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ABSTRACT

Mathematics professional communities were urged to make a strong, coordinated, and visible commitment to strengthening the mathematical preparation of elementary teachers. The Texas Statewide Systemic Initiative (Texas SSI) recommends that teachers complete a minimum of nine credit hours in mathematics. Texas SSI further recommends that these courses reflect the guidelines presented in this document. These guidelines were developed for the use of mathematics faculty who teach or design courses for preservice teachers at institutions of higher education in Texas, although they may be adapted for different audiences. The first section of this document outlines guidelines for the mathematics component of the elementary teacher education program and applies to the mathematics courses that are studied by elementary teacher candidates. The second section of the document describes the responsibilities and the role of assessment in supporting instruction and content. (Contains 23 references.) (ASK)

GUIDELINES
for the
MATHEMATICAL PREPARATION
of
PROSPECTIVE ELEMENTARY TEACHERS

THE TEXAS STATEWIDE SYSTEMIC INITIATIVE
THE CHARLES A. DANA CENTER

Guidelines
for the
Mathematical Preparation
of
Prospective Elementary Teachers

The Texas Statewide Systemic Initiative (Texas SSI)

Texas SSI Action Team
on
Strengthening the Mathematical Preparation of Elementary Teachers

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INTRODUCTION

A diverse group of K–16 educators was commissioned by the Texas Statewide Systemic Initiative, with additional funding by the Fund for the Improvement of Postsecondary Education, to develop a set of guidelines for the mathematics content courses required for the preparation of elementary teachers. The charge to this group is to affect the preparation of teachers in order to better prepare them to guide students in the uses of mathematics in today's world.

PURPOSE OF THIS DOCUMENT

Forward-looking visions of teaching and learning mathematics at elementary, secondary, and postsecondary levels are described in such recent publications as *Everybody Counts* (National Research Council [NRC], 1989), *The Curriculum and Evaluation Standards for School Mathematics* (National Council of Teachers of Mathematics [NCTM], 1989), *Professional Standards for Teaching Mathematics* (NCTM, 1991), *Crossroads for Mathematics: Standards for Introductory College Mathematics before Calculus* (American Mathematical Association of Two-Year Colleges [AMATYC], 1995), and *A Call for Change* (Mathematics Association of America [MAA], 1991). Accomplishing the vision is highly dependent upon the mathematical preparation of teachers. In *Mathematical Preparation of Elementary Teachers: Issues and Recommendations (Draft)*, a publication of the Mathematical Sciences Education Board (1995), mathematics professional communities were urged to make a strong, coordinated, and visible commitment to strengthening the mathematical preparation of elementary teachers. At the national meeting "On the Mathematics Preparation of Elementary School Teachers," the need to transform the mathematical preparation of preservice elementary teachers was clearly articulated.

> The mathematics community should establish a set of professional teaching guidelines at the college level [for preservice teachers] analogous to the NCTM, Teaching Standards for School Mathematics [*Professional Standards for Teaching Mathematics*].[1]

[1] Cipra, B., Ed., (1992), "On the Mathematical Preparation of Elementary School Teachers." Report of a conference held at The University of Chicago, p. 14.

These guidelines for the mathematical preparation of elementary teachers are the first steps to effect that change. They reflect and complement existing descriptions of new visions of mathematics instruction for elementary students, as well as postsecondary students. In order for the visions to be realized, preservice teachers must have substantive mathematics coursework as part of their preparation program. Many professional organizations have made recommendations for a minimum number of mathematics courses for preservice teachers.[2] In Texas, there are many variations in teacher preparation programs. Texas Statewide Systemic Initiative (Texas SSI) recommends that teachers complete a minimum of nine credit-hours in mathematics. Further, Texas SSI recommends that these courses reflect the guidelines presented in this document.

[2] See documents from Texas Association of Academic Administrators in the Mathematical Sciences, Mathematics Association of America, National Council of Teachers of Mathematics, Mathematical Sciences Education Board, and The American Mathematical Association of Two Year Colleges.

These guidelines may be used in different ways by different audiences. However, they have been developed for the use of mathematics faculty who teach or design courses for preservice teachers at institutions of higher education in Texas. Further, these guidelines are not intended to be a checklist for certification nor a compliance document; rather, they are general statements about what is important in order to:

- gauge the general quality of preservice mathematics courses, and

- support and promote requisite changes in institutions of higher learning.

THE CASE FOR REFORM

Mathematics changes as need dictates and as different tools become available. The requirements for understanding and applying mathematics today are much different from those necessary as little as ten years ago. Today's questions are more challenging and complex, and the mathematics used to investigate them is more diverse. Mathematics is increasingly seen as one way to help make sense of the world. Although there are many reasons for this change in the use of mathematics, much can be attributed to readily available, surprisingly affordable, and increasingly sophisticated technology. The reality of today's technology—and the promise of tomorrow's—changes the depth and breadth of the mathematics that can and needs to be studied and provides access to information that requires mathematical understanding for interpretation and decision making. Consequently, there is a change in the nature of the mathematics that can and must be taught—as well as the ways in which mathematics can be taught. These changes hold true at every level of mathematics teaching and learning from K–12 schools through institutions of higher education.

The change in the nature of the mathematics that needs to be learned and that can be taught implies changes in mathematics curriculum. These changes might be described as a shift in expected mathematical learning; that is, the focus is on the ability to use and apply mathematics and mathematical ideas in a manner that facilitates making relationships, generalizing, and predicting. Teaching for understanding implies teaching skills nested within the context of problem solving. Today's students have access to a rich and sophisticated level of mathematics, and applying this mathematics requires a deep understanding of mathematical principles and relationships.

In Texas, however, many elementary teachers are not adequately prepared to teach the mathematics of today. A recent study by the Texas SSI found that many institutions that certify elementary teachers require or offer a limited number of mathematics courses targeted for the elementary teacher; some offer none.[3] Additionally, many of the courses which are offered overlook some of the critical elements required to teach mathematics in today's schools, such as the use of technology.[4] In general, the mathematical preparation of elementary teachers in Texas is inconsistent with the large variations in programs and requirements.

[3] Texas Statewide Systemic Initiative (1995), *Strengthening the Mathematical Preparation of Prospective Teachers in Texas: Results of a Survey of Mathematics and Education Department Chairs of 2- and 4-Year Institutions of Higher Education in Texas.*

[4] Texas Education Agency (1992), *Report on the Statewide Assessment of Teacher Preparation Standards and Certification Requirements for Mathematics and Science.*

The lack of elementary teachers' mathematical preparation to teach today's mathematics may be further reflected by the performance of students in Texas. Scores on the Texas Assessment of Academic Skills (TAAS) are far below the state's goals for Texas students. TAAS is not solely an assessment of computational skills, algorithms, and procedures as were the state assessments of the past; it is a broader assessment of mathematical understanding that stresses concepts and mathematical ideas, with a focus on the ability to solve problems. Although student performance on this minimum measure is improving, the performance of students from historically underrepresented populations continues to be far below the state average. Access to mathematics for all students continues to be an issue, and the less than robust performance of students indicates that mathematics instruction may not be consistent with the state's vision for mathematics learning.

The need to improve the mathematical preparation of elementary teachers is not limited to Texas. At the national meeting "On the Mathematics Preparation of Elementary School Teachers," a central problem in mathematics education was discussed and highlighted:

> Plainly put, teachers tend to teach the way they themselves were taught. If teachers of our children are only exposed to mathematics as an arcane collection of rules and formulae taught in a dry lecture-drill format, we can hardly expect them to discard that model for unfamiliar alternatives.[5]

[5] Cipra, B. (1992), p. 4.

This restrictive approach to learning has generated a cycle that denies students at all levels true access to meaningful mathematical learning. However, in preparation programs for elementary teachers there is an opportunity to break this cycle and provide preservice teachers with alternatives for learning and teaching mathematics. To develop more effective teacher preparation programs, the critical factors of mathematics content, instruction, and assessment in the preparation of preservice teachers must be considered.

A goal of mathematics courses for preservice teachers is to help them learn how to learn mathematics in an environment of good teaching practices, which in turn can help them learn how to teach mathematics. Significant changes in K–12 mathematics instruction can be made by preparing teachers who themselves receive high quality mathematics teaching preparation. Faculty need to employ a wide variety of teaching styles while taking into consideration the needs of various learners. Preservice teachers should be presented with mathematically challenging tasks and encouraged to work through them in an environment where it is acceptable to take risks and where there is ample room for trial and error.[6]

To continue to provide K–12 students at all levels with an appropriate mathematics education that prepares them for tomorrow's world, it is essential that teacher preparation be reexamined and restructured. This examination process should focus on the mathematical preparation and mathematical understanding of the preservice teacher. Mathematics faculty at institutions of higher education are challenged to be active participants in addressing changes in mathematics teaching and learning for prospective teachers. These guidelines are intended to facilitate this process.

[6] Billstein, R. (1993), "Improving K–8 Preservice Mathematics Education in Departments of Mathematics," in *Proceedings of the National Science Foundation Workshop on the Role of Faculty from the Scientific Disciplines in the Undergraduate Education of Future Science and Mathematics Teachers*, pp. 146–149.

Assumptions

In developing this set of guidelines, the writing team has operated under the following set of assumptions:

- ☐ All people can benefit from experiences with mathematical ideas and the understanding of those ideas.

Mathematics, in its many forms and representations, is everywhere. It is a genuine, important, and evident part of the world. A deep, broad range of mathematical experiences and understanding provides a person an enhanced appreciation for and perspective about the world. Every piece of mathematical understanding, no matter how elementary, allows one to view the world a little differently and a little more completely. This enhanced perspective of the world is benefit enough. However, it also serves to foster mathematical power and literacy, which maximize opportunities and choice.

☐ The process of teaching mathematics and learning mathematics is iterative: the way preservice elementary teachers are taught influences their understanding of and beliefs about mathematics; their understanding of and beliefs about mathematics influence the way they teach; and the way they teach influences their students' understanding of and beliefs about mathematics.

Faculty who teach preservice elementary teachers can have a significant impact on elementary classroom practice through the instructional strategies they model. The teaching of mathematics can be approached in such a way as to foster mathematical understanding for everyone regardless of prior experience. The challenge for higher education faculty is to ensure that excellent instructional practices enable preservice elementary teachers to build mathematical understanding. Hence, higher education faculty significantly influence the education of young children.

☐ The teaching of mathematics not only requires knowledge of content and pedagogy, but also requires an understanding of the relationship and interdependence between the two.

Teaching mathematics to children requires significant content knowledge, i.e., knowledge of and an understanding about mathematics. However, teachers must also have pedagogical knowledge—knowledge about the different ways children take in, process, and retain information, as well as the teaching strategies that are appropriate for the varied ways children learn. Yet, having content and pedagogical knowledge is not enough. For teachers to be effective at teaching mathematics, there are aspects of mathematics that a teacher must know that are related to but different from pedagogy and content. This is pedagogical content knowledge—knowledge about the interaction between the content and pedagogy; it is the knowledge that allows for instructional decisions to be made in light of the mathematical content to be taught.[7] Pedagogical content knowledge creates a depth of understanding that provides the classroom teacher with the capability to make instructional decisions that are appropriate—decisions that will lead to rich mathematical experiences for their students.

- ☐ The responsibility for shaping the way teachers teach mathematics is a shared responsibility among all institutions of higher education, particularly mathematics departments and teacher preparation units, elementary and secondary schools, and the community at large.

[7] For information on Pedagogical Content Knowledge, see Shulman (1987).

The education of well-prepared mathematics teachers is crucial not only for the students they teach but also for the economic, civic, and cultural well-being of our society. There are many stakeholders that share responsibility for the preparation of teachers. Just as the problems in teacher preparation are not attributable to any one stakeholder, solving the problems cannot be assigned to any single group. Rather, everyone must come together to seek solutions. To begin to solve the problems, all institutions of higher education, elementary and secondary schools, and the community at large must reshape the way teachers are prepared to teach mathematics. Changing the way teachers are prepared is a shared responsibility and needs to be recognized as a long-term commitment. This implies that between the stakeholders there must exist continual and open communication as well as a willingness to serve as a mutual support structure. Acting alone, any one stakeholder can influence local changes in teacher preparation. However, it is only in working together that lasting, systemic change can occur.

Organization of This Document

This document is organized around two central themes. The first section outlines guidelines for the mathematics component of the elementary teacher education program and applies to the mathematics courses that are studied by elementary teacher candidates. It describes the mathematics content that should be valued, instructional practices that facilitate the expected mathematical understanding, and the role of assessment in supporting instruction and content. The second section of the document describes the responsibilities of the preservice teachers who take these courses, of the faculty who teach these courses, and of the institutions that house these courses.

In the context of this document, the term *preservice teachers* refers to those students in higher education seeking elementary teacher certification. *Students* refers to those students, present and future, in an elementary classroom. *Institutions* refers to institutions of higher education—public and private, two- or four-year, while *schools* refers to the public K–12 systems. *Faculty*, whether tenured, tenure-track, adjunct, or graduate student, refers to personnel who teach mathematics courses for preservice teachers. *Teachers* refers to the teachers in the K–12 schools.

INSTRUCTION ASSESSMENT CONTENT

GUIDELINES

Preservice teachers' own experiences in learning mathematics greatly influence the way they will teach mathematics to their future elementary students. Thus, mathematics courses for prospective elementary teachers must be designed to provide them with the positive experiences in mathematical content, instruction, and assessment as described in these guidelines. These courses should be rich in mathematics and should focus on big ideas that connect and describe all mathematics. It is essential that faculty who teach these courses choose appropriate mathematical tasks and establish environments in which preservice teachers can explore mathematics. Development and practice of important skills should be embedded in a variety of problem-solving contexts. Assessment should be an ongoing, integral component of instruction, primarily focused on supporting and enhancing the learning process. Although content, instruction, and assessment cannot be separated in classroom practice, there are aspects of each that need to be addressed. These are detailed in the following sections.

Content

The content suggested in these guidelines represents the mathematics that is important for preservice teachers to know. The task of faculty is to design experiences based on these guidelines that encourage preservice teachers to explore mathematics from different perspectives. In this way, they can interact with mathematical ideas in ways that help them build a more substantial understanding of mathematics and provide a variety of approaches to mathematics.

The content described in these guidelines is not intended as a review of elementary arithmetic. It is intended to strengthen the preservice teachers' content knowledge as well as develop the connections between mathematics content and pedagogy that will lead to their making effective instructional decisions in the K–8 classroom. The content should reflect the Texas Essential Knowledge and Skills for K–8 mathematics, as well as the mathematics reflected in the Examination for the Certification of Educators in Texas (ExCET). Individuals who are proficient in mathematics, as well as those who have arithmetic deficiencies or limited mathematical experiences, can benefit from interacting with mathematics from a perspective that fosters both content and pedagogical content understanding. All preservice teachers should find the recommended content challenging, engaging, and informative.

The content guidelines are organized into five broad strands, with each strand subdivided into several descriptive areas. Each descriptive area contains examples of mathematical tasks that can promote the suggested level of thinking and understanding. It is important to emphasize that the broad strands described in the content guidelines are neither discrete, complete, nor sequential. They are not intended to represent a specific course nor a preferred way to organize or compartmentalize the mathematics content. Rather, this is one of many ways to organize a description of the necessary and meaningful mathematical experiences for preservice teachers. These guidelines are provided as a framework so that faculty at any institution can develop the mathematical experiences that best suit the needs of their preservice teachers.

Investigating Numbers and Their Properties

Most students of mathematics, including preservice teachers, are familiar with many definitions and procedures related to the concept of number and number properties. The elements outlined in this section are designed to help preservice teachers focus on patterns and relationships that provide the foundations of the definitions and procedures they already know. It is important for preservice teachers to explore these patterns and relationships through activities which include descriptive, concrete, pictorial, and symbolic perspectives. Suggested experiences include opportunities to:

- Create definitions of number and number properties through experiences which emphasize sorting and classifying.

- definitions
  number theory
  algebraic thinking
  counting
  whole numbers
  integers
  rational numbers
  real numbers

In their own elementary and secondary mathematics experiences, many preservice teachers have been exposed to an extensive mathematics vocabulary. Exposure to or memorization of a definition, however, does not guarantee understanding of the term nor its relationships and connections within the greater structure of mathematics. Preservice teachers need opportunities to explore this structure in depth, observe essential numerical characteristics and relationships within a variety of contexts, experiment with classification strategies, and construct definitions. These experiences help preservice teachers better understand the elements and properties of mathematics and the language that describes them.

Square Numbers

Use tiles, sticks, graph paper, and ideas about geometry and number to develop three ways to identify a square number.

CONTENT

NUMBER PATTERNS AS FUNCTIONS

Find patterns in the following sets of numbers and use algebraic notation to describe the pattern in a general form. Use a graphing calculator to compare the graphs of the functions that you have described algebraically to the sets of numbers.

a) 1, 3, 5, 7, 9, 11, ...
b) 2, 5, 10, 17, 26, ...

☐ Analyze the common algorithms used within the context of number relationships and justify those algorithms using sound arguments based on number properties.

○ number theory
factors and multiples
logical reasoning

There are many algorithms associated with numbers and their properties. Examples include the many divisibility rules and algorithms for finding prime factors, common factors, and least common multiples. Preservice teachers need to use their knowledge of number properties and relationships to explore, compare, analyze, and justify these algorithms. These experiences help the preservice teachers develop an understanding of the patterns, relationships, and structure of "number."

LEAST COMMON MULTIPLES

Analyze two different methods for finding the least common multiple for 27 and 36, and explain why each method reliably provides the least common multiple for the pair. Next, compare the two methods and describe how they are related.

DIVISIBILITY RULES

How would you explain to your students why it is sufficient to check only for a 0, 2, 4, 6, or 8 in the units place to determine if a number is even? Does this "rule" generalize to all bases? Justify your answer.

☐ Compare and contrast characteristics of numeration systems.

○ number sense
place value
history (of mathematics)

The characteristics of the base ten numeration system are so familiar that most individuals are unaware that other valid numeration systems exist. In addition, most individuals are unaware that many of the common arithmetic algorithms are directly related to the properties of place value. Exploring the characteristics of a variety of number systems helps to develop a better understanding of the base ten system and the role of place value. In order to put the base ten numeration system in context, preservice teachers need the opportunity to explore the historical development of ancient number systems, the development of a zero symbol, the reasons for the selection of the base ten system, and contexts in which other numeration systems and bases are used.

Numeration Systems

For each of the following characteristics, describe two numeration systems in which it is found and discuss the advantages it brings to those numeration systems.

- *place value*
- *a symbol for zero*

Patterns in Multiples of 10

One of your students knows a nice little "trick" that says to multiply a number by 10 all you have to do is tack a zero onto the end of the original number (e.g., 10 times 123 equals 123 with a zero on the end: 1230). How would you explain to your student why this works?

☐ Compare and contrast multiple representations of rational numbers.

○ number sense
technology
ratio
proportional reasoning

Many precollege mathematics textbooks, as well as elementary text series, organize different forms of the rational numbers (integers, fractions, decimals, and percents) into separate topics and connect them to each other only for the purpose of converting from one form to another. Consequently, many preservice teachers have had little or no opportunity to compare and contrast different forms of rational numbers. It has been sufficient to use fractions in the fraction lesson, decimals in the decimal lesson, and percents in the percent lesson without ever needing to think about why one might choose one form over another. Rational numbers have a variety of representations, some of which are more useful in some contexts than others. Understanding the relationship among the forms of rational numbers facilitates making a choice. For example, understanding the relationship between fractions and decimals helps one to use technology more efficiently and interpret solutions appropriately. Preservice teachers need opportunities to analyze whole-to-whole, part-to-whole, and part-to-part relationships from a variety of perspectives. These types of experiences will help preservice teachers develop a better understanding of how equivalent representations of rational numbers are related, how to choose the representation most appropriate for a particular context, and how to interpret the results within that context.

Decimals versus Remainders

Suppose you are teaching fourth grade and a student comes to you and says, "I worked the problem 188 divided by 8 and got 23 remainder 4. My partner worked it on the calculator and got 23.5. Which answer is right?" How would you explain to your student the ways in which these two answers are the same? How would you explain the ways in which they are different?

PREDICTIONS WITH FRACTIONS AND PERCENTS

This statement appeared in a newsletter on health: "About one in 10 Americans eventually develops a kidney stone. Four out of five people who develop kidney stones are men." What fraction of American men are expected to develop a kidney stone? What percent of American women are expected to develop a kidney stone? What is the probability that an American man will develop a kidney stone? Justify your answers.[8]

[8] Stanley, D. (1994, October 10), "Kidney Stones" problem from the Balanced Assessment Project (Task ID H2070a).

REPRESENTING OPERATIONS AND DEVELOPING COMPUTATIONAL ALGORITHMS

Preservice teachers need opportunities to expand their understanding of each arithmetic operation through exploration of physical contexts in which the operations naturally arise and of the relationship between the contexts and the symbolic algorithms. They also need opportunities to analyze and compare algorithms in order to understand the similarities and relationships among them. Suggested experiences include opportunities to:

- Explore the multiple interpretations and contexts for each arithmetic operation.

- problem solving
  addition
  subtraction
  multiplication
  division

The choice of an arithmetic operation is dependent on context, and for each arithmetic operation there are many contexts which may seem distinct and unrelated. Also, there are different interpretations for each operation. For example, multiplication can be interpreted as a collection of equivalent sets or as the area of a rectangular region. A situation also determines the reasonableness of an operation or a solution. Although the commutative property holds true in the abstract sense, 2×3 represents a different physical situation than 3×2. Preservice teachers need an opportunity to explore and carefully analyze a variety of contexts and interpretations for which a given operation may be appropriate.

THE MEANING OF QUOTIENTS

Write a problem situation in which 23 r4 is a more appropriate answer than 23.5 to the problem 188 ÷ 8. Describe a situation in which 24 is a more appropriate answer than 23.5. Describe a situation in which 23 is a more appropriate answer than 23.5. In what types of settings would 23.5 be the appropriate answer?

RELATIONSHIPS BETWEEN MULTIPLICATION AND DIVISION

How could you use pictures to help a student understand why $\dfrac{10}{\frac{1}{2}}$ *is equivalent to* $10 \cdot 2$ *and different from* $\dfrac{10}{2}$*?*

☐ Analyze and describe relationships between physical and pictorial representations and the associated symbolic algorithms.

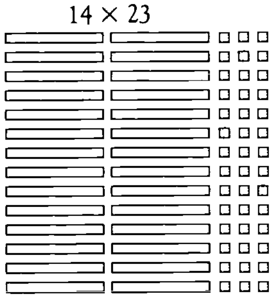
○ multiple representations
algorithms
algebraic thinking

The arithmetic operations have their roots in action: adding, subtracting, multiplying, and dividing represent physical actions–physical operations. Preservice teachers need to have an opportunity to explore and analyze the ways in which an abstract manipulation of symbols (i.e., an algorithm) for a given operation can be a result of the physical manipulation of concrete objects. The transition to the abstract is facilitated by drawing illustrations of these operations and by using the operations in context. Connecting these different representations builds an understanding of symbolic algorithms and a strong basis for the use of these algorithms in numerical and algebraic contexts.

MODELS FOR MULTIPLICATION

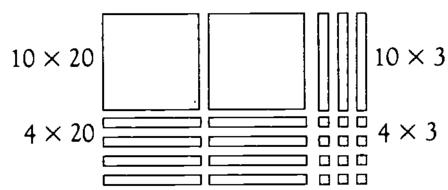
How are the repeated addend and area models of the product 14 × 23 connected to the partial products algorithm?

14 × 23



Repeated Addends Model

14 sets of 23



Area Model

Partial Product Algorithm

23	
× 14	
12	(4 × 3)
80	(4 × 20)
30	(10 × 3)
200	(10 × 20)

BINOMIAL MULTIPLICATION

How can you use the partial products algorithm to illustrate the product of $(x + y)^2$?

☐ Develop, compare, and contrast multiple algorithms for the arithmetic operations.

○ algorithms
logical reasoning
algebraic thinking

There are many valid algorithms for each arithmetic operation. For example, algorithms that individuals use for mental calculations are different from those that they would use for paper-and-pencil computation. When given the opportunity to compare arithmetic strategies, individuals are often surprised to discover that there exists a variety of sophisticated algorithms. Preservice teachers need opportunities to explore the basic operations; develop algorithmic strategies based on their understandings of the operations; compare, contrast, and analyze new and existing algorithms; and justify the validity of each algorithm within the context of its use.

WHOLE NUMBER ALGORITHMS

How is the partial products algorithm for 14 × 23 related to the traditional paper-and-pencil algorithm?

DECIMAL ALGORITHMS

How could you use an understanding of fractions and of the multiplication of fractions to help your students develop an algorithm for multiplying decimals?

EXPLORING SPACE AND SHAPE

Many preservice teachers have been exposed to definitions and formulas in geometry. However, few may have had the opportunity to explore carefully the geometric structures from which these definitions and formulas evolved. Preservice teachers need opportunities to explore the characteristics of space and shape, analyze geometric relationships, and use geometry to solve problems. Suggested experiences include opportunities to:

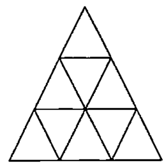
- Explore the geometric attributes of physical objects in order to classify and to form definitions.

- 2-D and 3-D figures
  definitions
  (figures and their) properties
  relationships

Preservice teachers need opportunities to physically manipulate, explore, and analyze geometric figures in order to build understandings of characteristics and relationships. These explorations should involve comparative observations of two- and three-dimensional figures. Characteristics to be explored should include congruence, symmetry, similarity, unique properties, and shared properties. Preservice teachers need opportunities to use their observations to construct organizational and classifying strategies for geometric figures. These observational and organizational activities will provide opportunities for preservice teachers to uncover relationships among figures.

Tessellations

Use geometry exploration software to test which shapes tessellate and which do not. Make a conjecture about shapes that tessellate.



Quadrilateral Classifications

There are many special types of quadrilaterals. Draw a Venn diagram showing the relationships among these different types of quadrilaterals. Label each set with its defining characteristics.

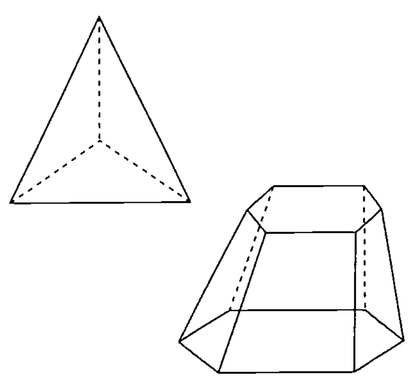
- ☐ Analyze spatial characteristics such as direction, orientation, and perspective.

○ congruence
rotations
reflections
translations
similarity
dilation
perspective
proportional reasoning

Three-dimensional geometric experiences which include direction, orientation in space, transformation, and perspective are often overlooked in precollege mathematics classes. Preservice teachers need opportunities to expand their spatial visualization skills so that they, in turn, will be able to help their students develop visualization skills. The skills to explore include drawing simple figures in perspective and interpreting two-dimensional representations of three-dimensional objects. Preservice teachers also need opportunities to make connections between physical transformations of objects, two-dimensional representations of those physical transformations, and "imagined" transformations that predict the results of physical transformations.

Transformations in Space

Apply your understanding of transformations in a plane to determine which characteristics of a rectangular prism are preserved under each of the following transformations in space: expansion, contraction, reflection, rotation, translation. Test the generalizability of your conjectures to other figures by applying them to a regular tetrahedron and an oblique truncated hexagonal pyramid.



Properties of Polyhedra

Use concrete models to find as many polyhedra as you can with 25 or fewer faces that are congruent equilateral triangles. What is the fewest number of triangles required? Which of your polyhedra are regular? Which are concave? Which are convex? Is there more than one way to construct distinct polyhedra that have the same number of faces? What are the unique characteristics of each of the polyhedra?

CONTENT

☐ Justify properties of and relationships among geometric figures.

○ logical reasoning
2-D and 3-D figures

Properties of and relationships among geometric figures are often provided in the form of definitions and theorems. However, preservice teachers also need opportunities to explore these properties and relationships from a physical point of view. They need to manipulate figures physically in order to develop strong intuitive understandings of the validity of properties and relationships. Their activities should include explorations of both two- and three-dimensional figures, thus providing examples that lead to the elemental axioms and verification of the resulting theorems. For example, they may explore and justify the properties of parallel lines, angles formed by transversals through parallel lines, relationships among polygons, relationships among platonic and semiplatonic solids, and properties of diagonals in polygons.

QUADRILATERAL SYMMETRY

Quadrilaterals are often classified by sides and angles. For example, a parallelogram is a quadrilateral with two pairs of opposite sides parallel, or a square is a rhombus with a right angle. Classify the set of quadrilaterals according to the number of lines of symmetry each one has. How do these classifications compare to the ones made using sides and angles? How do lines of symmetry work for classifying triangles?

PROPERTIES OF QUADRILATERALS

Use geometry exploration software to draw a quadrilateral. Construct the midpoints of the sides of the quadrilateral. These midpoints form another quadrilateral. Use the software to change the shape of the original quadrilateral and observe the effects on the quadrilateral formed by the midpoints. Make a conjecture.

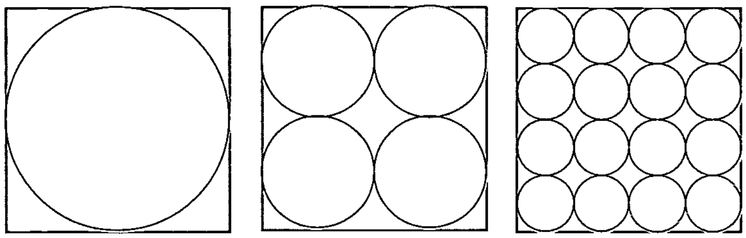
☐ Connect geometric ideas to number, measurement, probability, and algebra.

○ algebra
problem solving
probability & statistics

One application of geometry is to represent and study that which is known or being explored in other areas of mathematics. Preservice teachers need opportunities to explore the connections between geometry and quantity (e.g., triangular numbers and square numbers), between geometry and measurement (e.g., patterns in the dimensions and measurements of similar figures), between geometry and probability (e.g., representing probability with area models), and between geometry and algebra (e.g., symbolic representations of transformations).

CHANCES ON A DARTBOARD

The following three figures represent three dartboards of the same size where the diameters of the middle-sized circles are $\frac{1}{2}$ the diameter of the large circle, and the diameters of the small circles are $\frac{1}{2}$ the diameter of a middle-sized circle.



Which dartboard would you choose to give you the best chance of throwing a dart to land in the interior of a circle? Use mathematics to justify your answer.[9]

[9] Rice Bag Toss and Random Distribution (1991, January), NCTM *Student Math Notes*.

SIMILARITY AND MEASUREMENT

Select a tall structure on campus or in the community. Use properties of similar figures to develop a process for determining the height of the structure. Test your process. Compare your process and result to those of your classmates.

☐ Use geometric models to solve problems.

○ problem solving
algebra
probability & statistics
proportional reasoning

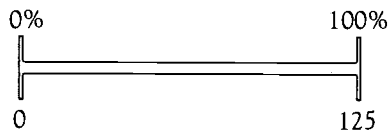
Because many mathematical ideas have geometric representations, preservice teachers need opportunities to explore the use of geometric models as a solution strategy in problem situations. Geometric models provide a visual or illustrative approach to problems, and they also encourage connections by highlighting certain attributes or properties that may not be as evident in nongeometric models. Examples include looking at the product of two factors as the area of a rectangular region, using a hundreds bar to estimate percent, using an area model to represent probability, and using the geometric representation of the Pythagorean Theorem.

MULTIPLE SOLUTIONS

If there are ten people in a room and each person shakes hands with every other person exactly once, how many handshakes will there be? How could you use triangular numbers to model the solution? How could you use polygons? How could you use an algebraic formula and a spreadsheet to solve for any number of people in the room?

Models for Percent

How could you use a hundreds bar to help a student estimate a reasonable amount for 36% of 125?



Using Measurement

The characteristics and complexities of measurement are often overlooked in precollege and college mathematics classes, and measurement experiences are often simplified to choosing a unit or formula for a particular situation. Preservice teachers need to explore measurement as a quantifying process for describing the physical world. In order to do this they need opportunities to develop understandings about the properties of the attributes to be measured; the role of error in measurement; the importance of the unit; the relationships among linear, area, and volume measures; and the types of information which can be communicated using measurement. Suggested experiences include opportunities to:

- ☐ Explore measurement as a process of identifying the attribute to be measured, quantifying the attribute by selecting and using an appropriate unit, and communicating information about the attribute using the unit of measure.

- ❍ non-standard and standard units
  measurement process

Measurement describes an attribute of an object or event. It is a process that involves isolating the attribute and quantifying it. Preservice teachers need experience identifying and isolating attributes, and comparing like attributes among unlike items. This can be accomplished with formal or informal measures and comparisons. A carefully chosen unit is a key idea in measurement. Preservice teachers need the opportunity to experiment with a variety of standard and nonstandard units in a given situation in order to discover the ways in which the choice of unit influences the information they have about the attribute being measured. Extensions of this understanding will underscore the need for standard units as a way to facilitate clarity of communication.

THE MEASUREMENT PROCESS

With three other classmates, design a method for measuring the strength of a magnet. In your design, what issues did you have to consider? Discuss with your group how those same issues are reflected in the ways we measure volume, area, length, mass, and temperature.

AREA AND VOLUME

Why are square units, as opposed to triangular or other rectangular units, used to measure area? Why are cubic units used to measure volume?

☐ Recognize the roles of approximation, estimation, and the effects of error in measurement.

○ approximation
estimation
error
proportional reasoning

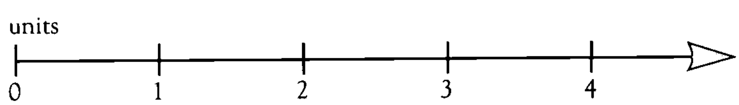
Measurements are approximations, and every measurement involves error. Because measurement is an inexact process, it is critical to recognize the existence of error in measurement and how the choice of unit affects the error. Preservice teachers need a variety of opportunities to develop these critical concepts in order to build a more complete understanding of measurement.

ACCURACY AND PRECISION IN MEASUREMENT

Measure the length of the table to the nearest meter, to the nearest decimeter, to the nearest centimeter, and to the nearest millimeter. Repeat, this time measuring the length of a paper clip. Compare each measurement to its greatest possible error, and discuss the effects on error as you measure with smaller units.

APPROXIMATION IN MEASUREMENT

On the scale below, draw segments of three different lengths that all measure 3 units when measured to the nearest unit. Justify your choices.



☐ Use measurement to collect data from which to recognize relationships and develop generalizations, including formulas.

○ perimeter
area
surface area
volume
measurement
formulas
algebraic thinking
functions

Preservice teachers need opportunities to experiment with different measurements in physical settings. These experiments should be designed to help preservice teachers build their understandings of relationships in measurement, how those relationships can be modeled by algebraic functions, and how those functions can then be used to draw conclusions and test predictions. In this way, preservice teachers can build a rationale for the need for formulas and a basis for understanding a formula's origin.

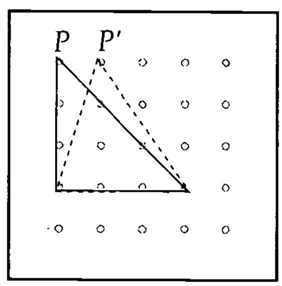
CONTENT

Surface Area Formulas

If you covered a yellow Cuisenaire rod with centimeter grid paper, how many squares would it take to cover the entire rod? What are the squares measuring? If you glued two yellow rods together along a long face to make a new block, how many squares would it take to cover the entire block? If you glued a third yellow rod to the other two along a long face, how many squares would it take to cover the block of three rods? If you glued "n" yellow rods together, how many squares would it take to cover it? Write a numerical expression to describe how you would determine the surface area of "n" yellow rods glued together.

Area of a Triangle

On a geoboard, use one rubber band to outline a triangle. Count the squares in the interior of the triangle to determine its area in square units. Move point P to the next peg (P′) to make a new triangle and determine its area. Continue to do this across the geoboard. How does the formula for the area of a triangle relate to your results?



☐ Apply measurement to solve problems.

○ problem solving
proportional reasoning
scale
rates of change

Measurement offers an abundant source of problem-solving opportunities and contextual settings. Preservice teachers need opportunities to develop understandings of the nature of measurement and the relationships among measurements, as well as opportunities to apply this knowledge in real-life situations. For example, preservice teachers need to be provided with opportunities to explore how changing an attribute affects the related measurements, how a change in unit affects the number of units needed to measure the same object, and how smaller units provide more precise measurements. Explorations should also include investigations of rates of change and the effects of scale transformations.

Proportional Reasoning and Indirect Measurement

Find the length of the Statue of Liberty's nose if you know that her forearm is 42 ft long. Compare your procedure with others' procedures, and discuss the advantages and disadvantages of each.

PIZZA PRICES

Should a personal-sized pizza that is 5″ in diameter cost half as much as a large pizza of the same type that is 10″ in diameter? Use mathematics to write a justification to explain why or why not.

CONTENT

MAKING GENERALIZATIONS, DRAWING CONCLUSIONS, AND MAKING PREDICTIONS

Patterns exist within mathematics, and mathematics allows us to describe situations. Because of this, one can generalize, conclude, predict, and hence, make well-informed decisions based on patterns. Preservice teachers need opportunities to analyze situations in order to generalize, conclude, and predict. This requires modeling situations, as well as recognizing and analyzing patterns. Suggested experiences include opportunities to:

☐ Explore various ways to organize data, and use the organization in order to make generalizations, draw conclusions, and make predictions.

○ sorting and classifying
patterns
data analysis

In precollege mathematics courses, the organization of data in problem situations is often provided, and as a result, preservice teachers may not possess well-developed organizational strategies. Preservice teachers need opportunities to freely explore, manipulate, and organize data. It is through these explorations that preservice teachers discover how different ways of organizing data can illuminate patterns and relationships. Once these patterns and relationships are identified, the preservice teachers can use them to make generalizations, draw conclusions, and make predictions.

DEFINITIONS OF PRIMES AND COMPOSITES

For each natural number from 1 to 15, make as many rectangular arrays of that number of squares as possible. Organize the data in order to keep track of the observations you make about the arrays for each number. Look for patterns, characteristics, and relationships among the numbers based on your observations of the arrays.

REPRESENTATION OF PHYSICAL CONDITION

In a recent study, a comparison of a person's height to that person's waist size was used as a measure of the person's general physical condition. Collect some data from the class about height and waist size. Use a graphing calculator to design a way to organize the data to determine whether or not a useful relationship exists between these two physical attributes. If such a relationship exists, find a mathematical representation of it. If not, make a conjecture about some other pair of characteristics that might better measure general physical condition and explain how you would test your conjecture.

☐ Recognize mathematical relationships, express those relationships with appropriate mathematical representations, and use those representations to make generalizations, draw conclusions, and make predictions.

○ algebraic thinking
functions
multiple representations

Preservice teachers need to experience abstract representations of mathematical relationships as efficient descriptions of physical relationships. However, for a given mathematical relationship, they first must have the opportunity to explore the physical characteristics of the relationship and then describe those characteristics in words and in pictures. Only after preservice teachers are comfortable with explorations using concrete and pictorial representations should they begin to experiment with the abstract symbols of algebra and coordinate geometry. Once preservice teachers are firmly grounded in the relationship between the physical world and the abstract representations of the physical world, they will then be able to use the abstractions as a tool for describing the world around them.

EULER'S FORMULA

Count the number of vertices, edges, and faces on several polyhedra. Organize this data in such a way that you can look for patterns. Use algebraic notation to write a general description of the patterns you find.

GENERALIZATIONS OF FORMULAS

Suppose new blocks are to be formed by gluing together two or more Cuisenaire rods of the same color along one of their long faces. Write a mathematical expression you could use to find the number of centimeter squares needed to cover a block made from any number of Cuisenaire rods of any one color. What variables are involved? What components of the blocks do you need to represent? How does your expression compare with others in the class? What types of questions can be answered with your mathematical expression?

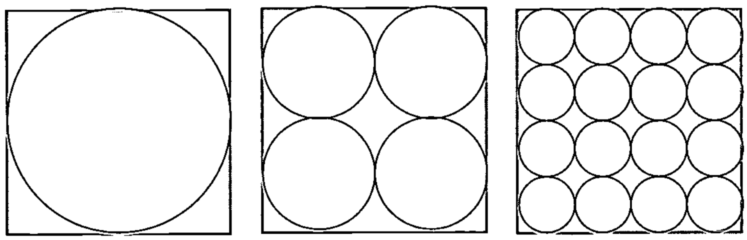
☐ Use probability and statistics to make generalizations, draw conclusions, and make predictions by collecting data, looking for relationships, and representing those relationships with appropriate statistical measures, tables, graphs, and charts.

○ data analysis
probability
statistics
multiple representations

People everywhere are being inundated with information that claims to be grounded in statistics. They need opportunities to scrutinize and evaluate the validity of conclusions based on statistical analyses and probability data. Particularly, preservice teachers need opportunities to build an understanding of the usefulness as well as the potential deceptiveness of probability and statistical analyses. They need opportunities to collect data, use the data to make and test conjectures, and make comparisons to the corresponding probability and statistical theory.

CHANCES ON A DARTBOARD-REVISITED

The following three figures represent three dartboards of the same size where the diameters of the middle-sized circles are $\frac{1}{2}$ the diameter of the large circle, and the diameters of the small circles are $\frac{1}{2}$ the diameter of a middle-sized circle.



On which dartboard do you have a higher probability of hitting a point in the interior of a circle? Justify your answer.[10]

[10] Rice Bag Toss and Random Distribution (1991, January).

REPRESENTATIONS OF DATA

Use a graphing calculator or statistical software package to view a set of continuous data in various forms—circle graph, bar graph, line graph, box-and-whisker graph. Evaluate which representation is most informative for this data. Why?

☐ Support generalizations, conclusions, and predictions with sound mathematical arguments using appropriate language and logical structures.

○ logical reasoning
proof

It is one thing to find an answer to a problem and quite another to justify convincingly that the answer is reasonable and accurate. Preservice teachers need to develop accurate and effective reasoning and communication skills so that they are able to support (reliably and convincingly) the predictions they make, the processes they use, and the conclusions they present. As a result of understanding the power of logical reasoning, they will have higher expectations for this type of reasoning from their own students.

PRIMES AND COMPOSITES

After looking at the rectangular arrays for the numbers 1 through 15, predict how many rectangular arrays could be made for 1000. Justify your prediction.

ALTERNATIVE SOLUTIONS

Now that you have shown different representations (e.g., triangular numbers, polygons, algebraic equations) that can be used for solving the handshake problem, explain how each of the representations is related to the others.

Instruction

INSTRUCTION

Preservice teachers themselves must be engaged in worthwhile mathematical tasks within an environment that supports exploration, problem solving, reasoning, and communication if they are to be prepared to make effective instructional decisions in teaching mathematics. The mathematical tasks chosen by faculty should engage preservice teachers in explorations that uncover the connections and relationships among big ideas around which elementary mathematics curricula are organized. Sample tasks from the *Content* section are repeated here as examples of the environment and the tasks described.

Instructional Environment

Instruction for preservice teachers must occur in a learning environment that accommodates the needs of all learners and supports them in their efforts to use mathematics as a tool for describing situations, making predictions, and evaluating decisions.

- ☐ A supportive instructional environment provides preservice teachers with opportunities to ask questions and contribute ideas for investigation or discussion in order to build their mathematical competence and confidence.

Preservice teachers often have natural insight and intuition for mathematics that have been overlooked or undernurtured in their earlier years of schooling. For some, however, their past experiences with mathematics may not always have been pleasant or successful ones, and many of them consequently have developed concerns and fears about mathematics. Unless preservice teachers have experiences that help them overcome these fears as learners and teachers of mathematics, their future stu-

Representation of Physical Condition

In a recent study, a comparison of a person's height to that person's waist size was used as a measure of the person's general physical condition. Collect some data from the class about height and waist size. Use a graphing calculator to design a way to organize the data to determine whether or not a useful relationship exists between these two physical attributes. If such a relationship exists, find a mathematical representation of it. If not, make a conjecture about some other pair of characteristics that might better measure general physical condition and explain how you would test your conjecture.

dents are likely to develop the same concerns and fears. They need an environment in which the questions they ask and the insights and intuitions they express are considered valid and worthy of investigation. This begins the process of building the mathematical competence and confidence preservice teachers need to make good instructional decisions when teaching mathematics.

- ☐ A supportive instructional environment leads preservice teachers to recognize and revise previously formed misconceptions in order to better make sense of mathematics.

Because of the emphasis on rote memorization of mathematical rules, often without understanding, that begins as early as the lower-elementary grades, many preservice teachers have developed some basic misconceptions or limiting definitions of important mathematical ideas. Preservice teachers who have developed the misconception that a square is not a rectangle or who think that the tens can never be added first in 25 + 48 will pass these mathematical misunderstandings and limitations to their students. These faulty ideas must be challenged within an environment where the faculty and preservice teachers work together to find and correct misconceptions and form accurate definitions. In this way, preservice teachers may be better prepared to help their future students develop mathematical strength.

QUADRILATERAL CLASSIFICATIONS

There are many special types of quadrilaterals. Draw a Venn diagram showing the relationships among these different types of quadrilaterals, and label each set with its defining characteristics.

- ☐ A supportive instructional environment provides experiences that reflect mathematics as a process of observing patterns or regularities, generalizing these observations to create rules or algorithms, verifying the generalizations with valid reasoning, and using the generalizations to predict outcomes and make decisions.

A common approach to teaching mathematical formulas or principles is to present the formula initially in its symbolic form, then have students evaluate or apply the formula in several exercises. This approach encourages early memorization without much understanding. To develop an appreciation of the descriptive and interpretative power of mathematics, preservice teachers need experiences in forming their own generalizations. Preservice teachers can use technology (including handheld graphing calculators, data collection devices, and computers) as they collect data, organize the data in various ways, look for patterns and

SURFACE AREA FORMULAS

If you covered a yellow Cuisenaire rod with centimeter grid paper, how many squares would it take to cover the entire rod? What are the squares measuring? If you glued two yellow rods together along a long face to make a new block, how many squares would it take to cover the entire block? If you glued a third yellow rod to the other two along a long face, how many squares would it take to cover the block of three rods? If you glued "n" yellow rods together, how many squares would it take to cover it? Write a numerical expression to describe how you would determine the surface area of "n" yellow rods glued together.

trends among the data, and identify relationships that appear. The observed relationships should be described by the preservice teachers orally and in writing. If the relationships are numerical, they can then be described in abbreviated word-equation form, followed by the traditional symbolic form. In this way, preservice teachers, and the students they will eventually teach, can become as comfortable with mathematical language as they are with their everyday language. For example, they will be able to think not only "the perimeter of a square is four times as long as one of the sides" but also "$p = 4s$." The ability to use mathematics to describe patterns in the functional world is fundamental to an appreciation of mathematics.

INSTRUCTION

- A supportive instructional environment provides a variety of instructional materials, including manipulative materials and technological tools, with which preservice teachers can explore mathematical ideas in order to increase their understanding of mathematics.

Through the use of concrete models, diagrams, calculators, and computers, preservice teachers can explore the patterns that lead to abstract mathematical concepts. In addition, the various learning styles that exist within every group of preservice teachers make it necessary to provide tactile, as well as visual and auditory, experiences. Manipulative materials such as place-value blocks and geoboards can provide physical and visual models of computational processes and geometric relationships. Computers and calculators are excellent tools for conducting numerical or graphical pattern searches by quickly producing or processing large amounts of data or simply by removing the computational burdens so that other mathematical ideas can be explored. Preservice teachers should be provided with and encouraged to select appropriate tools from technology, concrete materials, drawings, and diagrams based on what they find most useful for a given problem.

PROPERTIES OF QUADRILATERALS

Use geometry exploration software to draw a quadrilateral. Construct the midpoints of the sides of the quadrilateral. These midpoints form another quadrilateral. Use the software to change the shape of the original quadrilateral and observe the effects on the quadrilateral formed by the midpoints. Make a conjecture.

- A supportive instructional environment provides a variety of instructional settings within which preservice teachers can explore mathematical ideas in order to strengthen their communication and reasoning skills.

People learn in a variety of ways and through a variety of means. It is not uncommon for faculty to approach instruction only in a style with which they are comfortable. It is also not uncommon for preservice teachers to become frustrated at their inability to "get" a concept or idea. This frustration, and resultant anxiety, is what greatly affects their perspectives about mathematics and, hence, their own eventual mathematics teaching. For these reasons, faculty must support preservice teachers in their learning of mathematics by using a variety of instructional strategies to reach students with different learning styles.

One strategy for which there is evidence of success is using cooperative groups to explore bigger problems. The preservice teachers benefit greatly from partner or team groupings because they must communicate with each other about their mathematical thinking. Articulating mathematical understanding allows preservice teachers to reinforce their understandings and build confidence in doing mathematics. Working with partners and in groups can help preservice teachers begin to form the sense of professional responsibility and collegiality that results in a greater commitment to learning.

Another example is to use an inductive approach to teaching. By producing and analyzing sets of examples, preservice teachers can form generalizations, rules, and algorithms. This approach allows preservice teachers to construct their own mathematical understanding as it builds on the mathematical process of using patterns, forming generalizations, and making predictions and decisions. Experiencing different instructional strategies as they learn mathematics helps preservice teachers prepare to use a variety of instructional strategies as they teach mathematics to their own groups of diverse learners.

- ☐ A supportive instructional environment provides time for guided reflection and class discussion in order for preservice teachers to analyze their own learning.

Although preservice teachers should experience a variety of mathematics activities, the completion of an activity will not in itself guarantee an understanding of the intended concept. Preservice teachers must be allowed the time to reflect on their actions and thoughts and to verbalize what they have learned in response to carefully selected and sequenced questions. Sharing their approaches with others in class is a very effective way for preservice teachers to accomplish an analysis of their own learning and to recognize the overall mathematical purpose of the activity.

The Measurement Process

With three other classmates, design a method for measuring the strength of a magnet. In your design, what issues did you have to consider? Discuss with your group how those same issues are reflected in the ways we measure volume, area, length, mass, and temperature.

Similarity and Measurement

Select a tall structure on campus or in the community. Use properties of similar figures to develop a process for determining the height of the structure. Test your process, and compare your process and result to those of your classmates.

Instructional Tasks

In order to maximize each preservice teacher's participation in mathematical thinking and growth in mathematical competence and confidence, the instructional tasks in which they are asked to engage must be carefully selected.

☐ Worthwhile mathematical tasks involve investigations, applications, analyses, inductive reasoning, or deductive reasoning in order to help preservice teachers make sense of mathematics.

Worthwhile mathematical tasks engage preservice teachers in a variety of mathematical processes. They can investigate why algorithms work by generating their own algorithms, comparing them to traditional ones, determining why specific formats were developed, and evaluating their usefulness. Preservice teachers can experience inductive thinking through activities that allow them to generate their own empirical data and to look for regularities within the data. From such observations they learn to develop generalizations that can then be used to generate new examples, make predictions, or verify other generalizations. Preservice teachers also need opportunities to apply number-sense ideas and techniques for mental computation, evaluating whether each technique produces an exact answer or an estimate and when such estimations are acceptable.

The Meaning of Quotients

Write a problem situation in which 23 r4 is a more appropriate answer than 23.5 to the problem 188 ÷ 8. Describe a situation in which 24 is a more appropriate answer than 23.5. Describe a situation in which 23 is a more appropriate answer that 23.5. In what types of settings would 23.5 be the appropriate answer?

☐ Worthwhile mathematical tasks enhance preservice teachers' problem-solving skills.

Preservice teachers must have ample opportunity to solve problems and, as a regular part of their mathematical experiences and preparation for teaching, to practice, develop, and refine their problem-solving skills. As preservice teachers solve problems, they should discuss the different approaches used. It should never be assumed that all people attack a given problem in the same way or that a particular approach is best for all. For example, some people rely heavily on the direct use of formulas and equations, while others use diagrams or make tables. One important facet of problem solving is acknowledging that several different approaches to a problem can be used and that there are benefits to each approach. Exposure to a variety of ways to solve a problem increases a preservice teacher's repertoire of strategies. This allows preservice teachers to test their strategies for efficiency and generalizability against other possible solution strate-

Multiple Solutions

If there are ten people in a room and each person shakes hands with every other person exactly once, how many handshakes will there be? How could you use triangular numbers to model the solution? How could you use polygons? How could you use an algebraic formula and a spreadsheet to solve for any number of people in the room?

gies. Faculty and preservice teachers should make themselves and each other aware of their thoughts and questions as they move through the solution process, so that all can become better problem solvers.

- Worthwhile mathematical tasks engage preservice teachers in using everyday language to speak and write about mathematical ideas, as well as in using mathematical language and notation to describe given situations.

When preservice teachers try to communicate their mathematical ideas and observations to others, they find the weaknesses that may exist in their own thinking. Faculty also become aware of any misunderstandings that the preservice teachers may have and can take measures to correct such misconceptions through focused questioning and additional activities. Brief written summaries of newly formed generalizations or descriptions of algorithmic procedures often reveal the correct or incorrect reasoning being applied by the preservice teachers. Writing about mathematics and using mathematical language and notation allow preservice teachers to reinforce their understandings and build confidence in doing mathematics.

PIZZA PRICES

Should a personal-sized pizza that is 5" in diameter cost half as much as a large pizza of the same type that is 10" in diameter? Use mathematics to write a justification to explain why or why not.

- Worthwhile mathematical tasks encourage preservice teachers to compare and contrast multiple representations for each mathematical concept.

Mathematical concepts are best taught through a wide variety of representations, including concrete models, illustrations, and abstract symbols. If a new concept is presented through only one representation, preservice teachers may focus on the insignificant attributes of the representation or may not recognize the importance of key elements. Focusing on the relationships among the different physical, pictorial, and symbolic representations of a mathematical concept helps preservice teachers develop a stronger understanding of the significant aspects of the concept and its relationship to other mathematical ideas.

ALTERNATIVE SOLUTIONS

Now that you have shown different representations (e.g., triangular numbers, polygons, algebraic equations) that can be used for solving the handshake problem, explain how each of the representations is related to the others.

- Worthwhile mathematical tasks highlight for preservice teachers the connections among mathematical concepts.

Preservice teachers must learn to see mathematics as a related whole connected through big ideas such as data analysis, number sense, and spatial relationships rather than as a linear progression of topics and skills. For example, the use of fractions to describe probability does not necessarily mean that fractions must be completely understood before engaging in activities about probability. In fact, probability experiences can enhance a student's understanding of fraction notation. By developing a strong sense of the connections among the various mathematical ideas in the elementary curriculum, preservice teachers can provide meaningful mathematical experiences for their students.

- ☐ Worthwhile mathematical tasks make preservice teachers aware of the historical contributions of a diverse mathematics community.

Educators realize that public school curricula across all subject areas have often ignored the contributions of many cultural groups. Mathematics certainly is no exception. Preservice teachers must have the opportunity to explore the mathematical and historical contributions from men and women of various cultures, and these explorations should be an integral part of the mathematical content.

PROPORTIONAL REASONING AND INDIRECT MEASUREMENT

Find the length of the Statue of Liberty's nose if you know that her forearm is 42 ft long. Compare your procedure with others' procedures, and discuss the advantages and disadvantages of each.

Assessment

ASSESSMENT

Assessment is a complex aspect of teaching. It reflects as well as measures the mathematical knowledge and understanding that is important for preservice elementary teachers. Additionally, assessment is a tool for guiding both the instructor in making instructional decisions and the preservice teachers in evaluating their own learning.

The term assessment is not always defined consistently. However, two principal documents define assessment in ways that are consistent with and reflect the guidelines for assessment addressed in this document. *The Assessment Standards for School Mathematics* defines assessment as "the process of gathering evidence about a student's knowledge of, ability to use, and disposition toward mathematics and of making inferences from that evidence for a variety of purposes."[11] *Measuring What Counts* defines assessment as "a way of measuring what students know and expressing what students should learn."[12] These definitions provide the basis for the guidelines on assessment. These guidelines center on the mathematics content reflected in assessment and on the purposes and uses of assessment.

Content, instruction, and assessment are integrally linked, and changes in assessment must reflect the changes made in content and instruction. This link implies moving away from the familiar sources of evaluation—particularly, traditional testing, quizzes, and homework—as the primary means of assessment, and moving toward less familiar forms of assessment, such as projects, interviews, journals, and portfolios. Faculty will need to consider the purpose for any particular assessment and carefully select appropriate methods; preservice teachers will need to be informed of what is important for them to know, how they will be assessed, and what the benefits are of the new assessment methods. These guidelines are intended to facilitate and support this transition.

[11] National Council of Teachers of Mathematics (1995), *Assessment Standards for School Mathematics*, p. 87.

[12] Mathematical Sciences Education Board (1993), *Measuring What Counts: A Conceptual Guide for Mathematics Assessment*, p. 1.

In previous sections, sample tasks are provided to help clarify and exemplify the spirit and intent of this document. Similarly, assessment tasks are offered at the end of this section for the same purpose. Although these guidelines and sample tasks provide a starting point for reevaluating methods of assessment, they are not intended to be comprehensive nor prescriptive. Rather, the following guidelines represent a vision of what is valued in the assessment of the mathematical understanding of preservice teachers.

☐ Assessment reflects the knowledge and understanding of the mathematical concepts and skills that are important for every preservice teacher to know.

The important concepts and skills that preservice teachers need to know should be determined before designing the ways in which preservice teachers are assessed. Assessment tasks should involve quality mathematics and require preservice teachers to demonstrate their understanding of mathematical concepts, as well as their proficiency in mathematical skills. In order to assess what preservice teachers understand as well as what they can do, they should be asked to demonstrate their knowledge in a variety of ways: through the design and use of physical representations, through the selection and use of appropriate tools, and through narrative descriptions as well as symbolic representations of what they know. By demonstrating their knowledge in these ways—together with the use of appropriate mathematical language, including mathematical symbolism—they synthesize what they know about concepts, skills, and the relationships among them, and they strengthen their understanding and build confidence in their own mathematical knowledge. Assessment must address mathematical understanding using each of these perspectives so that it reflects both the concepts and the skills that are important. Then, when making future instructional decisions, preservice teachers will be able to draw on their understanding of and confidence in significant mathematical concepts and skills.

☐ Assessment provides a balanced picture of mathematical knowledge through a variety of methods, including the use of appropriate technological tools and physical models.

Many traditional assessment tasks are one-dimensional; they assess only the proficiency of skills associated with a mathematical idea, such as multiplication. These one-dimensional tasks fall far short of evaluating a preservice teacher's mathematical knowledge and understanding. Providing preservice teachers with a variety of assessment tasks will improve the validity of the inferences made about preservice teachers' learning.[13] Engaging preservice teachers in a variety of assessment tasks will allow them to demonstrate more fully their knowledge and understanding of the content, as well as provide opportunities for the uncovering of misconceptions and areas of incomplete knowledge. Types of assessment tasks include:

[13] NCTM (1995).

- open-ended problems that allow for different responses to a question that may have more than one correct answer in order to accommodate preservice teachers' differing perspectives and unique experiences;

- journals in which preservice teachers can record a summary of their learning in order to develop skills of self-assessment;

- research projects and presentations by groups or individuals in order to encourage special interests in mathematics;

- interviews in which preservice teachers discuss mathematics in a nonthreatening environment in order to solidify their understanding; and

- portfolios that display collections of preservice teachers' best work in order to measure growth of mathematical understanding over time.

A scoring guide may help organize and interpret evidence of preservice teachers' performance on assessment tasks. It might be composed of a continuum of performance levels (a holistic scoring guide) or a list of specific criteria that are required to be in the solution of a problem (an analytic scoring guide). Faculty might choose to utilize a holistic scoring guide where they would rate each preservice teacher's solution on a scale against criteria as to what constitutes a score for each level. For example, on a scale of 1 to 6 (lowest to highest), the highest score might be assigned to a response that is computationally correct, is clearly and thoroughly explained, and contains both a symbolic and a diagrammatic solution. The lowest score would be assigned to a response that not only is computationally incorrect, but also is incomplete, is not well thought out, and shows no evidence of understanding. An analytic scoring guide would be more prescriptive in that it would, for example, assign one point to the inclusion of a diagram, three points to the correct answer, two points for a symbolic solution, four points for a clear explanation, etc. Both alternatives have merit depending on the task and the purpose of the assessment. Although a complete discussion of the various types of assessment tasks falls outside the scope of this document, there are various sources describing assessment that serve as starting points for research on this issue.[14]

- ☐ **Assessment provides access to mathematics by providing information on both learning and teaching for both the preservice teacher and the faculty member.**

Assessment is an integral part of the learning process. It should be not only an opportunity for preservice teachers to demonstrate their growth in mathematical understanding, but also a learning experience in itself. Preservice teachers can use appropriate assessment tasks as personal guideposts: Are they still struggling with understanding the procedure or concept, or are they comfortable with it? Can they express their understanding in pictures and words as well as symbols, or is their ability to arrive at a correct answer limited to the use of a memorized algorithmic routine? This reflection then determines a direction for their continued learning in that area of mathematics. Similarly, faculty can use appropriate assessment tasks to evaluate their instruction. Are the preservice teachers demonstrating sufficient understanding of probability at the concrete and pictorial, as well as symbolic, levels? Is their understanding of the operation of multiplication being reflected in their proficiency in finding products of numerical and algebraic factors? What additional experiences, if any, need to be provided to enhance their understanding? Faculty can use the results of these reflections to make necessary instructional decisions.

Sample Tasks

Earlier in this document, tasks were provided to exemplify worthwhile mathematical content and to promote supportive instructional environments. However, these tasks can also serve as good examples for assessment. The following examples show how these mathematical tasks also fit the guidelines for assessment tasks.

[14] *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark, 1991) and *Assessment Alternatives in Mathematics: An Overview of Assessment Techniques That Promote Learning* (Stenmark, 1989) describe different assessment strategies not previously used in mathematics classrooms. *Measuring Up: Prototypes for Mathematics Assessment* (MSEB, 1993) contains some justification for changing assessment practices as well as assessments recommended for fourth-grade students which could be used as model assessments for preservice teachers. *Measuring What Counts: A Conceptual Guide for Mathematics Assessment* (MSEB, 1993) and the *Assessment Standards for School Mathematics* (NCTM, 1995) explore, in depth, the goals and purposes of assessment. In addition, the *Assessment Standards* contain an extensive bibliography on assessment. *Assessment in the Mathematics Classroom: 1993 Yearbook* (Webb, 1993), *Research Agenda for Mathematics Education: The Teaching and Assessing of Mathematical Problem Solving* (Charles and Silver, 1989), and *Mathematics Assessment and Evaluation: Imperatives for Mathematics Educators* (Romberg, 1992) further explore assessment techniques and issues.

Assessing Predictions with Fractions and Percents

This statement appeared in a newsletter on health: "About one in 10 Americans eventually develops a kidney stone. Four out of five people who develop kidney stones are men." What fraction of American men are expected to develop a kidney stone? What percent of American women are expected to develop a kidney stone? What is the probability that an American man will develop a kidney stone? Justify your answers.[15]

[15] Stanley, D. (1994, October 10), "Kidney Stones" problem from the Balanced Assessment Project (Task ID H2070a).

- This task involves intricate and changing part-to-whole relationships as well as connections between fraction, decimal, and percent representations of probability in a meaningful, problem-solving context.

- The answers can be justified through symbolic representations, drawings, or appropriate geometric models. Justifications could also include preparing a poster summarizing the data and results in an organized manner or recording in a journal the solution procedure and narrative interpretations of the answers.

- The task engages preservice teachers in worthwhile mathematics, allows a variety of approaches to solving the problem, and provides opportunities for incorporating writing and presentations. The justifications designed by the preservice teachers will give evidence of their understanding of how mathematics can be used to make predictions.

Assessing the measurement Process

Design a method for measuring the strength of a magnet. In your design, what issues did you have to deal with? How are those same issues reflected in the ways we measure volume, area, length, mass, and temperature?

- This task requires preservice teachers to examine their own understanding of measurement as a process. What does it actually mean to measure something? What characteristic is measured? How is a unit selected? How are numbers and units used to quantify the characteristic?

- This open-ended task, with its many possible solutions, gives preservice teachers the opportunity to exhibit their individual levels of understanding of measurement.

- The solutions to this open-ended task would give faculty insight into the level of understanding each preservice teacher has of the process of measurement. Faculty could then use the misconceptions and incomplete understandings about measurement of an attribute like the strength of a magnet to design discussions exploring the kinds of measurement that most preservice teachers have learned to do by rote, such as measuring the length of a segment with a ruler. Instructional decisions based on preservice teachers' responses to the assessment task might include asking such questions as, "How is measuring a segment like measuring the strength of a magnet?" and, "What characteristics or procedures do all measurements have in common?" In addition, preservice teachers will become more aware that they may have learned to perform mathematical skills such as measurement in a rote manner without much understanding and how that lack of understanding affects their ability to communicate clearly about the mathematical ideas involved in measurement. Responses to this question made both before and after further experiences in measurement could be collected in a portfolio and analyzed for indications of growth and understanding.

As seen in these examples, assessment is a complex component of quality education. While improving and guiding preservice teachers' learning, it aids faculty in making sound instructional decisions, which also improve learning. Assessment is an ongoing process that can take on many forms, some of which will require considerable changes by faculty and preservice teachers to implement.

Conclusion

Traditional classroom assessments, while valuable in some ways, are not sufficient to garner a clear picture of preservice teachers' knowledge and understanding of mathematics. The types of assessment necessary to achieve a balanced picture will require more effort and time on the part of faculty, especially in the initial stages, but it is imperative that faculty expend that time and effort to ensure that assessment meets both their needs and the needs of preservice teachers. Different assessment strategies benefit the preservice teachers in ways that enhance their learning, provide them opportunities to self-assess their learning and therefore guide future learning, and raise self-confidence in their mathematical abilities. Assessment serves as a way for faculty to connect content and instruction and for preservice teachers to connect teaching and learning. These strategies can also provide preservice teachers with experiences that will affect the decisions they make in their own future classrooms.

Changing assessment practices will be difficult, but the benefits to change are enormous. Faculty development opportunities for adjusting to and learning about these changes will be necessary. Structured opportunities may not be readily available, but there is current literature on alternative assessment which will provide a starting point for change. In the educational triad of content, instruction, and assessment, balanced, equitable, quality assessment is the least understood and requires further work and study. It is important to acknowledge that the issues attached to assessment are highly complex and cannot be fully addressed within the scope of this document. However, implementation of broad, balanced assessment strategies will help provide mathematical competence and confidence to a new generation of elementary teachers.

FACULTY ☞ INSTITUTIONS ☞ PRESERVICE TEACHERS ☞

RESPONSIBILITIES

A discussion of these guidelines would be incomplete without also considering the people who are directly affected. If these guidelines are to be successful, both faculty and preservice teachers will need to acknowledge their responsibilities. However, success also requires that the needs of preservice teachers and faculty be acknowledged by the institution. When both the responsibilities of and the responsibilities toward faculty and preservice teachers are considered, acknowledged, and answered, then Texas will be closer to the vision described in this document.

Preservice Teachers

Restructuring the mathematical preparation of teachers will have, ultimately, a powerful effect on preservice teachers. They, along with their future students, stand to be the recipients of the greatest benefits from this process. However, preservice teachers also share responsibility for their preparation. They must be aware of and responsive to certain expectations if this effort is to be successful. To be a professional has certain implications. These guidelines address the personal responsibilities that preservice elementary teachers are expected to accept as part of becoming a professional.

☐ Preservice teachers are expected to take risks in order to learn mathematics, and they must recognize that taking risks is essential to learning mathematics.

Learning mathematics in such a way as to develop understanding at the levels described in this document may require confronting and stretching one's own mathematical boundaries. This may mean taking intellectual risks–experiencing and enduring a certain level of intellectual disequilibrium, and at times, frustration. Preservice teachers should be aware that this will happen and be prepared to cope with these challenging situations by seeking support from instructors and peers, as well as by persevering in their mathematical learning.

☐ Preservice teachers are expected to be active participants in all collaborative learning experiences in mathematics.

The mathematics content, instruction, and assessment as described in this document can benefit preservice teachers only if they accept the responsibility to participate actively in the mathematical learning opportunities afforded to them. Preservice teachers are expected to accept tasks and challenges presented to them; to question themselves, colleagues, and their instructors; and to read, explore, research, and write about their understanding of mathematics. Preservice teachers are also expected to share ideas regularly and to interact with other preservice teachers, with faculty, and with members of the professional community. These interactions provide opportunities to solve problems, to make mathematical connections, and to broaden understanding of mathematics. Collaborative experiences are important to preservice teachers' preparation in mathematics because they must communicate with each other about their mathematical understanding. The articulation of their understanding allows preservice teachers to reinforce what they know, to build confidence in doing mathematics, to rely on themselves as one source of mathematical knowledge, and to create similar learning environments in their future classrooms.

- Preservice teachers are expected to seek and identify connections within mathematics, between mathematics and other fields, and between the mathematics they study and the mathematics they will teach.

Making connections within mathematics is a critical component of the learning process because it provides a way to clarify mathematical understanding. Preservice teachers need help discovering connections within mathematics and between mathematics and other fields. They are expected to consciously seek out and demonstrate the connections between mathematics and their experiences, and between the mathematics they study and the mathematics they will teach. This ability to make connections strengthens preservice teachers' abilities to help their students learn to make connections for themselves.

- Preservice teachers are expected to demonstrate a commitment to the continued study of mathematics and educational issues.

As emerging professionals, preservice teachers acknowledge that their preparation for teaching does not end upon certification. Not all the mathematics that prospective teachers will need can be addressed in college coursework. Preservice teachers are expected to pursue experiences to strengthen their mathematical understanding throughout their professional career. This implies a commitment to participate in continuing professional development and to communicate with others about mathematics and mathematics education. Active participation in professional meetings, in professional readings, and in other professional activities strengthens knowledge of content, pedagogical content, and pedagogy. Participation also serves as a vehicle to keep preservice teachers in touch with the mathematics and education communities. Preservice teachers need to be aware of the resources available to support them as learners and teachers of mathematics. They need to be aware of education policies, guidelines and reform efforts at the local, state and national levels. They also need to be aware of the mechanisms for remaining informed. Ultimately, mathematics teachers who model such continued learning enable their students to accept responsibility for their own lifelong learning.

Faculty

The development of mathematics teachers for the elementary school depends largely upon the faculty who provide mathematical experiences for them during their preservice years. The preparation of preservice teachers in mathematics is part of the basic responsibility of an entire department; however, teaching preservice teachers should be a privilege earned by the best teachers in the department. The guidelines that apply to preservice teachers also apply to the faculty who prepare them, but additionally, *all* mathematics faculty have a responsibility to model these guidelines. Faculty who model their enjoyment, their excitement, and their appreciation of their own learning of mathematics will influence the attitudes of their preservice teachers.

- Faculty have a responsibility and commitment to help preservice teachers develop appropriate content knowledge of mathematics, its real-world applications, its historical context, and its roots in diverse cultures.

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Faculty should exhibit knowledge of mathematical concepts and their applications in business and industry, the sciences, and other real-world endeavors. Additionally, an important part of mathematical content must include an awareness of the historical contributions to mathematics from men and women of various cultures. It is important for faculty to possess mathematical knowledge which includes content appropriate for elementary school mathematics and to exhibit confidence in this knowledge. Faculty are also expected to model confidence in the classroom and, in turn, encourage preservice teachers to build their own confidence in mathematics.

☐ Faculty have a responsibility and commitment to choose and implement the appropriate instructional strategies for their preservice teachers.

The learning and teaching of mathematics is influenced by factors such as gender, culture, and styles of learning. It is essential that faculty possess a wide repertoire of instructional and assessment techniques in order to respond to these factors. Faculty should work to structure a supportive classroom environment and to model instructional and assessment techniques that enhance mathematical learning for all preservice elementary teachers. Faculty who pose worthwhile learning tasks and use a variety of effective strategies serve as exemplary role models for preservice teachers who will the use the teaching and assessment methods which they themselves have experienced.

☐ Faculty have a responsibility and commitment to their own continuing professional development.

A commitment to participate in continuing professional development and to share ideas about mathematics education is essential for faculty growth. Active participation in professional meetings, in professional readings, and in other professional activities strengthens knowledge of content, pedagogical content, and pedagogy. It also serves as a vehicle to keep faculty in touch with the mathematics and broader communities. Another aspect of professional growth is to conduct and utilize research on learning and teaching mathematics. Part of a faculty member's responsibility is to be well informed and to inform others about education policies, guidelines, and reform efforts at the local, state, and national levels. However, at the core of professional development is ongoing contact with the elementary classroom and other related field experiences. Contact with and experience in the elementary classroom enable faculty to maintain a current perspective about and to have an influence on public education.

☐ Faculty have a responsibility and commitment to serve as change agents in mathematics departments and as advocates in the community to sustain and improve quality mathematics preparation.

The future of effective mathematics instruction requires elementary teachers who are proficient in many areas of mathematics. It is vital that the faculty who teach the mathematics component for elementary teachers take the lead in refining how preservice teachers are prepared. In particular, mathematics for preservice teachers should serve as a vehicle to strengthen their preparation and to provide access to mathematics and mathematical understanding. In this way, preservice teachers, and ultimately their students, can build their own mathematical power. Systemic change begins with faculty effecting substantive change in their own classrooms. Faculty can build support for appropriate mathematics practice by serving as advocates of change in the broader community. They can be active participants in the community by looking for and responding to opportunities in schools, and mathematics departments can use these opportunities as a vehicle for professional service. Thus, as all interested stakeholders work collaboratively, systemic change can occur. It is the responsibility of faculty to provide the leadership necessary for this change to begin.

Institutions

In restructuring the way preservice teachers are prepared to teach mathematics, there are certain responsibilities and obligations that fall directly on the institution, which includes not only the institution itself, but also its respective and appropriate organizational substructures in academics, administration, and governance. In order to build more effective teacher preparation programs, institutions of higher education must be responsive to preservice teachers and to the faculty who work with them. There are certain issues that can be addressed only by the institution, and a response by the institution to these matters is a critical component in strengthening the mathematical preparation of teachers. When interpreting and responding to these guidelines, it is important that a substructure of an institution, be it a department, a dean's office, or a faculty committee, acknowledge those areas which it can effect and accept the responsibility to do so.

Preservice Teachers

The responsibilities of institutions to preservice teachers revolve around providing access to appropriate mathematical preparation and creating a supportive learning environment. These opportunities maximize the chances that prospective teachers will have the solid mathematical preparation needed to teach mathematics well to elementary students.

- The institution has a responsibility to provide access to the mathematics learning experiences described in this document to all students who are prospective elementary teachers.

Successful mathematics learning experiences described in this document should be the goal for all prospective elementary teachers. Any institution that prepares elementary teachers has

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a responsibility to offer mathematics courses designed to meet their specific needs. Geographic location should not be a barrier. Preservice teachers in both rural and urban areas, and those in community colleges, as well as four-year colleges and universities, must have equal access to sound mathematical preparation. Elementary teachers need a variety of mathematical experiences including content development, collaboration with peers, and opportunities to create mathematical connections which are unique to the profession. Thus, the component courses may not be appropriate for students in degree areas outside education. These courses are the beginning of a transitional period—the transition from learner of mathematics to teacher of mathematics—which will culminate in experiences focusing on the methods of teaching mathematics to children. Hence, these course should be designed principally for preservice teachers.

☐ The institution has a responsibility to provide preservice elementary teachers access to appropriate facilities and instructional equipment.

Proficiency in the use of a variety of tools and materials, including technology, for learning and teaching is crucial for the success of preservice teachers. Calculators, computers, physical models, and manipulative materials can enhance mathematics learning. Hence, it is essential that preservice teachers be provided access to appropriate equipment so that they can engage in learning experiences in mathematics that take advantage of such important resources.

☐ The institution has a responsibility to recruit preservice elementary teachers from a variety of backgrounds and cultures who exhibit the qualities requisite for success in teaching mathematics.

Institutions that set high standards for admission to elementary education programs, and actively recruit those who meet these standards, help provide elementary schools with teachers who are mathematically talented and well-equipped to provide quality mathematics instruction to students. These standards might include such qualities as good academic standing, a clear commitment to teaching and children, and an enthusiasm about learning, and in particular, learning mathematics. Moreover, because the student population of the state has become very diverse, with students from a variety of ethnic groups and cultures, it is essential that prospective teachers be recruited to reflect this diversity in the population of the state.

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- ☐ The institution has a responsibility to provide continued, appropriate advising concerning mathematics education to students who are prospective elementary teachers.

Advisors for preservice teachers can best serve if they themselves are familiar with current trends and reform efforts in mathematics education and are knowledgeable about courses offered which may meet or supplement these recommended mathematical experiences, both in content and in instruction. An advisor's understanding of certification requirements and willingness to work with preservice teachers are components of the support structure necessary to help retain teacher education candidates and to draw new candidates into the program.

- ☐ The institution has a responsibility to continue providing support for and communicating with teachers after graduation.

The commitment of an institution to the teachers it prepares does not end upon their graduation. As teachers are expected to continue their own learning, it is an institution's responsibility to encourage opportunities for continued learning. In addition to providing professional development opportunities, institutions can work with schools on cooperative grants and projects as well as use the Internet and other electronic networks as points of contact and discussion. This effort is imperative if institutions are to have a continued and lasting effect on public education. Ultimately, this continued involvement provides institutions with an area in which research can take place, and it serves as a vehicle for institutions to evaluate and improve their own programs.

Faculty

Institutions have unique responsibilities to faculty who teach the mathematics component of the teacher preparation program. For successful and lasting change in the mathematical preparation of teachers to occur, institutions must address faculty recruitment, faculty support, and the institutional reward structure.

- ☐ The institution has a responsibility to recruit mathematics faculty who are committed to the improvement of K–12 education.

A successful program in the preparation of elementary mathematics teachers begins with the acquisition of a talented pool of instructors who are knowledgeable about current research in mathematics education, are proficient in the application of the research, and have a demonstrated history of effective teaching. It is imperative that institutions recruit mathematics faculty who are committed to the improvement of K–12 education, allocate appropriate resources, and recognize the importance of the preparation of K–12 teachers in the mission of the mathematics department. These criteria should apply to both tenure-track and non-tenure-track faculty. Further, special care must be taken when placing graduate students and part- or full-time instructors in mathematics courses for preservice elementary teachers to ensure that they also meet these criteria. Mathematics departments should recruit tenure-track faculty in mathematics education in numbers sufficient to sustain a successful program for elementary teachers.

- The institution has a responsibility to allocate necessary and sufficient resources for faculty who teach mathematics for preservice elementary teachers.

An institution's commitment to excellence in teacher preparation must be made evident by its allocation of resources. Access to adequate instructional materials, textbooks, and resources which reflect current direction, as well as the latest innovations in technology for teaching, must be assured in order for faculty and preservice teachers to realize effective mathematics instruction. Institutional commitment to adequate office, classroom, and laboratory space; reasonable class size; and the appropriate support personnel is essential to establish and maintain a viable teacher preparation program in mathematics. Most importantly, the institution must demonstrate a commitment to ensure quality instruction by providing meaningful, ongoing professional development opportunities for all faculty.

- The institution has a responsibility to actively encourage faculty to become involved in partnerships with schools, businesses, and the community.

Partnerships with other stakeholders in teacher preparation provide opportunities for faculty to share their knowledge of research and practice and to gain information about the application of mathematics in other settings. Faculty who are informed about the issues, practices, and people in schools, businesses, and the community offer broader perspectives to preservice teachers as well as to their departmental peers. Institutions should encour-

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age faculty to make professional contributions by becoming actively involved with people and organizations from the greater community. Institutions should also reward such efforts accordingly.

- The institution has a responsibility to encourage faculty to become active partners in the processes through which teacher preparation is regulated and evaluated.

Since the faculty who teach these courses are at the focal point of teacher preparation in mathematics, they are uniquely qualified to speak to issues of curriculum, instruction, assessment, and program evaluation. As active researchers in the fields of education and mathematics, faculty involved in the education of teachers of mathematics can make a significant contribution to teacher certification and related policy issues. Institutions have a particular responsibility to encourage and support faculty to become involved in these processes as part of their obligation to the larger community.

- The institution has a responsibility to reward fairly and appropriately the efforts of faculty who are primarily concerned with teacher preparation.

Faculty who are primarily concerned with teacher preparation have every right to expect the institution to reward their efforts on a fair and appropriate scale. Those primarily concerned with teacher preparation should receive raises, promotions, and other visible signs of recognition consistent with other faculty in keeping with their role in this vital mission of the mathematics community. Furthermore, scholarship should be broadly defined to include efforts to improve mathematics education through grant acquisitions, professional presentations, and the development of curricular and instructional materials, as well as scholarly publications and a focused research agenda in mathematics education.

Institutions committed to recruiting, supporting, and rewarding quality mathematics faculty who prepare elementary teachers will be instrumental in building a community of mathematically literate citizens who can effectively participate in and contribute to a changing world.

CONCLUSIONS & IMPLICATIONS

The guidelines described in this document can be achieved. However, in order for this vision to be fully realized as described, it is necessary that all related facets be acknowledged. Although many of these facets fall outside the scope of the purpose and intent of this document, certain issues need to be discussed and addressed, and they are mentioned to indicate possible directions for next steps.

The Texas SSI Action Team on Strengthening the Mathematical Preparation of Elementary Teachers initiated this endeavor in response to forward-looking visions of teaching and learning mathematics at elementary, secondary, and postsecondary levels. The project was commissioned by the Texas Statewide Systemic Initiative (Texas SSI) in 1994 and was supported, in part, by the U.S. Department of Education's Fund for the Improvement of Postsecondary Education. The Action Team, composed of mathematics faculty and related professionals, began the task of writing and building consensus for a set of guidelines regarding the mathematics courses studied by preservice elementary teachers. This document is a product of a consensus-building process involving hundreds of mathematics faculty at both two- and four-year institutions, as well as the other stakeholders in teacher preparation. The guidelines offer insights into the teaching and learning of mathematics that preservice teachers should experience. These experiences are outlined in terms of content, instruction, and assessment. The guidelines also support that outline with a framework for successful interaction among the principals: the preservice teachers, the faculty, and the institution.

Policy Issues

This document recommends that state policymakers acknowledge the need to improve the mathematical preparation of preservice teachers and work toward this end. The preparation of elementary teachers should include the core mathematical content and instruction described in this document. Maintaining the status quo in the mathematical preparation of preservice elementary teachers will preclude any significant improvement in K–12 education. Substantive courses based on the experiences described in this document should be made available to every preservice elementary teacher in the state if Texas teachers are expected to significantly improve mathematics instruction.

Academic units, including academic departments and the structures that house them, will need to ensure that the policies they set reflect these recommendations. At the institutional level, encouraging and creating an atmosphere conducive to the necessary collaboration within and between academic units is essential in order to facilitate the implementation of these guidelines across campuses. Furthermore, institutions must support and encourage the academic units to implement these guidelines, and they should consider and implement policy-level decisions accordingly. It is imperative that teacher preparation units and mathematics departments work cooperatively to ensure that preservice teachers receive the best mathematical preparation possible.

Additionally, academic units of four-year institutions that certify teachers must acknowledge community colleges as important members of the teacher preparation program. Similarly, community colleges will need to acknowledge their role and responsibilities in teacher preparation. This acknowledgment implies that community colleges, four-year colleges, and universities should engage in a collaborative dialogue about issues important to the preparation of teachers, including but not limited to program development, funding opportunities, the transferability of courses and higher education students, professional development opportunities, and participation in influencing policy.

Other Issues

There are other issues affecting the mathematical preparation of preservice teachers, as well as the quality of mathematics instruction in the elementary school. These broader issues, like the preceding policy issues, must be addressed in order to improve the overall quality of K–12 mathematics instruction.

Certification Issues

As faculty and institutions work to implement the guidelines described in this document, they may encounter difficulties due to issues of teacher certification. Uniform and substantial certification requirements must be implemented at the state level in order for these guidelines to be met fully. The current legislative requirements for the mathematical preparation of elementary teachers is minimal and insufficient to meet these recommendations. The Texas SSI Action Team recognizes that not every institution can implement completely these guidelines with their present mathematics requirements for certification. This team recommends that a minimum of nine credit-hours in mathematics be required for certification, that those mathematics courses be designed principally for prospective elementary teachers, and that those courses reflect the guidelines in this document. However, while working toward changes in certification at the state level, institutions and faculty can strengthen their own requirements, supplement existing courses, and structure meaningful mathematical experiences for their preservice teachers, not merely require more mathematics. Additionally, care must be taken to ensure that candidates in alternative and field-based certification programs also have the opportunity to strengthen their mathematical preparation as described in these guidelines. As mathematics departments work to implement these suggestions, they must also be part of an effort, along with their teacher preparation units, to restructure teacher certification.

A recent report by the Texas Statewide Systemic Initiative documents studies that describe a discrepancy between the perceptions of higher education faculty and mathematics and science supervisors in the field concerning the preparedness of novice elementary teachers.[16] Higher education faculty (those from arts and sciences departments, as well as those from education departments) are generally satisfied with the Texas Education Agency's (TEA) standards and competencies for science and mathematics certification as codified in state law.[17] Furthermore, faculty believe that they are successful in helping preservice teachers meet these requirements. However, mathematics and science supervisors throughout the state believe the majority of elementary teachers in Texas have little or no college-level training in elementary mathematics content and methods.[18] This discrepancy between perceptions of those in the field compared to those in higher education indicates that the TEA standards and competencies do not reflect the preparedness required for those entering the profession. Meeting these standards and competencies does not insure that incoming teachers are well prepared nor well qualified.[19] Consequently, these requirements, exemplified on the Examination for Certification of Educators in Texas (ExCET) (Elementary Comprehensive), must be reexamined and revised.

[16] Texas Statewide Systemic Initiative (1995).

[17] Texas Education Agency (1992).

[18] Texas Education Agency (1993), *Evaluation of 1987 Standards for Teacher Preparation, Licensing, Certification, and Endorsement for Elementary and Secondary Mathematics and Science in Texas*.

[19] Texas Statewide Systemic Initiative (1995).

Mathematics Specialists in the Elementary Schools ....

Improving mathematics instruction in the elementary school can be further facilitated by mathematics specialists. Many institutions have programs for the preparation of mathematics specialists, but there are not nearly enough programs nor specialists to meet the needs in the schools. Implementing programs for mathematics specialists should be a high priority for mathematics departments. These preservice teachers should experience the same mathematical preparation as described in this document, along with other specialized mathematics courses. Courses exist at various institutions that can serve as models in developing specialist programs. A possible next step in strengthening the mathematical preparation of elementary teachers is to form a collaborative statewide effort to develop complementary guidelines for the mathematical preparation of elementary mathematics specialists.

Support for Practicing Teachers

Many novice and veteran teachers are not prepared to teach meaningful mathematics. Strengthening the mathematical preparation of elementary teachers includes providing the professional development opportunities for practicing teachers to have access to the mathematical experiences described in this document. This responsibility is shared by both institutions of higher education and public school systems. Along with other agencies, mathematics departments have the unique opportunity and responsibility to take the lead in establishing and implementing professional development programs. Systemic reform can only be achieved when both practicing and emerging teachers are provided with comprehensive mathematics programs reflecting these guidelines.

NEXT STEPS

Several other issues have not been addressed in this document but can serve as next steps in improving the mathematical preparation of prospective elementary teachers. Means of communication within departments, between departments of mathematics and education, and among various institutions should be explored, and networks comprised of those interested in these courses can become vital tools for reform efforts. Exemplary curriculum and assessment materials can be identified and samples can be collected and made readily available. Opportunities for research should be explored and findings utilized to strengthen programs for prospective teachers. Other next steps can be identified and addressed by institutions in collaboration with other stakeholders in the mathematical preparation of teachers.

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