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ABSTRACT

This report describes a program for increasing students' ability to simplify square roots. The targeted population consisted of high school students in a rural community in a midwestern plains state. The problem of the ability to understand the abstract algebraic process of simplifying square roots was documented through teacher-made tests, teacher observation, and poor/inadequate homework. Analysis of probable cause data revealed that early teens may not have the cognitive ability to simplify square roots using an algebraic method based upon the mathematical/logical intelligence. The understanding of abstract concepts appears to be related to the way in which students perceive their own thought processes. A review of solution strategies suggested by knowledgeable others, combined with an analysis of the problem setting, resulted in selecting an intervention consisting of a visual approach to simplifying square roots. The students were encouraged to use their choice of the algebraic or the visual method. Post intervention data indicated an increase in the targeted students' ability to simplify square roots through the use of the visual method presented in this report. Additional analysis of the data indicated a preference among the students for the visual method as compared to the algebraic method. (Contains 20 references.) (Author)

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A VISUAL APPROACH TO SIMPLIFYING SQUARE ROOTS APPLIED IN GEOMETRY AND ADVANCED ALGEBRA COURSES

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CHAPTER 1

PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

The students of the targeted advanced algebra and geometry classes exhibit a lack of ability to understand the algebraic algorithm for simplifying square roots. Evidence for the existence of the problem include test scores, teacher observations, and inadequate homework.

Immediate Problem Context

The site is a high school located in a rural area of a midwestern plains state and has a total enrollment of 282. All of the students come from the two elementary schools in the district. The following is a breakdown of the students' characteristics: 98.5% White, no Black, 1.5% Hispanic, no Asian/Pacific Islander, and no Native American. The site has no chronic truants, an attendance rate of 94.6% and a 7.3% mobility rate (State School Report Card, 1997).

The site is in a unit district which has a total of 63 teachers and there is a pupil to teacher ratio of 14 to 1. The average salary for the teachers is $35,570 while the administration's average salary is $62,391. The average teaching experience in the district is 14.2 years. Finally, 52.5% of the total number of teachers with a Master's Degree or above (State School Report Card, 1997).

The high school, built in 1956, is the location of the district superintendent's office as well as the building principal's office. The classrooms at the site are relatively small with two brick walls, one wall of bookshelves, and large windows on the fourth side. The targeted students are enrolled in entirely heterogeneous geometry and Algebra 2 classes. Algebra 2 is the title of the advanced algebra course; every student must pass geometry before enrolling in Algebra 2. Each of the four geometry classes are made up of students ranging in age from 14 to 18. The ability levels of two geometry classes range from those students eligible for support from the resource department to students who are taking the course for honors credit. The two Algebra 2 classes consist of students ranging in grade from tenth to twelfth and ability levels from average to honors. Only freshmen are eligible to take geometry for honors credit and Algebra 2 students must be sophomores to be honors students. The Algebra 2 curriculum is based on the 1993 Scott Foresman Algebra 2 with Trigonometry textbook and the geometry course follows the 1992 Houghton Mifflin Geometry text. Honors students are expected to complete the same curriculum as the rest of the students in addition to more difficult homework and test problems.

The Surrounding Community

As stated earlier, the site is located in a small midwestern town. The students attending this high school are from one of the two nearby townships. Township A has a population of 1682 residents. The median family income for Township A is $45,521 with

73% of the residents living in a home owned by a member of the household. One hundred seventy-one of the total 621 dwellings are rented in Township A at an average cost of $435 per month; this is approximately 22% of the average household's income. Sixty-nine percent of the women in Township A with children under six years old are employed (US Census Bureau, 1990).

The population of Township B is 1933 and the median family income for the residents is $46,250. Eighty-six percent of the homes are owned by a member of the household while renters pay an average of $611 per month. Twelve percent of the average households income in Township B is spent on rent and almost 75% of the women with children under six are employed (US Census Bureau, 1990).

More than 98% of the residents from both townships are white and English is the primary language spoken in over 97% of the households. About 88% of the residents from each township has reached an educational level of high school or above and approximately 16 % have at least a bachelor's degree. Both Township A and Township B have poverty levels near 4% and unemployment in each township is below 3%. Finally, the median household value in the townships is $109,290 (School District Data Book, 1996).

National Context of the Problem

Pedagogical reform in mathematics has been a popular topic for research in the last 15 years. Documents such as *Mathematics Counts* (Cockcroft, 1982), *A Nation at Risk* (National Commission on Excellence in Education, 1983), and the National Council of Teachers of Mathematics' *Standards* (NCTM, 1991) call for curriculum content change as well as changes in pedagogical practice. Indeed, problem solving, Standard 4, is a skill valued by today's employers and studies have been conducted regarding which types of mathematical reasoning affect a learner's problem solving ability (Dreyfus & Shama, 1994). In addition to pedagogy, many believe in changing curricular materials to suit the visual and kinesthetic learners (Glidden, 1996).

Dauterman, Dubinsky, and Zakis (1996) suggest that pedagogical approaches be shifted from emphasizing the differences between visual and analytic thought to the interaction between the modes of thought. With the renewed interest in Gardner's Multiple Intelligences theory, researchers are now exploring the unique tendencies of students and identifying the personal traits of learners (Clements, 1984). This new wave of reform is focusing more on students' mathematical reasoning processes and less on instructional materials aimed at improving student understanding (Research Advisory Committee of NCTM, 1996).

Recent studies have been conducted, however, testing the effectiveness of such instructional materials. Cobb, Wood, and Yackel (1995) stated that it is possible for students to learn mathematics through the use of manipulatives. Furthermore, a 1989 study conducted by Kaput established that "meanings are developed within or relative to particular representations" (p. 168). In other words, students are able to develop mathematical meanings through physical representations of mathematical concepts. An example of this is when students construct rectangles with areas representing algebraic expressions.

CHAPTER 2

PROBLEM DOCUMENTATION

Problem Evidence

In order to document the extent to which the targeted Geometry and Algebra 2 students do not understand the abstract algebraic process of simplifying square roots and to determine probable causes, the Simplifying Square Roots Pre-test (Appendix A), the Pretest of Factoring Skills (Appendix B), and the High School Mathematics Survey (Appendix C) were administered during the first week of the action plan. These instruments were all teacher-made. Teacher observations were also used to determine probable causes and possible solutions.

The High School Mathematics Survey (Appendix C), was used to determine an overall idea of how able the students considered themselves to be in performing certain operations. The students were given the surveys and asked to rate their own ability in the areas of arithmetic, algebra, and geometry by circling one, two, three, or four next to each category. A rating of one would indicate that the student believes he or she has a very low level of ability in that category and a four would indicate a high ability level. The categories in the Arithmetic section were: Multiplication and division, Addition and subtraction, Computing powers, Computing square roots, and Factoring integers. The categories in the Algebra section were: Adding and subtracting expressions, Multiplying and dividing expressions, and Using formulas. The categories in the Geometry section were: Measuring with a ruler, Computing areas of shapes, Visualizing a problem, and Drawing a diagram. Selected results of this survey are shown in Figure 1. The complete results of the survey can be seen in Appendix D.

Category	1	2	3	4
Computing square roots	12	28	56	38
Factoring integers	0	18	52	59
Visualizing a problem	9	35	53	33
Drawing a diagram	10	24	47	50

Figure 1: Number of Students Rating in Each Category out of 134 Students

For the purpose of analyzing the data, the selected categories were grouped into pairs. The first two categories shown in Figure 1, Computing square roots and Factoring integers, can be grouped together as the main components of an algebraic approach to simplifying square roots. To simplify a number which is not a perfect square, the students must first factor the number into the product of a perfect square and a number which is not a perfect square. After factoring the integers, the square root of the perfect square factor is then computed. For example, to simplify $\sqrt{32}$ (the square root of 32) the number 32 is factored into 16 times 2 to get $\sqrt{32} = \sqrt{(16)(2)}$. The following step is justified using a proven theorem for all real numbers: $\sqrt{(16)(2)} = (\sqrt{16})(\ 2)$. Finally, the square root of 16 is computed to get an answer of $(4)(\sqrt{2})$ and the simplified square root is written: $4\sqrt{2}$.

The second pair of categories is Visualizing a problem and Drawing a diagram. These two categories are assumed to be important components of most geometric strategies. More specifically, they are essential skills used in any visual method for completing an algebraic manipulation. Several well known publishers of educational materials provide ways to simplify algebraic expressions and solve equations by relating the variables to the lengths of sides of a rectangle. The curriculum includes classroom and homework assignments that require the students to draw rectangles. The rectangles have sides representing variables so there is certainly some visualization taking place before drawing the diagram. Similarly, a visual approach to simplifying square roots would require students to visualize and draw squares.

The results in Figure 1 show a predisposition of the students towards algebraic processes. Responses of three or four were given by 78% of the students on the factoring and computing categories while only 70% responded with a three or four on the drawing and visualizing categories. The tendency to rate themselves lower in ability in a visual setting may effect students' choices of solution strategies. For example, students who rated themselves highly in the algebraic categories would perceive themselves as more successful at performing an algebraic process and may be unwilling to attempt a visual strategy.

Furthermore, the average response in the factoring integers category was a 3.3 showing a high level of student confidence in factoring.

The students' actual factoring abilities were tested using the Factoring Quiz (Appendix B). On this quiz the students were asked to write the factorization of six integers. The students supported their high ratings in factoring with an overall average score of 87%. Fifty-seven percent of the students demonstrated mastery by scoring a perfect six out of six on the quiz. Finally, 82% of the students factored at least five of the integers correctly.

The Simplifying Square Roots Pre-test (Appendix A) was administered to document the students' ability to simplify square roots using the traditional algebraic method discussed previously in this chapter. This method is taught as part of the Geometry curriculum but not until March so it was necessary to show the Geometry students how to simplify square roots before administering the test. The Algebra 2 students, however, were given a review lesson over the algebraic method they had learned while taking the Geometry course. As expected, the increased experience of the Algebra 2 students is apparent in the results of the pre-test. The Algebra 2 students correctly simplified 79% of the square roots while the Geometry students averaged only 54% correct. Furthermore, 56% of the Algebra 2 students simplified at least eight out of ten square roots correctly and only 15% of the Geometry students correctly simplified at least eight out of ten. The average score of all Algebra 2 and Geometry classes combined was 64% correct.

Probable Causes

Site-based Causes

There are several probable reasons why the targeted students are unable to simplify square roots using the traditional algebraic method. Perhaps the most obvious difficulty stems from the fact that the simplification of square roots is not taught as part of the Algebra I course in the district. The concept is one which most algebra textbooks include, but it is not included in the course curriculum. Without any prior instruction, many students experience confusion and are likely to give up.

Boredom is another probable cause. Many students fail to understand why the concept is relevant so they have a difficult time being attentive. This problem is compounded by a failure to understand right triangle geometry. The Geometry students are taught that the ability to simplify square roots is needed to solve geometric problems involving special right triangle relationships and the Pythagorean Theorem. If they do not desire to solve such problems, however, the concepts needed to solve them remain irrelevant.

Finally, the students' inability to simplify square roots can probably be attributed to a misunderstanding of how they should write their answers. In fact, an analysis of the incorrect responses given on the Simplifying Square Roots Pre-test (Appendix A) showed that 44% of the wrong answers were merely written incorrectly. For example, when simplifying $\sqrt{18}$, many students wrote the answer as $2\sqrt{3}$, instead of the correct answer,

$3\sqrt{2}$. This indicated that the correct factorization was used, $\sqrt{18} = \sqrt{9}\sqrt{2}$, but they did not know how to write the answer.

Literature-based causes

The literature suggests several underlying causes for the students' inability to comprehend abstract algebra when taught with a traditional method. The current curricula used to teach the simplification of square roots relies heavily upon a student's ability to factor an integer into two new integers, one of which being a perfect square. This type of factorization is not unique for many integers causing students to begin these problems in a correct but incomplete manner. For example, to find the square root of 32, the students are encouraged to write 32 = 16 times 2. However, starting this problem by writing 32 = 4 times 8 is also acceptable if the student proceeds to factor the 8 into 4 times 2. According to Mason (1989), this ambiguity hinders the analogical reasoning process students use to learn from teacher examples and from their own previous attempts at simplifying radicals. If students are to learn through example and practice, they need to understand the reasoning behind the factorizations and not just the process of factoring a number.

Another possible cause for student misunderstanding may be that they are taught a specific process or algorithm for simplifying square roots. Cobb and Wheatley (1988) have shown that much of what students learn in a conventional classroom setting where most of their time is spent practicing numerical procedures imposed by others discourage students to develop meaningful ways of thinking about mathematics. In general terms, algebra needs to both relevant and meaningful for students (Haimes, 1996).

A final reason why students are unable to understand how to simplify square roots may be that they are weak in their ability to think analytically. According to Piaget (1980), some people appear to be highly visual while others are mainly verbal, motor, auditory, or analytical. The students who are not analytically inclined may have a very difficult time understanding an algebraic algorithm or process such as the one used to simplify square roots. In addition to the students who are primarily visual and those who are mainly verbal, there may be a third group who can be considered "mixers" (Dreyfus & Shama, 1996). These mixers may need to use a combination of visual and algebraic strategies. The fact that the current method is wholly analytical may exclude a large number of students from ever being able to grasp the concept.

CHAPTER 3

THE SOLUTION STRATEGY

Literature Review

Children's mathematics education begins from the moment they learn to count and proceeds through pre-secondary mathematics consisting of increasingly complex numerical computations. During this period of mathematical development, learners are progressing through three levels of concept development (Sfard, 1991). In the first stage, students are introduced to the most basic mathematical operations such as adding and subtracting whole numbers. Sfard's second stage involves mentally combining operations to perform computations with greater efficiency. Rather than adding 4 + 4 + 4, the learners eventually see that multiplying 4 times 3 is a more efficient method of computing the result. Students remain in stage two until the concepts are recognized as being separate from the processes being performed. This is where the third stage begins and the learner starts to build categories of concepts which become objects for new operations (Sfard, 1991). In other words, once arithmetic operations are mastered, students can study them in the context of the unknown, hence studying the operations themselves. An example from the third stage would be the ability to prove that integers are commutative.

Another theory was given by Mason (1989). He suggests that the development of mathematical concepts begins with the manipulation of objects such as blocks. According to Mason (1989), this manipulation does not necessarily have to occur in the physical sense; a diagram showing different perspectives and mental rotations can both be thought of as forms of manipulation. The learner must be able to express the general properties of the object and continue to search for new properties (Mason, 1989). For example, a student would first recognize that a cube has edges and continue from there to discover that it has exactly 12 edges. These processes of manipulating and expressing continue simultaneously until the mathematical construct is effectively developed (Mason, 1989). Carrying the cube manipulations to a higher level might include expressing the relationship between the number of edges and the number of faces using an algebraic formula.

Research conducted in the field of mathematics often focuses on the difference between visual and algebraic strategies. The literature suggested several definitions of visual thinking. Presmeg (1986) says that a visual strategy is one which "involves visual imagery, with or without a diagram, as an essential part of the method of solution" (p. 298). A student does not have to draw anything to use visualization to solve either a visual or non-visual problem (Presmeg, 1986). For example, picturing a rectangle divided equally can be considered a visual method for factoring integers. Visualization can also be thought of as the use of spacial properties inherent in a presented problem (Dreyfus & Shama, 1994). Such properties include, but are not limited to: area, perimeter, circumference of circles and volumes of three-dimensional objects.

Project Objectives and Processes

As a result of a visual/spacial approach to simplifying square roots, during the period of November 1997, the targeted advanced algebra and geometry students will increase their ability to simplify square roots, as measured by a teacher-constructed test.

In order to accomplish the terminal objective, the following processes are necessary:

1. Surveys will be created and administered to determine student attitudes.

2. Curricular materials which use a visual/spacial method for simplifying square roots will be developed.

3. A lesson for teaching the visual/spacial method will be developed.

4. Forms of assessing student understanding of the concept will be determined.

Project Action Plan

Stage One: Collect Preliminary Data (week 1)

- Administer High School Mathematics Survey
- Administer Factoring Quiz

Stage Two: Evaluate Existing Teaching Method (week 2)

- Present algebraic process for simplifying square roots
- Structure time for students to practice skill
- Assess understanding through Simplifying Square Roots Pre-test

Stage Three: Implement Intervention (week 3)

- Present visual method for simplifying square roots
- Structure time for students to practice skill

- Assess understanding through Simplifying Square Roots Post-test

Stage Four: Evaluate Action Plan (week 4)

- Analyze responses on the High School Mathematics Survey
- Analyze results of Factoring Quiz
- Compare results of Simplifying Square Roots Pre-test and Post-test
- Analyze responses on Post-test Strategy Survey
- Make appropriate adjustments to current curriculum
- Report on results of action plan

Methods of Assessment

The following assessment tools will be used:

1. High School Mathematics Survey
2. Factoring Quiz
3. Pre-test of Simplifying Square Roots
4. Post-test of Simplifying Square Roots
5. Post-test Strategy Survey
6. Teacher journaling

CHAPTER 4

PROJECT RESULTS

Historical Description of the Intervention

The objective of this project was to increase the targeted students' ability to simplify square roots. The desired effects were sought through the implementation of a curriculum designed to teach students a visual method for simplifying square roots.

It was discussed previously that an accepted way to teach algebra visually is to consider a rectangle with a given base and height. The product of the base and height is then related to the area of the rectangle. For example, a rectangle with an area of 20 square units and a base of 5 units must have a height of 4 units. Through the use of special colored tiles students are able to create rectangles with variable lengths, therefore teaching them to multiply, divide, and factor algebraic expressions. Since the objective of this project only included the simplification of the square roots of integers and not algebraic expressions the colored tiles were unnecessary. Students could use tiles of the same color or simply create diagrams on their paper. Furthermore, the concept of a square root is based upon the length of a side of a square (not just a rectangle) so the base and height of the constructed figures must be equal. For example, if a square has an area of 25 square units, then each side must have a length of 5 units, making the base and height of the square both equal to five. Thus the students in this study were instructed to use only square tiles or draw diagrams of squares subdivided into smaller squares of equal size. For example, to simplify the square root of 20, the students were taught to draw a square with an area of 20 square units making the length of one side equal to the square root of 20. The square is then divided into four squares each with areas of five square units each; the length of one side of these smaller squares is the square root of five. Therefore, the length of one side of the original square, $\sqrt{20}$, is simplified to $\sqrt{5} + \sqrt{5}$ which is written $2\sqrt{5}$. More complete examples of this visual method for simplifying square roots are provided in Appendix E.

All of the targeted students were presented the algebraic method for simplifying square roots and completed exercises to practice the skill. Many of the Algebra 2 students recalled learning the method while in Geometry but most agreed that a review was needed. Some of the Geometry students were aware of the concept only concerning integers which are perfect squares, such as 9 and 25. None of the Geometry students had any prior experience simplifying integers which are not perfect squares. All classes received identical instruction and practiced the skill for approximately 30 minutes. The Simplifying Square Roots Pre-test (Appendix A) was administered the following day.

Several days after the pre-test was administered, the students participated in an activity building squares out of square tiles. By the completion of the activity, the students compiled a list of the first ten perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. The list contains possible numbers to use when factoring to simplify a square root. After the square activity, the visual method for simplifying square roots was presented to the students and they were given approximately 20 minutes to practice the skill. The next day, the Simplifying Square Roots Post-test (Appendix F) was administered.

Finally, on the fifth day of the intervention, the last assessment tool was administered. The students completed the Post-test Strategy Survey (Appendix G) after seeing their scores on both the Simplifying Square Roots Pre-test and the post-test. The students were asked not to identify themselves on the survey. This was done to elicit the most honest responses possible.

The actual intervention did not take place exactly as described in the action plan. Of particular note is that the entire intervention was completed in one week instead of the allotted two weeks. Furthermore, the students were encouraged to use either the visual or algebraic method on the Simplifying Square Roots Post-test, whichever they preferred. This freedom of choice was not provided for in the action plan and is an important consideration as the post-intervention data is analyzed.

Presentation and Analysis of Results

The first step in analyzing the effects of the visual method on student understanding was to compare the results of the Simplifying Square Roots Pre-test (Appendix A) with the results of the post-test (Appendix F). The percentages of correct responses on the pre-test were stated in the Problem Evidence section in Chapter 2 and are compared to the percentages of correct responses on the post-test in Figure 2.

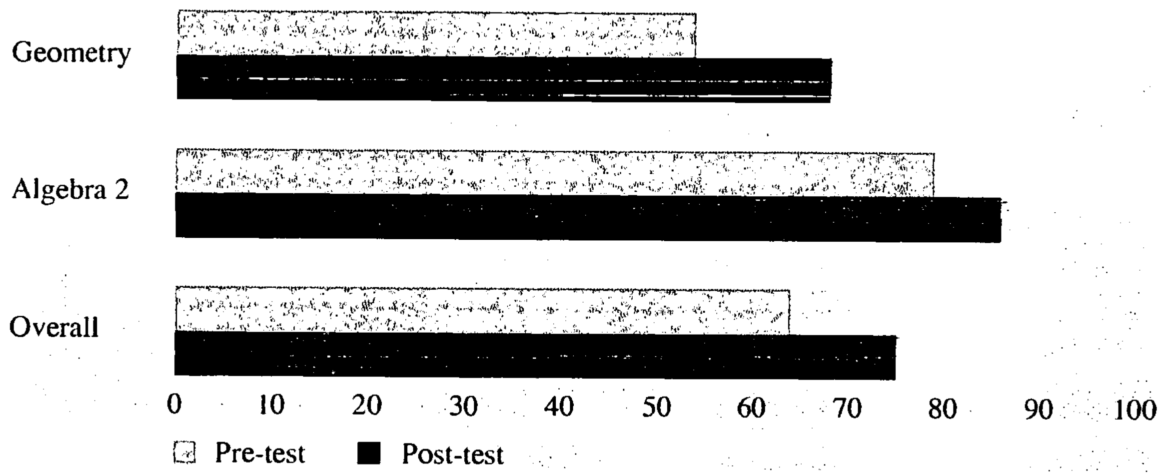

Figure 2: Percent Correct on the Simplifying Square Roots Pre-test and Post-test.

The Geometry students exhibited the largest increase of ability by scoring an average of 14% higher on the post-test than they did on the pre-test. The Algebra 2 students also showed improvement; they correctly simplified seven percent more of the square roots on the post-test. These individual improvements translate into an 11% overall increase in scores from 64% correct on the pre-test to 75% correct on the post-test.

Additional data collected from the Simplifying Square Roots Post-test (Appendix F) included the students' choice of method (Figure 3). As stated previously, the students were encouraged to use either the algebraic or visual method for simplifying square roots for each problem on the post-test. Some students used the algebraic method on all ten problems while others chose an entirely visual approach. The students who used the visual method on at least one problem but used the algebraic method on at least one other problem were said to have used a combination of the two methods.

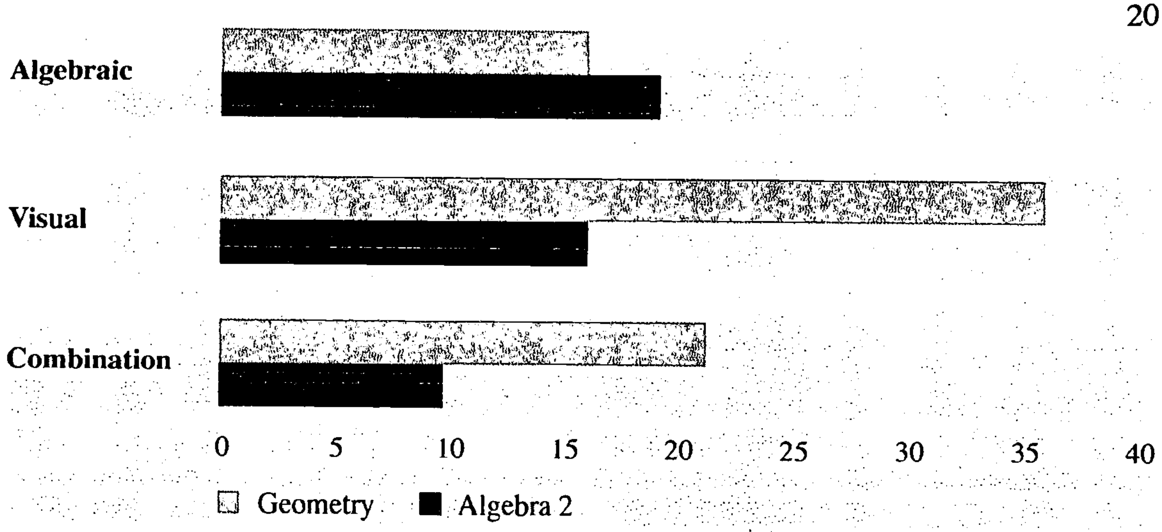

Figure 3: Number of Students Using Each Method on the Post-test.

The visual method was more popular among the Geometry students, while the Algebra 2 students were more likely to use the algebraic method. Forty-eight percent of the Geometry students used the visual method on all ten problems on the Simplifying Square Roots Post-test, but only 23% chose to use an entirely algebraic approach. Furthermore, 77% of the Geometry students used either the visual approach or a combination of the methods. The Algebra 2 students, on the other hand, used the visual method only 35% percent of the time and 42% did not approach any of the problems visually. An overall comparison of the strategies used on the Simplifying Square Roots Post-test resulted in finding that 70% of the targeted students chose either the visual method or a combination of the methods and 30% chose to simplify the square roots using only the algebraic method.

Results from the Post-test Strategy Survey (Appendix G) provides data showing the overall preferences of the students. When asked which method they preferred, 53 of the 117 students surveyed responded that they found the algebraic method easier and 64 students preferred the visual method. Five students wrote that their choice of strategy depends on the problem and five more stated that they did not like either of the methods.

The choice of strategy indicates the students' preference, but how did it affect performance? Figure 4 shows the average scores of the students depending on which method they used on the post-test. Probable cause data from Chapter 2 indicated that the students' inability to simplify square roots was due in part to knowing how to write the correct answer. An analysis of the Algebra 2 students' responses on the Simplifying Square Roots Post-test showed that the number of incorrect responses caused by a failure to merely write the answer correctly decreased by 15% to only three percent. Furthermore, 29% of the Algebra 2 students and 50% of the Geometry students improved their score by using either the visual method or a combination of visual and algebraic strategies.

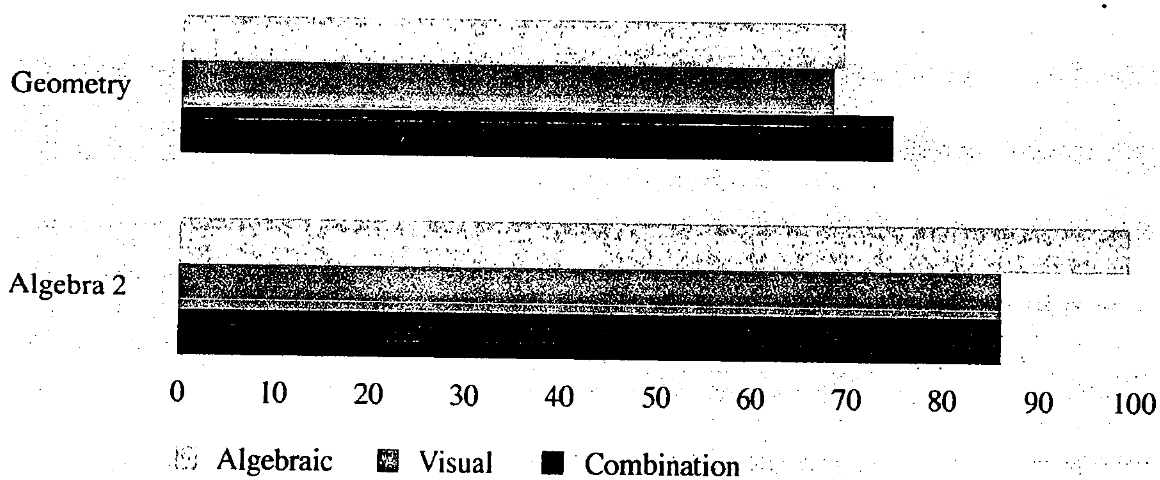

Figure 4: Percentage Correct by Method on the Simplifying Square Roots Post-test

Conclusions and Recommendations

Based on the presentation and analysis of data collected on the Simplifying Square Roots Post-test (Appendix F), the targeted students substantially improved their ability to simplify square roots. Although the greatest increase in ability was seen in the Geometry students, the Algebra 2 students showed some improvement as well. One would expect the Geometry students to exhibit more improvement because of their inexperience with the concept prior to the intervention. The main reason the Algebra 2 students did not improve their scores as much as the Geometry students is because more of them already knew how to simplify square roots. The initial advantage held by the Algebra 2 students over the Geometry students is evidenced by their superior scores on the Simplifying Square Roots Pre-test (Appendix A).

The data presented in this chapter consistently illustrates higher scores on both the pre-test and the post-test from the students who used the algebraic method. This data does not, however, support the conclusion that the algebraic method is a more effective means of simplifying square roots. What the data does support is that the students who were initially able to understand the algebraic method were unlikely to attempt the visual strategy on the post-test due to their previous success on the pre-test without knowing that a visual method existed. Therefore, the students whose strategy was defined as algebraic should be expected to score better than the students who may have embraced the visual method as a strategy for their own improvement. This conclusion is further supported by the increase in test scores by the Geometry students who implemented the visual method or a combination of the methods on the post-test. Many of the Algebra 2 students, on the other hand, were nearly perfect on the pre-test and remained that way throughout the intervention by choosing the strategy with which they enjoyed the most success.

Revising several of the assessments used to collect data for this project could provide an accurate description of how well all of the students were able to understand the visual method. First of all, the Pre-test of Factoring Skills (Appendix B) simply called for the ability to factor an integer in order to score highly. Although factoring is an essential component of simplifying square roots, it is not sufficient to factor into any numbers. To be able to simplify the square roots the students must also have a firm grasp on the concept of a perfect square. Therefore, it would be appropriate to modify the directions on the Pre-test of Factoring Skills to require the students to factor the numbers into one perfect square and one integer which is not a perfect square. The evidence in favor of this revision is that some of the students who attempted the visual method were inclined to make the same factoring "mistakes" as they did while using the algebraic method. These "mistakes" were not mistakes in the factoring sense, but they were such that it became impossible for the students to continue and get the correct answer. An example of this kind of mistake is factoring 20 into the product of 10 and 2 instead of the correctly factoring into 4 and 5 so that one could then compute the square root of 4. A diagnostic test of this kind would give an exacting measure of the students' ability to perform the type of factoring necessary to simplify square roots. Furthermore, an additional instrument assessing the students' understanding of perfect squares could be introduced into the project's action plan.

The problem of inappropriate factoring and misunderstanding perfect squares appeared on some incorrect responses on the post-test. When attempting to use the visual method some students divided the original square into rectangles instead of squares. For example, one student drew a square and labeled one edge 18 so he could simplify the square root of 18. Drawing the rest of the segments for the problem, the student created 6 smaller rectangles with areas of 3 each instead of correctly dividing the original square into 9 smaller squares each having an area of 2. This mistake parallels the error of inappropriately factoring while using the algebraic method. The students was able to maintain the size of the original square yet could not proceed successfully. It is possible that the students making these errors are not able to fully understand all of the physical attributes of squares and therefore may not be considered strong visualizers. With respect to the intervention, additional instruction regarding the concept of a perfect square and its relationship to the area of a physical square would have certainly improved the students' results on the project's assessments.

The Simplifying Square Roots Post-test could also be revised. Modifications in the directions of the post-test would enable future researchers to concisely determine the effectiveness of the visual method for simplifying square roots. One could ensure pure results in future testing by restricting the students from using the algebraic method. Finally, under these conditions, data could provide a correlation between a student's ability to perform arithmetic operations in an entirely analytical sense and their visual reasoning skills.

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APPENDICES

APPENDIX A

SIMPLIFYING SQUARE ROOTS PRE-TEST

DIRECTIONS: Simplify each square root. Show all your work. NAME:

1. $\sqrt{9}$
2. $\sqrt{12}$
3. $\sqrt{8}$
4. $\sqrt{18}$
5. $\sqrt{32}$
6. $3\sqrt{8}$
7. $\sqrt{20}$
8. $2\sqrt{12}$
9. $\sqrt{108}$
10. $\sqrt{63}$

APPENDIX B

PRE-TEST OF FACTORING SKILLS

DIRECTIONS: Factor each number completely. NAME:

1. 24
2. 9
3. 63
4. 32
5. 20
6. 108

APPENDIX C

HIGH SCHOOL MATHEMATICS SURVEY

DIRECTIONS: Place an X next to the appropriate response. NAME:

Grade level: ____ 9th ____10th ____11th ____12th

Age: ____13 ____14 ____15 ____16 ____17 ____18

Approximate grade earned in Algebra I: ____A ____B ____C ____D

Rate your own ability in each area. 1 is the lowest rating and 4 is the highest.

ARITHMETIC

1. Multiplication & Division	1	2	3	4
2. Addition & Subtraction	1	2	3	4
3. Computing powers(example: 7^2)	1	2	3	4
4. Computing square roots(example: $\sqrt{16}$)	1	2	3	4
5. Factoring (example: $18 = 2 \cdot 3 \cdot 3$)	1	2	3	4

ALGEBRA

1. Adding/Subtracting expressions(example: $2x + x$)	1	2	3	4
2. Multiplying/Dividing expressions(example: $x \cdot x^2$)	1	2	3	4
3. Using formulas(example: $A = b \cdot h$)	1	2	3	4

GEOMETRY

1. Measuring with a ruler	1	2	3	4
2. Computing areas of shapes	1	2	3	4
3. Visualizing a problem or situation	1	2	3	4
4. Drawing a diagram	1	2	3	4

APPENDIX E

SIMPLIFYING SQUARE ROOTS POST-TEST

DIRECTIONS: Simplify each square root. Show all your work. NAME:

1. $\sqrt{16}$
2. $\sqrt{18}$
3. $\sqrt{24}$
4. $\sqrt{28}$
5. $\sqrt{27}$
6. $\sqrt{40}$
7. $2\sqrt{8}$
8. $3\sqrt{12}$
9. $\sqrt{72}$
10. $\sqrt{125}$

APPENDIX F

POST-TEST STRATEGY SURVEY

Which method for simplifying square roots is the easiest?

Why do you think this method is the easiest?

In general, do you prefer to use a visual approach (such as drawing a diagram) to solve problems?

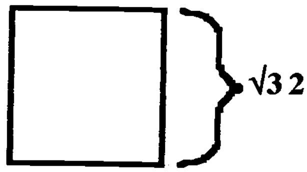
Why do you think visual strategies help some students?

APPENDIX G

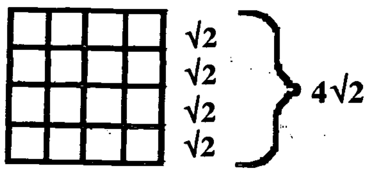
SIMPLIFYING SQUARE ROOTS USING THE VISUAL METHOD

#1 Simplify **√32**.

Step 1 - Construct a square with an area of **32** units (each side has length **√32**).



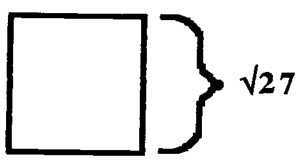
Step 2 - Divide the square into **16** squares with areas of **2** units each (each side has length **√2**).



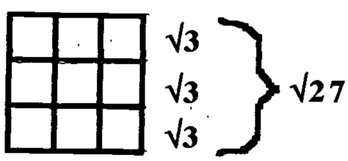
Step 3 - Express the length of one side of the original square in terms of the lengths of the sides of the smaller squares to get **√32 = 4√2** for the answer.

#2 Simplify **√27**

Step 1 - Construct a square with an area of **27** units (each side has length **√27**).



Step 2 - Divide the square into **9** squares with areas of **3** units each (each side has length **√3**).



Step 3 - Express the length of one side of the original square in terms of the lengths of the sides of the smaller squares to get **√27 = 3√3** for the answer.

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