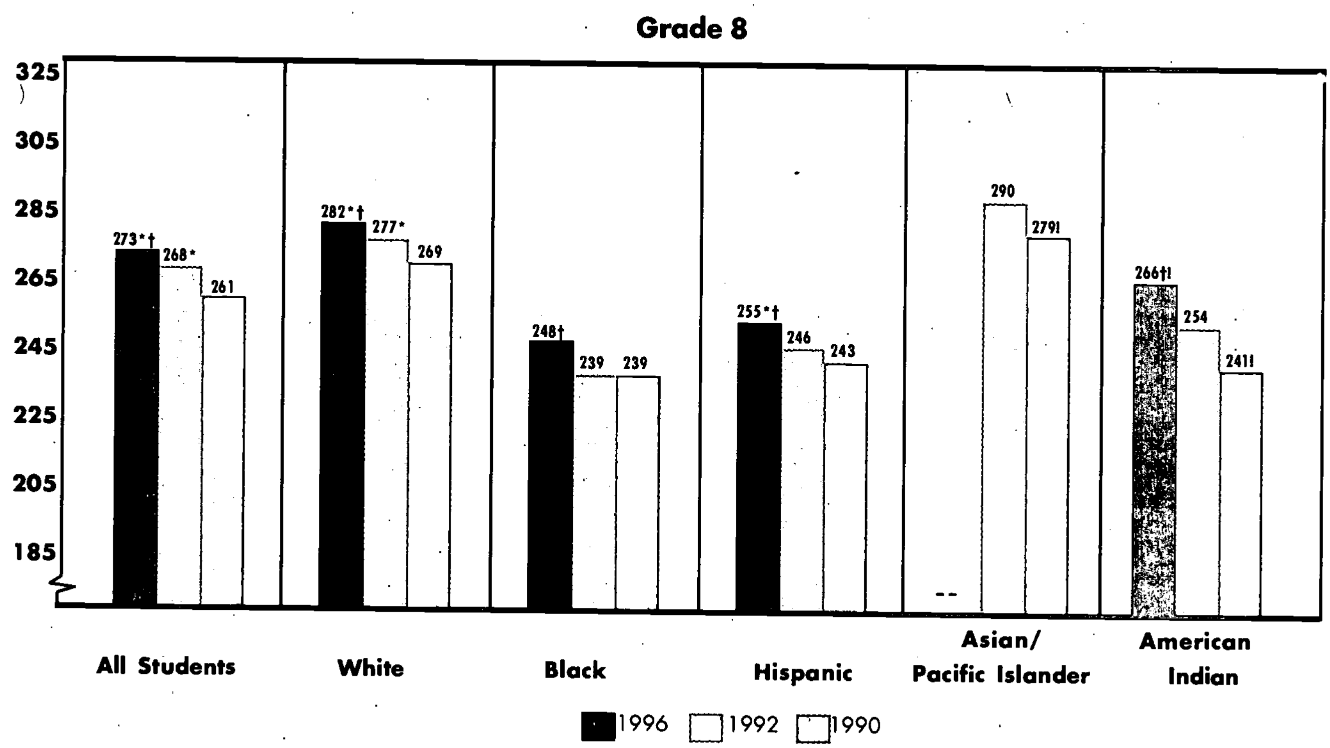
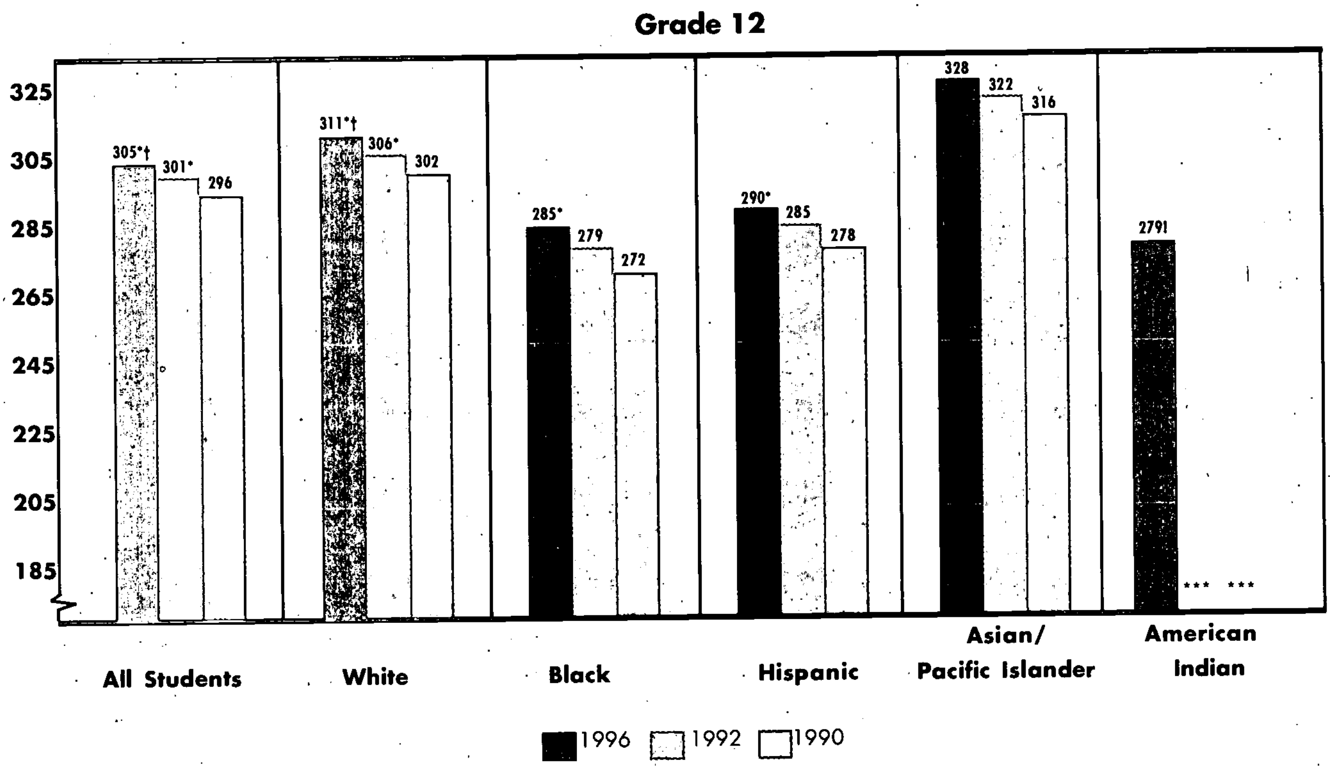


Figure 2.8 (cont)

Average Proficiency in Algebra and Functions by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep



* Significant difference from 1990.

† Significant difference from 1992.

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

I Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Average Proficiency in Mathematics Content Strands by Courses Taken

Performance in the different content strands is affected by the number and type of mathematics courses taken. For a discussion of the mathematics courses that eighth-grade students were enrolled in at the time of the NAEP 1996 assessment and the mathematics course-taking history of twelfth-grade students participating in the assessment, see Chapter 8.

Figures 2.9–2.13 show the average 1996 performance on content strands for eighth- and twelfth-grade students with different course-taking patterns. In general, taking more mathematics courses and more advanced mathematics courses was associated with improved mathematics performance in all content strands. Figure 2.9 shows average scale scores in each content strand for eighth-grade students enrolled in algebra, pre-algebra, or eighth-grade mathematics. Eighth-grade students enrolled in algebra performed better in all content strands than eighth-grade students enrolled in pre-algebra or eighth-grade mathematics. Similarly, eighth-grade students enrolled in pre-algebra performed better than students enrolled in eighth-grade mathematics in all content strands except Geometry and Spatial Sense. In the latter content strand, performance was not significantly different for the two groups.

Twelfth-grade results show a similar story. Figure 2.10 presents average scale scores for each content strand for twelfth-grade students according to the highest course they had taken in the algebra-through-calculus sequence.[3] The algebra-through-calculus sequence consists of elementary and intermediate algebra, followed by pre-calculus and calculus. Students at any given point in this sequence performed better than students whose mathematics exposure had stopped at the next lowest course in the sequence. The only exception was that students whose highest course had been pre-algebra did not score significantly higher than students who had taken neither pre-algebra nor algebra. There was no significant difference in the performance of these two groups of students.

Figures 2.11 and 2.12 report average scale scores for students who had taken at least one course in geometry or in probability or statistics, as well as for students who had not taken these courses. The results in Figure 2.11 show that students who had taken geometry performed better in all content strands than those who had not taken geometry. This overall higher performance might be explained by the fact that most of the students who took geometry also took at least 2 years of algebra, whereas most of the students who did not take geometry took 1 year or less of algebra. The performance results were different, however, when comparing students who had taken a course in probability or statistics with those who had not (Figure 2.12). Here, there were no significant differences between the two groups. It may be that students who take probability or statistics are students who do not take more advanced courses, or that the assessment questions did not provide sufficient breadth to allow students taking statistics courses to display their added knowledge.

[3] The twelfth-grade course sequence was defined as the algebra-through-calculus sequence, not including geometry, because variability in mathematics course sequencing from school to school makes the position of geometry in the curriculum ambiguous.

The final figure in this section (2.13) shows that taking more mathematics courses in high school is related to higher mathematics performance. (The only exception was in the Measurement content strand, where the apparent difference between students who took 3 to 4 semesters of mathematics and those who took only 1 to 2 semesters was not statistically significant.)

Figure 2.9 ***Average Proficiency in Mathematics Content Areas by Course Taking, Grade 8***

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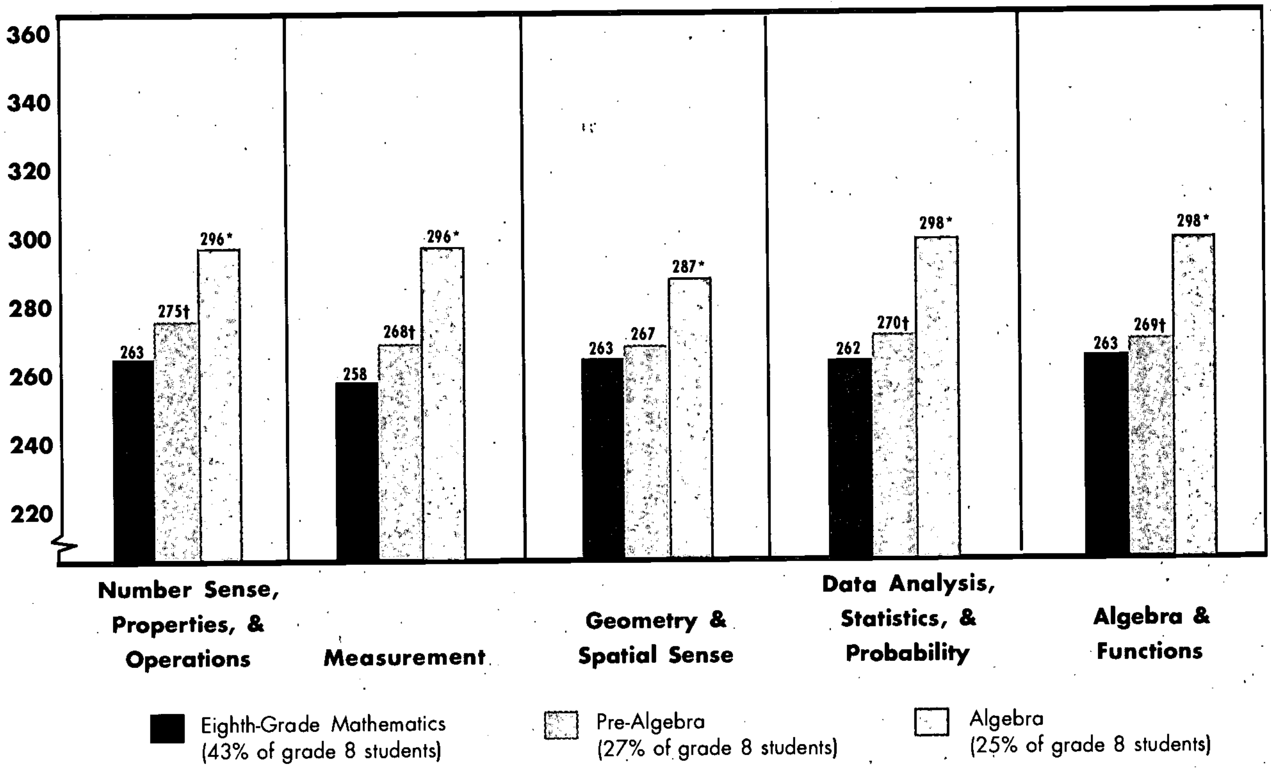


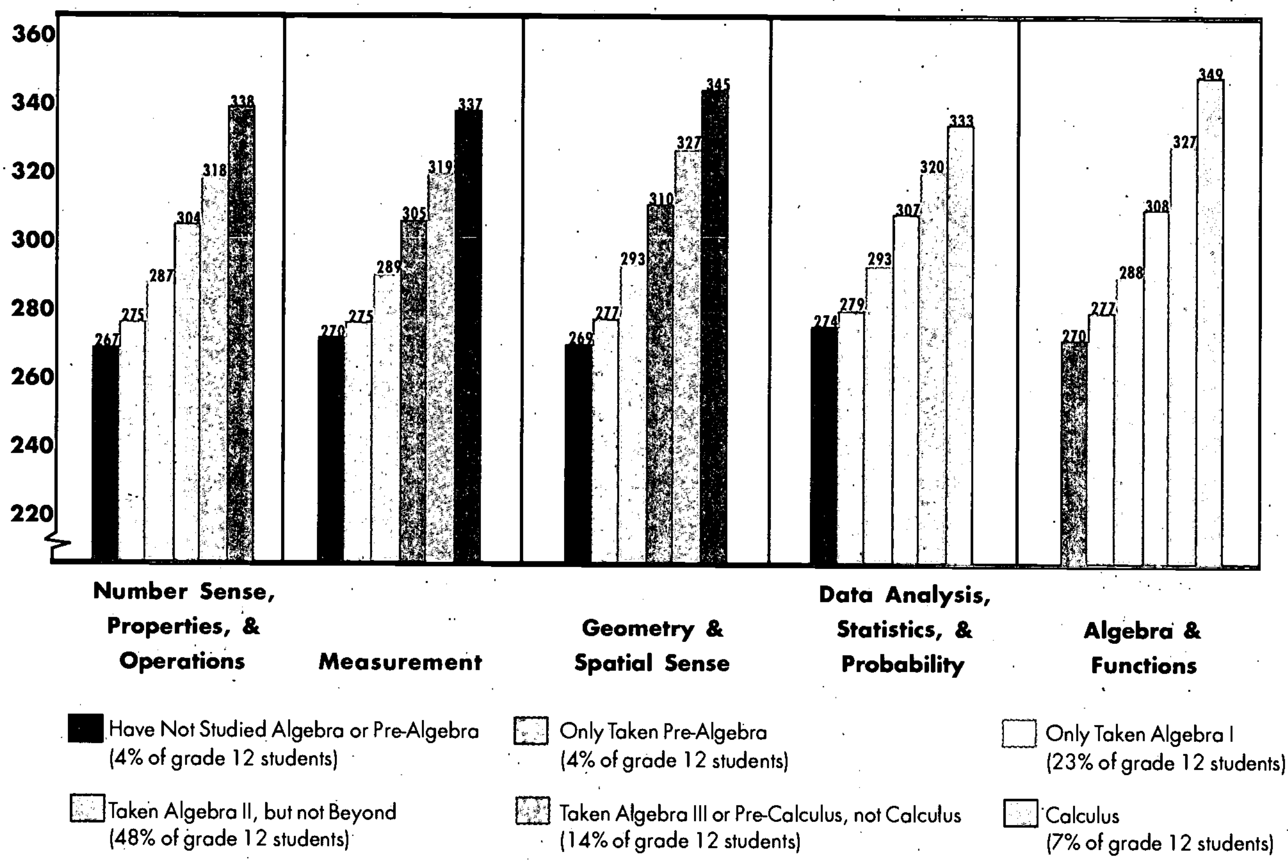
* Indicates a significant difference between algebra and pre-algebra group results and between algebra and eighth-grade mathematics group results.

† Indicates a significant difference between pre-algebra and eighth-grade mathematics group results.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure 2.10

*Average Proficiency in Mathematics Content Areas by Algebra and Calculus Courses Taken, Grade 12**



* Students at any given point in this sequence performed significantly better than students whose mathematics exposure had stopped at the next lowest course in the sequence. The only exception was that students whose highest course had been pre-algebra did not score significantly higher than students who had taken neither pre-algebra nor algebra.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure 2.11

Average Proficiency in Mathematics Content Areas by Geometry Course Taken, Grade 12

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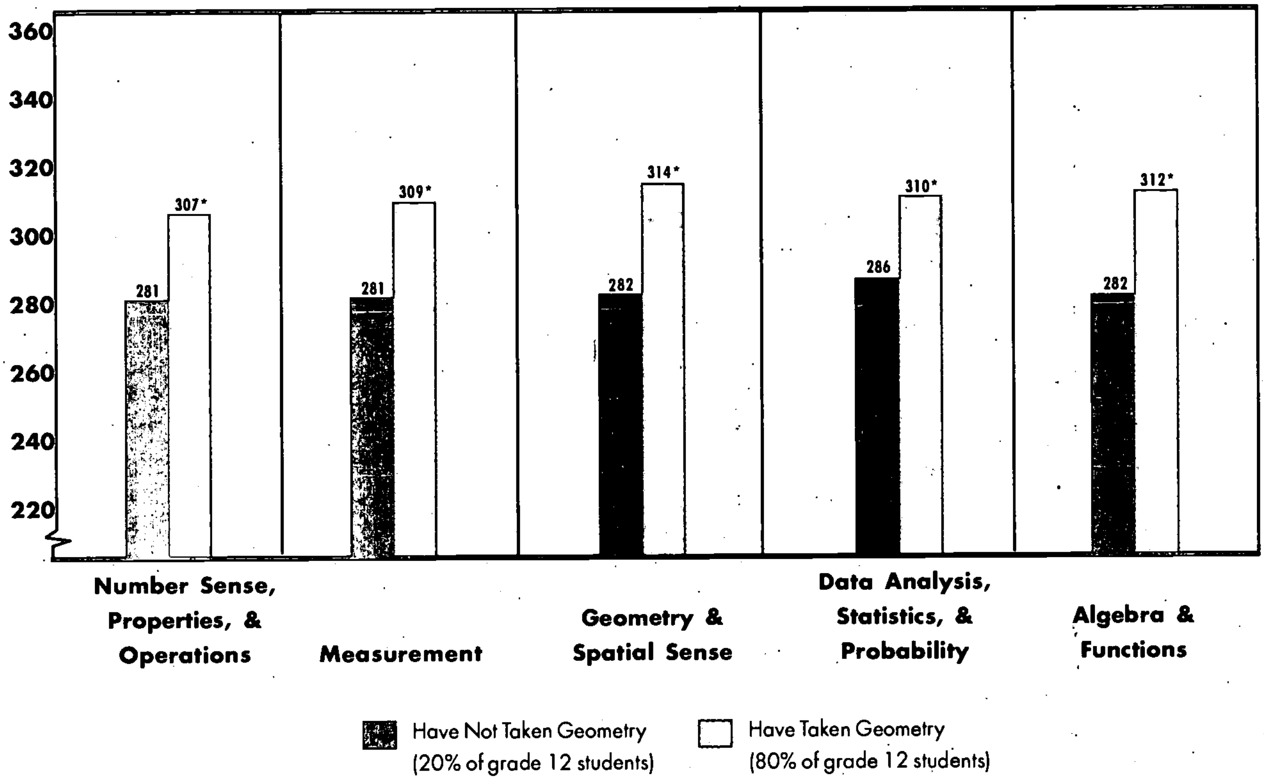

* Indicates a significant difference in results between those who had taken geometry and those who had not taken geometry.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Figure 2.12 **Average Proficiency in Mathematics Content Areas by Probability or Statistics Course Taken, Grade 12**

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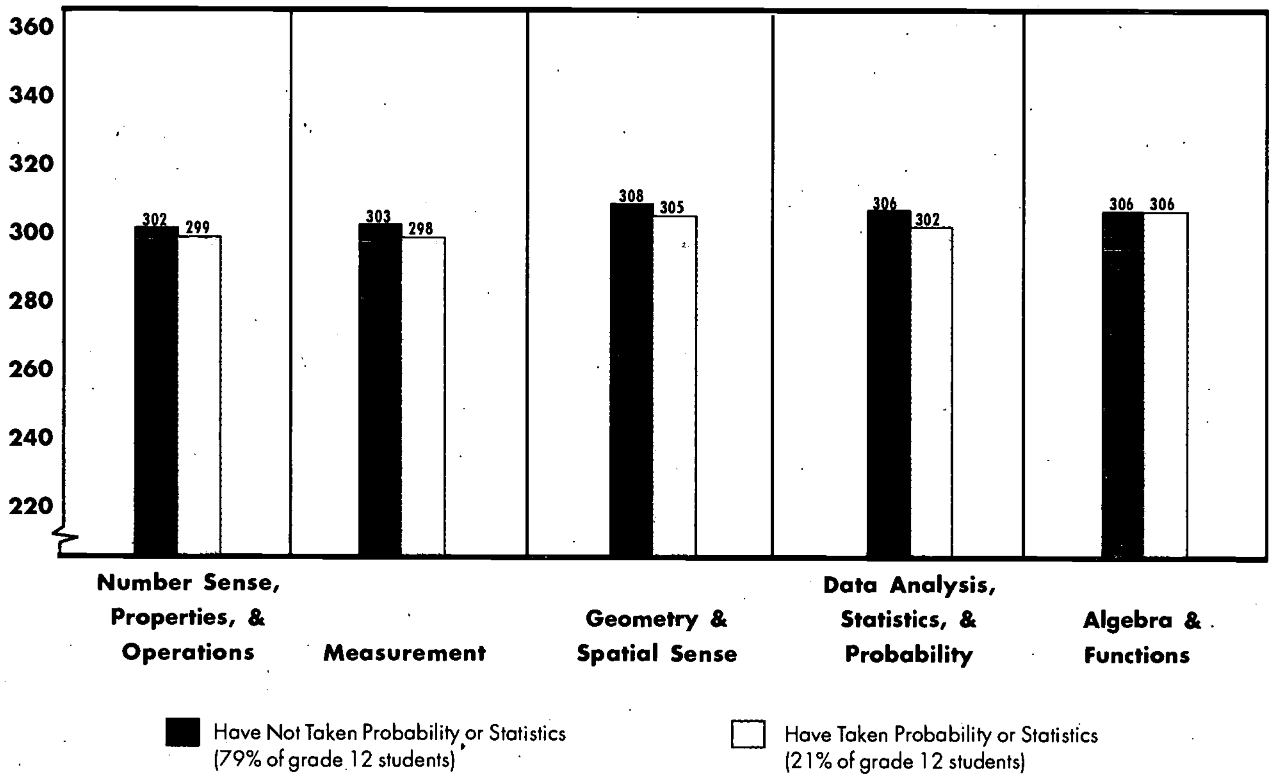

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Figure 2.13 **Average Proficiency in Mathematics Content Areas by Number of Semesters of Mathematics Courses Taken in Grades 9 through 12, Grade 12**

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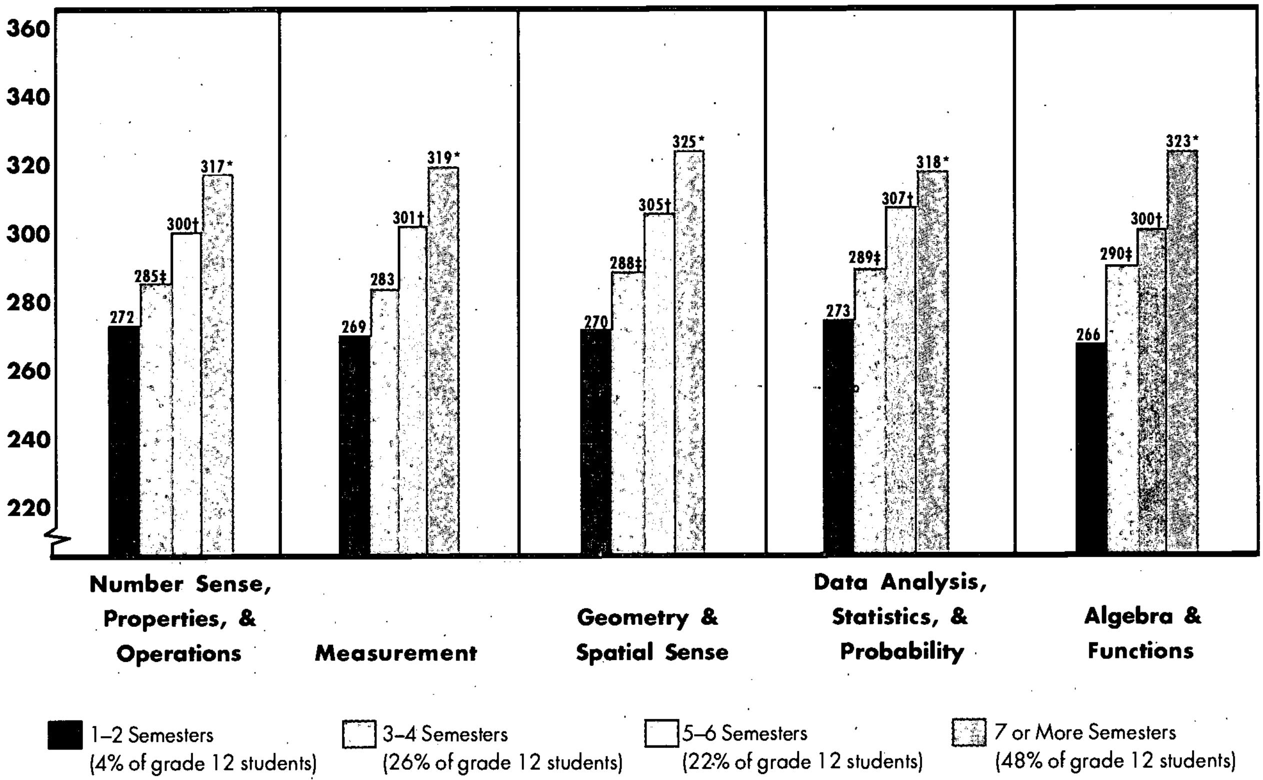

NOTE: Sample size for 0 semesters is insufficient to permit a reliable estimate.

* Indicates a significant difference between results for students with 7 or more semesters and students with 5–6 semesters of mathematics.

† Indicates a significant difference between results for students with 5–6 semesters and students with 3–4 semesters of mathematics.

‡ Indicates a significant difference between results for students with 3–4 semesters and students with 1–2 semesters of mathematics.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Introduction to Content Strand Chapters

So far, this report has provided an overall look at performance trends in the five mathematics content strands. Chapters 3–7 provide a detailed look at student performance *within* each of the five content strands and offer many examples of assessment questions and actual student work. The goal of Chapters 3–7 is to provide the reader with a general understanding of a) the range of skills assessed within each content strand, b) how 1996 performance varied within each content strand by grade, and c) how performance varied across grades. When reading these chapters, it is important to bear in mind the variety of factors that influence student performance and, therefore, the relative difficulty of particular questions. One source of performance variation is content, represented in the NAEP 1996 mathematics assessment by the content strands. Students' opportunities to learn content vary, as does their ability to retain what they have learned and to apply their content knowledge to assessment questions. However, other sources of performance variation are related to the questions themselves, irrespective of the content. Some of these sources — the type of question (e.g., multiple-choice, constructed-response), the extent to which the question draws upon the different mathematical abilities (e.g., conceptual understanding, procedural knowledge, problem solving) and mathematical power (e.g., communication, reasoning), and the use of manipulatives — were described in Chapter 1. Other characteristics of questions, such as how the question is presented (e.g., as a real-life problem, as a numerical equation, with a pictorial or graphical representation) and the number of steps required to reach a solution, also influence performance. Thus, two questions may be assessing the same content but may elicit different results by virtue of how the problem was presented to the students, what students were asked to do with the content, and how students were asked to respond. Understanding how and why student performance varies, therefore, entails more than just knowing what content a question was measuring; it also entails knowing *how* the question was measuring what it measured.

In order to provide a better understanding of the multiple ways in which different item characteristics can be combined, Table 2.2 lists a few questions that appear as examples in the chapters that follow. The device of "map number," used in the table, is described in more detail below, but questions with higher map numbers are generally more difficult than questions with lower numbers.

In each of the chapters that follow, several questions have been selected from a set of released questions to illustrate what is assessed in each content strand. To provide the reader with a ready visual reference for the relative difficulty of the questions, they have been mapped onto the NAEP composite mathematics scale, which is the measure of overall mathematics achievement. For each question, the item map provides a marker of the performance level at which students are relatively likely to answer the question correctly.[4] The questions for all three grade levels map onto the composite scale, whose possible values range between 0 and 500. Most fourth-grade questions map to the lower end, most eighth-grade questions map to the middle, and most twelfth-grade questions map to the higher end of the scale. Some questions

[4] The procedures used to develop the item maps are detailed in Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). *The NAEP 1996 technical report*. Washington, DC: National Center for Education Statistics.

were administered to students at more than one grade level and may map at slightly different values for each grade. That is, mathematics skills are generally cumulative in nature, justifying the use of a single scale to portray the growth in mathematics achievement across years. The relationship between performance on a specific question and overall mathematics performance, however, may not be entirely consistent across grades.

Table 2.2 ***Characteristics of Sample Questions from the NAEP 1996 Mathematics Assessment***

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Question	Map	Grade	Content Strand	Question Type[a]	Ability[b]	Power[c]	Other
Relate a Fraction to 1	248	4	Num.	SCR	CU		
Describe Measurement Task	332	4	Meas.	SCR	PS	RE/CM/CN	real-life problem
Compare Two Geometric Shapes	324	4	Geo.	ECR	PS		question contains a picture
Translate Words to Symbols	281	8	Alg.	MC	CU		real-life problem
Subtract Integers	335	8	Alg.	SCR	PK	RE/CN	real-life problem, multistep
Compare Mean and Median	463	12	Data	ECR	PS	RE/CM/CN	question contains a table
Draw a Parallelogram with Perpendicular Diagonals	356	12	Geo.	SCR	PS		solution requires drawing

[a] MC = Multiple-Choice, SCR = Short Constructed-Response, ECR = Extended Constructed-Response.
[b] CU = Conceptual Understanding, PK = Procedural Knowledge, PS = Problem Solving.
[c] RE = Reasoning, CN = Connecting, CM = Communicating.

To see how to interpret the item map, consider the first question in the table above, "Relate a Fraction to 1." This is a short constructed-response question for fourth graders that is scored right/wrong. It maps at a scale score of 248 (refer to item map, Figure 2.14). Mapping the question at a score of 248 on the NAEP composite mathematics scale implies that students whose overall mathematics proficiency scores are 248 or higher have at least a 65 percent chance of correctly answering this question.[5] Students whose overall mathematics proficiency scores are below 248 have less than a 65 percent chance of correctly answering the question. This does not mean that students at or above the 248 level always answer this question correctly, or that students below the 248 level always answer the question incorrectly. Rather, students have a higher or lower probability of successfully answering the question depending on their overall ability as measured on the NAEP mathematics scale.

[5] For constructed-response questions, a criterion of 65 percent was used. For multiple-choice questions with four or five alternatives, the criteria were 74 and 72 percent, respectively. The use of a higher criterion for multiple-choice questions reflects students' ability to guess the correct answer from among the alternatives.

Note that the item mapping refers to student performance on the *composite* mathematics scale (i.e., the scale for general mathematics performance) and not to performance on the separate scales for each content strand. Thus, in the example above, the map number for the question, which is from the Number Sense, Properties, and Operations content strand, locates the question in relation to the performance of fourth-grade students on the entire mathematics assessment, as opposed to their performance on the Number Sense, Properties, and Operations content strand alone.

Chapters 3–7 discuss student performance within each of the five content strands in more detail. Several assessment questions from the content strands of the NAEP 1996 assessment are shown in these chapters. The sample questions were selected from those released at each of the three grade levels to illustrate varying difficulty levels, a variety of question formats, and different mathematical abilities. Examples also were chosen to illustrate how questions tested students' conceptual understanding and procedural knowledge, as well as their abilities to reason, communicate, and make connections.

In each chapter, the content in a particular content strand is further organized into areas for ease of presentation. The organizing areas are not mutually exclusive, and many questions required students to use knowledge and skills from more than one area. At least one sample question is presented for each area, and information about students' performance on each sample question is given for all students, as well as for students classified by gender and race/ethnicity. For questions on the eighth-grade assessment, student performance also is examined with respect to the mathematics course students currently are taking. For questions on the twelfth-grade assessment, performance is examined with respect to the highest mathematics course students have taken in the algebra-through-calculus sequence.

The impact of taking geometry on student performance at the twelfth-grade level is not discussed, although the data are presented in the tables. Because more able students are likely to progress further in the mathematics course sequence, it is difficult to separate the impact of a particular curriculum from the impact of the student's overall strengths in mathematics. In addition, the pool of students on which the specific influence of geometry could be isolated is rather small: 90 percent of students report having 1 year or more of first-year algebra, 80 percent report having 1 year or more of geometry, and 70 percent report having 1 year or more of second-year algebra. Therefore, on the assumption that second-year algebra typically follows geometry, only about 10 percent of students could be classified as having had first-year algebra and geometry, but no further mathematics. For these reasons, discussion of the impact of mathematics courses on student performance at twelfth grade is limited to the algebra-through-calculus sequence.

Figure 2.14

Map of Selected Questions on the NAEP Composite Mathematics Scale (Item Map)

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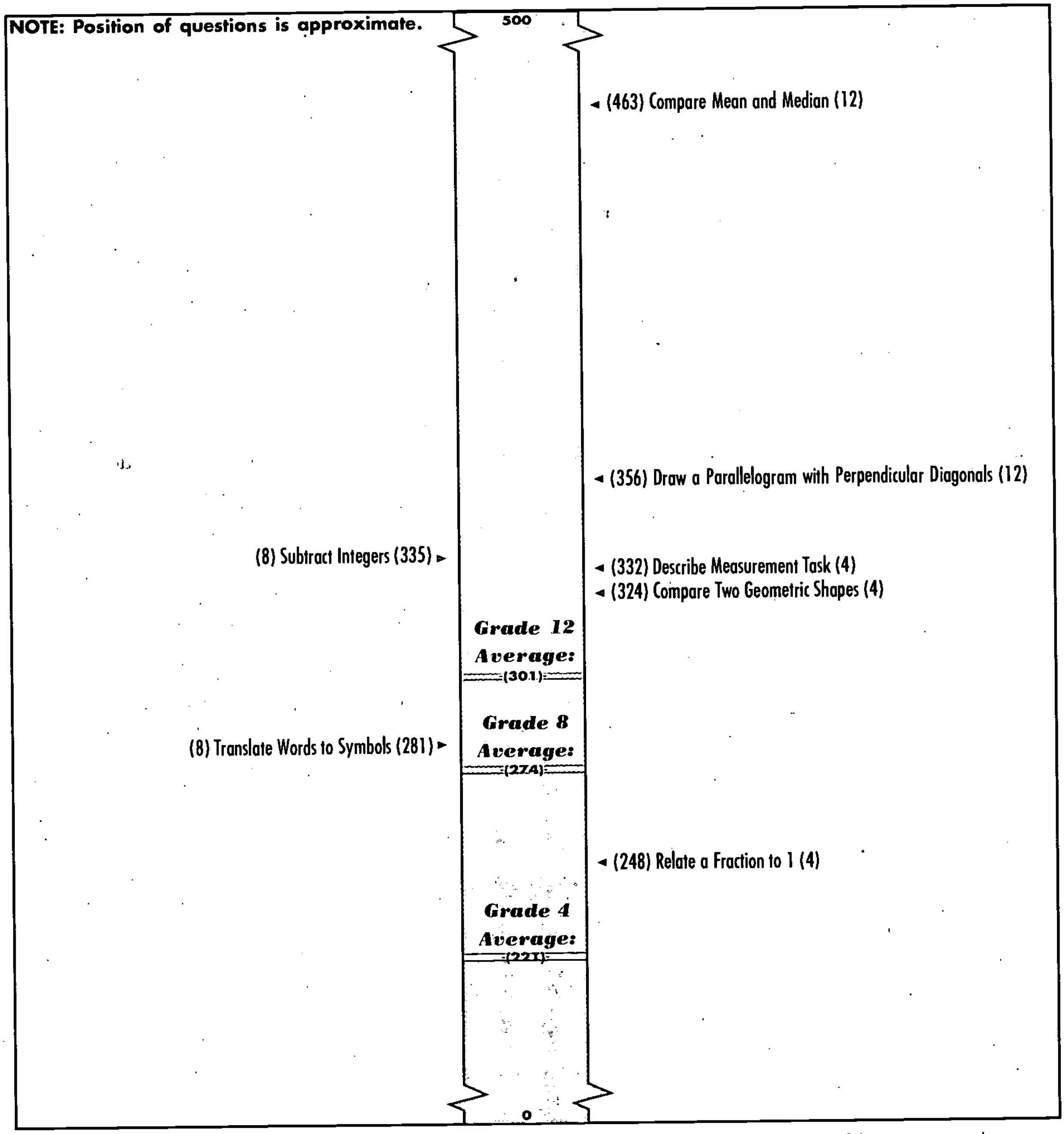
NAEP Scale



NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Chapter 3

Number Sense, Properties, and Operations

Content Strand Description

The questions included in the content strand of Number Sense, Properties, and Operations primarily covered basic arithmetic skills and concepts. These skills and concepts represent a significant part of the mathematics curriculum, particularly at the lower grade levels, at most American schools. Reflecting this emphasis, a large portion of the questions on the NAEP 1996 mathematics assessment fell under this content strand, although the portion was smaller than in 1990 and 1992. As shown in Table 1.1 in Chapter 1, 40 percent of the mathematics questions given to fourth-grade students, 25 percent of those given to eighth-grade students, and 20 percent of those given to twelfth-grade students fell into this content strand.

The Number Sense, Properties, and Operations content strand focused on students' understanding of numbers (whole numbers, fractions, decimals, integers, real numbers, and complex numbers), operations, and estimation, and on application of their understanding to real-world situations. Questions in this content strand required students to demonstrate an understanding of number properties and operations, to generalize from numerical patterns, and to verify results. The questions also assessed student understanding of numerical relationships as expressed in ratios, proportions, and percents.

At all grade levels, students were assessed on their comprehension of number concepts and properties as well as their skills in addition, subtraction, multiplication, and division of whole numbers, simple fractions, and decimals. This included their knowledge of correct mathematical procedures and their ability to apply this knowledge to solve problems. At the eighth-grade level, students were required to demonstrate skill with whole numbers, fractions, decimals, integers, and rational numbers. Eighth-grade students also were assessed on their ability to use ratios and proportions and on their ability to read, use, and apply scientific notation to represent large and small numbers. At grade 12, questions within this content strand covered real and complex numbers as well as operators such as powers and roots. Students at all grades were assessed on their ability to reason mathematically and to communicate the reasoning they used to solve problems involving number sense, properties, and operations.

Examples of Individual Questions and Student Performance

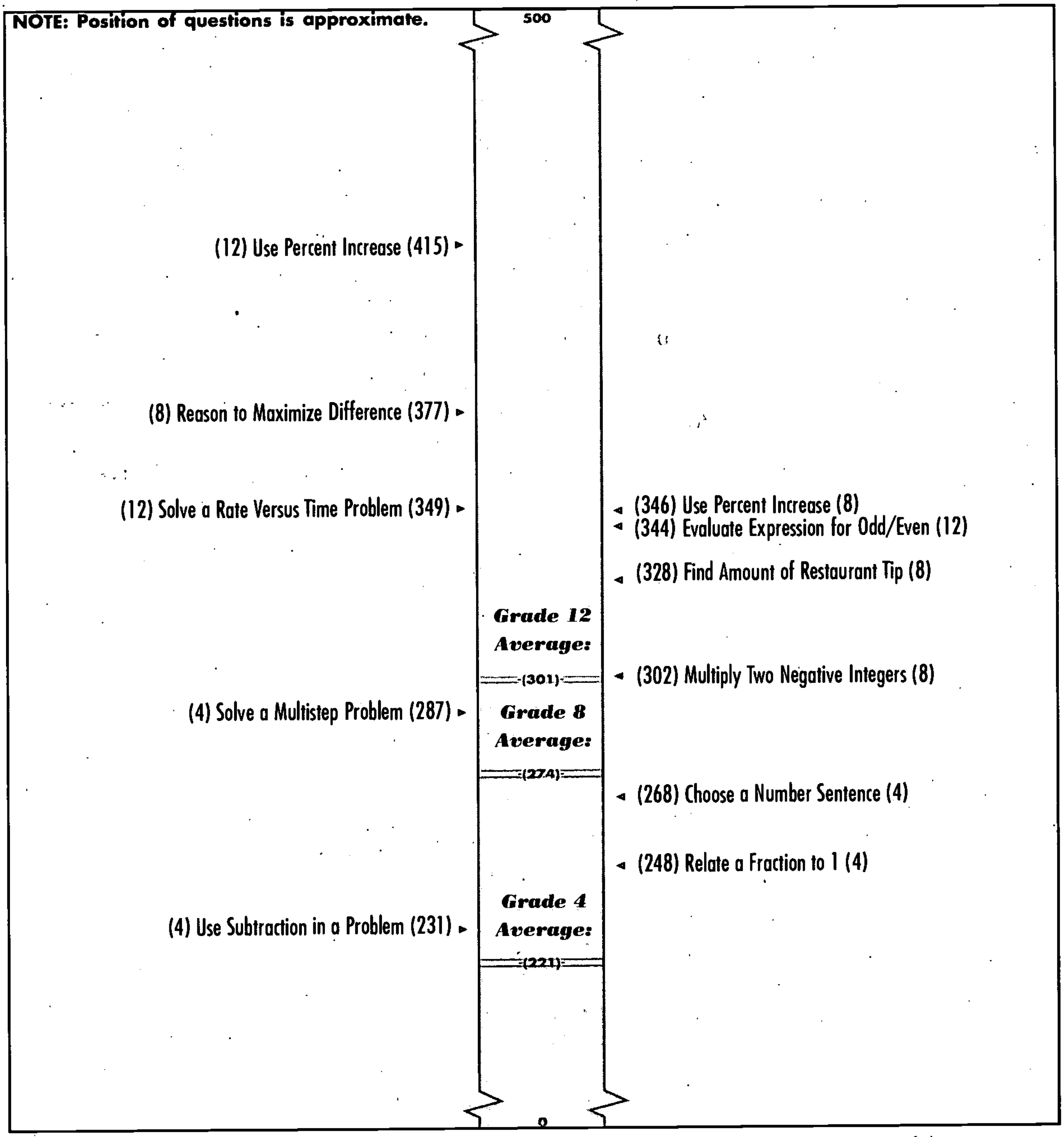
Some of the Number Sense, Properties, and Operations questions from the NAEP 1996 mathematics assessment follow. Presentation of the questions is organized around five areas of emphasis within the content strand of Number Sense, Properties, and Operations. The first three areas of emphasis are directly related to the mathematical abilities from the NAEP mathematics framework. Thus, the area of *number meanings, properties, and other number concepts* includes questions that assessed a student's conceptual understanding of numbers and related number concepts; the area of *computation* includes questions that assessed a student's procedural knowledge of number operations; and the area of *application of computations* includes questions that assessed a student's problem-solving abilities. Two additional areas, *rounding and estimation* and *fractions, ratios, and proportions,* include questions that measured students' abilities to use skills specifically related to these two topics. Questions within all five areas also required students to reason, communicate, and make connections.

All sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 3.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide a visual summary of the relative difficulty of each sample question and, thus, of the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. Keep in mind, however, that the difficulty of a question is influenced by many factors, including characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application) as well as the particular mathematics content associated with the question and student opportunities to learn this content. Remember also that overall performance on the Number Sense, Properties, and Operations content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

Figure 3.1 **Map of Selected Number Sense, Properties, and Operations Questions on the NAEP Composite Mathematics Scale (Item Map)**

THE NATION'S REPORT CARD naep

NAEP Scale



NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
NOTE: The map values for the question "Use Percent Increase" are very different for grades 8 and 12. This is because the question was treated differently in the estimation of achievement at the two grades. See discussion in text.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Number meanings, properties, and other number concepts

Questions in this area required students to demonstrate a conceptual understanding of numbers. A relatively small proportion of the questions in the content strand fell into this area.

The questions for fourth-grade students that fell into this area tapped concepts related to the relative size of numbers, place value of whole and decimal numbers, and basic multiples (e.g., of 5 and 10). The questions for eighth-grade students measured their understanding of odd and even numbers and the properties of these numbers. More difficult questions included concepts of scientific notation and power. Twelfth-grade questions asked students to perform manipulations of place value and to apply their understanding of numbers to mathematical problems.

The following example from the twelfth-grade assessment tested students' understanding of the attributes of odd and even numbers and required a short constructed response.

> If x and y are integers, then the expression $4x + 5y$ has a value that is odd or even depending on the values of x and y. For example, if x and y are each even, $4x$ is even and $5y$ is even. Therefore, $4x + 5y$ is also even. Fill in each of the blank spaces in the following table with either "odd" or "even" for the value of $4x + 5y$.

Value of x	Value of y	Value of $4x + 5y$
even	even	even
even	odd	
odd	even	
odd	odd	

The correct answers in descending vertical order are "odd," "even," and "odd."

This question assessed student understanding of what happens when odd and even numbers are multiplied and when they are summed. Students could have answered the question by trial and error (i.e., inserting various combinations of odd and even numbers into the equation). However, students could have responded more quickly if they had realized that odd numbers result only by multiplying two odd numbers or adding an even and an odd number. The question also incorporated symbolic (algebraic) notation and, thus, also evaluated a student's ability to make connections across content strands. If all three entries in the table were correctly listed as "odd," "even," "odd," the response was considered "correct." All other responses were considered "incorrect."

Student performance is reported in Tables 3.1 and 3.2. The title of the question (in quotation marks) can be used to locate the question on the item map in Figure 3.1. Thirty-eight percent of the students correctly responded to the question; that is, they had three correct entries in their tables. Fifty-seven percent had "incorrect" responses to the question, with 54 percent having one or two correct entries in their tables and 3 percent having no correct entries.[1] Six percent of students did not attempt the question. As might be expected, students who had taken more advanced mathematics courses were more likely to respond correctly to the question than students who had taken less advanced courses. For example, among twelfth-grade students who had taken or were taking calculus, 68 percent responded correctly to this question as opposed to only 16 percent of the students who had taken no algebra courses beyond pre-algebra.

Table 3.1 **Score Percentages for "Evaluate Expression for Odd/Even"**

THE NATION'S REPORT CARD naep

	Correct	Incorrect		Omit
	3 Correct Entries	1 or 2 Correct Entries	No Correct Entries	
Grade 12				
Overall	38	54	3	6
Males	36	54	4	7
Females	39	54	2	5
White	40	54	2	5
Black	36	51	6	7
Hispanic	22	63	5	10
Asian/Pacific Islander	57	35	2	6
American Indian	***	***	***	***
Geometry Taken	41	51	2	5
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	16	70	3	9
First-Year Algebra	26	62	5	6
Second-Year Algebra	41	53	2	4
Third-Year Algebra/Pre-Calculus	53	41	2	5
Calculus	68	23	1	8

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

When performance is disaggregated by achievement level, Table 3.2 shows that 17 percent of students below the *Basic* level, 40 percent of students at the *Basic* level, and 67 percent of those at the *Proficient* level filled in their tables correctly. The question mapped at score 344 on the NAEP composite mathematics scale, meaning that students who scored 344 or above on the overall NAEP scale could likely fill in the table correctly.

Table 3.2 ***Percentage Correct Within Achievement-Level Intervals for "Evaluate Expression for Odd/Even"***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	**Basic**	**Proficient**	**Advanced**
38	17	40	67	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Computational skills

Questions that fell within the area of computational skills assessed students' procedural knowledge of number operations. These questions ranged from those that presented students with a number sentence (e.g., 2 + 5 = __) and required them to solve for the missing number, to more complex questions that might have required one, two, or multiple steps (i.e., operations). In some cases, students might have needed to recognize the order in which the steps were to be completed. However, in all cases, the operations to be performed were made explicit for the student. Computation questions were inherently context free; that is, they were not tied to any real-life situation or problem. They required the student to perform more or less routine calculations.

Computation questions at fourth grade primarily tended to be one or two steps and required the student to add, subtract, multiply, or divide whole or, sometimes, decimal numbers. At times, students were asked to perform two operations, but the order in which the operations were performed typically did not matter. At eighth grade, the calculations began to include negative numbers and the use of parentheses to designate the order in which operations needed to be performed. Some twelfth-grade questions included the use of exponents or algebraic notation and typically involved larger numbers than did questions at lower grade levels.

The following example is an eighth-grade question. The question required the student to multiply two negative numbers. It was a multiple-choice question and tested procedural knowledge of multiplication. Additionally, in order to respond correctly to the question, students needed to understand that the use of parentheses in this question indicated multiplication, to recognize that the computation involved negative numbers, and to know that the product of two negative numbers is a positive number. The question mapped at a score of 302 on the NAEP composite mathematics scale.

3. (– 5)(– 7) =

(A) – 35
(B) – 12
(C) – 2
(D) 12
(E) 35

The correct option is E.

Performance data for this question are presented in Tables 3.3 and 3.4. Fifty percent of the students selected the correct option. Twenty percent of the students chose Option A, suggesting correct multiplication of the absolute value of the numbers but lack of knowledge of how to multiply negative numbers. Another 14 percent chose Option B, suggesting a lack of understanding of the arithmetic operation they were to perform.

Familiarity with negative numbers may depend on the student's curriculum. When student performance was examined by course enrollment, students in eighth-grade mathematics had the most difficulty with the question. Students in pre-algebra performed better than those in eighth-grade mathematics, and those in algebra performed better than students in both eighth-grade mathematics and pre-algebra.

Table 3.3 ***Percentage Correct for "Multiply Two Negative Integers"*** THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	50
Males	47
Females	54
White	55
Black	41
Hispanic	34
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	35
Pre-Algebra	51
Algebra	75

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

As shown in Table 3.4, 56 percent of those at the *Basic* level, 79 percent of those at the *Proficient* level, and 94 percent of those at the *Advanced* level selected the correct option. Only 25 percent of the students functioning below the *Basic* level responded correctly to this question.

Table 3.4 ***Percentage Correct Within Achievement-Level Intervals for "Multiply Two Negative Integers"***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
50	25	56	79	94

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Application of computational skills

Questions in this area assessed students' abilities to apply their computational skills to solve real-life problems. These questions were of the type traditionally referred to as word or story problems. What distinguished these questions from basic computation questions is that the questions were placed in a real-life context, requiring students to determine what operations they needed to perform and what numbers they needed to use in those operations. Sometimes the questions also presented extraneous or irrelevant information. Most often, students needed to identify and perform an arithmetic operation to arrive at an answer to the problem presented; however, at times they were simply asked to identify a number sentence that would lead to a correct solution. Again, arriving at a solution could entail one or several steps. Fourth-grade questions could often be solved in one or two steps, required simple computations, and presented little extraneous information. At grades 8 and 12, the questions involved more complex computations, required several steps, and presented more information for the student to synthesize. A fairly large proportion of the questions at all three grades fell into this area of emphasis, although they may have required the student to use skills in other areas such as rounding or proportional reasoning as well.

Three examples are presented for this area. The first two examples are fourth-grade multiple-choice questions, and the third is an eighth-grade extended constructed-response question.

The first question provided students with information about a partially completed driving trip and asked them to determine the remaining distance to be driven. In order to compute the number of miles left, students had to identify which numbers were extraneous and which were essential to the calculation, recognize that they needed to subtract, and know which number to subtract from the other; they then had to perform the subtraction correctly. Thus, the question also assessed mathematical reasoning and procedural knowledge in addition to problem-solving ability. It was a fairly easy question and mapped at a composite scale score of 231.

1. Kitty is taking a trip on which she plans to drive 300 miles each day. Her trip is 1,723 miles long. She has driven 849 miles. How much farther must she drive?

(A) 574 miles
(B) 874 miles
(C) 1,423 miles
(D) 2,872 miles

Did you use the calculator on this question?

◯ Yes ◯ No

The correct option is B.

Student performance data are presented in Table 3.5, and the percentage of students within each achievement-level interval who successfully answered the question is presented in Table 3.6. Sixty-four percent of the students answered the question correctly. Incorrect responses were evenly distributed across the other options. Seventy-five percent of students at the *Basic* level and more than 90 percent of students at the *Proficient* level selected the correct response.

Table 3.5 — ***Percentage Correct for "Use Subtraction in a Problem"***

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	64
Males	65
Females	63
White	71
Black	43
Hispanic	53
Asian/Pacific Islander	***
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

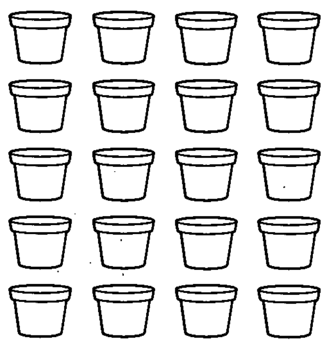
Table 3.6 *Percentage Correct Within Achievement-Level Intervals for "Use Subtraction in a Problem"*

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
64	34	75	94	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second question also is a multiple-choice question for fourth-grade students. It presented the student with a graphic of flowerpots arranged in five rows and four columns. The student needed to determine which of the four number sentences would enable "Kevin" to calculate the number of seeds needed if three seeds were to be placed in each pot. The question also assessed the student's understanding of operations in that the student needed to know that a correct answer required Kevin to multiply the number of seeds by the number of rows and the number of pots per row. The question mapped at a score of 268 on the composite mathematics scale.



5. The picture shows the flowerpots in which Kevin will plant flower seeds. He needs 3 seeds for each pot. Which of the following number sentences shows how many seeds Kevin will need for all of the pots?

Ⓐ $5 \times 4 \times 3 = \square$

Ⓑ $(5 \times 4) + 3 = \square$

Ⓒ $(5 + 4) \times 3 = \square$

Ⓓ $5 + 4 + 3 = \square$

The correct option is A.

Student data for this question are presented in Tables 3.7 and 3.8. Fifty percent of the students answered the question correctly; however, 25 percent of the students chose Option B as the correct response. These students may have recognized that they needed to multiply the number of rows by columns in order to determine the number of flowerpots, but failed to recognize they also needed to multiply by the number of seeds.

Table 3.7 **Percentage Correct for "Choose a Number Sentence"**

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	50
Males	50
Females	50
White	53
Black	42
Hispanic	45
Asian/Pacific Islander	45
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

According to Table 3.8, a correct response to this question was provided by approximately three-quarters of the fourth-grade students classified as *Proficient*, half of those classified as *Basic*, and 30 percent of those classified as below *Basic*.

Table 3.8 **Percentage Correct Within Achievement-Level Intervals for "Choose a Number Sentence"**

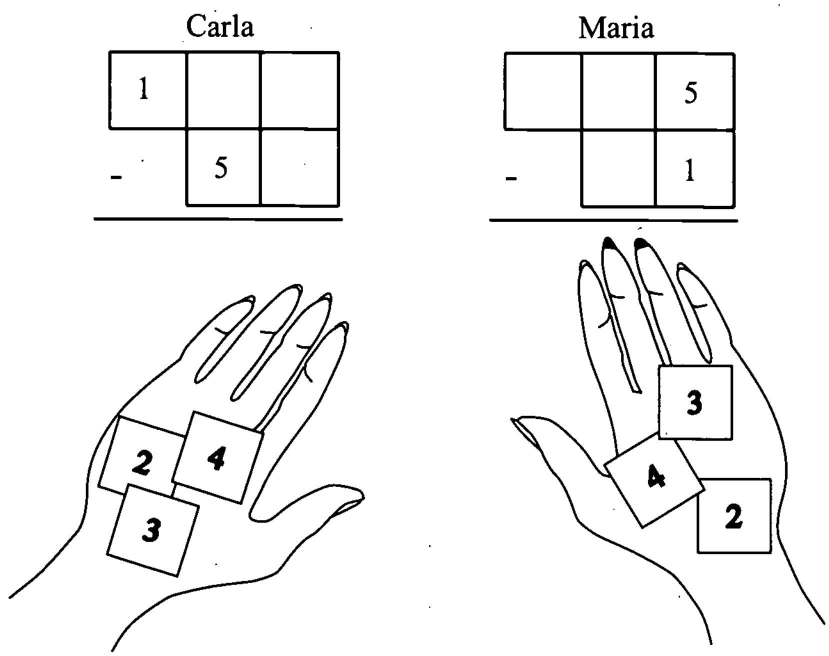
THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
50	30	53	74	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example is a problem-solving question for eighth-grade students that was set in the context of a mathematical game. Students in today's classrooms often are presented with such games, sometimes referred to as brain twisters, mind benders, or math challenges. This question involved an extended constructed response, requiring the student not only to reason, but also to communicate mathematically.

The question first presented students with some general directions explaining that it was important for them to show their work and explain their reasoning so that someone reading their response could understand their thinking. Next, students were shown pictures representing the ways two players had placed their numbered tiles for a subtraction problem and were told that the player with the largest answer would win the game. Students then were asked to state who would win the game and to explain how they knew that person would win.

> This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.
>
> **13.** In a game, Carla and Maria are making subtraction problems using tiles numbered 1 to 5. The player whose subtraction problem gives the largest answer wins the game.
>
> Look at where each girl placed two of her tiles.
>
> 
>
>
> Who will win the game? ______________________________
>
> Explain how you know this person will win.

The correct answer is Maria.

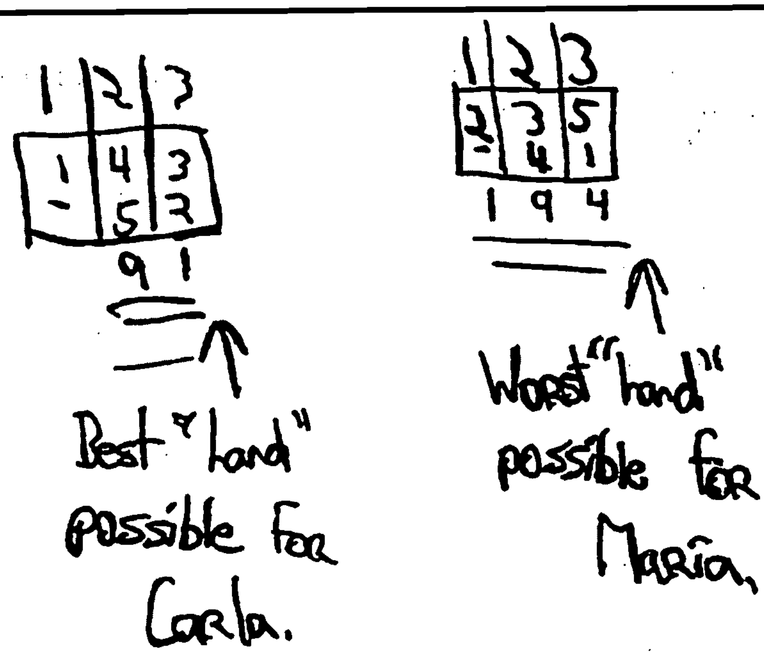
In rating student responses, readers could rate a response as "extended," "satisfactory," "partial," "minimal," or "incorrect." Students whose responses were considered to be "extended" correctly identified Maria as the winner by describing the answers to the subtraction problems. Examples of "extended" explanations included:

- The largest possible difference for Carla is less than 100, and the smallest possible difference for Maria is 194.

- Carla will only get a difference of 91 or less, but Maria will get several larger differences.

A sample "extended" response follows. This student displayed Carla's best possible hand and Maria's worst possible hand, labeled each as such, and explained that if these hands were played, Maria would win. This student clearly demonstrated to a reader the reasoning required to reach the correct conclusion.

Sample "extended" response



Who will win the game? Maria

Explain how you know this person will win.

Because, If Maria places the rest of her cards in the worst order and Carla places the Rest of her cards in the best order than Maria still wins.

A response was considered to be "satisfactory" if the student correctly identified Maria as the winner and gave an explanation that indicated the rudimentary elements of a correct generalization. Acceptable "satisfactory" explanations included:

- Carla can have only up to 143 as her top number, but Maria can have 435 as her largest number.
- Carla has only one hundred, but Maria can have two, three, or four hundreds.
- Maria can never take away as much as Carla.

In the sample "satisfactory" response that follows, the student recognized and stated that Maria would always win because her top number would always be higher than Carla's and her bottom number always lower.

Sample "satisfactory" response

Who will win the game? Maria

Explain how you know this person will win.

Maria will win because no matter which one she plays as her first digit in the top number it will be more than 1 and on the bottom it will be less than 5 so carla has no chance of winning.

A response was considered "partial" if the student correctly identified Maria as the winner of the game but provided a partially correct or incomplete explanation. For example, in the following sample response, the student explained that Carla "made a mistake" by putting the "1" in the hundreds place, but did not complete the explanation by telling why this was a mistake and, thus, did not explain why Maria would always win.

Sample "partial" response

Who will win the game? Maria

Explain how you know this person will win.

because I to on maria's side 431 on carla's side she

$$\begin{array}{r} 431 \\ -\ 21 \\ \hline 414 \end{array}$$

already made a Mistake by putting the 1 in first at the top. So then I took 143 on carla's

$$\begin{array}{r} 143 \\ -\ 52 \\ \hline 91 \end{array}$$

side, wich leaves Maria as the winner

69

A response was considered "minimal" if it correctly identified Maria as the winner of the game but included no explanation, an incorrect explanation, or some response that did not enable the reader to determine how the student reached the conclusion. The following "minimal" response provides an example of a student who correctly identified the winner of the game but failed to explain why Maria's score could never be lower than Carla's. Thus, a reader would be unable to determine if the student arrived at the conclusion simply by randomly placing numbers in the squares or whether the student truly understood why Maria had to win.

Sample "minimal" response

Who will win the game? M'IRA

Explain how you know this person will win.

Becaus she has 435
- 21
414
hers is hier
then Carla

To be evaluated as anything other than "incorrect," students' responses had to correctly identify Maria as the winner of the game. That is, "incorrect" answers were answers that indicated an outcome other than Maria winning the game. The following response is an example of an "incorrect" response.

Sample "incorrect" response

Who will win the game? tie

Explain how you know this person will win.

They both have the same numbers

71

Information on student performance in this question is presented in Tables 3.9 and 3.10. This question was quite difficult for students, and when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. While most eighth graders (95%) attempted to answer the question, only 15 percent provided a response that was rated at least "satisfactory." Another 16 percent provided responses rated "partial," and more than 60 percent provided responses rated "minimal" or "incorrect."

Table 3.9 — ***Score Percentages for "Reason to Maximize Difference"***

THE NATION'S REPORT CARD naep

Grade 8	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	1	14	16	32	31	5
Males	1	12	14	32	35	5
Females	1	17	19	32	26	4
White	1	17	18	29	29	5
Black	0!	7	10	41	36	5
Hispanic	0!	4	17	35	37	5
Asian/Pacific Islander	– –	– –	– –	– –	– –	– –
American Indian	***	***	***	***	***	***
Mathematics Course Taking:						
Eighth-Grade Mathematics	1	9	14	32	37	6
Pre-Algebra	0	12	19	31	32	5
Algebra	2	24	19	32	20	3

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Only 13 percent of students at the *Basic* level, 29 percent of students at the *Proficient* level, and 61 percent of students at the *Advanced* level submitted a response that was considered at least "satisfactory." The question mapped at 377 on the NAEP composite mathematics scale.

72

Table 3.10 — ***Percentage at Least Satisfactory Within Achievement-Level Intervals for "Reason to Maximize Difference"***

THE NATION'S REPORT CARD

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
15	4	13	29	61

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Rounding and estimation

Some questions in the Number Sense, Properties, and Operations content strand assessed students' abilities to round numbers and to estimate. Questions of this nature were either presented abstractly as a number or set of numbers for the student to round, or were presented within the context of a real-life type of problem. Students at each grade were asked to round whole as well as decimal numbers. Questions asking students to apply their rounding and estimation skills often involved money.

The example is a fourth-grade short constructed-response question presenting the student with prices for lunch items and asking the student to indicate the minimum number of one-dollar bills needed to pay for lunch for a week if the same items were purchased every day.

7. Sam can purchase his lunch at school. Each day he wants to have juice that costs 50¢ a sandwich that costs 90¢ and fruit that costs 35¢. His mother has only $1.00 bills. What is the least number of $1.00 bills that his mother should give him so he will have enough money to buy lunch for 5 days?

   Did you use the calculator on this question?

   ◯ Yes ◯ No

The correct response is 9.

Responses were rated as "correct," "partial," or "incorrect." In order for a response to be rated as "correct," a student needed to add the cost of the items for a single day ($1.75) and then multiply this cost by 5 to determine the cost for 5 days ($8.75). Finally, the student needed to round this number to $9.00 and recognize that 9 one-dollar bills would be needed to buy lunch for a week, as shown in the following sample "correct" response. Note that the students were permitted to use calculators and were not required to show their work or provide an explanation in order for a response to be considered "correct." Simply writing down the number "9" would have been considered "correct."

Sample "correct" response

> $9.00
>
> Sam's mother should give Sam $9.00, because if you add 50¢ + 90¢ + 35¢ that equals $8.75. You round $8.75 you would get $9.00 because Sam's mother only has dollar bills.
>
> Did you use the calculator on this question?
>
> ● Yes ○ No

Rounding the weekly total down to $8.00 or estimating $2.00 each day for a total of $10.00 resulted in a response rated as "partial," as did small errors in computation. In the following sample response, the student correctly calculated the cost of lunch per day, but indicated rounding this number to $2.00. The student's final answer of "10 bills" was rated "partial."

Sample "partial" response

> 90¢
> 50¢
> × 35¢
> $1.75 [illegible]
>
> 10 bills
>
> Did you use the calculator on this question?
>
> ○ Yes ● No

74

All other answers were considered to be "incorrect." In the next sample response, the student presumably calculated the cost per day and for the week on the calculator. The student reached the correct total of "875" but did not place the decimal correctly.

Sample "incorrect" response

$875,00

Did you use the calculator on this question?

● Yes ○ No

Student data are presented in Tables 3.11 and 3.12. This question was difficult for most students. Ten percent of the students did not respond to the question, and half of the students responded incorrectly. The remaining students' responses split almost evenly between "correct" and "partial." Omitting the question was more common among Black students than among students from other racial/ethnic groups.

THE NATION'S REPORT CARD naep

Table 3.11 ***Score Percentages for "Solve a Multistep Problem"***

Grade 4	Correct	Partial	Incorrect	Omit
Overall	17	20	51	10
Males	19	22	45	11
Females	15	18	57	8
White	21	23	47	7
Black	6	9	63	20
Hispanic	6	15	63	11
Asian/Pacific Islander	***	***	***	***
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Only 1 percent of grade 4 students classified as below *Basic* and 14 percent of those classified as *Basic* responded correctly to the question. Forty-four percent of those classified as *Proficient* responded correctly. The question mapped at 287.

Table 3.12 **Percentage Correct Within Achievement-Level Intervals for "Solve a Multistep Problem"**

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
17	1	14	44	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Fractions, ratios, and proportions

The largest proportion of the Number Sense, Properties, and Operations questions measured student skills and knowledge in the areas of fractions, decimal fractions, percentages, ratios, and proportions. Many of these questions were among the most difficult for students. They included questions that required students to identify appropriate representations of common and decimal fractions, to order or identify equivalent fractions, and to apply their skills to computations involving fractions and percentages or problems involving proportional reasoning.

Fourth-grade questions covered representation, equivalence, and ordering of common fractions such as $^1/_2$ or $^1/_3$. Some of the more difficult questions involved decimals. Eighth-grade questions involved manipulation of more complex fractions, sometimes requiring the student to identify a least common denominator or to simplify the representation (i.e., reduce the fraction). Some questions required an understanding of the relationship between common and decimal fractions or involved the use of percentages. The twelfth-grade questions required students to exhibit such skills as explaining the relationship between common and decimal fractions and percentages, calculating fractions of fractions or interest, and reasoning with proportions in complex situations.

Four sample questions are presented for this area: one fourth-grade question, one eighth-grade question, one question that was presented at both eighth and twelfth grades, and one twelfth-grade question. The fourth-grade question was a short-answer question involving common fractions. The eighth-grade question involved calculation of a percentage. The eighth- and twelfth-grade question assessed student understanding of and ability to calculate percent increase. The twelfth-grade question was a rate versus time question. The example for grade 4 students follows.

> **1.** How many fourths make a whole?
> Answer: ____________________

The correct answer is 4.

This question tested students' understanding of how fractions relate to a whole and required them to write a short response. The responses were rated "correct" or "incorrect," and a variety of responses such as "4," or "four fourths," or "4 fourths," etc., were accepted as "correct." Student performance data are presented in Table 3.13. Table 3.14 shows the percentage of students within each grade 4 achievement-level interval on the NAEP composite scale who successfully answered the question.

Table 3.13 ***Percentage Correct for "Relate a Fraction to 1"***

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	50
Males	50
Females	50
White	57
Black	29
Hispanic	33
Asian/Pacific Islander	53
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 3.14 Percentage Correct Within Achievement-Level Intervals for "Relate a Fraction to 1"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
50	22	56	81	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Overall, 50 percent of fourth-grade students answered this question correctly. Sixteen percent of the students omitted the question. When results are presented by achievement level, 56 percent of students at the *Basic* level on the NAEP composite mathematics scale provided a correct response, whereas 81 percent at the *Proficient* level answered the question correctly. The question mapped at a scale score of 248 on the NAEP composite mathematics scale.

The second example is an eighth-grade question that asked students for the closest approximation of a 15 percent tip on a given restaurant bill. It required an understanding of both percent and estimation.

5. Of the following, which is the closest approximation of a 15 percent tip on a restaurant check of $24.99?

Ⓐ $2.50
Ⓑ $3.00
Ⓒ $3.75
Ⓓ $4.50
Ⓔ $5.00

The correct option is C.

Student performance data for this question are presented in Table 3.15. This question was fairly difficult for eighth-grade students and mapped at a scale score of 328 on the NAEP composite mathematics scale. Only 38 percent of students chose the correct option, while approximately 20 percent of students chose Option A, and another 20 percent chose Option B. The performance suggests that students had difficulty calculating the requested percent, that they did not appreciate the level of precision required for a successful estimation, or that they simply responded with what they considered to be an appropriate tip without attending to the direction that the tip be 15 percent. Students currently taking pre-algebra or eighth-grade mathematics performed similarly, whereas those currently taking algebra performed better than students in the other two groups.

Table 3.15

Percentage Correct for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	38
Males	37
Females	39
White	38
Black	40
Hispanic	28
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	34
Pre-Algebra	33
Algebra	48

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who successfully answered the question is presented in Table 3.16. That the question was challenging for students can be seen by the fact that only 37 percent of eighth-grade students at the *Basic* level, 54 percent at the *Proficient* level, and 68 percent at the *Advanced* level on the NAEP composite mathematics scale answered the question correctly.

Table 3.16

Percentage Correct Within Achievement-Level Intervals for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
38	26	37	54	68

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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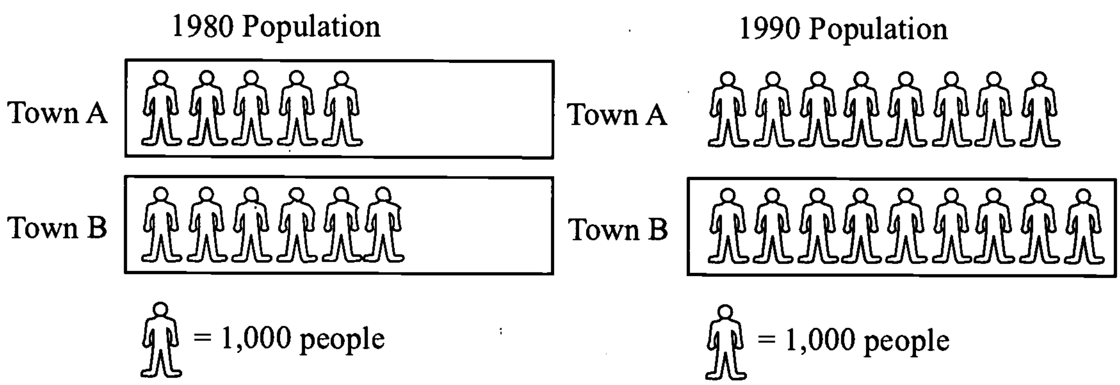
The third example in this area is a problem-solving question that was administered to students in grades 8 and 12. It described the population growth of two towns, both textually and graphically, and gave two opinions (Brian's and Darlene's) regarding the relative growth of the two towns. Students were asked to use mathematics to explain how either opinion might be justified.



5. In 1980, the populations of Town A and Town B were 5,000 and 6,000, respectively. The 1990 populations of Town A and Town B were 8,000 and 9,000, respectively.

   Brian claims that from 1980 to 1990 the populations of the two towns grew by the same amount. Use mathematics to explain how Brian might have justified his claim.

   Darlene claims that from 1980 to 1990 the population of Town A had grown more. Use mathematics to explain how Darlene might have justified her claim.

   Did you use the calculator on this question?

   ⬭ Yes ⬭ No

In the question, Brian offered a conclusion based on the fact that the absolute size of the population growth was the same for both towns. Acceptable mathematics for demonstrating Brian's conclusion included:

Town A	8,000 − 5,000 = 3,000		6,000 − 5,000 = 1,000
		or	
Town B	9,000 − 6,000 = 3,000		9,000 − 8,000 = 1,000

Darlene's conclusion was based on the proportional growth of the two towns, which was greater for Town A than Town B. Acceptable mathematics for demonstrating Darlene's conclusion included:

Town A $\quad \frac{8{,}000 - 5{,}000}{5{,}000} \times 100\% = 60\%$ $\quad$ or $\quad 8 \div 5 = 1.6$

Town B $\quad \frac{9{,}000 - 6{,}000}{6{,}000} \times 100\% = 50\%$ $\quad$ or $\quad 9 \div 6 = 1.5$

A response was rated as "correct" if the student provided a correct mathematical calculation (as illustrated above) for both Brian and Darlene. In the following "correct" example, the student provided correct mathematical explanations for both Brian's and Darlene's conclusions.

Sample "correct" response

Bk in 1980 the populations of A + B were 5000 + 6000 respectively and in 1990 the populations of A + B were 8000 + 9000 respectively so they both in creased by 3000 people
A 8000-5000=3000 + 9000-6000=3000

Town A increased by a greater percentage than town B 3000=__% of 5000 =60%
3000=__% of 6000 =50%

Did you use the calculator on this question?

● Yes ○ No

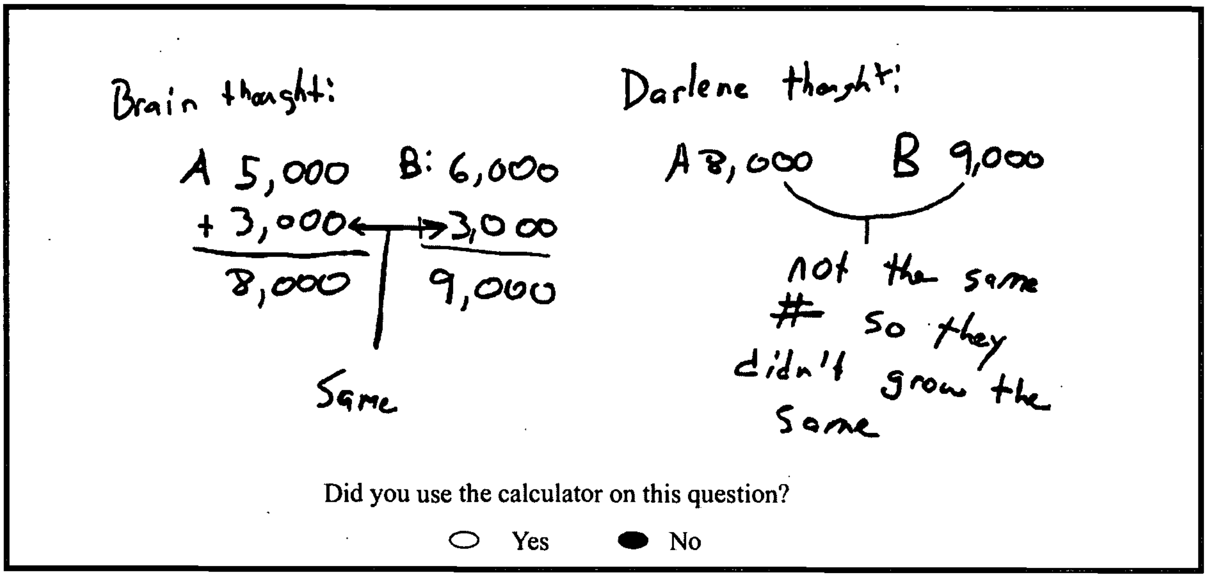
Student responses were rated as "partial" if they did one of the following:

- indicated Brian's solution (either 1,000 or 3,000) and Darlene's solution (60% and 50%) but did not show the mathematical explanation (calculation) that they used to arrive at these solutions; or

- indicated either Brian's solution or Darlene's solution with the correct mathematical explanation (calculation).

This next sample response was rated as a "partial" response. The student gave a variation of the 8,000 – 5,000 and 9,000 – 6,000 mathematical explanation presented above for Brian's conclusion. However, the student did not provide a correct mathematical explanation for Darlene's conclusion.

Sample "partial" response



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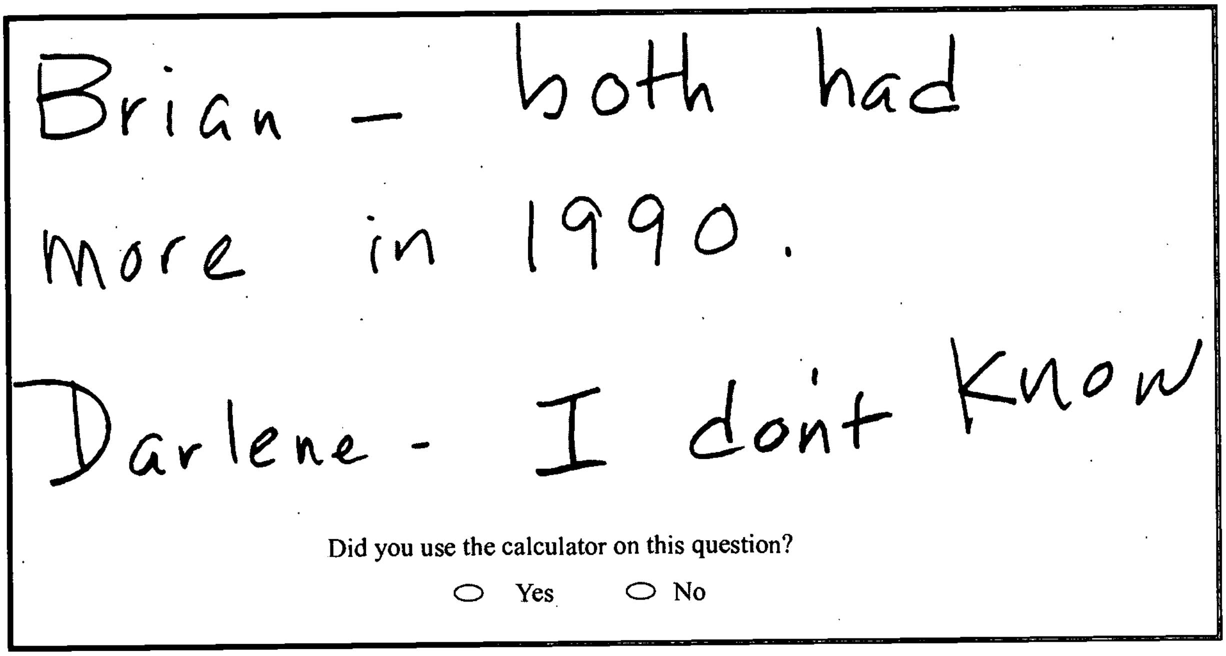
"Incorrect" responses were those that were not considered to be at least partially correct. In the following sample "incorrect" response, the student provided no mathematical explanation for either Brian's or Darlene's conclusion.

Sample "incorrect" response


Brian – both had more in 1990.

Darlene - I don't know

Did you use the calculator on this question?

○ Yes ○ No

Student performance data for both grades 8 and 12 are presented in Tables 3.17 and 3.18. Student performance on the question was similar across the two grades. One percent of eighth-grade students and 3 percent of twelfth-grade students provided responses that were rated "correct," and 21 percent of eighth-grade students and 24 percent of twelfth-grade students provided responses that were rated as "partial." Sixty percent of the responses at grade 8 and 56 percent of the responses at grade 12 were rated "incorrect."

Table 3.17 Score Percentages for "Use Percent Increase"

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
Overall	1	21	60	16
Males	0	17	62	19
Females	1	26	58	13
White	1	24	62	11
Black	0!	14	57	28
Hispanic	0!	17	52	31
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	0!	15	66	16
Pre-Algebra	0!	21	58	18
Algebra	2	33	53	11
Grade 12				
Overall	3	24	56	16
Males	4	22	56	18
Females	2	27	56	14
White	4	25	60	11
Black	0!	21	50	26
Hispanic	2	18	46	34
Asian/Pacific Islander	5	45	31	17
American Indian	***	***	***	***
Geometry Taken	3	27	56	14
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	1	15	61	22
Second-Year Algebra	3	24	57	14
Third-Year Algebra/Pre-Calculus	4	39	53	4
Calculus	12	47	33	8

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Few students at any of the achievement levels for either grade provided "correct" responses to the question. The best performance was by twelfth-grade students at the *Proficient* level. Eleven percent of twelfth-grade students classified as *Proficient* provided "correct" responses.

For grade 12, the question mapped at 415. However, at grade 8, when the question was anchored to the NAEP scale, the "correct" and "partial" rating categories were collapsed. The collapsed response category mapped at 346 for grade 8 on the NAEP composite mathematics scale. In other words, whereas the highest response category ("correct") was mapped for grade 12, the lower collapsed category ("correct" plus "partial") was mapped for grade 8.

Table 3.18 **Percentage Correct Within Achievement-Level Intervals for "Use Percent Increase"**

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	1	0!	0!	2!	4!
Grade 12	3	0!	1	11	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final example is a twelfth-grade multiple-choice question. The question involved rate and time and tested students' knowledge of procedures used to solve for rate per unit of time.

3. A certain machine produces 300 nails per minute. At this rate, how long will it take the machine to produce enought nails to fill 5 boxes of nails if each box will contain 250 nails?

Ⓐ 4 min
Ⓑ 4 min 6 sec
Ⓒ 4 min 10 sec
Ⓓ 4 min 50 sec
Ⓔ 5 min

The correct option is C.

This question was a multistep problem that a student could have solved in a number of ways. One possible approach was to determine how many nails were desired (5 boxes × 250 nails/box = 1,250 nails), then to solve for time required to produce 1,250 nails (1,250 nails/300 nails per minute). The solution is 4.16 minutes, which equals 4 minutes and 10 seconds. A proportional approach also could have been used. After determining the numbers of nails desired, a student could have solved the proportionality equation $300/60 = 1{,}250/x$ to get the time required.

Student performance data are presented in Tables 3.19 and 3.20. Almost half of the students answered the question correctly. Nineteen percent chose Option B, and 12 percent chose Option D. Male students performed better than females. This question mapped at 349 on the NAEP composite mathematics scale.

Table 3.19 ***Percentage Correct for "Solve a Rate Versus Time Problem"***

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	49
Males	56
Females	43
White	53
Black	36
Hispanic	41
Asian/Pacific Islander	63
American Indian	***
Geometry Taken	52
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	37
First-Year Algebra	49
Second-Year Algebra	48
Third-Year Algebra/Pre-Calculus	57
Calculus	65

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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The question was answered correctly by 73 percent of the students classified as *Proficient*, 51 percent of the students classified as *Basic*, and 34 percent of the students classified as below *Basic*.

Table 3.20 **Percentage Correct Within Achievement-Level Intervals for "Solve a Rate Versus Time Problem"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
49	34	51	73	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

Questions in this content strand assessed students' conceptual understanding of number meanings, properties, and other number concepts; procedural knowledge of number operations; and application of this understanding and knowledge to real-life problems. The understanding, knowledge, and application sometimes involved rounding, estimation, or proportional thinking. Questions assessing ratios and proportional thinking tended to be among the most difficult, and the computation questions tended to be among the easiest. Few questions required decontextualized computations. Rather, the questions often involved real-life situations presented either as a "story" or in graphics. Some questions asked students to round or estimate as one step in arriving at the solution.

The majority of students appeared to grasp many of the fundamental concepts of numbers, relationships between numbers, and properties of numbers, as well as to display the skills required for manipulating numbers and completing computations. Questions requiring multistep solutions or involving new concepts tended to be more difficult. Additionally, questions requiring students to solve problems and communicate their reasoning proved challenging, and often it was the communication aspect that provided the most challenge.

Chapter 4

Measurement

Content Strand Description

The Measurement content strand focuses on an understanding of the process of measurement and on the use of measurement to describe and compare mathematical and real-world objects. Students were asked to identify attributes of measurement, select appropriate units and tools, apply measurement concepts, and communicate measurement-related ideas.

At the fourth-grade level, the focus was on measurement of time, money, temperature, length, perimeter, area, weight/mass, and angles. At the eighth- and twelfth-grade levels, the measurement problems were more complex and involved volume and surface areas in addition to the aforementioned topics. Questions also involved reasoning with proportions, such as is required in scale drawing and map reading.

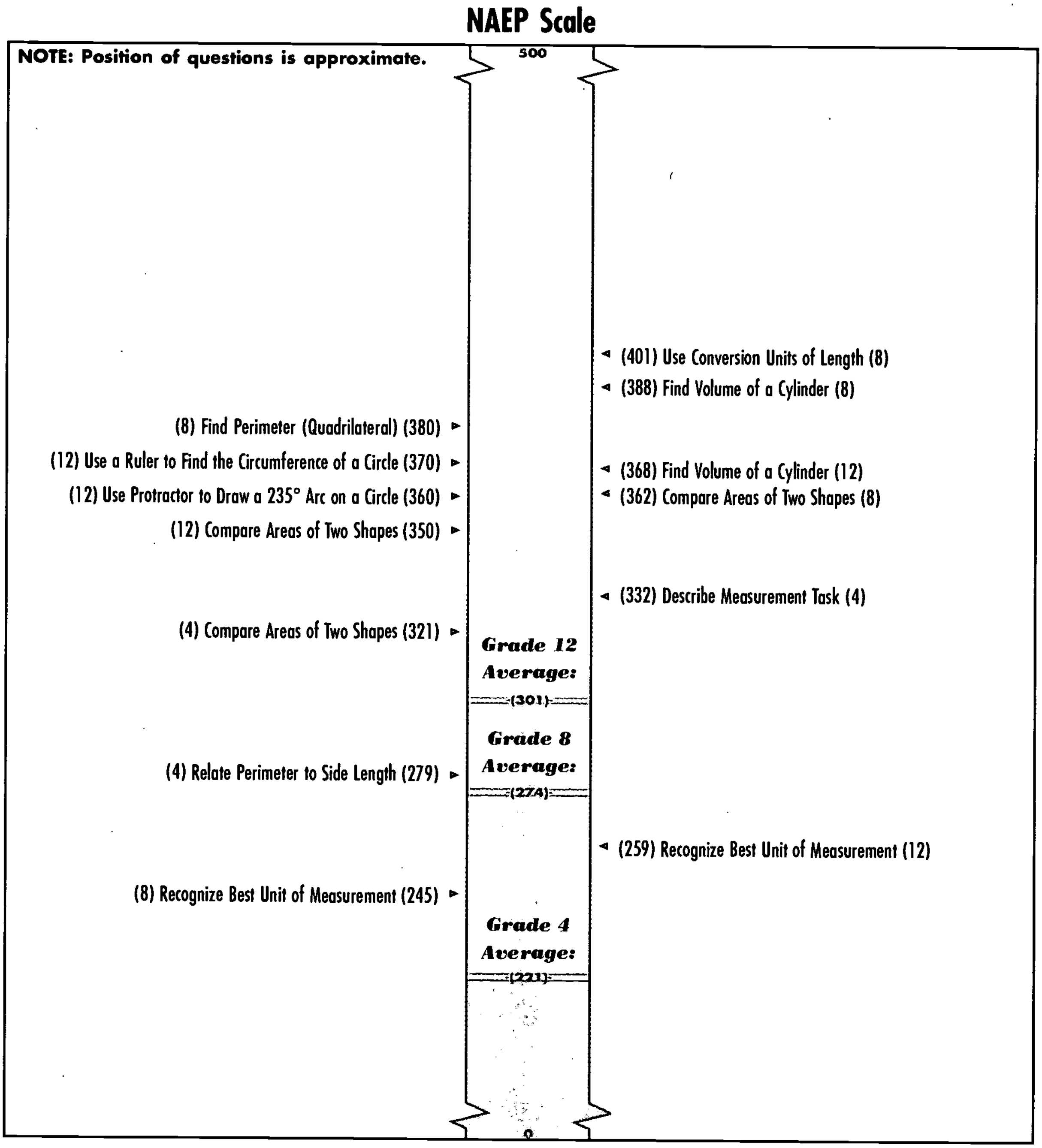
Examples of Individual Questions and Student Performance

Several assessment questions from the Measurement content strand of the NAEP 1996 assessment follow. For ease of discussion, presentation of the questions is organized around four areas of emphasis within the Measurement content strand: 1) *units of measurement*; 2) *measurement instruments*; 3) *perimeter, area, and volume*; and 4) *estimation of measurements*. Questions within all four areas tested students' conceptual understanding and procedural knowledge, as well as their abilities to reason, communicate, and make connections.

The sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 4.1. Specific instructions on how to interpret this map are detailed at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each sample question and, thus, to suggest the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. It should be remembered, however, that the difficulty of a question is influenced by many factors, including characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application) as well as the particular mathematics content associated with the question and student opportunities to learn this content. It also must be remembered that overall performance on the Measurement content strand is not determined solely by performance on the few examples presented here. These examples illustrate only some of what students know and can do.

Figure 4.1 ***Map of Selected Measurement Questions on the NAEP Composite Mathematics Scale (Item Map)***

THE NATION'S REPORT CARD



NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The performance of students on the questions in the Measurement content strand is examined with respect to gender, race/ethnicity, and, for grades 8 and 12, the types of mathematics courses taken. However, as described in Chapter 2, the impact of taking geometry on student performance is not discussed for several reasons. First, there is only a small pool of students on which the specific influence of geometry could be isolated, given that most students who have taken geometry also have taken at least two years of algebra. Moreover, because more able students are likely to progress further in the mathematics course sequence, it is difficult to separate the impact of a particular curriculum from the impact of a student's overall strength in mathematics. Although comments on the impact of geometry course taking on performance on the questions in this content strand might be expected, these confounding effects make it difficult to isolate the specific impact of geometry. The data, however, are presented in the tables.

Units of measurement

These questions primarily assessed students' conceptual understanding and procedural knowledge of measurement units. Students had to understand what various units of measurement represent and the relationships between units. Questions typically required students to choose the best unit for a particular problem, and questions for older students required finer distinctions. Units that were assessed included standard and metric units of length, distance, volume, speed, and temperature, as well as units of time. Some questions required students to convert from one unit of measurement to another within the same system of measurement (e.g., feet to yards, quarts to pints) or to make and read scale drawings.

The following sample question was administered to both eighth- and twelfth-grade students. It is a multiple-choice question that tested students' conceptual understanding of appropriate measurement units. The question asked for the best unit for measuring plant growth during a 2-week period. To answer the question correctly, students had to realize that the daily growth of a plant would be small, and they needed to be familiar with different units of length in order to recognize which of those listed was small enough to make such a measurement. The question was not difficult for students and mapped at 245 for grade 8 and at 259 for grade 12 on the NAEP composite mathematics scale.

3. Of the following, which is the best unit to use when measuring the growth of a plant every other day during a 2-week period?

Ⓐ Centimeter

Ⓑ Meter

Ⓒ Kilometer

Ⓓ Foot

Ⓔ Yard

Did you use the calculator on this question?

○ Yes ○ No

The correct option is A.

Student performance is reported in Tables 4.1 and 4.2. That the question was fairly easy can be seen by the fact that almost 80 percent of eighth-grade students and almost 90 percent of twelfth-grade students who answered the question selected the correct option. At the eighth-grade level, 54 percent of students classified as below the *Basic* level, 90 percent of students classified as *Basic*, and 97 percent of those classified as *Proficient* answered correctly. As might be expected, the percentage of twelfth-grade students at each achievement level who answered correctly was even higher: 72 percent of those classified as below *Basic*, 93 percent of those classified as *Basic*, and 98 percent of those classified as *Proficient*.

Eighth-grade students taking algebra were more likely to select the correct answer than those taking eighth-grade mathematics. The performance of students who were in pre-algebra, however, was not significantly different from the performance of students in either eighth-grade mathematics or algebra.

At the twelfth-grade level, students whose highest course was calculus in the algebra-through-calculus sequence performed better than those whose highest course was first- or second-year algebra.

Table 4.1 Percentage Correct for "Recognize Best Unit of Measurement"

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 8	
Overall	78
Males	78
Females	78
White	84
Black	63
Hispanic	66
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	74
Pre-Algebra	80
Algebra	87
Grade 12	
Overall	87
Males	87
Females	87
White	90
Black	77
Hispanic	83
Asian/Pacific Islander	92
American Indian	***
Geometry Taken	90
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	***
First-Year Algebra	85
Second-Year Algebra	89
Third-Year Algebra/Pre-Calculus	93
Calculus	96

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table 4.2 Percentage Correct Within Achievement-Level Intervals for "Recognize Best Unit of Measurement"

THE NATION'S REPORT CARD NAEP

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	78	54	90	97	100!
Grade 12	87	72	93	98!	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example in the *units of measurement* area also is a multiple-choice question for eighth-grade students that presented a real-world situation to assess students' problem-solving skills. The question gave a car odometer reading in miles, told the student there would be a detour some number of feet ahead, and then asked what the odometer would read when the car reached the detour. The conversion factor between feet and miles was given. To calculate the answer, students had to correctly convert the distance to the detour, given in feet, to a decimal fraction of a mile and then add that decimal fraction to the original odometer reading. Alternatively, students could have scanned the response options and selected the one response that corresponds to an increase of less than one mile.

7. A car odometer registered 41,256.9 miles when a highway sign warned of a detour 1,200 feet ahead. What will the odometer read when the car reaches the detour? (5,280 feet = 1 mile)

Ⓐ 42,456.9

Ⓑ 41,279.9

Ⓒ 41,261.3

Ⓓ 41,259.2

Ⓔ 41,257.1

Did you use the calculator on this question?

◯ Yes ◯ No

The correct option is E.

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Performance data are shown in Tables 4.3 and 4.4. Twenty-six percent of the students selected the correct option. Another 37 percent selected Option A, the option corresponding to simply adding the number of feet to the odometer reading without first converting the distance to miles. Students taking pre-algebra or eighth-grade mathematics performed similarly, whereas students currently taking algebra performed better than the other two groups. Males performed better than females. Twenty-five percent of students at the *Basic* level, 50 percent at the *Proficient* level, and 70 percent of students at the *Advanced* level answered correctly. Only 11 percent of students classified as below the *Basic* level answered the question correctly. The question mapped at 401 on the NAEP mathematics composite scale.

Table 4.3 — ***Percentage Correct for "Use Conversion Units of Length"***

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	26
Males	30
Females	21
White	30
Black	16
Hispanic	15
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	22
Pre-Algebra	21
Algebra	39

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.4 — ***Percentage Correct Within Achievement-Level Intervals for "Use Conversion Units of Length"***

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
26	11	25	50	70

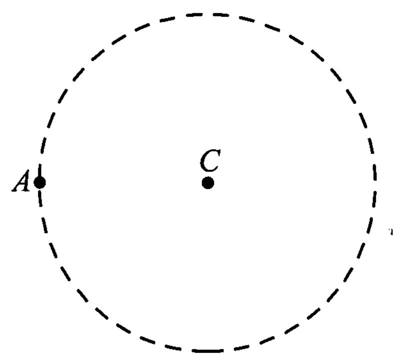
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Measurement instruments

Questions in this area assessed students' understanding of and ability to use measurement tools and instruments. Students had to identify appropriate instruments for certain situations. They also had to read representations of measurement instruments such as rulers, thermometers, gauges, and dials. Some questions in this area required students actually to use tools such as rulers, protractors, or compasses to measure and construct shapes. Questions for younger students involved more common instruments and required less accurate measurements than did questions for older students.

The following example is a short constructed-response question for grade 12 that required students to use a measurement instrument, provided with the assessment, to solve a problem. The question presented a dashed circle with center, *C*, and a point, *A*, marked on the circumference and asked students to use a protractor to draw and label an arc, *AB*, with a 235° angle. Because protractors only provide measurements up to 180 degrees, students needed to understand how to work with the difference between 235° and 180° to draw the obtuse angle required for the solution.

7. On the circle with center *C* shown below, use the protractor to locate and label a point *B* that creates an arc *AB* with measure 235°. Darken this arc.



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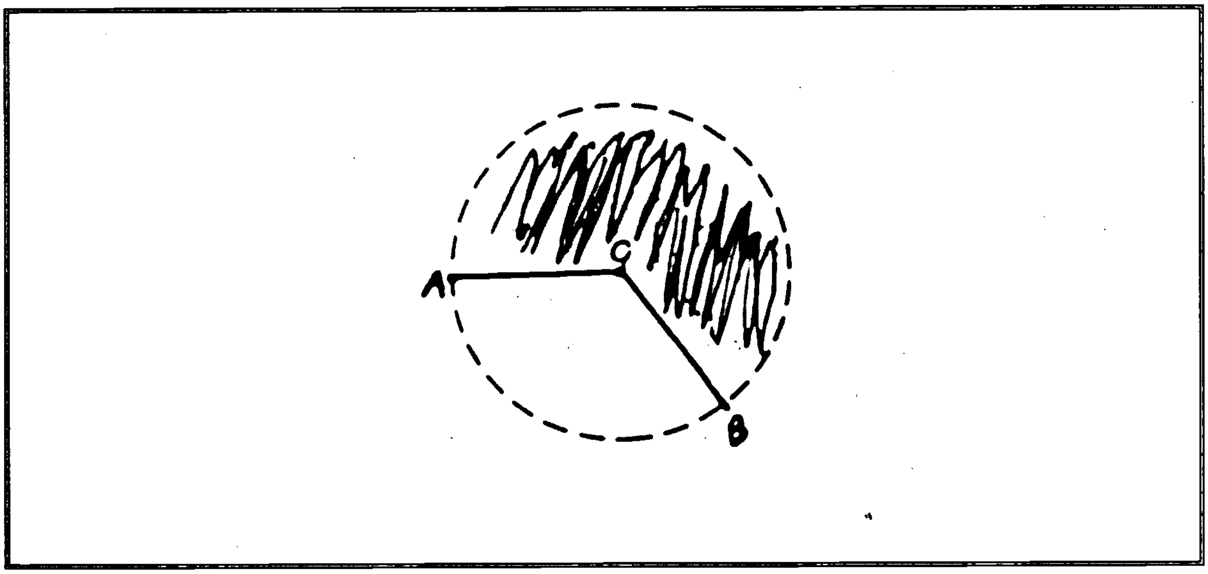
Responses were rated "correct" if they portrayed an obtuse angle *ACB* measuring within ±5° of 235°. More accurate responses, which were accurate within ±2°, were tabulated separately, as shown in Table 4.5.

Responses were rated "incorrect" if point *B* was placed correctly on the circle (i.e., within ±5° of the correct location) but the arc was not clearly indicated, or if a sector or arc of 235° (±5°) was shown that did not have an endpoint at point *A*. Responses also were rated "incorrect" if they were incorrect for any other reason (e.g., the angle was incorrect). Three sample responses follow: one rated "correct" and two rated "incorrect." The "correct" response shows an angle *ACB* of 235° with a shaded area of the circle corresponding to the arc *AB*.

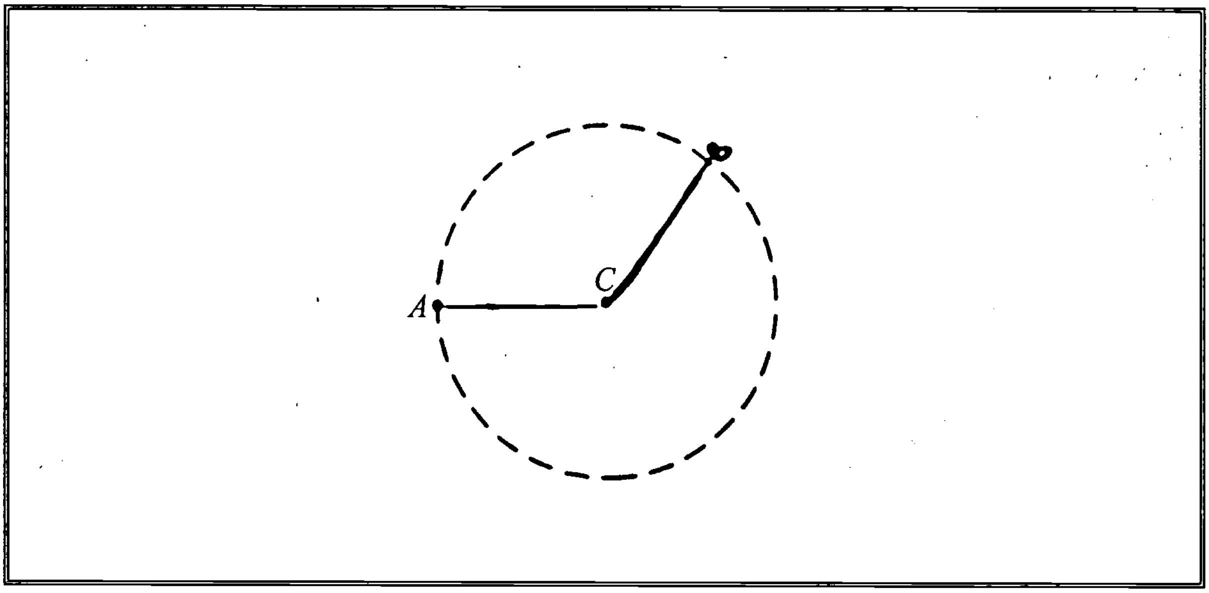
Sample "correct" response



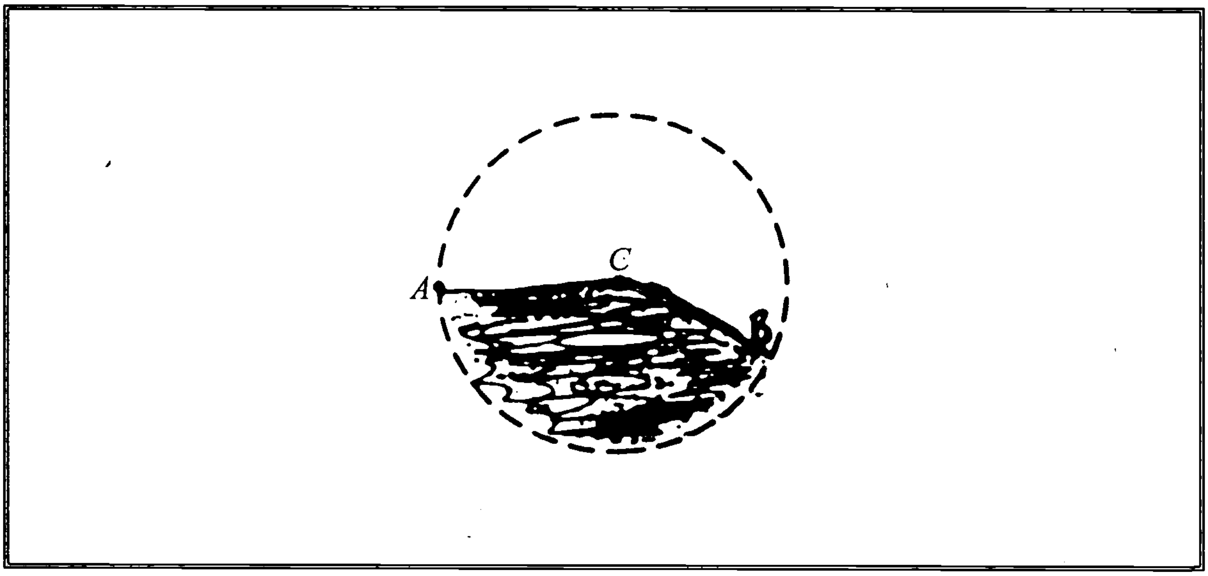
96

The first sample "incorrect" response has the angle ACB of 235° ±5° correctly drawn, but does not indicate the arc AB, whereas the second sample "incorrect" response has neither the angle nor the arc correctly indicated.

Sample "incorrect" response 1



Sample "incorrect" response 2



97

Information on student responses is presented in Table 4.5. One-fourth of the students provided responses rated "correct," 62 percent provided responses rated "incorrect," and approximately 12 percent did not respond to the question.[1] Students whose highest mathematics course was calculus were more likely to provide a response considered to be "correct" than those whose highest course was pre-calculus, and students who had taken pre-calculus were more likely to provide a response rated "correct" than those whose highest course was second-year algebra. Males were more likely than females to provide a response rated "correct."

Table 4.5 ***Score Percentages for "Use Protractor to Draw a 235° Arc on a Circle"***

THE NATION'S REPORT CARD naep

	Correct		Incorrect			Omit
	(± 2°)	(± 3–5°)	No "A" Endpoint	Arc Not Indicated	Other	
Grade 12						
Overall	15	10	0	4	58	12
Males	18	12	0!	5	55	10
Females	12	9	0	3	61	14
White	18	12	0	4	57	9
Black	5	5	0!	2	70	18
Hispanic	8	5	0!	4	57	27
Asian/Pacific Islander	24	23	0!	2	46	6
American Indian	***	***	***	***	***	***
Geometry Taken	17	12	0	4	57	10
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	12	4!	0	6	67	6
First-Year Algebra	10	9	0	5	64	10
Second-Year Algebra	14	10	1	3	58	14
Third-Year Algebra/Pre-Calculus	19	16	0!	5	54	6
Calculus	39	17	0	1	41	2

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

The percentage of students within each achievement level who provided a response that was considered "correct" is shown in Table 4.6. Twenty-six percent of students at the *Basic* level and 59 percent of those at the *Proficient* level provided responses rated "correct." The question was very difficult for students classified as below the *Basic* level; only five percent provided a response rated "correct." The question mapped at 360 on the composite mathematics scale.

Table 4.6 **Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw a 235° Arc on a Circle"**

THE NATION'S REPORT CARD naep

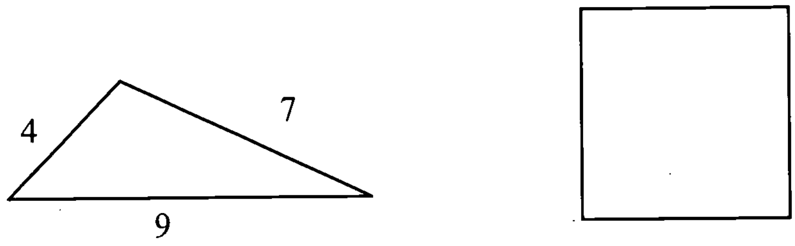
Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
26	5	26	59	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Perimeter, area, and volume

This area included questions measuring student procedural knowledge and problem-solving abilities applied to the concepts of perimeter, area, and volume. Questions at grade 4 asked students to calculate perimeters or areas of simple figures. Questions at grades 8 and 12 involved more complex figures, including three-dimensional figures. Some also required calculations of volume or circumference as well as an understanding of the relationship between perimeter, area, and volume. A number of the questions provided built-in aids, such as grids, to help students in their calculations. Assistance of this sort was especially common at the lower grade levels. As was true throughout the assessment, real-life problem situations were employed for many of the questions.

The first example in this area is a multiple-choice question for grade 4. The question presented two figures: a triangle with the lengths of the sides shown and a square. Students were asked what the length of each side of the square would be if the square and the triangle had the same perimeter. In order to answer correctly, students had to know that the perimeter is the distance around a figure. They needed to correctly sum the numbers shown on the triangle and then divide that sum by four to obtain the length of one side of the square.



8. If both the square and the triangle above have the same perimeter, what is the length of each side of the square?

- (A) 4
- (B) 5
- (C) 6
- (D) 7

The correct option is B.

Student performance data are presented in Table 4.7. Overall, 26 percent of the students selected the "correct" option, B, while 36 percent selected Option A, and 25 percent chose Option D. Only 10 percent selected Option C. Both Option A and Option D contain a number that is equal to one of the numbers on the sides of the triangle. Therefore, students who were unable to solve the problem may have been attracted to these options.

Table 4.7 ***Percentage Correct for "Relate Perimeter to Side Length"***

Grade 4	Percentage Correct
Overall	26
Males	29
Females	23
White	30
Black	19
Hispanic	15
Asian/Pacific Islander	27
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who successfully answered the question is presented in Table 4.8. Eleven percent of students classified as below *Basic*, 20 percent of those classified as *Basic,* and 58 percent of those classified as *Proficient* selected the correct response. The question mapped at 279 on the NAEP mathematics composite scale.

Table 4.8 **Percentage Correct Within Achievement-Level Intervals for "Relate Perimeter to Side Length"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	**Basic**	**Proficient**	**Advanced**
26	11	20	58	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The following example is a short constructed-response question administered to both grades 8 and 12. The question is a word problem that gave the dimensions of a cylindrical cereal box and asked for the volume of the box to the nearest cubic inch. The formula for the volume of a cylinder also was presented. To answer correctly, students had to know how to substitute the specified values for height and radius into the formula, solve the equation, and then round the answer correctly. It was not necessary to know the value of pi because the calculators provided for the assessment had pi keys on them.

6. A cereal company packs its oatmeal into cylindrical containers. The height of each container is 10 inches and the radius of the bottom is 3 inches. What is the volume of the box to the nearest cubic inch? (The formula for the volume of a cylinder is $V = \pi r^2 h$.)

Answer: ____________________ cubic inches

Did you use the calculator on this question?

◯ Yes ◯ No

The correct answer is 283 cubic inches.

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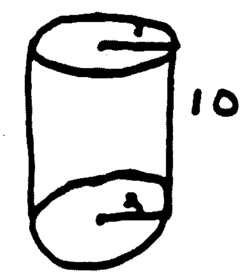
Responses were rated "correct," "partial," or "incorrect." A response was considered "correct" if the correct numerical answer of "283" was given with correct accompanying work or with no work shown. A response was considered "partial" if it showed any of the following: 1) correct substitutions into the formula but incorrect rounding; 2) 282.74334, suggesting multiplication by the pi key on the calculator, but with no work shown; 3) 282.6, suggesting multiplication by 3.14 on the calculator, but with no work shown; or 4) 282.8571, suggesting multiplication by 22/7 on the calculator, but again with no work shown. All other responses were considered "incorrect." Sample responses follow. The sample "correct" response contains the correct answer with accompanying work. The "partial" response shows correct substitution into the formula but no rounding, and the "incorrect" response shows no work and provides an incorrect answer. The question mapped at 388 for grade 8 and at 368 for grade 12.

Sample "correct" response


Answer: 283 cubic inches

$V = \pi(3)^2 10$

$V = \pi(9) 10$

$V = \pi(90)$

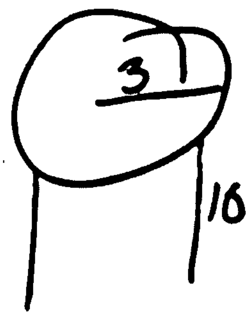
$V = 283 \text{ in}^3$

Did you use the calculator on this question?

● Yes ○ No

Sample "partial" response

Answer: 282.74 cubic inches



$V = \pi 3^2(10)$

$282.74334 = \pi 3^2(10)$

Did you use the calculator on this question?

● Yes ○ No

Sample "incorrect" response

Answer: 30 cubic inches

Did you use the calculator on this question?

○ Yes ● No

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Student performance is reported in Table 4.9. The question was fairly difficult for eighth-grade students, as can be seen by the fact that less than one-third of the students submitted responses considered to be at least partially correct. However, eighth-grade students enrolled in algebra performed better than their peers: 27 percent received full credit, and an additional 25 percent received partial credit.

As may be expected, twelfth-grade students had less trouble with the question; 55 percent of students submitted a response that was considered to be at least partially correct. Students who had taken second-year algebra as their highest mathematics course in the algebra-through-calculus sequence were more likely than those who had taken only first-year algebra to submit a response considered to be at least partially correct, and students whose highest course was calculus were more likely than those with less mathematics to submit a response rated "correct."

At both grade levels, female students were more likely than males to submit a response considered to be at least partially correct.

Table 4.9 **Score Percentages for "Find Volume of a Cylinder"**

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
Overall	13	17	57	12
Males	11	14	59	13
Females	14	19	54	11
White	16	19	56	8
Black	4	12	63	20
Hispanic	6	7	56	28
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	8	13	63	15
Pre-Algebra	8	15	63	10
Algebra	27	25	42	6
Grade 12				
Overall	29	26	36	8
Males	30	20	38	10
Females	28	31	33	7
White	32	27	34	6
Black	18	25	40	14
Hispanic	19	25	34	19
Asian/Pacific Islander	42	14	38	5
American Indian	***	***	***	***
Geometry Taken	32	28	34	6
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	16	22	46	13
Second-Year Algebra	31	28	33	7
Third-Year Algebra/Pre-Calculus	37	30	31	1
Calculus	56	26	18	0!

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who successfully answered the question is presented in Table 4.10. At the eighth-grade level, 9 percent of students classified as *Basic*, 34 percent of those classified as *Proficient*, and 62 percent of those classified as *Advanced* provided a response rated "correct," whereas at grade 12, 6 percent of students below the *Basic* level, 32 percent at the *Basic* level, and 60 percent at the *Proficient* level submitted a "correct" response.

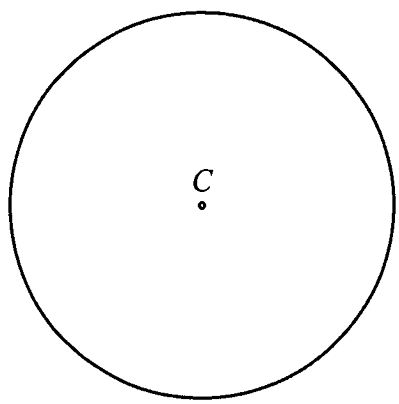
Table 4.10 **Percentage Correct Within Achievement-Level Intervals for "Find Volume of a Cylinder"**

THE NATION'S REPORT CARD

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	13	0	9	34	62
Grade 12	29	6	32	60	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example in the area of *perimeter, area, and volume* is a short constructed-response question for grade 12. In this question, students were shown a picture of a circle with the center, *C*, marked and were instructed to use a centimeter ruler to find the circumference of the circle. No calculator was available. They were told the value of pi but were not given the formula for circumference. The answer blank specified an answer in centimeters.



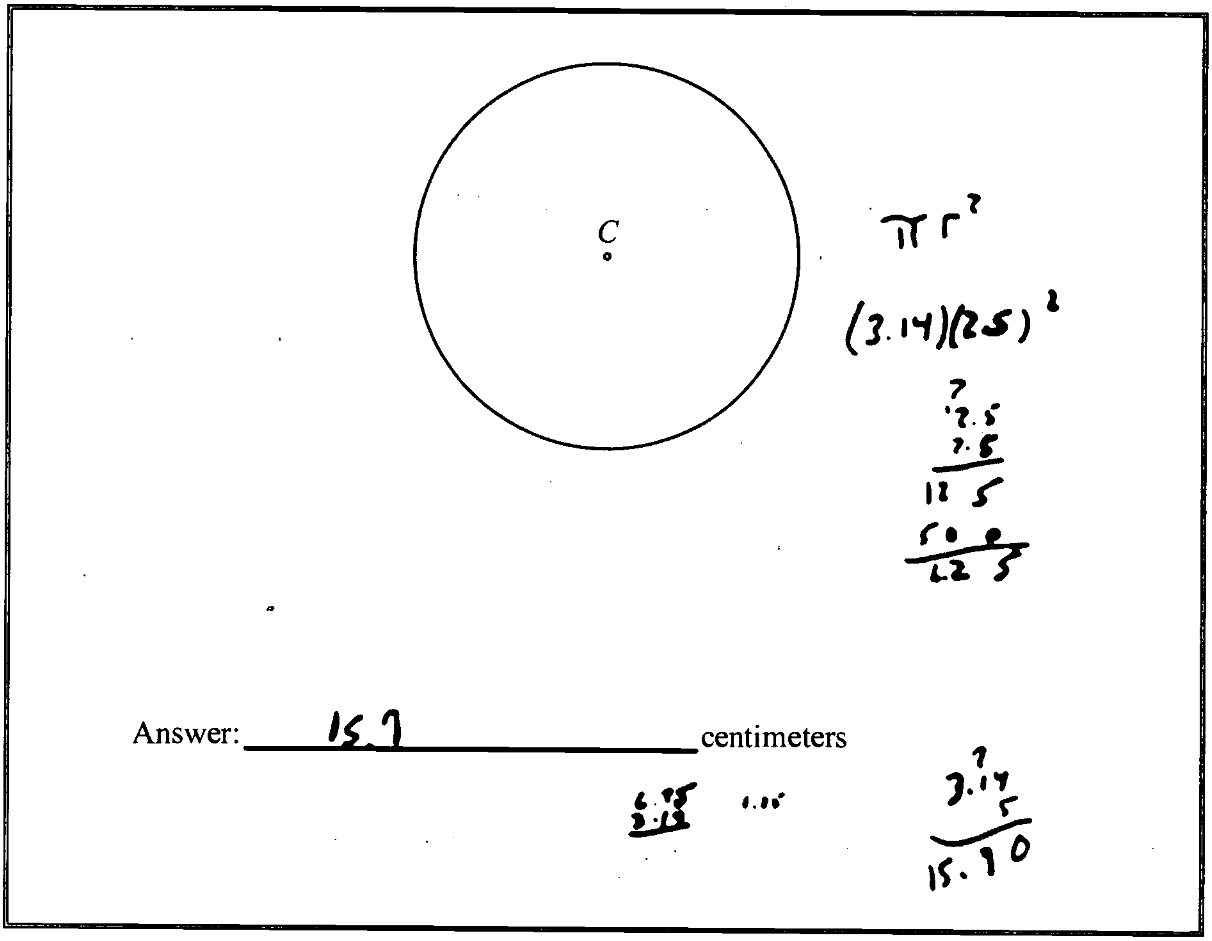
4. Using the centimeter ruler provided, find the circumference of the circle with center *C* above. (Use $\pi = 3.14$.)

Answer: ______________________________ centimeters

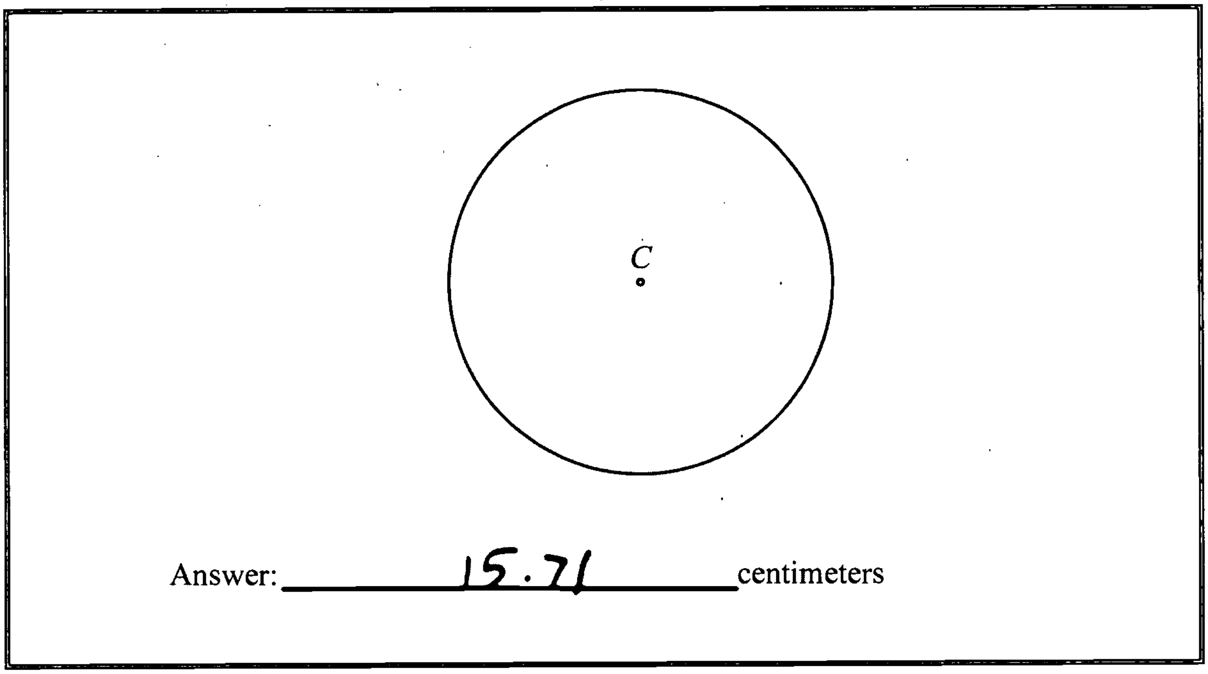
The correct answer is 15.70 centimeters.

In order to answer the question correctly, students had to know how the circumference of a circle is computed, make the correct measurement, and perform the multiplication correctly. Responses of either "15.7" or "15.70" centimeters were considered "correct," as were other answers between 15.0 and 16.4 centimeters. Any answer in centimeters outside this range, as well as any response in inches, was considered "incorrect." Two sample "correct" responses and one sample "incorrect" response are shown to illustrate these rating categories. In the first sample "correct" response, the student has given the correct answer of "15.7," while in the second sample, the student has given an answer within the acceptable range but not exactly 15.7.

Sample "correct" response 1

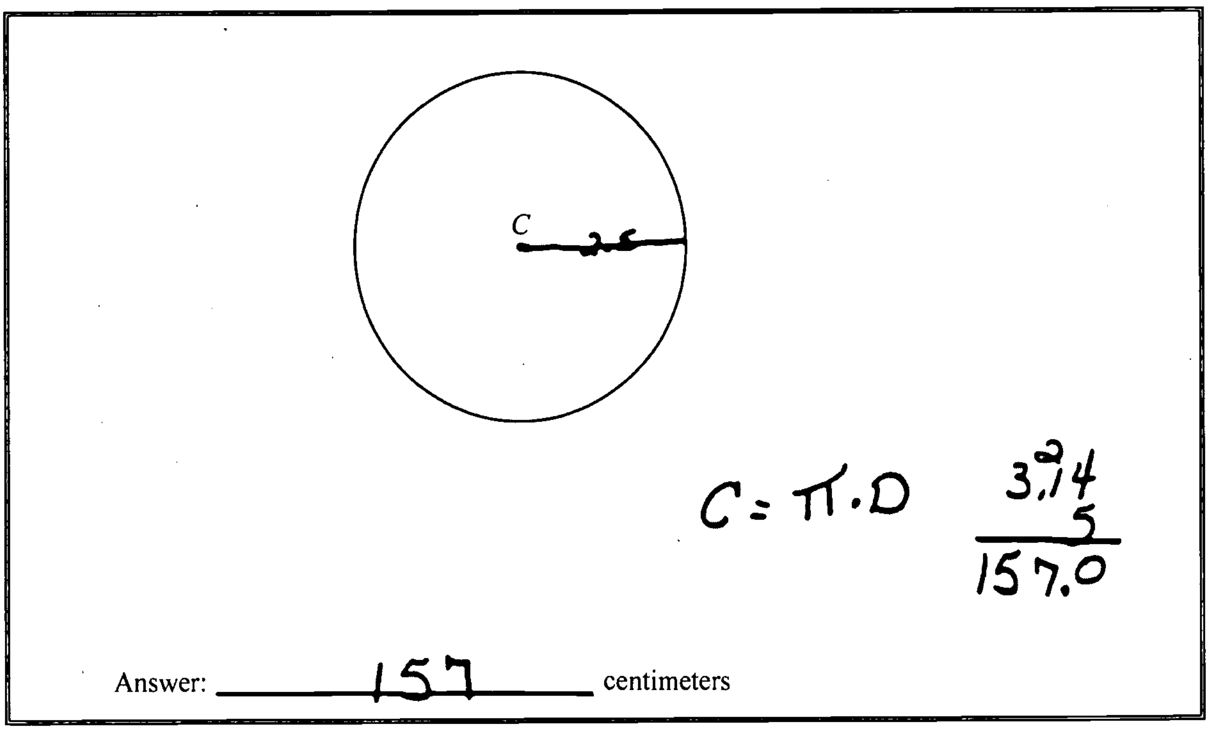


Sample "correct" response 2



In the following sample "incorrect" response, the student undertook the correct calculation of pi times the diameter of the circle, but made a decimal error.

Sample "incorrect" response



Information on student performance is presented in Table 4.11, and the percentage of students at each achievement level who provided a "correct" response is shown in Table 4.12. Overall, 29 percent of students provided a response rated "correct." Of those students, only a small percent (3%) did not get the exact answer of 15.7 cm. Only one percent had their responses rated "incorrect" because they were given in inches. Students who had taken at least pre-calculus were more likely than those in the less advanced mathematics classes to submit a response rated "correct." Ten percent of students classified as below the *Basic* level, 30 percent at the *Basic* level, and 62 percent at the *Proficient* level answered the question correctly. The question mapped at 370 on the composite scale.

Table 4.11 ***Score Percentages for "Use a Ruler to Find the Circumference of a Circle"***

THE NATION'S REPORT CARD naep

	Correct		Incorrect		Omit
	15.7 cm	15.0–16.4 cm Not Including 15.7 cm	Any Response in Inches	Other	
Grade 12					
Overall	26	3	1	57	13
Males	25	4	1	58	12
Females	27	3	1	56	14
White	29	3	1	58	8
Black	16	2	1	60	21
Hispanic	16	2	0!	52	30
Asian/Pacific Islander	42	6	1!	43	9
American Indian	***	***	***	***	***
Geometry Taken	29	4	1	57	9
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	13	1	2	68	1
First-Year Algebra	18	3	0!	60	16
Second-Year Algebra	24	2	1	61	11
Third-Year Algebra/Pre-Calculus	44	7	0!	43	5
Calculus	52	6	1!	41	0

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.12 **Percentage Correct Within Achievement-Level Intervals for "Use a Ruler to Find the Circumference of a Circle"**

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
29	10	30	62	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Estimation of measurements

Questions in this area assessed students' abilities to estimate absolute and relative measurements, including size, weight, time, and distance. Questions for older students frequently required more accurate estimates or finer comparisons.

The first example in this area is a short constructed-response question for grade 4 that assessed students' abilities to reason, make connections, and communicate in mathematics. The question presented a measurement task that "Brett" needed to do without using a measuring instrument and asked the students to write directions to tell "Brett" how to accomplish the task. In order to answer the question correctly, students had to understand what "four equal pieces of string" means (i.e., draw on their knowledge of fractions) and visualize a method for obtaining these pieces. Then they had to communicate their idea in writing.

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

____

____

____

____

Did you use the calculator on this question?

◯ Yes ◯ No

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A response was considered "correct" if it contained directions to fold the string in half and cut it and then to fold each of the resulting pieces in half and cut them. A response was considered "partial" if it mentioned folding the string in half once (e.g., "fold the string and cut") or mentioned cutting in the middle and doing that to the pieces. Partial credit also was given if the student only addressed the question of how to get three more equal pieces once the first piece was made. All other responses were considered "incorrect," including those that simply said to fold the string. Sample responses follow.

Sample "correct" response

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

Fold the string in half (cut it.)
Fold it again (cut it)

Did you use the calculator on this question?

○ Yes ● No

Sample "partial" response

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

Brett should fold the string up into four and cut the four pieces

Did you use the calculator on this question?

○ Yes ● No

Sample "incorrect" response

8. Brett needs to cut a piece of string into four equal pieces without using a ruler or other measuring instrument.

Write directions to tell Brett how to do this.

cut a little bit off 3 times

Did you use the calculator on this question?

○ Yes ● No

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Data on student performance are presented in Tables 4.13 and 4.14. This question was difficult for fourth-grade students and mapped at 332 on the composite scale. Overall, only six percent of students provided responses rated "correct." However, another 34 percent provided responses considered at least partially correct and rated "partial." Four percent of students at the *Basic* level and 14 percent at the *Proficient* level provided a response rated "correct."

Table 4.13 ***Score Percentages for "Describe Measurement Task"*** THE NATION'S REPORT CARD naep

Grade 4	Correct	Partial	Incorrect	Omit
Overall	6	34	50	9
Males	5	32	52	10
Females	6	36	48	9
White	7	40	44	8
Black	1	15	66	15
Hispanic	1	24	61	13
Asian/Pacific Islander	***	***	***	***
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to provide a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.14 ***Percentage Correct Within Achievement-Level Intervals for "Describe Measurement Task"*** THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
6	0!	4	14	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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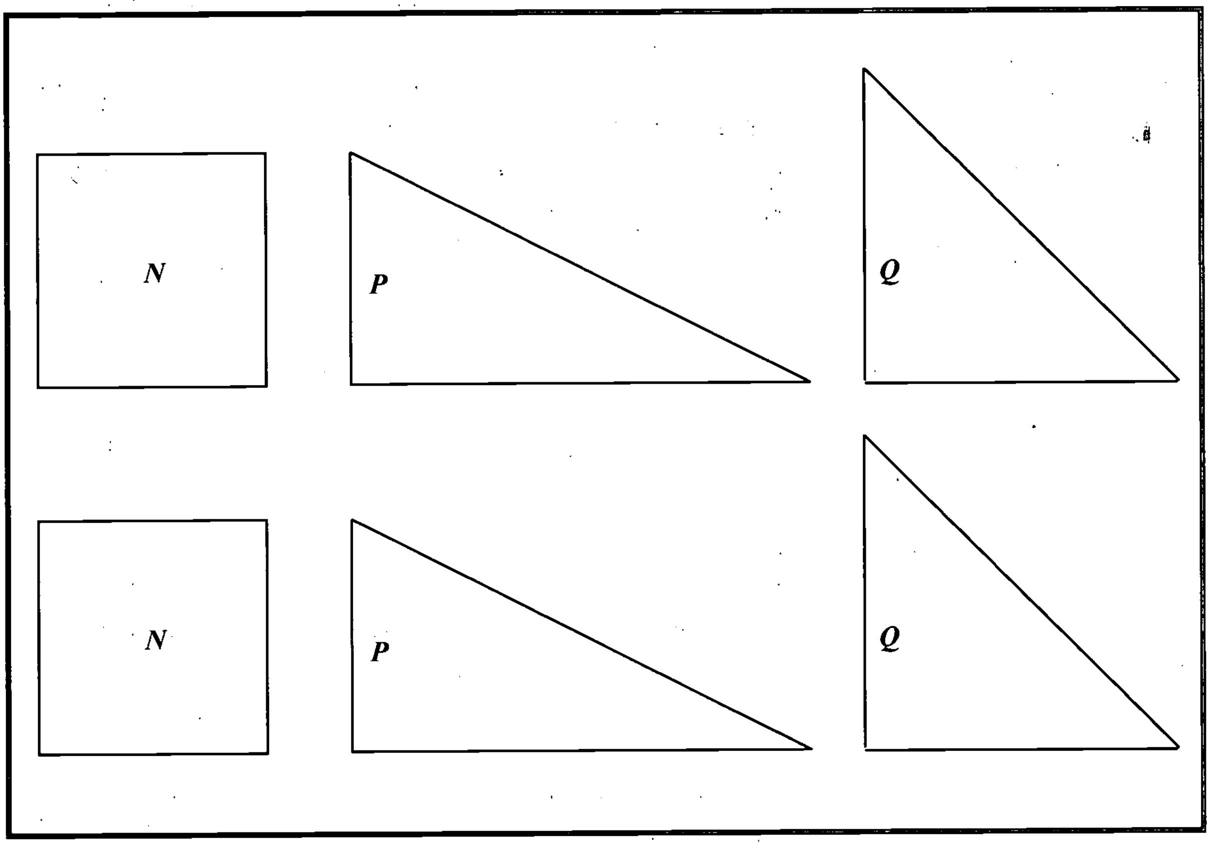
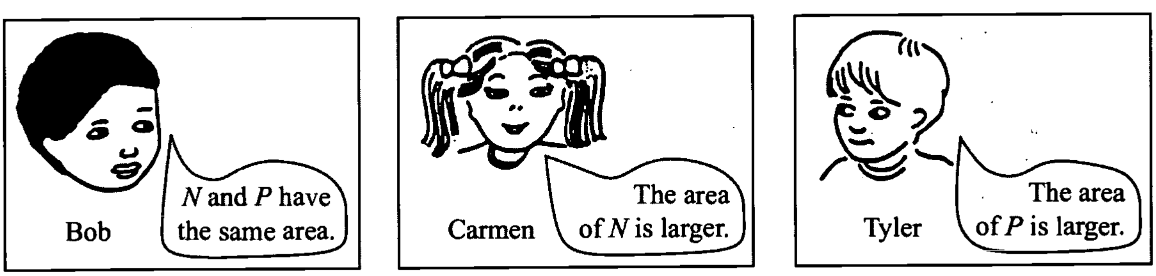
The following example is another fourth-grade short constructed-response question. For the block of questions in which this question appeared, students were provided with two cardboard cutouts of each of three different shapes: squares labeled N, and two different types of right triangle labeled P and Q, respectively.



Students were shown cartoons of three different children, "Bob," "Carmen," and "Tyler," who were making statements comparing the areas of N and P, and they were asked who made the correct statement. They were instructed to use pictures and words to explain their answers.

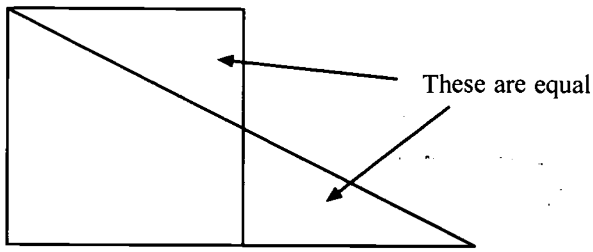
6. Bob, Carmen, and Tyler were comparing the areas of N and P.



Who was correct?

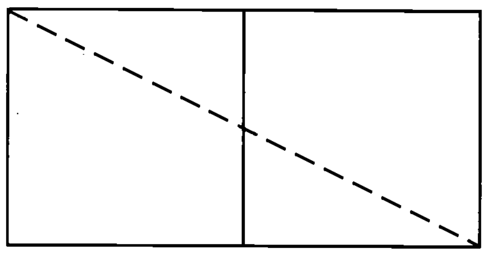
Use pictures and words to explain why.

A response was considered "correct" if an adequate explanation was presented with or without naming "Bob" as being correct. Adequate explanations included the following:



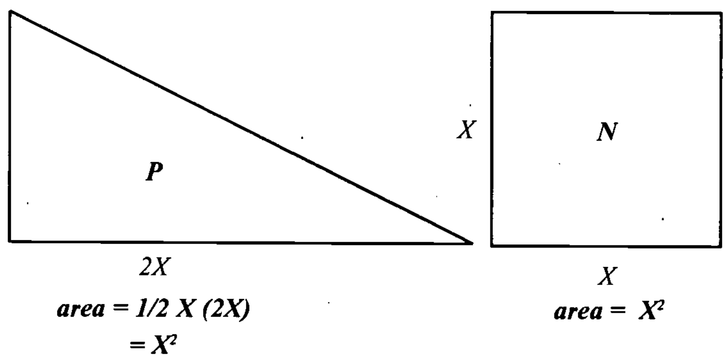
Part of P overlaps N, and part sticks out. The sticking-out part is equal to the left-out part of N.

OR



Two Ps match two Ns; therefore, they have the same area. (Therefore, one N has the same area as one P.)

OR



Areas are equal because the height of P is the same as the height of N, and the base of P is twice the base of N.

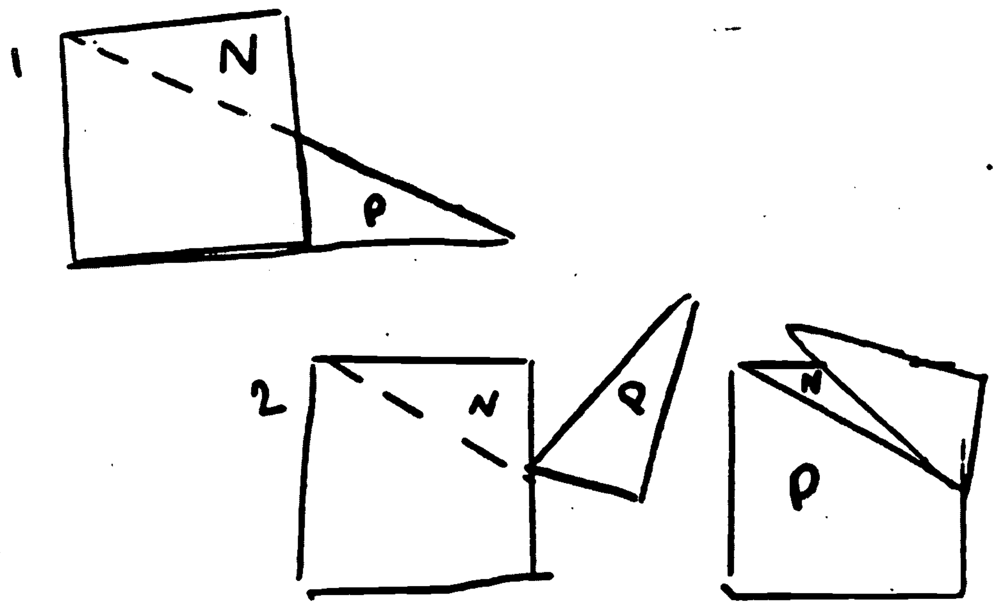
Responses were considered "incorrect" for two main reasons: 1) the student said "Bob" was correct but gave an inadequate or no explanation, or 2) the students named "Carmen" or "Tyler" as being correct or omitted a name and gave no satisfactory explanation. Sample student responses follow.

The first sample response was rated "correct" because the student conveyed a clear understanding of how the part of shape *P* that "sticks out" can be repositioned to form shape *N*. The drawings show that shapes *N* and *P* have the same area.

Sample "correct" response

Who was correct? Bob

Use pictures and words to explain why.

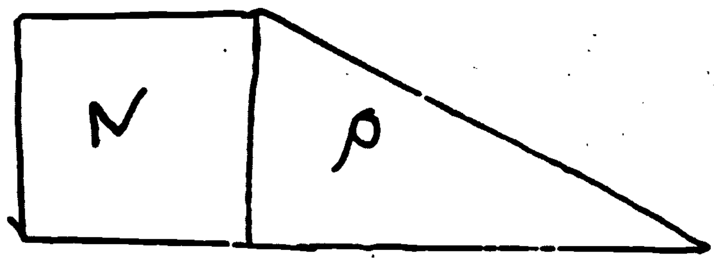


The next two sample responses were rated "incorrect." In the first sample "incorrect" response, the student named "Bob" as being correct, but gave an inadequate explanation. In the second sample "incorrect" response, the student said that "Carmen" was correct.

Sample "incorrect" response 1

Who was correct? Bob

Use pictures and words to explain why.



Sample "incorrect" response 2

Who was correct? Carmen

Use pictures and words to explain why.

N is faster

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Data on student performance are presented in Tables 4.15 and 4.16. This question also was difficult for fourth-grade students; as the table shows, only six percent of students provided responses rated "correct." Another 21 percent correctly answered "Bob" but could not provide an adequate explanation for their answer. The question mapped at 321 on the composite scale. Two percent of students at the *Basic* level and 14 percent at the *Proficient* level provided a response rated "correct."

Table 4.15 **Score Percentages for "Compare Areas of Two Shapes," Grade 4**

THE NATION'S REPORT CARD naep

Grade 4	Correct	Incorrect		Omit
		Bob—No Adequate Explanation	Not Bob	
Overall	6	21	74	0
Males	6	22	71	0
Females	5	19	76	0!
White	7	21	71	0
Black	0!	18	81	0!
Hispanic	0!	18	81	0!
Asian/Pacific Islander	6	21	73	0!
American Indian	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 4.16 **Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grade 4**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
6	1!	2	14	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example is essentially the same question as the one just shown for grade 4, but the question was presented as a word problem at grades 8 and 12 rather than in cartoon form. (The different presentation for grade 4 was used to reduce the amount of reading required for younger students.)

5. Bob, Carmen, and Tyler were comparing the areas of *N* and *P*. Bob said that *N* and *P* have the same area. Carmen said that the area of *N* is larger. Tyler said that the area of *P* is larger.

   Who was correct? ________________

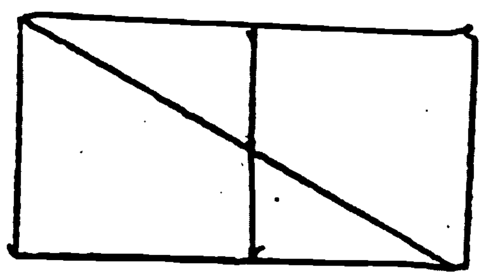
   Use words or pictures (or both) to explain why.

The question was rated according to the criteria described above for grade 4, and sample responses follow. Similar to the fourth-grade sample "incorrect" responses, the first "incorrect" response shows "Bob" named as correct but has an inadequate explanation, and the second "incorrect" response says "Tyler" is correct.

Sample "correct" response

Who was correct? Bob

Use words or pictures (or both) to explain why.



Sample "incorrect" response 1

Who was correct? Bob

Use words or pictures (or both) to explain why.

because N has 4 right sides

Sample "incorrect" response 2

Who was correct? Tyler

Use words or pictures (or both) to explain why.

Shape P is clearly larger than shape O I measured them all.

Information on student performance is presented in Table 4.17. Ninety-eight percent of eighth-grade students attempted the question, and 27 percent submitted a response that was rated "correct." Students currently taking eighth-grade mathematics or pre-algebra performed similarly, whereas students taking algebra performed better than students in the other two groups. At grade 8, the question mapped at 362 on the NAEP composite scale.

At grade 12, 95 percent of the students attempted the question, and 35 percent submitted a response that was rated "correct." Students who had calculus as their highest mathematics course performed better than those who had less mathematics. In addition, those whose highest course in the algebra-through-calculus sequence was second-year algebra outperformed those who had taken only first-year algebra. The question mapped at 350 on the composite scale.

Table 4.17 Score Percentages for "Compare Areas of Two Shapes," Grades 8 and 12

THE NATION'S REPORT CARD naep

	Correct	Incorrect: Bob–No Adequate Explanation	Incorrect: Not Bob	Omit
Grade 8				
Overall	27	16	54	2
Males	28	16	54	3
Females	26	16	56	2
White	32	16	50	2
Black	8	14	75	3
Hispanic	18	19	59	5
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	25	19	55	1
Pre-Algebra	23	14	60	2
Algebra	38	13	47	1
Grade 12				
Overall	35	14	46	5
Males	35	17	43	4
Females	35	12	48	5
White	40	15	42	3
Black	12	16	64	8
Hispanic	25	11	54	9
Asian/Pacific Islander	54	12	32	3
American Indian	***	***	***	***
Geometry Taken	38	15	43	4
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	18	8	68	4
First-Year Algebra	25	18	51	5
Second-Year Algebra	39	14	43	3
Third-Year Algebra/Pre-Calculus	44	15	36	4
Calculus	62	9	26	3

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement level who provided a "correct" response is shown in Table 4.18. Perhaps not surprisingly, performance was better at twelfth grade, with more than 70 percent of students at the *Proficient* level submitting a response considered "correct."

Table 4.18 **Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grades 8 and 12**

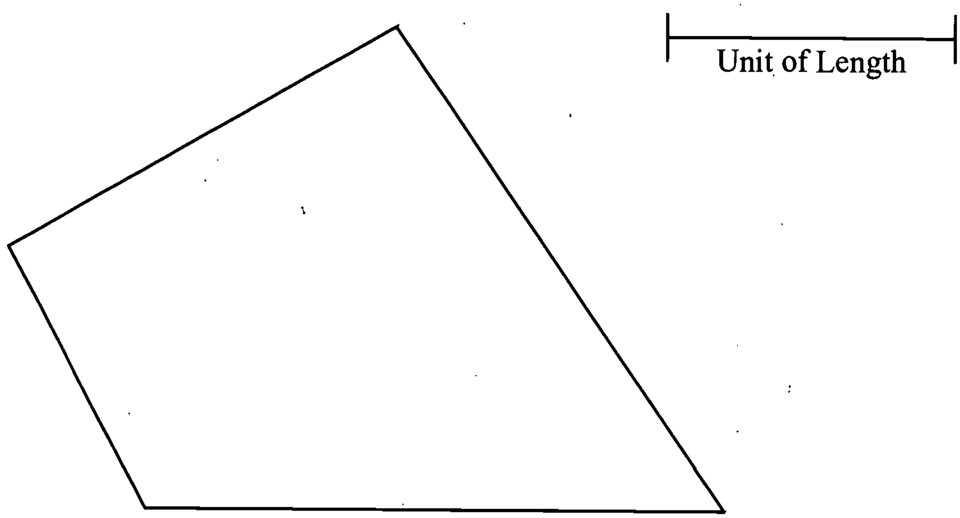
THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	27	8	26	58	***
Grade 12	35	10	39	72	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The last example in the area of *estimation of measurements* is a short constructed-response question for grade 8. Students were presented with a four-sided figure (quadrilateral) and a line labeled "unit of length" that did not give the dimensions of the unit. They were instructed to use the unit of length to estimate the perimeter of the figure and then to specify two consecutive whole-number units between which the length of the perimeter would lie. Since students did not have rulers for this question, they had to figure out other ways to estimate the number of "units" that were needed to go around the figure. They could do this by simple visual comparison or perhaps by adjusting the distance between the thumb and index finger to equal the unit of length given in the question and then applying that measure to the figure.

11. Use the unit of length below to estimate the perimeter of the figure shown. Between which two consecutive whole-number units does the perimeter lie?



Answer: Between ______________ and ______________

The correct answer is "between 6 and 7". Students needed to have both 6 and 7 in their answer for it to be rated "correct." Student responses of "between 5 and 6" and "between 7 and 8" were rated "incorrect," as were any other incorrect responses. Information on student performance is shown in Tables 4.19 and 4.20. This question had a high omit rate of 21 percent. Another 21 percent submitted a response that was considered "correct." Almost equal percentages of students (about 5%) submitted responses of "between 7 and 8" or "between 5 and 6," possibly representing a miscounting of units. Students currently taking eighth-grade mathematics or pre-algebra performed similarly, whereas students taking algebra performed better than students in the other two groups. The question mapped at 380 on the NAEP composite mathematics scale.

When performance is disaggregated by achievement level, Table 4.20 shows that 19 percent of students at the *Basic* level, 44 percent of students at the *Proficient* level, and 69 percent of students at the *Advanced* level answered the question correctly. Only five percent of students below the *Basic* level were able to answer correctly.

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Table 4.19

Score Percentages for "Find Perimeter (Quadrilateral)"

	Correct	Incorrect			Omit
	Between 6 and 7	Between 7 and 8	Between 5 and 6	Other	
Grade 8					
Overall	21	6	5	47	21
Males	22	6	5	47	20
Females	20	5	5	48	22
White	26	7	5	45	17
Black	10	2	2	53	33
Hispanic	12	1	5	55	26
Asian/Pacific Islander	--	--	--	--	--
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	16	4	5	50	23
Pre-Algebra	16	6	4	54	18
Algebra	34	8	7	37	12

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table 4.20

Percentage Correct Within Achievement-Level Intervals for "Find Perimeter (Quadrilateral)"

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
21	5	19	44	69

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

This content strand assessed students' conceptual understanding and procedural knowledge of measurement units, their ability to use measurement tools and instruments, and their problem-solving abilities applied to the concepts of perimeter, area, and volume. In addition, several questions assessed students' abilities to estimate absolute and relative measurements. Many of the questions shown here as examples were difficult for students, particularly those requiring unit conversions, calculations of volume and circumference, and estimation of measurements.

Eighth-grade algebra students tended to perform better on the questions than other eighth-grade students, whereas eighth-grade students in pre-algebra or regular mathematics tended to perform similarly. At the twelfth-grade level, those students whose highest course in the algebra-through-calculus sequence was second-year algebra tended to outperform those who had only reached first-year algebra, whereas there were not always significant differences in performance between students who had taken pre-calculus/third-year algebra and those who had stopped with second-year algebra. In addition, students who reported calculus as their highest mathematics course tended to perform better than those who had only taken the less advanced mathematics courses.

Chapter 5

Geometry and Spatial Sense

Content Strand Description

At the foundation of successful performance in the Geometry and Spatial Sense content strand is a conceptual understanding of geometric figures and their properties. However, the questions classified under this content strand extended well beyond low-level identification of geometric shapes. Some of the questions required students to visualize and draw geometric figures after transforming them or combining them with other figures, and many required them to apply their understanding of geometry to reason through and solve problems. A large number of the questions from this content strand were constructed-response questions, including questions requiring drawn responses.

At the fourth-grade level, students were asked to demonstrate an understanding of the properties of shapes and to visualize shapes and figures under simple combinations and transformations. Fourth-grade students also were asked to use their mathematical communication skills to translate verbal descriptions into drawn figures. At the eighth-grade level, some questions measured concepts related to properties of angles and polygons. These included symmetry, congruence and similarity, and the Pythagorean theorem. Students also had to apply reasoning skills to make and validate conjectures about combinations and transformations of shapes. At the twelfth-grade level, students were expected to demonstrate a knowledge of more sophisticated geometric concepts and formulas and more sophisticated reasoning processes than at earlier grade levels. Questions sometimes involved proportional reasoning or coordinate geometry.

Examples of Individual Questions and Student Performance

Several questions from the NAEP 1996 mathematics assessment follow. Presentation of the questions is organized around four areas of emphasis within the Geometry and Spatial Sense content strand. The area of *basic geometric concepts and properties* includes questions that assessed a student's conceptual understanding of geometry. The area of *geometric procedures* includes questions that assessed a student's procedural knowledge of geometric constructions and computations. The area of *geometric transformations and spatial sense* includes questions that assessed students' abilities to visualize shapes and figures as well as transformations and combinations of shapes and figures. Finally, the area of *geometric models and problems* includes questions that measured students' abilities to represent problem situations with geometric models and to apply an understanding of the properties of different figures to solve problems. Questions within all four areas also required students to reason, communicate, and make connections.

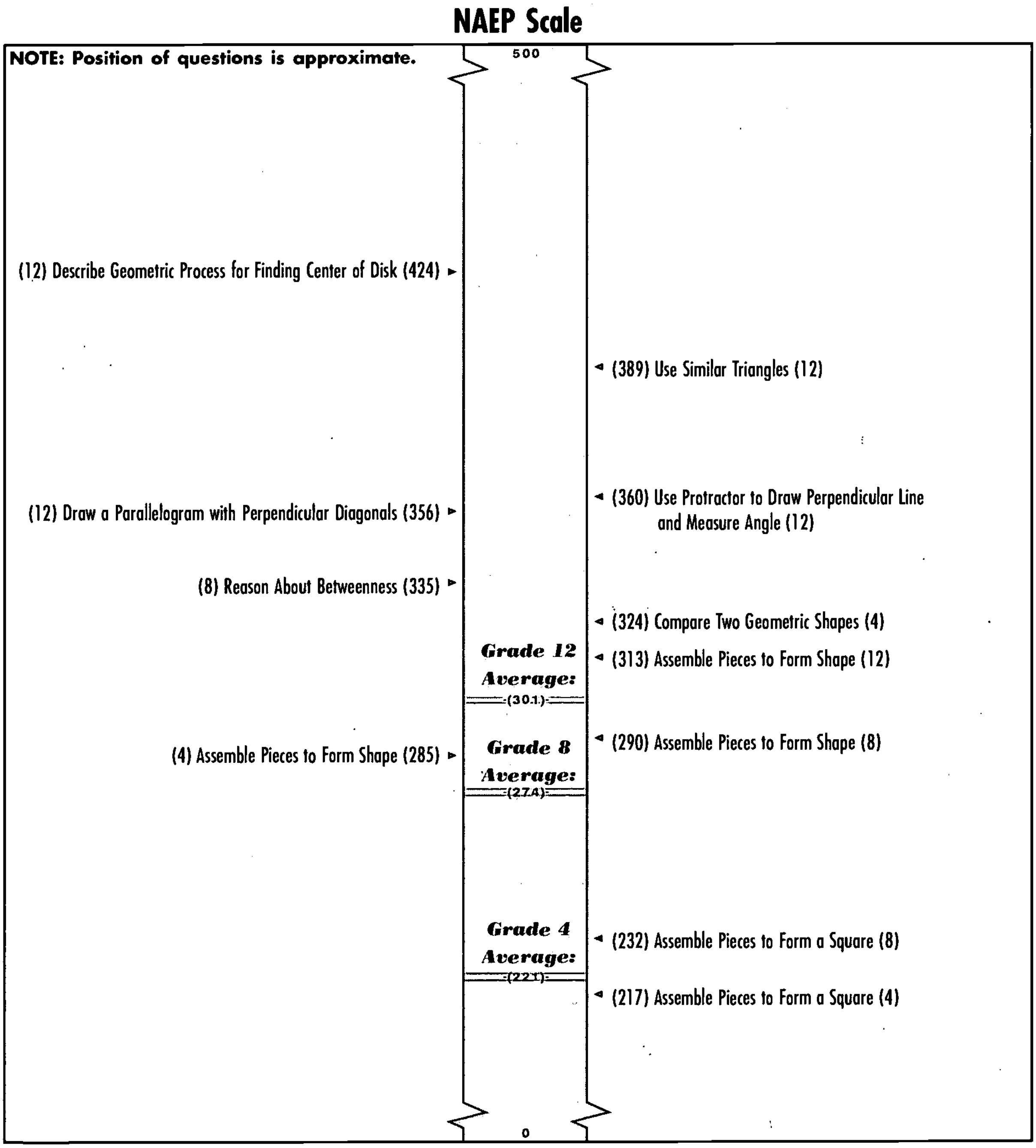
All sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 5.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each example question and, thus, to indicate the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. Keep in mind, however, that the difficulty of a question is influenced by many factors, including characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application) as well as the particular mathematics content associated with the question and student opportunities to learn this content. Also, remember that overall performance on the Geometry and Spatial Sense content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

The performance of students on the questions in the Geometry and Spatial Sense content strand is examined with respect to gender, race/ethnicity, and, for grades 8 and 12, the types of mathematics courses taken. However, as described in Chapter 2, the impact of taking geometry on student performance is not discussed for several reasons. First, there is only a small pool of students on which the specific influence of geometry could be isolated, given that most students who have taken geometry also have taken at least 2 years of algebra. Moreover, because more able students are likely to progress further in the mathematics course sequence, it is difficult to separate the impact of a particular curriculum from the impact of a student's overall strength in mathematics. Although comments on the impact of geometry course taking on performance on the questions in this content strand might be expected, these confounding effects make it difficult to isolate the specific impact of geometry. The data, however, are presented in the tables.

Figure 5.1

Map of Selected Geometry and Spatial Sense Questions on the NAEP Composite Mathematics Scale (Item Map)

THE NATION'S REPORT CARD



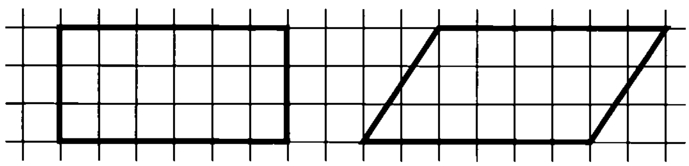
NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Basic geometric concepts and properties

Questions in this area required students to demonstrate a conceptual understanding of geometric figures, including an understanding of the properties of various figures and the definition of geometric terms. Some questions asked students to demonstrate their understanding by classifying or comparing various figures.

The example for this area is a fourth-grade extended constructed-response question. Students were presented with two four-sided figures (a rectangle and a parallelogram) and were asked to list the ways in which the figures are alike and the ways in which they are different. Students were instructed to list as many ways as they could.

Think carefully about the following question. Write a complete answer. You may use drawings, words, and numbers to explain your answer. Be sure to show all of your work.



10. In what ways are the figures above alike? List as many ways as you can.

In what ways are the figures above different? List as many ways as you can.

Some correct responses for how the figures are alike were the following:

- They both have four sides (or four corners or four angles).
- They both have parallel sides.
- They both have two sets of sides that are the same length.
- They have the same area.
- They have the same length (base).
- They have the same height.
- They have the same number of little squares.

A response of "they both have lines that are straight" was not accepted as correct. An answer that they both have four sides and four angles was considered to be only one reason.

Some correct responses for how the two figures are different were as follows:

- One has four equal angles, and the other does not.
- One has right angles or perpendicular lines, and the other does not.
- One is "slantier" than the other (or takes up full squares).
- They have different perimeters.

A response of "they are not both the same shape" was considered to be a rephrasing of the given information that the figures are different and was not accepted as correct. Furthermore, students did not need to make the comparisons in their responses; that is, they merely could have stated, for example, "one has four equal angles."

Student responses were rated as being either "extended," "satisfactory," "partial," "minimal," or "incorrect." However, when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. The rating guide for this question is presented below:

- "Extended": The student gave at least two valid reasons why the figures are alike and at least two valid reasons why they are different.
- "Satisfactory": The student gave two valid reasons why the figures are alike and at least one valid reason why they are different, *or* gave one valid reason why they are alike and two valid reasons why they are different.
- "Partial": The student gave one valid reason why the figures are alike and one valid reason why they are different, *or* gave two valid reasons why they are alike and no valid reasons why they are different, *or* gave two valid reasons why they are different and no valid reasons why they are alike.
- "Minimal": The student gave a nonspecific response (e.g., "the one on the right is skinnier") *or* gave only one correct reason why they are alike or why they are different.
- "Incorrect": Any response not fitting into the categories above.

Virtually no fourth-grade responses met the criteria for an "extended" response, that is, contained two valid reasons for why the figures are alike and two valid reasons for why they are different. However, the following is a sample of a "satisfactory" response.

Sample "satisfactory" response

10. In what ways are the figures above alike? List as many ways as you can.

they both are shapes
they both have four sides
they both have four courners
they both have the same amouht of smaller squares if you add peices together on one

In what ways are the figures above different? List as many ways as you can.

they aren't both the same shape
they don't both have full squares on both their courners

The student listed four reasons why the shapes are alike. However, one reason (they both are shapes) was not accepted as a valid reason, and the answers about both having four sides and four corners are considered to be the same reason. The student also listed two reasons why the shapes are different but was not given credit for, "they aren't both the same shape." Thus, the student was credited with providing two valid reasons why the shapes are alike and one valid reason why they are different. This met the criterion for a "satisfactory" response.

In the sample "partial" response below, the student listed three reasons why the shapes are alike and one reason why they are different, but among the reasons why they are alike, only one (they both have 18 squares) was considered correct. The response, therefore, was rated as "partial," because the student provided only one correct reason why the shapes are alike and one correct reason why they are different.

Sample "partial" response

10. In what ways are the figures above alike? List as many ways as you can.

- They both have 18 squares, they are both rectangles, they are both the same size.

In what ways are the figures above different? List as many ways as you can.

- one is slanted, one is not

The next sample response was rated as a "minimal" response. The student listed three reasons why the figures are alike and three reasons why they are different, but only one of these reasons (one is slanted/one is straight) was considered valid.

Sample "minimal" response

10. In what ways are the figures above alike? List as many ways as you can.

(1) They can both be square
(2) They can both be slanted
(3) They can both turn many ways

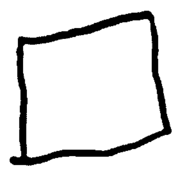
In what ways are the figures above different? List as many ways as you can.

(1) In the picture they are difrent.
(2) One is slanted.
(3) One is turn and strate.

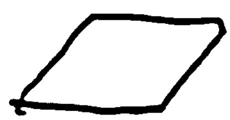
The next response, in which the student failed to list any information, was rated "incorrect."

Sample "incorrect" response

10. In what ways are the figures above alike? List as many ways as you can.



In what ways are the figures above different? List as many ways as you can.



Response rates for this question are reported in Tables 5.1 and 5.2. As stated earlier, virtually no students provided responses that were rated "extended." Only 11 percent of the students provided responses that were rated "satisfactory," and the remaining students' responses were divided fairly evenly among the "partial," "minimal," and "incorrect" categories.[1] Females were more likely than males to provide "satisfactory" or "partial" responses.

THE NATION'S REPORT CARD NAEP

Table 5.1 ***Percentage Correct for "Compare Two Geometric Shapes"***

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	0	11	29	31	23	5
Males	0	9	25	32	25	7
Females	0	13	33	30	21	3
White	0	13	32	30	20	4
Black	0!	5	21	33	28	11
Hispanic	0!	6	22	29	34	8
Asian/Pacific Islander	0!	8	21	33	33	4
American Indian	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

About 20 percent of the students whose overall mathematics performance put them at the *Proficient* achievement level provided responses that were considered to be at least "satisfactory." As would be expected, "satisfactory" responses were even less frequent at the lower achievement levels. The question mapped at a score of 324 on the NAEP composite mathematics scale.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

Table 5.2 *Percentage Satisfactory Within Achievement-Level Intervals for "Compare Two Geometric Shapes"*

THE NATION'S REPORT CARD

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
11	4	12	19	***

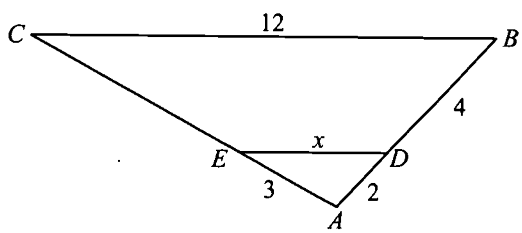
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Geometric procedures

This area included questions that assessed students' procedural knowledge in geometry, including their ability to use the Pythagorean theorem or the properties of ratio and proportion; to draw shapes; and to use such tools as straightedges, compasses, and protractors.

Three examples of twelfth-grade questions are provided for this area. One example is a multiple-choice question, and two are short constructed-response questions that required drawn responses.

The first question presented students with two similar triangles, one within the other. Measurements were provided for two sides of the small triangle and two sides of the large triangle, and students were asked to determine the length of the third side of the small triangle.



1. If triangles ADE and ABC shown in the figure above are similar, what is the value of x?

Ⓐ 4

Ⓑ 5

Ⓒ 6

Ⓓ 8

Ⓔ 10

Did you use a calculator on this question?

◯ Yes ◯ No

The correct option is A.

To respond correctly to this question, students needed to know the properties of similar triangles — specifically, how to find the length of the sides of one triangle given the length of the corresponding sides of a similar triangle. In this question, the sides of the smaller triangle were one-third the length of the corresponding sides of the larger triangle. Once students recognized this, they could compute the length of small triangle side x as $1/3$ of large triangle side 12.

Performance data for this question are shown in Tables 5.3 and 5.4. Thirty-seven percent of twelfth-grade students provided a correct response. Half of the students whose highest mathematics course was third-year algebra or pre-calculus and more than 60 percent of the students who had taken calculus responded correctly to the question. Both of these groups of students performed better than students who had taken less math.

Forty-seven percent of the students chose "6" (Option C) as the correct response. These students may have calculated the length of AB instead of x, may have thought the smaller triangle was $1/2$ the length of the larger triangle, or may have used, in their calculations, the 4 to 2 relationship of DB to AD instead of the 6 to 2 relationship of AB to AD.

Table 5.3 ***Percentage Correct for "Use Similar Triangles"***

THE NATION'S REPORT CARD NAEP

Grade 12	Percentage Correct
Overall	37
Males	38
Females	36
White	40
Black	30
Hispanic	25
Asian/Pacific Islander	44
American Indian	***
Geometry Taken	38
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	***
First-Year Algebra	34
Second-Year Algebra	32
Third-Year Algebra/Pre-Calculus	51
Calculus	62

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The question mapped at 389 on the NAEP composite mathematics scale. Over half of the students classified as *Proficient* responded correctly to the question compared with approximately one-third of those in the *Basic* category and approximately one-quarter of those whose overall performance was below *Basic*.

Table 5.4 **Percentage Correct Within Achievement-Level Intervals for "Use Similar Triangles"**

THE NATION'S REPORT CARD naep

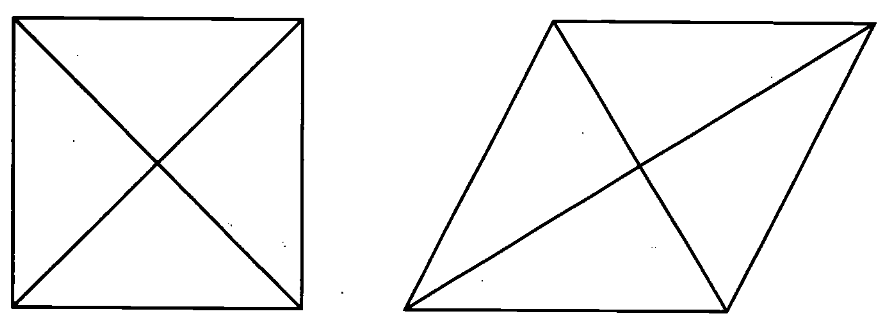
Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
37	26	37	56	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In the next sample question for this area, students were asked to draw a parallelogram with perpendicular diagonals. To respond correctly, students needed to know the definitions of parallelogram, perpendicular, and diagonal. They also needed to be able to transfer this knowledge to a drawing.

> **2.** In the space below, use your ruler to draw a parallelogram that has perpendicular diagonals. Show the diagonals in your sketch.

"Correct" responses included drawings of a square or another rhombus with diagonals shown:



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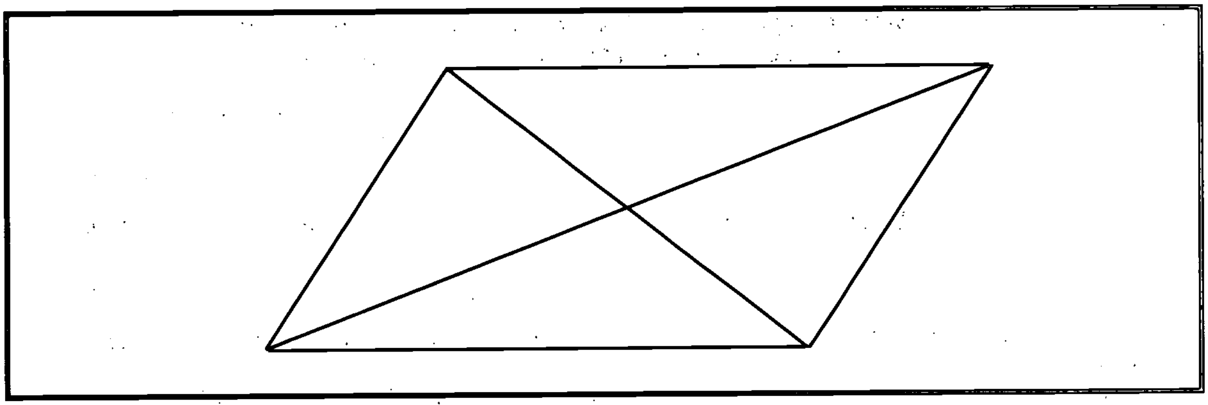
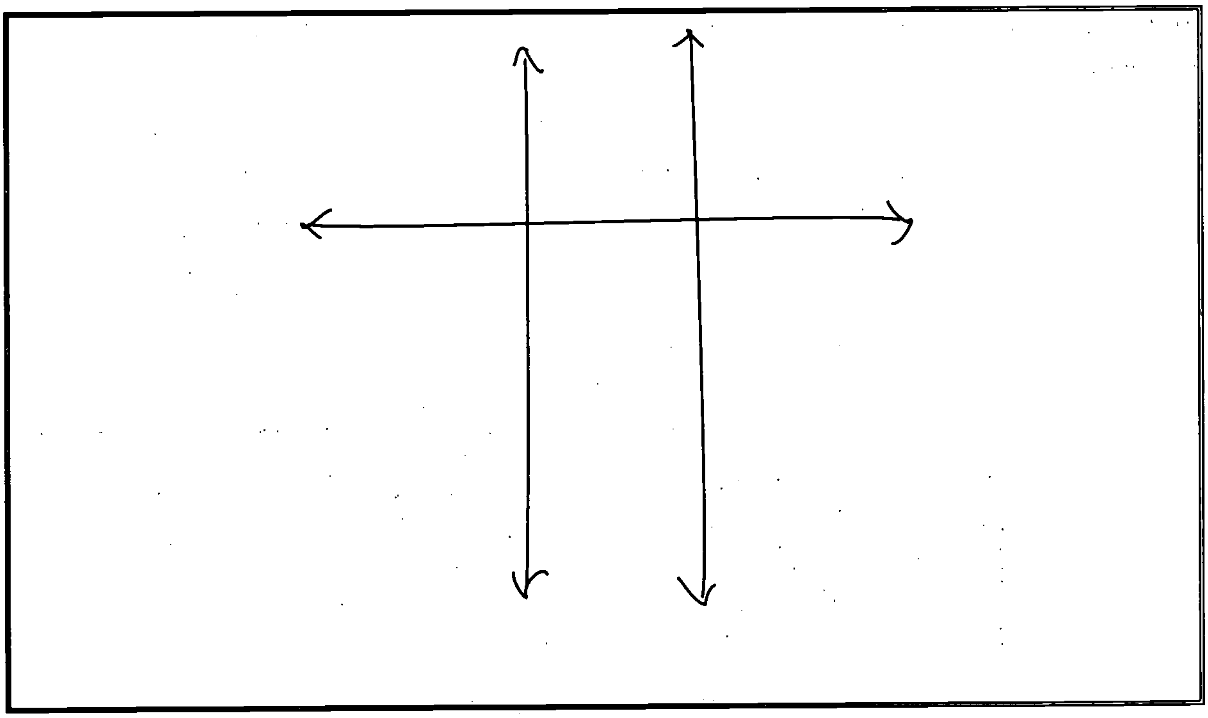
All other responses were considered "incorrect." A common "incorrect" response was a quadrilateral, with or without diagonals, that appeared to be a parallelogram other than those already described. One sample response of an incorrect quadrilateral follows:

Sample "incorrect" response 1



Another sample of an incorrect drawing is the following:

Sample "incorrect" response 2



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Student performance data are presented in Tables 5.5 and 5.6. Nineteen percent of the students responded correctly to the question, including seven percent who drew a correct rhombus that was not a square. Forty-four percent of the students drew an incorrect quadrilateral, and 22 percent of the students submitted other types of "incorrect" responses. Students who had taken calculus performed better than all other groups of students (56% responded correctly), and students whose highest course in the algebra-calculus sequence was third-year algebra or pre-calculus performed better than those with less mathematics.

Table 5.5 ***Score Percentages for "Draw a Parallelogram with Perpendicular Diagonals"***

THE NATION'S REPORT CARD naep

	Correct		Incorrect		Omit
	Rhombus that is Not a Square	Square	Quadrilateral with Incorrect Diagonals	Other	
Grade 12					
Overall	7	12	44	22	15
Males	7	13	42	23	15
Females	7	12	46	20	15
White	8	15	47	19	10
Black	2	4	38	31	26
Hispanic	3	3	37	30	27
Asian/Pacific Islander	16	20	35	14	15
American Indian	***	***	***	***	***
Geometry Taken	8	14	48	68	10
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	0!	4!	40	32	20
First-Year Algebra	4	6	44	26	18
Second-Year Algebra	6	12	48	21	12
Third-Year Algebra/Pre-Calculus	12	21	49	12	6
Calculus	27	29	25	15	5

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Fifteen percent of the students did not attempt the question. The tendency to omit the question was higher for students whose highest course was either pre-algebra or first-year algebra than it was for students who had taken more advanced courses.

Table 5.6 shows student performance disaggregated by NAEP achievement levels. Over half of the students classified as *Proficient* responded correctly to the question, whereas only 17 percent of those classified as *Basic* and 1 percent of those classified as below *Basic* responded correctly. The question mapped at a score of 356 on the NAEP composite mathematics scale.

Table 5.6 ***Percentage Correct Within Achievement-Level Intervals for "Draw a Parallelogram with Perpendicular Diagonals"***

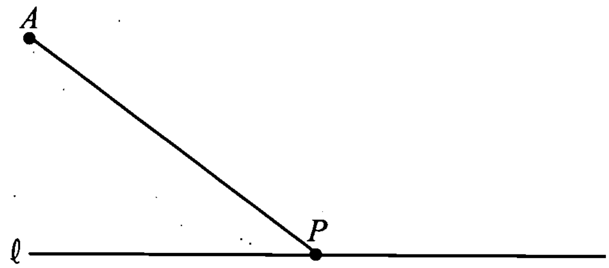
THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
19	1	17	57	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final sample question for this area follows. The question presented a line l containing a point P and an angle formed by line segment AP and line l. Students first were asked to draw a line (m) through point P that was perpendicular to segment AP. They then were asked to measure, with a protractor, the smaller angle formed by lines l and m. Correct responses required students to understand that perpendicular means 90 degrees and to know how to use the protractor to draw a 90-degree angle and measure another angle. Students also needed to recognize which angle to measure.

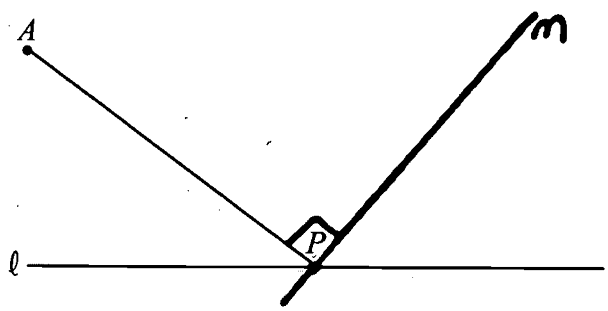
8. In the figure below, use the protractor to draw a line m through point P perpendicular to segment AP. In the answer space provided, give the measure of the smaller angle formed by lines ℓ and m.



Answer: ____________________

The correct response for the size of the angle was 50 degrees, but, to allow for measurement error, answers between 46 degrees and 54 degrees inclusive were accepted as "correct." All other answers were rated as "incorrect." The following is an example of a "correct" response.

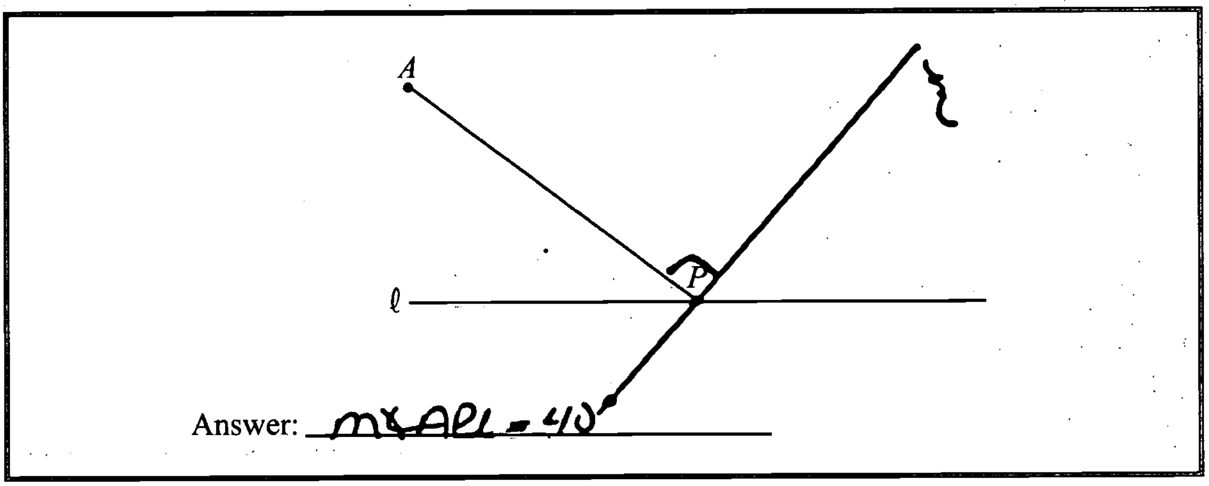
Sample "correct" response



Answer: 50°

Some "incorrect" responses gave correct angle measurements but had incorrectly drawn lines. The following sample of an "incorrect" response shows an instance in which the line was drawn correctly, but the angle measurement was incorrect.

Sample "incorrect" response 1



In the following sample of an "incorrect" response, the student incorrectly drew a line that was perpendicular to line *l*.

Sample "incorrect" response 2

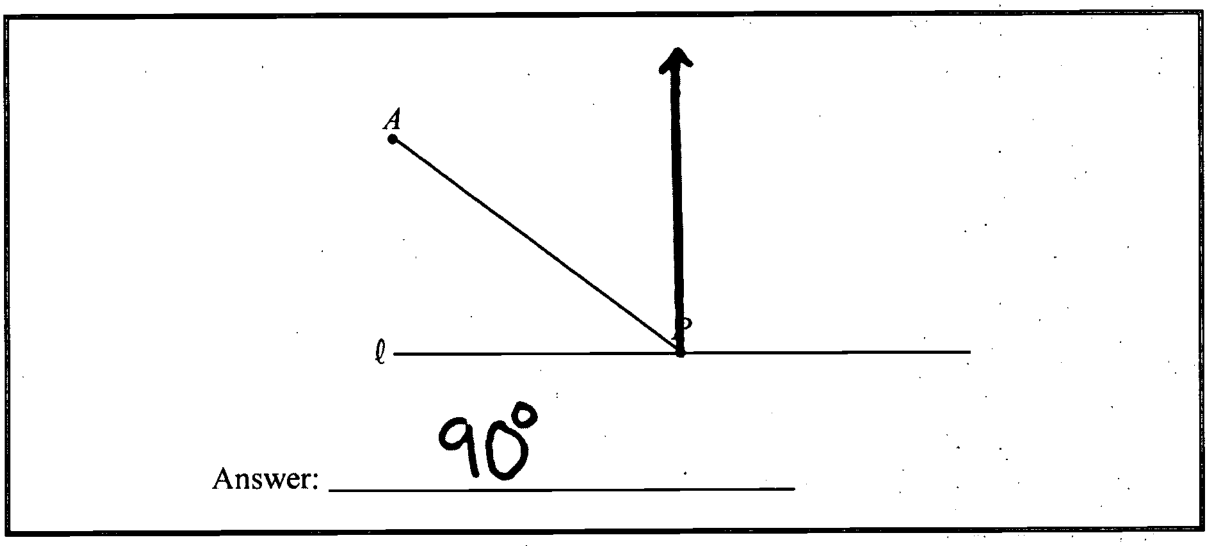


Table 5.7 shows student performance overall and by various subgroups. Twenty-three percent of the students drew line *m* correctly and provided a correct answer for the size of the angle; 4 percent drew incorrect lines but had correct angle measurements; 17 percent drew correct lines but had incorrect angle measurements; and half of the students had other "incorrect" responses. A higher percentage of males than females provided "correct" responses to the question, and the probability of a "correct" response was related to the student's mathematics preparation.

Table 5.7 ***Score Percentages for "Use Protractor to Draw Perpendicular Line and Measure Angle"***

THE NATION'S REPORT CARD NAEP

Grade 12	Correct	Incorrect: Line, Correct Angle	Incorrect: Angle, Correct Line	Incorrect: Other	Omit
Overall	23	4	17	50	7
Males	26	4	17	46	7
Females	19	4	17	54	7
White	27	4	17	45	7
Black	6	3	15	67	9
Hispanic	11	3	13	67	6
Asian/Pacific Islander	35	3	25	36	1
American Indian	***	***	***	***	***
Geometry Taken	26	4	18	48	4
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	***	***	***	***	***
First-Year Algebra	14	3	16	63	3
Second-Year Algebra	21	4	17	52	4
Third-Year Algebra/Pre-Calculus	34	5	21	32	8
Calculus	49	5	18	25	1!

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Performance by achievement levels, shown in Table 5.8, was similar to that of the previous question. The question mapped at a score of 360.

Table 5.8 **Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw Perpendicular Line and Measure Angle"**

THE NATION'S REPORT CARD NAEP

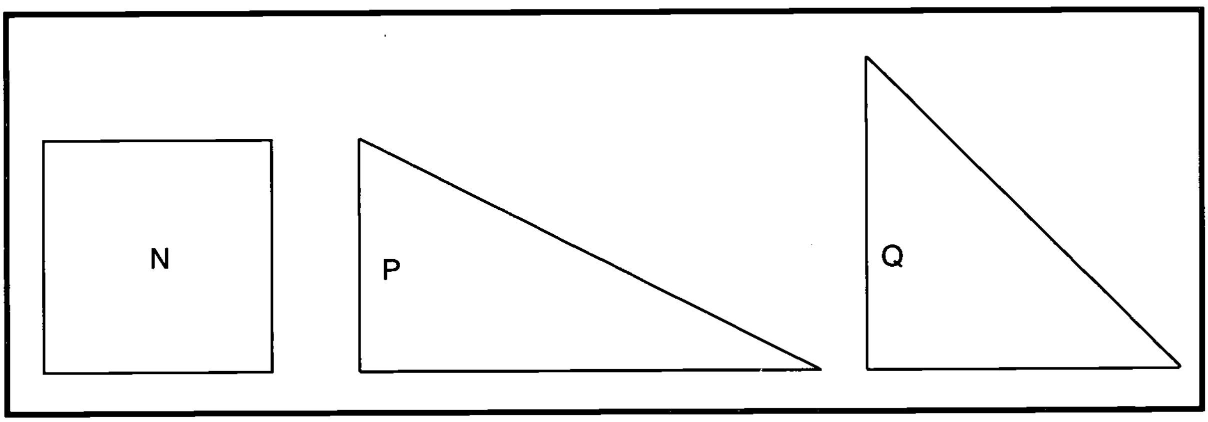
Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
23	1!	21	57	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Geometric transformations and spatial sense

The area of geometric transformations and spatial sense includes questions that tapped students' visual-spatial skills. In many of the questions, students were presented with a figure and asked what the figure would look like if it were flipped, rotated, folded, unfolded, pulled apart, combined with another figure, or transformed in some other manner.

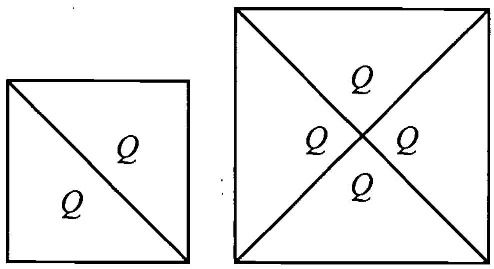
Three sample questions are presented for this area. The first two sample questions were included within a block of questions for which students were provided with two cardboard cutouts of each of the following shapes:



For the first question, students at grades 4 and 8 were asked to assemble the 2 pieces labeled Q (triangles) to make a square.

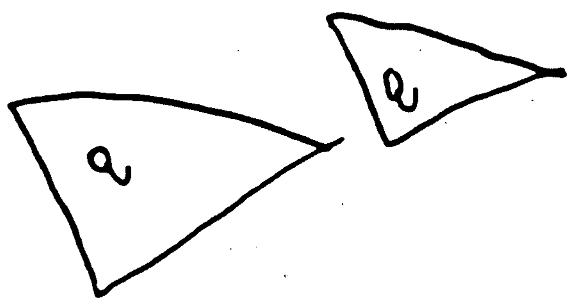
> **3.** You will need the 2 pieces labeled Q. Please find those 2 pieces now.
>
> Use the 2 pieces labeled Q to make a square. Trace the square and draw the line to show where the 2 pieces meet.

Responses that were rated as "correct" could include either two or four replicates of Q, as long as the final shape was a square.



Diagonals had to be shown, although a slight space between the shapes was acceptable. Freehand drawings also were rated as "correct." Squares without diagonals drawn were rated "incorrect," as were other types of drawings not resembling those above. A sample of an "incorrect" response follows:

Sample "incorrect" response



Student performance data are presented in Tables 5.9 and 5.10. As might be expected, the question was fairly easy for students at both grade levels. Seventy-three percent of fourth-grade students and 89 percent of eighth-grade students assembled and drew the pieces correctly. The question mapped at a score of 217 for grade 4 and 232 for grade 8. At least three-quarters of the students classified in each of the achievement levels at each grade responded correctly, except for fourth-grade students performing below the level of *Basic*.

Table 5.9 **Percentage Correct for "Assemble Pieces to Form a Square"**

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 4	
Overall	73
Males	75
Females	71
White	81
Black	46
Hispanic	55
Asian/Pacific Islander	81
American Indian	***
Grade 8	
Overall	89
Males	89
Females	89
White	93
Black	75
Hispanic	84
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	89
Pre-Algebra	91
Algebra	91

***Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 5.10 Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form a Square"

THE NATION'S REPORT CARD naep

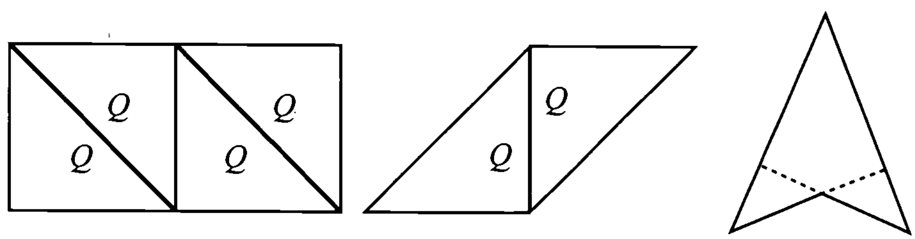
	Overall	NAEP Grades 4 and 8 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 4	73	38	85	98!	***
Grade 8	89	77	95	99	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next question, which was presented to students at all three grade levels, was more difficult. For this question, students had to assemble the same two pieces used in the previous question to make a four-sided shape that was not a square. Students again had to trace the figure and draw a line to show where the pieces met.

> 3. Use the 2 pieces labeled Q to make a 4-sided shape that is not a square. Trace the shape and draw the line to show where the 2 pieces meet.

The three types of drawings that were accepted as "correct" follow. As in the previous question, two or four replicates of Q could be used, but they had to be placed such that the resultant shape was rectangular or rhomboid, but not square. A four-sided figure composed by overlapping two Q shapes also was accepted as "correct." In the latter case, it was not necessary to show what would anyway have been an ambiguous concept, namely the line where the two pieces meet.



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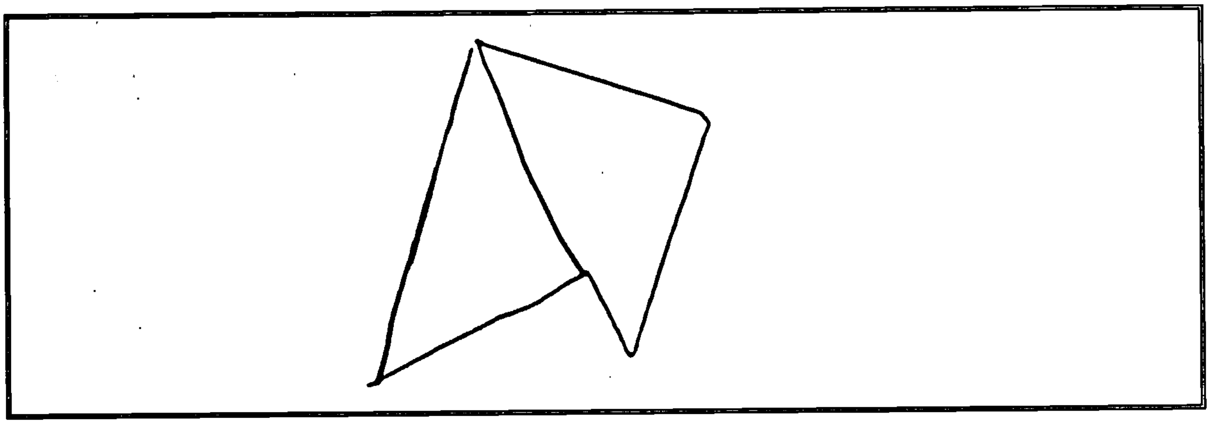
A sample "incorrect" response, in which the figure has more than four sides, is shown below.

Sample "incorrect" response



Tables 5.11 and 5.12 present performance results for this question. Only 16 percent of the students at grade 4 were able to provide a "correct" response to the question, while close to half of the eighth-grade students and over half of the twelfth-grade students provided a response rated "correct." Males were more likely than females to respond correctly. Furthermore, eighth-grade students in algebra performed better than those in pre-algebra or eighth-grade mathematics.

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Table 5.11 Score Percentages for "Assemble Pieces to Form Shape"

THE NATION'S REPORT CARD NAEP

	Correct		Incorrect	Omit
	Rhombus	Not a Rhombus		
Grade 4				
Overall	15	1	80	5
Males	18	1	75	6
Females	12	1	84	4
White	19	1	77	3
Black	3	0!	88	9
Hispanic	9	1!	80	11
Asian/Pacific Islander	17	2!	80	1!
American Indian	***	***	***	***
Grade 8				
Overall	45	3	49	2
Males	48	3	46	2
Females	42	3	52	3
White	52	3	43	2
Black	17	4	74	5
Hispanic	38	5	54	4
Asian/Pacific Islander	--	--	--	--
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	41	3	54	2
Pre-Algebra	42	2	54	2
Algebra	59	4	36	1
Grade 12				
Overall	53	5	39	3
Males	55	6	36	3
Females	51	4	42	3
White	59	5	34	2
Black	28	4	63	5
Hispanic	39	8	49	4
Asian/Pacific Islander	68	8	23	1!
American Indian	***	***	***	***
Geometry Taken	56	5	37	2
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	35	11	46	6
First-Year Algebra	51	4	42	2
Second-Year Algebra	54	5	39	2
Third-Year Algebra/Pre-Calculus	60	5	32	2
Calculus	60	5	34	1!

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Among eighth- and twelfth-grade students, over half of those classified as *Basic* or *Proficient* responded correctly to the question. At grade 4, "correct" responses were provided by 15 percent of those classified as *Basic* and one-third of those classified as *Proficient*. The question mapped at 285 for grade 4, 290 for grade 8, and 313 for grade 12.

Table 5.12 **Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form Shape"**

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 4, 8, and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 4	16	3	15	33	***
Grade 8	49	25	54	76	***
Grade 12	58	32	66	81	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final example for this area was a short constructed-response question for eighth-grade students. Students were given three facts about the spatial relationships among points *A*, *B*, and *C* and asked whether these facts supported the conclusion that *C* always had to fall between *A* and *B*. They also were asked to draw a diagram to explain their answer.

12. Jaime knows the following facts about points *A*, *B*, and *C*.

- Points *A*, *B*, and *C* are on the same line, but might not be in that order.
- Point *C* is twice as far from point *A* as it is from point *B*.

Jaime concluded that point *C* is always between points *A* and *B*.

Is Jaime's conclusion correct?

◯ Yes ◯ No

In the space provided, use a diagram to explain your answer.

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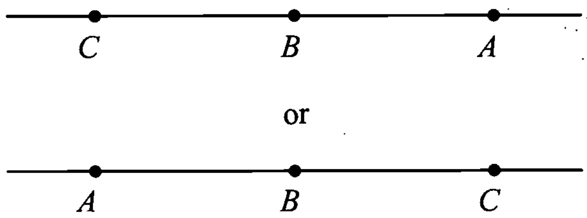
To be considered "correct," a response had to disagree with the conclusion and include one of the following diagrams in which *B* is shown to fall halfway between points *A* and *C*. All other responses were considered "incorrect."



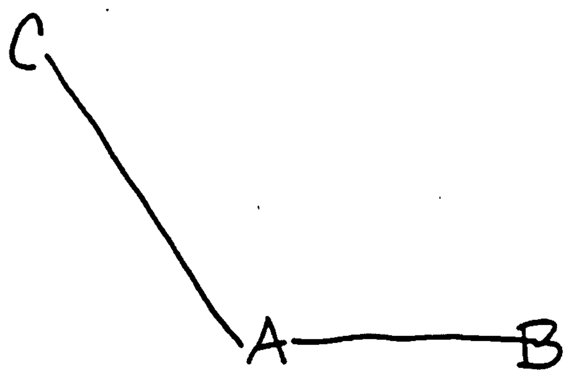
The following is an example of an "incorrect" response.

Sample "incorrect" response

Is Jaime's conclusion correct?

○ Yes ● No

In the space provided, use a diagram to explain your answer.



Student performance data are presented in Tables 5.13 and 5.14. The question was answered correctly by approximately one-quarter of the students. More females than males responded correctly. The percentage of eighth-grade students enrolled in algebra who answered the question correctly (34%) was greater than the percentage of those enrolled in pre-algebra or regular mathematics who answered correctly. Slightly under half of the students classified as *Advanced* or *Proficient* responded correctly. However, only 24 percent of the students classified at the *Basic* achievement level and 6 percent of those classified as below *Basic* responded correctly. The question mapped at a score of 335.

Table 5.13 **Percentage Correct for "Reason About Betweenness"**

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	23
Males	21
Females	26
White	27
Black	10
Hispanic	16
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	18
Pre-Algebra	19
Algebra	34

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 5.14 **Percentage Correct Within Achievement-Level Intervals for "Reason About Betweenness"**

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
23	6	24	44	48

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Geometric models and problems

Questions falling into the final area of geometric models and problems required students to apply their geometric skills and understanding in order to represent situations with geometric models or solve practical problems. Many questions requiring extended responses fell into this area. The example shown is a twelfth-grade extended constructed-response question. Students were asked to describe a procedure for locating the point that is the center of a circular paper disk. They were provided with an actual disk and told they could manipulate it in any way. Students were asked to use "geometric definitions, properties, or principles" to justify their procedure.

> This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all your work.
>
> **10.** Describe a procedure for loacting the point that is the center of a circular paper disk. Use geometric definitions, properties, or principals to explain why your procedure is correct. Use the disk provided to help you formulate your procedure. You may write on it or fold it in any way that you find helpful, but it will not be collected.

Responses were rated "extended," "satisfactory," "partial," "minimal," or "incorrect." However, when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. A description of the ratings and sample responses for each rating category follow.

An "extended" response was one that described locating the center of the circle by folding or by compass and straightedge construction and that clearly and completely explained what geometric properties of circles justified the method chosen (e.g., two diameters intersect in the center of the circle, the intersection of two perpendicular bisectors of two nonparallel chords is the center of the circle). So few students received a rating of "extended" that no samples are available to present.

A "satisfactory" response was one in which the student described a method of locating the center point by folding or by compass-and-straightedge construction but did not use appropriate geometry terminology in the explanation. For example, the following sample response was rated as "satisfactory." In it the student described a method of folding the circle in half and then in half again, but did not use any geometric terminology.

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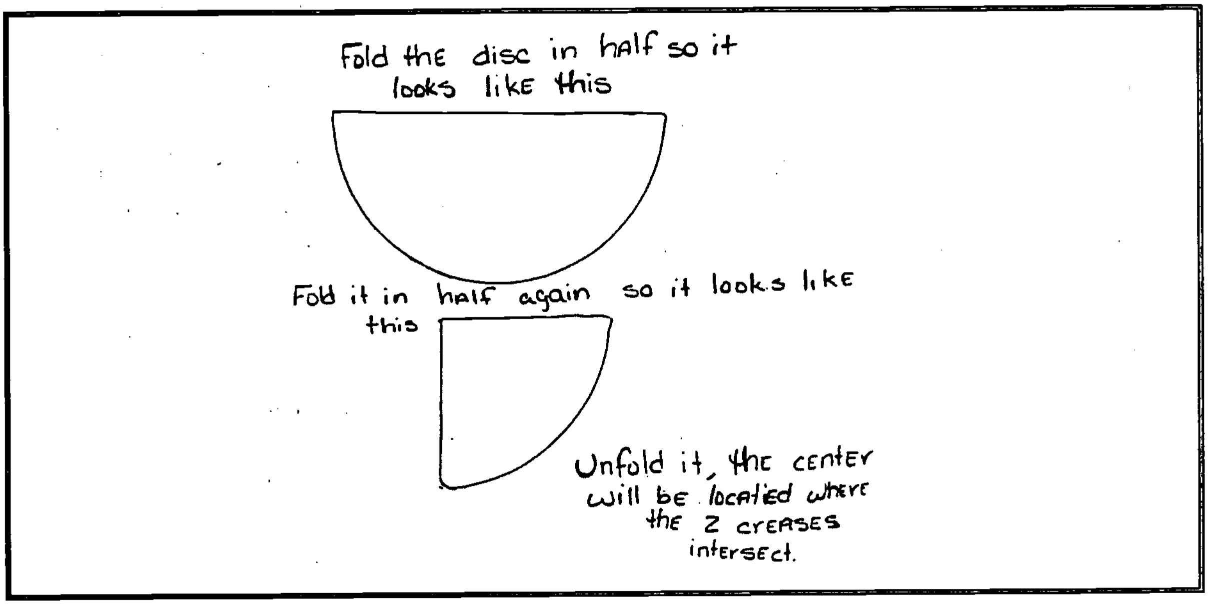
Sample "satisfactory" response



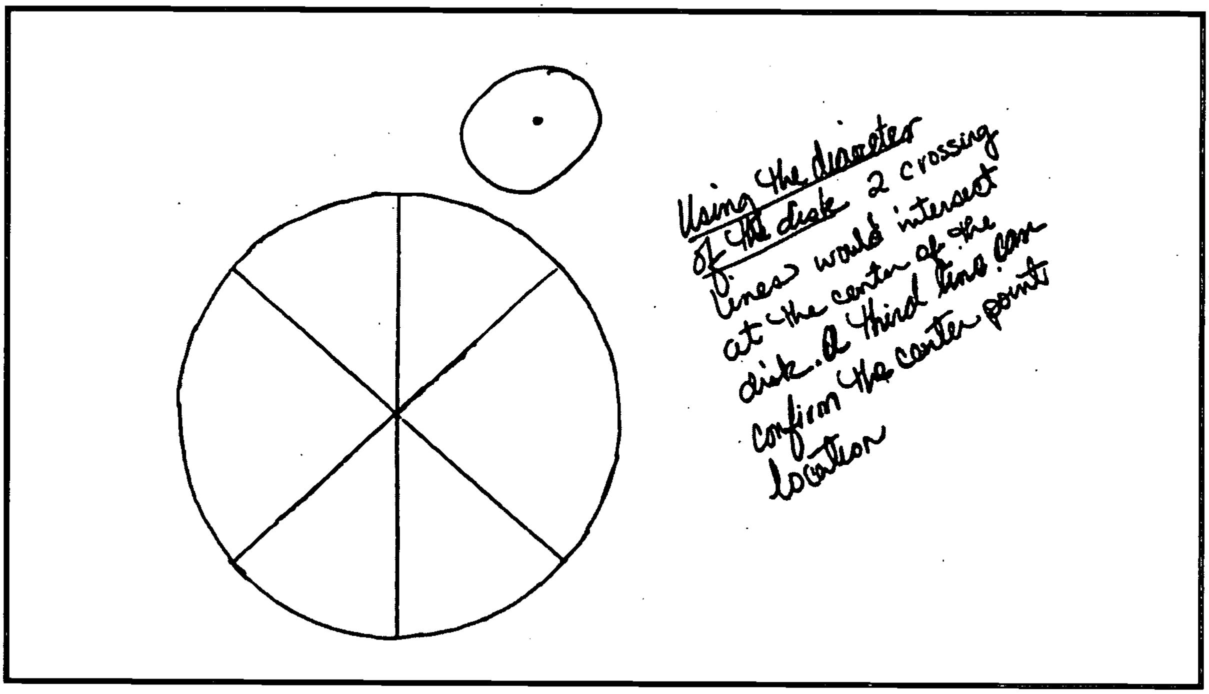
A "partial" response was one that located the center by folding or that described a compass and straightedge construction of the perpendicular bisectors of two nonparallel chords, but the explanation was incomplete. Responses that correctly explained a drawing of two diameters *or* the perpendicular bisectors of two nonparallel chords also were considered as "partial." The following response was rated "partial" because the student did not describe how to determine the diameter of the circle; thus, the response was incomplete.

Sample "partial" response

> You could take the ruler and draw a diameter across the disk. Draw another diameter perpendicular to the first diameter. Where the two diameters intersect is where the point is located.

"Minimal" responses showed a line that appeared to include the center of the circle (e.g., a diameter or the perpendicular bisector of a chord), but the explanation was inaccurate, as in the following example.

Sample "minimal" response



All other responses were considered to be "incorrect."

Sample "incorrect" response

> Measure around it and use that number for C in the equation.
>
> $C = 2\pi r$
>
> If the number is 6.5 for ex. it would be $6.5 = 2\pi r$
>
> $r = 1.04$ then you would measure to the middle w/a ruler

Tables 5.15 and 5.16 present student performance data on this question. Only 1 percent of the students provided a response that was considered to be "extended," and 13 percent provided "satisfactory" answers. Approximately one-quarter each provided answers that were rated as either "minimal" or "incorrect," and nearly as many did not respond at all. Students who had taken calculus were less likely than others to omit the question, and students who had taken at least third-year algebra, pre-calculus, or calculus were more likely to receive a rating of at least "satisfactory" than those who had not.

Table 5.15 ***Score Percentages for "Describe Geometric Process for Finding Center of Disk"***

THE NATION'S REPORT CARD naep

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	1	13	9	25	28	23
Males	1	13	10	25	28	22
Females	0!	13	7	26	29	24
White	1	15	9	28	27	19
Black	0!	4	5	13	36	42
Hispanic	0!	9	7	26	28	28
Asian/Pacific Islander	3	17	17	19	30	14
American Indian	***	***	***	***	***	***
Geometry Taken	1	14	9	26	29	22
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	0	4	8!	25	33	25
First-Year Algebra	0!	8	11	26	27	25
Second-Year Algebra	1	13	8	26	30	21
Third-Year Algebra/Pre-Calculus	2	20	5	25	25	23
Calculus	0!	26	10	25	24	16

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The question mapped at a score of 424. One-third of the students at the *Proficient* achievement level and lower percentages at the levels below *Proficient* were able to respond correctly.

Table 5.16 **Percentage Satisfactory Within Achievement-Level Intervals for "Describe Geometric Process for Finding Center of Disk"**

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
13	4	14	31	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

Many of the example questions for this content strand were difficult for students. The easier questions required students to construct or describe simple shapes, and more difficult questions required the application of knowledge about geometric properties to solve complex problems. Most of the questions required students to draw or explain a response. Questions in this content strand also relied upon students' visual-spatial skills. For many of the sample questions a significant difference between the performance of male and female students existed. Eighth-grade algebra students tended to perform better than other eighth-grade students, whereas eighth-grade students in pre-algebra or regular mathematics performed similarly. Also, an increase in performance was sometimes noted between twelfth-grade students who had taken at least second-year algebra and those who had not. Additionally, a further increase in performance was noted at times for students who had taken at least third-year algebra or pre-calculus.

Chapter 6

Data Analysis, Statistics, and Probability

Content Strand Description

Questions in this content strand assessed students' skills in collecting, organizing, reading, representing, and interpreting data. Also assessed were students' understanding of the basic elements of sampling, data analysis, and probability as well as their competence in calculating simple statistics and probabilities. Many questions required a constructed response and asked students to do a variety of tasks, such as completing or discussing charts and graphs or describing the best ways to collect or display data.

Students at grade 4 were expected to be familiar with a variety of types of graphs (typically pictorial), make predictions from data and explain their reasoning, and use the basic concept of chance. At grade 8, students were expected to analyze statistical claims and design experiments, demonstrate some understanding of sampling, and be able to make predictions based on complex data. Students at grade 12 were expected to use a wide variety of statistical techniques to model situations and solve problems. They also were expected to understand and apply concepts of probability to dependent and independent events and to have some knowledge of conditional probability.

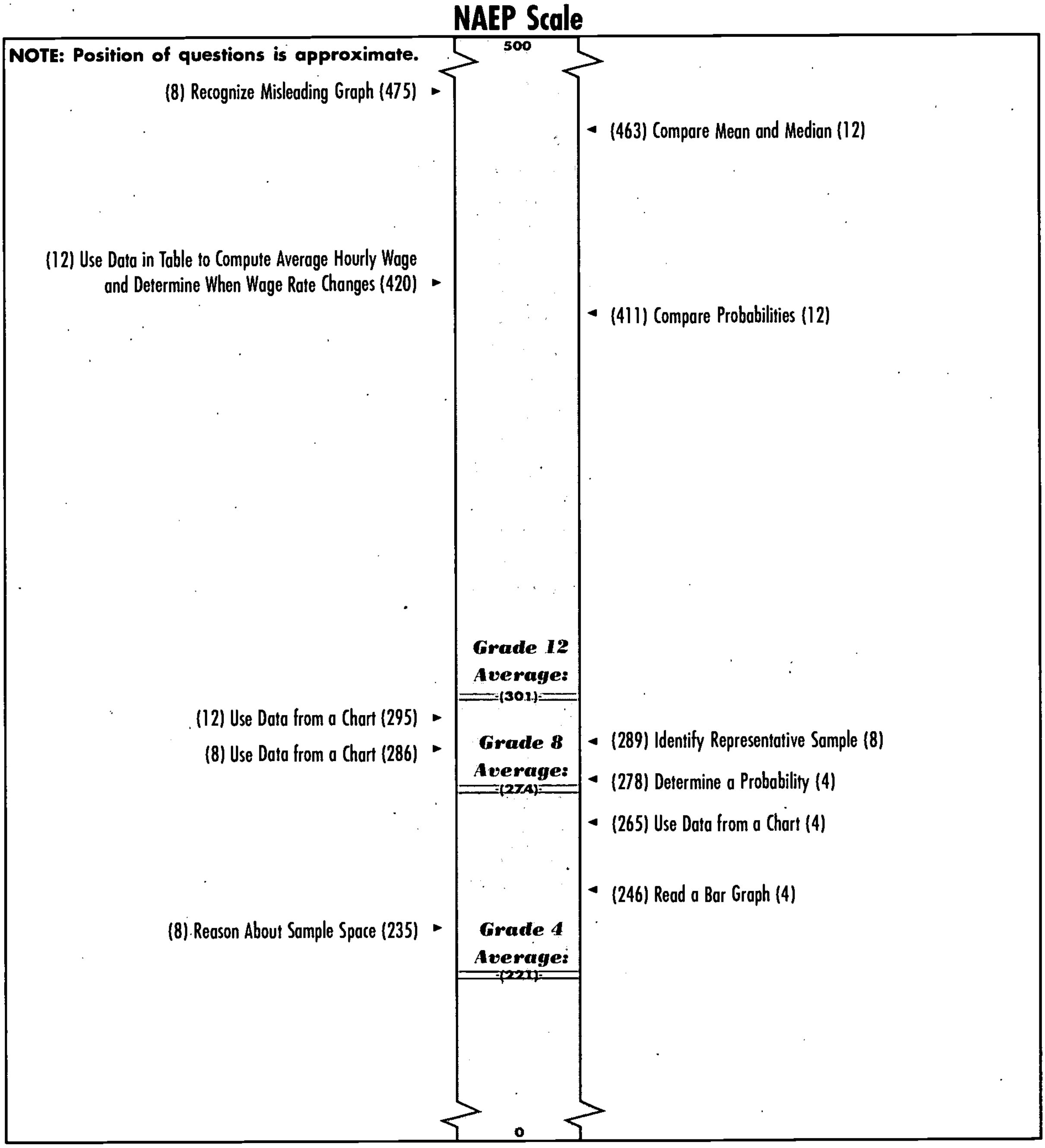
Examples of Individual Questions and Student Performance

A number of the Data Analysis, Statistics, and Probability questions from the NAEP 1996 mathematics assessment are shown in this chapter. Presentation of the questions is organized around three areas of emphasis. *Tables, graphs, and charts* includes questions that assessed students' abilities to interpret and display data; *sampling and statistics* includes questions that assessed students' knowledge and skills in these areas; and *probability* includes questions that assessed students' understanding of and ability to calculate the probability of simple and related events.

All sample questions from this content strand are mapped onto the NAEP mathematics scale as shown in Figure 6.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each example question and, thus, to indicate the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. As noted in previous chapters, however, the difficulty of any question is a function of the relationship between characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application), the specific mathematics content associated with the question, and students' opportunities to learn this content. It should be remembered also that overall performance on the Data Analysis, Statistics, and Probability content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

Figure 6.1 ***Map of Selected Data Analysis, Statistics, and Probability Questions on the NAEP Composite Mathematics Scale (Item Map)***

THE NATION'S REPORT CARD



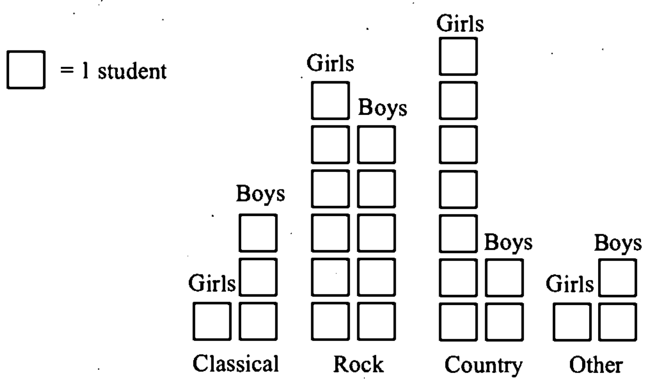
NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Tables, graphs, and charts

These questions assessed students' abilities to interpret and display data in tables, graphs, and charts. At all grade levels, students had to read and interpret data, make predictions, compute with data, and interpolate and extrapolate. They also had to translate data into tables and graphs. Questions for fourth-grade students often used pictographs, with symbols representing single or multiple units. Fourth-grade students also were evaluated on their ability to interpret simple pie charts. Questions for older students included stem-and-leaf and box-and-whisker plots. Graphs and charts often involved percents, and graphs often compared units on two dimensions. Students in eighth and twelfth grade were asked to make decisions about the best representation of data for certain situations or to compare data in two different tables, graphs, or charts.

Four examples of questions are presented here — one at each grade level and one that appeared at all three grade levels. The first example is a multiple-choice question that appeared on the assessment for fourth-grade students. The question presented students with a bar graph representing class votes on favorite types of music. Results for three types of music and a residual "other" category were displayed separately for boys and girls. A legend indicated that the square symbol used in the graph represented one student. Students were to determine the type of music preferred by most of the students in the class. In order to respond correctly, students had to add the number of votes for boys and girls together within categories and compare the totals.

4. Each boy and girl in the class voted for his or her favorite kind of music. Here are the results.



Which kind of music did most students in the class prefer?

Ⓐ Classical

Ⓑ Rock

Ⓒ Country

Ⓓ Other

Did you use a calculator on this question?

◯ Yes ◯ No

The correct option is B.

This question was not very difficult for fourth-grade students. It mapped at a score of 246 on the NAEP composite mathematics scale. Student performance data are presented in Tables 6.1 and 6.2. Nearly 60 percent of the students responded correctly to the question. Another 36 percent of the students chose Option C (country music) as the appropriate response. These students may not have understood that they had to sum the data for girls and boys and may have simply chosen the category with the longest bar. Table 6.2 shows that approximately two-thirds of the students at the *Basic* achievement level and more than 80 percent of those at the *Proficient* level responded correctly to the question.

Table 6.1 **Percentage Correct for "Read a Bar Graph"**

THE NATION'S REPORT CARD NAEP

Grade 4	Percentage Correct
Overall	59
Males	61
Females	57
White	67
Black	33
Hispanic	45
Asian/Pacific Islander	***
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 6.2 **Percentage Correct Within Achievement-Level Intervals for "Read a Bar Graph"**

THE NATION'S REPORT CARD NAEP

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
59	38	66	82	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second example for this area was a question that appeared on the assessment for all three grade levels. It is a short constructed-response question for which students had to interpret data from a table and then explain their interpretation. The data in this question again represented votes, this time regarding shapes that were being considered for a class symbol. (The question fell within a block for which students were supplied with cardboard shapes or manipulatives. The designations *N*, *P*, and *Q* that are used in the question refer to these shapes.) Based on the preference data from three classes, students were to determine which shape should be selected for the symbol and tell why. The correct response was shape *N* because it received more total votes than the other two shapes; students also could have stated that it was the first choice in one class and the second choice in the others.

5. This question refers to pieces *N*, *P*, and *Q*.

In Mr. Bell's classes, the students voted for their favorite shape for a symbol. Here are the results.

	Class 1	Class 2	Class 3
Shape *N*	9	14	11
Shape *P*	1	9	17
Shape *Q*	22	7	2

Using the information in the chart, Mr. Bell must select one of the shapes to be the symbol. Which one should he select and why?

The shape Mr. Bell should select: ____________________

Explain:

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A sample "correct" response follows. In this response, the student chose shape *N*, supporting this choice by adding up the total number of votes for each shape and showing that shape *N* received the most votes overall.

Sample "correct" response

The shape Mr. Bell should select: N

Explain:

N	P	Q
14	17	
11	9	
9	1	
34	27	31

more votes — 34

These next two samples are "incorrect" responses. In the first, the student correctly chose shape *N* but provided an incorrect explanation. It is followed by a sample response from a student who chose shape *Q*.

Sample "incorrect" response 1

The shape Mr. Bell should select: N

Explain: becaues it is a neat shape and becaues he likes it.

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Sample "incorrect" response 2

The shape Mr. Bell should select: Shape Q

Explain:

more students voted forit

This question was somewhat difficult for students in grade 4 but easier for students in grades 8 and 12. The performance results are displayed in Tables 6.3 and 6.4. Table 6.3 shows the percentage of students at each grade who 1) chose shape N and had a correct explanation, 2) who chose shape N but had no or an incorrect explanation, 3) who chose shape Q, and 4) who made some other incorrect response.[1] Only the responses of students who chose shape N and had a correct explanation were rated "correct."

Approximately one-third of the fourth-grade students, one-half of the eighth-grade students, and two-thirds of the twelfth-grade students chose shape N for the symbol and had correct explanations. At each of the three grades, the percentage of students who chose shape N but had no or incorrect explanations was between 12 and 17 percent. Perhaps the most interesting difference was in the percent of students who chose shape Q. Thirty-two percent of the fourth-grade students (equivalent to the percentage who answered correctly) chose shape Q. At grade 8, this percentage dropped by half, and at grade 12, only nine percent of the students chose shape Q. At the earlier grades, students may be more inclined than at later grades simply to base their response on the largest single number in the table rather than to sum the data across classes. Another possible explanation is that, at the fourth-grade level, students simply answered their favorite shape.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

Table 6.3 Score Percentages for "Use Data from a Chart"

THE NATION'S REPORT CARD naep

	Correct	Incorrect			Omit
	Shape N–Correct Explanation	Shape N–No, or Incorrect, Explanation	Shape Q	Other	
Grade 4					
Overall	32	12	32	21	3
Males	31	12	30	23	3
Females	33	12	33	18	3
White	38	12	32	14	3
Black	13	11	34	39	3
Hispanic	16	13	25	39	6
Asian/Pacific Islander	33	18	31	16	2!
American Indian	***	***	***	***	***
Grade 8					
Overall	58	15	16	10	0
Males	57	16	16	11	0
Females	60	14	16	10	0!
White	64	15	13	7	0!
Black	38	20	21	20	2
Hispanic	52	9	20	19	0!
Asian/Pacific Islander	--	--	--	--	--
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	56	16	19	10	0
Pre-Algebra	57	14	17	11	0
Algebra	67	15	9	10	0
Grade 12					
Overall	67	17	9	6	1
Males	66	20	7	6	2
Females	67	16	10	6	1
White	70	18	7	4	1
Black	58	15	16	8	2
Hispanic	55	21	9	12	4
Asian/Pacific Islander	67	14	10	7	2
American Indian	***	***	***	***	***
Geometry Taken	68	18	8	5	1
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	65	10	11	9	3
First-Year Algebra	61	20	10	6	2
Second-Year Algebra	69	18	8	5	0
Third-Year Algebra/Pre-Calculus	71	17	6	5	1
Calculus	75	10	10	4	0

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 6.4 shows, for each grade, the percentage of students within each of the NAEP achievement levels who responded correctly to the question. At grade 4, only students who were at least at the *Proficient* level had a greater than 50 percent chance of answering correctly. At grade 8, more than two-thirds of students at the *Basic* level provided "correct" responses, and at grade 12, even students below the *Basic* level had a 50 percent chance of correct response. The question mapped at a score of 265 on the NAEP composite mathematics scale for grade 4 students. At grade 8, the question mapped at 286, and at grade 12, the question mapped at 295.

Table 6.4 ***Percentage Correct Within Achievement-Level Intervals for "Use Data from a Chart"***

THE NATION'S REPORT CARD NAEP

		NAEP Grades 4, 8, and 12 Composite Scale Ranges			
	Overall	Below Basic	Basic	Proficient	Advanced
Grade 4	32	8	32	61	***
Grade 8	58	38	68	77	***
Grade 12	67	51	73	77	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The third sample for this area is an extended constructed-response question for eighth-grade students. Students were presented with two graphs displaying the number of riders for the Metro Rail Company over a 6-month period. The difference between the graphs was that one displayed the scale for the number of riders in increments of 2,000 while the other displayed the scale in increments of 100. The question itself had two components, although these were scored together to provide a single rating for the question. First, students were instructed to choose, and justify their choice for, the graph that would best convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October. Second, students were asked to explain why some people might consider the graph they chose to be misleading.

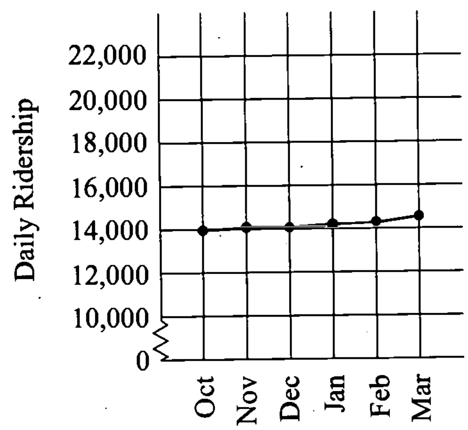
This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> of your work.

9. METRO RAIL COMPANY

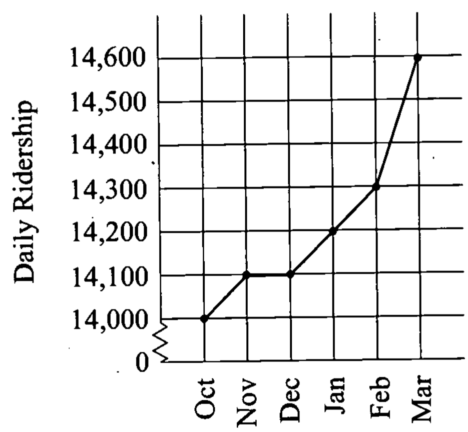
Month	Daily Ridership
October	14,000
November	14,100
December	14,100
January	14,200
February	14,300
March	14,600

The data in the table above has been correctly represented by both graphs shown below.

Graph A



Graph B



Which graph would be best to help convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October?

Explain your reason for making this selection.

Why might people who thought that there was little difference between October and March ticket sales consider the graph you chose to be misleading?

Did you use a calculator on this question?

◯ Yes ◯ No

The correct answer for the first part of the question was Graph B because it appeared to show a large increase from October to March. Acceptable variations on this explanation included:

- The line in Graph B goes up more, has a more dramatic rise, or climbs higher.
- Graph B climbs faster.
- Graph B is steeper.
- Graph B shows a larger visual increase.

For the second part of the question, students were expected to recognize that Graph B might be considered misleading because it exaggerated a relatively small increase in ridership (misuse of scale). Acceptable variations of this reason included:

- Graph B has a smaller scale.
- Graph A has a larger scale.
- The numbers on B are smaller than those on A (they increase by 100s not 1,000s).

The use of the term "range" instead of "scale" was not considered acceptable.

Student responses were considered "correct" if they identified B as the best graph and had a complete explanation for both parts of the question. An example of a "correct" response follows. In this example, the student used the term "range" in both answers, which would not, in itself, be considered "correct"; however, the student also said that Graph B should be chosen because it climbs faster and is misleading because it only has 100 at a time. Both of these statements were considered "correct" responses.

Sample "correct" response

Which graph would be best to help convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October?

Explain your reason for making this selection.

Graph B. It has a smaller range of numbers. Therefore it makes the graph climb faster.

Why might people who thought that there was little difference between October and March ticket sales consider the graph you chose to be misleading?

The range (of the numbers) isn't very big. They only have a 100 at a time.

Did you use a calculator on this question?

○ Yes ● No

Student responses were rated as "partial" if they chose Graph B and had an incomplete, but partially correct, explanation for one or both parts of the question. For example, in the following sample response the student gave a complete explanation of why B should be chosen, but an incomplete explanation of why Graph B could be considered misleading. When the question was anchored to the NAEP scale, the "correct" and "partial" rating categories were collapsed.

Sample "partial" response

Which graph would be best to help convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October?

Explain your reason for making this selection.

B, it looks like it's increasing more

Why might people who thought that there was little difference between October and March ticket sales consider the graph you chose to be misleading?

The #'s for Daily Ridership are more detailed.

Did you use a calculator on this question?

○ Yes ● No

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Students also could have had responses that identified Graph B as the best graph but offered no explanations or only incorrect explanations. Answers of this type, which are illustrated in the following example, were rated as "minimal."

Sample "minimal" response

Which graph would be best to help convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October?

Explain your reason for making this selection.

Graph B because it is more clear to read.

Why might people who thought that there was little difference between October and March ticket sales consider the graph you chose to be misleading?

Because they might not be able to read the graph I have choosen

Did you use a calculator on this question?

○ Yes ● No

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Student responses, such as the following, that did not identify B as the best graph were rated "incorrect."

Sample "incorrect" response

> Which graph would be best to help convince others that the Metro Rail Company made a lot more money from ticket sales in March than in October?
>
> Explain your reason for making this selection.
>
> graph A because the dot is very close to 15,000 and in graph B it is in 14,600.
>
> Why might people who thought that there was little difference between October and March ticket sales consider the graph you chose to be misleading?
>
> Because the one I chose dosen't look like much difference because it has a longer range of amounts
>
> Did you use a calculator on this question?
>
> ◯ Yes ⬤ No

Table 6.5 shows that only 2 percent of the students chose Graph B and gave complete explanations, while 19 percent chose Graph B and gave incomplete, but partially correct, explanations for at least one part of the question. However, 35 percent of the eighth-grade students who were taking algebra were able to provide at least partially correct explanations for choosing Graph B. This was a higher percentage than was obtained for students enrolled in pre-algebra or eighth-grade mathematics. Overall, 30 percent of the students did not even attempt the question.

THE NATION'S REPORT CARD

Table 6.5 Score Percentages for "Recognize Misleading Graph"

Grade 8	Correct Graph B–Complete Explanation	Partial Graph B–Incomplete but Partially Correct Explanation	Minimal Graph B–No or Incorrect Explanation	Incorrect	Omit
Overall	2	19	34	14	30
Males	2	19	32	15	31
Females	2	18	36	14	30
White	2	23	34	11	28
Black	2!	6	38	21	33
Hispanic	0!	7	24	25	43
Asian/Pacific Islander	--	--	--	--	--
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	1	16	32	19	32
Pre-Algebra	3	13	38	15	30
Algebra	3	32	31	6	28

NOTE: Row percentages may not total to 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 6.6 shows that 35 percent of the eighth-grade students who were classified as *Proficient* on the NAEP composite mathematics scale, 22 percent of the students classified as *Basic*, and only 7 percent of the students classified as performing below *Basic* chose Graph B and responded with at least partially correct explanations to the two parts of the question. The question mapped at a score of 475.

THE NATION'S REPORT CARD

Table 6.6 Percentage at Least Partial Within Achievement-Level Intervals for "Recognize Misleading Graph"

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
20	7	22	35	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final example for this area is a short constructed-response question that was used at grade 12. The question presented a table summarizing time of day and number of hours worked, average hourly wage, and daily earnings for an individual on each of 5 days. For the fourth day, the cells for average hourly wage and total earnings were left blank. As in the previous example, this question had two parts that were considered together in determining the student's score. In the first part, students were given the total earnings for all 5 days and asked to use this information, in conjunction with the table, to determine the average hourly wage for day 4. To respond correctly, students had to add the daily earnings for the 4 days presented ($119.00), subtract this from the total earnings of $153.50, and divide by the total number of hours worked on the fourth day. This yielded the correct answer of $5.75. For the second part of the question, students were to use the information on time of day and number of hours worked, along with the average hourly rate, to determine the time of day at which the hourly rate changed. The correct answer was 5:00 p.m.

TIME CARD Name: J. Jasmine	Number of Hours	Average Hourly Wage	Total Daily Earnings
Mon. 10:00 a.m. — 3:00 p.m.	5	5.50	27.50
Tue. 9:00 a.m. — 4:00 p.m.	7	5.50	38.50
Wed. 3:00 p.m. — 7:00 p.m.	4	5.75	23.00
Thur. 2:00 p.m. — 8:00 p.m.	6		
Fri. 5:00 p.m. — 10:00 p.m.	5	6.00	30.00

2. According to the information above, what is the average hourly wage for Thursday's earnings if the total earnings for five days was $153.50?

Answer: ______________

The hourly wage rate changes at some hour during the day. At what time does the hourly wage rate change?

Answer: ______________

Did you use a calculator on this question?

◯ Yes ◯ No

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Student responses were rated "correct," "partial," or "incorrect." "Correct" responses identified both the correct hourly wage and the correct time of the rate change. "Partial" responses identified either the correct average hourly wage or the correct time of the rate change, and "incorrect" responses did not correctly identify either. Following are three sample responses. The first two responses were rated "partial"; each student correctly computed the average hourly wage for the fourth day, but in the second part of the question the first respondent entered the total daily earnings for day 4 instead of the time of the rate change, and the second respondent entered 2:00 (the time the individual started work on day 4).

Sample "partial" response 1

2. According to the information above, what is the average hourly wage for Thursday's earnings if the total earnings for five days was $153.50?

Answer: 5.75

The hourly wage rate changes at some hour during the day. At what time does the hourly wage rate change?

Answer: 34.50

Did you use a calculator on this question?

● Yes ○ No

Sample "partial" response 2

2. According to the information above, what is the average hourly wage for Thursday's earnings if the total earnings for five days was $153.50?

Answer: 5.75

The hourly wage rate changes at some hour during the day. At what time does the hourly wage rate change?

Answer: 2:00

Did you use a calculator on this question?

● Yes ○ No

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The third sample response was rated "incorrect." This student entered the total daily earnings for the fourth day as a response to the first question and identified 3:00 p.m. as the time of the rate change.

Sample "incorrect" response

2. According to the information above, what is the average hourly wage for Thursday's earnings if the total earnings for five days was $153.50?

   Answer: $34.50

   The hourly wage rate changes at some hour during the day. At what time does the hourly wage rate change?

   Answer: 3pm

   Did you use a calculator on this question?

   ● Yes ○ No

Performance data are presented in Tables 6.7 and 6.8. Thirteen percent of students answered both parts of the question correctly, whereas 43 percent responded correctly to one of the two parts. Students who had taken calculus were more likely to respond correctly than students who had not taken calculus, and students who had taken at least third-year algebra or pre-calculus were more likely than those who had taken less mathematics to provide at least a partially correct response.

Table 6.7

Score Percentages for "Use Data in Table to Compute Average Hourly Wage and Determine When Wage Rate Changes"

THE NATION'S REPORT CARD NAEP

Grade 12	Correct	Partial	Incorrect	Omit
Overall	13	43	40	3
Males	13	43	40	3
Females	13	43	40	3
White	16	46	36	2
Black	4	38	51	7
Hispanic	11	36	46	7
Asian/Pacific Islander	15	44	37	4
American Indian	***	***	***	***
Geometry Taken	14	46	36	3
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	8	36	51	4
Second-Year Algebra	15	45	37	3
Third-Year Algebra/Pre-Calculus	14	53	31	2
Calculus	25	59	14	2

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The question mapped at a score of 420. Twenty-three percent of the students classified as *Proficient* and 14 percent of those classified as *Basic* were credited with a fully correct response.

Table 6.8

Percentage Correct Within Achievement-Level Intervals for "Use Data in Table to Compute Average Hourly Wage and Determine When Wage Rate Changes"

THE NATION'S REPORT CARD NAEP

	NAEP Grade 12 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
13	4	14	23	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Sampling and statistics

Questions in this area, which appeared primarily on the instruments for eighth- and twelfth-grade students, assessed students' understanding of and ability to apply sampling theory and statistical analyses. Students were asked questions regarding sampling, data representation, and data summarization. They were evaluated on their understanding of the various measures of central tendency as well as on their ability to calculate these measures. Students also needed to understand concepts related to correlation. Three questions are presented here. Two were eighth-grade multiple-choice questions, and one was a twelfth-grade extended constructed-response question.

The first eighth-grade question assessed students' understanding of what can happen when a sample is taken. Students were told that a bag contained two red candies and one yellow candy and that each of two persons took one candy out of the bag, without replacement. The question then listed four combinations of candy colors and asked which combinations could have been drawn by these two people, given the candies in the bag. Only the fourth combination, both picking yellow candies, was not possible.

4. A bag contains two red candies and one yellow candy. Kim takes out one candy and eats it, and then Jeff takes out one candy. For each sentence below, fill in the oval to indicate whether it is possible or not possible.

Possible	Not Possible	
Ⓐ	Ⓑ	Kim's candy is red and Jeff's candy is red.
Ⓐ	Ⓑ	Kim's candy is red and Jeff's candy is yellow.
Ⓐ	Ⓑ	Kim's candy is yellow and Jeff's candy is red.
Ⓐ	Ⓑ	Kim's candy is yellow and Jeff's candy is yellow.

Table 6.9 presents the percentages of students responding correctly to none, one, two, three, or all four of the statements of sampling possibilities. Nearly 80 percent responded correctly to all four statements, and another 10 percent responded correctly to at least three of the statements. More females than males responded correctly to all four questions. When the question was anchored to the NAEP scale, the categories of none, one, or two correct responses to statements were collapsed.

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Table 6.9

Score Percentages for "Reason About Sample Space"

Grade 8	Number Correct: 4	3	2	1	None	Omit
Overall	79	10	3	6	1	0
Males	75	11	4	8	2	0
Females	85	9	2	4	1	0
White	86	8	2	2	0	0!
Black	61	14	7	14	4	0!
Hispanic	65	13	3	16	3	0!
Asian/Pacific Islander	– –	– –	– –	– –	– –	– –
American Indian	***	***	***	***	***	***
Mathematics Course Taking:						
Eighth-Grade Mathematics	76	12	5	6	1	1
Pre-Algebra	82	11	2	5	1	0
Algebra	86	7	1	4	2	0

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

This question was very easy for eighth-grade students. Table 6.10 shows that at least three-quarters of the students who performed below the *Basic* level on the NAEP mathematics assessment gave the correct response to at least three statements, as did nearly all of the students in the other achievement level categories. The question mapped at a score of 235.

THE NATION'S REPORT CARD NAEP

Table 6.10

Percentage with at Least Three Correct Within Achievement-Level Intervals for "Reason About Sample Space"

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
89	75	97	99!	100!

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The second example for this area is also an eighth-grade multiple-choice question. This question assessed students' understanding of what constitutes a representative sample. Students were told that a poll was being taken at a junior high school to determine the school mascot and were asked where they could find a sample of students to interview that was most representative of the students in the school.

6. A poll is being taken at Baker Junior High School to determine whether to change the school mascot. Which of the following would be the best place to find a sample of students to interview that would be most representative of the entire student body?

Ⓐ An algebra class

Ⓑ The cafeteria

Ⓒ The guidance office

Ⓓ A French class

Ⓔ The faculty room

The correct option is B.

This question also was fairly easy for eighth-grade students. Table 6.11 shows that 65 percent of the students answered correctly. One percent of the students chose Option D, the French class, while approximately 10 percent of the students chose each of the remaining three options. The question mapped at a score of 289, and most of the students whose performance was classified as *Basic* or above chose the correct option.

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Table 6.11 Percentage Correct for "Identify Representative Sample"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	65
Males	64
Females	66
White	73
Black	48
Hispanic	47
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	59
Pre-Algebra	67
Algebra	74

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 6.12 Percentage Correct Within Achievement-Level Intervals for "Identify Representative Sample"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
65	43	72	87	96!

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final example for this area is an extended constructed-response question for grade 12 in which students were asked to determine whether the mean or median better represented the typical daily attendance in each of two theaters and to justify their answers. They were provided with data on each theater's daily attendance over a 5-day period, along with the median and the mean of the 5 days. Theater A had a nontypical, or outlier, value for attendance on day 4. The attendance for Theater B was bimodal.

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show all of your work.

10. The table below shows the daily attendance at two movie theaters for 5 days and the mean (average) and the median attendance.

	Theater A	Theater B
Day 1	100	72
Day 2	87	97
Day 3	90	70
Day 4	10	71
Day 5	91	100
Mean (average)	75.6	82
Median	90	72

(a) Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater A? Justify your answer.

(b) Which statistic, the mean or the median, would you use to describe the typical daily attendance for the 5 days at Theater B? Justify your answer.

Did you use the calculator on this question?

○ Yes ○ No

The correct answer for Theater A was the median, and the correct answer for Theater B was the mean. The appropriate explanation for the choice of the median for Theater A conveyed the idea that the attendance on day 4 was significantly different from attendance on the other days. Appropriate explanations for the choice of the mean for Theater B were variations on the following:

- There are two clusters of data;
- The median is representative of one of the clusters, while the mean is representative of both; and
- 82 is a better indicator of where the "center" of the five data points is located.

Responses were rated as "extended," "satisfactory," "partial," "minimal," and "incorrect." However, when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. A description of the ratings and sample responses for each rating category follow.

In order to have been rated as "extended," a student's response had to identify the appropriate measure for each theater and provide a correct explanation for at least one of the choices. The following is a sample of an "extended" response. After correctly identifying each statistic, the student explained that the median is better for Theater A because the mean is pulled down by 1 day, and that the mean is better for Theater B because it is closer to the middle.

Sample "extended" response

(a) The median because its more accurate. The mean is a lot lower than the majority of attendance just because of one day.

(b) The mean because it is closer to the middle as an average. The median is rather low when you have days with 97 – 100 people attending.

Did you use the calculator on this question?

○ Yes ○ No

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In the following sample, rated as "satisfactory," the student indicated the better measure for both theaters, but only provided a complete explanation for Theater A.

Sample "satisfactory" response

(a) ~~Mean- it is the mathematical average, where you add them and divide by the number of days. The Median is just the 3rd largest (or smallest)~~ day.

Median - one small day of 10 threw off the mean.

(b) Mean- it is the average (add all + divide by # of days). - All days are about the same.

Did you use the calculator on this question?

○ Yes ● No

Students' responses also could be rated as "partial" or "minimal." "Partial" responses either correctly identified the better measure for both theaters but did not provide appropriate explanations for either, or correctly identified and explained only one measure. Two examples of "partial" responses follow.

Sample "partial" response 1

(a)

Median

Because the numbers are closer to 90 all week

(b) Mean

they are too jumpy and not accurate.

Did you use the calculator on this question?

○ Yes ● No

Sample "partial" response 2

(a)

The median because Day 4 was an outlier and threw off the mean

(b)

The median - 3 of the 5 days were in the 70's - closest to the median of 72

Did you use the calculator on this question?

○ Yes ● No

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The next two sample responses were rated as "minimal." Both students correctly identified the better measure for only one of the theaters (the first for Theater A and the second for Theater B); however, neither had an appropriate explanation. The first student came close with the explanation of the median for Theater A, but failed to complete the thought that the attendance for day 4 was an outlier compared to the other 4 days.

Sample "minimal" response 1

(a) Median ~~it is the average amt. of people it's correct Median is only the middle number.~~ b/c for 3 days 100, 90, 91, 87 people came in which is closer to 90.

(b) Median. For 3 days, 72, 70, 71 people came in each day.

Did you use the calculator on this question?

○ Yes ● No

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Sample "minimal" response 2

(a) The mean is the average not what occurred at the middle

(b) The mean (average). We want to know the typical daily attendence which is the average

Did you use the calculator on this question?

○ Yes ● No

All other responses were considered "incorrect." The final example is of an "incorrect" response.

Sample "incorrect" response

(a) The mean because not always do 100 people show up to theaters And giving the median would be wrong statistics because it's not a definate number.

(b) The median for the same reason as question A

Did you use the calculator on this question?

○ Yes ● No

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Tables 6.13 and 6.14 present student performance data for this question. The question was fairly difficult, and only four percent of the students chose the better measure for both theaters and gave a complete explanation for at least one of their choices. Slightly more than 30 percent of the students omitted the question, and over half of the students produced responses that were rated "incorrect" or "minimal." Students who had taken at least third-year algebra or pre-calculus were more likely than other students to choose the better measures for both theaters and offer at least one complete explanation; however, even among this group the percentage of responses that were at least "satisfactory" was small.

Table 6.13

Score Percentages for "Compare Mean and Median"

THE NATION'S REPORT CARD naep

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
	Better Measure Both Theaters; Complete Explanation	Better Measure Both Theaters; Complete Explanation for 1 Theater	Better Measure and Complete Explanation 1 Theater; or Better Measure Both Theaters with No or Incomplete Explanation	Better Measure 1 Theater; No or Incomplete Explanation		
Grade 12						
Overall	1	3	10	28	25	31
Males	1	3	12	25	23	32
Females	0!	2	9	31	27	30
White	1	4	12	30	25	27
Black	0!	0!	7	25	24	42
Hispanic	0!	1!	6	18	24	48
Asian/Pacific Islander	1!	4	7	25	24	34
American Indian	***	***	***	***	***	***
Geometry Taken	1	3	10	29	27	30
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	***	***	***	***	***	***
First-Year Algebra	0	2	8	28	26	33
Second-Year Algebra	0	2	10	29	26	30
Third-Year Algebra/Pre-Calculus	0	6	18	31	18	25
Calculus	6	10	11	26	25	21

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.

*** Sample size is insufficient to permit a reliable estimate.

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 6.14 shows that few of the students within any of the achievement level classifications received full credit on this question, and when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. The question mapped at 463 on the NAEP composite mathematics scale.

Table 6.14 — ***Percentage at Least Satisfactory Within Achievement-Level Intervals for "Compare Mean and Median"***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
4	0!	2	13	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Probability

This area included questions measuring students' understanding of probabilistic events and their ability to determine the probability of simple and compound events. Questions for fourth-grade students used less advanced terminology than those for older students, and probabilities were simpler to calculate. Questions for older students required them to predict outcomes given two or more dependent events. Some questions also involved percents and proportions. Two questions are presented as examples for this area. One is a fourth-grade multiple-choice question, and the other is a twelfth-grade extended constructed-response question.

The following example question asked fourth-grade students to determine the chances that the person randomly chosen to be the captain of a swim team would be a fifth grader, given that the membership of the swim team was divided between fifth- and sixth-grade students in a specified manner. The language used in this example is typical of the probability questions presented in the fourth-grade assessment.

9. There are 3 fifth graders and 2 sixth graders on the swim team. Everyone's name is put in a hat and the captain is chosen by picking one name. What are the chances that the captain will be a fifth grader?

(A) 1 out of 5

(B) 1 out of 3

(C) 3 out of 5

(D) 2 out of 3

The correct option is C.

Tables 6.15 and 6.16 display student performance data on this question. Approximately one-third of the students responded correctly. Twenty-two percent of the students chose Option A, the probability that any individual student would be chosen, whereas 16 percent chose Option B, and 28 percent chose Option D. The appeal of the latter option may have been that it contained both of the numbers specified in the stem of the question. Clearly, many fourth-grade students did not know how to determine chance.

Table 6.15 ***Percentage Correct for "Determine a Probability"***

THE NATION'S REPORT CARD NAEP

Grade 4	Percentage Correct
Overall	31
Males	32
Females	30
White	34
Black	22
Hispanic	26
Asian/Pacific Islander	35
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table 6.16 shows the percentage of students within each of the achievement level intervals who responded correctly to this question. Half of the students classified as *Proficient* responded correctly compared with approximately one-quarter of the students at each of the lower two levels. The question mapped at 278.

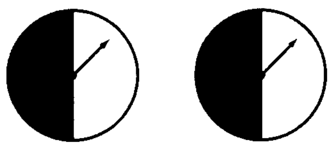
Table 6.16 ***Percentage Correct Within Achievement-Level Intervals for "Determine a Probability"***

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	**Basic**	**Proficient**	**Advanced**
31	23	24	54	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The final example is a twelfth-grade extended constructed-response question on joint probabilities. The question showed two spinners that were half black and half white, and students were told that to "win," both arrows had to land on black when the spinner was spun once. They then were asked whether they agreed that there was a 50-50 chance of this happening and instructed to justify their answers. The correct response was "no" because the possibility of either event happening was 1 in 2; therefore, the possibility of both happening was 1 in 4, or 25 percent.



9. The two fair spinners shown above are part of a carnival game. A player wins a prize only when both arrows land on black after each spinner has been spun once.

James thinks he has a 50-50 chance of winning. Do you agree?

Ⓐ Yes Ⓑ No

Justify your answer.

Did you use the calculator on this question?

◯ Yes ◯ No

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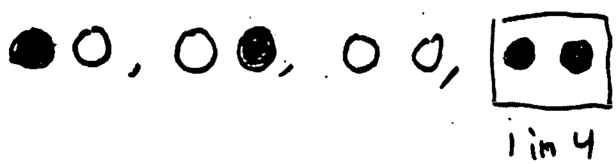
Students' explanations were rated "correct," "partial," or "incorrect." A "correct" explanation had to indicate that the actual chances were 1 in 4, or 25 percent, and correctly justify this conclusion. Both of the following responses were considered "correct."

Sample "correct" response 1

> James thinks he has a 50-50 chance of winning. Do you agree?
>
> Ⓐ Yes ● No
>
> Justify your answer.
>
> he has a 50-50 chance of winning on one of the spinners then on the other one it's the same chances. .50 x .50 = .25 he has a
>
> 25% chance of winning
>
> Did you use the calculator on this question?
>
> ○ Yes ● No

Sample "correct" response 2

> James thinks he has a 50-50 chance of winning. Do you agree?
>
> Ⓐ Yes ● No
>
> Justify your answer.
>
> The chance that each spinner lands on black is one in two, or $\frac{1}{2}$, the probability that two $\frac{1}{2}$ spinners get a certain result is $\frac{1}{2} \cdot \frac{1}{2}$ or $\frac{1}{4}$.
>
> possible combinations:
>
> 
>
>
> Did you use the calculator on this question?
>
> ○ Yes ● No

Students also could have simply drawn a diagram similar to the one presented in the second sample above and still have been considered as giving a "correct" response. For a response to have been rated as "partial," students had to do one of the following:

- list the sample space correctly, but with less than a complete explanation;
- draw a correct tree diagram, but with less than a complete explanation; or
- simply state that the chance would be 1 in 4.

In the following example of a "partial" explanation, the student described the sample space but did not tell what the actual chances of winning were.

Sample "partial" response

> James thinks he has a 50-50 chance of winning. Do you agree?
>
> Ⓐ Yes ● No
>
> Justify your answer.
>
> no because, the spinners could either both go on black or bothon white, or one on white and one on black or one on black and one on white.
>
> Did you use the calculator on this question?
>
> ○ Yes ● No

"Incorrect" explanations included all explanations that did not meet the criteria stated above. Note that students who responded correctly to the initial "yes/no" question but were not able to provide at least a partially adequate explanation received a rating of "incorrect." Two examples of responses that were rated as "incorrect" follow.

Sample "incorrect" response 1

James thinks he has a 50-50 chance of winning. Do you agree?

Ⓐ Yes ● No

Justify your answer.

they start at the same place but it depends on how hard or light each spinner was spun

Did you use the calculator on this question?

○ Yes ● No

Sample "incorrect" response 2

James thinks he has a 50-50 chance of winning. Do you agree?

Ⓐ Yes ● No

Justify your answer.

1 w 1B
2 w
2 B

There are 3 different chances

Did you use the calculator on this question?

○ Yes ● No

Student performance data are presented in Tables 6.17 and 6.18. Forty-four percent of the students provided an "incorrect" response to the initial question, meaning that they did not answer "no" to whether there was a 50-50 chance of the spinners both landing on black. Approximately one-quarter of the students answered the initial question correctly but provided an incorrect explanation. The remainder were able to give an explanation that was at least partially correct. Students whose highest course was calculus were substantially more likely than other students to provide a fully correct explanation: 34 percent of these students provided a response rated "correct." Males were more likely than females to provide at least a partial explanation to the question.

Table 6.17 ***Score Percentages for "Compare Probabilities"***

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect		Omit
	Correct Answer to "Yes/No" Question; Correct Explanation	Correct Answer to "Yes/No" Question; Partial Explanation	Correct Answer to "Yes/No" Question; Incorrect Explanation	Incorrect Answer to "Yes/No" Question	
Grade 12					
Overall	8	20	24	44	4
Males	9	23	25	40	3
Females	7	16	24	49	4
White	9	23	26	39	3
Black	4	8	18	62	7
Hispanic	5	7	18	65	4
Asian/Pacific Islander	11	20	29	38	11!
American Indian	***	***	***	***	***
Geometry Taken	8	22	25	41	4
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	***	***	***	***	***
First-Year Algebra	5	12	24	56	3
Second-Year Algebra	7	21	28	42	3
Third-Year Algebra/Pre-Calculus	8	37	21	26	8
Calculus	34	24	16	19	6

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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The question mapped at a score of 411. Almost two-thirds of the students who were classified as being *Proficient* on the NAEP mathematics assessment responded with at least a partial explanation. However, only 27 percent of those classified as *Basic* and 5 percent of those classified as below *Basic* performed as well.

Table 6.18 **Percentage Correct Within Achievement-Level Intervals for "Compare Probabilities"**

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
28	5	27	65	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

This content strand included questions that assessed students' understanding of data, including how to best collect, display, and interpret data. Questions also assessed students' understanding of statistics and probability and their competence in calculating statistics and determining probabilities. Statistics included mean, median, mode, and standard deviation of distributions, and probabilities could be simple, dual, or conditional.

As might be expected, straightforward interpretations of graphs, charts, and tables were easier for students than questions that asked them to perform calculations with displayed data. Students also had difficulty explaining why one method of reporting or displaying data was better than another, even though they may have recognized which was the better method. Questions asking students to determine chance or probability were generally difficult for them.

Chapter 7

Algebra and Functions

Content Strand Description

The Algebra and Functions content strand extends from work with simple patterns at grade 4 to basic algebra concepts at grade 8 to more advanced questions at grade 12. The strand includes not only algebra, but also pre-calculus and some topics from discrete mathematics. On the NAEP 1996 mathematics assessment, students were expected to use algebraic notation and to solve mathematical problems set in real-world contexts. Across the grades, students also were expected to demonstrate an increasing understanding of the use of algebraic functions and geometry as representational tools.

At grade 4, the assessment involved informal demonstration of students' abilities to generalize from patterns and justify such generalizations, translate between mathematical representations, use simple equations, and construct basic graphs. At grade 8, the assessment included more algebraic notation, stressing the meaning of variables and an informal understanding of the use of symbolic representation in problem-solving contexts. Students also were expected to use basic concepts of functions as a way of describing relationships and to solve simple equations and inequalities. At grade 12, students were expected to be adept at appropriately choosing and applying algebraic representations in a variety of problem-solving situations, including using functions in representing and describing more complex relationships.

Examples of Individual Questions and Student Performance

Several questions from the Algebra and Functions content strand of the NAEP 1996 mathematics assessment are shown in this chapter. Presentation of the questions is organized around four areas of emphasis within the Algebra and Functions content strand: 1) *patterns and functional relationships;* 2) *number lines and graphs;* 3) *equations and inequalities*, which includes algebraic representations and solving equations and inequalities; and 4) *advanced functions topics and trigonometry.* As was true for the other content strands, questions within all four areas tested students' conceptual understanding and procedural knowledge, as well as their abilities to reason, communicate, and make connections.

All sample questions from this content strand are mapped onto the NAEP composite mathematics scale as shown in Figure 7.1. Specific instructions on how to interpret this map are given at the end of Chapter 2. The map is included to provide an indication of the relative difficulty of each sample question and, thus, to indicate the type of material mastered within this content strand by students with varying degrees of mathematics proficiency. As mentioned in previous chapters, it is important to remember that the difficulty of a question is a function of the characteristics specific to the question (e.g., format, absence or presence of graphics, real-world application), as well as the specific mathematics content associated with the question, and students' opportunities to learn this content. Remember also that overall performance on the Algebra and Functions content strand is not determined solely by performance on the examples presented here. These examples illustrate only some of what students know and can do.

Patterns and functional relationships

Most of the questions in this area that required students to solve problems related to patterns of numbers, letters, or figures were found at grade 4. In simpler patterns, all elements changed in the same way (e.g., adding 5, rotating figure one unit). In more complex patterns, elements changed in different ways. The most difficult questions covered relationships between patterns. Questions asked students to identify the next element(s) in the pattern, fill in missing elements, perform computations with missing elements, or explain patterns.

Two sample questions are presented for this area — an eighth-grade short constructed-response question and a twelfth-grade extended constructed-response question. Both questions assess students' problem-solving skills.

The question selected for the first example listed the number of diagonals that can be drawn from any vertex of various polygons and then asked how many diagonals could be drawn from any vertex of a 20-sided polygon. In order to answer the question correctly, students had to analyze the information presented and determine the pattern present, that is, that the number of diagonals is always three less than the number of sides of the polygon. Then they needed to apply that pattern to the 20-sided polygon to compute the correct answer of 17. Any other answer was considered "incorrect."

From any vertex of a 4-sided polygon, 1 diagonal can be drawn
From any vertex of a 5-sided polygon, 2 diagonals can be drawn
From any vertex of a 6-sided polygon, 3 diagonals can be drawn
From any vertex of a 7-sided polygon, 4 diagonals can be drawn

10. How many diagonals can be drawn from any vertex of a 20-sided polygon?

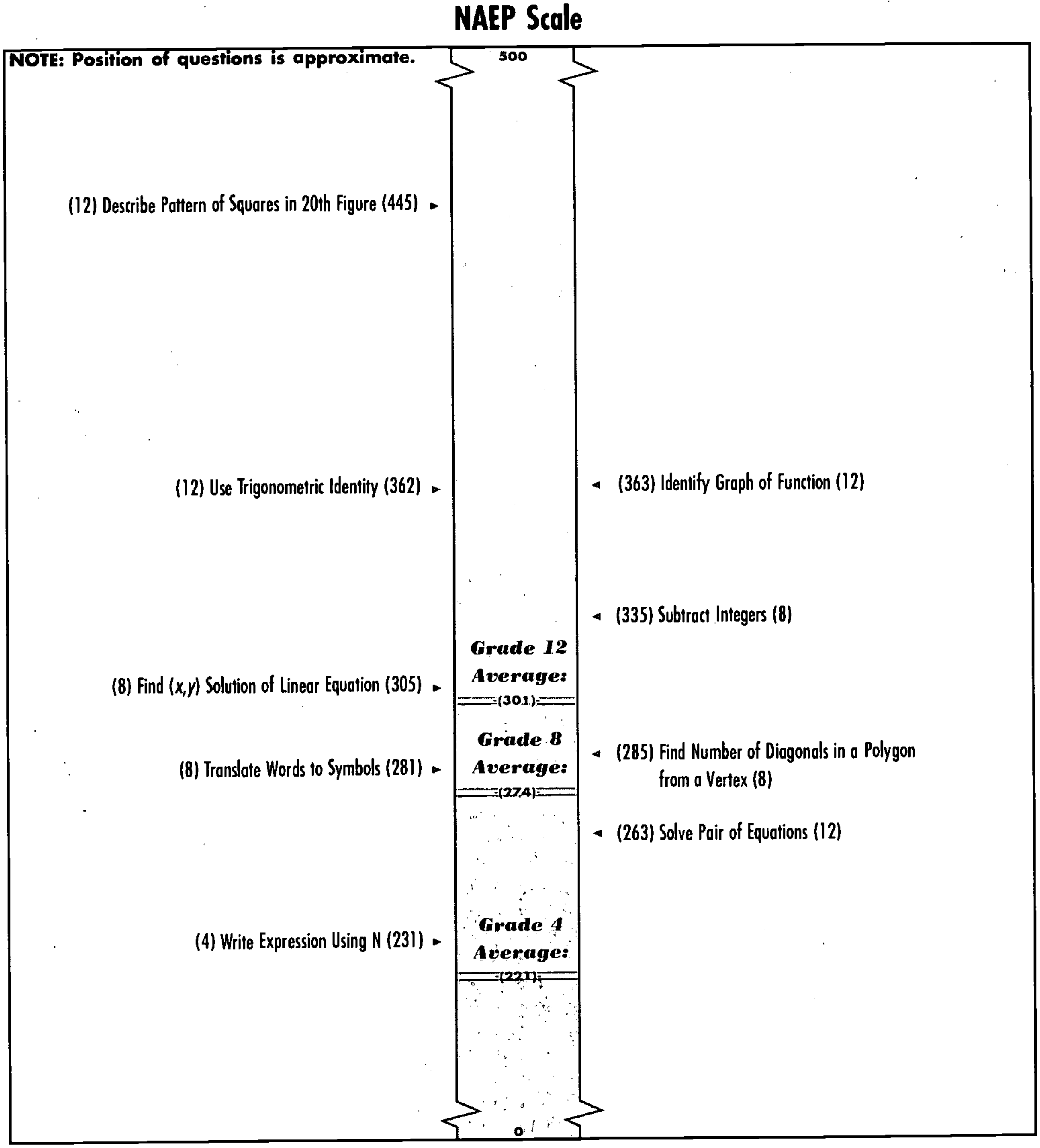
Answer: ______________________

The correct answer is 17.

Figure 7.1

Map of Selected Algebra and Functions Questions on the NAEP Composite Mathematics Scale (Item Map)

THE NATION'S REPORT CARD



NOTE: Each mathematics question was mapped onto the NAEP 0 to 500 mathematics scale. The position of the question on the scale represents the scale score obtained by students who had a 65 percent probability of successfully answering the question. (The probability was 74 percent for a 4-option multiple-choice question and 72 percent for a 5-option multiple-choice question.) Only selected questions are presented. The number 4, 8, or 12 in parentheses is the grade level at which the question was asked.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Student performance data are presented in Table 7.1, and the percentage of students within each achievement-level interval who successfully answered the question is presented in Table 7.2. Fifty-four percent of the students answered the question correctly. Female students outperformed males on this question. Students taking eighth-grade mathematics or pre-algebra performed similarly, while those taking algebra performed better than the other two groups. Sixty percent of students at the *Basic* level, 84 percent at the *Proficient* level, and more than 90 percent of students at the *Advanced* level gave the "correct" response. The question mapped at a composite scale score of 285.

Table 7.1 **Percentage Correct for "Find Number of Diagonals in a Polygon from a Vertex"**

THE NATION'S REPORT CARD NAEP

Grade 8	Percentage Correct
Overall	54
Males	50
Females	59
White	61
Black	35
Hispanic	41
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	47
Pre-Algebra	51
Algebra	69

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.2 **Percentage Correct Within Achievement-Level Intervals for "Find Number of Diagonals in a Polygon from a Vertex"**

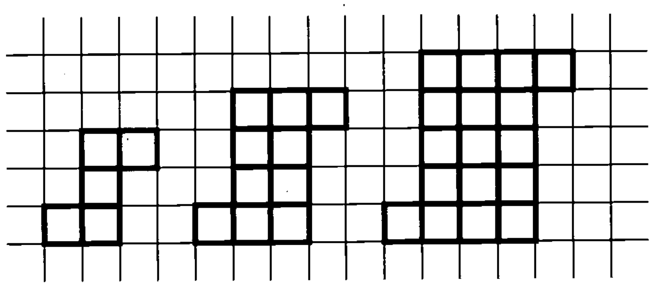
THE NATION'S REPORT CARD naep

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
54	27	60	84	96!

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example is an extended-constructed response question for twelfth-grade students. It included directions to the students to show all of their work and explain their reasoning. Students were told that they could use drawings, words, or numbers in their explanations and that the answer needed to be clear enough that another person could read it and understand their thinking. In the problem, students were shown the first three figures in a pattern of tiles that they were told contained a total of 50 figures. They were asked to describe the 20th figure in the pattern and to explain the reasoning they used to determine this solution. They then were asked to write a general description that could be used to define any of the 50 figures in the pattern.

9. The first 3 figures in a pattern of tiles are shown below. The pattern of tiles contains 50 figures.



Describe the 20th figure in this pattern, including the total number of tiles it contains and how they are arranged. Then explain the reasoning that you used to determine this information. Write a description that could be used to define any figure in the pattern.

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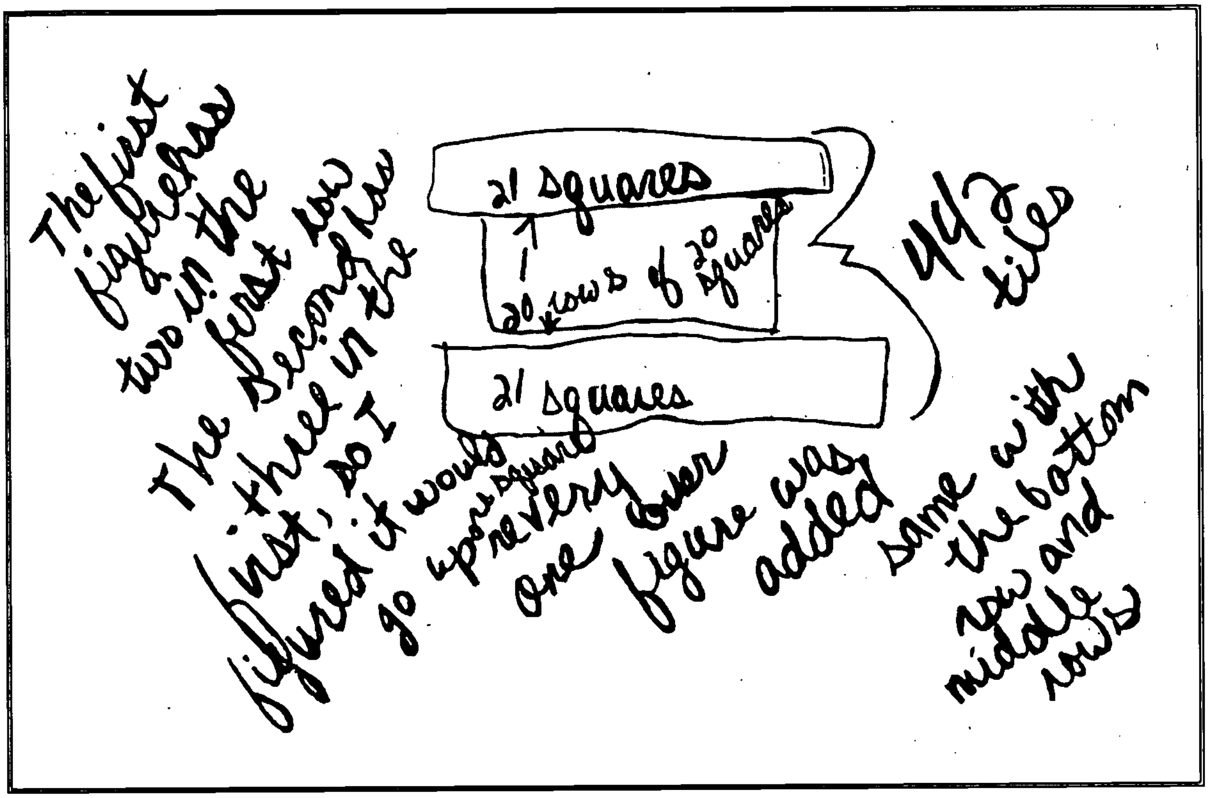
In rating student responses, readers could rate a response as "extended," "satisfactory," "partial," "minimal," or "incorrect." A response was considered "extended" if it included the following elements: 1) a correct count of 442 tiles for the 20th figure; 2) a verbal or graphical explanation of the reasoning the student used (e.g., a description of a figure with a row of 21 tiles across the top, a row of 21 across the bottom, and a 20 x 20 square between these rows with the top row extending one tile to the right of the square, and the bottom row extending one tile to the left); and 3) an accurate generalization, based on inductive reasoning. The following sample "extended" response contains all of these elements.

Sample "extended" response

Each figure increases 1 layer in height and one middle layer in width for every succession, relative to the first. For example, for the nth section the figure will be n+1 units across at the base, n units wide, n+1 units across at the top, and n+2 units high. This is the pattern
The 20th figure will be 21 units across on the bottom length, 20 units wide in the middle, 22 units high, and 21 units wide at the top. The increase is linear.
Total number of tiles it contains:
21 + (20x20) + 21 = 442
The inner square is always (n×n) units in area (n²)

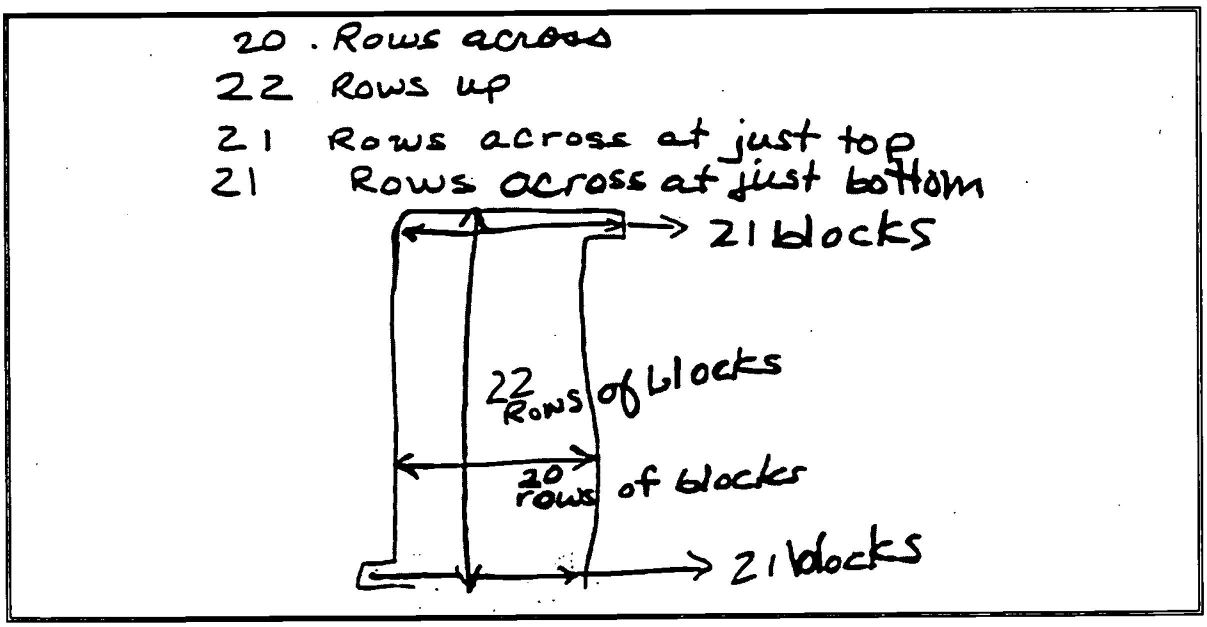
A response was considered "satisfactory" if the student described the 20th figure, gave the number of tiles, and provided some evidence of sound reasoning. However, "satisfactory" responses either included errors in computation or lacked clarity in the explanation. The following sample "satisfactory" response is an example of a response that contained most of the elements asked for in the question but that lacked a clear explanation or generalization.

Sample "satisfactory" response



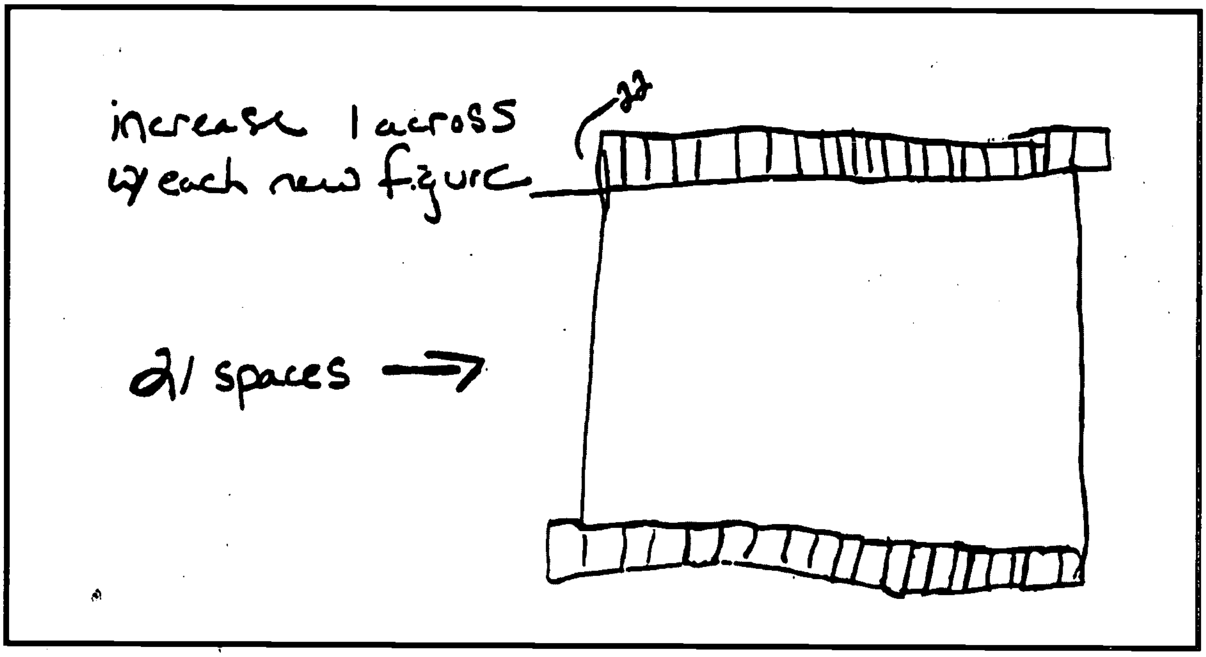
A response was considered "partial" if the student illustrated or described at least one additional figure in the pattern correctly or stated that there are 442 tiles in the 20th figure but did no more. In the following sample "partial" response, the student correctly diagrammed the 20th figure but did not tell how many tiles were in the figure, explain his or her reasoning, or provide a generalization.

Sample "partial" response



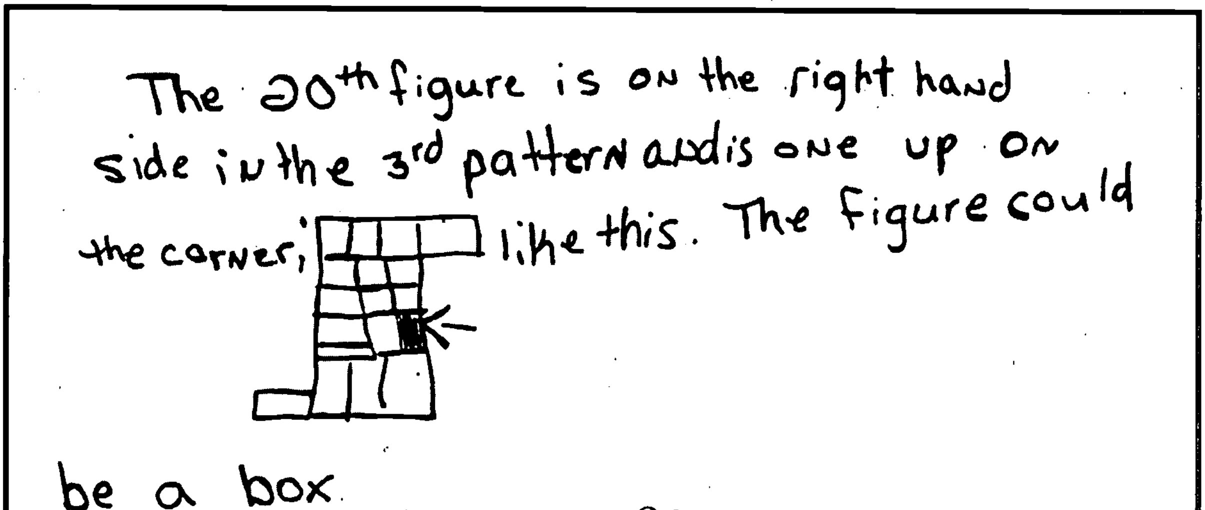
A response was considered "minimal" if the student attempted to draw or describe the given pattern or an additional figure in the pattern or made some attempt to go beyond what was shown in the question. The following "minimal" response is an example of a student who made an attempt to draw the 20th figure but did not correctly describe it or tell the number of tiles it contains. The student's reasoning was not clearly explained, and there was no description that could be generalized to any figure in the pattern.

Sample "minimal" response



A response was considered "incorrect" if the student showed no attempt to go beyond what was shown in the question. The following sample "incorrect" response is an example of a student who just repeated one of the figures shown in the original question and gave a verbal answer that appeared to show lack of comprehension of the question.

Sample "incorrect" response



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Information on student performance on this question is presented in Table 7.3. This question was quite difficult for students, and when the question was anchored to the NAEP scale, the "extended" and "satisfactory" rating categories were collapsed. While 80 percent of twelfth-graders attempted to answer the question, only 4 percent provided a response that was rated as "satisfactory" or higher. Another 18 percent provided "partial" responses, and more than 50 percent provided responses rated as "minimal" or "incorrect."[1] Students whose highest mathematics course was calculus were more likely to provide a response considered to be at least partially correct than students with fewer courses in the algebra-through-calculus sequence, and students whose highest course was third-year algebra were more likely to provide a response considered to be at least partially correct than those whose highest course was first-year algebra.

Table 7.3 ***Score Percentages for "Describe Pattern of Squares in 20th Figure"***

THE NATION'S REPORT CARD naep

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	2	2	18	28	25	20
Males	2	2	19	26	27	19
Females	1	2	17	31	23	21
White	3	2	19	33	23	15
Black	0!	0!	9	17	35	36
Hispanic	0!	1!	18	19	26	33
Asian/Pacific Islander	2!	3	30	20	20	22
American Indian	***	***	***	***	***	***
Geometry Taken	2	3	20	30	26	19
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	***	***	***	***	***	***
First-Year Algebra	2	1	13	24	26	28
Second-Year Algebra	2	2	18	30	24	21
Third-Year Algebra/Pre-Calculus	1!	7	22	32	22	15
Calculus	7	3	38	28	12	8

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[1] Student responses for this and all other constructed-response questions also could have been scored as "off task," which means that the student provided a response, but it was deemed not related in content to the question asked. There are many examples of these types of responses, but a simple one would be "I don't like this test." Responses of this sort could not be rated. In contrast, responses scored as "incorrect" were valid attempts to answer the question that were simply wrong.

The percentage of students within each achievement-level interval who provided a response that was rated at least satisfactory is shown in Table 7.4. Only 2 percent of students at the *Basic* level and 15 percent of students at the *Proficient* level submitted responses that were considered at least "satisfactory." The question mapped at 445.

Table 7.4 ***Percentage at Least Satisfactory Within Achievement-Level Intervals for "Describe Pattern of Squares in 20th Figure"***

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
4	0!	2	15	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

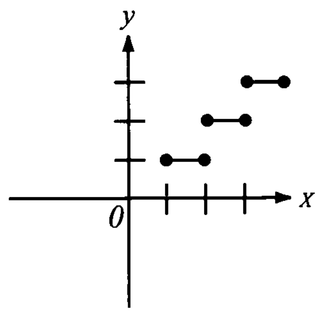
Number lines and graphs

Questions in this area asked students to use number lines and rectangular coordinate systems. At grade 4, students primarily were asked to locate points on graphs, trace paths, and identify points on number lines. At grade 8, many questions asked students to identify coordinates, and at grade 12, questions asked students to represent equations, inequalities, and functions on number lines and graphs.

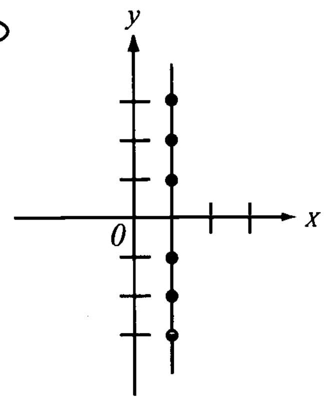
The following example is a twelfth-grade question that asked students to identify which of five figures shown could be the graph of a function. To answer the question correctly students needed to understand the definition of a function and be able to test each of the figures against that definition. The question mapped at a score of 363 on the NAEP composite mathematics scale.

9. Which of the following could be the graph of a function?

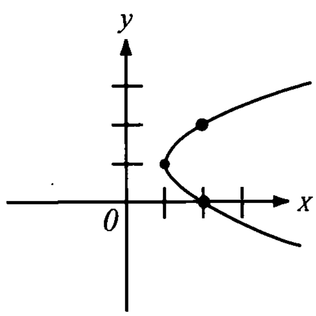
(A)



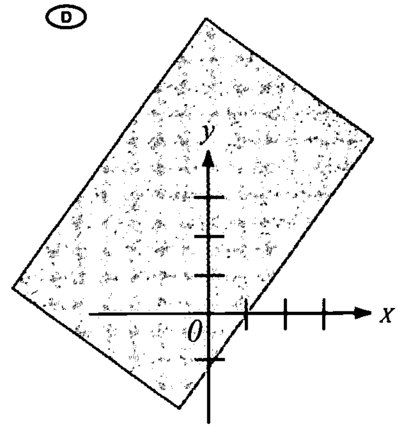
(B)



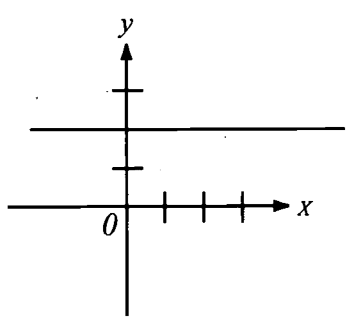
(C)



(D)



(E)



The correct option is E.

Performance data for this question are presented in Tables 7.5 and 7.6. The question was fairly difficult for students and mapped at a score of 363 on the composite scale. Twenty percent of the students selected the correct option, E, while another approximately 20 percent chose Option D, and 32 percent chose Option C. It is possible that students selected Option C because they did not know the definition of a function, or could not recognize Option E as representative of a function because y does not vary with values of x, and simply selected the figure that appeared most complicated.

As might be expected, familiarity with functions appears to depend on a student's curriculum. Students who had taken more advanced mathematics courses were more likely to respond correctly to the question than students who had not taken these courses. Students who cited at least third-year algebra/pre-calculus as their highest mathematics course taken performed better than those who had taken fewer courses in the algebra-through-calculus sequence. That the question was difficult for students can be seen by the fact that only 17 percent of students at the *Basic* level and 56 percent of those at the *Proficient* level answered correctly.

Table 7.5 ***Percentage Correct for "Identify Graph of Function"***

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	20
Males	21
Females	20
White	20
Black	16
Hispanic	22
Asian/Pacific Islander	43
American Indian	***
Geometry Taken	22
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	9
First-Year Algebra	10
Second-Year Algebra	17
Third-Year Algebra/Pre-Calculus	36
Calculus	55

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.6

Percentage Correct Within Achievement-Level Intervals for "Identify Graph of Function"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
20	8	17	56	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Equations and inequalities

Questions assessing student knowledge of, and skill in using, equations and inequalities ranged from those asking for simple algebraic representations of situations and problems to those asking students to solve systems of equations and inequalities. At grade 4, students had to identify simple algebraic representations involving number sentences or pictures. They also were asked to identify missing numbers in simple equations and inequalities. Eighth-grade students were asked to solve more complex equations and inequalities, often involving two missing variables. At times they were asked to predict the resultant effect on the value of one variable when the value of another variable had been changed. Questions for students in grade 12 sometimes involved exponents and square roots as well as systems of equations and inequalities.

The next two sample questions assessed students' conceptual understanding of algebraic representations. The first question is a multiple-choice question for grade 4. It presented a short word problem about stamps that included a variable, *N*. Students were asked to identify the symbolic representation of the correct answer.

3. *N* stands for the number of stamps John had. He gave 12 stamps to his sister. Which expression tells how many stamps John has now?

   (A) $N + 12$

   (B) $N - 12$

   (C) $12 - N$

   (D) $12 \times N$

The correct option is B.

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To answer the question correctly, students needed to understand that "giving stamps away" corresponds to subtracting them from the total, *N*, and is correctly represented by a minus sign. The question was fairly easy for fourth-grade students and mapped at a composite scale score of 231. Student performance is shown in Tables 7.7 and 7.8. Two-thirds of fourth-grade students selected the correct option; the remaining students were fairly evenly divided between Options A and C, with only around three percent selecting Option D. Female students performed better than males. Seventy-three percent of students at the *Basic* level and 90 percent of students at the *Proficient* level selected the correct response. Fewer than half of the students classified as below *Basic* were able to choose the correct response.

Table 7.7 ***Percentage Correct for "Write Expression Using N"***

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	67
Males	64
Females	70
White	71
Black	56
Hispanic	58
Asian/Pacific Islander	70
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.8 ***Percentage Correct Within Achievement-Level Intervals for "Write Expression Using N"***

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
67	44	73	90	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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The next example in this area is a grade 8 multiple-choice question similar to the previous one. It presented a word problem and then asked students to identify the symbolic representation of the solution. In this question, a plumber's hourly rate was given, plus travel charges. Students also were told to let *h* represent the number of hours worked and then were asked which expression could be used to calculate the plumber's total charge. In order to answer correctly, they needed to know what computations were required to solve the word problem and how those computations should be expressed in an equation. The question was fairly easy and mapped at 281 on the composite scale.

9. A plumber charges customers $48 for each hour worked plus an additional $9 for travel. If *h* represents the number of hours worked, which of the following expressions could be used to calculate the plumber's total charge in dollars?

(A) $48 + 9 + h$

(B) $48 \times 9 \times h$

(C) $48 + (9 \times h)$

(D) $(48 \times 9) + h$

(E) $(48 \times h) + 9$

The correct option is E.

Student performance data are presented in Table 7.9, and the percentage of students within each achievement-level interval who successfully answered the question is presented in Table 7.10. Fifty-eight percent of the students answered the question correctly. Incorrect responses were fairly evenly distributed across the other options. Students currently enrolled in algebra performed better than those in pre-algebra or eighth-grade mathematics, whereas students in the latter two courses performed similarly. Sixty-six percent of students at the *Basic* level and more than 90 percent of students at the *Proficient* level selected the correct response. Only one-fourth of the students below the *Basic* level were able to respond correctly.

Table 7.9 Percentage Correct for "Translate Words to Symbols"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	58
Males	55
Females	60
White	64
Black	39
Hispanic	46
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	48
Pre-Algebra	54
Algebra	76

*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.10 Percentage Correct Within Achievement-Level Intervals for "Translate Words to Symbols"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
58	24	66	94	99!

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The following three sample questions assess students' knowledge of procedures for solving equations and inequalities. The first question is a grade 8 multiple-choice question that asked students to identify a solution to a linear equation with two unknowns, x and y. To answer correctly, students could solve the equation by trial and error, working their way through the pairs of x and y values given in the response options and, in each case, determining whether the resultant expression equaled 6, as specified by the equation. Alternatively, a student could graph the equation and test which of the points specified by the (x, y) coordinates in the response options fell onto the graphed line. The question mapped at 305 on the NAEP composite scale.

8. Which of the following ordered pairs (x, y) is a solution to the equation $2x - 3y = 6$?

Ⓐ (6, 3)
Ⓑ (3, 0)
Ⓒ (3, 2)
Ⓓ (2, 3)
Ⓔ (0, 3)

The correct option is B.

Student performance data are presented in Table 7.11. Over 40 percent of students answered the question correctly. The remaining students were distributed fairly evenly among Options A, C, and D, with less than five percent selecting Option E. Students currently taking pre-algebra or eighth-grade mathematics performed similarly, whereas those currently taking algebra performed better than students in the other two groups.

Table 7.11 ***Percentage Correct for "Find (x, y) Solution of Linear Equation"***

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	42
Males	42
Females	40
White	46
Black	30
Hispanic	29
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	31
Pre-Algebra	36
Algebra	64

*** Sample size is insufficient to permit a reliable estimate.
-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The percentage of students within each achievement-level interval who correctly answered the question is presented in Table 7.12. More than 90 percent of students classified as *Advanced*, 75 percent of those classified as *Proficient*, and 44 percent of those classified as *Basic* selected the correct response.

Table 7.12

Percentage Correct Within Achievement-Level Intervals for "Find (x, y) Solution of Linear Equation"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
41	15	44	75	93!

! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

The next example in this area is a short constructed-response question for grade 8 students. Students were asked to find the difference between a high point above sea level and a low point below sea level and to show their work. In order to answer the question correctly, students had to recognize that the correct procedure would be to sum the two numbers, equivalent to subtracting a negative number, rather than to subtract one from the other.

2. The lowest point of the St. Lawrence River is 294 feet below sea level. The top of Mt. Jacques Cartier is 1,277 feet above sea level. How many feet higher is the top of Mt. Jacques Cartier than the lowest point of the St. Lawrence River? Show your work.

The correct response is 1,571 feet.

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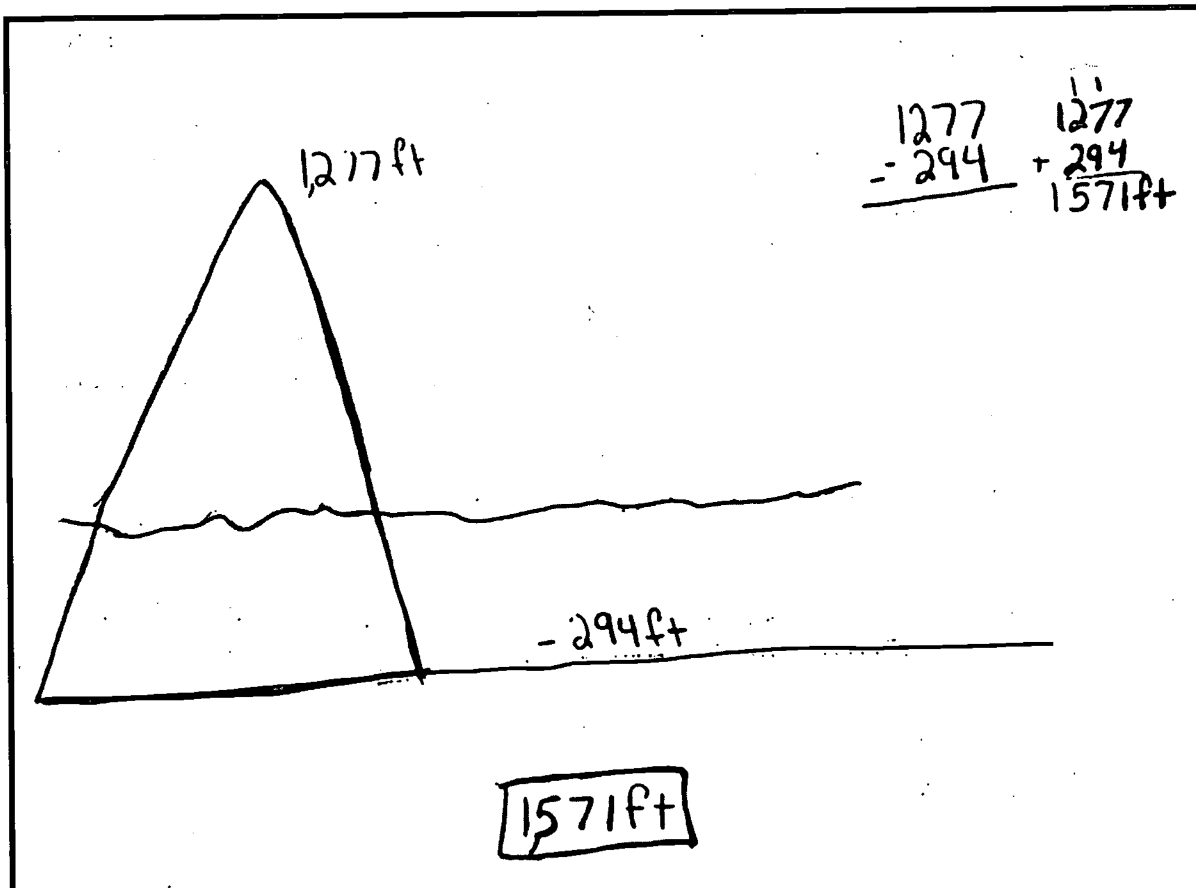
Readers could rate responses as "correct," "partial," or "incorrect." A response was considered "correct" if the answer given was 1,571 feet, even if the student's work was not shown. A response was considered "partial" if it showed the correct procedure, either 1,277 − (−294) or 1,277 + 294, but did not have the correct answer. Anything else was considered "incorrect." Sample student responses follow. In the sample "partial" response, the student showed the correct procedure but made an arithmetic error. In the "incorrect" response, the student subtracted the two numbers even after drawing a figure that indicated some understanding of the relationship between the top of the mountain and the bottom of the river.

Sample "correct" response



Did you use the calculator on this question?

○ Yes ● No

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Sample "partial" response

1277
− −294

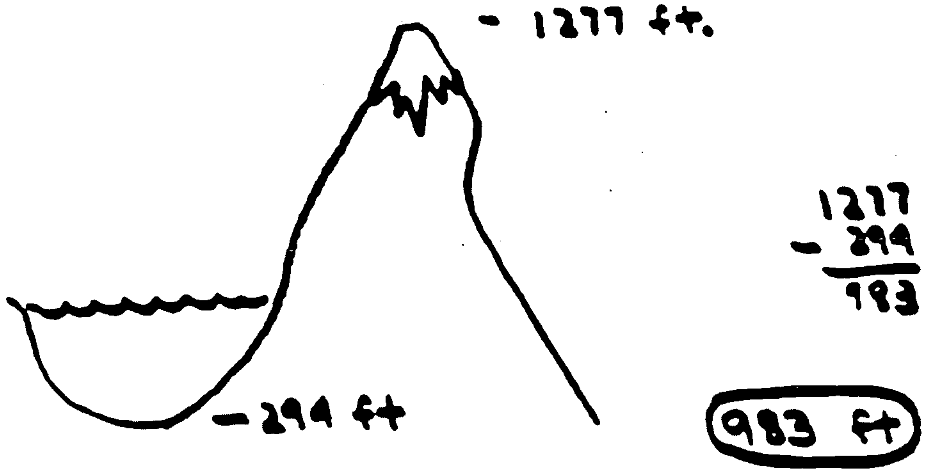
1277
+ +294

1541

Did you use the calculator on this question?

● Yes ○ No

Sample "incorrect" response



Did you use the calculator on this question?

○ Yes ● No

Table 7.13 shows student performance on this question. While more than 95 percent of eighth-grade students attempted to answer the question, only 25 percent provided a response that was rated at least "partial." When the question was anchored to the NAEP scale, the "correct" and "partial" rating categories were collapsed. The question mapped at 335 on the composite scale. As may be expected, students currently taking algebra outperformed those taking eighth-grade mathematics or pre-algebra. Male students performed better than females.

Table 7.13

Percentage Correct for "Subtract Integers"

THE NATION'S REPORT CARD NAEP

Grade 8	Correct	Partial	Incorrect	Omit
Overall	22	3	70	4
Males	25	3	65	6
Females	19	2	76	2
White	26	3	68	2
Black	10	3	78	9
Hispanic	14	2	72	12
Asian/Pacific Islander	--	--	--	--
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	18	2	75	5
Pre-Algebra	15	2	81	2
Algebra	38	5	54	2

NOTE: Row percentages may not total 100 due to rounding. Responses that could not be rated were excluded.
*** Sample size is insufficient to permit a reliable estimate.
-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.14 presents the percentages of students whose responses were rated "correct" overall and within each of the achievement-level intervals. That this question was fairly difficult for students can be seen by the fact that only 18 percent of those classified as *Basic* and 46 percent of those classified as *Proficient* answered the question correctly.

Table 7.14

Percentage Correct Within Achievement-Level Intervals for "Subtract Integers"

THE NATION'S REPORT CARD NAEP

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
22	9	18	46	81

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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The following example is a multiple-choice question for grade 12 that required students to solve a pair of equations. The question showed two equations, each with two boxes for missing numbers, and asked what single number could be placed in all four boxes to make both equations true. In order to answer the question correctly, students had to realize that the only number that could be multiplied by both 4 and 3 and remain unchanged is 0. However, even if students did not realize this immediately and set about answering the question by trial and error (i.e., substituting the numbers presented in the options into the equations to solve for the answer), they would quickly obtain the correct answer, as it was presented in the first option. Presumably, this would be the first number tried by the students.

6. $4 \times \square = \square$ and $\square \times 3 = \square$

What number if placed in each box above would make both equations true?

(A) 0

(B) 1

(C) 2

(D) 3

(E) 4

The correct option is A.

Student performance data are presented in Tables 7.15 and 7.16. The question was not difficult for students, as can be seen by the high percentage of students (88%) overall who answered correctly. Ninety-eight percent of students classified at the *Proficient* level, 96 percent of those classified at the *Basic* level, and 69 percent of those classified as below the *Basic* level answered correctly. Performance on this question appears less dependent on advanced curriculum than does performance on some of the more difficult questions; it appears that the concepts assessed in this question are taught in the lower level algebra courses. Thus, students in calculus, third-year algebra/pre-calculus, and second-year algebra performed similarly, whereas students in second-year algebra performed better than those in first-year algebra, and those in first-year algebra performed better than those in pre-algebra. Female students performed better than males on this question. The question mapped at 263 on the NAEP composite mathematics scale.

Table 7.15 Percentage Correct for "Solve Pair of Equations"

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	88
Males	85
Females	90
White	91
Black	79
Hispanic	78
Asian/Pacific Islander	90
American Indian	***
Geometry Taken	92
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	57
First-Year Algebra	83
Second-Year Algebra	92
Third-Year Algebra/Pre-Calculus	96
Calculus	95

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table 7.16 Percentage Correct Within Achievement-Level Intervals for "Solve Pair of Equations"

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
88	69	96	98!	***

*** Sample size is insufficient to permit a reliable estimate.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Advanced functions topics and trigonometry

Questions testing advanced algebraic concepts asked students to describe functions and their properties, apply properties of functions, and apply functions to real-world situations. There also were questions that assessed students' familiarity with trigonometry. The following grade 12 example is a multiple-choice question that assessed students' knowledge of a trigonometric identity. To answer the question correctly, students had to know, or be able to derive, the identity that demonstrates that the value of the expression $cos^2 x + sin^2 x$ equals 1 for any real number x. The question mapped at 362 on the composite scale.

8. $\cos^2(3x) + \sin^2(3x) =$

Ⓐ 0

Ⓑ 1

Ⓒ 3

Ⓓ 6

Ⓔ 9

The correct option is B.

Student performance data are presented in Table 7.17. Overall, 27 percent of the students who attempted the question answered correctly. Approximately 18 percent selected each of Options C and D, 15 percent selected Option E, and 9 percent chose Option A. Almost 13 percent of the students omitted the question. Students whose highest course was pre- or first-year algebra performed similarly. However, above that level, each additional course in the algebra-through-calculus sequence was associated with an increase in the proportion of students who could answer the question correctly.

Table 7.17 Percentage Correct for "Use Trigonometric Identity"

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	27
Males	26
Females	27
White	28
Black	17
Hispanic	25
Asian/Pacific Islander	56
American Indian	***
Geometry Taken	30
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	13
First-Year Algebra	15
Second-Year Algebra	26
Third-Year Algebra/Pre-Calculus	43
Calculus	64

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

When performance is disaggregated by achievement level, Table 7.18 shows that 11 percent of students below the *Basic* level, 26 percent of students at the *Basic* level, and 60 percent of those at the *Proficient* level answered the question correctly. As might be expected, this question was difficult for students performing at the *Basic* level and below.

Table 7.18 Percentage Correct Within Achievement-Level Intervals for "Use Trigonometric Identity"

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
27	11	26	60	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Summary

Questions in this content strand assessed students' knowledge of and ability to solve problems in four areas: *patterns and functional relationships, number lines and graphs, equations and inequalities,* and *advanced functions topics and trigonometry*. The majority of students at all grade levels appeared to understand basic algebraic representations and simple equations, as well as how to find simple patterns. Students at grades 8 and 12 had difficulty with questions requiring knowledge of linear equations, algebraic functions, and trigonometric identities. Students in these grades also found that questions requiring them to identify and generate complex patterns and solve real-world problems were challenging. In general, for eighth- and twelfth-grade students, those with more advanced coursework performed better on this content strand.

Chapter 8

Course-Taking Patterns

When students do well in mathematics they are likely either to select or be placed in more advanced courses earlier in their school careers than students experiencing less success. This allows them to take a greater number of increasingly difficult courses as they progress through high school — courses that expose them to more advanced content, as well as provide them the opportunities to practice and apply more powerful mathematical techniques in problem settings. In contrast, lower performing students may select, or be assigned to, less demanding curricular offerings — placements that provide them with fewer challenging opportunities, offer slower progress toward more advanced coursework, or even increase the likelihood that they will terminate their study of mathematics earlier in their school careers than more successful students.

This chapter is about student course-taking patterns. It includes information on the types of mathematics courses in which eighth-grade students were enrolled at the time of the NAEP 1996 assessment and on the mathematics course-taking histories of twelfth-grade students participating in the assessment. It also presents course-taking information for different gender and racial/ethnic subgroups. One of the reasons for monitoring course taking by gender and racial/ethnic groups is that research indicates that males and White students are likely to study algebra before females and some minority students.[1] Perhaps more importantly, taking algebra early appears to be related to student outcomes of taking more mathematics overall as well as more advanced coursework in mathematics.[2]

Eighth-Grade Course Taking

In 1996, less than one percent (0.2%) of eighth-grade students indicated that they were not taking a mathematics course. Table 8.1 presents self-reported information on mathematics course taking by eighth-grade students. The average mathematics scores of students with different course enrollments also are shown.

[1] Fennema, E., & Leder, G. C. (Eds.) (1990). *Mathematics and gender*. New York: Teachers College Press; Kifer, E. (1992). Opportunities, talents, and participation. In L. Burstein (Ed.), *The IEA student of mathematics III: Student growth and classroom processes*. (pp. 279–307). New York: Pergamon Press.

[2] Smith, J. B. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics attainment. *Educational Evaluation and Policy Analysis*, *18*(2), 141–153.

Table 8.1

Average Scale Score by Mathematics Course Enrollment and by Gender, Race/Ethnicity, and Whether School Offers Algebra for High School Credit or Placement, Grade 8

THE NATION'S REPORT CARD

Grade 8	Assessment Year	Mathematics Course: Algebra — Percentage of Students	Algebra — Average Scale Score	Pre-Algebra — Percentage of Students	Pre-Algebra — Average Scale Score	Eighth-Grade Mathematics — Percentage of Students	Eighth-Grade Mathematics — Average Scale Score	Other Mathematics — Percentage of Students	Other Mathematics — Average Scale Score
All Students	1996	25*†	295	27*	270	43*	262*†	5	270*
	1992	20	299	28*	273	49*	256	3	257
	1990	16	295	20	271	61	252	3	257
Females	1996	26*†	294	27	271	42*	261*	5	278*†
	1992	21	300	28	272	48	255	3	251
	1990	16	293	21	268	60	252	4	***
Males	1996	25*	297	27*	269	43*	264*†	5	262
	1992	19	299	28*	273	49*	256	4	250
	1990	16	298	19	275	62	253	3	***
White	1996	27*	305	29	277	40*	271*	4	284†
	1992	22	306	31	278	45	266	3	259
	1990	18	300	21	276	57	260	3	265
Black	1996	20*	258	25	240	48*	237	7	254
	1992	13	259	23	247	60	231	4	***
	1990	9	***	16	246	72	234	3	***
Hispanic	1996	20*†	262	22	260	52*	249	6	***
	1992	12	277	21	256	62	241	5	***
	1990	7	***	14	260	76	240	4	***
Asian/Pacific Islander	1996	– –	– –	– –	– –	– –	– –	– –	– –
	1992	42	313	24	***	32	265	2	***
	1990	39	***	22	***	33	***	6	***
American Indian	1996	14	***	18	***	63	***	6	***
	1992	7	***	30	***	57	253	5	***
	1990	6	***	8	***	84	***	3	***
School Offers Algebra for High School Credit or Placement:									
Yes	1996	28*	298	28	271	39*	262*	5	276*†
	1992	23	302	29	274	45*	256	4	249
	1990	18	301	20	271	58	254	4	254
No	1996	16	279	24	268	56	266	5	***
	1992	10	285	28	270	60	257	2	***
	1990	7	***	19	272	73	252	1	***

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
† Significantly different from 1992.
*** Sample size is insufficient to permit a reliable estimate.
– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Among the eighth-grade students who provided usable answers about the mathematics course in which they were enrolled, 43 percent indicated that they were in a basic eighth-grade mathematics class; 27 percent were in a pre-algebra class; 25 percent were in an algebra class; and 5 percent were in some other mathematics course.[3] These "other mathematics" courses included applied mathematics (also referred to as technical preparation mathematics) and integrated or sequential mathematics. Perhaps not surprisingly, students in algebra classes outperformed students in pre-algebra, eighth-grade mathematics, and "other mathematics" courses on the NAEP 1996 assessment, and students in pre-algebra outperformed students in eighth-grade mathematics classes.

In 1996, the eighth-grade course-taking patterns and NAEP mathematics performance of *females and males* in the same mathematics courses were similar to each other. For example, 42 percent of female students and 43 percent of male students were enrolled in eighth-grade mathematics. Similarly, the percentages of female students enrolled in pre-algebra, algebra, and "other mathematics" classes were similar to the percentages of male students enrolled in the same type of class. Furthermore, within each mathematics course, female students and male students performed similarly on the NAEP 1996 mathematics assessment; for example, female students in algebra classes had an average scale score of 294, while the average scale score for male students in algebra classes was 297.

However, when one looks at performance differences across mathematics courses, the pattern differs by gender group. For female students, those taking algebra outperformed female students in pre-algebra classes, and female students in algebra, pre-algebra, and "other mathematics" courses outperformed those in eighth-grade mathematics. For male students, those taking algebra outperformed those in pre-algebra, eighth-grade mathematics, and "other mathematics" courses, while male students in any of the courses other than algebra performed similarly to each other.

An examination of the 1996 percentages of the different racial/ethnic groups enrolled in each type of mathematics course shows no significant differences, except in algebra, where the percentage of White students was higher than the percentage of American Indian students.[4] There were, however, some differences in the overall performance of different racial/ethnic groups in specific mathematics courses. In algebra, White students outperformed Black and Hispanic students. In pre-algebra, White and Hispanic students outperformed Black students, and White students also outperformed Hispanic students. In eighth-grade mathematics, White and Hispanic students outperformed Black students. In "other mathematics" courses, White students outperformed Black students.

Comparisons of percentages of students enrolled in different mathematics courses by *whether or not their school offered algebra for high school credit or placement* show that offering algebra for high school credit appears to make some difference. The percentage of eighth-grade

[3] About two percent of eighth-grade students taking the NAEP 1996 assessment either omitted this question or provided multiple responses. These were considered nonlegitimate answers, and, therefore, these students were excluded from analyses involving eighth-grade course-taking patterns.

[4] The reader is reminded that statements about significant differences are based on statistical tests that consider the magnitude of the difference among the percentages or averages and the standard errors of those statistics. Therefore, differences that appear to be large may turn out to be statistically nonsignificant. More details on statistical inferences using NAEP data are available in Appendix A.

students enrolled in algebra in schools that offered algebra for high school credit was higher than the percentage of students enrolled in algebra in schools that did not offer this option. Because course enrollments were self-reported, students in schools that did not offer algebra for high school credit, but who indicated that they were taking algebra, may have erroneously reported their enrollment status. However, in this report, we have assumed that the responses of these students were correct and either their schools offered a nontransferable algebra course or that they were taking algebra at an alternative site such as a local high school or community college.

As might be expected, the pattern of eighth-grade mathematics enrollment by whether or not the school offered algebra for high school credit was the converse of the algebra enrollment pattern. That is, the percentage of students enrolled in eighth-grade mathematics in schools that offered algebra for high school credit was lower than the percentage enrolled in eighth-grade mathematics in other schools. Comparisons of students' performance on the NAEP 1996 assessment show that algebra students from schools that offered algebra for high school credit performed better than algebra students from schools that did not.

Because some current mathematics reform efforts advocate that students take more difficult mathematics courses earlier in their school careers and specifically suggest that students be prepared to take algebra in eighth grade, we examined *enrollment patterns over time* to determine whether enrollment in eighth-grade algebra was increasing. Indeed, the data indicate that a higher percentage of eighth-grade students was enrolled in algebra in 1996 (25%) than had been enrolled in algebra in 1992 or in 1990 (20% and 16%, respectively). However, despite these increases in the percentages enrolled, students enrolled in algebra in 1996 performed similarly on the NAEP mathematics assessment to students enrolled in algebra in 1992 and 1990.

The percentage of eighth-grade students enrolled in pre-algebra in 1996 did not increase from 1992 but was higher than the percentage of students enrolled in pre-algebra in 1990. As with students in algebra, performance on the mathematics assessment for students in pre-algebra in 1996 was similar to the performance for pre-algebra students in 1992 and 1990. The percentage of eighth-grade students enrolled in eighth-grade mathematics in 1996 was similar to the percentage enrolled in 1992, but was lower than the percentage enrolled in 1990. It is possible that the curriculum of eighth-grade mathematics had also been changing over this period because, in 1996, students in eighth-grade mathematics performed better in mathematics than eighth-grade mathematics students did in 1992 and 1990.

The small percentage of students enrolled in "other mathematics" courses in 1996 was similar to the percentages enrolled in 1992 and 1990. Students in "other mathematics" courses in 1996 outperformed students in "other mathematics" courses in 1992.

Comparisons of gender groups over time show that a higher percentage of female students was enrolled in algebra in 1996 than was enrolled in algebra in 1992 and 1990. However, for male students, the percentage enrolled in algebra in 1996 was only significantly higher than the percentage enrolled in 1990.

Enrollment patterns in algebra over time differed among racial/ethnic subgroups. The percentages of White students and Black students enrolled in algebra were significantly higher in 1996 than they were in 1990, but not significantly higher than they were in 1992, while the percentage of Hispanic students enrolled in algebra in 1996 was higher than the percentage enrolled in 1992 or 1990.

In schools that offered algebra for high school credit, student enrollment in algebra increased from 1990 to 1996. A significant increase in enrollment in algebra was not observed for schools that did not offer algebra for high school credit.

Mathematics Course Taking in High School

The NAEP background survey of twelfth-grade students collected considerable detail about students' current and past course-taking patterns. In 1996, three percent of the nation's twelfth-grade students attended schools that required 4 years of mathematics (taken in grades 9–12) for high school graduation, and 51 percent attended schools with a 3-year requirement. In schools that have less than a 4-year requirement, students generally take their mathematics classes earlier in their high school careers. This means that when these students graduate from high school, many have not been involved in the formal study of mathematics on a regular basis for a year or more. The chances, therefore, are likely that by the time they graduate and enter the work world or go on to higher education, many students probably will have forgotten much of what they learned or at least will be less facile with what they remember.

Table 8.2 shows that, in 1996, slightly less than two-thirds of the nation's twelfth-grade students (64%) were enrolled in a mathematics class. Being enrolled in mathematics, however, did not necessarily mean that these students had all had 4 years of high school mathematics or were enrolled in advanced courses. For some individuals, taking mathematics in their senior year might have been the result of having either failed previous classes or delayed taking a required class.[5] Nevertheless, taken as a group, students enrolled in mathematics in their twelfth-grade year outperformed students who were not enrolled in mathematics on the NAEP 1996 mathematics assessment. The average scale score of those who were taking mathematics was 311, while those who were not had an average scale score of 292.[6]

Similar percentages of female and male twelfth-grade students were enrolled in mathematics in 1996, 63 percent and 66 percent respectively. In terms of racial/ethnic groups, a higher percentage of Asian/Pacific Islander twelfth-grade students (77%) was enrolled in mathematics classes compared with White students (63%), Hispanic students (63%), and American Indian students (56%).

The percentage of students enrolled in a mathematics class their senior year was higher in 1996 than it was in 1990. Similarly, the percentage of female students enrolled in a mathematics class their senior year was higher in 1996 than in 1990.

[5] Here and throughout this report, the term "senior year" refers to students' twelfth-grade year.

[6] The source of these data is the NAEP 1996 mathematics assessment. The data are available on the World Wide Web at: <http://nces.ed.gov/naep/>.

Table 8.2 Percentage of Students Currently Enrolled in a Mathematics Course by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

Grade 12	Assessment Year	Percentage of Students
All Students	1996	64*
	1992	63
	1990	59
Females	1996	63*
	1992	61*
	1990	52
Males	1996	66
	1992	66
	1990	66
White	1996	63
	1992	62
	1990	58
Black	1996	70
	1992	64
	1990	62
Hispanic	1996	63*
	1992	62
	1990	53
Asian/Pacific Islander	1996	77
	1992	85
	1990	76
American Indian	1996	56
	1992	***
	1990	***

* Significantly different from 1990.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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First-Year Algebra

As shown by the data on mathematics course taking, in 1996 over a third of twelfth-grade students appear to have chosen to opt out of mathematics before their senior year of high school. Especially for these students, but for other students as well, to have taken any advanced mathematics courses, they would have had to take algebra as early as possible in their school careers. Information about when students initially took first-year algebra is an indicator of students' preparedness to enter a mathematics sequence that would lead to advanced courses. Data on eighth-grade course taking in 1996 shows that one-fourth of our nation's eighth-grade students were enrolled in algebra, which (in addition to geometry) is a prerequisite for higher level mathematics courses.[7] The responses of twelfth-grade students, who were asked to provide information on when they initially took first-year algebra, are presented in Table 8.3.

Table 8.3 **Percentage of Students by Year They Initially Took a First-Year Algebra Course, Grade 12**

THE NATION'S REPORT CARD NAEP

Grade 12	Assessment Year	Before 9th Grade	9th Grade	10th Grade	11th or 12th Grade	Not Taken
All Students	1996	29†	51	13	3	3†
	1992	23	51	15	5	6
Females	1996	29†	52	13	3	3†
	1992	23	52	15	5	5
Males	1996	30†	49	13	4	4†
	1992	24	49	15	5	7
White	1996	30†	52	12	3	3†
	1992	24	52	14	4	6
Black	1996	27	48	17	5	4
	1992	18	48	19	9	7
Hispanic	1996	21	51	16	7	5
	1992	17	45	23	9	7
Asian/Pacific Islander	1996	50	37	8	2	2
	1992	40	44	10	4	2
American Indian	1996	13	52	27	8	1
	1992	***	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

[7] This is not to imply that algebra is the most advanced mathematics course available to eighth-grade students. In fact, students may be in an integrated-mathematics course that substitutes for algebra, or some students may even be in courses more advanced than algebra. However, as shown in Table 8.1, only five percent of students were in mathematics courses other than algebra, pre-algebra, or eighth-grade mathematics.

In 1996, the majority of twelfth-grade students (51%) indicated that they initially took first-year algebra in the ninth grade and 29 percent took it before the ninth grade.[8] Regardless of whether twelfth-grade students were currently taking a mathematics class or not, about half of them indicated that they had initially taken algebra in the ninth grade.

The patterns for initially taking first-year algebra were similar for male and female students. The percentages by racial/ethnic groups, however, show some differences. For example, although the modal response of White, Black, Hispanic, and American Indian students indicated initially taking first-year algebra in the ninth grade, the modal response of Asian/Pacific Islander students indicated initially taking first-year algebra *before* the ninth grade.

Comparisons over time show that for all twelfth-grade students, for female and male students, and for White students, the percentages of students who initially took first-year algebra before the ninth grade were higher in 1996 than they were in 1992. Moreover, for all of those groups, the percentages of twelfth-grade students who had not taken a first-year algebra course at all were lower in 1996 than they were in 1992. These numbers appear to signify a positive trend in light of current mathematics reform efforts.

Number and Types of Mathematics Courses Taken

As shown in Table 8.4, in 1996 almost half of the nation's twelfth-grade students indicated having taken seven or more *semesters of mathematics* during their high school career (i.e., grades 9 to 12). Seven or more semesters of mathematics translates into more than 3 years of mathematics courses. This appears encouraging, given that only three percent of twelfth-grade students were enrolled in schools that required more than 3 years of mathematics courses for high school graduation. That is, students appear to be taking more mathematics than schools require for graduation. However, the reader should keep in mind that some of these semesters of coursework may reflect repeats of courses for students who failed to reach levels of performance that would have allowed them to move forward.

It also is encouraging that, in 1996, the percentage of female students with seven or more semesters of mathematics was similar to the percentage of male students.

[8] Discrepancies between these data on grade 12 students in 1996 and data reported in Table 8.1 on grade 8 students in 1992 (the same population of students) may be explained by the fact that these data are based on students' self-reports. Memory limitations or confusion about the different levels of algebra may influence the accuracy of students' responses, especially when students are asked about their course-taking experiences retrospectively at grade 12.

Table 8.4 Percentage of Students by Number of Semesters of Mathematics Taken (Grades 9 through 12) by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

	Assessment Year	7 or More Semesters	5–6 Semesters	3–4 Semesters	1–2 Semesters	No Semesters
Grade 12						
All Students	1996	48	22	26	4	1
	1992	48	23	25	3	1
	1990	45	23	27	6	0
Females	1996	47	23	26	4	0
	1992	46	25	25	3	0
	1990	40	27	28	5	0
Males	1996	49	20	26	5	1
	1992	50	21	25	3	1
	1990	50	18	26	6	1
White	1996	50	23	23	4	0
	1992	50	24	23	3	1
	1990	46	23	25	5	0
Black	1996	37	17	40	6	1
	1992	38	18	38	5	1
	1990	33	21	39	7	1
Hispanic	1996	44	23	28	5	1
	1992	38	28	28	5	1
	1990	38	22	35	5	0
Asian/Pacific Islander	1996	66	16	17	1	1
	1992	69	18	12	2	0
	1990	66	23	7	3	0
American Indian	1996	22	28	38	10	2
	1992	***	***	***	***	***
	1990	***	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Comparisons across racial/ethnic groups by semester categories show that, in 1996, the percentage of Asian/Pacific Islander students taking seven or more semesters of mathematics was higher than the percentages of students in the other racial/ethnic groups; and the percentage of White students was higher than the percentage of Black and American Indian students. For most of the racial/ethnic groups, the modal number of semesters was seven or more. However, for Black students and American Indian students, the modal number was 3–4 semesters.

Reform efforts to improve the mathematics achievement of the nation's students advocate taking more mathematics, as well as taking courses that are at more advanced levels of mathematics. In 1996, students were asked to indicate the level of exposure they had to different *types of mathematics courses* during their high school years. This information is presented in Table 8.5 along with student data from 1992 and 1990 where available.

In 1996, the relative percentages of students who indicated that they had taken one school year or more of each type of mathematics course were not unexpected. In terms of what are generally considered lower-level courses, 63 percent of twelfth-grade students indicated having taken a year or more of pre-algebra, and 53 percent indicated having taken a year or more of general mathematics. Among the higher level mathematics courses, the highest percentage of twelfth-grade students indicated having taken a year or more of first-year algebra (90%), followed by 80 percent who indicated having taken geometry. Two relatively new mathematics courses were added after the 1990 administration for students' consideration. In 1992, "unified, integrated, or sequential mathematics" was added to the list of mathematics courses, and, in 1996, nine percent of twelfth-grade students indicated having taken a year or more of that course. In 1996, students also were asked about "applied mathematics," also known as "technical preparation mathematics"; 15 percent of students indicated having taken a year or more of such work.

Given the belief that more students should be taking higher level mathematics, in general, the course-taking patterns of twelfth-grade students have improved over time. For example, between 1990 and 1996, there were increases in the percentages of students who had taken a full year of pre-algebra, first-year algebra, second-year algebra, pre-calculus (also known as third-year algebra), calculus, or probability or statistics. On the other hand, the percentage of students who reported having taken more than one year of general mathematics was also higher.

Table 8.5

Percentage of Students by Mathematics Courses and Years of Study, Grade 12

THE NATION'S REPORT CARD naep

Grade 12	Assessment Year	Years of Study: More Than One Year	One School Year	One-Half Year or Less	Not Studied
General Mathematics	1996	33*†	20	3	44
	1992	27	23	3	48
	1990	27	23	3	47
Business or Consumer Mathematics	1996	4	16†	9	71†
	1992	5	21*	9	66
	1990	5	17	9	6
Introduction to Algebra or Pre-Algebra	1996	11*†	52*	7	30*†
	1992	9	48	7	37
	1990	9	43	7	41
First-Year Algebra	1996	9	81*	4	6*†
	1992	8	79*	4	9*
	1990	8	73	4	14
Geometry	1996	5†	75	7	13†
	1992	4*	72	5	19
	1990	5	66	5	25
Second-Year Algebra	1996	4	66*†	7	23*†
	1992	3	58	8	32*
	1990	3	53	6	38
Trigonometry	1996	2	20	23	55*
	1992	2	19	22	58
	1990	2	15	19	64
Pre-Calculus, Third-Year Algebra	1996	2	22*	12	65*†
	1992	1	18	10	70
	1990	2	14	10	75
Calculus	1996	1	11*	4	84*
	1992	1	9	3	87
	1990	1	7	3	88
Probability or Statistics	1996	2	6*†	13*	79*
	1992	1	4	12	83
	1990	1	3	9	88
Unified, Integrated, or Sequential Mathematics	1996	4	5	4	87
	1992	2	4	5	89
Applied Mathematics (Technical Preparation)	1996	5	10	6	79

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
† Significantly different from 1992.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Algebra and Calculus Coursework in High School

In Table 8.6, twelfth-grade students are categorized according to the highest level mathematics course they indicated having taken in an algebra-through-calculus sequence. The algebra-through-calculus sequence for this analysis was created in accordance with the typical sequential order of these courses in high schools in the United States.[9] The lowest level in this sequence is "Not having taken at least a pre-algebra or introduction-to-algebra course" and the highest level is calculus, with intermediate steps as follows: pre-algebra, first-year algebra, second-year algebra, and pre-calculus (also referred to as third-year algebra or analysis). Students were credited with having taken a particular course only if they indicated that they had taken one school year or more of that course.

In 1996, almost half of the twelfth-grade students indicated that second-year algebra was the highest course in an algebra-through-calculus sequence that they had taken for one school year or more. Second-year algebra was the modal response of both female students and male students; however, a higher percentage of female students than male students indicated second-year algebra as the highest algebra-through-calculus course they had taken. Comparisons between the percentages of female students and male students at each course level showed no other significant difference.

There were few significant differences in the course-taking patterns of different racial/ethnic groups. For example, the percentages of White, Black, Hispanic, Asian/Pacific Islander, and American Indian students who indicated that second-year algebra was the highest algebra-through-calculus course they had taken were similar to each other. The only differences found were for first-year algebra and calculus. The percentage of American Indian students indicating first-year algebra as their highest level course taken was higher than the percentage of any other racial/ethnic group, and the percentage of Hispanic students indicating this was higher than the percentage of Asian/Pacific Islander students. For calculus, the percentage of White students was higher than the percentages of Black and Hispanic students.

The trend toward students taking more advanced-level courses also is apparent when we focus on the highest level courses students have taken in the algebra-through-calculus sequence. For example, in 1996, four percent of twelfth-grade students indicated not having taken pre-algebra; this was lower than the nine percent in 1990. In addition, four percent of students indicated pre-algebra as their highest level algebra-through-calculus course, and this was lower than the six percent of students who so indicated in 1992. Even the 23 percent of students who indicated that first-year algebra was their highest level algebra-through-calculus course was lower than the 29 percent in 1992. At the other end of the spectrum, seven percent of students in 1996 indicated that calculus was the highest algebra-through-calculus course taken; this was higher than the five percent of students in 1992, or the three percent of students in 1990, who indicated that calculus was their highest course.

[9] Chaney, B., Burgdorf, K., & Atash, N. (1997). Influencing achievement through high school graduation requirements. *Educational Evaluation and Policy Analysis, 19*(3), 229–244.

THE NATION'S REPORT CARD naep

Table 8.6 — *Percentage of Students by Highest Algebra-Through-Calculus Course Taken, Grade 12*

Grade 12	Assessment Year	Not Taken Pre-Algebra	Pre-Algebra	First-Year Algebra	Second-Year Algebra	Pre-Calculus or Third-Year Algebra	Calculus
All Students	1996	4*	4†	23*†	48*	14	7*†
	1992	6*	6	29	44	11	5*
	1990	9	8	28	43	9	3
Females	1996	3*†	4*†	21†	51*	14*	6*
	1992	5	6	28	45	11	5*
	1990	8	8	28	45	9	3
Males	1996	5*	5*	24	45	13	8*
	1992	6*	6	29	42	11	5
	1990	10	7	27	41	10	4
White	1996	3*	4*	22	49	15	8
	1992	5	5	27	45	12	5
	1990	8	7	27	44	10	4
Black	1996	5*	5*	24*†	52*†	11	3
	1992	8	8	37	37	7	3
	1990	13	10	31	39	6	0
Hispanic	1996	8	6	27	46	10*	4
	1992	7	9	34	40	6	4
	1990	17	10	31	37	3	1
Asian/Pacific Islander	1996	3	4	14	39	19	20
	1992	1	4	20	45	12	17
	1990	5	10	24	42	13	5
American Indian	1996	5	6	46	38	4	2
	1992	***	***	***	***	***	***
	1990	***	***	***	***	***	***

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
† Significantly different from 1992.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

The patterns for both female students and male students over time are encouraging, especially for female students. In 1996, there were declines from 1990 and/or 1992 in the percentages of females who reported that their highest level course in the algebra-through-calculus sequence was none, pre-algebra, or first-year algebra. At the same time, there were increases in the percentages of female students who reported having taken pre-calculus or calculus. For male students, there were declines in the percentages who reported having stopped with no year-long courses in the algebra-through-calculus sequence; there was also an increase in the percentage who reported having taken calculus.

Differences over time for the racial/ethnic groups were as follows: among White students, there was a significant increase in the percentage of students taking calculus, whereas declines were observed in the percentages of students who did not take pre-algebra or who did not advance beyond pre-algebra. For Black students, there were increases in the percentages who reported having taken second-year algebra or calculus as their highest course; there were also declines in the percentages who reported stopping with no pre-algebra, only pre-algebra, or only first-year algebra. Among Hispanic students, there was an increase in the percentage who reported having taken pre-calculus.

Geometry Coursework in High School

Although researchers have found that geometry is generally taken after first-year algebra and before second-year algebra, this sequence is not always the rule; therefore, we chose to examine course taking in geometry separate from algebra-calculus courses.[10] Furthermore, the role of geometry in the American educational system has changed over the years. Some educational researchers have cited geometry as the new "gatekeeper" course for access to higher education, because most colleges are now requiring the completion of a course in geometry prior to entrance.[11] Therefore, it seemed important to examine geometry course taking apart from course taking in an algebra-through-calculus sequence. The data in Table 8.7 indicate that in 1996, over 80 percent of twelfth-grade students had taken a year or more of geometry during their high school years. Similar percentages of female and male students in the twelfth grade reported having taken a school year or more of geometry. Over half of the students in all racial/ethnic groups indicated having taken geometry.

The 1996 percentage of twelfth-grade students who indicated having taken geometry was higher than the percentage in 1990. This pattern was similar for female and male students and for White, Black, and Hispanic students.

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[10] Ibid; National Assessment of Educational Progress (NAEP) 1996 mathematics assessment.

[11] Pelavin, S., & Kane, M. (1990). *Changing the odds: Factors increasing access to college.* New York: College Board Publications; U.S. Department of Education. (1997). *Getting ready for college early: A handbook for parents of students in the middle and junior high school years.* Available on the Word Wide Web at: <http://www.ed.gov/pubs/GettingReadyCollegeEarly/>.

Table 8.7

Percentage of Students by Whether They Have Taken a Geometry Course and by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

Grade 12	Assessment Year	Taken a Geometry Course: Yes	Taken a Geometry Course: No
All Students	1996	80*	20*
	1992	76*	24*
	1990	71	29
Females	1996	82*	18*
	1992	77*	23*
	1990	71	29
Males	1996	78*	22*
	1992	75	25
	1990	70	30
White	1996	81*	19*
	1992	78	22
	1990	73	27
Black	1996	82*	18*
	1992	72	28
	1990	61	39
Hispanic	1996	77*	23*
	1992	67	33
	1990	58	42
Asian/Pacific Islander	1996	84	16
	1992	86	14
	1990	85	15
American Indian	1996	58	42
	1992	***	***
	1990	***	***

NOTE: Row percentages may not total 100 due to rounding.
* Significantly different from 1990.
*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Summary

This chapter examined the course-taking patterns of the nation's eighth- and twelfth-grade students in 1996 and over time. In 1996, over half of the eighth-grade students were enrolled in pre-algebra or algebra, while most of the remaining students were enrolled in eighth-grade mathematics. The percentages of female students enrolled in each of these three mathematics courses were similar to the percentages of male students enrolled in them. Enrollment percentages in each of these three types of mathematics classes were also similar for the different racial/ethnic groups except that the percentage of White students enrolled in algebra was higher than the percentage of American Indian students enrolled in that course. Trends over time appear to show that more eighth-grade students were taking more advanced mathematics courses. For example, the percentage of female students enrolled in algebra in 1996 was higher than the percentage enrolled in algebra in 1992 and 1990, and the percentage of male students, White students, and Black students enrolled in algebra in 1996 was higher than the percentage enrolled in 1990.

In 1996, approximately two-thirds of twelfth-grade students reported being enrolled in a mathematics class. Similar percentages of female and male students were taking mathematics, while the percentage of Asian/Pacific Islander students taking mathematics was higher than the percentages of White, Hispanic, and American Indian students. The percentage of twelfth-grade students enrolled in mathematics in 1996 was higher than in 1990; this was also true for the percentage of female students enrolled in mathematics.

In 1996, 29 percent of twelfth-grade students reported that they initially took first-year algebra before the ninth grade. This was true of both female and male students. In terms of racial/ethnic groups, half of the Asian/Pacific Islander students initially took first-year algebra before the ninth grade, which was higher than the percentage of White or Hispanic students taking first-year algebra this early. Information appears to show that over time, more students were taking first-year algebra and taking it earlier in their school careers.

In 1996, nearly half of all twelfth-grade students, both female and male students, reported taking seven or more semesters of mathematics. Large majorities of students reported having taken first-year algebra, geometry, and second-year algebra. In addition, there have been significant increases over time in the percentages of students taking courses at all levels of the algebra-through-calculus sequence, including the most advanced mathematics courses. Almost half of the twelfth-grade students indicated second-year algebra as the highest course taken in the algebra-through-calculus sequence. Twenty-one percent of students indicated taking a higher level course (such as pre-calculus or calculus) and 31 percent of students indicated taking a lower-level course (such as first-year algebra or pre-algebra) as their highest course in this sequence. With the exception of second-year algebra, where the percentage of female students was higher than the percentage of male students, there were no significant gender differences in the highest algebra-through-calculus course taken.

Comparisons over time also indicate a rise in the percentage of twelfth-grade students who have taken geometry. In 1996, four out of five twelfth-grade students indicated that they had taken a year or more of geometry. This was true of female and male students as well as students from the different racial/ethnic groups, with the exception of American Indian students.

Chapter 9

Classroom Practices

The NAEP 1996 mathematics assessment sought to embody many of the curricular emphases and objectives laid out in the curriculum and evaluation standards developed by the National Council of Teachers of Mathematics (NCTM).[1] Among the key features of the NAEP 1996 mathematics assessment were the following:

- *movement away from earlier assessments emphasizing only number properties and operations to also measure ability in number sense and estimations, as well as problem solving, communication, reasoning, and connections;*
- *inclusion of questions that require students to work through an extended problem and explain their reasoning through writing, giving examples, or drawing diagrams;*
- *increased use of calculators; and*
- *increased use of manipulatives such as geometric shapes to provide students with concrete representations to use in problem-solving situations.*[2]

The importance of these key features in current mathematics reform efforts, as well as the prominence given to them in the NAEP mathematics assessments since 1990, invites the question of the extent to which the mathematics instruction offered in our nation's classrooms reflects these same features. Background questions asked of students who participated in NAEP and of their teachers and principals were used to gather information about the instructional practices students were experiencing in their mathematics classrooms. For example, teachers were asked about the emphasis they placed on different mathematics content strands and on different mathematics skills. Teachers and students also were asked about the frequency with which students engaged in a variety of pedagogical and assessment practices in their mathematics classes, including questions about the use of calculators to do mathematics schoolwork.

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[1] National Assessment Governing Board (1996). *Mathematics framework for the 1996 National Assessment of Educational Progress.* Washington, DC: National Assessment Governing Board.

[2] White, S. (1994). *Overview of NAEP assessment frameworks.* Washington, DC: National Center for Education Statistics, p. 51.

Chapters 2 through 7 focused on student performance in mathematics overall, in the content strands, and on individual mathematics questions. This chapter focuses on information gathered by NAEP about the mathematics instruction our nation's students are experiencing in their classrooms. The information presented in this chapter about fourth- and eighth-grade students was provided either by their mathematics teachers or by the students themselves. Teachers of twelfth-grade students were not surveyed; therefore, information about twelfth-grade students was obtained solely through students' self-reports. In addition, because the questions focused on practices directly related to mathematics instruction, most of the information about twelfth-grade students was limited to those students who reported that they were presently enrolled in a mathematics class.

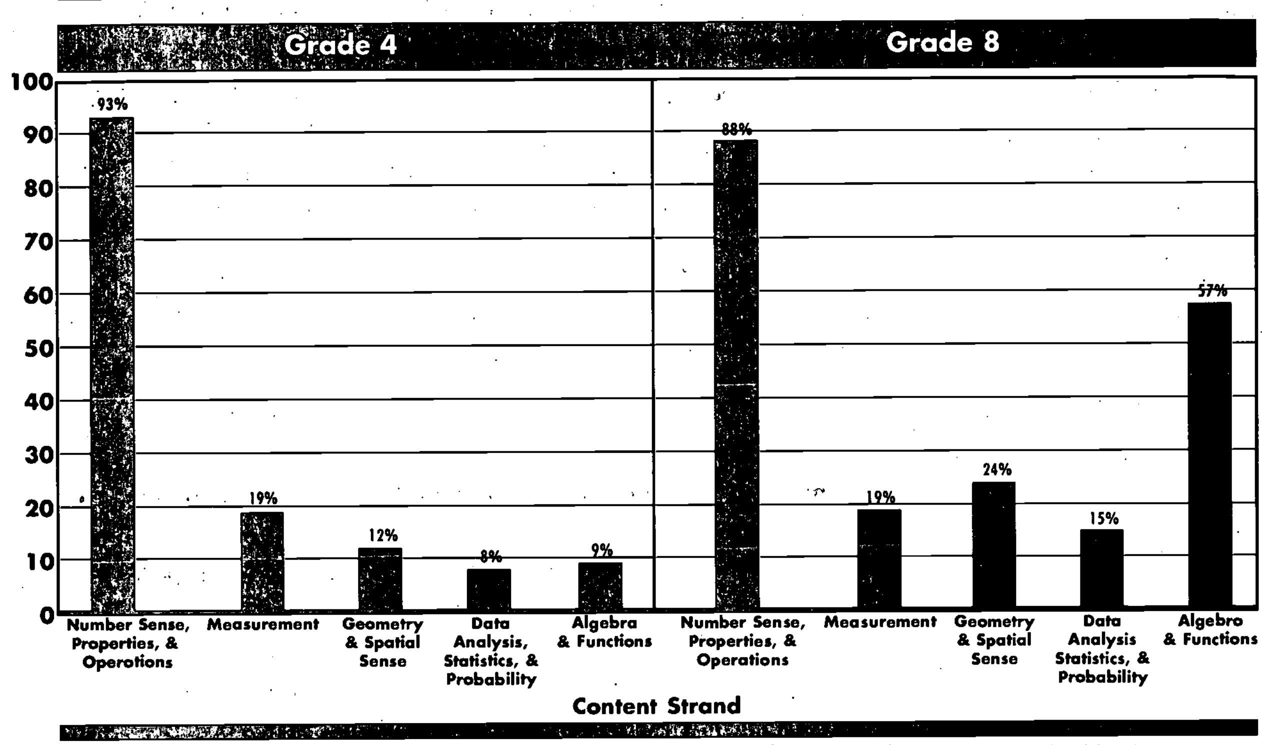
Emphasis on Content Strands

In the 1996 mathematics assessment, teachers of mathematics were asked about the level of emphasis they placed in their mathematics curriculum on each of the five mathematics content strands that are part of the NAEP mathematics framework: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. Because the data are based on written self-reports using a three-category response scale ("a lot," "some," or "a little or no" emphasis), there is a certain inherent ambiguity in the findings in that one teacher's reading of "a lot" may be another teacher's "some," and so on. Nevertheless, patterns do emerge and provide an important picture of the state of mathematics instruction in our nation's classrooms. Figure 9.1 shows the percentages of fourth- and eighth-grade students whose teachers reported placing "a lot" of emphasis on each of the five content strands. More detailed information on teachers' responses for each of the different content strands is presented in Tables 9.1–9.5.

The data in Figure 9.1 show that at both grades 4 and 8, a large percentage of students had teachers who placed "a lot" of emphasis on Number Sense, Properties, and Operations. At the fourth-grade level, fewer than one in five students had teachers who placed "a lot" of emphasis on any one of the remaining four content strands. In contrast, at the eighth-grade level, in addition to the prominence of the Number Sense, Properties, and Operations strand, teachers of over half of the students reported placing "a lot" of emphasis on Algebra and Functions. "A lot" of emphasis on the other content strands, however, was still infrequent. In addition, as the data in Tables 9.1–9.5 show, with the exception of Algebra and Functions, the emphasis placed on the different content strands did not differ by type of eighth-grade mathematics course in which students were enrolled.

Figure 9.1 ***Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Content Strands by Grade and Content Strand***

THE NATION'S REPORT CARD naep



SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Number Sense, Properties, and Operations

As shown in Figure 9.1, and again in Table 9.1, "a lot" of emphasis on Number Sense, Properties, and Operations was very common in mathematics classes for both grade 4 and grade 8.

Table 9.1 **Percentage of Students by Teachers' Reports on Emphasis Placed on Number Sense, Properties, and Operations, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

	A Lot	Some	Little or None
Grade 4			
All Students	93	7	0
Grade 8			
All Students	88	10	2
Students Enrolled in:			
Eighth-Grade Mathematics	87	11	1
Pre-Algebra	92	6	1
Algebra	87	10	3

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Measurement

The data in Table 9.2 show that the modal response from teachers of both fourth- and eighth-grade mathematics was "some" emphasis on the Measurement content strand. Nearly two-thirds of fourth-grade students were being taught mathematics by teachers who reported placing "some" emphasis on this strand and over one-half of eighth-grade students were in classes with "some" emphasis on this strand.

Table 9.2 **Percentage of Students by Teachers' Reports on Emphasis Placed on Measurement, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

	A Lot	Some	Little or None
Grade 4			
All Students	19	64	17
Grade 8			
All Students	19	58	23
Students Enrolled in:			
Eighth-Grade Mathematics	22	60	18
Pre-Algebra	19	57	24
Algebra	16	54	30

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Geometry and Spatial Sense

Data on the curricular emphasis given to Geometry and Spatial Sense are presented in Table 9.3. As with the Measurement content strand, mathematics teachers of the majority of students at grades 4 and 8 reported placing "some" emphasis on Geometry. Fifty-eight percent of fourth-grade students had mathematics teachers who indicated placing "some" emphasis on Geometry, and at the eighth-grade level, 54 percent of students had teachers who placed "some" emphasis on Geometry in their mathematics classes.

Table 9.3 — **Percentage of Students by Teachers' Reports on Emphasis Placed on Geometry and Spatial Sense, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	12	58	30
Grade 8			
All Students	24	54	22
Students Enrolled in:			
Eighth-Grade Mathematics	30	50	20
Pre-Algebra	16	59	25
Algebra	19	56	25

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Data Analysis, Statistics, and Probability

Although the Data Analysis, Statistics, and Probability content strand has received substantial attention in mathematics reform at all grade levels in recent years,[3] classroom emphasis placed on this strand appears to be less than the emphasis on Number Sense, Properties, and Operations; Measurement; or Geometry and Spatial Sense. However, the data, which appear in Table 9.4, indicate that there may be somewhat more emphasis at the eighth-grade level than at the fourth-grade level.

In 1996, only eight percent of fourth-grade students were taught mathematics by teachers who reported placing "a lot" of emphasis on Data Analysis, Statistics, and Probability and 41 percent of students had teachers who reported "some" emphasis. At the eighth-grade level, 15 percent of students had teachers of mathematics who reported placing "a lot" of emphasis on this content strand, and 47 percent had teachers who reported "some" emphasis.

Table 9.4 **Percentage of Students by Teachers' Reports on Emphasis Placed on Data Analysis, Statistics, and Probability, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	8	41	50
Grade 8			
All Students	15	47	38
Students Enrolled in:			
Eighth-Grade Mathematics	17	52	31
Pre-Algebra	12	45	43
Algebra	17	42	41

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[3] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards*. Reston, VA: Author.

Algebra and Functions

The data on emphasis on Algebra and Functions are presented in Table 9.5. Teachers of eighth-grade students reported placing much more emphasis on this content strand than did teachers of fourth-grade students. In 1996, only nine percent of fourth-grade students had teachers who reported "a lot" of emphasis on Algebra and Functions, while the majority of eighth-grade students (57%) had teachers who indicated "a lot" of emphasis on this content area.

An examination across types of eighth-grade mathematics courses by level of emphasis shows some significant, and perhaps expected, differences. Eighty-five percent of algebra students had teachers who reported "a lot" of emphasis on Algebra and Functions; this percentage was higher than the percentage of pre-algebra students (58%) or the percentage of eighth-grade mathematics students (40%) whose teachers reported placing "a lot" of emphasis on this content strand. Thirteen percent of algebra students were in mathematics classes with "some" emphasis on Algebra and Functions; this percentage was lower than the percentage of pre-algebra students (36%) or the percentage of eighth-grade mathematics students (45%). Finally, 15 percent of students in eighth-grade mathematics had teachers who reported "little or no" emphasis on Algebra and Functions, which was higher than the five percent of pre-algebra students and the two percent of algebra students.

Table 9.5 **Percentage of Students by Teachers' Reports on Emphasis Placed on Algebra and Functions, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	9	30	60
Grade 8			
All Students	57	34	9
Students Enrolled in:			
Eighth-Grade Mathematics	40	45	15
Pre-Algebra	58	36	5
Algebra	85	13	2

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Emphasis on Mathematical Processes

In addition to learning disciplinary content, students are expected to acquire mathematical skills and abilities that cut across content strands. Teachers of mathematics at grades 4 and 8 were asked questions about the extent to which they emphasized the following mathematical processes:

- learning mathematics facts and concepts;
- learning skills and procedures to solve routine problems;
- developing reasoning abilities to solve unique problems; and
- learning how to communicate ideas in mathematics.

Together, these mathematical skills provide students with the ability to do mathematics successfully. They also reflect the mathematical abilities and the construct of mathematical power described in the NAEP mathematics framework. Figure 9.2 presents data on students whose teachers reported placing "a lot" of emphasis on the different mathematical processes. Tables 9.6 through 9.9 provide more detailed information on teachers' responses regarding the level of emphasis they place on these processes in their mathematics instruction.

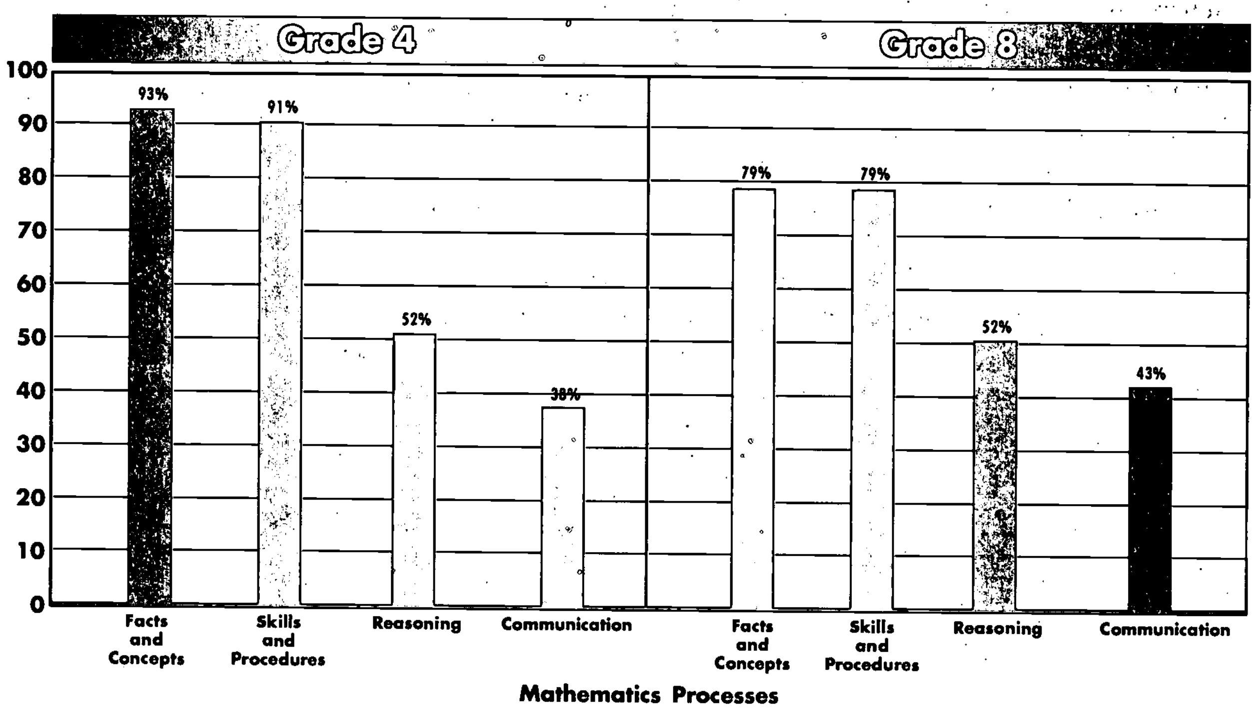
The data in Figure 9.2 show that teachers of the majority of fourth- and eighth-grade students reported placing "a lot" of emphasis on learning facts and concepts, learning skills and procedures to solve routine problems, and developing reasoning ability to solve unique mathematics problems. Although fewer students had teachers who reported placing "a lot" of emphasis on communicating ideas in mathematics effectively, over one-third of fourth-grade students and 43 percent of eighth-grade students had such teachers.

As with mathematics content, with the exception of developing reasoning abilities to solve unique problems, the emphasis placed on the different mathematical processes was not found to be related to the mathematics class in which students were enrolled.

Figure 9.2

Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Mathematics Processes by Grade and Mathematics Processes

THE NATION'S REPORT CARD naep



SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Learning mathematics facts and concepts

To do mathematics successfully, students must have knowledge of basic mathematics facts and a reasonable understanding of different mathematical concepts. The information provided by teachers on learning facts and concepts does not allow us to determine the relative focus on facts compared with concepts. Nevertheless, the data, which are presented in Table 9.6, provide a picture of the importance teachers of fourth- and eighth-grade students appear to place on learning mathematics facts and concepts. In 1996, 93 percent of fourth-grade students and 79 percent of eighth-grade students were taught mathematics by teachers who reported placing "a lot" of emphasis in their mathematics classes on learning mathematics facts and concepts.

Table 9.6 Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Mathematics Facts and Concepts, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	**Some**	**Little or None**
Grade 4			
All Students	93	7	0!
Grade 8			
All Students	79	16	5
Students Enrolled in:			
Eighth-Grade Mathematics	77	18	5
Pre-Algebra	82	15	3
Algebra	79	15	6

NOTE: Row percentages may not total 100 due to rounding.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Learning skills and procedures needed to solve routine problems

Knowing facts and concepts is an essential beginning. To use this knowledge to solve problems, students must acquire procedural knowledge and problem-solving skills.[4] Information from teachers on the emphasis they place on learning skills and procedures to solve routine problems is presented in Table 9.7.

Teachers of both fourth- and eighth-grade students place similar emphasis on learning skills and procedures to solve routine problems as on learning mathematics facts and concepts. In 1996, 91 percent of fourth-grade students were taught mathematics by teachers who reported placing "a lot" of emphasis on learning these skills and procedures, whereas 79 percent of eighth-grade students had such teachers.

Table 9.7 ***Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Skills and Procedures Needed to Solve Routine Problems, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD NAEP

	Level of Emphasis		
	A Lot	**Some**	**Little or None**
Grade 4			
All Students	91	8	0!
Grade 8			
All Students	79	18	3
Students Enrolled in:			
Eighth-Grade Mathematics	80	19	1
Pre-Algebra	79	18	3
Algebra	78	16	6

NOTE: Row percentages may not total to 100 due to rounding.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[4] Wakefield, A. P. (1997). Supporting math thinking. *Phi Delta Kappan*, *79*(3), 233–236.

Developing reasoning ability to solve unique problems

Doing mathematics successfully means a lot of things. It means being able to follow procedures and solve computational problems, and it means having the ability to solve classes of problems that become relatively routine through repeated exposure. In addition, it means being able to use one's knowledge and reasoning ability to solve mathematical problems in contexts that have not been encountered previously. The NAEP 1996 data on developing reasoning ability suggest that the task of helping students develop these capabilities may be somewhat more difficult to incorporate into mathematics instruction than the tasks of teaching students facts and concepts or how to apply more routine skills and procedures. That is, as the data in Table 9.8 show, compared with the mathematics processes discussed above, fewer students have teachers who reported placing "a lot" of emphasis on developing reasoning abilities. This is true in both fourth- and eighth-grade mathematics classes. Nevertheless, the majority of both fourth- and eighth-grade students still had teachers who reported "a lot" of emphasis on developing students' reasoning ability to solve unique mathematics problems, while most of the remainder had teachers who reported "some" emphasis.

Table 9.8 ***Percentage of Students by Teachers' Reports on Emphasis Placed on Developing Reasoning Ability to Solve Unique Problems, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD NAEP

	Level of Emphasis		
	A Lot	**Some**	**Little or None**
Grade 4			
All Students	52	41	8
Grade 8			
All Students	52	40	8
Students Enrolled in:			
Eighth-Grade Mathematics	42	47	12
Pre-Algebra	53	41	6
Algebra	68	29	3

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Overall, the reported emphasis at the two grades was nearly identical. That is, 52 percent of fourth-grade students and 52 percent of eighth-grade students were taught mathematics with "a lot" of emphasis on developing reasoning ability to solve unique problems. Forty-one percent of students at grade 4 and 40 percent students at grade 8, were taught by teachers who reported "some" emphasis.

However, there were some differences in the emphasis experienced by eighth-grade students in different mathematics courses. The percentage of algebra students (68%) in classes with "a lot" of emphasis on developing reasoning abilities was higher than the percentage of eighth-grade mathematics students (42%) in such classes. Reciprocally, the percentages of eighth-grade mathematics students in classes with "some" (47%) or "little or no" emphasis (12%) on developing reasoning skills were both higher than the percentages of algebra students in such classes (29% for "some" and 3% for "little or no" emphasis). That is, students perceived to be more advanced mathematically appear to get more exposure to higher level processes.

Learning how to communicate ideas in mathematics effectively

Not only do students need to acquire knowledge and be able to reason and solve problems, but they also need to be able to communicate ideas in mathematics effectively. More and more, NAEP and other mathematics assessments are assessing students' ability to explain how they solve problems. In addition, in more classrooms, students are being asked to discuss and, either verbally or in writing, to explain solutions to problems. Information about the emphasis teachers place on learning how to communicate ideas in mathematics effectively is presented in Table 9.9.

In 1996, similar percentages of fourth-grade students were taught mathematics by teachers who reported "some" or "a lot" of emphasis on communicating ideas in mathematics — 45 percent and 38 percent, respectively. At the eighth-grade level similar percentages of students also had teachers who reported "a lot" or "some" emphasis on communicating ideas in mathematics — 43 percent and 42 percent, respectively.

Table 9.9

Percentage of Students by Teachers' Reports on Emphasis Placed on Learning How to Communicate Ideas in Mathematics Effectively, Grades 4 and 8, 1996

THE NATION'S REPORT CARD

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	38	45	18
Grade 8			
All Students	43	42	16
Students Enrolled in:			
Eighth-Grade Mathematics	41	40	20
Pre-Algebra	39	47	14
Algebra	50	39	11

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Instructional Practices

Current mathematics reform efforts promote the use of a variety of instructional practices that can help students achieve academically.[5] This section includes teachers' reports on the frequency of use of selected classroom practices at the fourth- and eighth-grade levels and students' reports at the twelfth-grade level. Their responses provide a general picture of some instructional practices that students currently are experiencing in our nation's classrooms.

Use of manipulatives

Since the mid-1960s, mathematics educators have been promoting the use of manipulative materials to facilitate mathematics learning.[6] Such materials include Cuisenaire™ rods, geometric shapes, geoboards, Base 10 place value blocks, and a host of measuring instruments. Starting with the NAEP 1990 mathematics assessment, students were provided with rulers and protractors for use in some tasks on the assessments. With the 1992 assessment, students also received some geometric shapes to use in responding to questions requiring the analysis of relationships between these simple shapes and more complex shapes that could be formed from the pieces. The 1996 assessment expanded the practice of including manipulative materials. In order for students to use these manipulatives most appropriately and effectively in the NAEP mathematics assessment, they must have had previous experience with them; one of the best ways to provide such exposure is through classroom instruction.

[5] National Council of Teachers of Mathematics (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

[6] Bohan, H. J., & Shawaker, P. B. *Using manipulatives effectively: A drive down rounding road.* Available on the World Wide Web at: <http://www.enc.org/classroom/lessons/docs04083/4083.htm>.

As part of the NAEP 1996 assessment, teachers of fourth- and eighth-grade students were asked two separate questions about the frequency with which they used specifically named manipulatives in their mathematics instruction. Their responses are presented in Tables 9.10 and 9.11. Twelfth-grade students also were asked about their use of specific manipulatives, chosen to be more appropriate to their grade level. Information from twelfth-grade students who were taking mathematics is presented in Table 9.12. The data appear to show that working with these types of manipulatives is more common at lower grade levels and for lower level mathematics courses taken by eighth-grade students.

Table 9.10 ***Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Objects Like Rulers, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD NAEP

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4				
All Students	8	36	51	5
Grade 8				
All Students	7	18	53	21
Students Enrolled in:				
Eighth-Grade Mathematics	7	23	56	14
Pre-Algebra	5	16	58	22
Algebra	9	13	45	33

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In 1996, a substantial portion of fourth- and eighth-grade students were reported to work with objects like rulers in their mathematics classes at least "once or twice a month." Teachers of just over half of fourth-grade students reported using such objects "once or twice a month," while teachers of another third reported using such objects "once or twice a week." At the eighth-grade level, 53 percent of students worked with objects like rulers "once or twice a month," and 18 percent of students worked with them "once or twice a week."

Frequency of use of objects such as rulers differed slightly depending on the mathematics classes in which eighth-grade students were enrolled. Students in eighth-grade mathematics were more likely to use such objects than students in algebra; that is, only 14 percent of students in eighth-grade mathematics "never or hardly ever" used such objects, which was significantly lower than the 33 percent of algebra students who were in mathematics classes in which they "never or hardly ever" used such objects.

In addition to objects such as rulers, teachers were asked about the frequency of use of manipulatives and teaching aids such as counting blocks and geometric shapes. The information about these manipulatives is presented in Table 9.11. The use of these types of manipulatives appears less common than the use of objects such as rulers.[7] In 1996, although teachers of 47 percent of fourth-grade students reported using counting blocks and geometric shapes "once or twice a month," teachers of 26 percent of students reported "never or hardly ever" using such manipulatives.

At the eighth-grade level, the use of these manipulatives appears even less common. The majority of students (54%) had teachers who reported "never or hardly ever" having their students work with counting blocks or geometric shapes.

The frequency of use of counting blocks and geometric shapes differed slightly depending on the type of mathematics class eighth-grade students were taking. Students in eighth-grade mathematics classes were reported to use these types of manipulatives more frequently than those in algebra classes. That is, the percentage of students in algebra classes (66%) whose teachers reported "never or hardly ever" working with counting blocks or geometric shapes was higher than the percentage of students in eighth-grade mathematics classes (44%) whose teachers reported this.

Table 9.11 **Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Counting Blocks and Geometric Shapes, Grades 4 and 8, 1996**

THE NATION'S REPORT CARD NAEP

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4				
All Students	5	22	47	26
Grade 8				
All Students	1	7	38	54
Students Enrolled in:				
Eighth-Grade Mathematics	1	9	46	44
Pre-Algebra	1	6	36	58
Algebra	0!	5	29	66

NOTE: Row percentages may not total 100 due to rounding.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

[7] It is possible that some of this difference could be due to the wording of the two questions. The question about rulers asked about working with "objects like rulers," while the other question only mentioned working with "counting blocks and geometric shapes" rather than, for example, "manipulatives such as counting blocks and geometric shapes."

Twelfth-grade students were asked a single question that combined the use of measuring instruments and geometric solids. Of twelfth-grade students who indicated they were currently taking a mathematics class, the majority (53%) reported "never or hardly ever" working with measuring instruments or geometric solids in their mathematics classes.

Table 9.12 — Percentage of Students by Frequency with Which They Work with Measuring Instruments or Geometric Solids, Grade 12, 1996

THE NATION'S REPORT CARD naep

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 12				
Students Taking Mathematics	7	14	26	53

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Working in small groups or with a partner

One of the pedagogical strategies recommended to foster increased learning and understanding of mathematics is the use of small-group activities. By working in small groups or with a partner, students are expected to be more actively involved in the learning process, and this is believed to increase student learning.[8] Information about the frequency with which students were reported to work with other students to solve problems is presented in Table 9.13.

Teachers of a large majority of students in both grades 4 and 8 reported that their students worked at least once a week with other students to solve mathematics problems. However, although the percentages were relatively small, seven percent of fourth-grade students and eight percent of eighth-grade students "never or hardly ever" had this opportunity.

Twelfth-grade students enrolled in mathematics reported less frequency of working with other students to solve problems than did teachers of fourth- and eighth-grade students. In 1996, about one in five twelfth-grade students reported "never or hardly ever" working this way, while less than 10 percent of fourth- and eighth-grade students had teachers who reported "never or hardly ever" having their students work with a partner or in small groups.

[8] Lacampagne, C. B. (1993). *State of the art, transforming ideas for teaching and learning mathematics.* Washington, DC: U.S. Department of Education, Office of Educational Research and Improvement; Lotan, R. A., & Benton, J. H. (1990). Finding out about complex instruction: Teaching math and science in heterogeneous classrooms. In N. Davidson (Ed). *Cooperative learning in mathematics.* New York: Addison-Wesley.

Table 9.13 **Percentage of Students by Frequency with Which They Solve Problems in Small Groups or with a Partner, Grades 4, 8, and 12, 1996***

THE NATION'S REPORT CARD NAEP

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4				
All Students	25	50	18	7
Grade 8				
All Students	27	40	26	8
Students Enrolled in:				
Eighth-Grade Mathematics	24	44	25	6
Pre-Algebra	24	39	30	8
Algebra	34	33	25	9
Grade 12				
Students Taking Mathematics	26	32	21	21

NOTE: Row percentages may not total 100 due to rounding.

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Writing in mathematics and reports/projects

Writing across the curriculum is one of the current instructional strategies being advocated by many educators to increase student learning and communication skills.[9] Writing in mathematics not only helps students improve their language arts skills, but also places an expectation on them to be able to communicate mathematical thinking and understanding to others. Over the years, NAEP assessments have presented students with increasing numbers of questions that require them to write out responses and, often, to explain their answers in writing. Students have typically found these questions more challenging than multiple-choice questions.[10] However, it is reasonable to assume that if students are not being exposed to content or processes that are assessed by NAEP, they cannot be expected to answer those questions correctly. Therefore, whether students, in fact, are writing more in their mathematics classes is of interest to interpreters of NAEP assessment results, as well as to mathematics educators more generally. Information about the frequency with which students were reportedly asked to write a few sentences or to write larger reports in mathematics classes is presented in Tables 9.14 and 9.15.

[9] Miller, L. D. (1991). Writing to learn mathematics. *Mathematics Teacher, 84*(7), 516–521.

[10] Dossey, J. A., Mullis, I. V. S., & Jones, C. O. (1993). *Can students do mathematical problem solving?* Washington, DC: National Center for Education Statistics; Hawkins, E., Stancavage, F., Mitchell, J., Goodman, M., & Lazer, S. (1998). *Learning about our world and our past: Using the tools and resources of geography and U.S. history–A report of the 1994 NAEP assessment.* Washington, DC: National Center for Education Statistics.

In 1996, the majority of fourth-grade students had teachers who indicated that students wrote a few sentences about how to solve a mathematics problem "once or twice a month" or less. However, the percentages of students who were asked to write about solving problems "almost every day" or "once or twice a week" in 1996 were higher than the percentages in 1992, and the percentage of students who "never or hardly ever" wrote about solving problems decreased from 1992 to 1996.

The frequency with which eighth-grade students wrote about solving mathematics problems in 1996 appeared to be similar to that of fourth-grade students. However, percentage changes from 1992 to 1996 for eighth-grade students, although in the same direction as the changes for fourth-grade students, were not statistically significant.

The majority (61%) of twelfth-grade students taking mathematics reported that they "never or hardly ever" wrote a few sentences about how to solve a mathematics problem.

Table 9.14 **Percentage of Students by Frequency with Which They Write a Few Sentences about How to Solve a Mathematics Problem, Grades 4, 8, and 12***

THE NATION'S REPORT CARD NAEP

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	9†	26†	36	29†
	1992	2	17	36	45
Grade 8					
All Students	1996	5	25	37	33
	1992	3	18	37	41
Students Enrolled in:					
Eighth-Grade Mathematics	1996	4	27	35	34
	1992	2	16	38	44
Pre-Algebra	1996	4	27	37	33
	1992	2	18	42	37
Algebra	1996	5	20	39	36
	1992	7	24	31	38
Grade 12					
Students Taking Mathematics	1996	7	13	18	61

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

In 1996, fewer students were reported to be writing reports or doing mathematics projects than were reported to be writing a few sentences about how to solve a mathematics problem. As shown in Table 9.15, teachers of 66 percent of fourth-grade students and 64 percent of eighth-grade students reported "never or hardly ever" asking their students to write reports or do projects in their mathematics classes. Most of the remaining students had teachers who reported assigning reports or projects "once or twice a month." Responses indicating daily or weekly frequency were quite uncommon, but this may reflect the fact that such assignments, by their nature, have longer time spans associated with them and so would be less frequently assigned. At the twelfth-grade level, 71 percent of students taking mathematics reported that they "never or hardly ever" wrote reports or did mathematics projects.

Table 9.15 ***Percentage of Students by Frequency with Which They Write Reports or Do Mathematics Projects, Grades 4, 8, and 12****

THE NATION'S REPORT CARD NAEP

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	1	4†	29†	66†
	1992	0!	1	17	82
Grade 8					
All Students	1996	0!	3	33†	64†
	1992	0!	1	21	78
Students Enrolled in:					
Eighth-Grade Mathematics	1996	0!	3	35	63
	1992	0!	0	23	76
Pre-Algebra	1996	0!	5	34	61†
	1992	0!	1	20	79
Algebra	1996	0!	2	30†	68†
	1992	0!	1	16	83
Grade 12					
Students Taking Mathematics	1996	2	4	24	71

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Changes in percentages over time appear to show increases in the frequency with which the practices of writing reports or doing mathematics projects are being implemented in mathematics classrooms. For example, the percentages of fourth-grade students in 1996 whose teachers reported having students write reports or do projects "once or twice a week" or "once or twice a month" were both higher than the percentages at those frequencies in 1992. In addition, the percentage of fourth-grade students who "never or hardly ever" wrote reports or did projects in 1996 was 66 percent, which was significantly lower than the 82 percent in 1992.

At the eighth-grade level, the percentage of students (33%) who were reported to be writing reports and doing projects "once or twice a month" in 1996 was higher than the percentage of students (21%) doing so in 1992. Additionally, in 1996, the percentage of students (64%) who "never or hardly ever" wrote reports or did projects was lower than the percentage of students (78%) in this category in 1992. When eighth-grade students were grouped by type of mathematics course, changes over time also were apparent. For students in pre-algebra and algebra, the 1996 percentages of students who "never or hardly ever" wrote reports or did projects were lower than the 1992 percentages. For algebra students, the 1996 percentage of students who wrote reports and did projects "once or twice a month" was higher than the 1992 percentage.

Communicating and connecting mathematics

To reflect what is happening in mathematics classrooms across the nation, NAEP has attempted to develop an assessment that presents students with questions that represent real-life problems and require students to use their abilities to communicate mathematically. Information in Tables 9.16 and 9.17 shows that in 1996, substantial proportions of fourth-, eighth-, and twelfth-grade students were regularly involved in discussing solutions to mathematics problems with other students. Similarly large proportions of fourth- and eighth-grade students were working or discussing mathematics problems that reflected real-life situations. On average for the different grade levels, the frequency with which students were engaged in these practices had not changed significantly from 1992 to 1996, except for eighth-grade students in the less advanced mathematics classes, where there were indications of increased frequency.

As shown in Table 9.16, in 1996, over one-third of fourth-grade students and almost half of eighth-grade students were being taught mathematics by teachers who reported that their students had discussions with other students about mathematics solutions "almost every day." Similarly, almost half of twelfth-grade students taking mathematics also reported that they discuss mathematics solutions with other students "almost every day."

Between 1992 and 1996, for most of the response categories, the frequency with which fourth- and eighth-grade students were reported to discuss mathematics solutions with other students did not change significantly. However, for students in eighth-grade mathematics, the numbers suggest an upward trend, and the percentage in 1996 who "never or hardly ever" had such discussions was lower than the 1992 percentage. Also, for students in pre-algebra, the 1996 percentage of students who had such discussions "once or twice a month" was higher than the 1992 percentage.

Table 9.16 — **Percentage of Students by Frequency with Which They Discuss Solutions to Mathematics Problems with Other Students, Grades 4, 8, and 12***

THE NATION'S REPORT CARD NAEP

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	35	37	22	6
	1992	33	39	22	6
Grade 8					
All Students	1996	49	3	37	2
	1992	43	1	32	
Students Enrolled in:					
Eighth-Grade Mathematics	1996	44	3	39	2†
	1992	37	0	33	9
Pre-Algebra	1996	52	5	37	2
	1992	44	1	33	4
Algebra	1996	54	2	32	1
	1992	58	1	29	2
Grade 12					
Students Taking Mathematics	1996	48	28	11	14

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

It is important that students are able to apply the mathematics they learn in the classroom to solve real-life problems. And solving mathematics problems that reflect real-life situations in the classroom can facilitate mathematics learning and understanding.[11] In 1996, substantial proportions of students from grades 4 and 8 were working and discussing mathematics that reflected real-life situations at least "once or twice a week." Teachers of 29 percent of fourth-grade students reported that their students did this "almost every day," while teachers of 45 percent reported that their students did this "once or twice a week."

[11] National Council of Teachers of Mathematics. (1989). op. cit.; Usiskin, Z. (1993). Lessons from the Chicago mathematics project. *Educational Leadership*, *50*, 14–18.

The percentages were similar for eighth-grade students: teachers of 27 percent reported that students worked and discussed mathematics problems that reflected real-life situations "almost every day," and teachers of 47 percent reported working and discussing these types of problems "once or twice a week."

Table 9.17 **Percentage of Students by Teachers' Reports on Frequency with Which Students Work and Discuss Mathematics Problems That Reflect Real-Life Situations, Grades 4 and 8**

THE NATION'S REPORT CARD NAEP

	Assessment Year	Frequency			
		Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	29	45	23	4
	1992	26	48	23	4
Grade 8					
All Students	1996	27	47	22	4
	1992	19	51	24	6
Students Enrolled in: Eighth-Grade Mathematics	1996	26	48	23	4
	1992	19	51	25	5
Pre-Algebra	1996	28	48	21	3
	1992	19	52	24	5
Algebra	1996	28	45	22	5
	1992	20	53	19	8

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Calculator Use

The use of calculators as an instructional strategy is being highlighted because of the emphasis placed on appropriate use of calculators in mathematics education by NCTM curriculum and evaluation standards as well as other mathematics reform efforts. The increasing accessibility to a variety of calculators suggests an expectation that students have the ability to use them appropriately in the workplace and in everyday life. Although there is concern that wider use of calculators in mathematics instruction may interfere with students' mastery of basic skills in mathematics, there is research that shows that the proper use of calculators can enhance learning at all stages.[12] Furthermore, the NAEP 1996 mathematics framework recommends the inclusion of more mathematics questions that require the use of a calculator for successful completion of those questions.[13]

In the NAEP 1996 and 1992 assessments, teachers and students were asked about their use of calculators for schoolwork and on mathematics tests. Their responses are reported in this section. We also report findings in this section regarding the extent to which students used calculators appropriately in the 1996 assessment. The basis for the latter data is as follows: the assessment was subdivided into separately timed sections, or "blocks," and students were allowed to use calculators on some of these blocks. When students were allowed to use calculators, they also were asked to indicate if, in fact, they had used a calculator for each question. Each of the questions was in turn identified as to whether the use of a calculator to solve the question was warranted. That is, each question was characterized as: (a) calculator neither required nor useful, (b) calculator not required but some students might choose to use it; and (c) calculator required. By combining these two types of information, it is possible to examine data on the extent to which students used the calculators appropriately during the assessment.

Students' access to calculators

Increasing student use of calculators in mathematics assessment is most appropriate when all students have access to calculators for instruction. In 1996, teachers of 80 percent of fourth-grade students and 80 percent of eighth-grade students reported that their students had access to school-owned calculators to do their school work, and 95 percent of twelfth-grade students taking mathematics reported having a calculator available to do mathematics schoolwork.[14] In 1996, teachers of fourth- and eighth-grade students also were asked about the frequency with which they used calculators in their mathematics classes. As the data in Table 9.18 show, teachers of eighth-grade students reported much greater frequency of calculator use than teachers of fourth-grade students.

Teachers of 68 percent of fourth-grade students reported that their students used calculators in class "once or twice a month" or less. In contrast, 76 percent of eighth-grade students had teachers who reported that they used calculators at least "once or twice a week." Comparisons of percentages of eighth-grade students by mathematics class show that the percentage of algebra students (68%) whose teachers reported use of calculators "almost every day" was higher than the percentage of eighth-grade mathematics students (48%) whose teachers reported similar usage.

In 1996, over three-fourths of twelfth-grade students taking mathematics indicated that they used calculators for class work in mathematics "almost every day," and 14 percent reported using them "once or twice a week."

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[12] Lacampagne, C. B. (1993). op. cit.

[13] National Assessment Governing Board. (1996). op.cit.

[14] The source of these data is the NAEP 1996 mathematics assessment.

Data over time appear to show increased frequency of use of calculators; this is true for all students at grades 4 and 8 as well as for eighth-grade students in each of the three different types of mathematics classes. For fourth-grade students, the 1996 percentages reported to be using calculators either "almost every day," "once or twice a week," or "once or twice a month" were all higher than the corresponding 1992 percentages. In addition, the 1996 percentage of fourth-grade students whose teachers reported "never or hardly ever" using calculators was lower than the 1992 percentage.

For eighth-grade students overall, the 1996 percentage who used calculators "almost every day" was higher than the 1992 percentage, and the 1996 percentage who used calculators "never or hardly ever" was lower than the 1992 percentage. This same pattern held true for students in eighth-grade mathematics, pre-algebra, and algebra.

Table 9.18 ***Percentage of Students by Frequency with Which Students Use Calculators in Class, Grades 4, 8, and 12**** THE NATION'S REPORT CARD NAEP

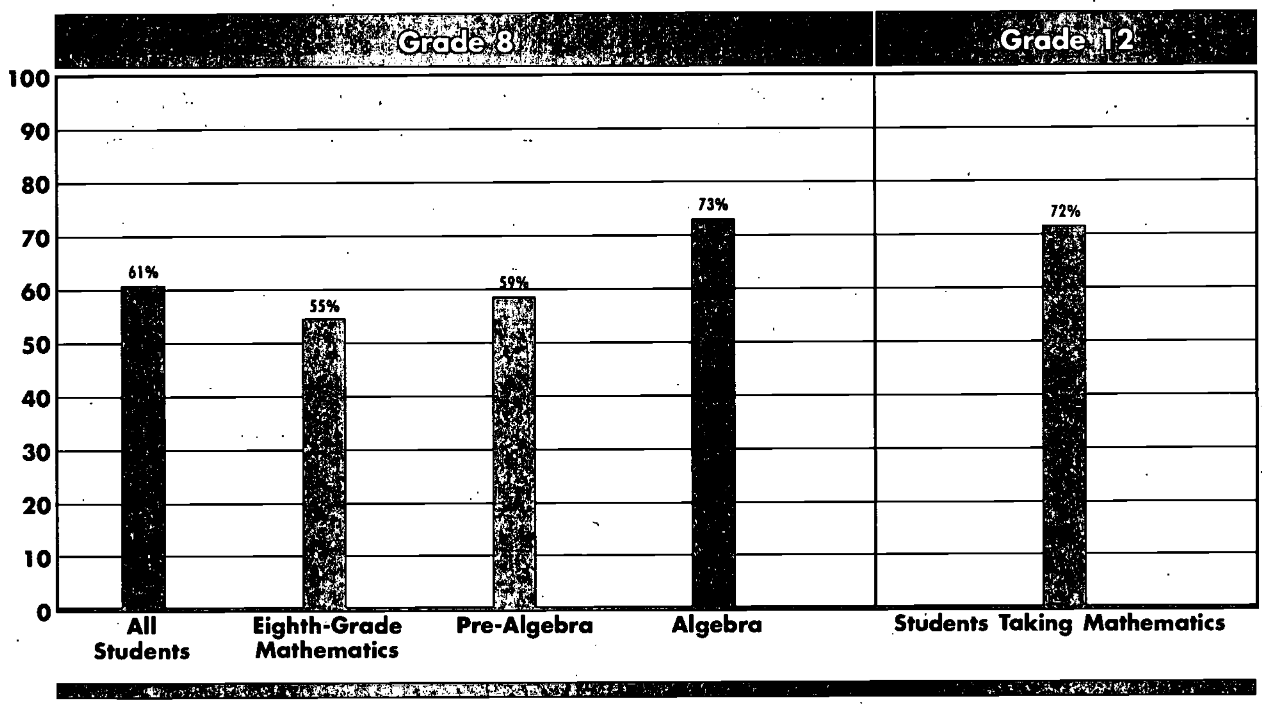
		Frequency			
	Assessment Year	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	5†	28†	42†	26†
	1992	1	15	32	51
Grade 8					
All Students	1996	55†	21	14	9†
	1992	34	22	21	24
Students Enrolled in: Eighth-Grade Mathematics	1996	48†	24	16	12†
	1992	27	25	24	24
Pre-Algebra	1996	57†	20	16	7†
	1992	36	21	18	25
Algebra	1996	68†	16	9	7†
	1992	49	18	13	19
Grade 12					
Students Taking Mathematics	1996	78	14	3	5

NOTE: Row percentages may not total 100 due to rounding.
† Significantly different from 1992.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Students in grades 8 and 12 were asked whether they use *scientific or graphing calculators* for their mathematics schoolwork, and data on their responses are presented in Figures 9.3 and 9.4. As perhaps expected, higher percentages of both eighth-grade students and twelfth-grade students taking mathematics reported using scientific calculators than reported using graphing calculators. In addition, the percentage of twelfth-grade students taking mathematics who use scientific calculators was higher than the percentage of eighth-grade students overall who do so; this also was true for the use of graphing calculators. At grade 8, the percentages of algebra students who indicated using scientific and graphing calculators were higher than the percentages of pre-algebra or eighth-grade mathematics students who reported using them.

Figure 9.3 ***Percentage of Students Who Report Using Scientific Calculators, Grades 8 and 12, 1996***

THE NATION'S REPORT CARD naep

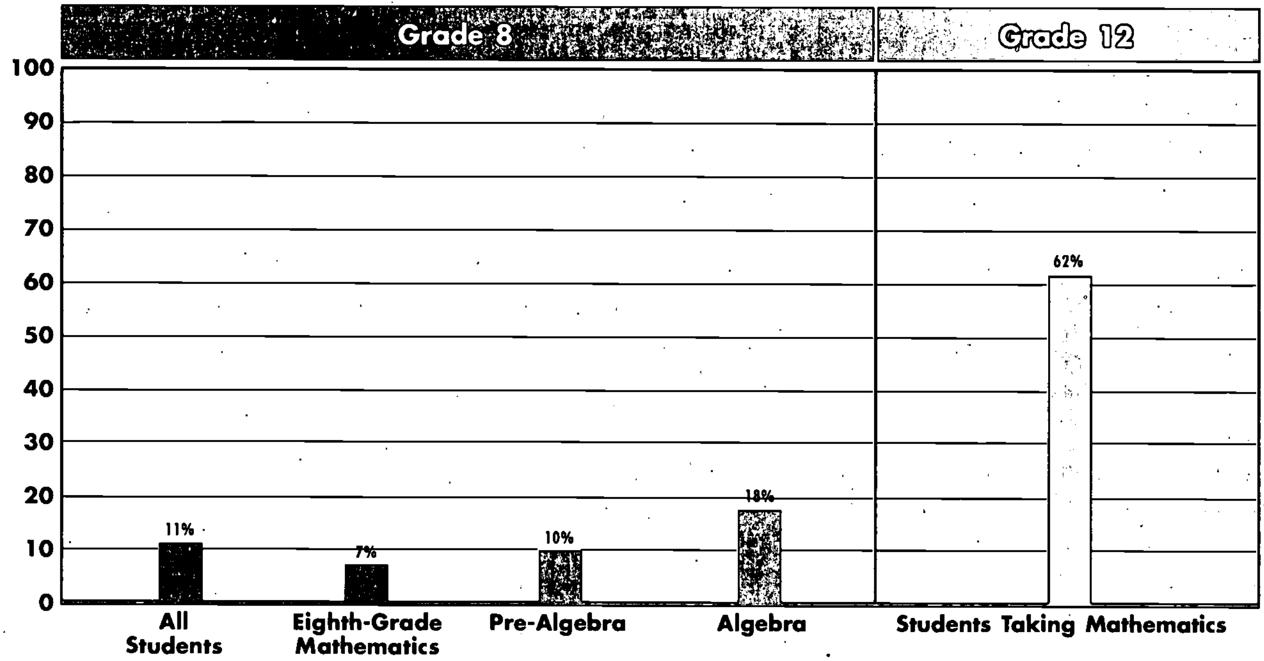


SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Figure 9.4

Percentage of Students Who Report Using Graphing Calulators, Grades 8 and 12, 1996

THE NATION'S REPORT CARD



SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

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Policies for using calculators in mathematics class

Classroom policies regarding the use of calculators can help students learn to use them appropriately and effectively.[15] In NAEP assessments, teachers of mathematics were asked if they allowed unrestricted use of calculators in their classes and also whether they allowed calculators on mathematics tests. Information based on their responses is provided in Table 9.19.

In 1996, 13 percent of fourth-grade students had teachers who reported that they allowed unrestricted use of calculators, and 10 percent of fourth-grade students had teachers who reported that they allowed calculators to be used on mathematics tests. A higher percentage of eighth-grade (47%) than fourth-grade students was allowed unrestricted use of calculators in mathematics classes, and a higher percentage (67%) also was allowed to use calculators on mathematics tests. Higher percentages of students taking algebra than students taking eighth-grade mathematics or pre-algebra had teachers who reported allowing unrestricted use of calculators and allowing calculators to be used on mathematics tests.

Between 1992 and 1996, there appears to have been an increase in the percentage of students being allowed unrestricted classroom use of calculators and use of calculators on mathematics tests. At the fourth-grade level, there were increases in both practices. This also was true for eighth-grade students and students taking eighth-grade mathematics. The differences between 1992 and 1996 for students in pre-algebra and algebra classes were significant only for the percentages being permitted to use calculators on mathematics tests.

Table 9.19 — ***Percentage of Students by Teacher Reported Uses of Calculators, Grades 4 and 8***

THE NATION'S REPORT CARD naep

	Assessment Year	Teachers Allow Unrestricted Use in Classroom	Teachers Allow Use on Mathematics Tests
Grade 4			
All Students	1996	13†	10†
	1992	5	5
Grade 8			
All Students	1996	47†	67†
	1992	30	48
Students Enrolled in:			
Eighth-Grade Mathematics	1996	42†	62†
	1992	23	43
Pre-Algebra	1996	42	66†
	1992	28	45
Algebra	1996	62	79†
	1992	50	65

† Significantly different from 1992.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

[15] National Council of Teachers of Mathematics (1991, February). *Calculators and the education of youth.* NCTM Position Statement. Reston, VA: Author.

As noted earlier, several blocks of questions in the NAEP 1996 assessment allowed students to use calculators. For these questions, students were asked to indicate if, in fact, they used a calculator in solving the problem or not. Students' responses were used in conjunction with information on whether or not the question was calculator-appropriate to categorize students into two groups: an "Appropriate calculator use" group and an "Other" group. Students in the "Appropriate calculator use" group used the calculator for at least 65 percent of the calculator-suitable questions and for no more than one of the calculator-unsuitable questions. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable questions and/or for more than one of the calculator-unsuitable questions. Information on calculator use by different instructional practices is presented in Table 9.20. Student mathematics performance information also is presented in the table.

Table 9.20

Percentage of Students by Calculator Use, Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD naep

	Calculator Use			
	Appropriate Calculator Use Group		Other Group	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4				
All Students	21	221	79	224
Unrestricted Classroom Use	19	217	81	226
Restricted Classroom Use	21	222	79	224
Allowed Use on Classroom Tests	16	224	84	225
Not Allowed Use on Classroom Tests	21	221	79	225
Grade 8				
All Students	20	285	80	269
Unrestricted Classroom Use	24	293	76	277
Restricted Classroom Use	17	278	83	265
Allowed Use on Classroom Tests	22	292	78	276
Not Allowed Use on Classroom Tests	17	271	83	261
Grade 12				
All Students	27	318	73	299
Use in Classwork:				
Almost Every Day	32	321	68	304
Once or Twice a Week	22	317	78	296
Once or Twice a Month	13	***	87	286
Never or Hardly Ever	16	293	84	285
Use on Tests or Quizzes:				
Almost Every Day	34	323	66	309
Once or Twice a Week	26	318	74	297
Once or Twice a Month	28	322	72	300
Never or Hardly Ever	16	292	84	285

NOTE: Students in the "Appropriate Calculator Use" group used the calculator for at least 65 percent of the calculator-suitable questions and for no more than one of the calculator-unsuitable questions. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable questions and/or used it for more than one of the calculator-unsuitable questions.

NOTE: Row percentages may not total 100 due to rounding.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In the NAEP 1996 mathematics assessment, 21 percent of fourth-grade students, 20 percent of eighth-grade students, and 27 percent of twelfth-grade students used the calculator appropriately, as we have defined appropriate use. The average mathematics scale score for fourth-grade students who used the calculator appropriately was similar to the average scale score of students who did not. However, at the eighth- and twelfth-grade levels, students who appropriately used calculators outperformed students who did not.

At the fourth-grade level, the appropriateness of students' use of calculators on the assessment was not related to whether they were allowed unrestricted use of calculators in the classroom or whether they were allowed to use calculators on classroom tests. On the other hand, at the eighth-grade level, students in classrooms that allowed unrestricted use of calculators were more likely than others to use calculators on NAEP appropriately. Furthermore, students in classrooms that allowed unrestricted use outperformed students in classrooms that did not allow unrestricted use. The findings with regard to use of calculators on classroom tests were similar. That is, the percentage of students who used calculators appropriately on NAEP was higher in classrooms where calculators were used on mathematics tests. In addition, students from classrooms in which calculators were used on classroom tests performed better on the 1996 mathematics NAEP than did students from classrooms in which calculators were not used on tests.

At the twelfth-grade level, it appeared that the more often students used calculators for class work and on classroom tests, the more likely they were to be appropriate users of calculators on the 1996 mathematics assessment. For example, the percentage who applied calculators appropriately on NAEP was higher among those who used calculators for class work "almost every day" than among those who used calculators for class work less often. Additionally, appropriate usage was more frequent among students who reported using calculators "once or twice a week" than among those who reported using calculators "once or twice a month." Among students who were able to use the calculator appropriately on NAEP, twelfth-grade students who reported "never or hardly ever" using calculators in the classroom performed lower on the NAEP 1996 assessment than students in other frequency-of-use groups.

In terms of frequency of use on classroom tests, the percentage of twelfth-grade students who used calculators appropriately on NAEP was higher among those who reported "almost every day" use of calculators on tests than among those who reported using calculators on tests less frequently.

Assessment Methods

The dialogue about assessment of students' academic achievement in mathematics continues to be an important one.[16] Most of the arguments focus on the inadequacies and inappropriateness of the format of assessment questions. For example, opponents of multiple-choice questions

[16] Cain, R. W., & Kenney, P. A. (1992). A joint vision for classroom assessment. *Mathematics Teacher, 85*(8), 612–615; Herman, J. L. (1997). *Large-scale assessment in support of school reform: Lessons in search of alternative measures.* Los Angeles: National Center for Research on Evaluation, Standards, and Student Testing; Glaser, R., & Silver, E. (1994). *Assessment, testing and instruction: Retrospective and prospect.* Los Angeles: National Center for Research on Evolution, Standards, and Student Testing; Romberg, T. A. (Ed.) (1995). *Reform in school mathematics and authentic assessment.* Albany, NY: State University of New York Press.

argue that these questions do not often provide students with the opportunity to show all that they know, and encourage movement to alternative methods of assessment such as performance-based assessments or project-based assessments.

In addition to arguments about the *validity* of current assessment formats, the education community has debated the *usefulness* of different forms of assessments for informing teachers and students about how to improve their teaching and learning. This section includes information from teachers' reports on the frequency with which they assess students and use different forms of assessment in mathematics.

In 1996, as shown in Table 9.21, the teachers of 64 percent of fourth-grade students reported that they gave mathematics tests "once or twice a month," and teachers of 32 percent of fourth-grade students reported that they gave mathematics tests "once or twice a week." At the eighth-grade level, the frequency of weekly tests increased somewhat, with 55 percent of students reportedly given tests "once or twice a month" and 45 percent reportedly given tests "once or twice a week."

Forty-one percent of twelfth-grade students who were taking mathematics reported that they took mathematics tests "once or twice a month," and 54 percent reported that they took mathematics tests "once or twice a week." Twelfth-grade students in mathematics reported taking mathematics tests with greater frequency than reported by teachers of eighth-grade students.

Table 9.21 **Percentage of Students by Frequency with Which Students Take Mathematics Tests, Grades 4, 8, and 12, 1996***

THE NATION'S REPORT CARD NAEP

	Frequency							
	Almost Every Day		Once or Twice a Week		Once or Twice a Month		Never or Hardly Ever	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4								
All Students	1	214	32	221	64	226	4	230
Grade 8								
All Students	1	***	45	273	55	275	0	***
Students Enrolled in:								
Eighth-Grade Mathematics	0	***	43	261	57	266	0	***
Pre-Algebra	0	***	47	272	53	271	0	***
Algebra	1	***	46	295	53	300	0	***
Grade 12								
Students Taking Mathematics	4	297	54	308	41	318	2	***

NOTE: Row percentages may not total 100 due to rounding.
*** Sample size is not sufficient to permit a reliable estimate.
* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Tables 9.22 through 9.25 provide information from fourth- and eighth-grade teachers on the types of assessments they used to assess students' progress. In 1996, teachers appeared to be responding to mathematics reform calls for less multiple-choice testing and more constructed-response testing. Teachers of nearly one-third of fourth-grade students reported that they "never or hardly ever" used multiple-choice tests to assess their students' progress in mathematics, although the modal response was to report using such tests "once or twice a month" (reported by teachers of 42 percent of grade 4 students). At the eighth-grade level, there was greater variability in the reported use of multiple-choice tests. Teachers of just over one-third of eighth-grade students indicated that they "never or hardly ever" used multiple-choice tests, 31 percent of students had teachers who indicated using multiple-choice tests "once or twice a year," and another 31 percent of students had teachers who indicated using such tests "once or twice a month."

Table 9.22 ***Percentage of Students by Teachers' Reports on the Frequency with Which They Use Multiple-Choice Tests to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	6	42	20	32
Grade 8				
All Students	3	31	31	34
Students Enrolled in:				
Eighth-Grade Mathematics	4	35	26	35
Pre-Algebra	3	30	36	32
Algebra	2	28	35	35

NOTE: Raw percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

In 1996, as shown in Table 9.23, 26 percent of fourth-grade students were taught by teachers who indicated that they used short and long written responses to assess students' progress in mathematics "once or twice a week," and 36 percent of students had teachers who reported using written responses to assess progress "once or twice a month." The pattern of percentages was only slightly different for eighth-grade students.

Table 9.23 ***Percentage of Students by Teachers' Reports on the Frequency with Which They Use Short and Long Written Responses to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD NAEP

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	26	36	19	18
Grade 8				
All Students	17	41	21	21
Students Enrolled in:				
Eighth-Grade Mathematics	17	40	20	22
Pre-Algebra	20	44	18	19
Algebra	14	35	28	23

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

As the data in Table 9.24 show, the use of individual or group projects or presentations for assessment appears less common than the use of short or long written responses. In 1996, teachers of over half of fourth-grade students indicated using projects or presentations only "once or twice a year" or less. The percentage of eighth-grade students whose teachers reported very limited use was even higher: 66 percent of eighth-grade students had teachers who indicated using such methods to assess students' progress in mathematics only "once or twice a year" or less frequently.

Table 9.24 ***Percentage of Students by Teachers' Reports on the Frequency with Which They Use Individual or Group Projects or Presentations to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996***

THE NATION'S REPORT CARD NAEP

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	16	30	31	24
Grade 8				
All Students	7	27	43	23
Students Enrolled in:				
Eighth-Grade Mathematics	7	29	44	19
Pre-Algebra	8	26	42	23
Algebra	6	25	38	30

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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The data in Table 9.25 show that, in 1996, the use of portfolios appeared to be more frequent at the fourth-grade level than at the eighth-grade level. Forty-five percent of fourth-grade students had teachers who reported using portfolios for assessing students' progress in mathematics "once or twice a month" or more often, whereas 29 percent of eighth-grade students had teachers who used portfolios at least "once or twice a month."

Table 9.25

Percentage of Students by Teachers' Reports on the Frequency with Which They Use Portfolio Collections of Each Student's Work to Assess Students' Progress in Mathematics, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	15	30	17	39
Grade 8				
All Students	10	19	21	50
Students Enrolled in:				
Eighth-Grade Mathematics	10	17	23	50
Pre-Algebra	11	19	18	51
Algebra	10	20	21	50

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Summary

This chapter provided a picture of the instructional practices students in grades 4, 8, and 12 were experiencing in 1996 in their mathematics classrooms. In terms of *disciplinary content*, the majority of fourth- and eighth-grade students were receiving mathematics instruction with "a lot" of emphasis on Number Sense, Properties, and Operations and "some" emphasis on Measurement and Geometry and Spatial Sense. Higher percentages of eighth-grade students compared with fourth-grade students had mathematics instruction with somewhat more emphasis on Data Analysis, Statistics, and Probability, and Algebra and Functions. Except for Algebra and Functions, eighth-grade students in different types of mathematics classes were not experiencing differing levels of emphasis on the different content strands. For Algebra and Functions, a higher percentage of students in algebra classes had instruction with "a lot" of emphasis on this content strand compared with the percentage of pre-algebra and eighth-grade mathematics students receiving such emphasis.

With regard to *mathematical processes*, in 1996, high percentages of fourth- and eighth-grade students had teachers who reported placing "a lot" of emphasis on learning mathematics facts and concepts, and learning skills and procedures needed to solve routine problems. A slight majority of fourth- and eighth-grade students were in classes with "a lot" of emphasis on developing reasoning ability. At both grades 4 and 8, the percentage of students with "a lot" of emphasis on how to communicate ideas in mathematics effectively was similar to the percentage of students with "some" emphasis. Only for the process of developing reasoning ability was the percentage of algebra students whose instruction had "a lot" of emphasis higher than the percentage of eighth-grade mathematics students.

Data on specific *instructional practices* in 1996 show differences by grade level and a few by eighth-grade course taking. Additionally, there were a few changes over time. For example, working with objects like rulers and other manipulatives was more common at the lower grade levels and in less advanced mathematics courses taken by eighth-grade students. The majority of fourth- and eighth-grade students work at least once a week with other students to solve mathematics problems, while twelfth-grade students taking mathematics report working with other students to solve problems less frequently.

Writing a few sentences about how to solve a mathematics problem was relatively rare among fourth- and eighth-grade students; however, the percentages of fourth-grade students who were asked to write about solving problems "almost every day" or "once or twice a week" in 1996 was higher than the percentages in 1992. On average, fewer students were writing reports or doing mathematics projects than were writing a few sentences about how to solve a mathematics problem. However, changes over time appear to show increases in the frequency with which the practices of writing reports or doing mathematics projects are being implemented in mathematics classrooms.

In 1996, substantial proportions of students — over one-third of fourth-grade students, almost half of eighth-grade students, and almost half of twelfth-grade students taking mathematics — were discussing solutions to mathematics problems with other students "almost every day." Furthermore, substantial proportions of students from grades 4 and 8 were working on and discussing mathematics that reflected real-life situations at least "once or twice a week."

As Table 9.18 indicates, in 1996, the frequency with which *calculators were used* increased with increasing grades and with more advanced mathematics courses at the eighth-grade level. The data across time show increases in the frequency of use by fourth- and eighth-grade students, regardless of mathematics course. A majority of eighth-grade students and twelfth-grade students taking mathematics reported using scientific calculators to do schoolwork. Although a majority of twelfth-grade students taking mathematics also reported using graphing calculators, only 11 percent of eighth-grade students did. At the eighth-grade level, for both scientific and graphing calculators, the percentage of algebra students who indicated using them was higher than the percentage of pre-algebra or eighth-grade mathematics students.

As Table 9.19 shows, in 1996, smaller percentages of fourth- than eighth-grade students had teachers who reported allowing unrestricted use of calculators and use of calculators on mathematics tests. Higher percentages of students taking algebra than students taking eighth-grade mathematics or pre-algebra had teachers who reported allowing unrestricted use of calculators and use of calculators on mathematics tests. Between 1992 and 1996, there appears to have been an increase in both the percentage of students allowed unrestricted use of calculators and the percentage of students allowed use of calculators on mathematics tests.

In the NAEP 1996 assessment, the majority of fourth-grade, eighth-grade, and twelfth-grade students did not use calculators appropriately (see Table 9.20). Appropriate calculator use is defined as using a calculator on questions for which a calculator is either required or useful. Although the average mathematics scale score for fourth-grade students who used the calculator appropriately was similar to the average scale score of students who did not, at the eighth- and twelfth-grade levels, students who appropriately used calculators outperformed students who did not.

In 1996, the majority of students in grades 4 and 8 were *assessed* in mathematics classes "once or twice a month," while the majority of twelfth-grade students were assessed "once or twice a week." Teachers of grades 4 and 8 reported less testing with multiple-choice questions and more with constructed-response questions. The use of individual or group projects or presentations was less common than the use of written responses. Teachers' use of portfolios was more common with fourth- than with eighth-grade students.

Chapter 10

Student Attitudes Toward Mathematics

Having the necessary content knowledge and skills is essential to being successful in using mathematics. However, some support also exists for the notion that students' attitudes and beliefs about mathematics can influence their persistence and achievement in the subject.[1] Over the years, in NAEP assessments, students have been presented with statements pertaining to their attitudes toward mathematics. To each of these statements students were asked to indicate whether they agreed with, disagreed with, or were undecided about the statement. Students' responses to the following three statements are discussed in this chapter:

- "I like mathematics";
- If I had a choice, I would not take any more mathematics"; and
- "Everyone can do well in mathematics if they try."

As shown in Table 10.1, in 1996, over half of fourth- and eighth-grade students agreed with the statement "I like mathematics." However, the percentage of fourth-grade students who agreed was significantly higher than the percentage of eighth-grade students who agreed. An examination of data by mathematics course showed that the percentage of algebra students who disagreed with the statement "I like mathematics" was significantly lower than the percentage of pre-algebra or eighth-grade mathematics students who disagreed.

Among twelfth-grade students, 50 percent indicated liking mathematics. This percentage was lower than the percentages of eighth-grade and fourth-grade students. As might be expected, the frequency of positive responses was greater among twelfth-grade students who were currently taking mathematics than among those who were not taking mathematics. Furthermore, positive responses increased in frequency among students who reported having taken more advanced mathematics coursework. For example, the percentage of twelfth-grade students who had taken geometry and agreed with the statement "I like mathematics" was 53 percent, which was higher than the 38 percent among students who had not taken geometry.

[1] Kohn, A. (1994). The truth about self-esteem. *Phi Delta Kappan, 76*(4), 272–283.

Similarly, when responses are examined by highest level algebra-through-calculus course taken, one observes that the percentage of students agreeing that they like mathematics was higher among those who had progressed to calculus or pre-calculus than among those whose highest course was second-year algebra, first-year algebra, or pre-algebra. There also was a higher rate of agreement among those whose highest algebra-through-calculus course was second-year algebra than among those whose highest course was first-year algebra or pre-algebra.

Table 10.1 **Percentages of Students by Their Response to the Statement: "I Like Mathematics," Grades 4, 8, and 12, 1996**

THE NATION'S REPORT CARD naep

	Agreement		
	Agree	Disagree	Undecided
Grade 4			
All Students	69	14	17
Grade 8			
All Students	56	23	21
Students Enrolled in:			
Eighth-Grade Mathematics	55	24	21
Pre-Algebra	54	24	22
Algebra	60	20	20
Grade 12			
All Students	50	33	17
Students Who Are:			
Enrolled in Mathematics	57	26	16
Not Enrolled in Mathematics	37	45	18
Students Who Have:			
Taken Geometry	53	30	17
Not Taken Geometry	38	44	18
Highest Algebra-Calculus Course Taken:			
Pre-Algebra	39	42	19
First-Year Algebra	39	42	20
Second-Year Algebra	51	33	16
Third-Year Algebra/Pre-Calculus	62	22	16
Calculus	74	11	14

NOTE: Row percentages may not total 100 due to rounding.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

There were no changes in the percentages agreeing or disagreeing with the statement "I like mathematics" from 1990 or 1992 to 1996 for either fourth-grade or eighth-grade students. This also was true for eighth-grade students regardless of the mathematics class they were taking. At the twelfth-grade level, however, the 1996 percentage of students who agreed with the statement "I like mathematics" (50%) was less than the 1990 percentage (54%).[2]

A second question addressed to the students was whether they agreed or disagreed with the statement, "If I had a choice, I would not study any more mathematics." In 1996, 72 percent of fourth-grade students disagreed with the statement, implying that, given a choice, they would choose to continue their studies in mathematics. The data presented in Table 10.2, suggest that, as students progress through their school careers, more students become disenchanted with mathematics and, if given a choice, would choose not to take any more mathematics. For example, the percentage of eighth-grade students who agreed that they would choose not to study any more mathematics (16%) was higher than the percentage of fourth-grade students who agreed (12%) and lower than the percentage of twelfth-grade students who agreed (31%).

Not surprisingly, students who had taken more mathematics were more likely to express interest in taking even more mathematics classes. Among eighth-grade students, the percentage of algebra students (70%) who indicated that they would choose to take more mathematics was higher than the percentage of pre-algebra (63%) or eighth-grade mathematics (63%) students who so indicated. Twelfth-grade students who were taking mathematics were more likely to indicate that they would take more mathematics (56%) than were those who were not taking mathematics (33%). Students who had taken geometry also were more likely to indicate that they would take more mathematics (50%) than those who had not taken geometry (38%). Students whose highest algebra-through-calculus class was calculus or pre-calculus were more likely to indicate that they would take more mathematics (70% and 62%, respectively) than students whose highest course was pre-algebra, first-year algebra, or second-year algebra (38%, 39%, and 46%, respectively). Students whose highest course was second-year algebra were more likely to indicate that they would take more mathematics than were students whose highest course was first-year algebra.

The 1996 percentage of fourth-grade students who disagreed with the statement (i.e., who implied they would take more mathematics; 72%) was lower than the 1992 percentage (76%).[3] This is somewhat discouraging, given current reform efforts to increase the accessibility of the mathematics curriculum as well as the amount of mathematics children take. Of course, fourth-grade students are not usually given the choice of taking or not taking mathematics. Nevertheless, this attitudinal trend does not reflect well on efforts to increase mathematics course taking. Between 1992 and 1996, there were no significant differences in the percentages of all eighth-grade students indicating agreement or disagreement. Similarly, over this time period, the opinions of eighth-grade students in the different mathematics classes did not change.

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[2] Sources of trend data are the NAEP 1996, 1992, and 1990 mathematics assessments. These data are available on the World Wide Web at: <http://nces.ed.gov/NAEP>.

[3] Sources of trend data are the NAEP 1996 and 1992 mathematics assessments. Fourth- and eighth-grade students were not asked to respond to this statement in the NAEP 1990 mathematics assessment; twelfth-grade students were not asked to respond to this statement in the NAEP 1992 and 1990 mathematics assessments.

Table 10.2 **Percentages of Students by Their Response to the Statement: "If I Had a Choice, I Would Not Study Any More Mathematics," Grades 4, 8, and 12, 1996**

THE NATION'S REPORT CARD NAEP

	Agreement		
	Agree	Disagree	Undecided
Grade 4			
All Students	12	72	16
Grade 8			
All Students	16	65	19
Students Enrolled in:			
Eighth-Grade Mathematics	16	63	21
Pre-Algebra	18	63	19
Algebra	13	70	17
Grade 12			
All Students	31	47	22
Students Who Are:			
Enrolled in Mathematics	24	56	21
Not Enrolled in Mathematics	42	33	25
Students Who Have:			
Taken Geometry	29	50	22
Not Taken Geometry	38	38	24
Highest Algebra-Calculus Course Taken:			
Pre-Algebra	40	38	22
First-Year Algebra	37	39	24
Second-Year Algebra	31	46	23
Third-Year Algebra/Pre-Calculus	20	62	18
Calculus	13	70	17

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

A potential motivator for students to persist in mathematics and to continue to work at improving their mathematics achievement is the belief that everyone can do well in mathematics. In 1996, students were asked whether they agreed with the statement, "Everyone can do well in mathematics if they try."[4] The data in Table 10.3 show that the nation's children were much more likely to agree than to disagree with the statement; 89 percent of fourth-grade students, 73 percent of eighth-grade students, and 50 percent of twelfth-grade students agreed with the statement. However, as the data also make clear, the percentage agreeing declined with grade level. Furthermore, increasing percentages of older students were unsure about how they

[4] Students were not asked to respond to this statement in the NAEP 1992 or 1990 mathematics assessments.

felt about the statement: 21 percent of twelfth-grade students, 15 percent of eighth-grade students, and 8 percent of fourth-grade students indicated that they were undecided in their opinion.

Perhaps surprisingly, an examination by course taking at the eighth-grade level shows that a higher percentage of students in eighth-grade mathematics (77%) than in algebra (67%) agreed with the statement. There were no significant differences in percentages by course taking at the twelfth-grade level.

Table 10.3 ***Percentage of Students by Their Response to the Statement: "Everyone Can Do Well in Mathematics If They Try," Grades 4, 8, and 12, 1996***

THE NATION'S REPORT CARD naep

	Agree	Disagree	Undecided
Grade 4			
All Students	89	3	8
Grade 8			
All Students	73	12	15
Students Enrolled in:			
Eighth-Grade Mathematics	77	10	13
Pre-Algebra	72	11	17
Algebra	67	15	18
Grade 12			
All Students	50	29	21
Students Who Are:			
Enrolled in Mathematics	51	28	21
Not Enrolled in Mathematics	47	31	22
Students Who Have:			
Taken Geometry	49	30	22
Not Taken Geometry	53	28	19
Highest Algebra-Calculus Course Taken:			
Pre-Algebra	54	25	20
First-Year Algebra	51	29	20
Second-Year Algebra	49	29	22
Third-Year Algebra/Pre-Calculus	47	32	22
Calculus	46	30	24

NOTE: Row percentages may not total 100 due to rounding.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Summary

This chapter included information on student attitudes and beliefs about mathematics. In particular, it reported on students' agreement with three specific statements: "I like mathematics"; "If I had a choice, I would not study any more mathematics"; and "Everyone can do well in mathematics if they try." In general, the majority of students at each grade level rendered a response that was favorable to mathematics. However, the percentage offering a favorable response declined with grade level. For example, 72 percent of fourth graders, but only 65 percent of eighth graders and 47 percent of twelfth graders disagreed with the statement "If I had a choice, I would not study any more mathematics." Liking mathematics, and a willingness to study more mathematics, were both positively associated with the students' mathematics course taking. That is, favorable responses were more frequent among eighth-grade students enrolled in algebra, twelfth-grade students enrolled in any mathematics class, and twelfth-grade students who had completed more advanced course work. These associations with course taking were not, however, apparent in students' opinions on the relationship between effort and mathematics achievement. In fact, eighth-grade students enrolled in algebra were *less* likely than those enrolled in eighth-grade mathematics to agree that "everyone can do well in mathematics if they try."

Chapter 11

Summary

This report has presented three types of information derived from the NAEP 1996 mathematics assessment: 1) information on what students know and can do in mathematics, 2) information on course-taking patterns and current classroom practices in this subject area, and 3) information on student attitudes about mathematics. The first portion of this information is derived from an analysis of student performance on the actual assessment exercises; the latter two portions draw upon the questionnaires completed by the students who participated in the assessment and their mathematics teachers.

The chapters on student work were organized around the five content strands assessed by NAEP: Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; and Algebra and Functions. Within these chapters, the discussion also highlighted students' proficiency on a number of cognitive skills that cut across the different content areas. These include conceptual understanding, procedural knowledge, and problem solving, as well as the ability to reason in mathematical situations, to communicate perception and conclusions drawn from a mathematical context, and to connect the mathematical nature of a situation with related mathematical knowledge and information gained from other disciplines or through observation.

Student Work

Trend comparisons

In 1990, NAEP gathered baseline achievement data for fourth-, eighth-, and twelfth-grade students, using a newly developed mathematics framework. Two subsequent assessments, based on the same framework and administered in 1992 and 1996, offered the opportunity to track trends in achievement. The results have been promising, indicating statistically significant improvements in overall mathematics performance at all three grade levels and in each of the five content strands. The gains were largest between 1990 and 1992, but additional gains also were evident between 1992 and 1996 on the overall composite scale and for some of the content strands. Specifically, student performance in Geometry and Spatial Sense and in Algebra and Functions improved at all grade levels; performance in Number Sense, Properties, and Operations and in Data Analysis, Statistics, and Probability improved at fourth grade; and student performance in Measurement and in Data Analysis, Statistics, and Probability improved at twelfth grade. When the achievement trends were disaggregated by race and gender, the direction of change still was generally positive for most comparisons. However, trend comparisons for some of the smaller or more diverse groups did not achieve statistical significance; as a result, one cannot say with certainty that these gains did not simply reflect chance variation due to sampling.

Subgroup comparisons

Gender. In 1996, gender differences in performance favoring males were observed for overall proficiency and three content strands at grade 4 (Number Sense, Properties, and Operations; Measurement; and Algebra and Functions) and for two content strands at grade 12 (Measurement, and Geometry and Spatial Sense).

Race/Ethnicity. In 1996, White and Asian/Pacific Islander students at grades 4 and 12 and White students at grade 8 performed better than other racial/ethnic groups overall and in each of the content strands of mathematics.[1] Hispanic students performed better than Black students in Geometry and Spatial Sense at grade 4; in Measurement and in Geometry and Spatial Sense at grade 8; and in Measurement and in Data Analysis, Statistics, and Probability at grade 12. American Indian students performed better than Black and Hispanic students in all strands at grade 4 and outperformed Black students in all content strands and Hispanic students in all strands but Geometry and Spatial Sense at grade 8. At grade 12, Asian/Pacific Islander students performed better than White students in Algebra and Functions.

Course Taking. In general, taking more mathematics courses and more advanced mathematics courses were associated with improved mathematics performance in all content strands. Eighth-grade students enrolled in algebra performed better in all content strands than eighth-grade students enrolled in pre-algebra or eighth-grade mathematics, and eighth-grade students enrolled in pre-algebra performed better than students enrolled in eighth-grade mathematics in all but one of the content strands (Geometry and Spatial Sense).

Twelfth-grade results show a similar story. Students at any given point in the algebra-through-calculus sequence performed better than students whose mathematics exposure had stopped at the next lowest course in the sequence with one exception: students whose highest course had been pre-algebra did not perform significantly better than students who had taken neither pre-algebra nor algebra. Similarly, students who had taken geometry performed better in all content strands than those who had not taken geometry.

In addition, taking more mathematics courses in high school was related to higher mathematics performance, with one exception: students who took 3–4 semesters of mathematics did not perform significantly better in Measurement than students who took only 1–2 semesters.

[1] Results for eighth-grade Asian/Pacific Islander students are not included in the body of this report. See Appendix A for details.

Content strands

Number Sense, Properties, and Operations. Students scoring in the *Basic* achievement level or above appeared to grasp many of the fundamental concepts and properties of and relationships between numbers, and displayed the skills required for manipulating numbers and completing computations. Questions assessing proportional thinking, requiring multistep solutions, or involving new concepts tended to be more difficult. Additionally, questions requiring students to solve problems and communicate their reasoning proved challenging, and often it was the communication aspect that provided the most challenge.

Measurement. Many of the measurement questions were difficult for students, particularly those requiring unit conversions, calculations of volume and circumference, and estimation.

Eighth-grade algebra students tended to perform better than other eighth-grade students, whereas eighth-grade students in pre-algebra or eighth-grade mathematics tended to perform similarly. At the twelfth-grade level, students whose highest course was second-year algebra tended to outperform those who had only reached first-year algebra, and students who reported calculus as their highest mathematics course tended to perform better than those who had taken less advanced mathematics courses.[2]

Geometry and Spatial Sense. Most of the questions in this content strand required a drawn or written response, and many were difficult for students. Questions in this content strand also relied upon students' visual-spatial skills. In several of the sample questions, a significant difference was found between the performance of male and female students. Here also, eighth-grade algebra students tended to outperform other eighth-grade students, whereas eighth-grade students in pre-algebra and those in eighth-grade mathematics performed similarly. In addition, on some of the questions, twelfth-grade students who had taken at least second-year algebra outperformed those who had not and, similarly, students who had taken at least third-year algebra or pre-calculus outperformed those who had not.

Data Analysis, Statistics, and Probability. In this content strand, students seemed to perform better on questions that asked them to make straightforward interpretations of graphs, charts, and tables as opposed to those requiring them to perform calculations with displayed data. Students had difficulty explaining why one method of reporting or displaying data was better than another, even though they may have recognized which was the better method. Questions asking students to determine chance or probability also were difficult.

Algebra and Functions. The majority of students at all grade levels appeared to understand basic algebraic representations and simple equations, as well as how to find simple patterns. The more proficient students at grades 8 and 12 were able to demonstrate knowledge of linear equations, algebraic functions, and trigonometric identities, but even those students found that questions requiring them to identify and generalize complex patterns and solve real-world problems were challenging. In general, for eighth- and twelfth-grade students, those with more advanced coursework performed better in this content strand.

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[2] Performance in Measurement and in Geometry and Spatial Sense was not analyzed with respect to whether students had taken a course in geometry because of the variability in mathematics course sequencing, the small percentage of students for whom the impact of geometry can be isolated, and the difficulty associated with identifying the effect of a particular curriculum on the performance of students in advanced mathematics. See discussion in Chapter 2.

Classroom Teaching

Course-taking patterns

In 1996, the modal group, but not the majority, of eighth-grade students, regardless of whether they were male or female, were enrolled in eighth-grade mathematics, and most of the remaining students were enrolled in pre-algebra or algebra. Trends over time show increases in the percentage of eighth-grade students taking more advanced mathematics courses.

These positive trends also were evident at the twelfth-grade level. For example, the 1996 percentage of twelfth-grade students enrolled in mathematics was significantly higher than the 1990 percentage. In addition, over time more students appear to be initially taking first-year algebra earlier in their school careers. Examination of the highest course taken by twelfth-grade students in an algebra-through-calculus sequence showed that in 1996, almost half of the twelfth-grade students indicated second-year algebra as their highest course taken. In the remaining half, fewer students indicated a course higher than second-year algebra as their highest course taken than indicated a lower level course as their highest course taken.

Classroom practices

In 1996, teachers of fourth- and eighth-grade students were asked about the emphasis they placed on different mathematics content and processes in their mathematics instruction. The majority of fourth- and eighth-grade students were receiving mathematics instruction with more emphasis on Number Sense, Properties, and Operations; Measurement; and Geometry and Spatial Sense than on Data Analysis, Statistics, and Probability; and Algebra and Functions. Perhaps as expected, more emphasis was placed on Data Analysis, Statistics, and Probability and on Algebra and Functions at the eighth-grade level than at the fourth-grade level. In all of the eighth-grade mathematics classes, students experienced similar levels of emphasis on the mathematics content strands, except for Algebra and Functions, which was more heavily emphasized in the algebra classes. Mathematics instruction at grades 4 and 8 placed more emphasis on learning mathematics facts and concepts and on learning skills and procedures needed to solve routine problems than on developing reasoning ability or on learning how to communicate ideas in mathematics effectively.

Teachers of fourth- and eighth-grade students, as well as twelfth-grade students, were asked about a variety of instructional practices that were being implemented in their mathematics classes. In 1996, results showed differences in the frequencies of implementation of some practices at different grade levels. For example, working with objects like rulers and other manipulatives was more common at the fourth-grade level and in less advanced mathematics courses taken by eighth-grade students. Similarly, the majority of fourth- and eighth-grade students worked at least once a week with other students to solve mathematics problems, while this type of structured interaction was less frequent among twelfth-grade students.

Reports on these practices over time show some significant changes. For example, while the practice of writing a few sentences about how to solve a mathematics problem was relatively rare among fourth-grade students, there have been increases in frequency over time. On average, few students at grades 4 and 8 were writing reports or doing mathematics projects, but changes over time show increases in the frequency of implementation of this practice also.

In 1996, the frequency with which calculators were used increased with increasing grade level and with mathematics content at the eighth-grade level. Furthermore, the use of calculators has increased over time. The majority of eighth- and twelfth-grade students taking mathematics reported using scientific calculators to do schoolwork. At the eighth-grade level, the use of scientific and graphing calculators was more common in the higher level mathematics courses than in the lower level courses. A majority of the twelfth-grade students taking mathematics reported using graphing calculators, although only about one in ten eighth-grade students did. In addition, the unrestricted use of calculators and the use of calculators on mathematics tests were more common among eighth-grade than fourth-grade students and among eighth-grade students in higher level mathematics courses than among those in lower level courses.

Finally, students in grade 12 reported being tested more frequently in mathematics than teachers reported that fourth- and eighth-grade students were tested. Teachers of grades 4 and 8 reported less testing with multiple-choice questions than with constructed-response questions and less use of individual or group projects than of written responses. Teachers' use of portfolios was more common with fourth- than with eighth-grade students.

Student Attitudes Toward Mathematics

The NAEP 1996 mathematics assessment probed student attitudes and beliefs about mathematics. In particular, it examined students' agreement with three specific statements: "I like mathematics"; "If I had a choice, I would not study any more mathematics"; and "Everyone can do well in mathematics if they try." In general, the majority of students at each grade level rendered a response that was favorable to mathematics. However, the percentage offering a favorable response declined with grade level.

Liking mathematics and being willing to study more mathematics were both positively associated with students' mathematics course taking. That is, favorable responses were more frequent among eighth-grade students enrolled in algebra, twelfth-grade students enrolled in any mathematics class, and twelfth-grade students who had completed more advanced coursework. These associations with course taking were not, however, apparent in students' opinions on the relationship between effort and mathematics achievement. In fact, eighth-grade students enrolled in algebra were *less* likely than those enrolled in eighth-grade mathematics to agree that "everyone can do well in mathematics if they try."

Conclusions

Performance of U.S. students in mathematics continues to improve. Since 1990, improved performance overall at all three grade levels and in each of the five content strands has been observed. When the achievement trends observed in 1996 were disaggregated by race and gender, improvement in performance continued to be observed for most groups. In addition, taking more, and more advanced, coursework in mathematics was associated with improved performance in all content strands.

Examination of student work revealed that certain types of questions were harder for some students than others. In particular, questions involving new concepts or requiring multistep solutions, written (or drawn) explanations of students' reasoning, problem solving, estimation, or the use of spatial skills were difficult for students. Straightforward questions that required simple (decontextualized) calculations were easier.

While examination of 1996 course-taking patterns revealed that more students appear to be taking more, and more advanced, mathematics courses than before, a look at classroom practices indicated that students still need more exposure to communicating effectively about mathematics. In particular, students need more practice writing about how to solve mathematical problems and discussing how to solve problems reflecting real-life situations. Activities of this sort invite students to engage more fully with the content of mathematics, can serve to increase students' ability to think analytically, and are necessary for improving performance on more difficult cognitive questions.

Appendix A

Procedures

The NAEP 1996 Mathematics Assessment

The 1996 assessment utilized the first update of the NAEP mathematics assessment framework since the release of the National Council of Teachers of Mathematics (NCTM) *Curriculum and Evaluation Standards for School Mathematics*.[1] This update sought to incorporate new knowledge about the teaching and learning of mathematics while also ensuring comparability of results across the 1990, 1992, and 1996 assessments.

The Assessment Design

Each student participating in the assessment received a booklet containing three 15-minute segments, or blocks, of cognitive questions. NAEP uses an adaptation of matrix sampling called balanced incomplete block (BIB) spiraling — a design that enables broad coverage of mathematics content while minimizing the burden for any one student. The balanced incomplete block part of the design assigns blocks of questions to booklets; each pair of blocks appears together in at least one booklet, and each pair of booklets shares at least one block of questions. The spiraling part of the method cycles the booklets for administration, so that typically only a few students in any assessment session receive the same booklet.

Of the 17 blocks in the national sample at grade 4 and 19 blocks in the national sample at grades 8 and 12, three were carried forward from the 1990 assessment, and five were carried forward from the 1992 assessment, to allow for the measurement of trends across time. The remaining blocks of questions at each grade level contained new questions that were developed for the 1996 assessment as specified by the updated framework.

Each cognitive block of math questions consisted of multiple-choice and constructed-response questions. In addition, five to seven of the blocks at each grade allowed for the use of calculators. For several blocks, students were given manipulatives (including geometric shapes, three-dimensional models, and spinners). For two of the blocks, students were given rulers at grade 4 and rulers and protractors at grades 8 and 12.

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[1] National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

Each student booklet also included three sets of student background questions. The first set included general background questions such as questions about the student's race or ethnicity, mother's and father's level of education, number and type of reading materials in the home, amount of time spent on homework, and student's academic expectations. The second set was directed specifically at the student's mathematics background and included questions about mathematics instructional activities, mathematics courses taken, use of specialized resources such as calculators in mathematics classes, and views on the utility and value of mathematics. These first two sets of background questions preceded the cognitive blocks in the assessment. The third set of questions followed the cognitive question blocks and contained five questions about students' motivation to do well on the assessment, their perception of the difficulty of the assessment, and their familiarity with the types of cognitive questions included. Students were given 5 minutes to complete each set of background questions, with the exception of fourth graders, who were given more time on the initial set of general background questions to allow those questions to be read aloud to them.

In addition to the student assessment booklets, two other instruments relevant to this report provided data relating to the assessment — a mathematics teacher questionnaire and a school characteristics and policy questionnaire.

The teacher questionnaires were administered to the mathematics teachers of each of the fourth- and eighth-grade students participating in the assessment. Because twelfth-grade students were not necessarily enrolled in mathematics, no questionnaires were administered to twelfth-grade mathematics teachers. The teacher questionnaire consisted of three sections and took approximately 20 minutes to complete. The first section focused on the teacher's general background and experience; the second section focused on the teacher's background related to mathematics; and the third section focused on classroom information about mathematics instruction. Because the sampling for the teacher questionnaire was based on participating students, the responses to the mathematics teacher questionnaire do not necessarily represent all fourth- or eighth-grade mathematics teachers in the nation or in a state. Rather, they represent teachers of the representative sample of students assessed. It is important to note that in this report, as in all NAEP reports, the student is always the unit of analysis, even when information from the teacher or school questionnaire is being reported. Using the student as the unit of analysis makes it possible to describe the educational context experienced by representative samples of students. Although this approach may provide a different perspective from that obtained by simply collecting information from teachers or schools, it is consistent with NAEP's goals of providing information about the educational context and performance of students.

The school characteristics and policy questionnaires were given to the principals or other administrators in each participating school and took about 20 minutes to complete. The questions asked about the principal's background and experience, school policies, programs, facilities, and the demographic composition and background of the students and teachers in that school.

National Samples

The national results presented in this report are based on nationally representative probability samples of fourth-, eighth-, and twelfth-grade students. The samples were selected by Westat using a complex multistage sampling design that involved sampling students from selected schools within selected geographic areas across the country. For a more detailed description of the sampling procedures, see the *NAEP 1996 Mathematics Report Card for the Nation and the States.*[2]

Students with Disabilities (SD) and Limited English Proficient (LEP) Students

It is NAEP's intent to assess all selected students. However, some students with disabilities or limited English proficiency are not capable of taking the assessment, or not capable of taking it under standard conditions. NAEP provides written guidelines in an effort to standardize local school decisions about which students will participate in the assessment and under what conditions.

The 1996 assessment marked a transition in NAEP guidelines for the inclusion of students with disabilities or limited English proficiency. New guidelines were developed in an effort to 1) increase inclusion rates, 2) be applied more consistently across states and jurisdictions, and 3) ensure that inclusion decisions would be related to the subject matter instruction given to the student rather than less relevant considerations. Under the new guidelines, students with disabilities should participate unless:

- the student's Individualized Education Plan (IEP) team (or equivalent) determined that the student cannot participate in assessments such as NAEP; *or*
- the student's cognitive functioning is so severely impaired that he or she cannot participate; *or*
- the student's IEP requires an accommodation or adaptation that NAEP and the school do not provide, and the student cannot demonstrate his or her knowledge without that accommodation.

The guidelines indicate that students with limited English proficiency should participate unless:

- the student has received language arts instruction primarily in English for less than three school years including the current year; *and*
- the student cannot demonstrate his or her knowledge of the subject being assessed in English even with an accommodation permitted by NAEP.

In all cases, schools are encouraged to include the student in instances of doubt.

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[2] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). *NAEP 1996 mathematics report card for the nation and the states*. Washington, DC: National Center for Education Statistics.

In order to determine the impact of the change in criteria on the measurement of trends, the 1996 national mathematics sample was subdivided into three parts: S1, S2, and S3. Schools in S1 received the old inclusion guidelines, and schools in S2 and S3 received the new guidelines. In addition, schools in S3 were instructed to offer a series of specified accommodations to students who normally receive such accommodations for testing.

Initial analyses of the 1996 results demonstrated that the change in written inclusion guidelines did not adversely impact the cross-sectional or trend estimation of achievement. Therefore the S1 and S2 samples were combined for reporting. Data from students in S3, however, were held aside for further analysis of the impact of accommodations on the measurement of trend.[3]

Data Collection and Scoring

As with all NAEP assessments, data collection was conducted by trained field staff. For the national assessment, this was accomplished by Westat staff. Materials collected as part of the 1996 assessment were shipped to National Computer Systems, where trained staff evaluated the responses to the constructed-response questions using scoring rubrics or guides prepared by the Educational Testing Service (ETS).

Each constructed-response question had a unique scoring rubric that defined the criteria used to evaluate students' responses. The extended constructed-response questions were evaluated with four- or five-level rubrics (e.g., no evidence of understanding, evidence of minimal understanding, evidence of partial understanding, and evidence of satisfactory or extended understanding), while the short constructed-response questions first appearing in the 1996 assessment were rated according to three-level rubrics that permitted partial credit (e.g., evidence of little or no understanding, evidence of partial understanding, and evidence of full understanding). Other short constructed-response questions that appeared in previous assessments were scored as either correct or incorrect. For more information, see *The NAEP 1996 Technical Report*.[4]

Student responses for constructed responses also could have been scored as "off task," which meant that the students provided a response that was deemed unrelated in content to the question asked. A simple example of this type of response is, "I don't like this test." Responses of this sort could not be rated. By contrast, responses scored as incorrect were valid attempts to answer the question that were simply wrong.

Scoring of the NAEP 1996 assessment included rescoring to monitor interrater reliability and trend reliability. In other words, scoring reliability was calculated within year (1996) and across years (1990, 1992, and 1996). The overall within-year percentages of agreement for the 1996 national reliability samples were 96 percent at grade 4, 96 percent at grade 8, and 96 percent at grade 12. For information on trend reliability, see the *NAEP 1996 Mathematics Report Card for the Nation and the States*.[5]

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[3] For further details, see Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.; and Mazzeo, J., Carlson, J., Voekl, K., & Lutkus, A. (forthcoming). *Increasing the participation of students with disabilities and limited English proficient students in the National Assessment of Educational Progress: A special report on 1996 research activities.* Washington, DC: U.S. Department of Education.

[4] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). *The NAEP 1996 technical report.* Washington, DC: National Center for Education Statistics.

[5] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.

Data Analysis and IRT Scaling

Subsequent to the professional scoring, all information was transcribed to the NAEP database at ETS. Each processing activity was conducted with rigorous quality control. After the assessment information had been compiled in the database, the data were weighted according to the population structure. The weighting for the national and state samples reflected the probability of selection for each student as a result of the sampling design, adjusted for nonresponse. Through stratification, the weighting assured that the representation of certain subpopulations corresponded to figures from the U.S. Census and the Current Population Survey.[6]

Analyses then were conducted to determine the percentages of students who gave various responses to each cognitive and background question. Item response theory (IRT) was used to estimate average scale score proficiency for the nation, various subgroups of interest within the nation, and for the states. IRT models the probability of answering a question correctly as a mathematical function of proficiency or skill. The main purpose of IRT analysis is to provide a common scale on which performance can be compared across groups, such as those defined by grades and subgroups (e.g., gender or race/ethnicity). Because of the BIB spiraling design used by NAEP, students do not receive enough cognitive questions about a specific content area to provide reliable information about individual performance. Traditional test scores for individual students, even those based on IRT, would lead to misleading estimates of population characteristics, such as subgroup means and percentages of students at or above a certain proficiency level. Instead, NAEP constructs sets of plausible values designed to represent the distribution of proficiency in the population. A plausible value for an individual is not a scale score for that individual but may be regarded as a representative value from the distribution of potential scale scores for all students in the population with similar characteristics and identical patterns of item (question) responses. Statistics describing performance on the NAEP proficiency scale are based on these plausible values. They estimate values that would have been obtained had individual proficiencies been observed — that is, had each student responded to a sufficient number of cognitive questions so that proficiency could be precisely estimated.[7]

A score scale ranging from 0 to 500 was created to report performance for each content strand (Number Sense, Properties, and Operations; Measurement; Geometry and Spatial Sense; Data Analysis, Statistics, and Probability; Algebra and Functions). The scales summarize examinee performance across all three question types used in the assessment (multiple-choice, short constructed-response, and extended constructed-response). Each content area scale was based on the distribution of student performance across all three grades assessed in the 1996 national assessment (grades 4, 8, and 12) and had a mean of 250 and a standard deviation of 50. A composite score was created as an overall measure of students' mathematics proficiency. The composite scale was a weighted average of the five content-strand scales, where the weight for each content strand was proportional to the relative importance assigned to the content strands in the specifications developed by the Mathematics Objectives Panel.

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6 For additional information about the use of weighting procedures in NAEP, see Johnson, E. G. (December 1989). *Journal of Education Statistics*, *14*(4), pp. 303–334.

7 For theoretical justification of the procedures employed, see Mislevy, R. J. (1988). Randomization-based inferences about latent variables from complex samples. *Psychometrika*, *56*(2), pp. 177–196.

The NAEP proficiency scales make it possible to examine relationships between students' performance and a variety of background factors measured by NAEP. The fact that a relationship exists between achievement and another variable, however, does not reveal the underlying cause of the relationship, which may be influenced by a number of other variables. Similarly, the assessments do not capture the influence of unmeasured variables. The results are most useful when they are considered in combination with other knowledge about the student population and the educational system, such as trends in instruction, changes in the school-age population, and societal demands and expectations.

Most of the data analyses were conducted by ETS. However, some of the results presented in this report are based on additional analyses conducted by the American Institutes for Research using data sets provided by ETS.

More detailed information about data analysis and item response theory is presented in *The NAEP 1996 Technical Report.*[8]

Reporting Groups

In this report, some of the results are provided for subgroups of students with shared characteristics: gender, race/ethnicity, course-taking patterns. Based on criteria described later in this appendix, results are reported for subpopulations only when sufficient numbers of students and adequate school representation are present. The minimum requirement is at least 62 students in a particular subgroup from at least five primary sampling units (PSUs).[9] Regardless of whether the subgroup was reported separately, the data for all students were included in computing overall results. Definitions of the subpopulations referred to in this report are presented below.

Gender

Results are reported separately for males and females.

Race/Ethnicity

The race/ethnicity variable is derived from two questions asked of students and school records, and it is used for race/ethnicity subgroup comparisons. Two questions from the set of general student background questions were used to determine race/ethnicity:

If you are Hispanic, what is your background?

- I am not Hispanic
- Mexican, Mexican American, or Chicano
- Puerto Rican
- Cuban
- Other Spanish or Hispanic background

[8] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

[9] For the national assessment, a PSU is a geographic region (a county, a group of counties, or metropolitan statistical areas).

Students who responded to this question by selecting "Mexican, Mexican American, or Chicano," "Puerto Rican," "Cuban," or "Other Spanish or Hispanic background" were considered Hispanic. Students who selected "I am not Hispanic," did not respond to the question, or provided information that was illegible or could not be classified were further classified based on their responses to the following question:

Which best describes you?

- White (not Hispanic)
- Black (not Hispanic)
- Hispanic ("Hispanic" means someone who is from a Mexican, Mexican American, Chicano, Puerto Rican, Cuban, or other Spanish or Hispanic background.)
- Asian or Pacific Islander ("Asian or Pacific Islander" means someone who is from a Chinese, Japanese, Korean, Filipino, Vietnamese, or other Asian or Pacific Islander background.)
- American Indian or Alaskan Native ("American Indian or Alaskan Native" means someone who is from one of the American Indian tribes or one of the original people of Alaska.)
- Other (specify) ______________________________

Students' race/ethnicity was then assigned on the basis of their responses. For students who selected "Other" and provided illegible information or information that could not be classified or who did not respond at all, race/ethnicity was assigned as determined by school records.

Race/ethnicity could not be determined for students who did not respond to either of the demographic questions and whose schools did not provide information about race/ethnicity.

Details of how race/ethnicity classifications were derived is presented so that readers can determine how useful the results are for their particular purposes. Also, some students indicated that they were from a Hispanic background (e.g., Puerto Rican or Cuban) and that a racial/ethnic category other than Hispanic best described them. These students were classified as Hispanic based on the rules described above. Furthermore, the information from the schools did not always correspond to how students described themselves. Therefore, the racial/ethnic results presented in this report attempt to provide a clear picture based on several sources of information.

As noted in Chapter 2, scale score and achievement level results for eighth-grade Asian/Pacific Islander students are not included in the main body of this report. The decision not to publish these results is discussed in detail at the end of this appendix.

Eighth-grade course taking

Eighth-grade students responded to a question about what mathematics course they were taking. Students were provided with seven response options that included the following:

- I am not taking mathematics this year
- Eighth-grade mathematics
- Pre-algebra
- Algebra
- Integrated or sequential mathematics
- Applied mathematics (technical preparation)
- Other mathematics class

The course-taking grouping variable used in this report is based on the subset of students who responded that they were taking eighth-grade mathematics, pre-algebra, or algebra. Students who marked some other response are not included in the subpopulation analysis.

Twelfth-grade highest algebra-calculus course taken

At the twelfth-grade level, the course-taking subpopulations are based on the highest level mathematics course students reported having taken in an algebra-through-calculus sequence. The grouping of students was based on students' reports on the amount of time they took the following mathematics courses:

- Introduction to algebra or pre-algebra
- First-year algebra
- Second-year algebra
- Pre-calculus, third-year algebra, elementary functions, or analysis
- Calculus

Students' responses were edited for consistency with the standard course-taking sequence. That is, the student was not credited as having taken a certain course unless his or her responses also indicated completion of the course prerequisites.

The twelfth-grade grouping variable has six categories:

1. Not Taken Pre-Algebra: These are students who had less than a year of introduction to algebra or pre-algebra.
2. Pre-Algebra: These are students who had a year or more of introduction to algebra or pre-algebra, but not first-year algebra.
3. First-Year Algebra: These are students who had a year or more of first-year algebra, but not second-year algebra.
4. Second-Year Algebra: These are students who had a year or more of second-year algebra, but not pre-calculus.

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5. Pre-Calculus: These are students who had a year or more of pre-calculus, but not calculus.

6. Calculus: These are students who had a year or more of calculus.

Guidelines for Analysis and Reporting

This report describes students', teachers', and principals' responses to background questions as well as mathematics performance for fourth-, eighth-, and twelfth-grade students. The report also compares the performance results for various groups of students within these populations (e.g., subgroups formed of those who responded to a specific background question in a particular way or by individual course-taking groups as described above). However, it does not include an analysis of the relationships among combinations of these subpopulations or background questions.

Estimating variability

The statistics presented in this report are estimates of group and subgroup performance based on samples of students, and they therefore differ from statistics that could be calculated if every student in the nation answered every question. The degree of uncertainty associated with these sample-based estimates should, therefore, be taken into account. Two components of uncertainty are accounted for in the variability statistics based on student ability: 1) the uncertainty due to sampling only a relatively small number of students, and 2) the uncertainty due to sampling only a relatively small number of cognitive questions per student. The first component alone accounts for the variability associated with the estimated percentages of students who had certain background characteristics or who answered a certain cognitive question correctly.

Because NAEP uses complex sampling procedures, conventional formulas for estimating sampling variability that assume simple random sampling are inappropriate. NAEP uses a jackknife replication procedure to estimate standard errors. The jackknife standard error provides a reasonable measure of uncertainty for any student information that can be observed without error. However, because each student typically responds to only a few questions within any content strand, the scale score for any single student would be imprecise. In this case, plausible values technology can be used to describe the performance of groups or subgroups of students, but the underlying imprecision involved in this step adds another component of variability to statistics based on NAEP scale scores.[10]

Typically, when the standard error is based on a small number of students or when the group of students is enrolled in a small number of schools, the amount of uncertainty associated with the standard error may be quite large. Throughout this report, estimates of standard errors subject to a large degree of uncertainty are designated by a "!" symbol. In such cases, the standard errors — and any confidence intervals or significance tests involving these standard errors — should be interpreted cautiously. Additional details concerning procedures for identifying such standard errors are discussed in *The NAEP 1996 Technical Report.*[11]

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[10] For more details, see Johnson, E. G. & Rust, K. F. (1992). Population inferences and variance estimation for NAEP data. *Journal of Educational Statistics, 17*(2), pp. 175–190.

[11] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

The reader is reminded that, like findings from all surveys, NAEP results are subject to other kinds of error, including the effects of imperfect adjustments for student and school nonresponse and unknown effects associated with the particular instrumentation and data collection methods. Nonsampling errors can be attributed to a number of sources: inability to obtain complete information about all selected schools in the sample (some students or schools refused to participate, or students participated but answered only certain questions); ambiguous definitions; differences in interpreting questions; inability or unwillingness to give correct information; mistakes in recording, coding, or scoring data; and other errors in collecting, processing, sampling, and estimating missing data. The extent of nonsampling error is difficult to estimate, and because of their nature, the impact of such errors cannot be reflected in the data-based estimates of uncertainty provided in NAEP reports.

Drawing inferences from the results

As noted, the percentages of students and average scale scores used in reporting NAEP results are based on samples rather than on the entire population of fourth-, eighth-, or twelfth-graders in the nation or a jurisdiction. Consequently, the numbers reported are estimates and are subject to a measure of uncertainty, reflected in the standard error of the estimate. When the percentages or average scale scores of certain groups are compared, the standard error should be taken into account, and observed similarities or differences should not be relied on solely. Therefore, the comparisons discussed in this report are based on statistical tests that consider the standard errors of those statistics as well as the magnitude of the difference among the averages or percentages.

The results from the sample, taking into account the uncertainty associated with all samples, are used to make inferences about the population. Using confidence intervals based on the standard errors provides a way to make inferences about the population averages and percentages in a manner that reflects the uncertainty associated with the sample estimates. An estimated sample average scale score ± 2 standard errors approximates a 95 percent confidence interval for the corresponding population quantity. This statement means that one can conclude with approximately a 5 percent level of significance that the average performance of the entire population of interest (e.g., all fourth-grade students in public schools in a jurisdiction) is within ± 2 standard errors of the sample average.

As an example, suppose that the average mathematics scale score of the students in a particular group was 256, with a standard error of 1.2. A 95 percent confidence interval for the population quantity would be as follows:

Average ± 2 standard errors
$256 \pm 2 \times 1.2$
256 ± 2.4
253.6, 258.4

Thus, one can conclude with a 5 percent level of confidence that the average scale score for the entire population of students in that group is between 253.6 and 258.4.

Similar confidence intervals can be constructed for percentages, if the percentages are not extremely large or extremely small. For extreme percentages, confidence intervals constructed in the above manner may not be appropriate, and accurate confidence intervals can be constructed only by using procedures that are quite complicated.

Extreme percentages, defined by both the magnitude of the percentage and the size of the sample from which it was derived, should be interpreted with caution. *The NAEP 1996 Technical Report* contains a more complete discussion of extreme percentages.[12]

Analyzing group differences in averages and percentages

Statistical tests are used to determine whether the evidence, based on the data from the groups in the sample, is strong enough to conclude that the averages or percentages are actually different for those groups in the population. If the evidence is strong (i.e., the difference is statistically significant), the report describes the group averages or percentages as being different (e.g., one group performed higher than or lower than another group), regardless of whether the sample averages or percentages appear to be approximately the same. If the evidence is not sufficiently strong (i.e., the difference is not statistically significant), the averages or percentages are described as being not significantly different, regardless of whether the sample averages or percentages appear to be approximately the same or widely discrepant.

The reader is cautioned to rely on the results of the statistical tests rather than on the apparent magnitude of the difference between sample averages or percentages when determining whether the sample differences are likely to represent actual differences among the groups in the population.

To determine whether a real difference exists between the average scale scores (or percentages of a certain attribute) for two groups in the population, one needs to obtain an estimate of the degree of uncertainty associated with the difference between the averages (or percentages) of these groups for the sample. This estimate of the degree of uncertainty, called the standard error of the difference between the groups, is obtained by taking the square of each group's standard error, summing the squared standard errors, and taking the square root of that sum.

$$\text{Standard Error of the Difference} = SE_{A-B} = \sqrt{(SE_A^2 + SE_B^2)}$$

Similar to how the standard error for an individual group average or percentage is used, the standard error of the difference can be used to help determine whether differences among groups in the population are real. The difference between the averages or percentages of the two groups ± 2 standard errors of the difference represents an approximate 95 percent confidence interval. If the resulting interval includes zero, there is insufficient evidence to claim that a real difference between the groups is statistically significant (different) at the five percent level. In this report, differences among groups that involve poorly defined variability estimates or extreme percentages are not discussed.

[12] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

As an example, to determine whether the average mathematics scale score of Group A is higher than that of Group B, suppose that the sample estimates of the average scale score and standard errors were as follows:

Group	Average Scale Score	Standard Error
A	218	0.9
B	216	1.1

The difference between the estimates of the average scale scores of Groups A and B is two points (218–216). The standard error of this difference is:

$$\sqrt{(0.9^2 + 1.1^2)} = 1.4$$

Thus, an approximate 95 percent confidence interval for this difference is:

$$\begin{gathered}\text{Difference} \pm 2 \text{ standard errors of the difference}\\ 2 \pm 2 \times 1.4\\ 2 \pm 2.8\\ -0.8, 4.8\end{gathered}$$

The value zero is within the confidence interval; therefore, there is insufficient evidence to claim that Group A outperformed Group B.

The procedures described in this section and the certainty ascribed to intervals (e.g., a 95 percent confidence interval) are based on statistical theory that assumes that only one confidence interval or test of statistical significance is being performed. However, in this report, many different groups are being compared (i.e., multiple sets of confidence intervals are being analyzed). In sets of confidence intervals, statistical theory indicates that the certainty associated with the entire set of intervals is less than that attributable to each individual comparison from the set. To hold the significance level for the set of comparisons at a particular level (e.g., 0.05), adjustments (called multiple comparison procedures) must be made to the methods described in the previous section. One such procedure, the Bonferroni method, was used in the analyses described in this report to determine confidence intervals for the differences among groups when sets of comparisons were considered.[13] Thus, the confidence intervals for the sets of comparisons in the text are more conservative than those described on the previous pages.

Most of the multiple comparisons in this report pertain to relatively small sets or families of comparisons. For example, for discussions concerning comparisons of parents' level of education, six comparisons were conducted — all pairs of the four parental education levels. In these situations, Bonferonni procedures were appropriate. A detailed description of the Bonferroni procedure appears in *The NAEP 1996 Technical Report*.[14]

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[13] Miller, R. G. (1996). *Simultaneous statistical inference*. New York: Wiley.

[14] Allen, N. L., Carlson, J. E., & Zelenak, C. A. (1999). op. cit.

Revisions to the NAEP 1990 and 1992 Mathematics Findings

After the NAEP 1994 assessment was conducted, a technical problem was discovered in the procedures used to develop the NAEP mathematics scale used to report the 1992 mathematics assessment. This error affected the mathematics scale scores reported in 1992. The technical error has been corrected, and the revised national and state scale score results for 1992 are presented in the NAEP 1996 mathematics reports. The technical problem is described in greater detail in *The NAEP 1996 Technical Report*.[15] A brief summary of the problem is presented in the *NAEP 1996 Mathematics Report Card for the Nation and the States*.[16]

Discussion of the Grade 8 Asian/Pacific Islander Sample

As noted earlier, scale score and achievement level results for eighth grade Asian/Pacific Islander students are not included in the main body of this report. The decision to exclude these results was made following a thorough investigation by the current NAEP grantees (Westat and ETS)[17,18] into the quality and credibility of these results, as well as an independent review by a committee of statisticians from the National Institute of Statistical Sciences (NISS).[19] Collateral results from the grade 8 state assessment program in mathematics suggested that the 1996 national results may substantially underestimate actual achievement of the Asian/Pacific Islander group. Because of its potential to misinform, NCES decided to omit the national grade 8 Asian/Pacific Islander results from the body of the report. The results are, however, included in this appendix along with a description of the findings that led to this decision.

[15] Ibid.

[16] Reese, C. M., Miller, K. E., Mazzeo, J., & Dossey, J. A. (1997). op. cit.

[17] Carlson, J., & Williams, P. (1996, October 29) ETS/NAEP Technical Memorandum on 1996 Mathematics Grade 8 results for Asian/Pacific Island Subpopulation.

[18] Rust, K. (1996, November 1) Westat Memorandum to Gary Phillips on 1996 Mathematics Grade 8 Results for Asian and Pacific Islander Students.

[19] Letter from Jerome Sacks to Gary Phillips, dated November 21, 1996.

Concerns about the accuracy of the grade 8 Asian/Pacific Islander results were initially noted during routine quality control of the NAEP 1996 mathematics assessment results. Despite statistically significant gains from 1992 to 1996 in average scale scores for the nation as a whole at all three grade levels, a large apparent decline in average scores was observed for the grade 8 Asian/Pacific Islander subgroup. Table A.1 contains average mathematics scale score estimates, and their standard errors, for the Asian/Pacific Islander subgroup for the 1990, 1992, and 1996 assessment years. From 1992 to 1996, the estimated decline in average scores for this subgroup was approximately 14 scale score points (about .4 within-grade standard deviation units) on the NAEP 500-point scale. Despite the large magnitude of this apparent decline, it is not statistically significant at the .05 level, after controlling for multiple comparisons.

Table A.1 ***Average Mathematics Scale Scores for the Grade 8 Asian/Pacific Islander Subgroup***

THE NATION'S REPORT CARD naep

	1990		1992		1996	
	Percentage	Average Scale Score	Percentage	Average Scale Score	Percentage	Average Scale Score
All Students	100	263 (1.3)	100	268 (0.9)*	100	272 (1.1)*†
Students Who Indicated Their Race/Ethnicity as...						
Asian/Pacific Islander	2 (0.5)!	279 (4.8)!	3 (0.2)	288 (5.4)	3 (0.2)	274 (3.9)

The standard errors of the estimated percentages and average scale scores appear in parentheses.
* Indicates a significant difference from 1990.
† Indicates a significant difference from 1992.
! Statistical tests involving this value should be interpreted with caution. Standard error estimates may not be accurately determined and/or the sampling distribution of the statistic does not match statistical test assumptions (see Appendix A).
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

The data from the NAEP state assessment program in mathematics provided an independent data source to aid in evaluating the accuracy of the national grade 8 NAEP results for Asian/Pacific Islander students as well as for other subgroups. Forty states and the District of Columbia participated in the state assessment. Results based on the combined data from these jurisdictions are quite stable in that they are based on a sample of approximately 4,000 schools and over 100,000 students. Because of the voluntary nature of the state assessment program, these aggregated state results are not nationally representative. They can, however, be compared to restricted national results, calculated using public-school data from only those states participating in the state assessment, to obtain valuable insight into the quality of the national estimates for the grade 8 race/ethnicity subgroups.

Table A.2 contains restricted national results. Results are presented separately for four of the race/ethnicity subgroups: White, Black, Hispanic, and Asian/Pacific Islander. Aggregated state results are also presented for these same four subgroups. For three of the four subgroups, the difference between the restricted national estimates and aggregated state estimates are quite small. However, for the Asian/Pacific Islander subgroup, the difference between the two estimates, though again within reasonable bounds of sampling variability, is of considerably greater magnitude and the restricted national estimates are substantially lower than those obtained from the aggregated state data. These results suggest that the national grade 8 Asian/Pacific Islander results may substantially underestimate the performance of this subgroup. NCES was concerned that publishing the national results in the absence of the kind of discussion included in this appendix was potentially misinforming. Hence, NCES made the decision to omit the results from the body of the report and to include them in this appendix.

Table A.2 ***Average Mathematics Scale Scores by Race/Ethnicity for Restricted National and Aggregated State Samples***

THE NATION'S REPORT CARD NAEP

	Restricted National Sample	Aggregated State Sample	Difference
Grade 8			
Students Who indicated Their Race/Ethnicity as...			
White	280.7	280.0	0.7
Black	242.8	242.3	0.5
Hispanic	250.4	250.3	0.1
Asian/Pacific Islander	272.0	281.7	−9.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

It is important to note that all NAEP results are estimates and are subject to some degree of sampling variability. If different samples of schools or students had been obtained, results for some subgroups would be higher than reported here and some would be lower. In most subgroups, particularly large subgroups or subgroups for which special sampling procedures are employed, estimates of performance are likely to remain similar from one sample to another. However, the national population of Asian/Pacific Islander students is small (about 3 percent of the national population), heterogeneous with respect to academic achievement, and highly clustered in certain locations and schools — factors that are associated with large sampling variability in survey results and reflected in the large standard errors associated with performance estimates for this subgroup. Furthermore, the sampling plan for the national assessment does not include explicit stratification procedures designed to mitigate these factors. It was the judgment of all three organizations (ETS, Westat, and NISS) that investigated these results that the occurrence of this large, but statistically nonsignificant, change in the grade 8 Asian/Pacific Islander results was a consequence of these three factors: (1) the heterogeneous nature of the Asian/Pacific Islander population, (2) the current NAEP sampling design, and (3) the sample sizes that were assessed.

NCES, working with its current NAEP contractors and other advisory groups, will continue to investigate cost-effective ways of improving the accuracy and stability of NAEP results beginning with the 1998 assessment. NCES will also continue to seek improvements as part of an ongoing redesign of NAEP for the year 2000 and beyond.

Appendix B

Standard Errors

The comparisons presented in this report are based on statistical tests that consider the magnitude of the difference between group averages or percentages and the standard errors of those statistics. The following appendix contains the standard errors for the averages and percentages discussed in Chapters 2 through 10. For ease of reference, the format and headings of each table in this appendix match the corresponding chapter table, although the numbers that appear are actually standard errors.

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Figure B2.2

Standard Errors for Average Proficiency in Mathematics Content Strands, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	1996	1992	1990
Grade 4			
Overall Proficiency	0.9	0.7	0.9
Number Sense, Properties, & Operations	1.0	0.8	1.1
Measurement	1.1	0.8	1.0
Geometry & Spatial Sense	0.8	0.6	0.9
Data Analysis, Statistics, & Probability	1.1	0.9	–
Algebra & Functions	1.0	0.9	0.9
Grade 8			
Overall Proficiency	1.1	0.9	1.3
Number Sense, Properties, & Operations	1.0	0.8	1.3
Measurement	1.4	1.2	1.6
Geometry & Spatial Sense	1.1	0.9	1.3
Data Analysis, Statistics, & Probability	1.5	1.0	1.6
Algebra & Functions	1.1	1.0	1.2
Grade 12			
Overall Proficiency	1.0	0.9	1.1
Number Sense, Properties, & Operations	1.2	0.9	1.1
Measurement	1.1	0.9	1.3
Geometry & Spatial Sense	1.1	1.0	1.3
Data Analysis, Statistics, & Probability	1.0	1.0	1.2
Algebra & Functions	1.2	1.0	1.2

– 1990 data are not available.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Table B2.1	Standard Errors for Average Proficency in Mathematics Content Strands by Gender, Grades 4, 8, and 12

THE NATION'S REPORT CARD

	1996			1992			1990		
	All Students	Male	Female	All Students	Male	Female	All Students	Male	Female
Grade 4									
Overall Proficiency	0.9	1.1	1.0	0.7	0.8	1.0	0.9	1.2	1.1
Number Sense, Properties, & Operations	1.0	1.2	1.1	0.8	0.9	1.1	1.1	1.4	1.3
Measurement	1.1	1.2	1.2	0.8	1.0	1.0	1.0	1.3	1.3
Geometry & Spatial Sense	0.8	1.1	1.0	0.6	0.8	0.9	0.9	1.2	1.2
Data Analysis, Statistics, & Probability	1.1	1.4	1.3	0.9	0.9	1.2	–	–	–
Algebra & Functions	1.0	1.1	1.2	0.9	1.1	1.5	0.9	1.3	1.1
Grade 8									
Overall Proficiency	1.1	1.4	1.1	0.9	1.1	1.0	1.3	1.6	1.3
Number Sense, Properties, & Operations	1.0	1.4	1.1	0.8	1.0	1.0	1.3	1.6	1.3
Measurement	1.4	1.7	1.6	1.2	1.4	1.5	1.6	2.0	1.5
Geometry & Spatial Sense	1.1	1.3	1.3	0.9	1.1	1.0	1.3	1.6	1.3
Data Analysis, Statistics, & Probability	1.5	1.8	1.5	1.0	1.3	1.2	1.6	1.9	1.6
Algebra & Functions	1.1	1.5	1.1	1.0	1.2	1.2	1.2	1.6	1.3
Grade 12									
Overall Proficiency	1.0	1.1	1.1	0.9	1.1	1.0	1.1	1.4	1.3
Number Sense, Properties, & Operations	1.2	1.3	1.3	0.9	1.0	1.0	1.1	1.3	1.2
Measurement	1.1	1.3	1.3	0.9	1.2	1.1	1.3	1.5	1.5
Geometry & Spatial Sense	1.1	1.2	1.4	1.0	1.2	1.2	1.3	1.5	1.6
Data Analysis, Statistics, & Probability	1.0	1.2	1.1	1.0	1.1	1.1	1.2	1.4	1.5
Algebra & Functions	1.2	1.3	1.2	1.0	1.2	1.1	1.2	1.4	1.3

– 1990 data are not available.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.3

Standard Errors for Average Mathematics Proficiency, Composite Scale by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD NAEP

	1996 Average Scale Score	1992 Average Scale Score	1990 Average Scale Score
Grade 4			
All Students	0.9	0.7	0.9
White	0.9	0.9	1.1
Black	2.3	1.3	1.8
Hispanic	2.1	1.4	2.0
Asian/Pacific Islander	4.1	2.3	3.5
American Indian	2.3	3.1	3.9
Grade 8			
All Students	1.1	0.9	1.3
White	1.2	1.0	1.4
Black	2.0	1.3	2.7
Hispanic	2.0	1.2	2.8
Asian/Pacific Islander	– –	5.4	4.8!
American Indian	3.0!	2.8	9.4!
Grade 12			
All Students	1.0	0.9	1.1
White	1.0	0.9	1.2
Black	2.2	1.7	1.9
Hispanic	1.8	1.7	2.8
Asian/Pacific Islander	4.8	3.5	5.2
American Indian	8.9!	***	***

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.4

Standard Errors for Average Proficiency in Number Sense, Properties, and Operations by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD NAEP

	1996 Average Scale Score	1992 Average Scale Score	1990 Average Scale Score
Grade 4			
All Students	1.0	0.8	1.1
White	1.0	0.9	1.3
Black	2.7	1.3	1.9
Hispanic	2.2	1.8	2.2
Asian/Pacific Islander	4.8	2.5	3.6
American Indian	2.6	3.3	4.0
Grade 8			
All Students	1.0	0.8	1.3
White	1.2	0.9	1.3
Black	2.3	1.3	2.8
Hispanic	1.9	1.5	2.7
Asian/Pacific Islander	– –	5.2	4.5!
American Indian	3.9!	2.7	10.1!
Grade 12			
All Students	1.2	0.9	1.1
White	1.2	0.9	1.2
Black	2.4	1.5	1.8
Hispanic	1.7	1.8	2.9
Asian/Pacific Islander	5.1	3.8	4.8
American Indian	10.6!	***	***

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.5 Standard Errors for Average Proficiency in Measurement by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	1996 Average Scale Score	1992 Average Scale Score	1990 Average Scale Score
Grade 4			
All Students	1.1	0.8	1.0
White	1.2	1.0	1.3
Black	2.5	1.7	2.3
Hispanic	2.5	1.6	2.3
Asian/Pacific Islander	4.6	3.4	4.8
American Indian	2.8	3.5	4.8
Grade 8			
All Students	1.4	1.2	1.6
White	1.5	1.3	1.7
Black	2.6	1.9	3.2
Hispanic	2.8	1.7	3.3
Asian/Pacific Islander	--	7.1	6.4!
American Indian	4.5!	4.1	10.2!
Grade 12			
All Students	1.1	0.9	1.3
White	1.1	1.0	1.4
Black	2.2	1.8	2.2
Hispanic	2.1	1.8	3.0
Asian/Pacific Islander	6.6	4.0	6.1
American Indian	11.9!	***	***

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.6

Standard Errors for Average Proficiency in Geometry and Spatial Sense by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	1996 Average Scale Score	1992 Average Scale Score	1990 Average Scale Score
Grade 4			
All Students	0.8	0.6	0.9
White	0.9	0.8	1.1
Black	1.5	1.4	1.6
Hispanic	2.3	1.3	1.9
Asian/Pacific Islander	4.2	2.5	4.6
American Indian	2.8	3.4	4.0
Grade 8			
All Students	1.1	0.9	1.3
White	1.3	1.1	1.4
Black	2.5	1.7	3.1
Hispanic	2.4	1.2	2.5
Asian/Pacific Islander	– –	5.1	5.0!
American Indian	3.5!	3.3	8.5!
Grade 12			
All Students	1.1	1.0	1.3
White	1.2	1.1	1.5
Black	2.4	1.8	2.1
Hispanic	2.6	2.4	2.9
Asian/Pacific Islander	4.3	3.5	5.8
American Indian	7.9!	***	***

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.7

Standard Errors for Average Proficiency in Data Analysis, Statistics, and Probability by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	1996 Average Scale Score	1992 Average Scale Score	1990 Average Scale Score
Grade 4			
All Students	1.1	0.9	–
White	1.1	1.1	–
Black	3.5	1.6	–
Hispanic	2.4	1.4	–
Asian/Pacific Islander	4.7	3.0	–
American Indian	2.5	3.2	–
Grade 8			
All Students	1.5	1.0	1.6
White	1.8	1.1	1.6
Black	2.2	1.7	3.2
Hispanic	2.5	1.5	3.3
Asian/Pacific Islander	– –	6.3	5.4!
American Indian	4.5!	2.9	11.5!
Grade 12			
All Students	1.0	1.0	1.2
White	0.9	1.0	1.3
Black	2.4	1.9	2.3
Hispanic	2.0	2.2	3.7
Asian/Pacific Islander	5.7	4.4	5.5
American Indian	8.3!	***	***

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

– 1990 data are not available.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Figure B2.8

Standard Errors for Average Proficiency in Algebra and Functions by Race/Ethnicity, Grades 4, 8, and 12

THE NATION'S REPORT CARD naep

	1996 Average Scale Score	1992 Average Scale Score	1990 Average Scale Score
Grade 4			
All Students	1.0	0.9	0.9
White	1.0	1.0	1.1
Black	2.5	1.6	1.8
Hispanic	2.4	1.7	2.2
Asian/Pacific Islander	3.8	3.1	3.4
American Indian	2.3	3.4	3.8
Grade 8			
All Students	1.1	1.0	1.2
White	1.2	1.2	1.4
Black	2.0	2.0	2.6
Hispanic	2.0	1.4	2.9
Asian/Pacific Islander	– –	5.3	5.1!
American Indian	3.2!	2.9	8.3!
Grade 12			
All Students	1.2	1.0	1.2
White	1.2	1.0	1.3
Black	2.8	2.1	2.0
Hispanic	1.9	1.7	2.8
Asian/Pacific Islander	5.0	3.4	5.1
American Indian	8.0!	***	***

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

! Statistical tests involving this value should be interpreted with caution. Standard error estimate may not be accurately determined and/or the sampling distribution of the statistics does not match statistical test assumptions (see Appendix A).

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B2.9

Standard Errors for Average Proficiency in Mathematics Content Areas by Course Taking, Grade 8

THE NATION'S REPORT CARD NAEP

Content Area	Assessment Year 1996		
	Eighth-Grade Mathematics	Pre-Algebra	Algebra
	Average Proficiency	Average Proficiency	Average Proficiency
Number Sense, Properties, & Operations	1.4	1.4	1.6
Measurement	2.0	2.6	2.3
Geometry & Spatial Sense	1.5	1.5	1.7
Data Analysis, Statistics, & Probability	1.7	2.0	2.6
Algebra & Functions	1.4	1.4	1.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure B2.10

Standard Errors for Average Proficiency in Mathematics Content Areas by Algebra and Calculus Courses Taken, Grade 12

THE NATION'S REPORT CARD NAEP

Content Area	Course Taken For Assessment Year 1996					
	Have Not Studied Algebra or Pre-Algebra	Only Taken Pre-Algebra	Only Taken Algebra I	Taken Algebra II But Not Beyond	Taken Algebra III or Pre-Calculus But Not Calculus	Calculus
Number Sense, Properties, & Operations	2.7	2.2	1.6	1.2	1.6	2.3
Measurement	4.3	3.4	1.8	1.1	1.4	3.1
Geometry & Spatial Sense	4.2	2.7	1.9	1.1	1.5	2.0
Data Analysis, Statistics, & Probability	3.5	3.1	1.6	0.9	1.4	2.7
Algebra & Functions	3.1	2.4	1.8	1.1	1.6	2.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure B2.11

Standard Errors for Average Proficiency in Mathematics Content Areas by Geometry Course Taken, Grade 12

THE NATION'S REPORT CARD naep

	Assessment Year 1996	
	Have Not Taken Geometry	Have Taken Geometry
Content Area	Average Proficiency	Average Proficiency
Number Sense, Properties, & Operations	1.6	1.1
Measurement	2.2	0.9
Geometry & Spatial Sense	1.9	0.9
Data Analysis, Statistics, & Probability	2.1	0.8
Algebra & Functions	2.0	1.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure B2.12

Standard Errors for Average Proficiency in Mathematics Content Areas by Probability or Statistics Course Taken, Grade 12

THE NATION'S REPORT CARD naep

	Assessment Year 1996	
	Have Not Taken Probability & Statistics	Have Taken Probability & Statistics
Content Area	Average Proficiency	Average Proficiency
Number Sense, Properties, & Operations	1.2	2.7
Measurement	1.1	3.2
Geometry & Spatial Sense	1.1	2.7
Data Analysis, Statistics, & Probability	0.9	2.8
Algebra & Functions	1.2	2.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure B2.13

Standard Errors for Average Proficiency in Mathematics Content Areas by Number of Semesters of Mathematics Courses Taken in Grades 9 through 12, Grade 12

THE NATION'S REPORT CARD naep

	Average Proficiency by Number of Semesters, Assessment Year 1996			
Content Area	1–2 Semesters	3–4 Semesters	5–6 Semesters	7 or More Semesters
Number Sense, Properties, & Operations	2.3	1.1	1.9	1.2
Measurement	5.5	1.7	1.8	1.0
Geometry & Spatial Sense	3.2	1.1	1.5	0.9
Data Analysis, Statistics, & Probability	3.1	1.3	1.4	1.1
Algebra & Functions	2.3	1.2	1.4	1.3

NOTE: Sample size for 0 semesters is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.1 Standard Errors for Score Percentages for "Evaluate Expression for Odd/Even"

THE NATION'S REPORT CARD NAEP

Grade 12	Correct: 3 Correct Entries	Incorrect: 1 or 2 Correct Entries	Incorrect: No Correct Entries	Omit
Overall	1.6	1.6	0.5	0.8
Males	2.1	2.1	1.0	1.4
Females	2.0	2.1	0.5	0.9
White	1.7	1.9	0.4	0.8
Black	4.2	3.5	2.6	2.1
Hispanic	3.3	3.5	2.1	3.3
Asian/Pacific Islander	7.2	6.8	1.4	2.9
American Indian	***	***	***	***
Geometry Taken	1.7	1.7	0.5	0.8
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	6.6	5.5	2.2	3.4
First-Year Algebra	3.1	3.5	1.5	1.5
Second-Year Algebra	2.1	2.2	0.4	0.7
Third-Year Algebra/Pre-Calculus	3.7	3.8	0.7	2.4
Calculus	7.1	5.5	0.8	3.9

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.2 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Evaluate Expression for Odd/Even"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 12 Composite Scale Range: Below Basic	Basic	Proficient	Advanced
1.6	2.3	2.1	4.8	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.3 Standard Errors for Percentage Correct for "Multiply Two Negative Integers"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	2.1
Males	2.6
Females	2.8
White	3.0
Black	3.2
Hispanic	3.4
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.4
Pre-Algebra	4.0
Algebra	2.6

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.4 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Multiply Two Negative Integers"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
2.1	2.4	3.6	3.6	3.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table B3.5

Standard Errors for Percentage Correct for "Use Subtraction in a Problem"

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	1.4
Males	1.9
Females	2.2
White	1.6
Black	4.6
Hispanic	3.7
Asian/Pacific Islander	***
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.6

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Subtraction in a Problem"

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.4	2.5	2.4	1.6	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.7

Standard Errors for Percentage Correct for "Choose a Number Sentence"

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	1.5
Males	2.2
Females	2.0
White	1.9
Black	4.0
Hispanic	3.2
Asian/Pacific Islander	6.8
American Indian	***

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.8

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Choose a Number Sentence"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.5	2.4	2.4	3.8	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.9 — Standard Errors for Score Percentages for "Reason to Maximize Difference"

THE NATION'S REPORT CARD

Grade 8	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	0.3	1.0	1.0	1.4	1.3	0.9
Males	0.4	1.2	1.2	1.9	1.5	1.1
Females	0.5	1.5	1.8	2.1	2.1	1.0
White	0.4	1.4	1.2	1.8	1.7	1.2
Black	---	1.3	2.0	2.7	3.0	1.6
Hispanic	---	1.2	2.8	3.9	4.0	1.4
Asian/Pacific Islander	– –	– –	– –	– –	– –	– –
American Indian	***	***	***	***	***	***
Mathematics Course Taking:						
Eighth-Grade Mathematics	0.5	1.5	1.7	2.2	2.4	1.9
Pre-Algebra	---	1.7	2.0	2.9	3.2	1.0
Algebra	0.8	2.5	2.0	2.0	2.3	0.9

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.10 — Standard Errors for Percentage at Least Satisfactory Within Achievement-Level Intervals for "Reason to Maximize Difference"

THE NATION'S REPORT CARD

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.0	1.0	1.8	3.2	8.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.11

THE NATION'S REPORT CARD naep

Standard Errors for Score Percentages for "Solve a Multistep Problem"

Grade 4	Correct	Partial	Incorrect	Omit
Overall	1.4	1.2	1.6	0.9
Males	2.1	1.9	2.3	1.3
Females	1.6	1.6	1.9	1.1
White	1.8	1.8	2.0	1.1
Black	1.7	2.1	4.4	3.0
Hispanic	2.0	2.9	3.6	2.0
Asian/Pacific Islander	***	***	***	***
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.12

THE NATION'S REPORT CARD naep

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Solve a Multistep Problem"

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.4	0.6	1.6	4.0	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.13 Standard Errors for Percentage Correct for "Relate a Fraction to 1"

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	1.7
Males	2.0
Females	2.3
White	2.3
Black	4.0
Hispanic	3.8
Asian/Pacific Islander	6.7
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.14 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Relate a Fraction to 1"

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.7	2.9	2.6	3.4	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.15

Standard Errors for Percentage Correct for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	1.9
Males	2.3
Females	2.4
White	2.4
Black	4.3
Hispanic	3.2
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.5
Pre-Algebra	3.5
Algebra	2.2

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.16

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Amount of Restaurant Tip"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.9	2.4	2.9	3.6	9.4

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.17

Standard Errors for Score Percentages for "Use Percent Increase"

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
Overall	0.2	1.3	1.5	0.9
Males	0.3	1.4	1.9	1.4
Females	0.4	2.0	2.0	1.1
White	0.3	1.9	1.8	1.0
Black	---	3.2	3.8	2.2
Hispanic	---	3.0	4.0	3.5
Asian/Pacific Islander	--	--	--	--
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	---	1.8	2.1	1.6
Pre-Algebra	---	2.5	2.9	1.7
Algebra	0.8	3.2	3.1	1.8
Grade 12				
Overall	0.5	1.7	1.6	0.9
Males	0.8	2.1	2.0	1.4
Females	0.5	1.9	2.1	1.1
White	0.7	1.8	2.0	1.4
Black	---	3.6	4.7	4.1
Hispanic	0.8	3.3	4.2	3.0
Asian/Pacific Islander	2.7	6.5	6.2	6.7
American Indian	***	***	***	***
Geometry Taken	0.5	1.8	1.5	1.0
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	0.7	2.2	3.2	2.3
Second-Year Algebra	0.5	1.5	1.6	1.1
Third-Year Algebra/Pre-Calculus	2.4	6.2	4.3	1.4
Calculus	3.4	6.7	8.0	2.8

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.18 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Percent Increase"

THE NATION'S REPORT CARD NAEP

		NAEP Grades 8 and 12 Composite Scale Ranges			
	Overall	Below Basic	Basic	Proficient	Advanced
Grade 8	0.2	---	---	---	---
Grade 12	0.5	---	0.5	3.5	***

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.19 Standard Errors for Percentage Correct for "Solve a Rate Versus Time Problem"

THE NATION'S REPORT CARD NAEP

Grade 12	Percentage Correct
Overall	1.4
Males	2.2
Females	2.0
White	1.5
Black	3.3
Hispanic	3.3
Asian/Pacific Islander	6.0
American Indian	***
Geometry Taken	1.5
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	6.8
First-Year Algebra	3.0
Second-Year Algebra	2.3
Third-Year Algebra/Pre-Calculus	3.6
Calculus	5.4

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B3.20 **Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Solve a Rate Versus Time Problem"**

THE NATION'S REPORT CARD naep

		NAEP Grade12 Composite Scale Range			
	Overall	Below Basic	Basic	Proficient	Advanced
	1.4	2.5	2.1	4.8	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.1 Standard Errors for Percentage Correct for "Recognize Best Unit of Measurement"

THE NATION'S REPORT CARD NAEP

	Percentage Correct
Grade 8	
Overall	1.5
Males	2.1
Females	2.1
White	1.9
Black	3.5
Hispanic	4.0
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.0
Pre-Algebra	2.5
Algebra	2.7
Grade 12	
Overall	1.0
Males	1.5
Females	1.2
White	1.1
Black	3.7
Hispanic	3.6
Asian/Pacific Islander	4.1
American Indian	***
Geometry Taken	1.2
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	***
First-Year Algebra	2.3
Second-Year Algebra	1.3
Third-Year Algebra/Pre-Calculus	2.9
Calculus	1.3

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.2 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Recognize Best Unit of Measurement"

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	1.5	3.0	2.2	1.4	---
Grade 12	1.0	2.8	1.0	---	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.3 Standard Errors for Percentage Correct for "Use Conversion Units of Length"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	1.6
Males	2.6
Females	2.1
White	2.3
Black	2.6
Hispanic	2.9
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.3
Pre-Algebra	2.7
Algebra	2.8

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.4 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Conversion Units of Length"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.6	1.5	2.9	3.6	8.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.5 Standard Errors for Score Percentages for "Use Protractor to Draw a 235° Arc on a Circle"

THE NATION'S REPORT CARD naep

	Correct		Incorrect			Omit
	(± 2°)	(± 3–5°)	No "A" Endpoint	Arc Not Indicated	Other	
Grade 12						
Overall	1.1	0.8	0.2	0.5	1.4	1.0
Males	1.6	1.6	---	0.7	2.2	1.3
Females	1.5	1.1	0.3	0.7	1.6	1.4
White	1.4	1.1	0.2	0.6	1.8	1.0
Black	1.6	1.7	---	1.6	3.4	3.2
Hispanic	2.2	1.9	---	1.6	4.4	5.2
Asian/Pacific Islander	7.7	4.3	---	1.5	7.0	2.5
American Indian	***	***	***	***	***	***
Geometry Taken	1.3	1.0	0.2	0.6	1.6	1.0
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	5.6	---	0.0	3.5	6.5	2.7
First-Year Algebra	1.9	2.4	0.0	1.5	3.2	2.0
Second-Year Algebra	1.7	1.1	0.3	0.6	1.8	1.4
Third-Year Algebra/Pre-Calculus	2.5	2.8	---	2.0	3.7	1.5
Calculus	4.8	3.5	0.0	0.7	5.2	1.2

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.6 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw a 235° Arc on a Circle"

THE NATION'S REPORT CARD

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.6	1.8	2.3	4.5	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.7 Standard Errors for Percentage Correct for "Relate Perimeter to Side Length"

THE NATION'S REPORT CARD

	Percentage Correct
Grade 4	
Overall	1.4
Males	1.6
Females	2.0
White	1.8
Black	2.9
Hispanic	2.8
Asian/Pacific Islander	7.1
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.8 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Relate Perimeter to Side Length"

THE NATION'S REPORT CARD

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.4	2.0	2.5	4.2	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.9

Standard Errors for Score Percentages for "Find Volume of a Cylinder"

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 8				
Overall	1.1	1.2	1.4	1.0
Males	1.4	1.5	2.2	1.4
Females	1.7	1.9	2.1	1.5
White	1.4	1.7	2.0	1.3
Black	1.2	2.1	2.9	2.9
Hispanic	1.8	1.8	3.9	3.8
Asian/Pacific Islander	--	--	--	--
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	1.4	1.9	2.3	1.9
Pre-Algebra	1.7	2.2	2.7	1.6
Algebra	2.2	2.7	2.6	1.4
Grade 12				
Overall	1.5	1.2	1.6	0.9
Males	2.1	1.4	2.5	1.4
Females	2.1	2.0	2.3	1.0
White	1.9	1.6	2.2	1.0
Black	3.6	3.4	3.8	2.9
Hispanic	4.1	3.4	4.0	4.2
Asian/Pacific Islander	6.3	3.4	5.5	1.8
American Indian	***	***	***	***
Geometry Taken	1.6	1.3	1.7	0.7
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	2.0	2.3	2.8	2.1
Second-Year Algebra	1.8	1.6	1.8	0.9
Third-Year Algebra/Pre-Calculus	3.7	3.7	4.0	0.9
Calculus	6.5	6.6	3.5	---

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.10

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Volume of a Cylinder"

THE NATION'S REPORT CARD NAEP

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges: Below Basic	Basic	Proficient	Advanced
Grade 8	1.1	---	1.6	3.6	10.1
Grade 12	1.5	1.6	2.1	4.6	***

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.11

Standard Errors for Score Percentages for "Use a Ruler to Find the Circumference of a Circle"

THE NATION'S REPORT CARD NAEP

	Correct		Incorrect		Omit
	15.7 cm	15.0–16.4 cm Not Including 15.7 cm	Any Response in Inches	Other	
Grade 12					
Overall	1.6	0.6	0.2	1.6	0.9
Males	1.9	1.0	0.3	2.1	1.4
Females	1.9	0.6	0.3	1.9	1.2
White	2.1	0.8	0.2	2.4	1.0
Black	2.6	0.8	0.5	2.7	3.0
Hispanic	2.6	1.3	---	4.0	3.4
Asian/Pacific Islander	6.2	3.6	---	5.2	2.7
American Indian	***	***	***	***	***
Geometry Taken	1.7	0.7	0.2	1.8	0.7
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	4.6	0.9	---	7.7	5.3
First-Year Algebra	2.1	1.8	0.4	3.0	2.0
Second-Year Algebra	1.7	0.6	0.3	1.8	1.3
Third-Year Algebra/Pre-Calculus	4.6	1.9	---	4.5	1.7
Calculus	5.2	2.5	---	5.8	0.0

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.12 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use a Ruler to Find the Circumference of a Circle"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.8	1.6	1.9	4.6	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.13 Standard Errors for Score Percentages for "Describe Measurement Task"

THE NATION'S REPORT CARD naep

	Correct	Partial	Incorrect	Omit
Grade 4				
Overall	0.7	1.5	1.6	1.0
Males	0.9	1.9	1.9	1.6
Females	0.9	2.3	2.4	1.2
White	1.0	1.9	2.1	1.0
Black	0.8	2.8	2.7	2.2
Hispanic	0.7	2.8	3.5	2.7
Asian/Pacific Islander	***	***	***	***
American Indian	***	***	***	***

*** Sample size is insufficient to provide a reliable estimate

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.14

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Describe Measurement Task"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
0.7	---	1.1	2.4	***

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.15

Standard Errors for Score Percentages for "Compare Areas of Two Shapes," Grade 4

THE NATION'S REPORT CARD naep

	Correct	Incorrect		Omit
		Bob–No Adequate Explanation	Not Bob	
Grade 4				
Overall	0.7	1.3	1.3	0.1
Males	1.0	1.8	1.7	0.2
Females	0.8	1.6	1.8	---
White	0.9	1.6	1.6	0.1
Black	---	3.6	3.6	---
Hispanic	---	2.6	2.6	---
Asian/Pacific Islander	3.8	4.2	5.0	---
American Indian	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.16

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grade 4

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
0.7	---	1.1	2.9	***

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.17 Standard Errors for Score Percentages for "Compare Areas of Two Shapes," Grades 8 and 12

THE NATION'S REPORT CARD

	Correct	Incorrect: Bob–No Adequate Explanation	Incorrect: Not Bob	Omit
Grade 8				
Overall	1.4	0.9	1.4	0.5
Males	2.2	1.5	2.1	0.8
Females	1.6	1.5	1.7	0.5
White	1.9	1.1	1.7	0.4
Black	1.7	2.8	2.9	1.2
Hispanic	2.6	3.5	4.3	2.0
Asian/Pacific Islander	--	--	--	--
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	2.2	1.6	2.2	0.4
Pre-Algebra	2.0	1.7	1.9	0.7
Algebra	2.9	1.9	2.8	0.7
Grade 12				
Overall	1.3	1.1	1.3	0.5
Males	2.0	2.0	2.3	0.8
Females	1.5	1.0	1.7	0.6
White	1.5	1.4	1.7	0.5
Black	1.8	2.2	2.5	1.9
Hispanic	5.2	2.7	5.1	2.5
Asian/Pacific Islander	8.2	4.1	7.2	2.2
American Indian	***	***	***	***
Geometry Taken	1.3	1.2	1.3	0.6
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	5.1	3.2	6.0	2.3
First-Year Algebra	2.9	2.5	2.6	1.0
Second-Year Algebra	1.9	1.1	1.7	0.6
Third-Year Algebra/Pre-Calculus	3.4	2.2	3.3	1.6
Calculus	5.4	3.4	5.6	1.6

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.18

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Compare Areas of Two Shapes," Grades 8 and 12

THE NATION'S REPORT CARD

	Overall	NAEP Grades 8 and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 8	1.4	1.7	2.8	3.2	***
Grade 12	1.3	2.1	1.8	3.9	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.19

Standard Errors for Score Percentages for "Find Perimeter (Quadrilateral)"

THE NATION'S REPORT CARD

	Correct	Incorrect			Omit
	Between 6 and 7	Between 7 and 8	Between 5 and 6	Other	
Grade 8					
Overall	1.3	0.8	0.6	1.6	1.2
Males	1.7	1.0	0.9	2.0	1.6
Females	1.7	1.1	0.9	2.4	1.8
White	1.7	1.1	0.7	2.2	1.5
Black	1.9	0.9	0.6	3.3	3.3
Hispanic	2.7	0.5	1.8	3.7	2.8
Asian/Pacific Islander	--	--	--	--	--
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	2.2	1.0	0.8	2.4	2.2
Pre-Algebra	2.3	1.2	1.1	3.4	2.1
Algebra	2.8	1.5	1.3	2.7	1.8

*** Sample size is insufficient to permit a reliable estimate.
-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B4.20

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Perimeter (Quadrilateral)"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.3	1.4	2.9	4.0	5.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.1

Standard Errors for Percentage Correct for "Compare Two Geometric Shapes"

THE NATION'S REPORT CARD NAEP

Grade 4	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	0.1	1.1	1.6	1.2	1.4	0.6
Males	0.2	1.3	1.8	1.8	2.0	1.1
Females	0.1	1.3	2.1	1.8	1.8	0.7
White	0.2	1.2	2.1	1.4	1.6	0.6
Black	---	1.9	3.1	3.1	3.5	2.3
Hispanic	---	2.0	3.5	3.7	3.9	2.1
Asian/Pacific Islander	---	4.4	3.7	4.9	5.9	2.3
American Indian	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.2

Standard Errors for Percentage Satisfactory Within Achievement-Level Intervals for "Compare Two Geometric Shapes"

THE NATION'S REPORT CARD NAEP

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.1	1.1	1.6	2.6	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.3 — Standard Errors for Percentage Correct for "Use Similar Triangles"

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	1.4
Males	2.1
Females	1.6
White	1.6
Black	4.6
Hispanic	3.0
Asian/Pacific Islander	6.6
American Indian	***
Geometry Taken	1.4
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	***
First-Year Algebra	3.4
Second-Year Algebra	2.0
Third-Year Algebra/Pre-Calculus	3.6
Calculus	5.2

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.4 — Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Similar Triangles"

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.4	2.4	1.8	4.2	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.5	Standard Errors for Score Percentages for "Draw a Parallelogram with Perpendicular Diagonals"

THE NATION'S REPORT CARD

Grade 12	Correct		Incorrect		Omit
	Rhombus that is Not a Square	Square	Quadrilateral with Incorrect Diagonals	Other	
Overall	0.8	0.9	1.5	1.4	1.1
Males	1.0	1.3	1.7	2.0	1.5
Females	1.4	1.1	2.1	1.4	1.6
White	1.0	1.2	1.6	1.4	1.2
Black	0.9	0.9	3.5	3.5	3.5
Hispanic	1.3	1.7	4.6	4.8	3.7
Asian/Pacific Islander	4.9	4.8	8.8	4.0	4.1
American Indian	***	***	***	***	***
Geometry Taken	1.0	1.0	1.6	1.4	1.0
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	---	---	6.8	6.0	4.9
First-Year Algebra	1.2	1.6	2.5	2.3	2.1
Second-Year Algebra	1.2	1.4	2.2	1.8	1.5
Third-Year Algebra/Pre-Calculus	2.3	3.1	3.6	2.0	1.5
Calculus	5.6	5.0	4.3	4.1	2.3

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.6	Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Draw a Parallelogram with Perpendicular Diagonals"

THE NATION'S REPORT CARD

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.3	0.7	1.6	3.7	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.7 Standard Errors for Score Percentages for "Use Protractor to Draw Perpendicular Line and Measure Angle"

THE NATION'S REPORT CARD naep

Grade 12	Correct	Incorrect: Line, Correct Angle	Incorrect: Angle, Correct Line	Incorrect: Other	Omit
Overall	1.4	0.5	0.9	1.5	0.7
Males	1.6	0.8	1.4	2.1	1.1
Females	1.9	0.7	1.3	2.0	1.0
White	1.8	0.7	1.2	2.0	0.9
Black	1.8	0.9	2.8	3.1	2.1
Hispanic	2.9	1.4	2.3	4.1	2.1
Asian/Pacific Islander	7.0	1.5	4.5	8.0	0.8
American Indian	***	***	***	***	***
Geometry Taken	1.6	0.6	1.2	1.6	0.5
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	***	***	***	***	***
First-Year Algebra	1.7	1.0	2.0	2.7	1.1
Second-Year Algebra	2.0	0.7	1.5	2.5	0.7
Third-Year Algebra/Pre-Calculus	4.5	1.5	3.1	2.8	2.3
Calculus	7.0	2.8	4.3	4.2	---

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.8 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Protractor to Draw Perpendicular Line and Measure Angle"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range: Below Basic	Basic	Proficient	Advanced
1.4	---	1.6	4.0	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.9 Standard Errors for Percentage Correct for "Assemble Pieces to Form a Square"

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 4	
Overall	1.3
Males	1.6
Females	1.7
White	1.3
Black	3.8
Hispanic	4.2
Asian/Pacific Islander	4.8
American Indian	***
Grade 8	
Overall	0.8
Males	1.3
Females	1.1
White	0.7
Black	3.0
Hispanic	3.1
Asian/Pacific Islander	– –
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	1.3
Pre-Algebra	1.6
Algebra	2.4

***Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table B5.10

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form a Square"

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 4 and 8 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 4	1.3	3.1	1.5	---	***
Grade 8	0.8	1.8	1.1	0.7	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.11

Standard Errors for Score Percentages for "Assemble Pieces to Form Shape"

THE NATION'S REPORT CARD naep

	Correct: Rhombus	Correct: Not a Rhombus	Incorrect	Omit
Grade 4				
Overall	0.9	0.2	1.2	0.7
Males	1.4	0.4	1.7	1.0
Females	1.3	0.3	1.7	0.8
White	1.1	0.3	1.4	0.7
Black	1.7	- - -	3.0	2.2
Hispanic	2.2	- - -	3.1	2.9
Asian/Pacific Islander	4.9	- - -	6.0	- - -
American Indian	***	***	***	***
Grade 8				
Overall	1.2	0.7	1.3	0.4
Males	1.9	0.9	2.0	0.5
Females	1.7	0.8	1.8	0.6
White	1.7	0.8	1.8	0.4
Black	2.7	1.4	2.9	1.0
Hispanic	3.2	2.6	3.7	1.8
Asian/Pacific Islander	– –	– –	– –	– –
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	2.2	0.9	2.0	0.5
Pre-Algebra	2.4	0.9	2.2	0.6
Algebra	2.6	1.8	2.5	0.8
Grade 12				
Overall	1.2	0.6	1.1	0.4
Males	1.9	0.8	2.1	0.7
Females	1.8	0.8	1.6	0.6
White	1.4	0.6	1.5	0.4
Black	3.1	1.3	3.0	1.5
Hispanic	3.9	3.4	3.5	1.7
Asian/Pacific Islander	5.7	3.0	4.7	– –
American Indian	***	***	***	***
Geometry Taken	1.5	0.6	1.5	0.3
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	6.0	6.8	5.4	2.9
First-Year Algebra	3.0	1.1	2.6	0.8
Second-Year Algebra	1.9	0.9	1.9	0.5
Third-Year Algebra/Pre-Calculus	4.2	1.5	3.8	0.8
Calculus	5.2	2.8	5.0	- - -

*** Sample size is insufficient to permit a reliable estimate.

– – Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

- - - Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.12 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Assemble Pieces to Form Shape"

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 4, 8, and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 4	0.9	1.0	1.8	4.6	***
Grade 8	1.4	2.3	2.8	4.2	***
Grade 12	1.1	2.7	1.8	3.6	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics SAssessment.

Table B5.13 Standard Errors for Percentage Correct for "Reason About Betweenness"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	1.4
Males	2.0
Females	1.5
White	1.8
Black	2.5
Hispanic	2.7
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.0
Pre-Algebra	2.3
Algebra	2.8

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.14 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Reason About Betweenness"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.4	1.9	2.6	4.6	8.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.15 Standard Errors for Score Percentages for "Describe Geometric Process for Finding Center of Disk"

THE NATION'S REPORT CARD NAEP

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	0.2	1.2	1.2	1.7	1.2	1.3
Males	0.4	1.6	2.3	2.6	1.7	2.0
Females	---	1.5	1.0	1.9	1.7	1.9
White	0.3	1.4	1.7	2.2	1.4	1.5
Black	---	1.7	1.6	2.8	4.1	4.5
Hispanic	---	2.5	2.7	3.8	3.2	3.5
Asian/Pacific Islander	1.7	7.4	6.5	5.0	5.4	4.3
American Indian	***	***	***	***	***	***
Geometry Taken	0.2	1.4	1.4	1.7	1.5	1.5
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	0.0	2.4	---	5.7	6.1	5.4
First-Year Algebra	---	1.9	1.7	3.7	2.9	3.5
Second-Year Algebra	0.3	1.9	1.1	1.7	1.8	1.9
Third-Year Algebra/Pre-Calculus	1.0	2.7	2.0	3.3	2.8	3.1
Calculus	---	4.4	3.3	6.4	7.3	4.9

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B5.16 Standard Errors for Percentage Satisfactory Within Achievement-Level Intervals for "Describe Geometric Process for Finding Center of Disk"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.2	1.0	1.6	4.3	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.1 Standard Errors for Percentage Correct for "Read a Bar Graph"

THE NATION'S REPORT CARD NAEP

	Percentage Correct
Grade 4	
Overall	1.4
Males	2.1
Females	1.8
White	1.7
Black	3.4
Hispanic	3.2
Asian/Pacific Islander	***
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.2 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Read a Bar Graph"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 4 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.4	2.0	2.4	3.0	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.3	Standard Errors for Score Percentages for "Use Data from a Chart"

THE NATION'S REPORT CARD naep

	Correct	Incorrect			Omit
Grade 4	Shape N–Correct Explanation	Shape N–No, or Incorrect, Explanation	Shape Q	Other	
Overall	1.4	1.3	1.7	1.1	0.5
Males	1.8	1.6	2.0	1.7	0.8
Females	2.0	1.6	2.5	1.5	0.8
White	1.8	1.4	2.0	1.1	0.6
Black	2.7	2.5	3.9	3.3	0.8
Hispanic	2.6	3.5	4.0	5.0	2.2
Asian/Pacific Islander	5.7	4.1	5.8	4.6	---
American Indian	***	***	***	***	***
Grade 8					
Overall	2.0	1.5	1.2	1.1	0.2
Males	2.9	2.1	1.7	1.6	0.3
Females	2.2	1.6	1.2	1.2	---
White	2.2	1.8	1.3	1.0	---
Black	3.8	3.4	3.0	3.1	1.2
Hispanic	4.0	1.9	3.4	2.6	---
Asian/Pacific Islander	--	--	--	--	--
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	2.5	2.2	1.9	1.3	0.0
Pre-Algebra	4.2	2.7	2.6	1.8	0.0
Algebra	3.3	1.9	1.6	2.9	0.0
Grade 12					
Overall	1.5	1.3	0.7	0.8	0.3
Males	2.1	2.1	1.1	1.0	0.5
Females	1.7	1.4	1.0	0.9	0.3
White	1.7	1.4	0.8	0.7	0.3
Black	3.7	2.6	3.0	1.8	1.2
Hispanic	4.3	3.9	2.1	2.7	1.5
Asian/Pacific Islander	5.6	2.9	3.6	4.6	1.2
American Indian	***	***	***	***	***
Geometry Taken	1.4	1.3	0.9	0.8	0.3
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	6.0	4.9	3.1	3.1	1.8
First-Year Algebra	3.0	3.3	2.0	1.6	0.9
Second-Year Algebra	1.7	1.5	1.1	1.0	0.3
Third-Year Algebra/Pre-Calculus	3.8	3.7	1.6	1.6	0.3
Calculus	4.9	4.0	3.4	2.4	---

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.4 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Data from a Chart"

THE NATION'S REPORT CARD naep

	Overall	NAEP Grades 4, 8, and 12 Composite Scale Ranges			
		Below Basic	Basic	Proficient	Advanced
Grade 4	1.4	1.4	2.7	4.3	***
Grade 8	2.0	3.0	2.8	3.6	***
Grade 12	1.5	2.4	1.8	5.0	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.5 Standard Errors for Score Percentages for "Recognize Misleading Graph"

THE NATION'S REPORT CARD naep

	Correct	Partial		Incorrect	Omit
	Graph B–Complete Explanation	Graph B–Incomplete but Partially Correct Explanation	Graph B–No or Incorrect Explanation		
Grade 8					
Overall	0.4	1.3	1.5	1.5	1.6
Males	0.5	2.2	2.2	1.9	1.9
Females	0.7	1.8	2.2	1.7	2.0
White	0.5	1.8	1.8	1.9	1.7
Black	---	1.7	4.5	4.2	4.5
Hispanic	---	2.0	4.8	4.1	5.4
Asian/Pacific Islander	--	--	--	--	--
American Indian	***	***	***	***	***
Mathematics Course Taking:					
Eighth-Grade Mathematics	0.4	2.2	2.5	2.8	3.0
Pre-Algebra	1.1	1.7	2.9	1.9	2.8
Algebra	0.8	2.1	2.3	1.4	2.5

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.6 Standard Errors for Percentage at Least Partial Within Achievement-Level Intervals for "Recognize Misleading Graph"

THE NATION'S REPORT CARD NAEP

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.5	1.9	2.8	3.6	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.7 Standard Errors for Score Percentages for "Use Data in Table to Compute Average Hourly Wage and Determine When Wage Rate Changes"

THE NATION'S REPORT CARD NAEP

	Correct	Partial	Incorrect	Omit
Grade 12				
Overall	1.0	1.7	1.6	0.6
Males	1.3	2.2	2.1	0.7
Females	1.6	2.3	2.0	0.8
White	1.3	2.1	2.1	0.4
Black	0.9	5.2	5.2	2.0
Hispanic	1.8	3.5	5.0	3.5
Asian/Pacific Islander	4.5	5.4	7.5	1.7
American Indian	***	***	***	***
Geometry Taken	1.3	1.9	1.8	0.6
Highest Algebra-Calculus Course Taken:				
Pre-Algebra	***	***	***	***
First-Year Algebra	1.9	3.0	3.1	1.3
Second-Year Algebra	1.6	1.9	2.0	0.5
Third-Year Algebra/Pre-Calculus	2.7	3.8	3.6	0.8
Calculus	5.4	4.1	3.1	1.8

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.8 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Data in Table to Compute Average Hourly Wage and Determine When Wage Rate Changes"

THE NATION'S REPORT CARD

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.0	1.1	1.6	4.1	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.9 Standard Errors for Score Percentages for "Reason About Sample Space"

THE NATION'S REPORT CARD

	Number Correct					
Grade 8	4	3	2	1	None	Omit
Overall	1.1	0.9	0.4	0.5	0.3	0.1
Males	1.6	1.2	0.7	0.9	0.5	0.3
Females	1.5	1.3	0.5	0.6	0.3	0.1
White	1.1	1.0	0.5	0.5	0.2	---
Black	2.8	2.4	1.3	2.2	1.3	---
Hispanic	4.1	2.5	0.7	3.0	1.6	---
Asian/Pacific Islander	--	--	--	--	--	--
American Indian	***	***	***	***	***	***
Mathematics Course Taking:						
Eighth-Grade Mathematics	1.9	1.3	0.9	1.0	0.4	0.3
Pre-Algebra	2.4	2.1	0.7	1.1	0.4	0.0
Algebra	1.9	1.4	0.4	1.3	0.6	0.0

*** Sample size is insufficient to permit a reliable estimate.
-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.10

Standard Errors for Percentage with at Least Three Correct Within Achievement-Level Intervals for "Reason About Sample Space"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
0.8	2.0	1.0	---	---

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.11

Standard Errors for Percentage Correct for "Identify Representative Sample"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	1.6
Males	1.8
Females	2.4
White	1.8
Black	3.7
Hispanic	3.9
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.5
Pre-Algebra	2.6
Algebra	---

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.12

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Identify Representative Sample"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.6	2.7	2.7	2.6	---

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.13	Standard Errors for Score Percentages for "Compare Mean and Median"

THE NATION'S REPORT CARD naep

	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
	Better Measure Both Theaters; Complete Explanation	Better Measure Both Theaters; Complete Explanation for 1 Theater	Better Measure and Complete Explanation 1 Theater; or Better Measure Both Theaters with No or Incomplete Explanation	Better Measure 1 Theater; No or Incomplete Explanation		
Grade 12						
Overall	0.3	0.5	1.0	1.4	1.4	1.5
Males	0.5	0.7	1.7	1.9	2.2	2.5
Females	---	0.5	1.1	2.0	1.9	1.6
White	0.4	0.7	1.4	1.7	1.3	1.7
Black	---	---	1.3	4.0	3.9	4.4
Hispanic	---	---	2.2	3.3	7.1	7.4
Asian/Pacific Islander	---	2.1	3.0	6.0	5.3	6.2
American Indian	***	***	***	***	***	***
Geometry Taken	0.4	0.6	1.1	1.6	1.6	1.9
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	***	***	***	***	***	***
First-Year Algebra	---	0.8	1.8	2.8	3.4	3.1
Second-Year Algebra	0.2	0.5	1.3	2.0	2.1	2.2
Third-Year Algebra/Pre-Calculus	---	2.0	3.8	4.6	3.4	4.6
Calculus	2.7	2.1	3.7	4.1	4.4	3.5

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.14

Standard Errors for Percentage at Least Satisfactory Within Achievement-Level Intervals for "Compare Mean and Median"

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	**Below Basic**	**Basic**	**Proficient**	**Advanced**
0.6	---	0.9	3.9	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.15

Standard Errors for Percentage Correct for "Determine a Probability"

THE NATION'S REPORT CARD naep

	Percentage Correct
Grade 4	
Overall	1.5
Males	2.2
Females	1.6
White	1.8
Black	3.0
Hispanic	4.0
Asian/Pacific Islander	3.7
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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THE NATION'S REPORT CARD naep

Table B6.16 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Determine a Probability"

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.5	2.3	2.3	3.6	***

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

Table B6.17 Standard Errors for Score Percentages for "Compare Probabilities"

	Correct	Partial	Incorrect		Omit
	Correct Answer to "Yes/No" Question; Correct Explanation	Correct Answer to "Yes/No" Question; Partial Explanation	Correct Answer to "Yes/No" Question; Incorrect Explanation	Incorrect Answer to "Yes/No" Question	
Grade 12					
Overall	0.8	1.5	1.7	1.7	0.7
Males	1.3	2.3	2.4	2.1	0.9
Females	0.9	1.9	2.1	2.1	1.0
White	0.9	1.9	2.2	2.1	0.9
Black	1.9	2.9	3.0	3.4	2.0
Hispanic	2.9	2.7	5.9	6.9	3.0
Asian/Pacific Islander	3.6	5.7	6.1	7.5	---
American Indian	***	***	***	***	***
Geometry Taken	1.0	1.7	1.9	1.8	0.7
Highest Algebra-Calculus Course Taken:					
Pre-Algebra	***	***	***	***	***
First-Year Algebra	1.9	2.5	2.5	3.9	1.1
Second-Year Algebra	1.3	1.7	2.4	2.6	0.7
Third-Year Algebra/Pre-Calculus	2.2	4.8	3.3	4.4	3.1
Calculus	3.6	3.7	4.0	4.1	4.2

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B6.18 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Compare Probabilities"

THE NATION'S REPORT CARD NAEP

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.6	1.7	2.6	4.9	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.1 Standard Errors for Percentage Correct for "Find Number of Diagonals in a Polygon from a Vertex"

THE NATION'S REPORT CARD NAEP

Grade 8	Percentage Correct
Overall	1.6
Males	2.0
Females	2.1
White	1.7
Black	2.9
Hispanic	4.0
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.9
Pre-Algebra	3.3
Algebra	2.5

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.2

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find Number of Diagonals in a Polygon from a Vertex"

THE NATION'S REPORT CARD

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.6	2.2	2.1	2.6	---

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.3

Standard Errors for Score Percentages for "Describe Pattern of Squares in 20th Figure"

THE NATION'S REPORT CARD

Grade 12	Extended	Satisfactory	Partial	Minimal	Incorrect	Omit
Overall	0.4	0.5	1.2	1.4	1.4	1.3
Males	0.7	0.6	1.9	1.6	2.0	1.5
Females	0.5	0.9	1.8	2.2	1.9	1.9
White	0.5	0.7	1.5	1.7	1.6	1.2
Black	---	---	2.4	2.6	3.4	4.5
Hispanic	---	---	5.1	3.9	4.8	5.0
Asian/Pacific Islander	---	1.6	6.6	4.7	4.3	4.9
American Indian	***	***	***	***	***	***
Geometry Taken	0.4	0.6	1.3	1.7	1.6	1.3
Highest Algebra-Calculus Course Taken:						
Pre-Algebra	***	***	***	***	***	***
First-Year Algebra	0.8	0.4	2.4	2.7	3.1	2.8
Second-Year Algebra	0.6	0.5	1.8	2.3	1.9	1.7
Third-Year Algebra/Pre-Calculus	---	2.8	3.4	3.7	3.6	2.4
Calculus	3.0	1.9	5.0	5.1	3.6	2.5

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.4

Standard Errors for Percentage at Least Satisfactory Within Achievement-Level Intervals for "Describe Pattern of Squares in 20th Figure"

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
0.8	---	0.7	3.6	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP)1996 Mathematics Assessment.

Table B7.5

Standard Errors for Percentage Correct for "Identify Graph of Function"

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	1.4
Males	1.8
Females	1.9
White	1.8
Black	2.8
Hispanic	4.0
Asian/Pacific Islander	6.4
American Indian	***
Geometry Taken	1.6
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	2.7
First-Year Algebra	1.9
Second-Year Algebra	2.0
Third-Year Algebra/Pre-Calculus	3.9
Calculus	7.9

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.6

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Identify Graph of Function"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.4	1.9	2.0	6.0	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.7

Standard Errors for Percentage Correct for "Write Expression Using N"

THE NATION'S REPORT CARD naep

Grade 4	Percentage Correct
Overall	1.2
Males	1.7
Females	1.9
White	1.8
Black	3.0
Hispanic	4.6
Asian/Pacific Islander	4.4
American Indian	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.8 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Write Expression Using N"

THE NATION'S REPORT CARD naep

	NAEP Grade 4 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.2	2.4	2.2	2.1	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.9 Standard Errors for Percentage Correct for "Translate Words to Symbols"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	1.6
Males	2.1
Females	2.2
White	2.0
Black	3.1
Hispanic	3.5
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	3.1
Pre-Algebra	3.4
Algebra	2.1

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.10 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Translate Words to Symbols"

THE NATION'S REPORT CARD naep

Overall	NAEP Grade 8 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.6	2.4	2.5	1.3	---

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.11 Standard Errors for Percentage Correct for "Find (x, y) Solution of Linear Equation"

THE NATION'S REPORT CARD naep

Grade 8	Percentage Correct
Overall	1.7
Males	2.3
Females	2.0
White	2.1
Black	3.7
Hispanic	3.2
Asian/Pacific Islander	--
American Indian	***
Mathematics Course Taking:	
Eighth-Grade Mathematics	2.6
Pre-Algebra	2.8
Algebra	2.5

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996

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THE NATION'S REPORT CARD naep

Table B7.12 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Find (x, y) Solution of Linear Equation"

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.7	1.9	3.2	3.6	---

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD naep

Table B7.13 Standard Errors for Percentage Correct for "Subtract Integers"

Grade 8	Correct	Partial	Incorrect	Omit
Overall	1.4	0.6	1.5	0.6
Males	2.0	1.0	2.2	0.9
Females	1.4	0.6	1.4	0.6
White	1.9	0.8	2.1	0.4
Black	2.1	1.8	4.0	2.3
Hispanic	2.9	0.7	3.5	3.2
Asian/Pacific Islander	--	--	--	--
American Indian	***	***	***	***
Mathematics Course Taking:				
Eighth-Grade Mathematics	1.9	0.8	2.0	1.1
Pre-Algebra	2.2	0.6	2.4	0.6
Algebra	3.0	1.4	2.7	0.9

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.14 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Subtract Integers"

THE NATION'S REPORT CARD naep

	NAEP Grade 8 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.4	1.4	2.3	3.4	7.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.15 Standard Errors for Percentage Correct for "Solve Pair of Equations"

THE NATION'S REPORT CARD naep

Grade 12	Percentage Correct
Overall	1.0
Males	1.9
Females	1.1
White	1.1
Black	3.0
Hispanic	4.5
Asian/Pacific Islander	3.6
American Indian	***
Geometry Taken	0.9
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	6.1
First-Year Algebra	2.1
Second-Year Algebra	1.1
Third-Year Algebra/Pre-Calculus	1.6
Calculus	3.3

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.16

Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Solve Pair of Equations"

THE NATION'S REPORT CARD NAEP

Overall	NAEP Grade 12 Composite Scale Range			
	Below Basic	Basic	Proficient	Advanced
1.0	2.8	1.3	---	***

*** Sample size is insufficient to permit a reliable estimate.
--- Standard error estimate cannot be accurately determined.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B7.17

Standard Errors for Percentage Correct for "Use Trigonometric Identity"

THE NATION'S REPORT CARD NAEP

Grade 12	Percentage Correct
Overall	1.6
Males	1.8
Females	2.4
White	2.0
Black	3.0
Hispanic	3.1
Asian/Pacific Islander	6.7
American Indian	***
Geometry Taken	1.8
Highest Algebra-Calculus Course Taken:	
Pre-Algebra	5.1
First-Year Algebra	2.4
Second-Year Algebra	2.6
Third-Year Algebra/Pre-Calculus	4.1
Calculus	6.3

*** Sample size is insufficient to permit a reliable estimate.
SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table B7.18 Standard Errors for Percentage Correct Within Achievement-Level Intervals for "Use Trigonometric Identity"

THE NATION'S REPORT CARD naep

	NAEP Grade 12 Composite Scale Range			
Overall	Below Basic	Basic	Proficient	Advanced
1.6	1.8	2.4	5.1	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B8.1

Standard Errors for Average Scale Score by Mathematics Course Enrollment and by Gender, Race/Ethnicity, and Whether School Offers Algebra for High School Credit or Placement, Grade 8

THE NATION'S REPORT CARD NAEP

Grade 8	Assessment Year	Mathematics Course: Algebra — Percentage of Students	Algebra — Average Scale Score	Pre-Algebra — Percentage of Students	Pre-Algebra — Average Scale Score	Eighth-Grade Mathematics — Percentage of Students	Eighth-Grade Mathematics — Average Scale Score	Other Mathematics — Percentage of Students	Other Mathematics — Average Scale Score
All Students	1996	1.5	1.7	1.8	1.5	2.2	1.3	0.6	4.7
	1992	1.0	1.8	2.2	1.5	2.6	1.3	0.4	4.1
	1990	1.1	2.6	1.8	2.3	2.0	1.4	0.4	5.3
Females	1996	1.6	1.8	1.9	1.9	2.2	1.4	0.7	5.4
	1992	1.3	2.1	2.2	1.8	2.7	1.5	0.5	5.4
	1990	1.6	2.8	2.1	2.8	2.4	1.5	0.7	***
Males	1996	1.8	2.0	1.9	1.8	2.5	1.6	0.6	5.6
	1992	1.0	2.1	2.4	1.7	2.6	1.4	0.4	5.6
	1990	1.2	3.0	1.7	2.8	2.1	1.6	0.5	***
White	1996	2.1	1.4	2.3	1.7	2.9	1.4	0.7	4.5
	1992	1.3	1.6	2.5	1.2	3.1	1.3	0.4	5.4
	1990	1.5	2.5	2.2	2.1	2.3	1.6	0.6	6.9
Black	1996	2.0	2.9	2.4	2.2	2.2	1.9	1.0	8.2
	1992	1.7	4.8	3.7	3.1	3.9	1.4	1.2	***
	1990	2.1	***	2.9	6.0	4.6	3.2	0.8	***
Hispanic	1996	2.3	4.9	2.1	2.7	3.0	2.5	1.1	***
	1992	1.2	4.3	2.5	2.6	2.8	1.5	0.8	***
	1990	1.5	***	3.5	4.9	4.1	2.7	0.9	***
Asian/Pacific Islander	1996	--	--	--	--	--	--	--	--
	1992	5.1	5.0	3.4	***	5.1	4.6	0.8	***
	1990	6.6	***	6.1	***	6.3	***	2.3	***
American Indian	1996	2.9	***	4.8	***	7.8	***	3.2	***
	1992	2.7	***	6.1	***	6.2	3.8	1.1	***
	1990	2.7	***	6.8	***	5.8	***	1.9	***
School Offers Algebra for High School Credit or Placement: Yes	1996	2.2	1.9	2.2	2.1	2.6	1.5	0.7	5.3
	1992	1.3	1.7	2.5	1.5	2.7	1.8	0.5	4.9
	1990	1.5	2.7	2.3	3.2	2.5	1.7	0.7	6.6
No	1996	2.8	5.6	4.4	4.6	6.4	4.2	1.2	***
	1992	1.6	5.8	3.8	2.2	4.5	2.8	0.3	***
	1990	2.1	***	3.3	5.3	4.0	4.0	0.5	***

*** Sample size is insufficient to permit a reliable estimate.

-- Data for grade 8 Asian/Pacific Islanders are not reported due to concerns about the accuracy and precision of the national estimates. See Appendix A for further detail.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B8.2

Standard Errors for Percentage of Students Currently Enrolled in a Mathematics Course by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

Grade 12	Assessment Year	Percentage of Students
All Students	1996	1.2
	1992	1.2
	1990	2.0
Females	1996	1.4
	1992	1.3
	1990	2.2
Males	1996	1.6
	1992	1.4
	1990	2.4
White	1996	1.5
	1992	1.4
	1990	2.5
Black	1996	2.0
	1992	2.4
	1990	3.4
Hispanic	1996	2.6
	1992	2.1
	1990	2.3
Asian/Pacific Islander	1996	2.9
	1992	2.8
	1990	4.9
American Indian	1996	6.0
	1992	***
	1990	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B8.3

Standard Errors for Percentage of Students by Year They Initially Took a First-Year Algebra Course, Grade 12

THE NATION'S REPORT CARD

Grade 12	Assessment Year	Before 9th Grade	9th Grade	10th Grade	11th or 12th Grade	Not Taken
All Students	1996	1.2	1.1	0.9	0.3	0.4
	1992	1.0	1.4	0.8	0.5	0.5
Females	1996	1.3	1.4	1.1	0.3	0.5
	1992	1.1	1.6	1.0	0.6	0.5
Males	1996	1.5	1.4	1.0	0.5	0.5
	1992	1.1	1.5	1.0	0.6	0.6
White	1996	1.3	1.4	1.1	0.3	0.4
	1992	1.1	1.5	0.8	0.5	0.5
Black	1996	2.9	3.2	1.8	1.4	0.8
	1992	1.6	3.4	2.4	1.2	1.3
Hispanic	1996	2.0	3.3	1.8	1.4	1.6
	1992	2.0	3.0	2.4	1.4	2.3
Asian/Pacific Islander	1996	4.1	3.1	2.0	0.4	0.7
	1992	4.9	3.8	2.6	1.4	0.8
American Indian	1996	6.5	3.7	8.2	2.8	---
	1992	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B8.4

Standard Errors for Percentage of Students by Number of Semesters of Mathematics Taken (Grades 9 through 12) by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

	Assessment Year	7 or More Semesters	5–6 Semesters	3–4 Semesters	1–2 Semesters	No Semesters
Grade 12						
All Students	1996	1.3	0.9	0.8	0.5	0.1
	1992	1.2	0.8	1.0	0.4	0.1
	1990	1.8	1.2	1.1	0.9	0.1
Females	1996	1.3	0.9	0.9	0.5	0.1
	1992	1.5	1.1	1.1	0.5	0.1
	1990	2.1	1.5	1.3	1.2	0.1
Males	1996	1.7	1.1	1.1	0.6	0.2
	1992	1.3	1.0	1.2	0.4	0.2
	1990	2.1	1.5	1.5	0.9	0.2
White	1996	1.5	1.0	0.9	0.5	0.1
	1992	1.3	1.0	1.1	0.3	0.1
	1990	2.2	1.4	1.0	1.0	0.1
Black	1996	2.5	2.0	2.9	1.4	0.3
	1992	2.7	1.5	2.3	1.4	0.4
	1990	4.2	2.6	4.1	1.9	0.4
Hispanic	1996	2.6	1.7	2.2	1.2	0.3
	1992	2.9	2.2	2.6	1.5	0.5
	1990	3.8	3.5	4.3	1.9	---
Asian/Pacific Islander	1996	4.7	3.1	2.8	0.4	---
	1992	4.4	2.7	3.0	0.8	---
	1990	8.8	7.2	2.9	---	---
American Indian	1996	9.7	3.0	6.3	4.4	1.4
	1992	***	***	***	***	***
	1990	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

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Table B8.5 Standard Errors for Percentage of Students by Mathematics Courses and Years of Study, Grade 12

THE NATION'S REPORT CARD naep

	Assessment Year	Years of Study: More Than One Year	One School Year	One-Half Year or Less	Not Studied
Grade 12					
General Mathematics	1996	1.3	0.9	0.3	1.3
	1992	1.0	0.8	0.3	1.1
	1990	1.8	1.2	0.3	2.3
Business or Consumer Mathematics	1996	0.4	0.7	0.4	0.9
	1992	0.4	1.0	0.7	1.0
	1990	0.5	1.0	1.0	1.3
Introduction to Algebra or Pre-Algebra	1996	0.5	1.1	0.6	1.1
	1992	0.5	1.0	0.4	1.2
	1990	0.6	1.7	0.7	1.8
First-Year Algebra	1996	0.4	0.8	0.5	0.5
	1992	0.4	0.9	0.3	0.7
	1990	0.7	1.1	0.5	1.0
Geometry	1996	0.4	1.5	0.6	1.1
	1992	0.4	1.3	0.4	1.2
	1990	0.5	1.7	0.7	1.5
Second-Year Algebra	1996	0.3	1.4	0.6	1.2
	1992	0.2	1.5	0.7	1.4
	1990	0.4	2.1	0.7	1.9
Trigonometry	1996	0.3	1.5	1.2	1.6
	1992	0.2	1.2	1.1	1.5
	1990	0.2	1.5	1.4	1.7
Pre-Calculus, Third-Year Algebra	1996	0.2	1.4	0.8	1.4
	1992	0.2	1.1	0.8	1.2
	1990	0.2	1.5	0.8	1.7
Calculus	1996	0.2	0.8	0.3	0.9
	1992	0.2	0.7	0.3	0.8
	1990	0.2	0.7	0.5	0.7
Probability or Statistics	1996	0.4	0.7	0.7	1.4
	1992	0.2	0.3	0.6	0.7
	1990	0.2	0.4	1.0	1.2
Unified, Integrated, or Sequential Mathematics	1996	1.1	0.5	0.4	1.7
	1992	0.4	0.4	0.4	0.9
Applied Mathematics (Technical Preparation)	1996	0.4	0.7	0.5	0.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B8.6

Standard Errors for Percentage of Students by Highest Algebra-through-Calculus Course Taken, Grade 12

THE NATION'S REPORT CARD naep

Grade 12	Assessment Year	Not Taken Pre-Algebra	Pre-Algebra	First-Year Algebra	Second-Year Algebra	Pre-Calculus or Third-Year Algebra	Calculus
All Students	1996	0.5	0.3	1.0	1.3	0.9	0.5
	1992	0.5	0.5	1.3	1.7	0.8	0.6
	1990	0.8	0.7	1.6	1.6	1.1	0.5
Females	1996	0.4	0.3	1.2	1.5	1.2	0.6
	1992	0.6	0.5	1.5	1.9	1.0	0.6
	1990	1.0	1.0	1.9	1.7	1.2	0.5
Males	1996	0.7	0.4	1.2	1.5	1.1	0.8
	1992	0.7	0.6	1.3	1.8	0.9	0.6
	1990	1.2	0.7	1.7	1.9	1.2	0.6
White	1996	0.5	0.4	1.1	1.6	1.1	0.6
	1992	0.6	0.5	1.5	2.0	0.9	0.6
	1990	0.9	0.8	1.7	2.0	1.4	0.5
Black	1996	0.9	0.7	2.2	2.6	1.3	0.9
	1992	1.5	1.2	1.8	2.8	1.2	0.6
	1990	1.8	1.5	2.6	2.5	1.6	0.4
Hispanic	1996	1.7	1.0	1.8	3.3	1.6	1.0
	1992	2.1	1.2	2.6	4.0	1.0	0.8
	1990	3.3	2.2	3.2	3.4	0.9	0.7
Asian/Pacific Islander	1996	1.2	2.0	3.0	3.6	2.8	5.4
	1992	0.5	1.6	3.3	4.7	3.2	4.0
	1990	1.9	6.1	4.8	4.3	3.9	2.9
American Indian	1996	2.1	1.9	5.1	4.3	4.1	1.8
	1992	***	***	***	***	***	***
	1990	***	***	***	***	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Table B8.7 Standard Errors for Percentage of Students by Whether They Have Taken a Geometry Course and by Gender and Race/Ethnicity, Grade 12

THE NATION'S REPORT CARD naep

	Assessment Year	Taken a Geometry Course: Yes	No
Grade 12			
All Students	1996	1.4	1.4
	1992	1.3	1.3
	1990	1.7	1.7
Females	1996	1.5	1.5
	1992	1.3	1.3
	1990	1.9	1.9
Males	1996	1.5	1.5
	1992	1.6	1.6
	1990	1.9	1.9
White	1996	1.6	1.6
	1992	1.3	1.3
	1990	1.8	1.8
Black	1996	2.4	2.4
	1992	3.5	3.5
	1990	3.0	3.0
Hispanic	1996	2.7	2.7
	1992	5.6	5.6
	1990	3.8	3.8
Asian/Pacific Islander	1996	3.9	3.9
	1992	2.7	2.7
	1990	4.7	4.7
American Indian	1996	15.4	15.4
	1992	***	***
	1990	***	***

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1990, 1992, and 1996 Mathematics Assessments.

Figure B9.1 Standard Errors for Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Content Strands by Grade and Content Strand

THE NATION'S REPORT CARD naep

Grade 4	
Number Sense, Properties, & Operations	1.1
Measurement	2.2
Geometry & Spatial Sense	1.7
Data Analysis, Statistics, & Probability	1.4
Algebra & Functions	1.9
Grade 8	
Number Sense, Properties, & Operations	1.9
Measurement	2.8
Geometry & Spatial Sense	2.4
Data Analysis, Statistics, & Probability	2.0
Algebra & Functions	3.3

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.1 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Number Sense, Properties, and Operations, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	1.1	1.2	0.1
Grade 8			
All Students	1.9	1.8	0.6
Students Enrolled in:			
Eighth-Grade Mathematics	3.1	3.1	0.7
Pre-Algebra	1.8	1.7	0.8
Algebra	3.1	2.8	1.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.2 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Measurement, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	2.2	2.3	1.8
Grade 8			
All Students	2.8	3.2	2.6
Students Enrolled in:			
Eighth-Grade Mathematics	4.1	4.9	3.6
Pre-Algebra	3.6	4.7	4.3
Algebra	3.6	4.7	4.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.3 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Geometry and Spatial Sense, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	1.7	2.2	2.4
Grade 8			
All Students	2.4	2.7	2.7
Students Enrolled in:			
Eighth-Grade Mathematics	4.3	4.1	4.4
Pre-Algebra	2.9	3.8	3.7
Algebra	3.0	3.9	3.1

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.4 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Data Analysis, Statistics, and Probability, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	1.4	2.6	2.5
Grade 8			
All Students	2.0	3.1	3.3
Students Enrolled in:			
Eighth-Grade Mathematics	3.5	4.4	3.9
Pre-Algebra	2.5	4.6	4.7
Algebra	3.3	5.4	4.7

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.5 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Algebra and Functions, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	1.9	2.7	3.0
Grade 8			
All Students	3.3	2.9	1.1
Students Enrolled in:			
Eighth-Grade Mathematics	5.2	4.9	2.5
Pre-Algebra	4.7	4.5	1.3
Algebra	3.0	2.9	0.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure B9.2

Standard Errors for Percentage of Students Whose Teachers Place "A Lot" of Emphasis on Specific Mathematics Processes by Grade and Mathematics Processes

THE NATION'S REPORT CARD naep

Grade 4	
Facts and Concepts	1.0
Skills and Procedures	1.2
Reasoning	2.4
Communication	2.4
Grade 8	
Facts and Concepts	2.7
Skills and Procedures	2.5
Reasoning	3.0
Communication	3.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.6

Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Mathematics Facts and Concepts, Grades 4 and 8, 1996

THE NATION'S REPORT CARD naep

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	1.0	1.0	---
Grade 8			
All Students	2.7	2.2	1.4
Students Enrolled in:			
Eighth-Grade Mathematics	4.2	3.7	2.7
Pre-Algebra	3.2	3.0	0.9
Algebra	3.7	2.9	1.9

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.7 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Learning Skills and Procedures Needed to Solve Routine Problems, Grades 4 and 8, 1996

THE NATION'S REPORT CARD NAEP

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	1.2	1.2	---
Grade 8			
All Students	2.5	2.4	0.9
Students Enrolled in:			
Eighth-Grade Mathematics	3.6	3.7	0.3
Pre-Algebra	3.3	3.0	1.2
Algebra	3.3	2.7	2.1

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.8 Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Developing Reasoning Ability to Solve Unique Problems, Grades 4 and 8, 1996

THE NATION'S REPORT CARD NAEP

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	2.4	2.4	1.1
Grade 8			
All Students	3.0	3.1	1.5
Students Enrolled in:			
Eighth-Grade Mathematics	4.0	4.8	2.9
Pre-Algebra	4.6	4.4	1.1
Algebra	3.7	3.7	0.8

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD NAEP

Table B9.9

Standard Errors for Percentage of Students by Teachers' Reports on Emphasis Placed on Learning How to Communicate Ideas in Mathematics Effectively, Grades 4 and 8, 1996

	Level of Emphasis		
	A Lot	Some	Little or None
Grade 4			
All Students	2.4	2.5	1.8
Grade 8			
All Students	3.0	3.0	1.9
Students Enrolled in:			
Eighth-Grade Mathematics	4.0	4.4	3.4
Pre-Algebra	4.3	4.0	1.9
Algebra	3.8	3.8	2.4

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

THE NATION'S REPORT CARD NAEP

Table B9.10

Standard Errors for Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Objects Like Rulers, Grades 4 and 8, 1996

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4				
All Students	1.0	2.3	2.6	0.9
Grade 8				
All Students	2.0	2.4	2.7	2.2
Students Enrolled in:				
Eighth-Grade Mathematics	2.0	3.4	3.7	2.6
Pre-Algebra	2.4	2.8	4.7	3.3
Algebra	2.9	2.5	4.7	3.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.11 Standard Errors for Percentage of Students by Teachers' Reports on Frequency with Which Students Work with Counting Blocks and Geometric Shapes, Grades 4 and 8, 1996

THE NATION'S REPORT CARD

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4				
All Students	0.9	1.8	2.8	2.4
Grade 8				
All Students	0.5	1.8	3.3	3.4
Students Enrolled in:				
Eighth-Grade Mathematics	0.7	3.0	5.0	5.0
Pre-Algebra	0.8	2.5	4.1	4.3
Algebra	---	1.5	4.2	4.2

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.12 Standard Errors for Percentage of Students by Frequency with Which They Work with Measuring Instruments or Geometric Solids, Grade 12, 1996

THE NATION'S REPORT CARD

	Frequency			
Grade 12	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Students Taking Mathematics	0.5	0.7	1.0	1.4

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.13

Standard Errors for Percentage of Students by Frequency with Which They Solve Problems in Small Groups or with a Partner, Grades 4, 8, and 12, 1996*

THE NATION'S REPORT CARD

	Frequency			
	Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4				
All Students	2.0	1.9	1.4	1.5
Grade 8				
All Students	2.9	3.2	3.3	1.4
Students Enrolled in:				
Eighth-Grade Mathematics	4.0	5.2	4.5	2.0
Pre-Algebra	3.8	4.8	4.3	1.7
Algebra	4.1	3.3	4.2	2.7
Grade 12				
Students Taking Mathematics	1.4	1.2	0.8	1.0

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B9.14

Standard Errors for Percentage of Students by Frequency with Which They Write a Few Sentences about How to Solve a Mathematics Problem, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	1.4	2.1	2.6	2.4
	1992	0.6	1.9	1.9	2.3
Grade 8					
All Students	1996	1.1	2.8	2.8	3.3
	1992	0.9	2.0	2.5	2.5
Students Enrolled in:					
Eighth-Grade Mathematics	1996	1.3	4.2	3.8	4.2
	1992	0.6	2.5	3.1	3.3
Pre-Algebra	1996	1.5	4.3	3.9	4.0
	1992	0.9	3.1	3.8	3.1
Algebra	1996	1.7	3.5	5.4	5.2
	1992	3.0	3.0	3.5	3.6
Grade 12					
Students Taking Mathematics	1996	0.5	0.6	0.8	1.0

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B9.15 Standard Errors for Percentage of Students by Frequency with Which They Write Reports or Do Mathematics Projects, Grades 4, 8, and 12*

THE NATION'S REPORT CARD NAEP

	Assessment Year	Frequency: Almost Every Day	Frequency: Once or Twice a Week	Frequency: Once or Twice a Month	Frequency: Never or Hardly Ever
Grade 4					
All Students	1996	0.5	0.8	2.3	2.4
	1992	---	0.4	2.0	2.1
Grade 8					
All Students	1996	---	1.1	3.2	3.3
	1992	---	0.3	1.9	2.0
Students Enrolled in: Eighth-Grade Mathematics	1996	---	1.0	4.6	4.8
	1992	---	0.2	2.6	2.7
Pre-Algebra	1996	---	2.3	4.7	4.9
	1992	---	0.7	2.9	3.1
Algebra	1996	---	1.2	4.0	4.1
	1992	---	0.7	2.3	2.5
Grade 12					
Students Taking Mathematics	1996	0.2	0.3	1.2	1.3

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

--- Standard error estimate cannot be accurately determined.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

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THE NATION'S REPORT CARD NAEP

Table B9.16 Standard Errors for Percentage of Students by Frequency with Which They Discuss Solutions to Mathematics Problems with Other Students, Grades 4, 8, and 12*

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	2.1	2.1	1.7	1.3
	1992	2.3	1.9	2.0	0.8
Grade 8					
All Students	1996	3.2	3.1	1.9	0.7
	1992	2.3	2.1	1.9	1.1
Students Enrolled in: Eighth-Grade Mathematics	1996	4.8	4.4	3.1	1.1
	1992	3.1	3.2	2.4	1.6
Pre-Algebra	1996	4.0	3.7	1.5	0.9
	1992	3.6	2.4	3.3	1.5
Algebra	1996	4.6	4.3	2.9	0.4
	1992	3.7	3.4	2.1	0.6
Grade 12					
Students Taking Mathematics	1996	1.1	0.9	0.5	0.7

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B9.17

Standard Errors for Percentage of Students by Teachers' Reports on Frequency with Which Students Work and Discuss Mathematics Problems That Reflect Real-Life Situations, Grades 4 and 8

THE NATION'S REPORT CARD NAEP

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	2.1	2.1	1.9	0.9
	1992	2.1	2.4	1.7	1.1
Grade 8					
All Students	1996	2.6	2.9	2.7	1.1
	1992	1.6	2.2	2.1	1.0
Students Enrolled in:					
Eighth-Grade Mathematics	1996	3.3	3.7	3.8	1.6
	1992	2.1	2.7	2.9	1.1
Pre-Algebra	1996	4.2	4.7	3.6	1.4
	1992	2.9	2.6	3.1	1.7
Algebra	1996	4.0	5.0	3.5	1.8
	1992	3.6	4.2	2.8	1.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B9.18 Standard Errors for Percentage of Students by Frequency with Which Students Use Calculators in Class, Grades 4, 8, and 12*

THE NATION'S REPORT CARD naep

	Assessment Year	Frequency: Almost Every Day	Once or Twice a Week	Once or Twice a Month	Never or Hardly Ever
Grade 4					
All Students	1996	0.9	2.2	2.4	2.4
	1992	0.4	1.9	2.0	2.5
Grade 8					
All Students	1996	2.7	2.5	2.1	1.5
	1992	2.7	2.1	2.0	2.4
Students Enrolled in:					
Eighth-Grade Mathematics	1996	3.9	3.7	2.8	2.5
	1992	3.3	2.7	3.4	2.9
Pre-Algebra	1996	4.0	3.3	3.4	1.8
	1992	4.6	3.2	2.4	3.9
Algebra	1996	4.2	3.4	1.6	1.6
	1992	3.9	3.1	2.4	2.7
Grade 12					
Students Taking Mathematics	1996	1.1	0.9	0.3	0.5

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Figure B9.3 Standard Errors for Percentage of Students Who Report Using Scientific Calculators, Grades 8 and 12, 1996

THE NATION'S REPORT CARD naep

Grade 8	
All Students	2.1
Eighth-Grade Mathematics	2.2
Pre-Algebra	2.9
Algebra	3.0
Grade 12	
Students Taking Mathematics	1.3

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Figure B9.4

Standard Errors for Percentage of Students Who Report Using Graphing Calulators, Grades 8 and 12, 1996

THE NATION'S REPORT CARD naep

Grade 8	
All Students	1.1
Eighth-Grade Mathematics	0.8
Pre-Algebra	2.3
Algebra	2.3
Grade 12	
Students Taking Mathematics	2.0

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.19

Standard Errors for Percentage of Students by Teacher Reported Uses of Calculators, Grades 4 and 8

THE NATION'S REPORT CARD naep

	Assessment Year	Teachers Allow Unrestricted Use in Classroom	Teachers Allow Use on Mathematics Tests
Grade 4			
All Students	1996	1.8	1.7
	1992	1.1	1.1
Grade 8			
All Students	1996	2.9	2.6
	1992	2.3	3.0
Students Enrolled in:			
Eighth-Grade Mathematics	1996	4.0	3.7
	1992	2.7	3.6
Pre-Algebra	1996	4.8	3.9
	1992	4.2	4.1
Algebra	1996	5.0	3.1
	1992	4.0	3.6

SOURCE: National Center for Education Statistic, National Assessment of Educational Progress (NAEP) 1992 and 1996 Mathematics Assessments.

Table B9.20

Standard Errors for Percentage of Students by Calculator Use, Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD naep

	Calculator Use			
	Appropriate Calculator Use Group		Other Group	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4				
All Students	0.8	1.5	0.8	1.1
Unrestricted Classroom Use	2.4	5.0	2.4	2.7
Restricted Classroom Use	0.9	1.6	0.9	1.4
Allowed Use on Classroom Tests	2.8	5.2	2.8	3.0
Not Allowed Use on Classroom Tests	0.9	1.6	0.9	1.2
Grade 8				
All Students	0.9	1.8	0.9	1.0
Unrestricted Classroom Use	1.6	3.0	1.6	1.8
Restricted Classroom Use	0.9	2.2	0.9	1.8
Allowed Use on Classroom Tests	1.3	2.5	1.3	1.5
Not Allowed Use on Classroom Tests	1.2	2.9	1.2	2.1
Grade 12				
All Students	0.7	1.2	0.7	1.0
Use in Classwork:				
Almost Every Day	0.7	1.2	0.7	1.1
Once or Twice a Week	1.6	3.4	1.6	1.7
Once or Twice a Month	2.4	***	2.4	3.0
Never or Hardly Ever	1.5	2.5	1.5	1.7
Use on Tests or Quizzes:				
Almost Every Day	0.8	1.4	0.8	1.3
Once or Twice a Week	1.4	1.7	1.4	1.4
Once or Twice a Month	1.7	2.5	1.7	1.9
Never or Hardly Ever	1.0	2.8	1.0	1.4

NOTE: Students in the "Appropriate Calculator Use" group used the calculator for at least 65 percent of the calculator-suitable questions and used the calculator for no more than one of the calculator-unsuitable questions. Students in the "Other" group used the calculator for less than 65 percent of the calculator-suitable questions and/or used it for more than one of the calculator-unsuitable questions.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.21 Standard Errors for Percentage of Students by Frequency with Which Students Take Mathematics Tests, Grades 4, 8, and 12, 1996*

THE NATION'S REPORT CARD NAEP

	Frequency							
	Almost Every Day		Once or Twice a Week		Once or Twice a Month		Never or Hardly Ever	
	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score	Percentage of Students	Average Scale Score
Grade 4								
All Students	0.4	4.7	2.3	1.7	2.5	1.0	1.1	4.9
Grade 8								
All Students	0.3	***	3.1	1.7	3.1	1.7	0.1	***
Students Enrolled in:								
Eighth-Grade Mathematics	0.2	***	3.9	2.7	3.8	2.0	0.0	***
Pre-Algebra	0.3	***	5.1	1.8	5.1	2.4	0.2	***
Algebra	0.8	***	5.8	2.7	5.9	2.2	0.1	***
Grade 12								
Students Taking Mathematics	0.4	3.6	1.3	1.2	1.2	1.5	0.2	***

* Data on fourth- and eighth-grade students are based on teachers' reports, and data on twelfth-grade students are based on students' reports.

*** Sample size is insufficient to permit a reliable estimate.

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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Table B9.22

Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Multiple-Choice Tests to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996

THE NATION'S REPORT CARD

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	1.0	2.7	2.1	2.2
Grade 8				
All Students	0.9	3.1	3.1	2.8
Students Enrolled in:				
Eighth-Grade Mathematics	1.1	4.1	4.2	4.5
Pre-Algebra	1.0	4.2	4.5	4.3
Algebra	0.8	4.1	4.0	3.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.23

Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Short and Long Written Responses to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996

THE NATION'S REPORT CARD

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	2.6	2.4	2.0	1.9
Grade 8				
All Students	2.8	3.7	2.5	2.3
Students Enrolled in:				
Eighth-Grade Mathematics	3.6	4.9	3.2	3.6
Pre-Algebra	4.3	5.2	2.9	3.0
Algebra	2.8	4.1	4.0	3.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.24

Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Individual or Group Projects or Presentations to Assess Their Students' Progress in Mathematics, Grades 4 and 8, 1996

THE NATION'S REPORT CARD

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	2.2	1.7	2.4	2.4
Grade 8				
All Students	1.5	3.2	3.5	3.0
Students Enrolled in:				
Eighth-Grade Mathematics	1.6	4.5	4.9	3.8
Pre-Algebra	3.1	4.4	4.2	3.5
Algebra	2.0	4.5	4.5	3.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B9.25

Standard Errors for Percentage of Students by Teachers' Reports on the Frequency with Which They Use Portfolio Collections of Each Student's Work to Assess Students' Progress in Mathematics, Grades 4 and 8, 1996

THE NATION'S REPORT CARD

	Frequency			
	Once or Twice a Week	Once or Twice a Month	Once or Twice a Year	Never or Hardly Ever
Grade 4				
All Students	1.8	2.4	1.9	2.2
Grade 8				
All Students	2.1	2.5	2.5	3.7
Students Enrolled in:				
Eighth-Grade Mathematics	3.2	2.5	4.1	5.1
Pre-Algebra	2.7	3.4	2.4	4.8
Algebra	2.9	4.2	2.8	5.2

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B10.1

Standard Errors for Percentage of Students by Their Response to the Statement: "I Like Mathematics," Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD NAEP

	Agreement		
	Agree	Disagree	Undecided
Grade 4			
All Students	0.9	0.8	0.6
Grade 8			
All Students	1.1	0.7	0.8
Students Enrolled in:			
Eighth-Grade Mathematics	1.6	1.2	1.1
Pre-Algebra	1.6	1.2	1.1
Algebra	1.5	1.0	1.4
Grade 12			
All Students	0.8	0.8	0.6
Students Who Are:			
Enrolled in Mathematics	1.1	0.8	0.6
Not Enrolled in Mathematics	1.1	1.4	0.9
Students Who Have:			
Taken Geometry	0.9	0.8	0.5
Not Taken Geometry	1.9	1.9	1.4
Highest Algebra-Calculus Course Taken:			
Pre-Algebra	3.4	3.5	2.5
First-Year Algebra	2.1	2.2	1.4
Second-Year Algebra	1.2	1.1	0.8
Third-Year Algebra/Pre-Calculus	2.4	1.8	1.5
Calculus	3.6	1.6	2.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B10.2

Standard Errors for Percentage of Students by Their Response to the Statement: "If I Had a Choice, I Would Not Study Any More Mathematics," Grades 4, 8, and 12, 1996

THE NATION'S REPORT CARD NAEP

	Agreement		
	Agree	Disagree	Undecided
Grade 4			
All Students	0.6	0.9	0.7
Grade 8			
All Students	0.6	0.9	0.6
Students Enrolled in:			
Eighth-Grade Mathematics	1.0	1.5	1.0
Pre-Algebra	1.1	1.3	1.3
Algebra	1.1	1.3	1.0
Grade 12			
All Students	0.8	0.9	0.6
Students Who Are:			
Enrolled in Mathematics	0.7	1.0	0.6
Not Enrolled in Mathematics	1.6	1.2	1.1
Students Who Have:			
Taken Geometry	0.9	1.1	0.7
Not Taken Geometry	1.8	2.0	1.7
Highest Algebra-Calculus Course Taken:			
Pre-Algebra	3.8	2.6	3.1
First-Year Algebra	1.7	1.8	1.2
Second-Year Algebra	1.2	1.3	0.9
Third-Year Algebra/Pre-Calculus	1.4	2.1	1.3
Calculus	1.8	3.8	2.6

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

Table B10.3: **Standard Errors for Percentage of Students by Their Response to the Statement: "Everyone Can Do Well in Mathematics If They Try," Grades 4, 8, and 12, 1996**

THE NATION'S REPORT CARD

	Agreement		
	Agree	**Disagree**	**Undecided**
Grade 4			
All Students	0.5	0.3	0.5
Grade 8			
All Students	0.8	0.6	0.5
Students Enrolled in:			
Eighth-Grade Mathematics	1.2	1.0	0.8
Pre-Algebra	1.4	0.8	1.0
Algebra	1.5	1.2	0.9
Grade 12			
All Students	0.8	0.7	0.6
Students Who Are:			
Enrolled in Mathematics	1.0	0.8	0.8
Not Enrolled in Mathematics	1.2	1.4	1.1
Students Who Have:			
Taken Geometry	1.0	0.8	0.7
Not Taken Geometry	1.6	1.6	1.0
Highest Algebra-Calculus Course Taken:			
Pre-Algebra	3.5	3.3	2.7
First-Year Algebra	1.5	1.3	1.3
Second-Year Algebra	1.1	1.0	1.0
Third-Year Algebra/Pre-Calculus	2.3	1.6	1.8
Calculus	2.7	2.7	1.9

SOURCE: National Center for Education Statistics, National Assessment of Educational Progress (NAEP) 1996 Mathematics Assessment.

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